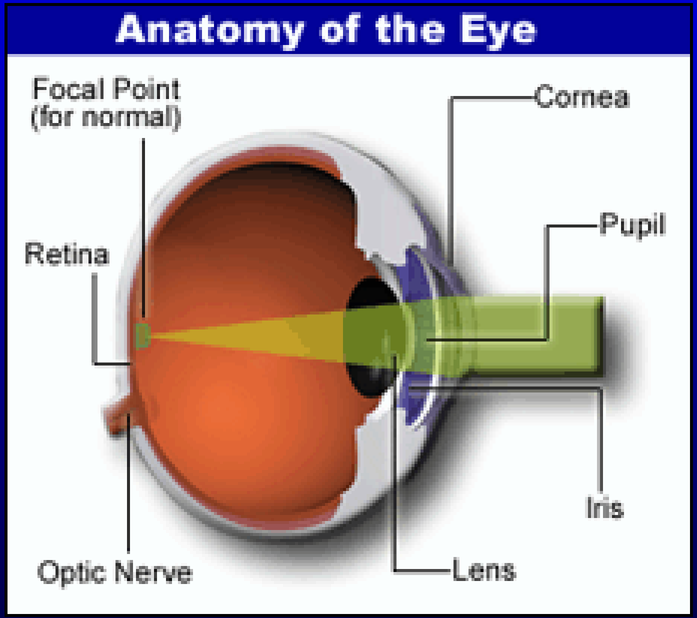
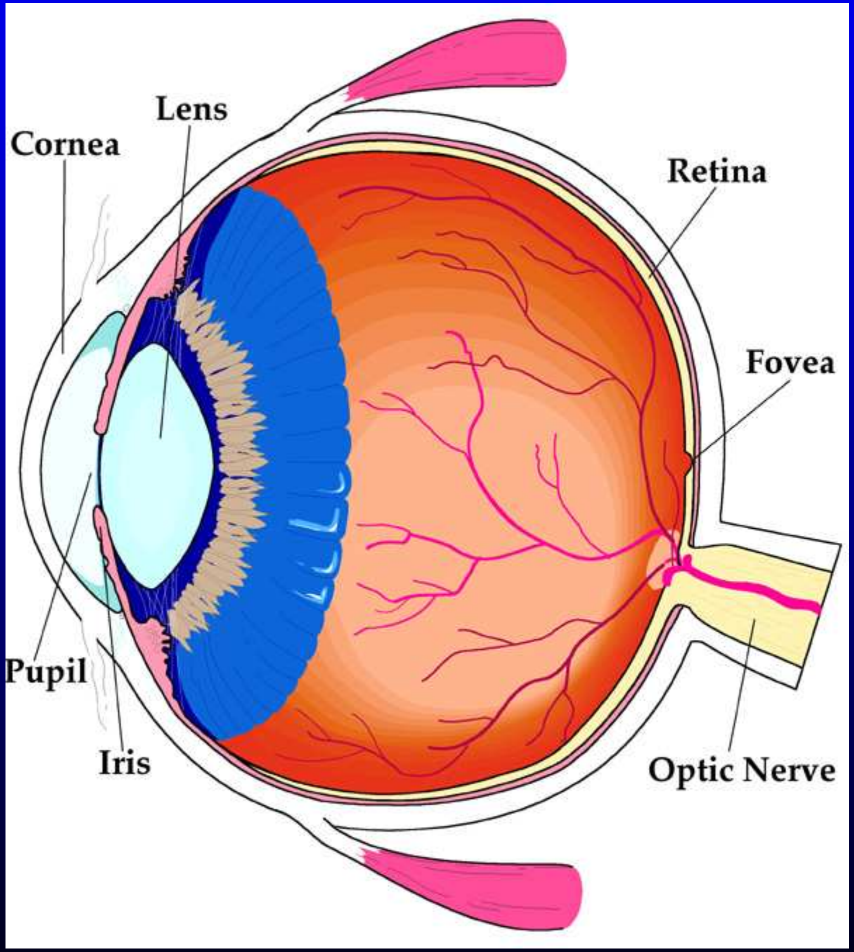
Observing the Universe

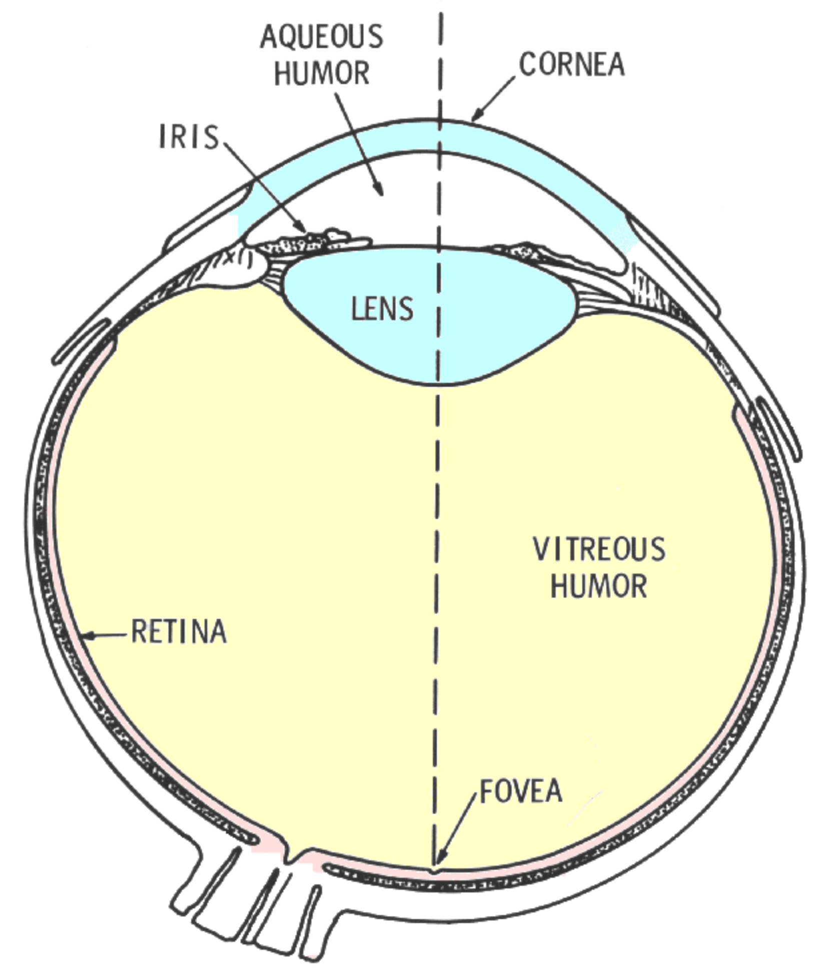
Optical Instruments

Our Eye


Anatomy of the Eye
Focal Point (for normal)
Cornea
Pupil
Retina
Iris
Optic Nerve
Lens


Lens
Cornea
Retina
Fovea
Pupil
Iris
Optic Nerve

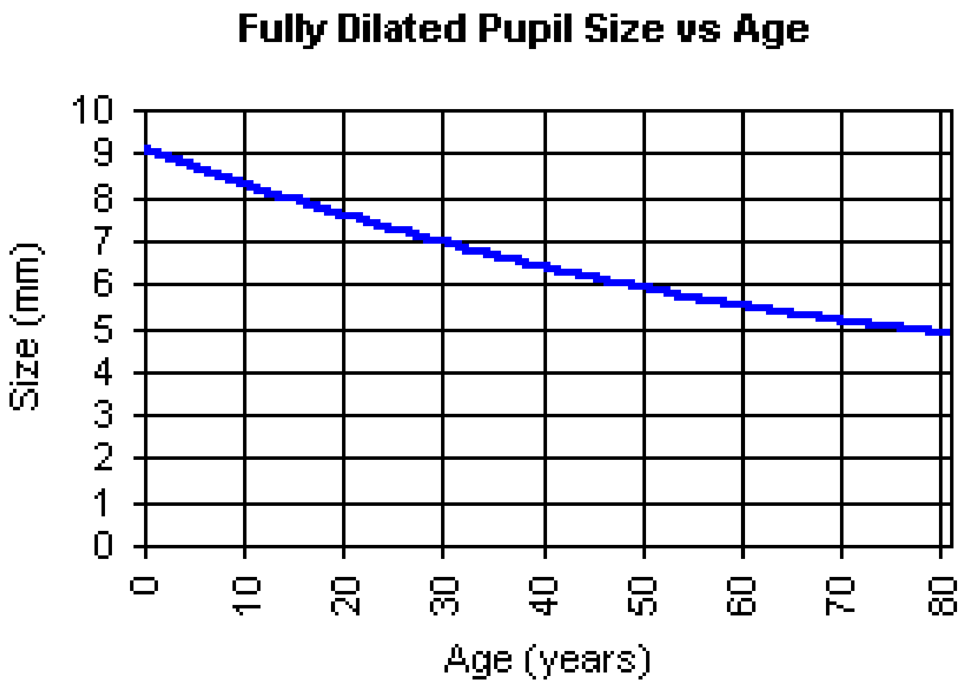
- The fovea has a high concentration of cones – sensitive to colour.
- Other parts of the retina have more rods these are not sensitive to colour, but have a higher sensitivity than cones.
  - We can see fainter objects by using AVERTED VISION



Dark Adaption 1

1) Pupil Dilation

- Quite rapid
- An older person’s eyes might only dilate to 5mm, whilst a young person’s might dilate to 7mm or more!
- A factor of at least 2 in area!



Dark Adaption 2

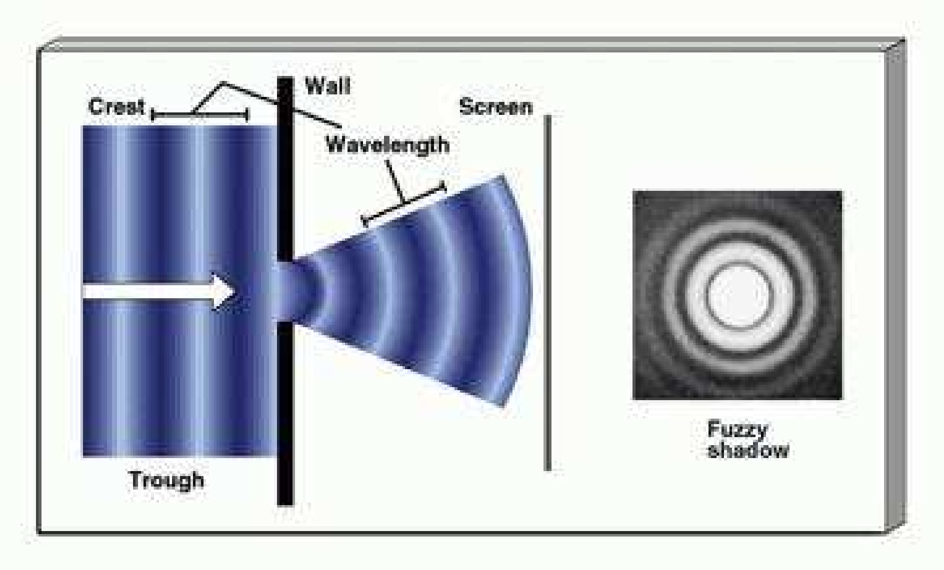
- Over a period of 20 minutes to 1 hour, Vitamin A converts into other chemicals which increase the sensitivity of the rods and cones.
- Bright white light quickly destroys this effect.
- So use a red torch to view sky charts.

What is the Eye’s Resolution?

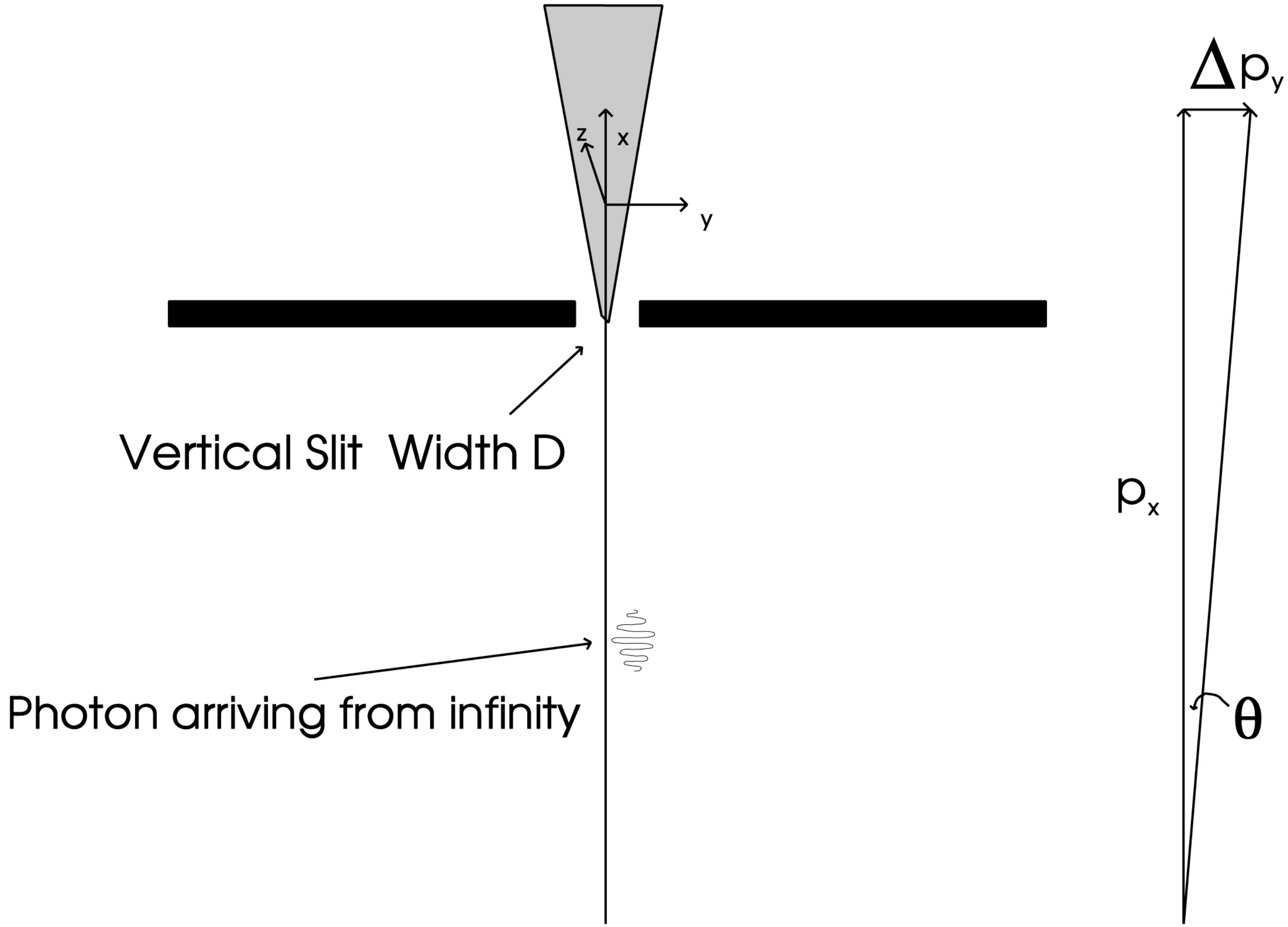
- It is generally said that the human eye can distinguish two objects that are separated in angle by about 1 minute of arc –1/60 degree

What limits this?

1) The size of the rods and cones – evolution has matched this to the fundamental resolution caused by the wave nature of light.
2) Diffraction.



Diffraction



Vertical Slit Width D

Photon arriving from infinity

Photon Momentum

- A photon reaches a slit of width D from a distant star. Its direction defines the x axis.
- All initial momentum is along x axis
- Quantum Mechanics states that the momentum p_x of the photon is given by

$$p_x = h/\lambda$$

Where h = Planck’s constant and λ is the wavelength

Heisenberg’s Uncertainty Principle

- The more precisely the position of an object is determined, the less precisely its momentum is known and vice versa.
  - Heisenberg 1927

- The product of these uncertainties is given by Planck’s constant, h.

Heisenberg's Uncertainty Principle

- By knowing that the photon passes through a slit of width D in the y direction, means that its momentum p in the y direction (right angles to the slit) is made uncertain by an amount Δp_y given by the uncertainty principle $\Delta p \Delta x = h$
- As $\Delta x = D$
  - So $\Delta p_y D = h$
  - But from the momentum equation, $h = p_x \lambda$
  - So $\Delta p_y D = p_x \lambda$
  - or $\Delta p_y / p_x = \lambda / D$

Passing through a slit

- The uncertainty in Δp_y gives an uncertainty in the direction in which the photon continues of $\Delta\theta = \Delta p_y / p_x$
- Substituting from above gives:

$$\Delta\theta = \lambda / D$$

Passing through a circular aperture

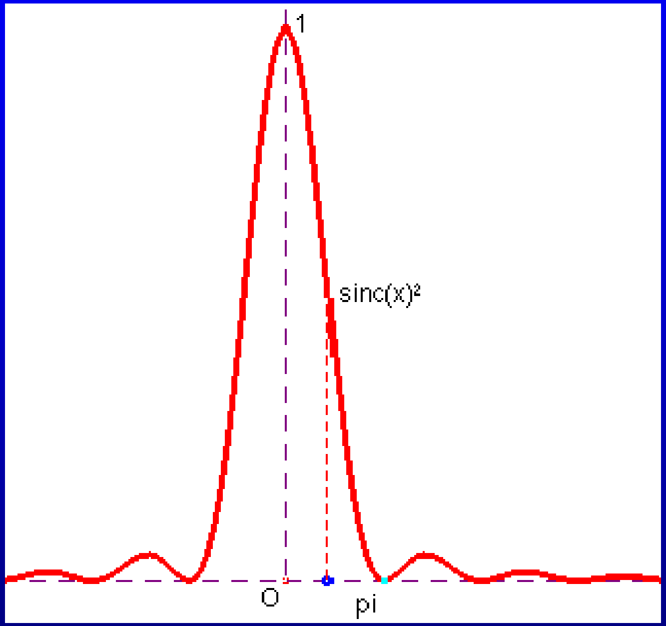
- The photon is now constrained in both y and z directions, so you would expect the uncertainty in angle to be greater.
- It is, and the equation is:

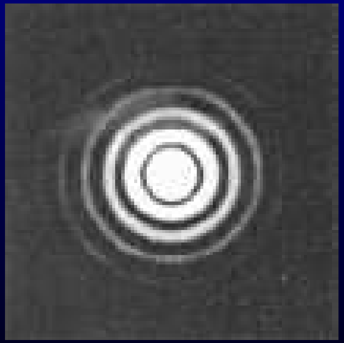
$$\Delta\theta = 1.22\,\lambda / D$$

- So a lens will not focus all the light passing through it (from infinity) to one point in space (the focus) but into a disk - called the Airy Disk – surrounded by concentric rings.

The Airy Disk

- A point object – like a star - will give an image where ~ 84% of the light is concentrated into a disk, called the Airy Disk, surrounded by a series of concentric rings.





The Size of the Airey Disk

- The angular diameter of the disc gets bigger for longer wavelengths, and smaller for larger apertures:

$$\theta = 1.22\ \lambda/d$$

Where θ is in radians and λ and d are in metres

Resolution of the Human Eye

- What would we expect the resolution of the human eye to be ?
  - For $\lambda = 0.5 \times 10^{-6}$ metres , $D = 3 \times 10^{-3}$ metres

    $\theta = 0.000202$ radians = 0.0115 degrees

    = 0.73 arc minutes

  i.e., just under one minute of arc.

Binoculars and Telescopes

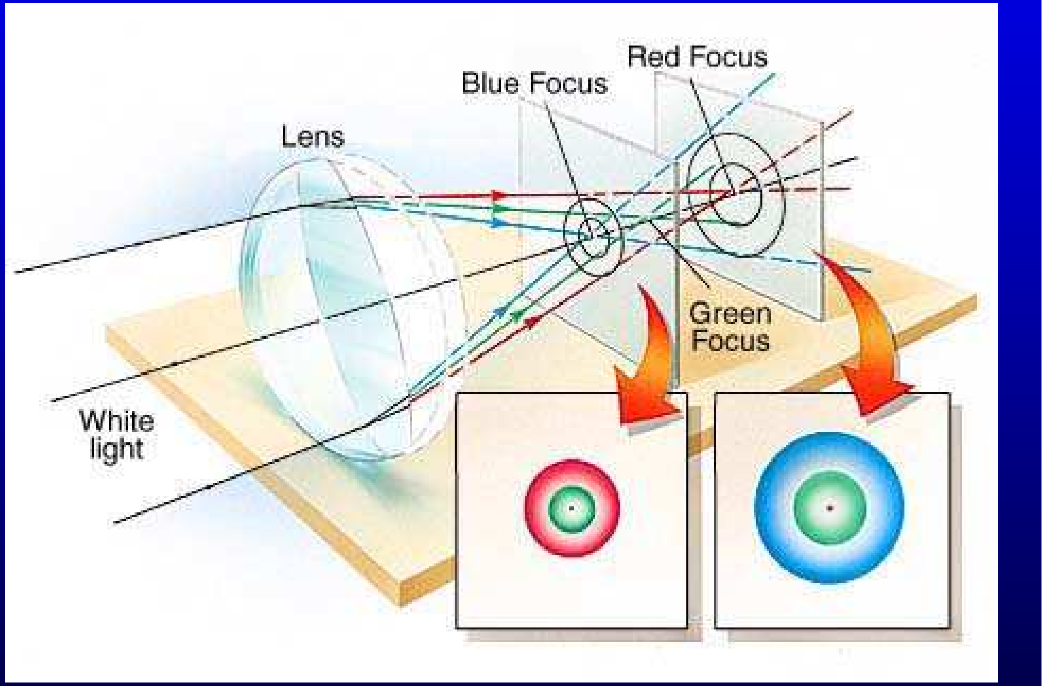
To see fainter objects and more detail

Refractors

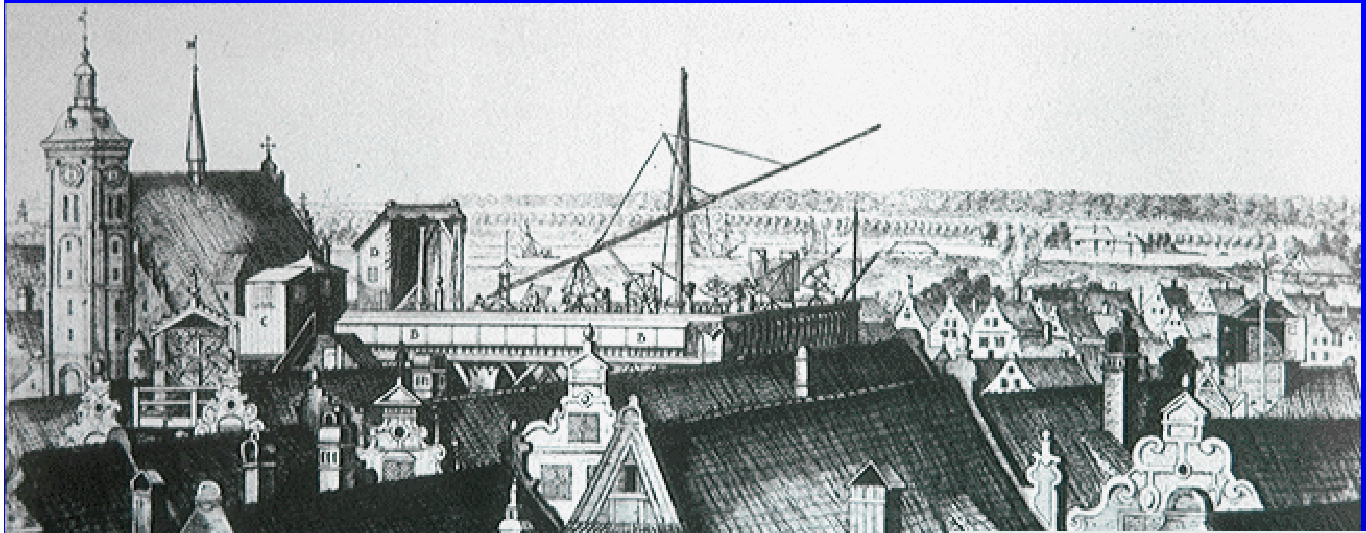
- These use an objective lens.

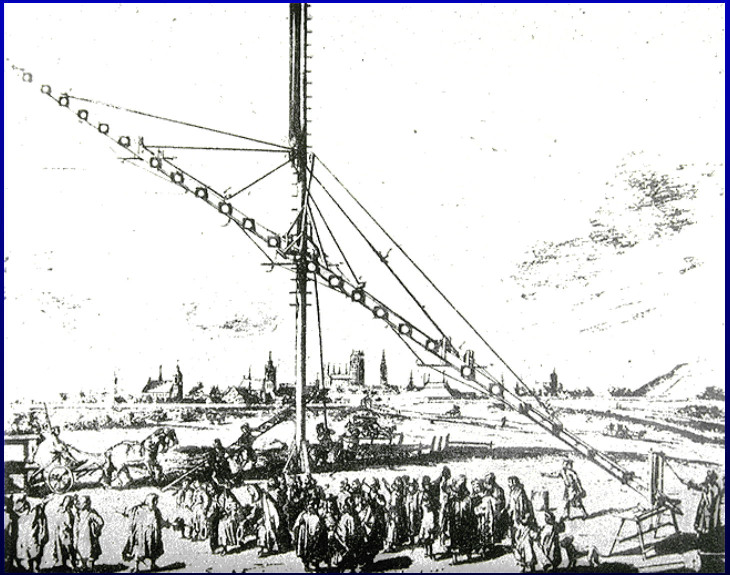
- The first lenses were single element, and produced colour fringing around bright objects – called chromatic aberration.

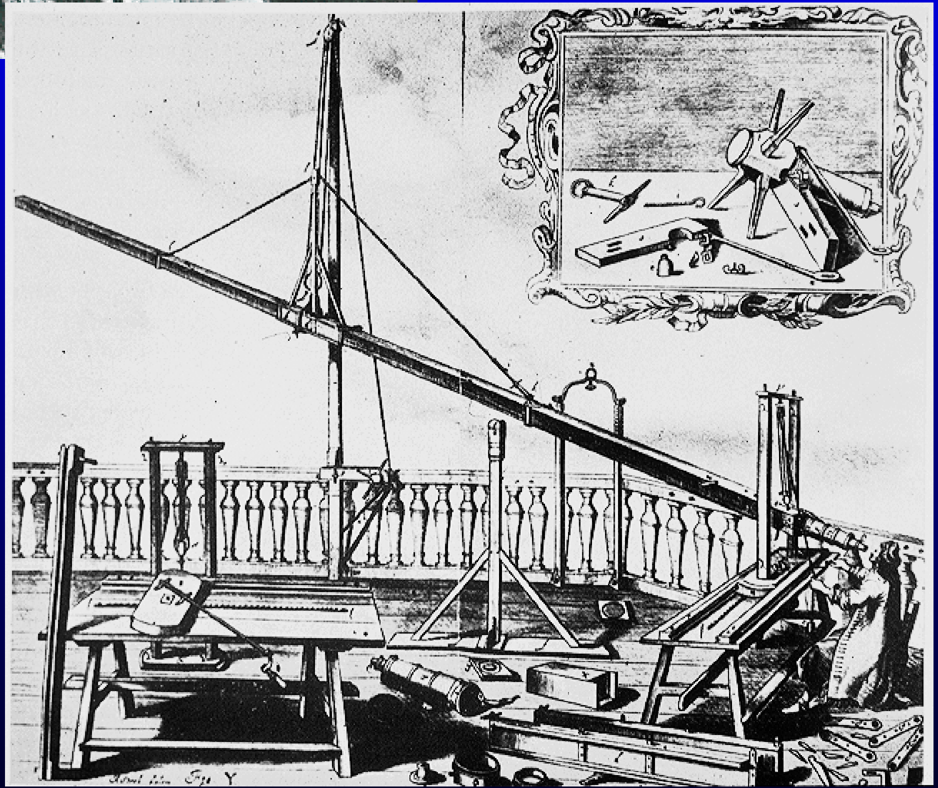
- Telescopes had to be very long to overcome its worst effects.



Helvelius

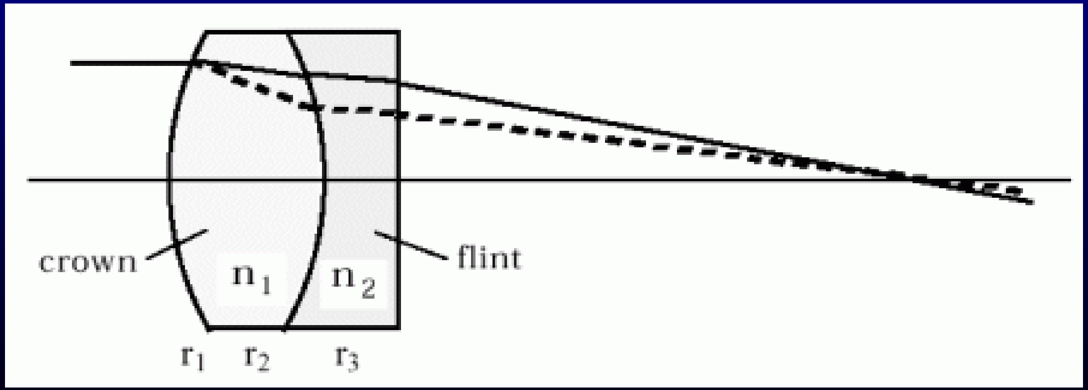






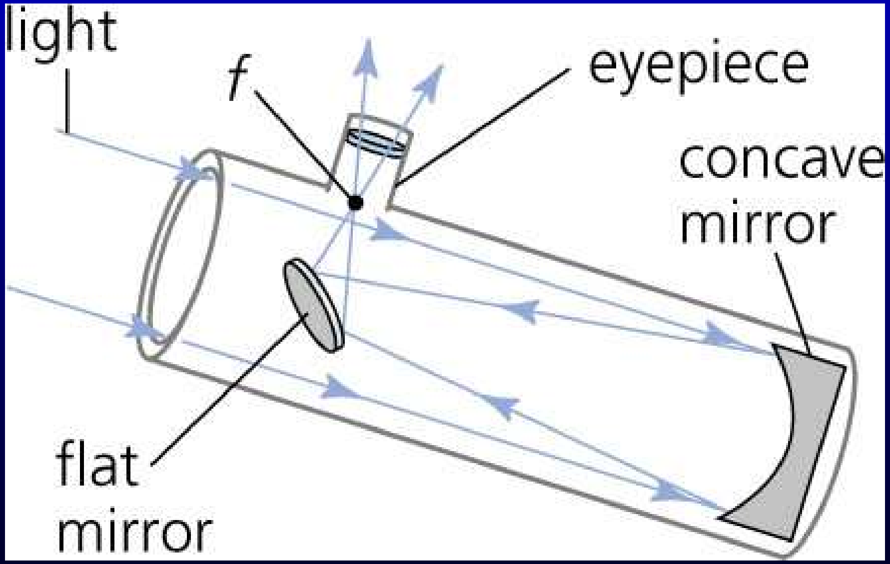
Achromatic Doublets

- In 1754 John Dolland (of Dolland and Aitchison) patented the Achromatic Doublet which uses a converging lens of crown glass coupled with a (less) diverging lens of flint glass to largely remove chromatic aberration.



Newton’s Reflecting Telescope

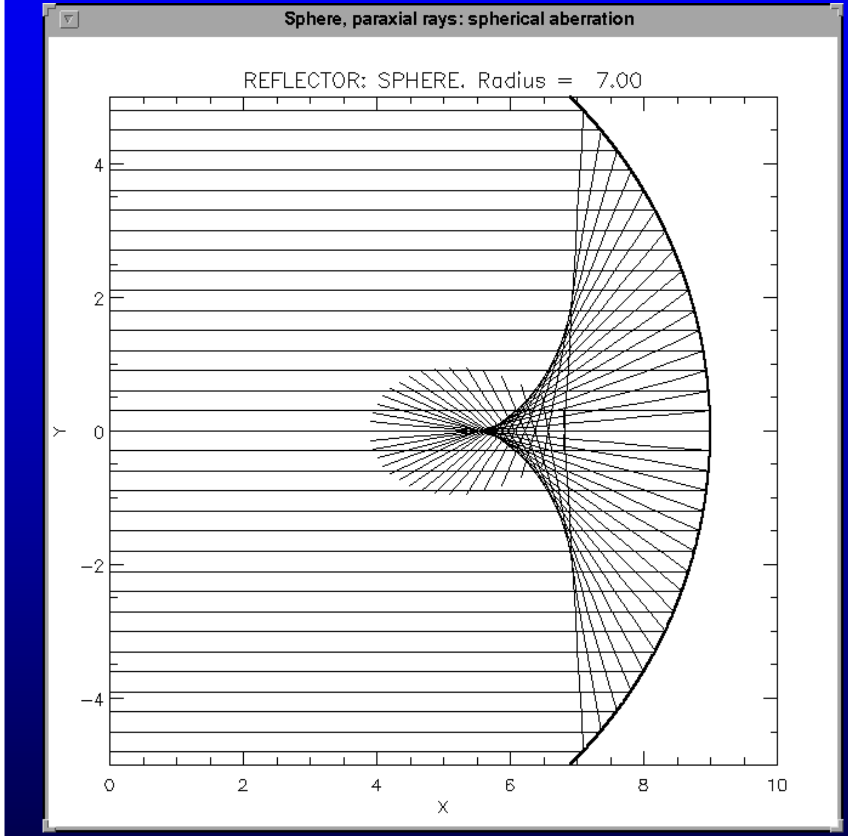
Newton believed that only a reflecting telescope could eliminate chromatic aberration



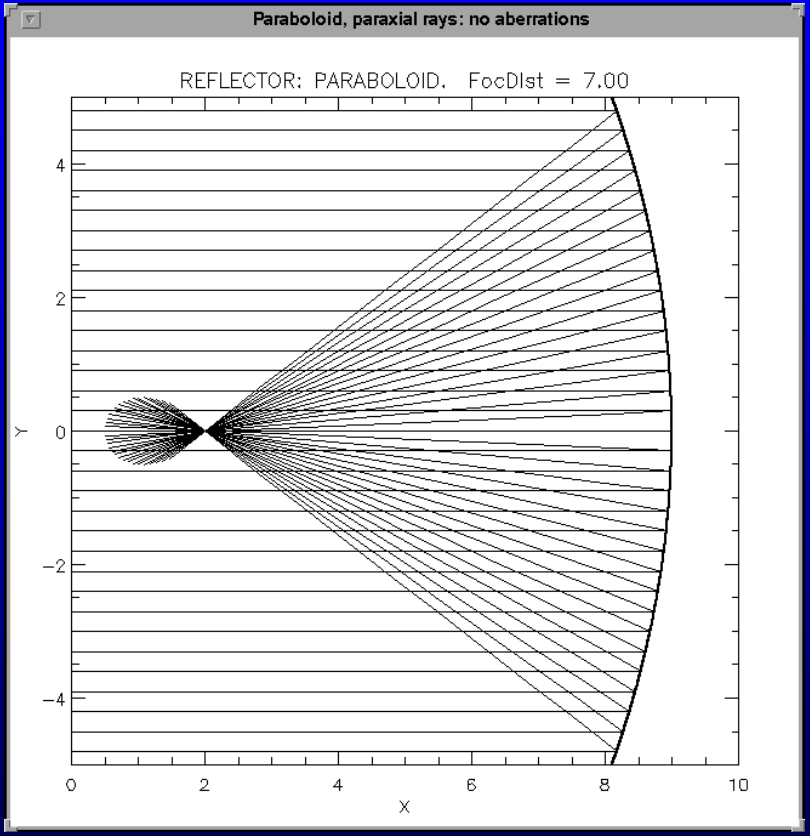
Wave optics

A better way to think about how a mirror or lens makes an image.

Ray Optics



Spherical Mirror: no focus.



Parabolic Mirror: perfect focus

- Unless all the rays pass through a single point there cannot be focus.
- BUT even if all rays do pass through a single point there may not be a focus.

A Fundamental Truth

The main purpose of a mirror or lens is to bring light that arrives over a wide area to a point – we call it the focus – where that light is concentrated and so enables us to see fainter objects.

For this light to form an image all the light, when it reaches the focus, must have left the object at the same instant of time.

It must be coherent

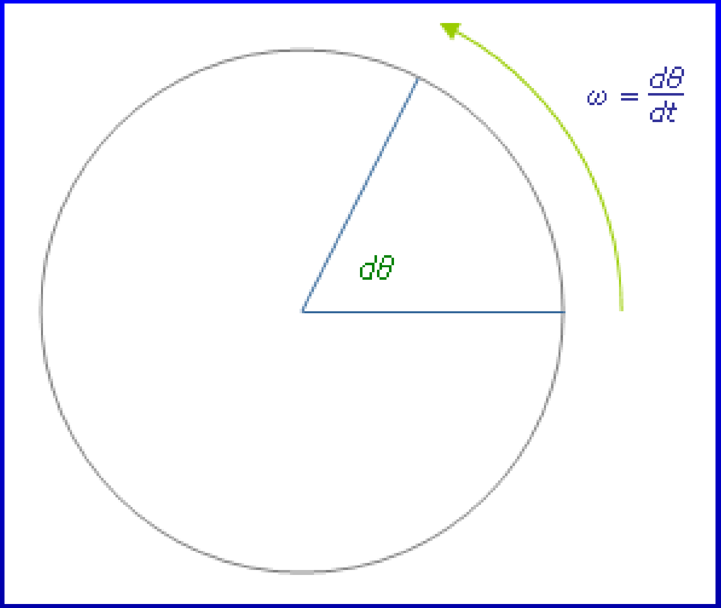
- This means that every possible path length from the source to the focus must have precisely the same length no matter which part of the lens it has passed through, or which part of the surface of a mirror it has been reflected by.

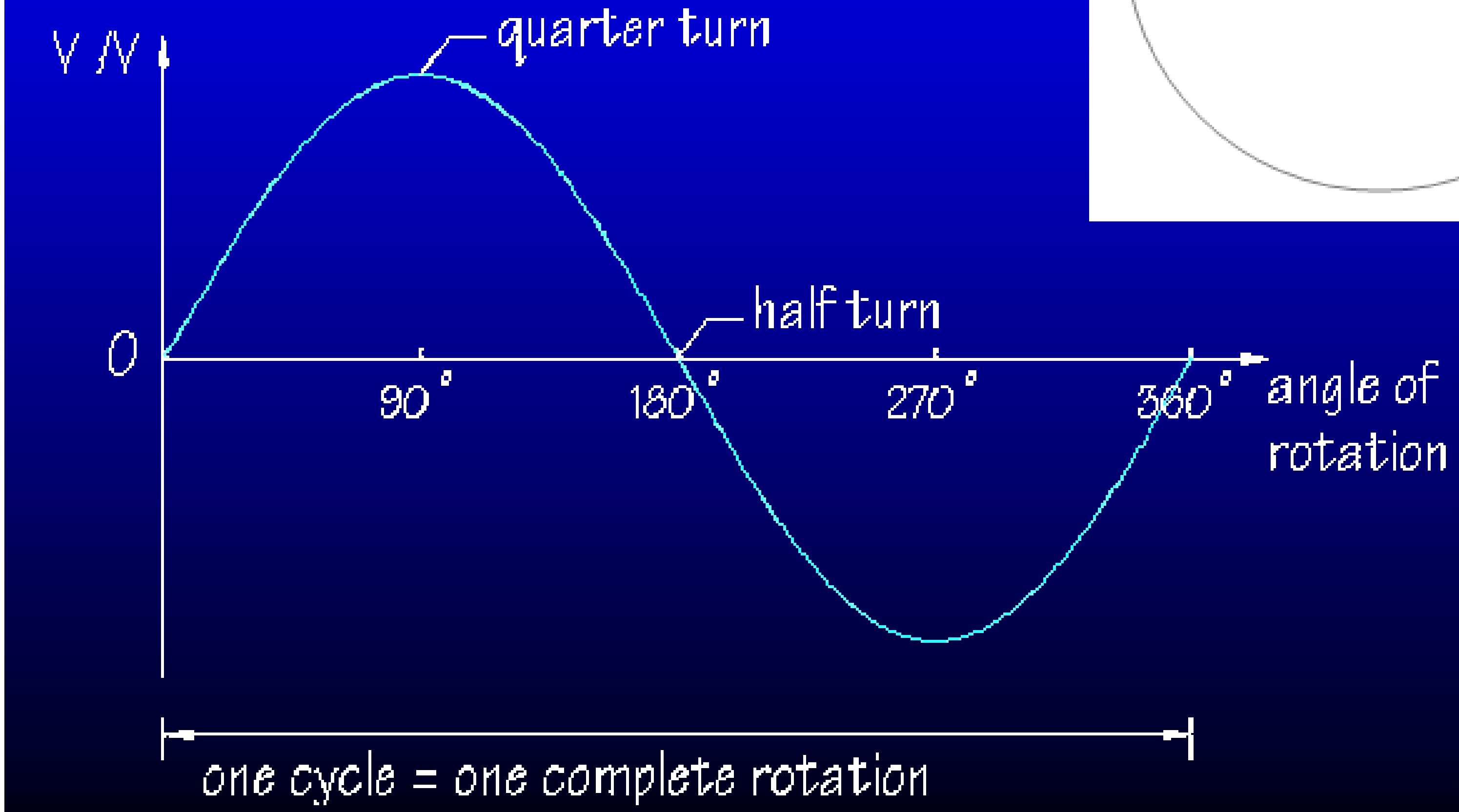
- It is, in fact, this property that defines where the focus of a lens or mirror will be in the space either in front of a mirror or beyond a lens.
- Ray optics, depending solely on the laws of refraction or reflection do derive the correct locations of the foci in the cases that are given but will not necessarily do so.
- The fact that the reflected or refracted rays pass through a single point in space is a necessary – but not sufficient – requirement

Wave motion

- For the purposes of what follows we can represent any point in a wavefront as having two properties, an amplitude and a phase.
- We can represent these as a rotating vector whose length is proportional to the amplitude with the angle relative to some arbitrary zero determining the phase (0 to 360 degrees)
- This can be called a “phasor”

Rotating vector

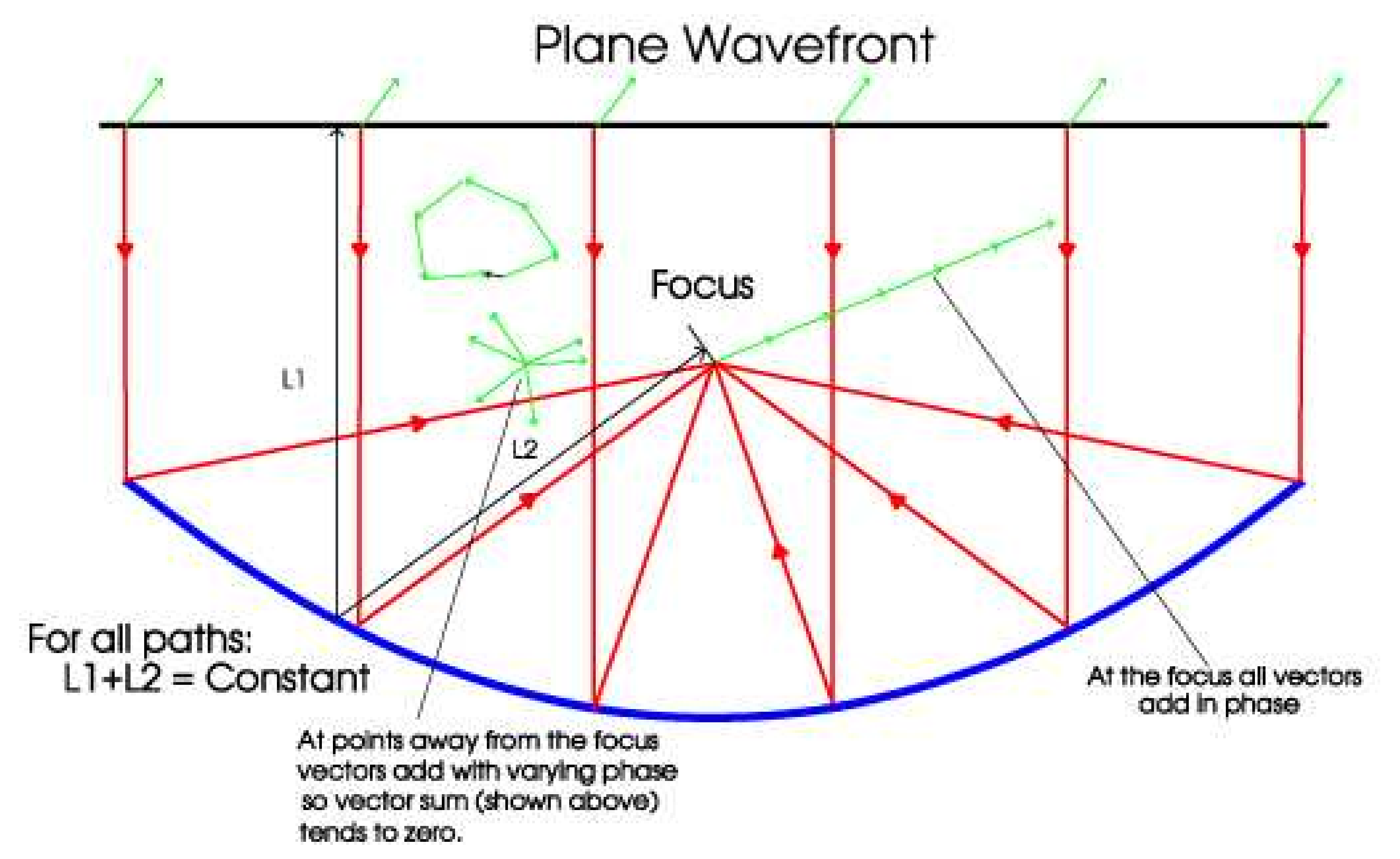




- As the wave moves through space the vector rotates: making one rotation in a time given by 1/f during which time it will have advanced one wavelength in distance.

- If two, or more, “wavelets” arrive at the same point in space, the resultant will be the vector addition of the individual “phasors”

Lets consider a Parabolic Mirror

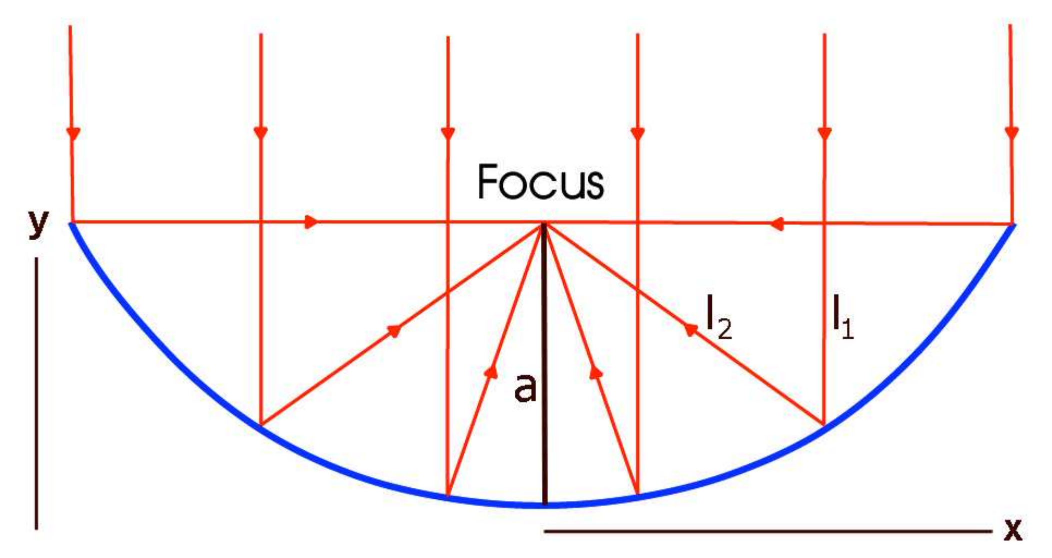


Lets first show that $l_1 + l_2$ is a constant

A parabola is defined by the formula:

$$y = (1/4a)\ x^2$$

a is the distance of the focus from the origin



It works

- Lets make a = 4, so $y = (1/16)\ x^2$

x	0	2	4	6	8
y	0	0.25	1	2.25	4
l_1	4	3.75	3	1.75	0
l_2	4	4.25	5	6.25	8
$l_1 + l_2$	8	8	8	8	8

So $l_1 + l_2 = 2a$

The Proof

$l_1 = a - y$

$l_2 = (x^2 + (a-y)^2)^{1/2}$

Substitute for x^2 ($= 4ay$) and expand the square term giving:

$$(4ay + a^2 -2ay +y^2)^{1/2}$$

$$(a^2 + 2ay +y^2)^{1/2}$$

$$((a + y)^{2})^{1/2} = a + y$$

So $l_1 + l_2 = (a - y) + (a + y) = 2a$

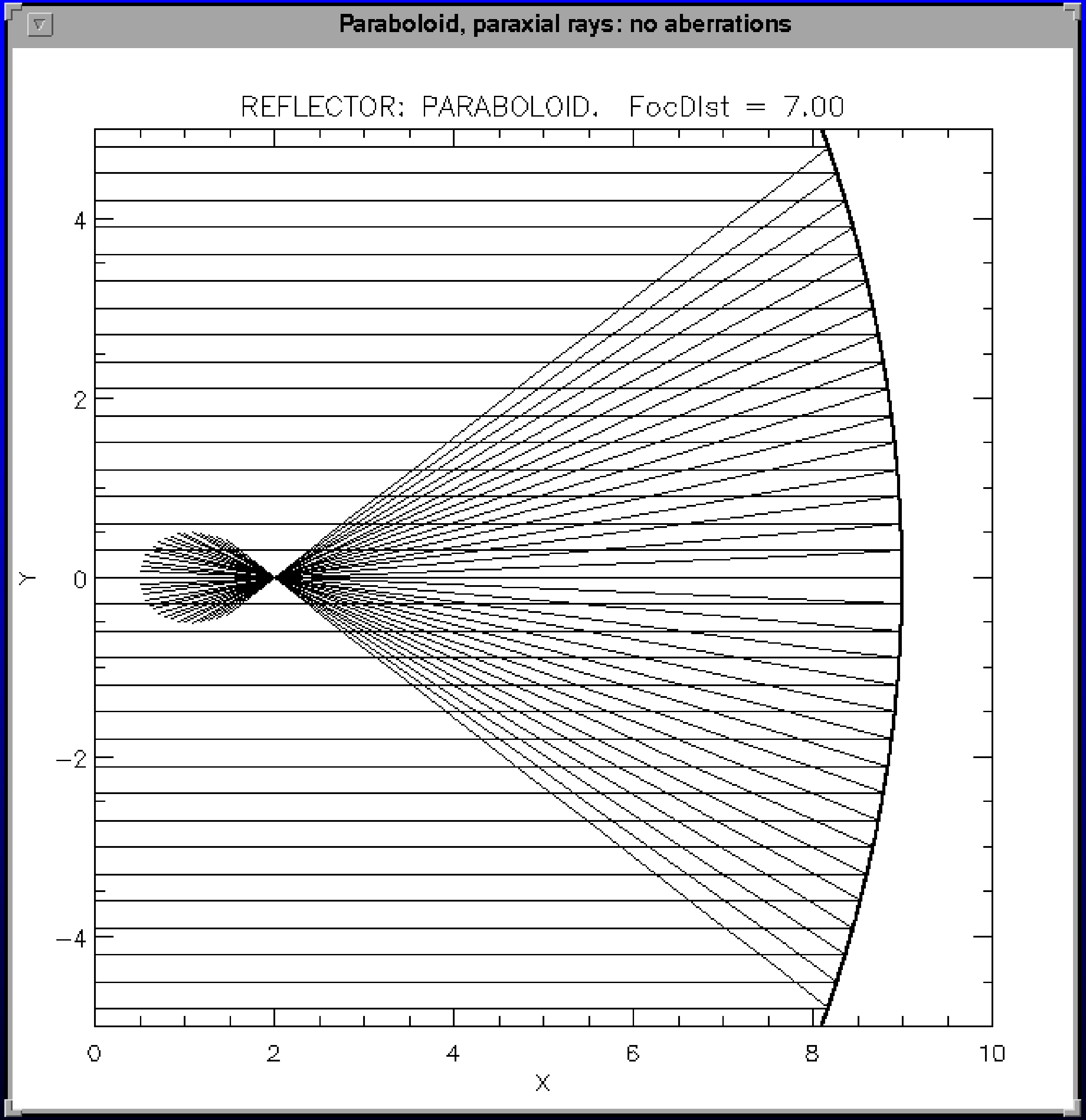
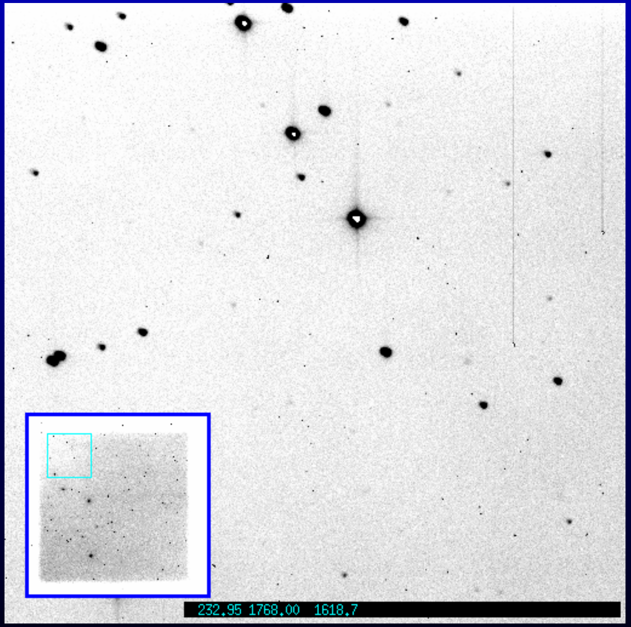
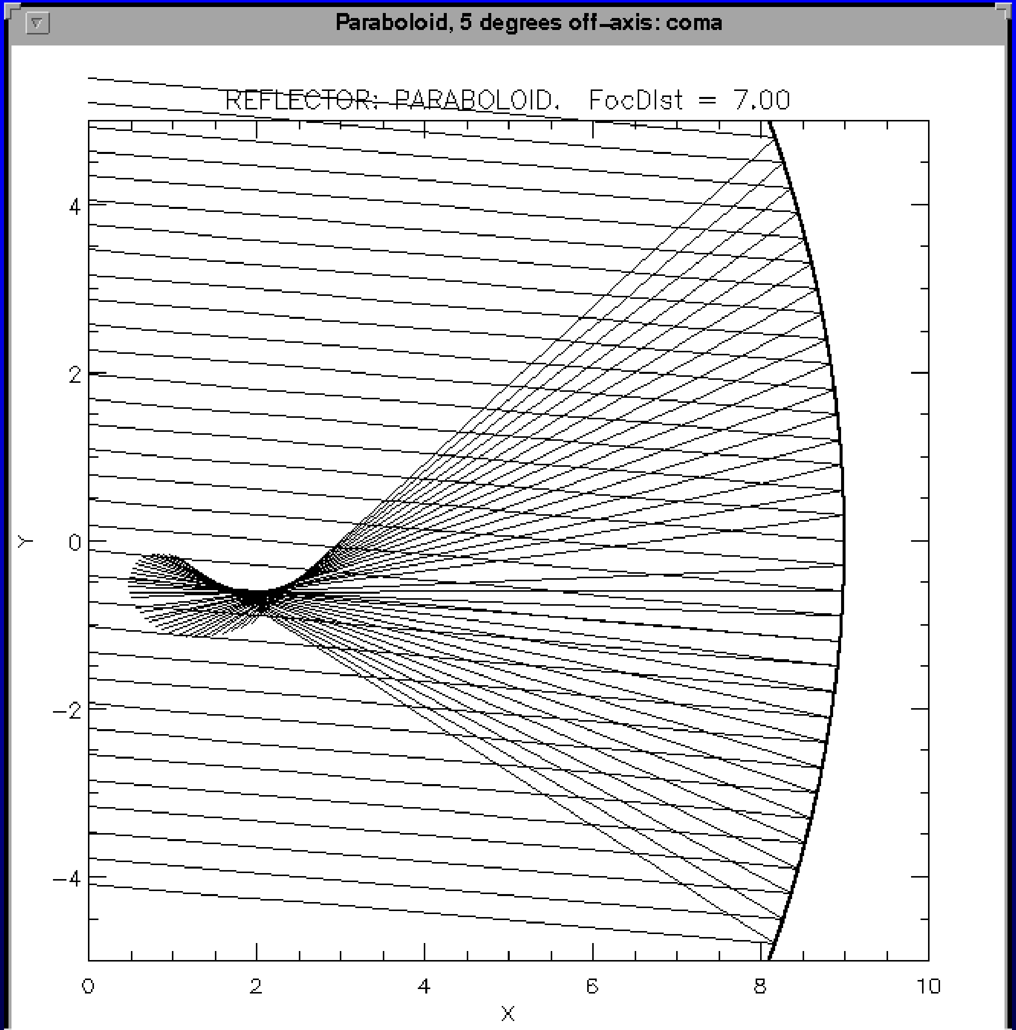

Paraboloid, paraxial rays: no aberrations
REFLECTOR: PARABOLOID. FocDIst = 7.00
Y
X
4
2
0
−2
−4
0
2
4
6
8
10

Image is not so
good
off-axis

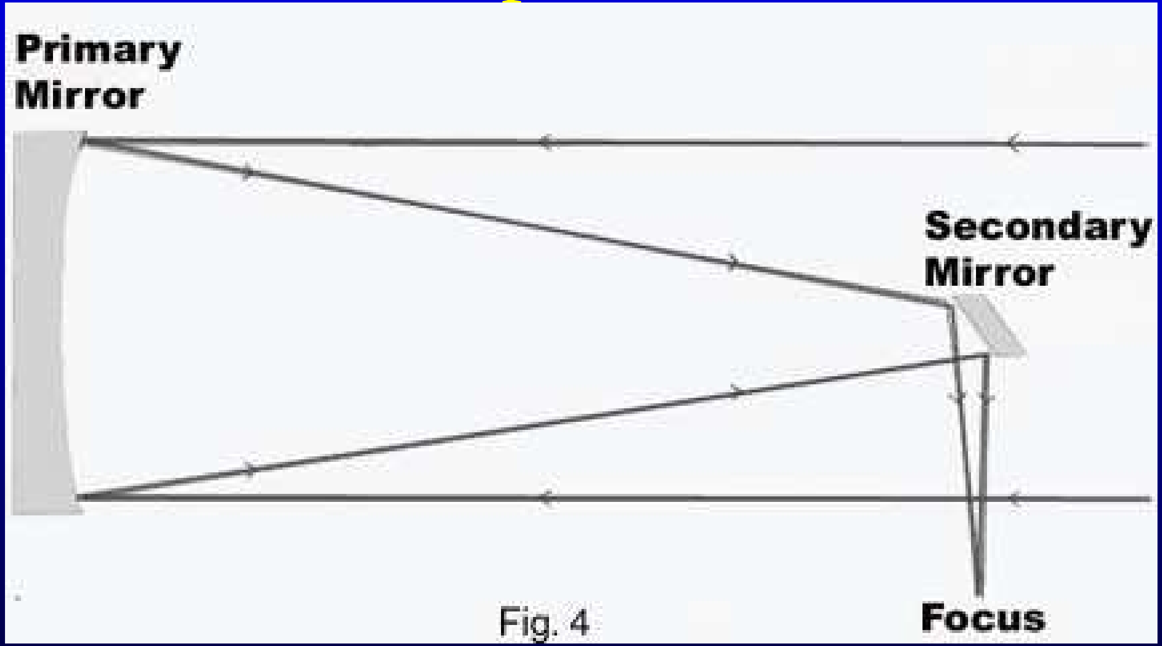
COMA
stars look like
little comets





Newtonian

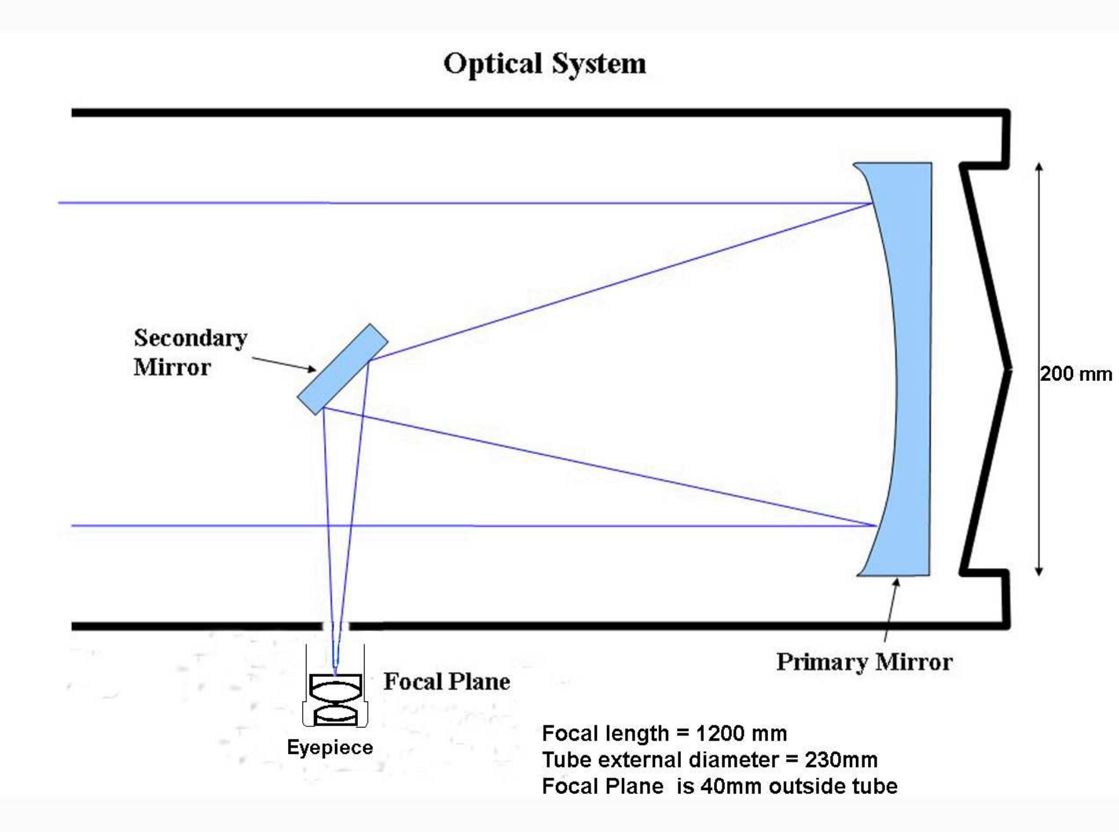
- Light is brought to a secondary focus just outside the telescope tube.



- Eyepiece placed at focus to observe image

Size of secondary mirror

- Lets make a telescope with a 200 mm primary mirror having a focal length 1200 mm. Assume that the telescope tube has an external diameter of 230 mm and that we wish the focal plane to be at a distance of 40 mm outside the tube.



The focal plane will be at a distance of

40 + 230/2 = 155 mm from the mirror axis.

For the secondary mirror (which must have an elliptical outline) to intercept the light cone from the primary it will need to have a minor axis of 200 x 155/1200 mm = 25.8 mm.

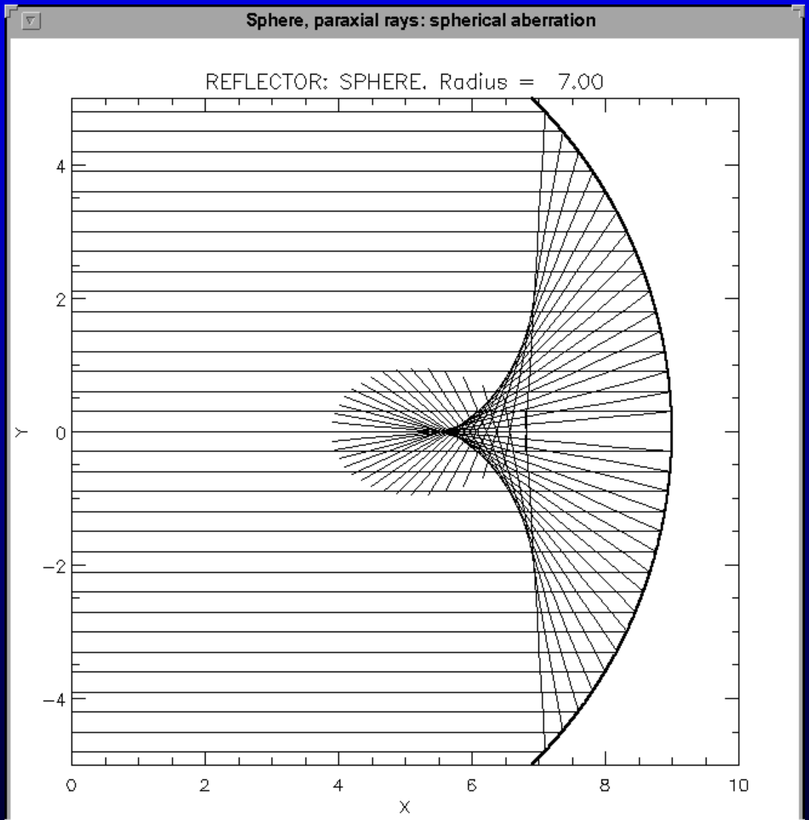
Its major axis will be this times the sqr root of 2 (1.414) = 36.5 mm.

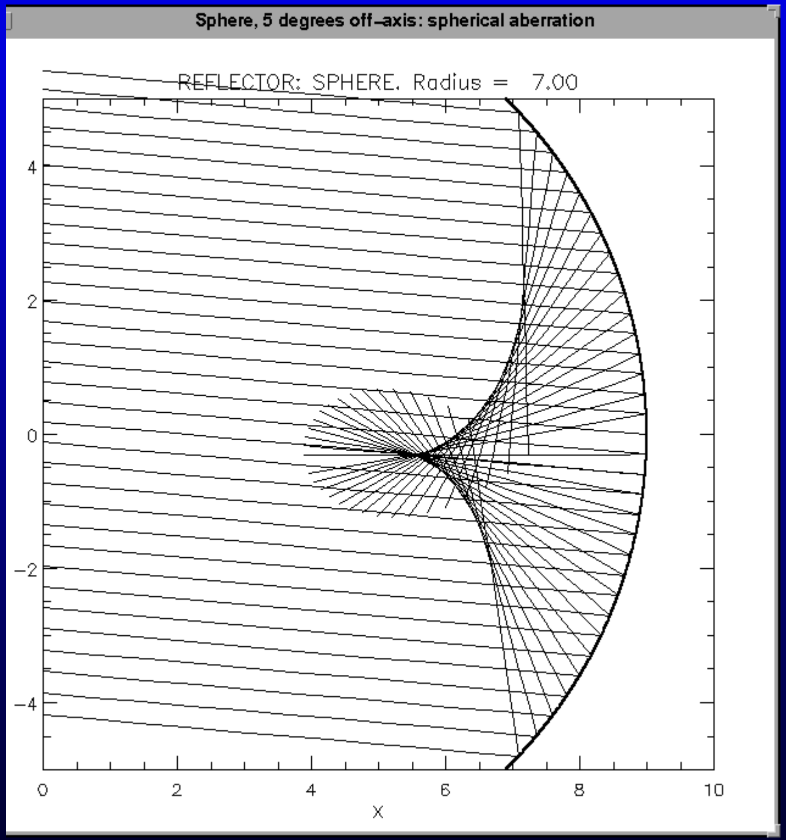
But this mirror will only give full illumination at the very centre of the field.

In practice

- The diameter of the secondary should be increased by field size of the eyepiece - lets say 25mm - so that the whole of the visible image is equally illuminated.
- We get :
  - Minor axis = 50.8 mm
  - Major axis = 71.8 mm

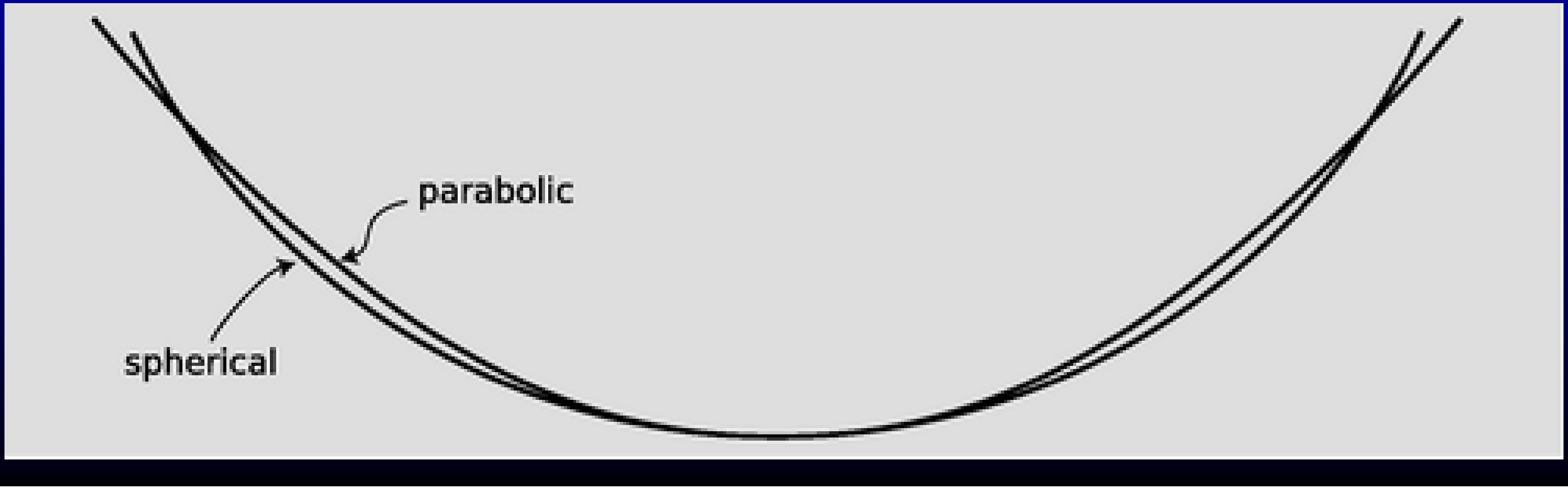
A spherical mirror suffers from “Spherical Aberration but this stays pretty much the same off axis





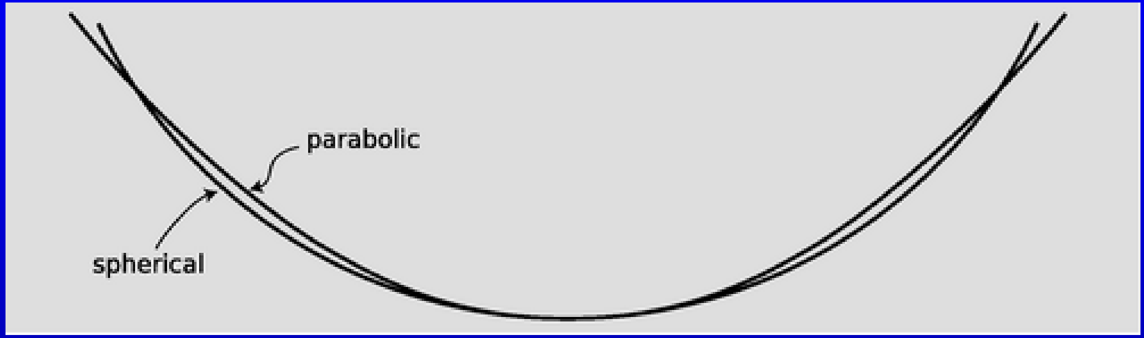
Can correct with a lens

- Has an arbitrary thickness in the centre where no correction is required.
- Is thinner in the middle section
- Is thicker in the outer section



Schmidt-Camera

- Designed by Bernhard Schmidt in 1920 for use as a wide field camera - so used a spherical mirror to eliminate Coma.
- Spherical aberration eliminated by a corrector plate.



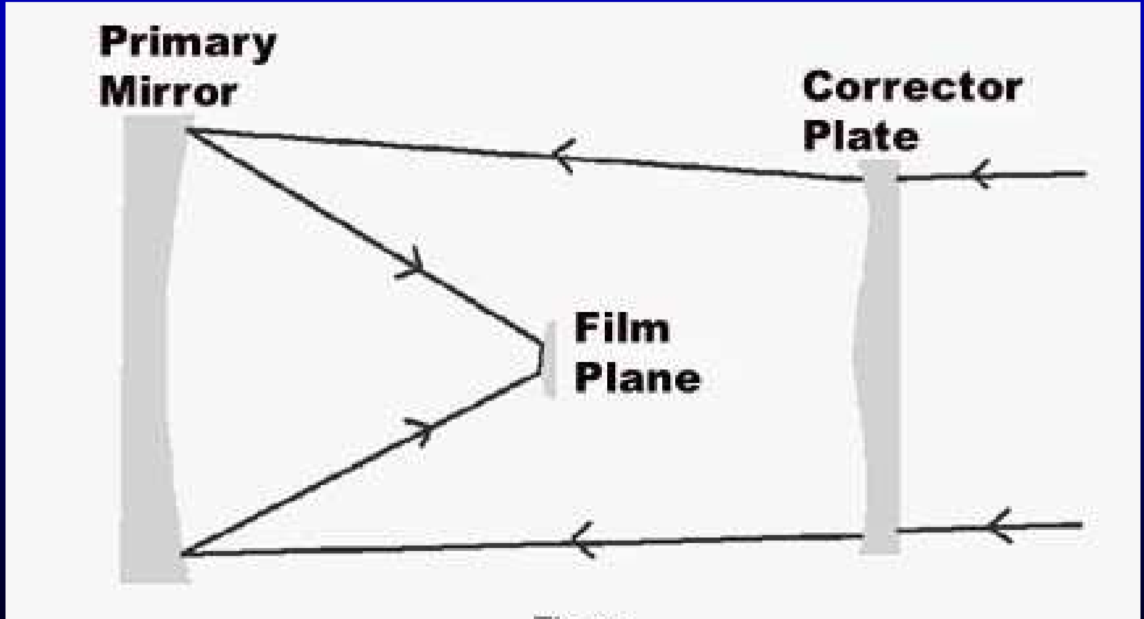
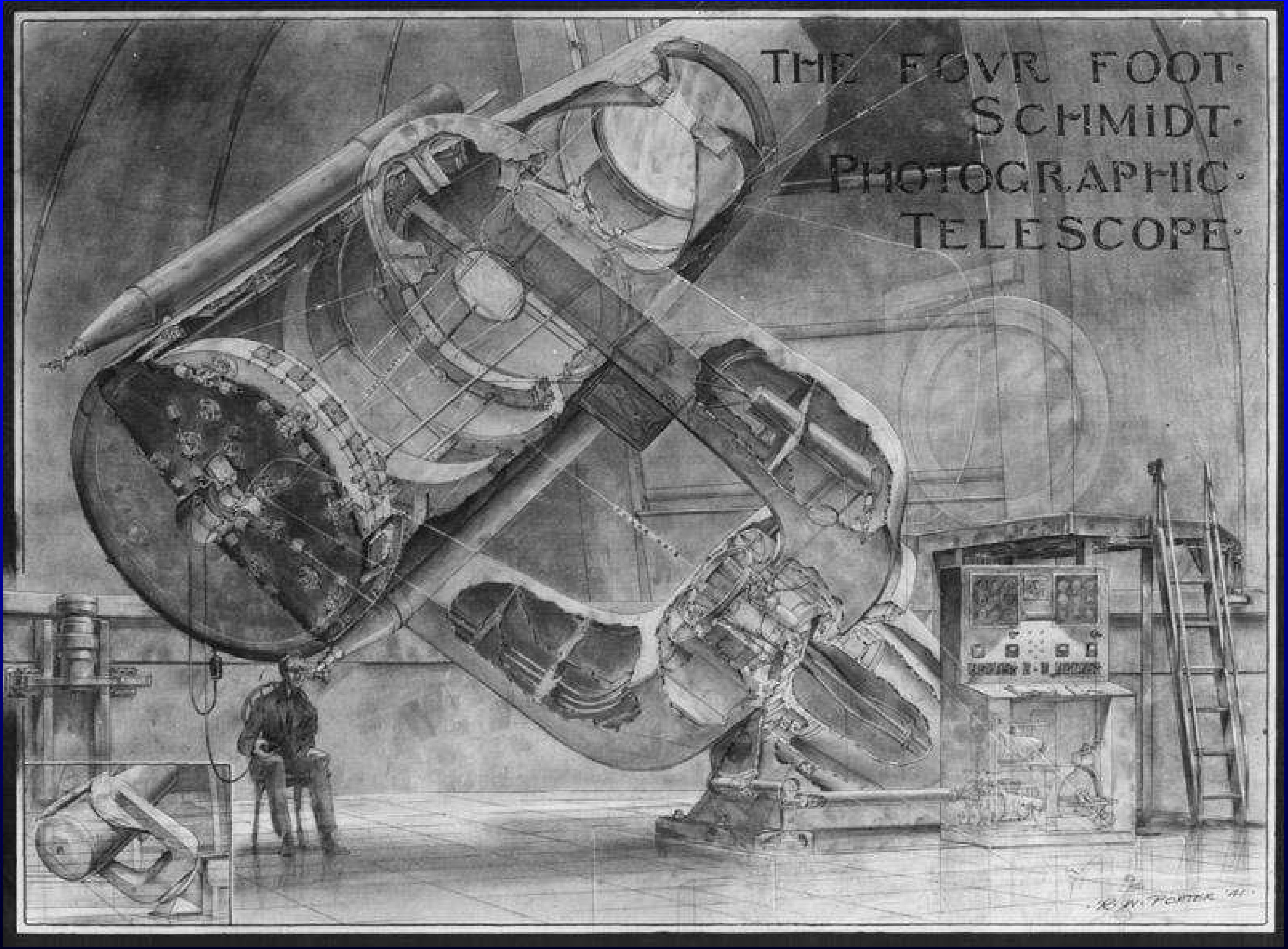
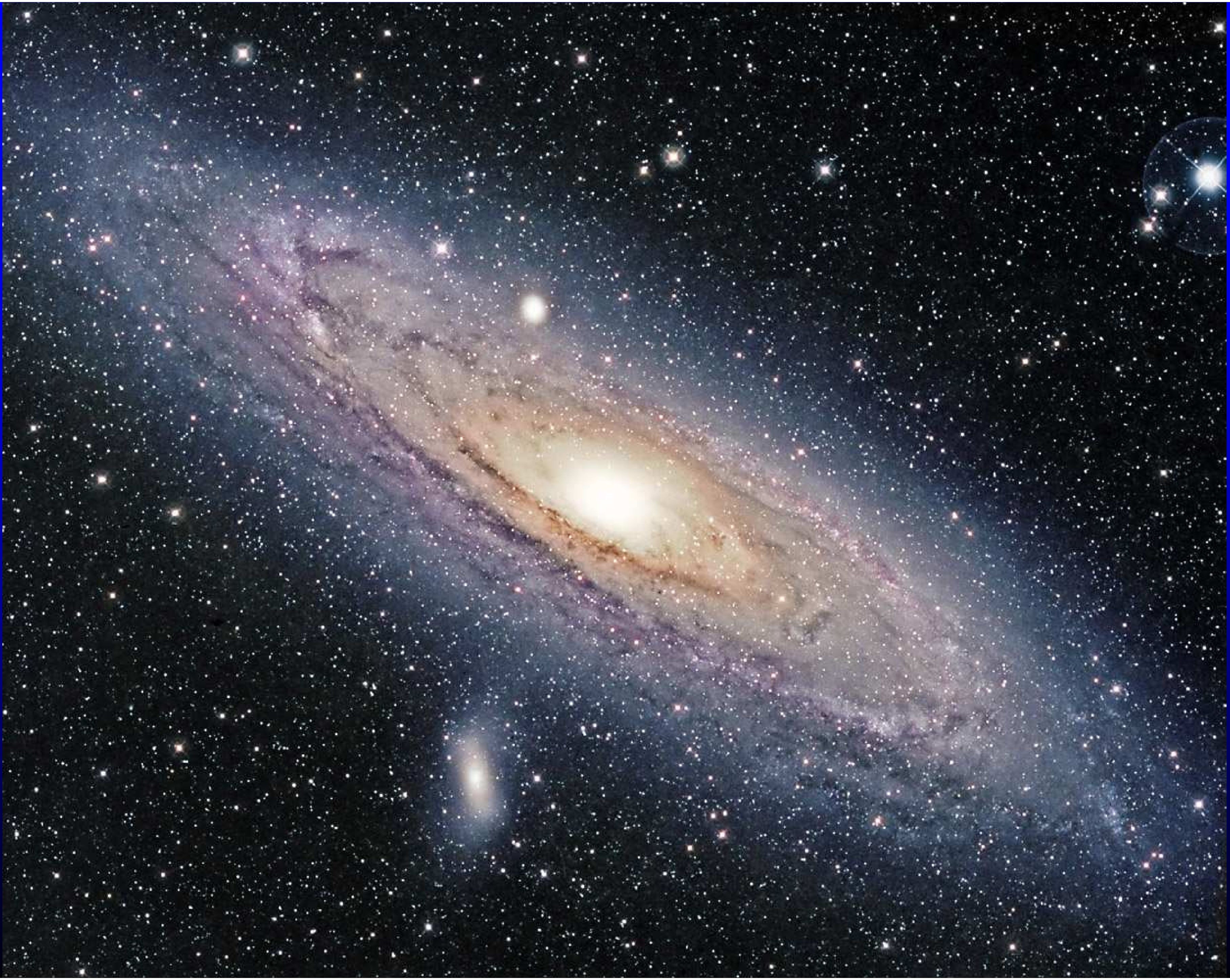


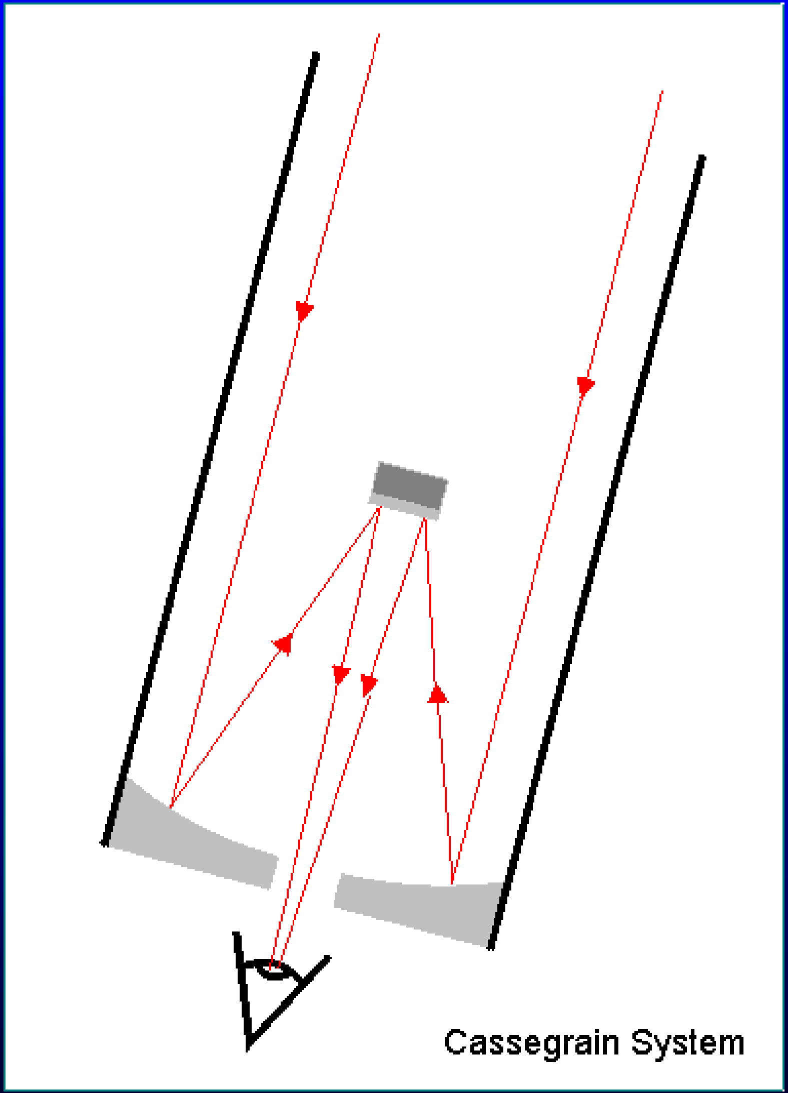
Fig. 5


THE FOVR FOOT·
SCHMIDT·
PHOTOGRAPHIC·
TELESCOPE·
R.W. PORTER '41



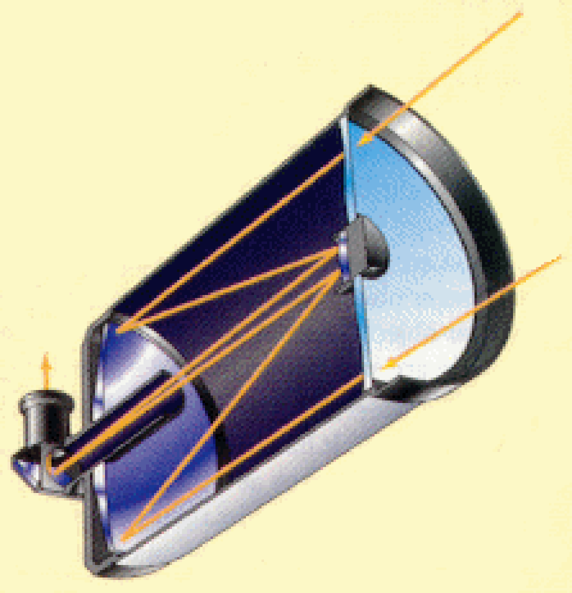
Cassegrain

- Light is reflected by a secondary mirror through the primary mirror to the focus.
- A better place to put instruments used with large telescopes.
- Primary is a parabolic mirror, secondary a hyperbolic mirror



Schmidt-Cassegrain

- A combination of a cassegrain focus – but using a spherical primary mirror and a Schmidt corrector plate.
- As spherical mirrors can be made more cheaply than parabolic ones, now in common use by amateur astronomers.
- Very compact for aperture.



Maksutov

- Uses a steeply curved correction plate.
- Secondary can be silvered on its back.
- Now quite popular.

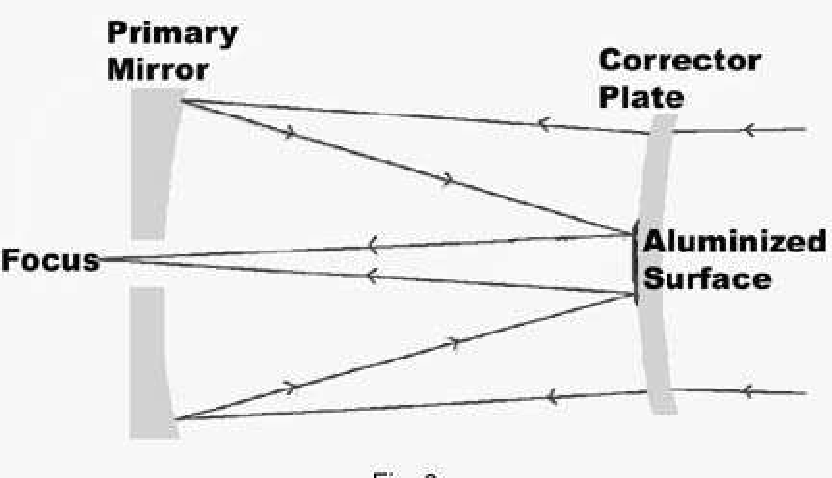
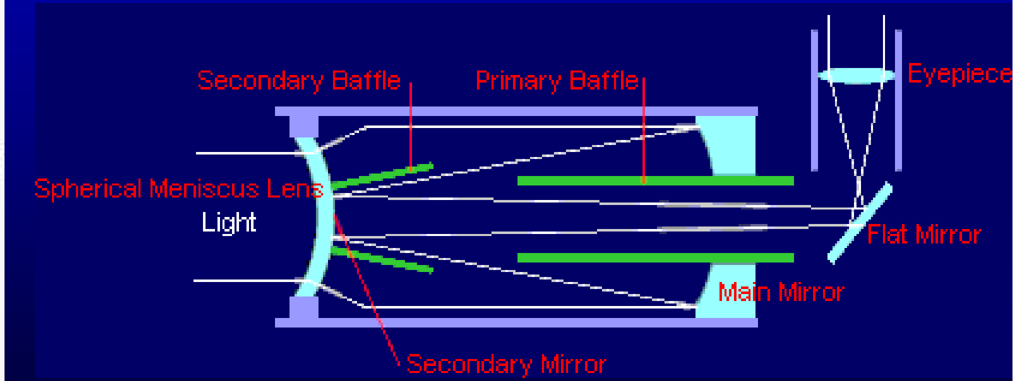


Fig. 3

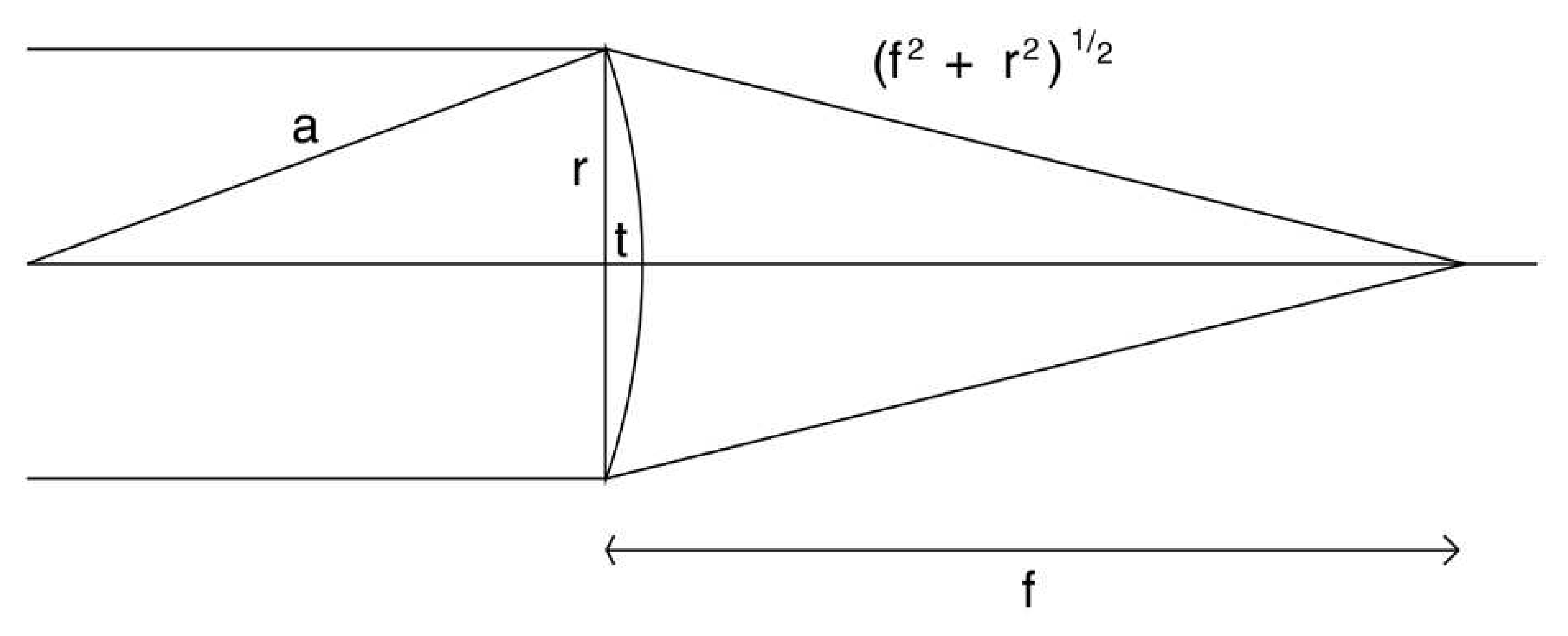


Lenses to form images

- Lenses, usually made of glass, can be used to form images due to the fact that light travels more slowly in glass so the effective path length that the light wave travels within the glass is greater than the actual distance by a factor that is called the Refractive Index, n.
- n for Crown Glass at a wavelength of 589.3 nm is 1.517
- In contrast, n for air is 1.0008 - essentially the same as for a vacuum for which n is defined as 1.

- I cannot prove that all path lengths through the lens are constant because they are not!
- But, for a thin lens, its pretty close. The lens will suffer some spherical aberration which can be removed by making it aspheric.
- We can, however, derive the lensmaker’s equation by equating the wave-path through the centre of the lens with one at its extreme edge
- We will take the simplest possible case and assume the lens is in a vacuum ($n = 1.0008$ for air so this is virtually true)

A Plano-convex lens



$$(f^2 + r^2)^{1/2} = (f - t) + nt$$
$$= f + t(n - 1)$$

Square both sides:

$$f^2 + r^2 = f^2 + 2ft(n - 1) + (t(n-1))^2$$

Ignoring the final term (t is very small so t^2 is even smaller) and cancelling out the f^2 term gives:

$$r^2 = 2ft(n - 1)$$

So

$$f = r^2 / 2t(n - 1)$$

If the radius of curvature of the lens surface is a, then:

$$a^2 = (a-t)^2 + r^2$$

$$a^2 = a^2 - 2at + t^2 + r^2$$

Cancelling out the a^2 term and ignoring t^2 gives:

$$2at = r^2$$

that is: $$a = r^2 / 2t$$

Substituting for $r^2 / 2t$ in the equation for the focal length gives:

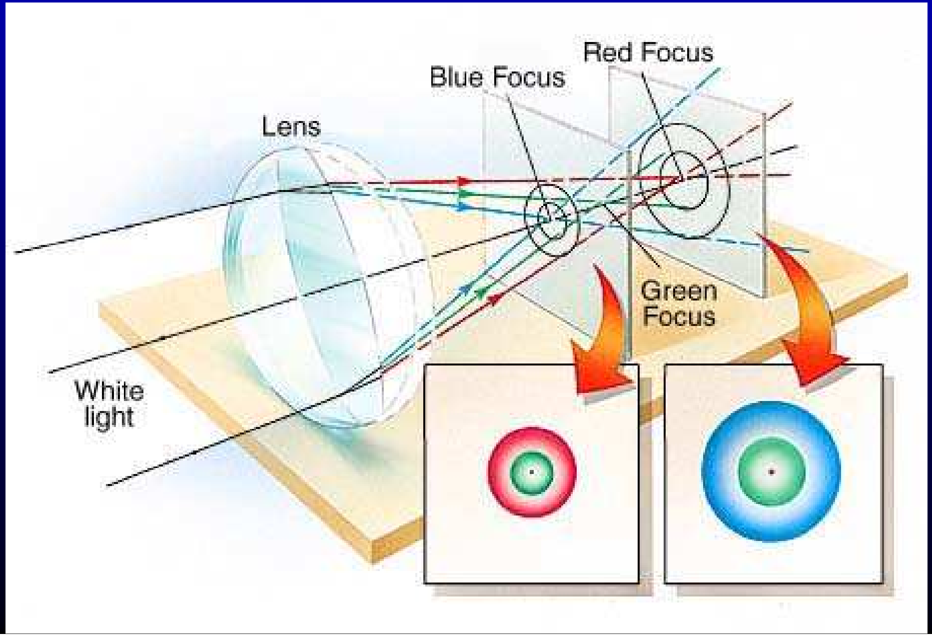
$$f = a / (n-1)$$

This is the lensmaker’s equation for this simplest case.

Chromatic Aberration

There is a problem:

The refractive index of a glass, n, varies with wavelength so the focus of each colour will be at different distances from the lens. Images focussed for green light will be fringed with red and blue light that is our of focus. (red + blue = purple)



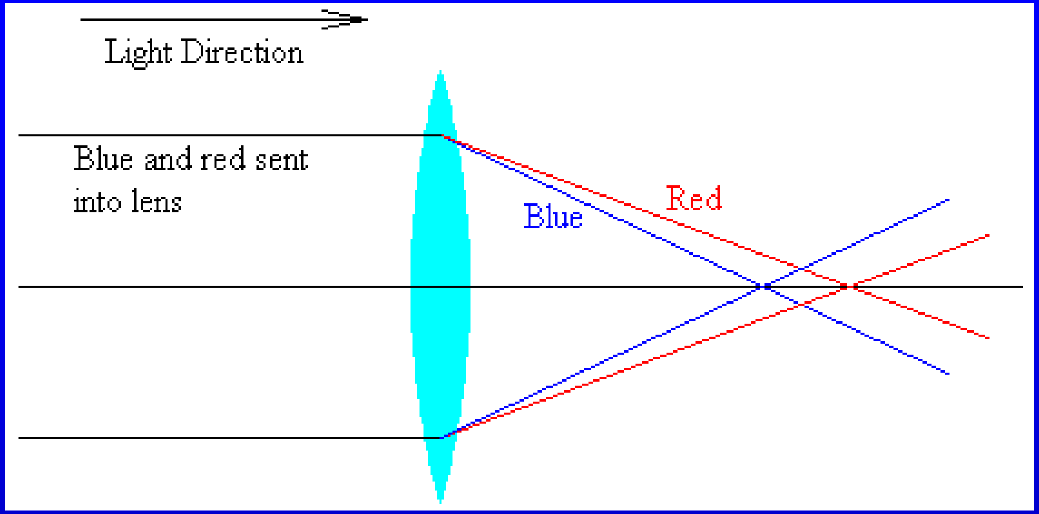
Refractive index as a function of wavelength.

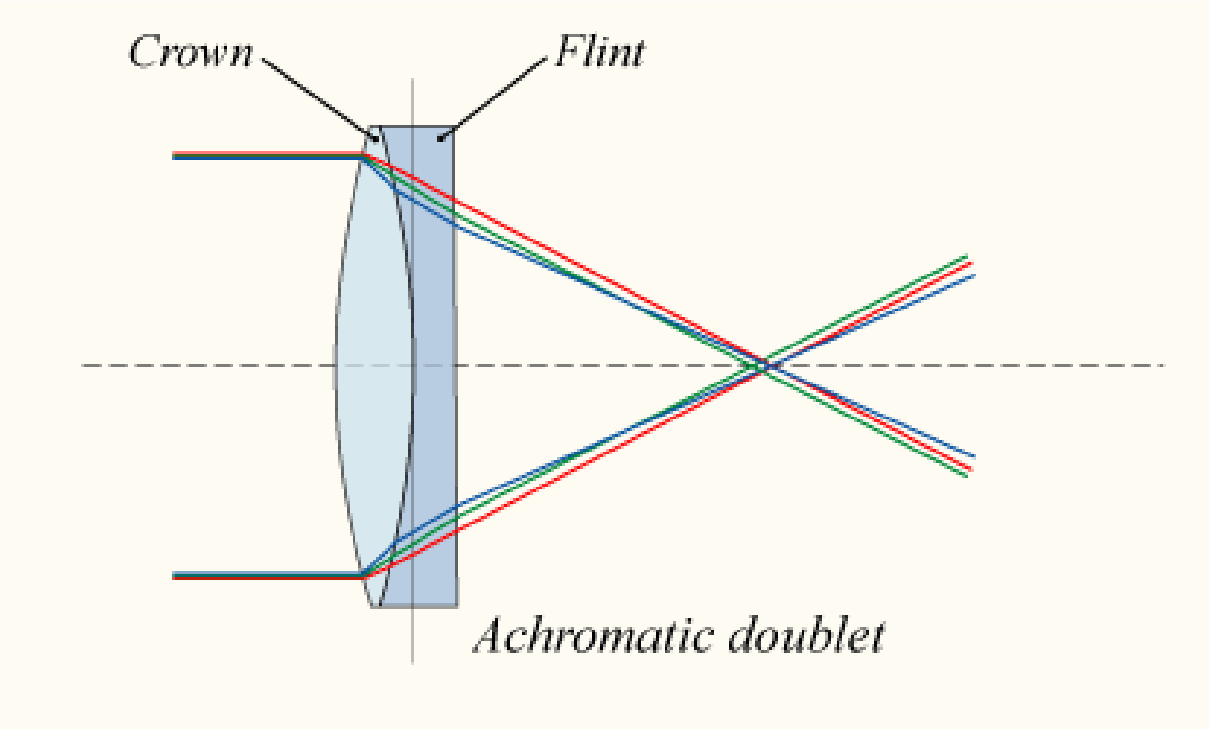
	Blue	Green	Red
	486.1 nm	589.3 nm	656.3 nm
Crown	1.524	1.517	1.515
Flint	1.639	1.627	1.622

Consider a plan-convex lens made of crown glass with radius of curvature of 2000 mm:

F_{blue} = 954 mm
F_{green} = 967 mm
F_{red} = 970 mm

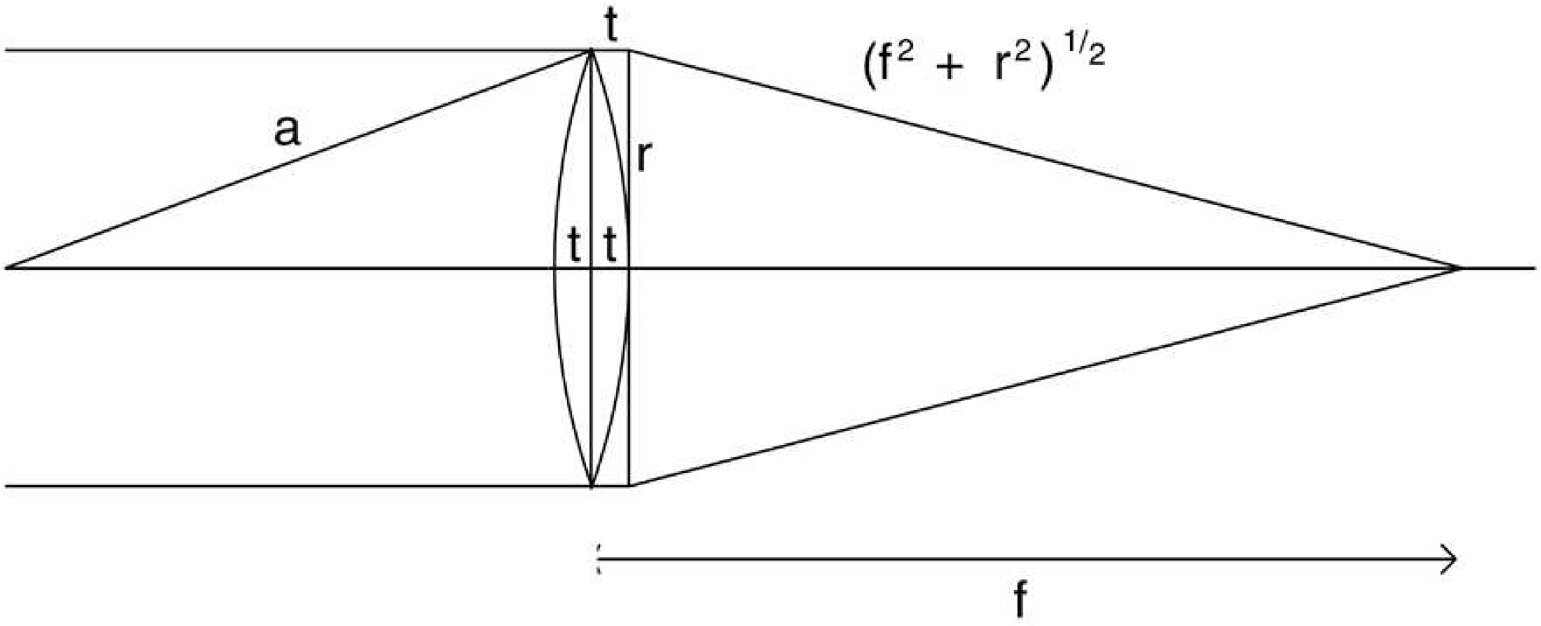
Achromatic Doublet





A doublet demonstration

- For simplicity of the mathematics we will make the minimum thickness of the Flint lens be zero. (additional thickness only adds a constant additional path length to all rays passing through the lens.



Let n_1 be the refractive index of the crown glass

Let n_2 be the refractive index of the flint glass

$$f + 2n_1t \quad = \quad (f^2 + r^2)^{1/2} + n_2t$$

$$f + t(2n_1 - n_2) = (f^2 + r^2)^{1/2}$$

Squaring both sides gives:

$$f^2 + 2ft(2n_1 - n_2) + (t(2n_1 - n_2))^2 = f^2 + r^2$$

Ignoring the term in t^2 and simplifying gives:

$$f = r^2 / 2ft\,(2n_1 - n_2)$$

Substituting for r^2 then gives:

$$f = a/(2n_1 - n_2)$$

We will use a value of “a” of 1360.5mm to give a yellow light focus of 967, the same as for as for the single crown glass lens. Given the values of n for Flint and Crown glass given above we get:

	$2n_1 - n_2$	f
Blue	1.409	965.6 mm
Green	1.407	967.0 mm
Red	1.408	966.3 mm

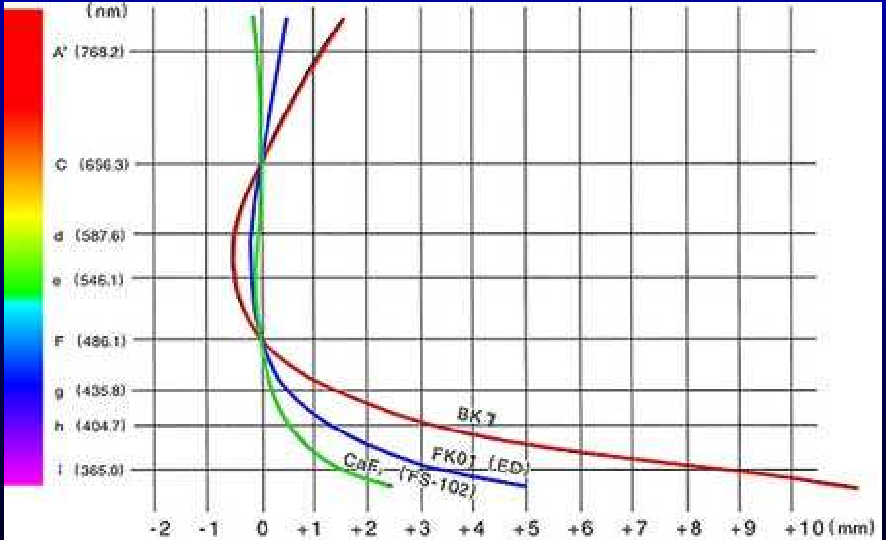
Whereas in the singlet lens the foci were spread over a distance of 16 mm, the doublet has reduced this to 1.4 mm - a factor of over 10 better.

There will still be some chromatic aberration but one can buy filters which remove the red and blue ends of the spectrum (where the eye is not very sensitive) to give almost colour free images.

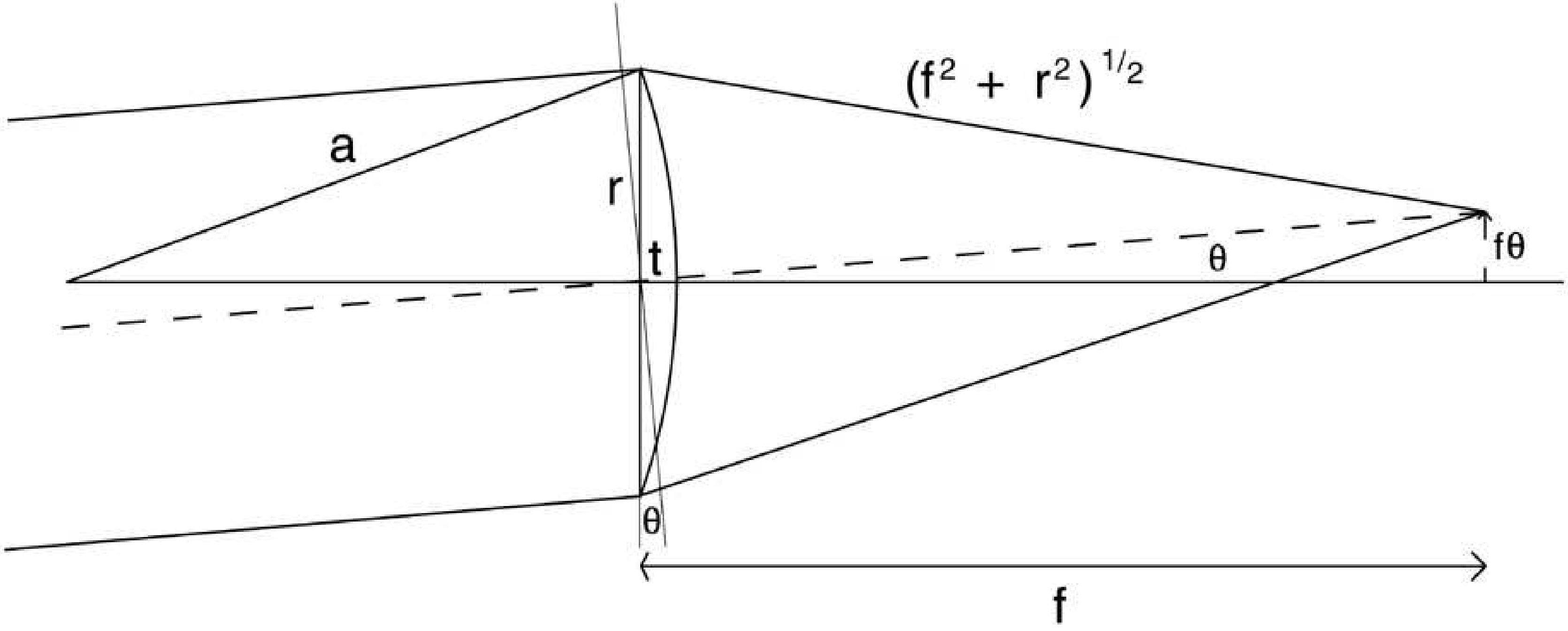
Most refracting telescopes use achromatic doublets

Apochromats

More sophisticated design and the use of either a flourite lens in the doublet or by the use of three lenses, one of which had a glass with a very high refractive index can essentially remove chromatic aberration completely. These are called Apochromatic lenses. They are very expensive but produce wonderful images.



Formation of an off axis image



- Imagine a plane wave front incident at an angle θ to the lens axis.
- The distance from the tips of the lens to the focus must be equal thus, by symmetry, the focus must lie on a line at an θ angle to the lens axis.
- To first order it will be at a distance of (fθ) from the lens axis at the same distance f from the lens. (NB θ is in radians)
- Thus the scale size of the primary image produced by a telescope is proportional to its focal length.

Magnification

- Our eye can resolve:
  - ~ 1 arc minute in the day time
  - ~ 2 arc minutes at night
    - (The lens at “full” aperture (~6mm) suffers from more aberrations than when “stopped down” in daylight (~2.5mm).
- So, with a magnification of x120, which makes the image appear 120 times bigger, we could resolve ~ 1 arc second.
- This is the effective resolution of a 102mm telescope.
- So we do not really need much more magnification than that!
- So rarely magnify more than x 300

- Larger aperture telescopes can theoretically resolve greater detail, BUT the atmosphere degrades the image quality.
- This is called the “seeing”
- In the UK it rarely better than 2 arc seconds due to turbulence in the atmosphere.
- To Improve:
  - 1) Get as high as you can – less atmosphere.
  - 2) Try to have a non turbulent airflow.
  - 3) Take VERY short exposure images.

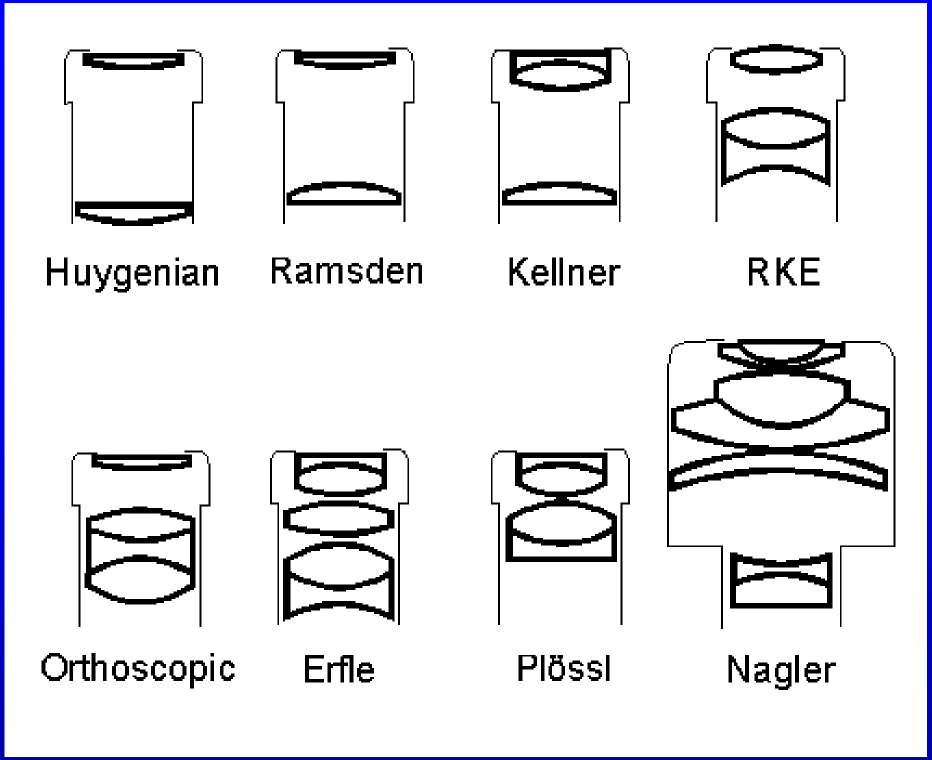
Telescope Parameters

- 1) Focal Length of Objective
- 2) Objective Diameter
- 3) Focal Ratio = Focal Length/Aperture.
  - So a 1000mm focal length, 125 mm aperture telescope has a focal ratio of f8.
- 4) Magnification = Focal Length of Objective / Focal Length of eyepiece.

Field of View

- Telescopes can only observe part of the sky at one time.
- In general, the greater the magnification, the smaller the field of view.
- It is determined by the aperture of the eyepiece – called the field stop.
- The largest field stop available for use in amateur telescopes is ~44 mm.

Eyepieces



- Eyepieces have focal lengths from ~ 2.5mm up to ~ 30mm
- So, given a 1000mm focal length telescope, we can get magnifications of from ~ 33 up to 400.
- Barlow lenses and Focal Reducers can be obtained to increase the focal ratio or reduce it – say from f8 to f16 or from f10 to f6.3 respectively. These can help give an appropriate image size on a CCD camera for planets when a Barlow is often used, or galaxies when a focal reducer is often used.

An example:

- What is the angular size of the field of view using an 44 mm field-stop eyepiece with a 1000 mm focal length telescope?
- It is the angle subtended by the field stop diameter (44 mm) at the objective.
- Using the small angle approximation

θ = 44/1000 radians = 44 x 57.3/1000 degrees

= 2.52 degrees.

- A short focal length eyepiece may only have a field stop diameter of 4mm. This is also the length of the side of a typical square CCD imaging array.
- What field of view would we get?

θ = 4 /1000 radians = 4 x 57.3/1000 degrees

= 0.23 degrees = 0.23 x 60 arc minutes

= 13.75 arc minutes.

Ideal Sites for Telescopes

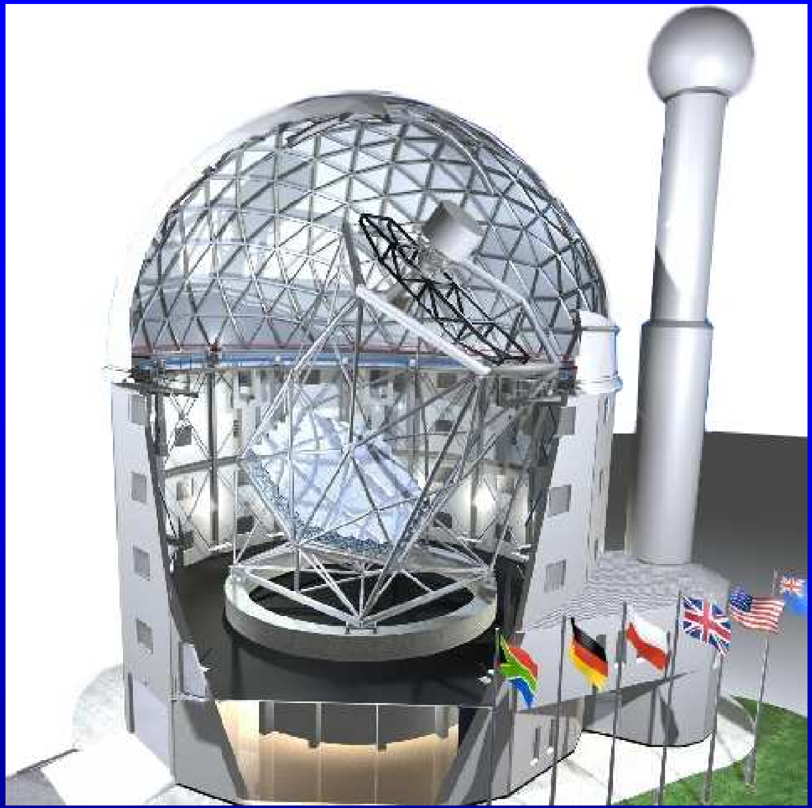
- On high peaks in the middle of oceans.
  - Above clouds and much of the atmosphere
  - In smooth airflow.
  - Where there is no light pollution!

- Mauna Kea on Big Island, Hawaii.
  - At a height of 4200m
- Island of La Palma, Canaries
  - At a height of 2350m
- Mountains in Chile – Paranal
  - At a height of 2635m
- The Northern Cape in South Africa
  - At a height of 1800m

Mauna Kea

VLT

Paranal, Chile

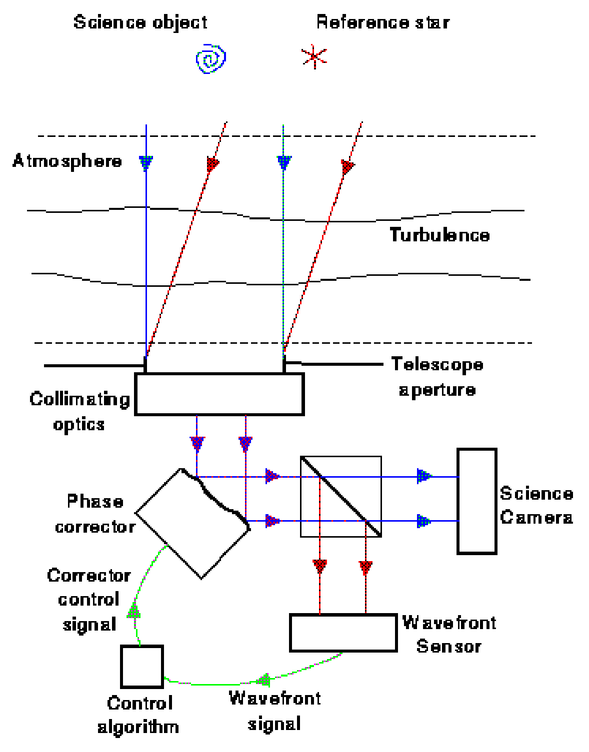


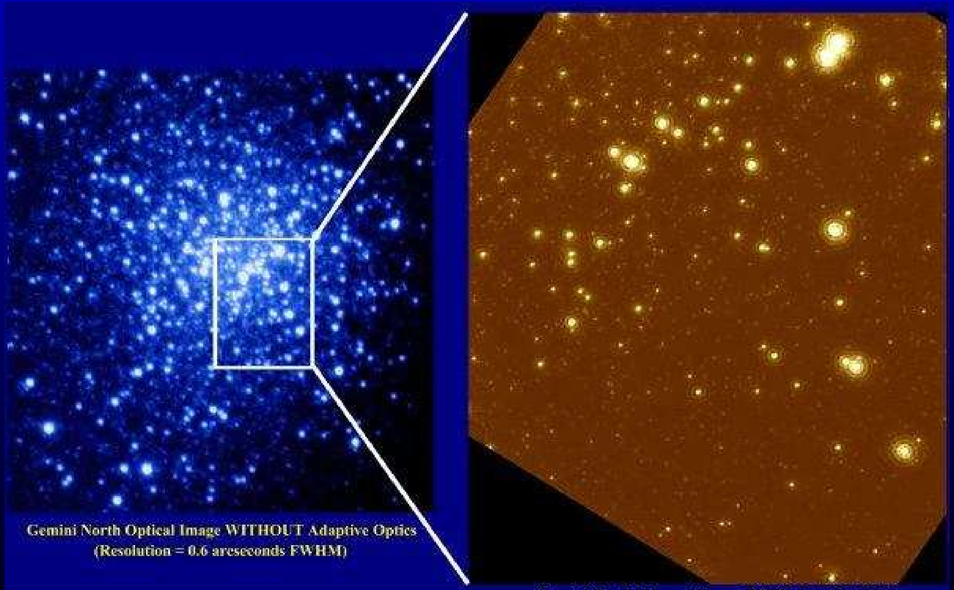
SALT
Southern Africa Large Telescope

William Herschell Telescope, La Palma

Image Resolution

- Here the seeing can be as good a ~ 1/3 arc second.
- At Infra-red wavelengths it is possible to use ADAPTIVE OPTICS to partially remove the atmospheric effects.



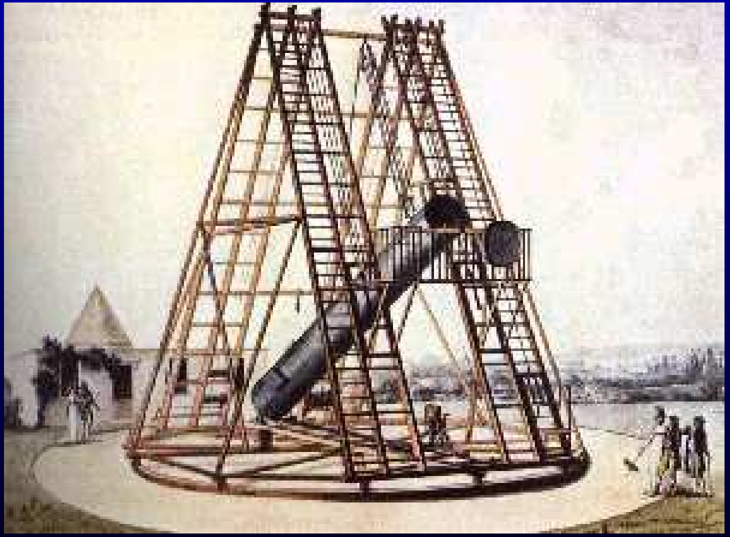


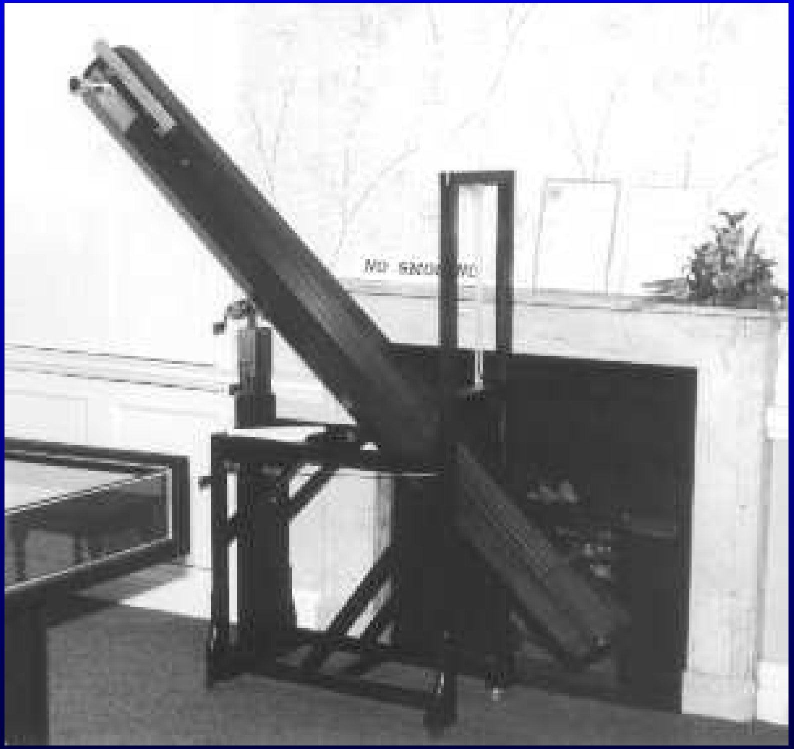
Gemini North Optical Image WITHOUT Adaptive Optics
(Resolution = 0.6 arcseconds FWHM)

Gemini North Infrared Image WITH Adaptive Optics
(Resolution = 0.09 arcseconds FWHM)

Great Optical Telescopes

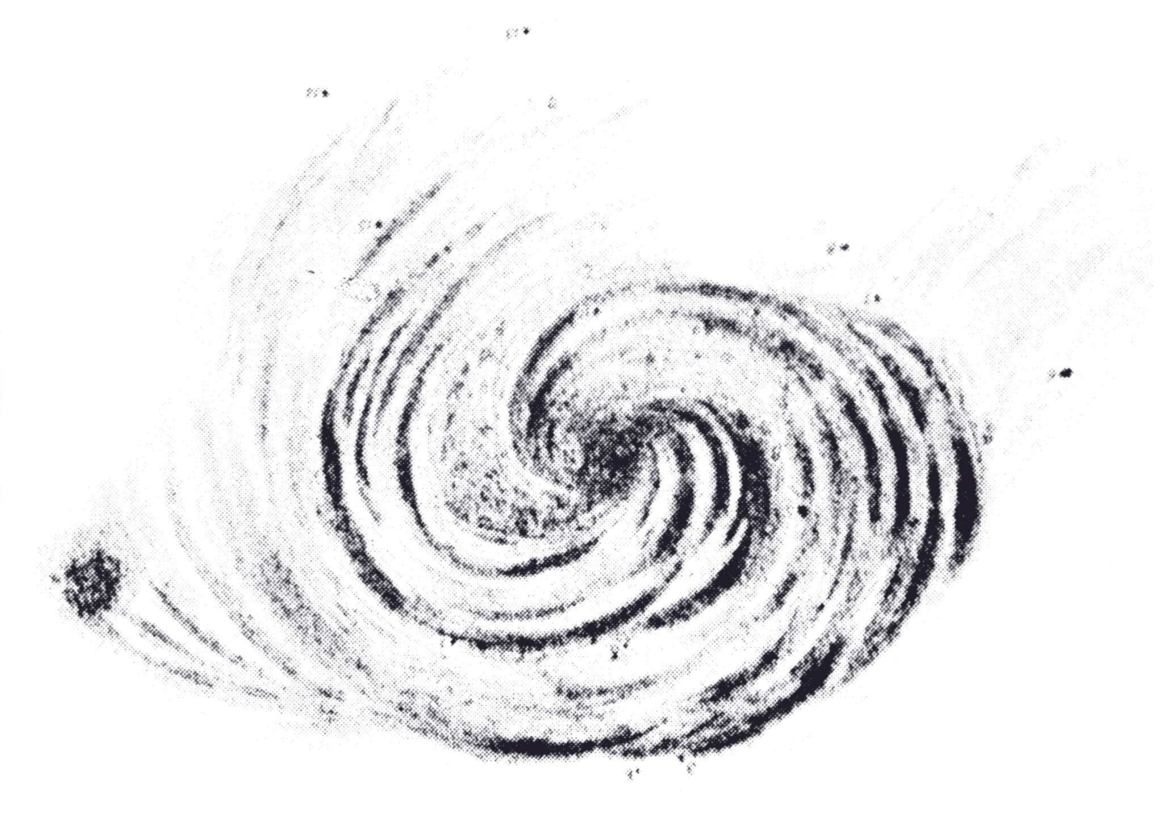
William Herschell and his Telescope



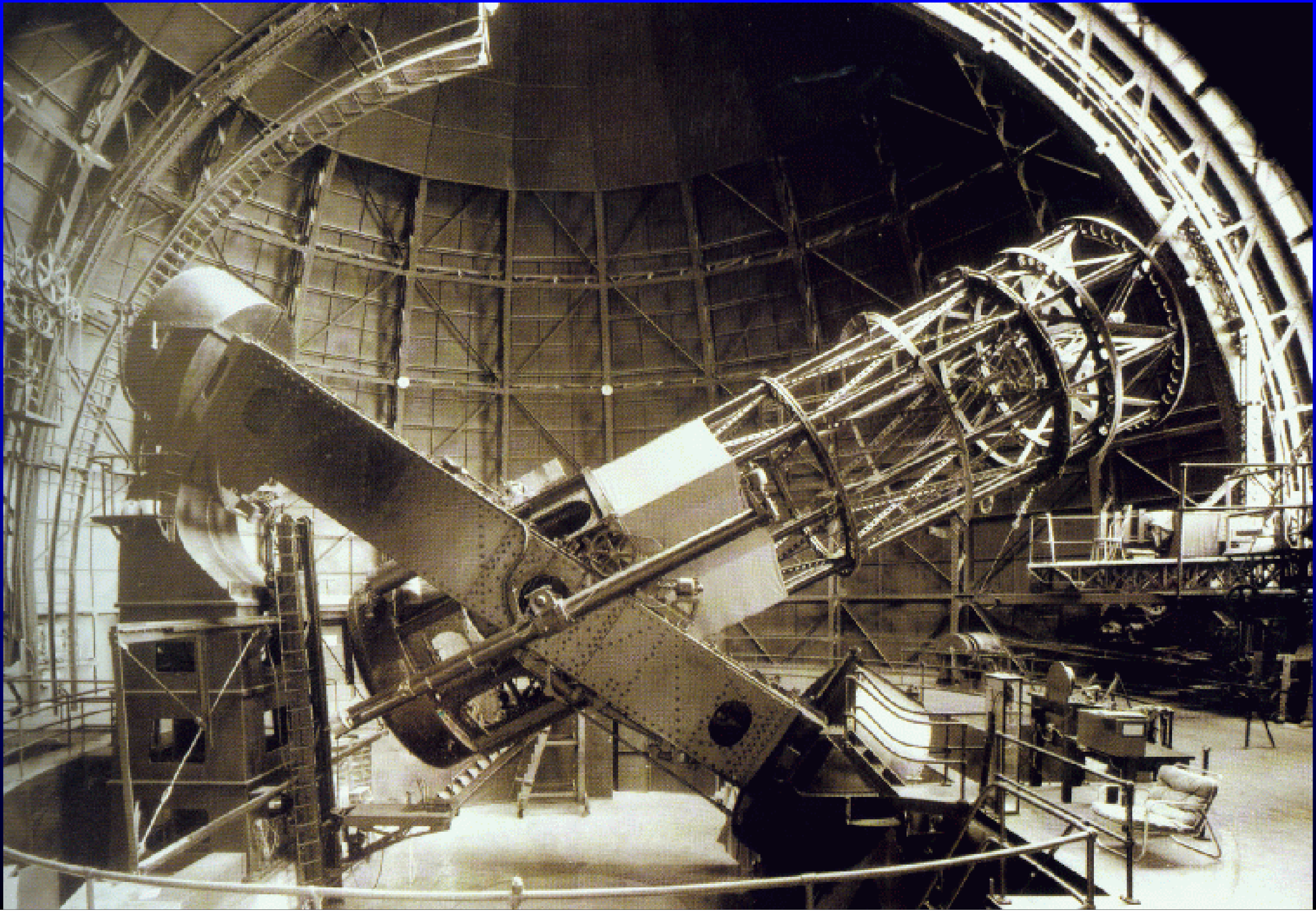


Birr Castle

M51 – The Whirlpool Galaxy



100” Hooker Telescope, Mt Wilson

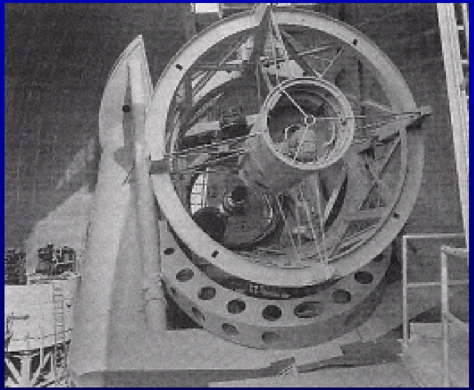


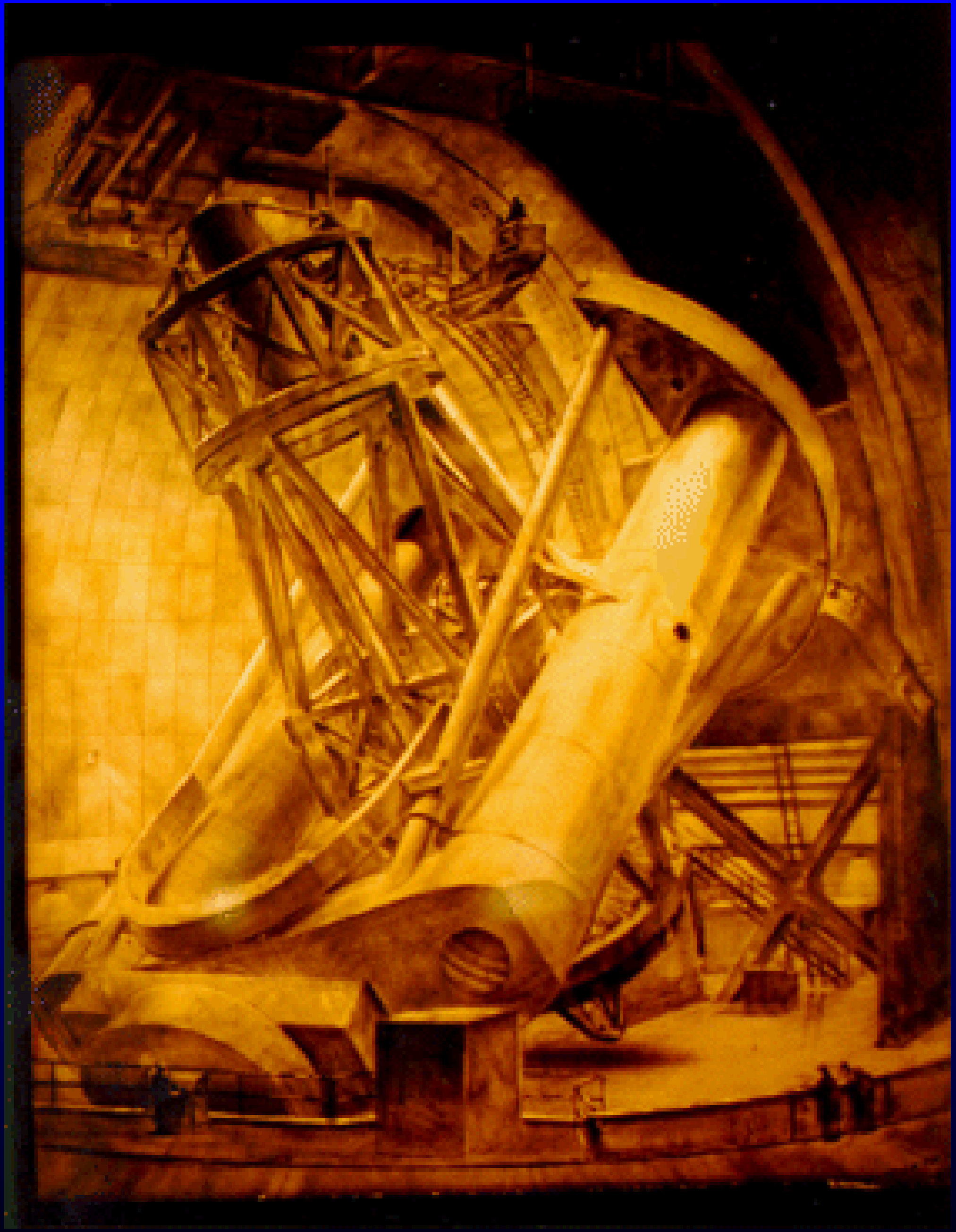
Edwin Hubble

Hale 200”

at

Mount Palomar





Hubble Space Telescope

Resolution of Hubble Space Telescope

- As outside the Earth's atmosphere, it is "diffraction limited" with an effectively perfect mirror.

theoretical resolution = 1.22 λ /D

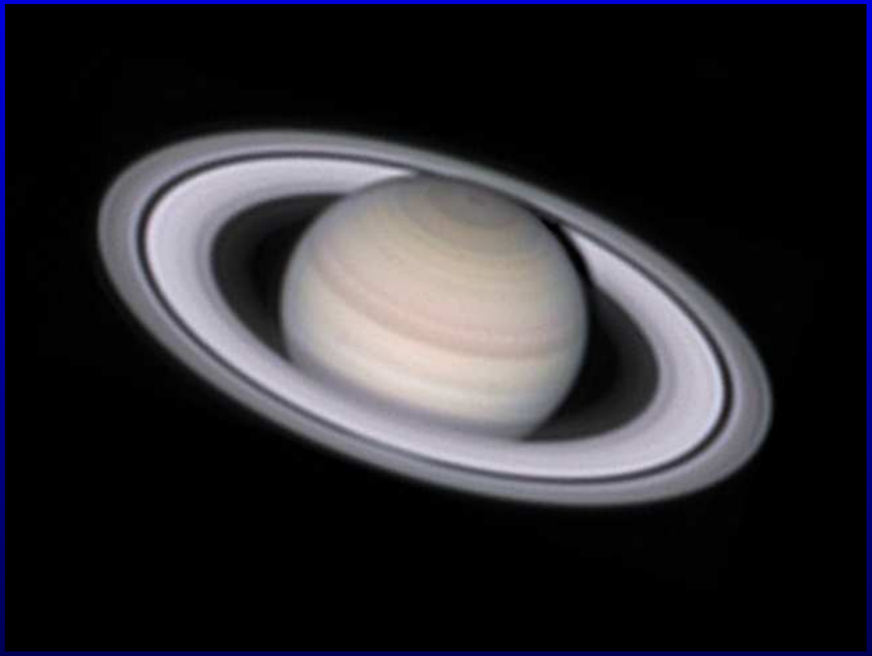
= 1.22 x 0.5E-6 m / 2.4 m radians

= 2.5E-7 radians

= 0.05 arc seconds

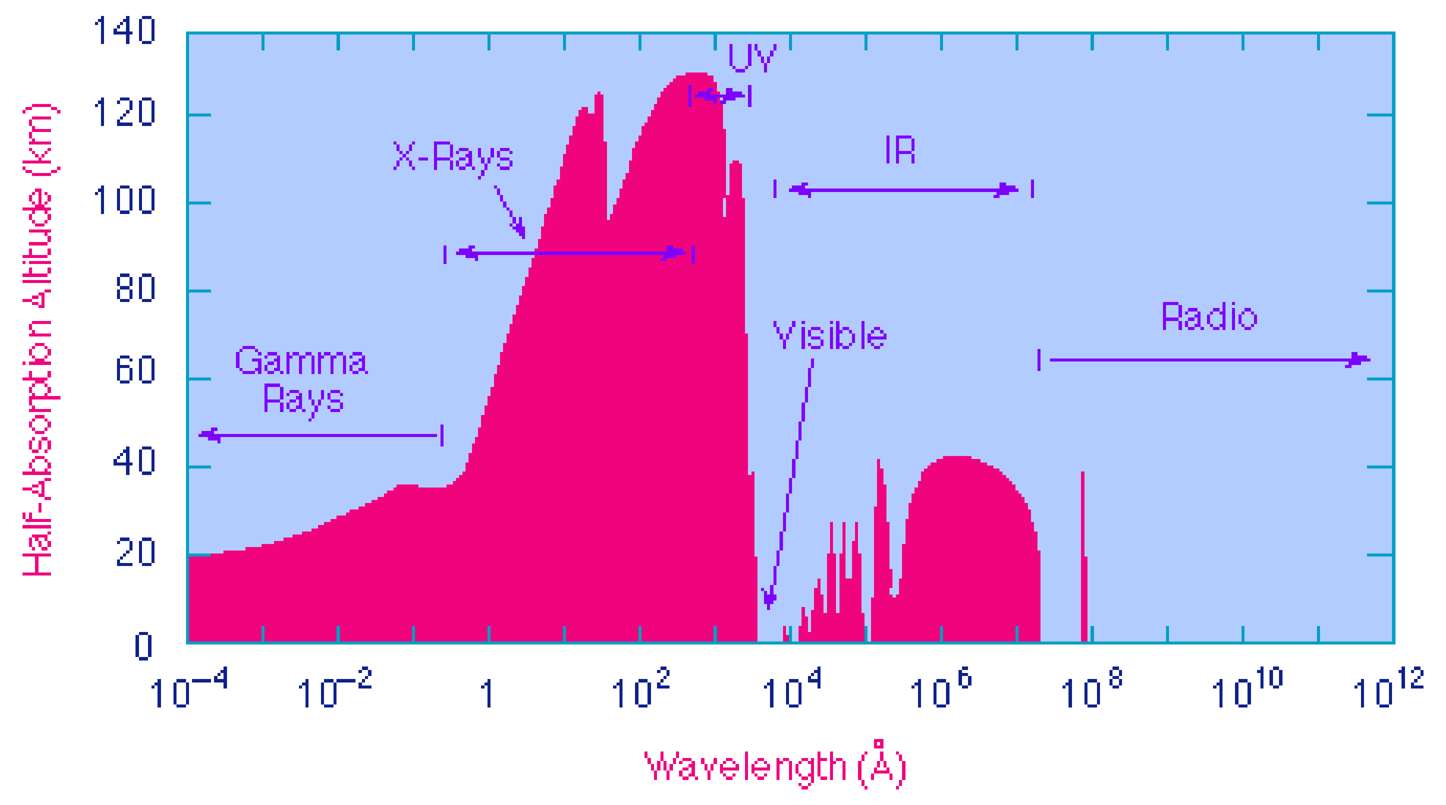
i.e., ~ 1/20th arc second.

Webcam Imaging



Radio Telescopes

A radio window too.
> 10mm wavelength (at ground)



1957 – The 76m MK 1

What resolution?

- At 21cm wavelength – the Hydrogen Line
- Theoretical resolution = 1.22 λ /D

= 1.22 x 0.21 m / 76 m rad

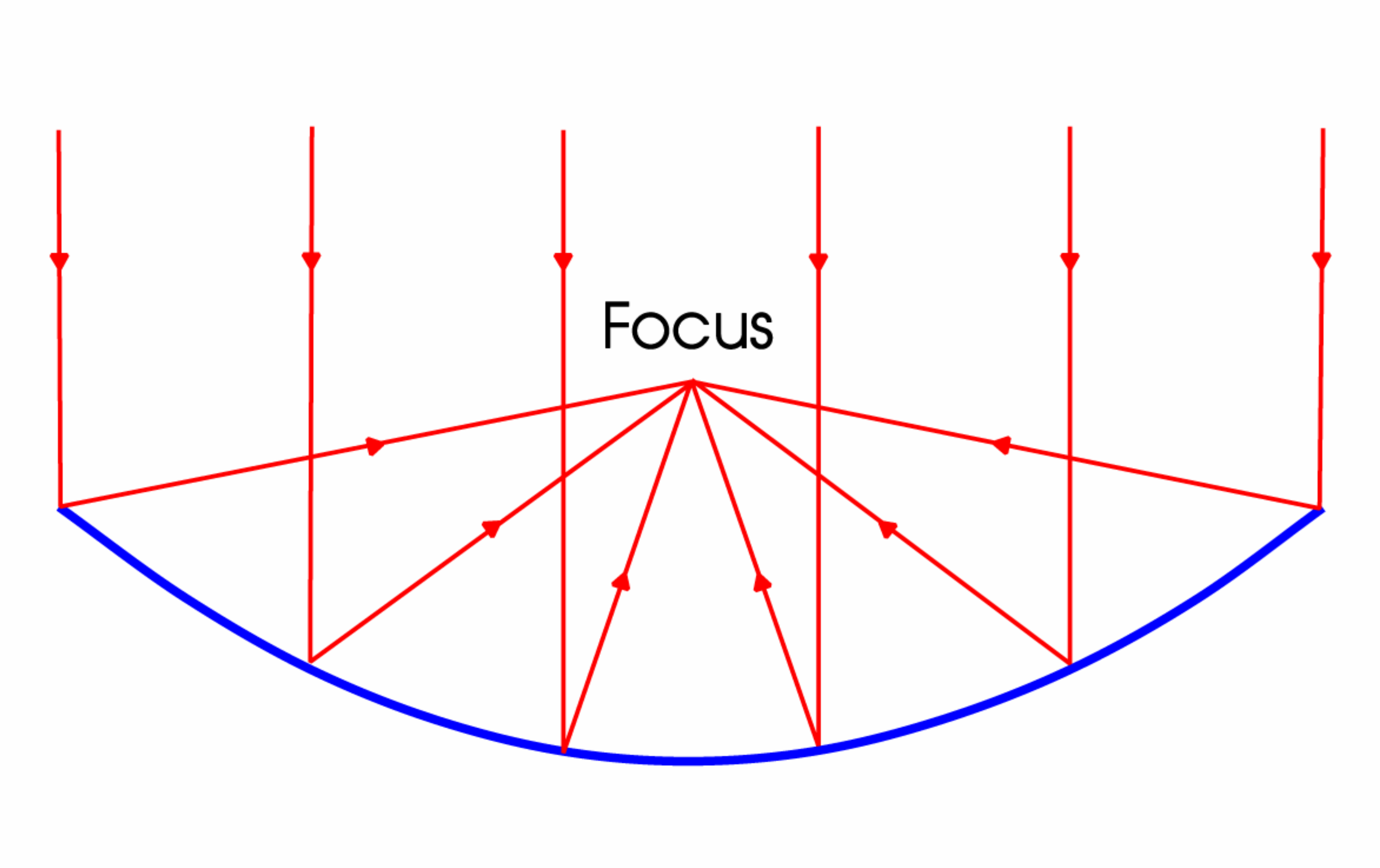
= 3.4E-3 radians

= 20 arc minutes

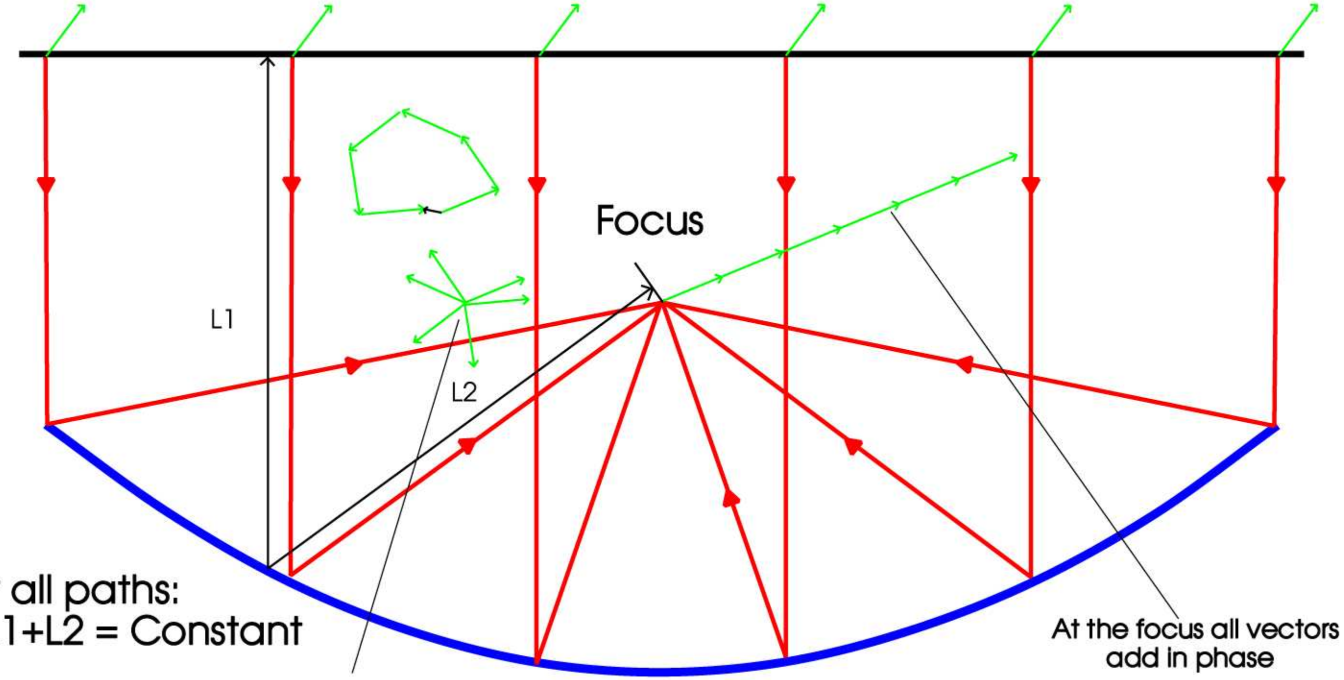
So the “beam” of even a large radio-telescope is very large in comparison to the size of the airy disc of an optical telescope.

1972 – The MK IA

A Parabolic Surface



Plane Wavefront

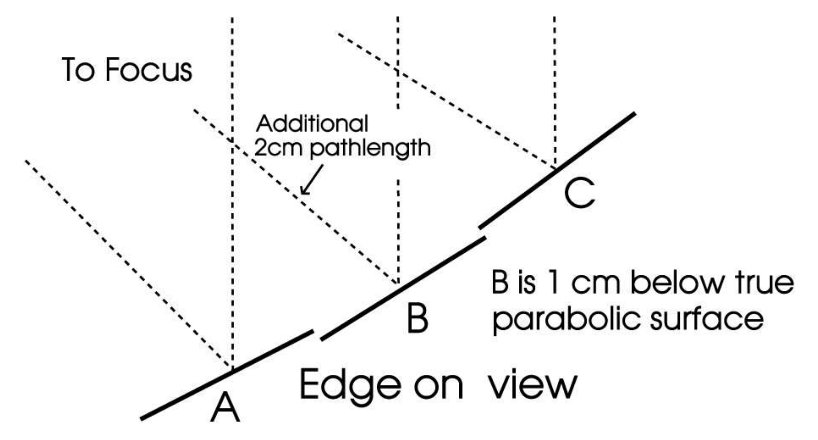


For all paths:
L1+L2 = Constant

At points away from the focus
vectors add with varying phase
so vector sum (shown above)
tends to zero.

At the focus all vectors
add in phase

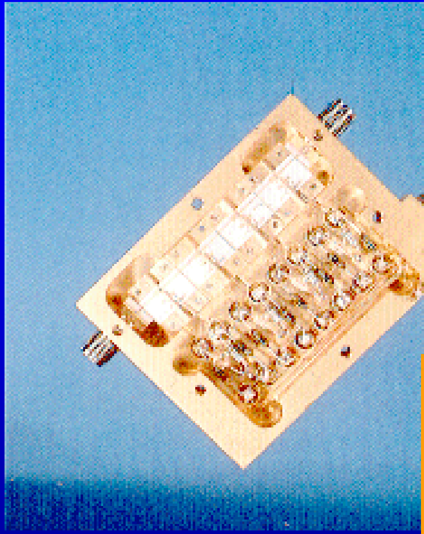
How accurate need the surface be?

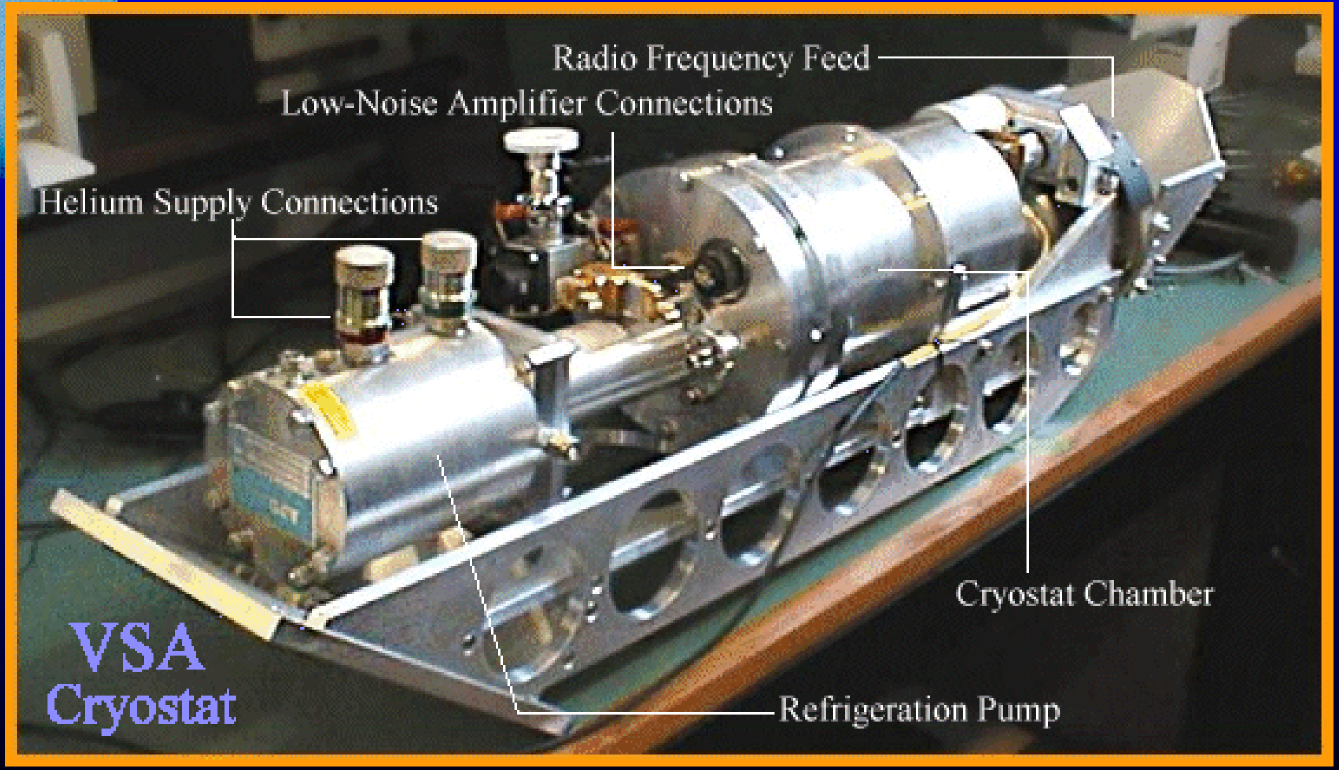


- Suppose one part of the mirror is 1 cm below the nominal parabolic curve. The wave will travel an additional ~ 2 cm through space.
- If its wavelength was 21cm, this would amount to a phase shift of 2/21 * 360 degrees = 34 degrees (~ 1/10th of a wavelength). When the energy in this part of the wave is vectorially added at the focus, it will not be fully coherent and the power received by the telescope reduced.
- For a mirror to perform well, the peak errors should not exceed 1/20th of a wavelength giving path length errors of 1/20th of a wavelength.
- Images produced by optical mirrors or lenses are acceptable if the errors do not exceed ¼ λ.

The Focus Box

The Receivers





Amplifier Noise

- Much of the noise produced at the output of the amplifier is thermal in origin.
- So we can reduce by making the amplifier very cold.
- Hence mounted inside a cryostat.

The cryostat

- Heat removed by a Helium Gas refrigerator
- Try to prevent heat reaching the amplifier inside
- Convection – evacuate the cryostat.
- Radiation – include a highly polished radiation shield
- Conduction – use very fine wires to carry current to the amplifiers and wrap around the refrigeration element to remove heat before it reached the amplifier.

The quest for high resolution.

The VLA

MERLIN


Jodrell Bank


Tabley


Knockin


Cambridge


Darnhall

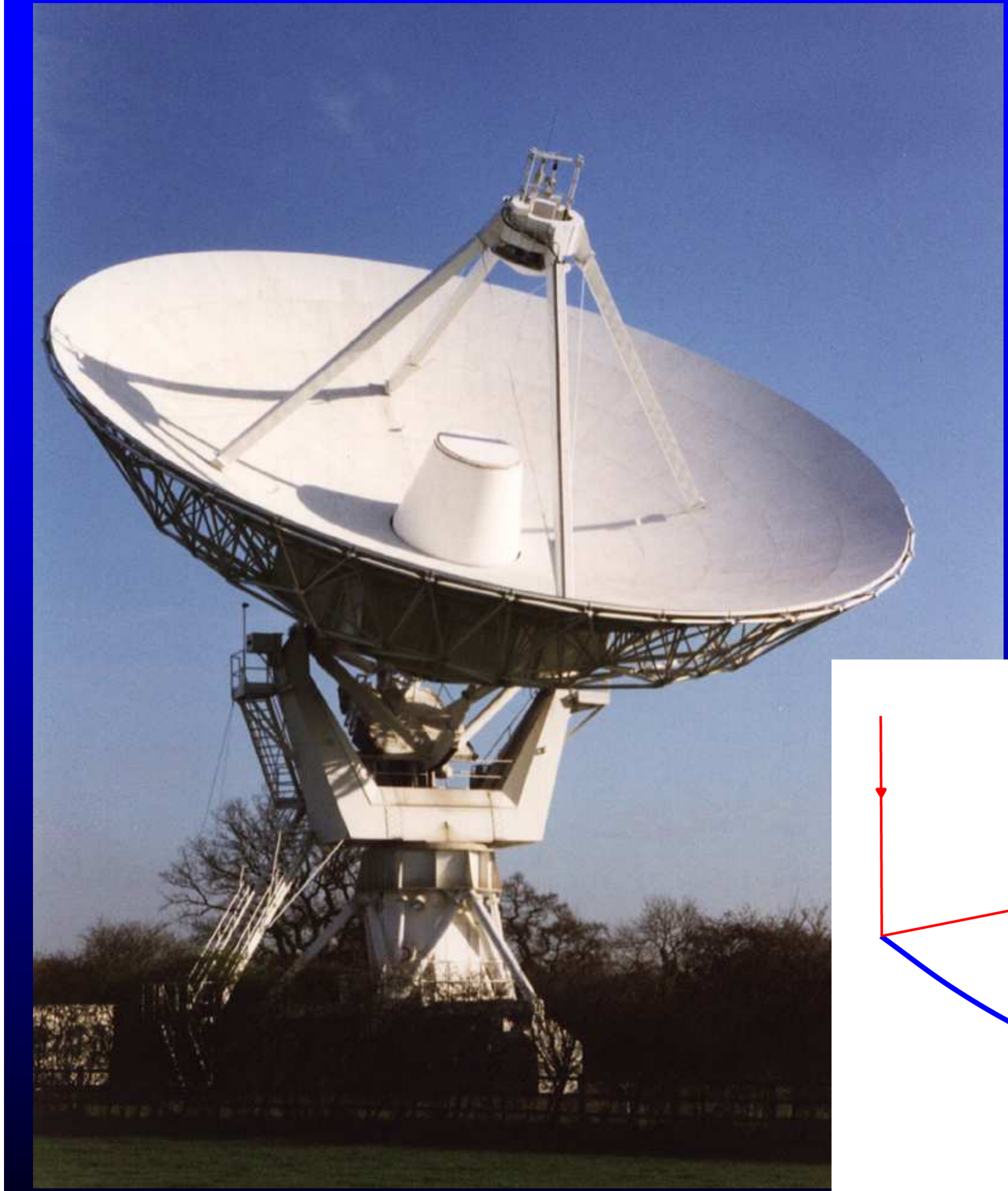

Defford

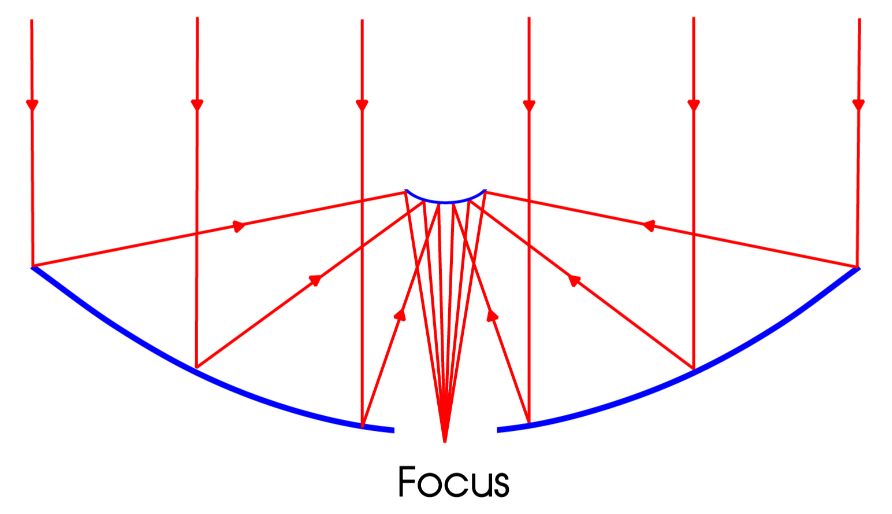
- MK II

- Knockin
- 25-m diameter
- One of three similar telescopes

Darnhall

Cassegrain Design





Pickmere

- MERLIN Telescope at Cambridge
- 32-m diameter

Resolution of MERLIN at 5cm λ

- λ is 0.05 m
- D is 217 km = 2.17 E5 m
- Q = 1.22 λ / D

= 1.22 x 0.05 / 2.17E5 radians

= 2.8E-7 radians

= 2.8E-7 x 57.3 x 3600 arc seconds

= 0.057 arc seconds

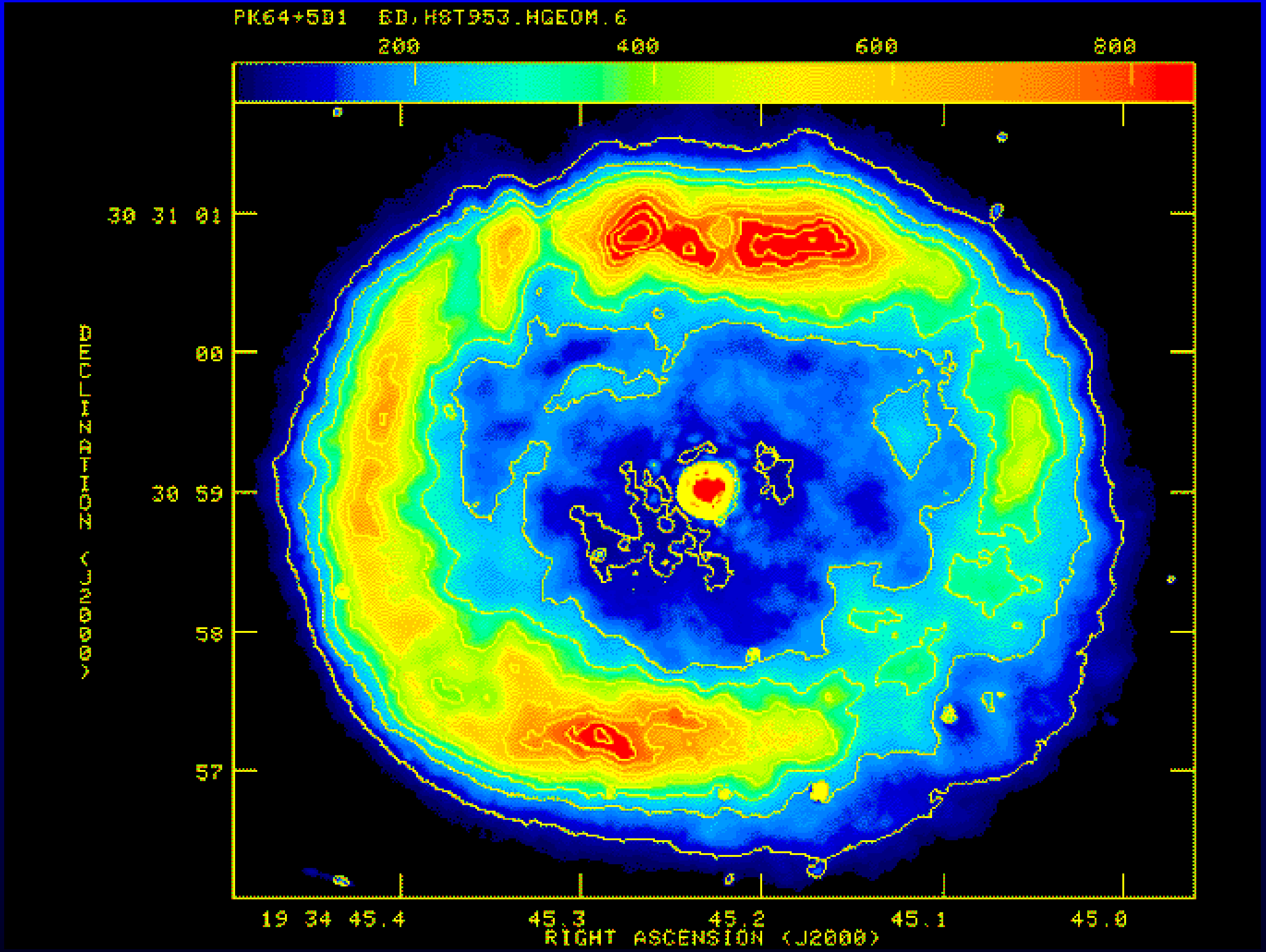
= 1/17th arc second

Thus comparable to Hubble Space Telescope.

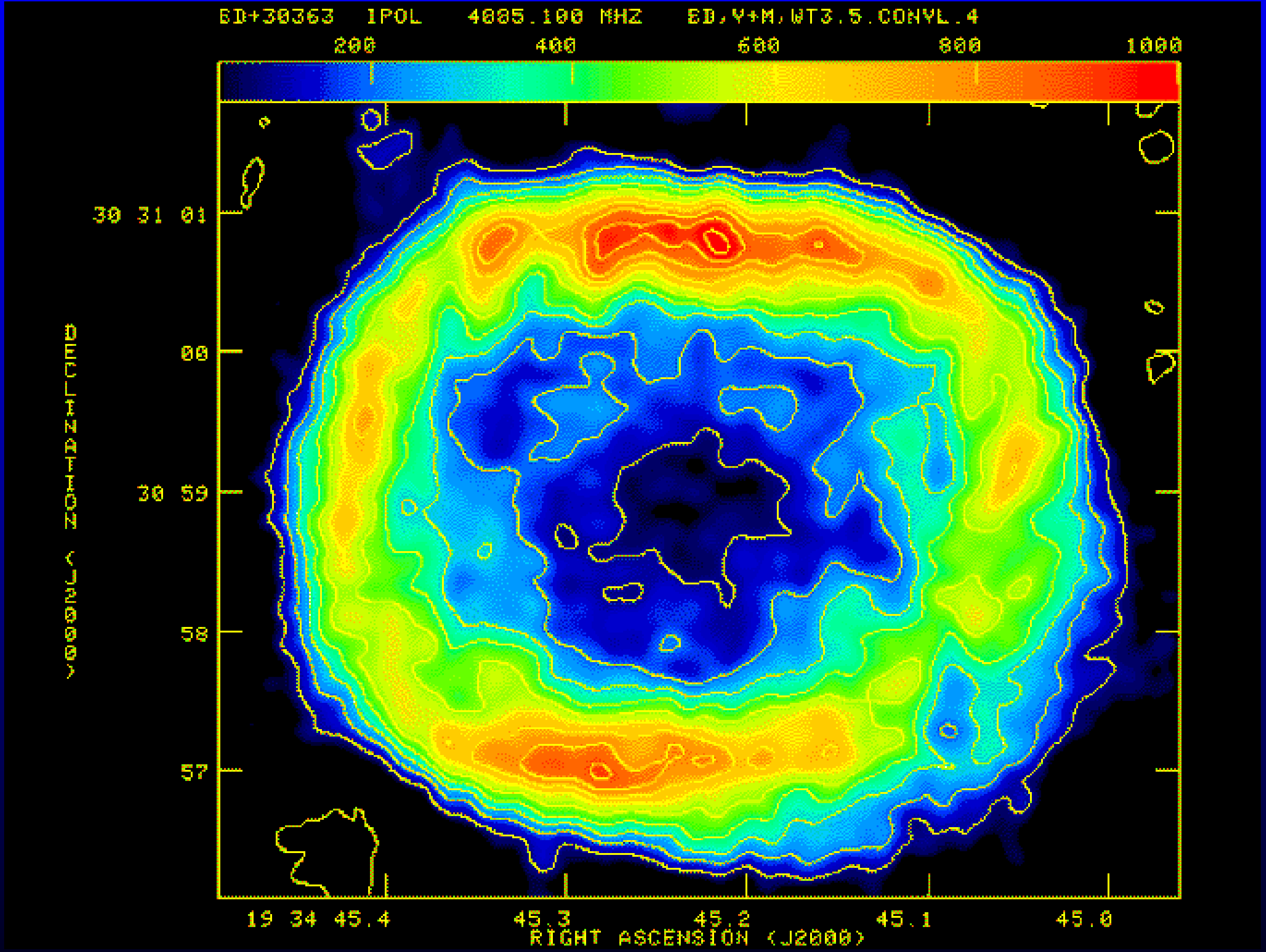
A Planetary Nebula

M57 in Lyra

BD+303639: HST image of planetary nebula



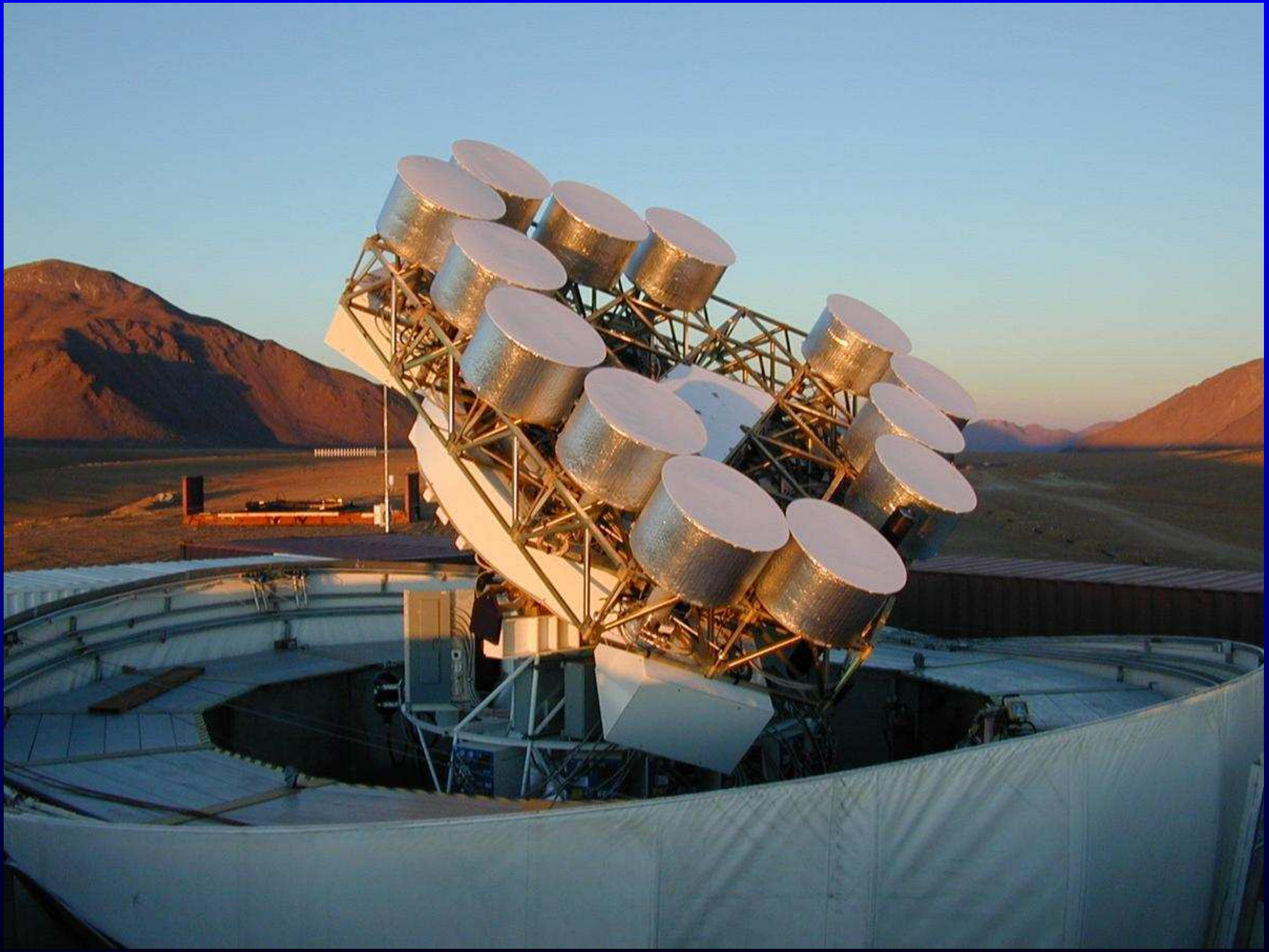
BD+303639: MERLIN & VLA 6cm image



Wavelengths < 10mm

- Is is possible to observe down to ~ 5mm from the ground BUT only where there is very little water vapour in the atmosphere as this both emits radiation at these wavelengths and absorbs signals passing through it.
- Need a high and dry site.
- Or a very cold one.

CBI on the Atacama Desert, Chile
5080m (16,700 ft)



VSA on Mount Teide
2400m

DASI at the South Pole

- It is so cold that the water-vapour is frozen out of the atmosphere

At shorter wavelength radio and infra-red

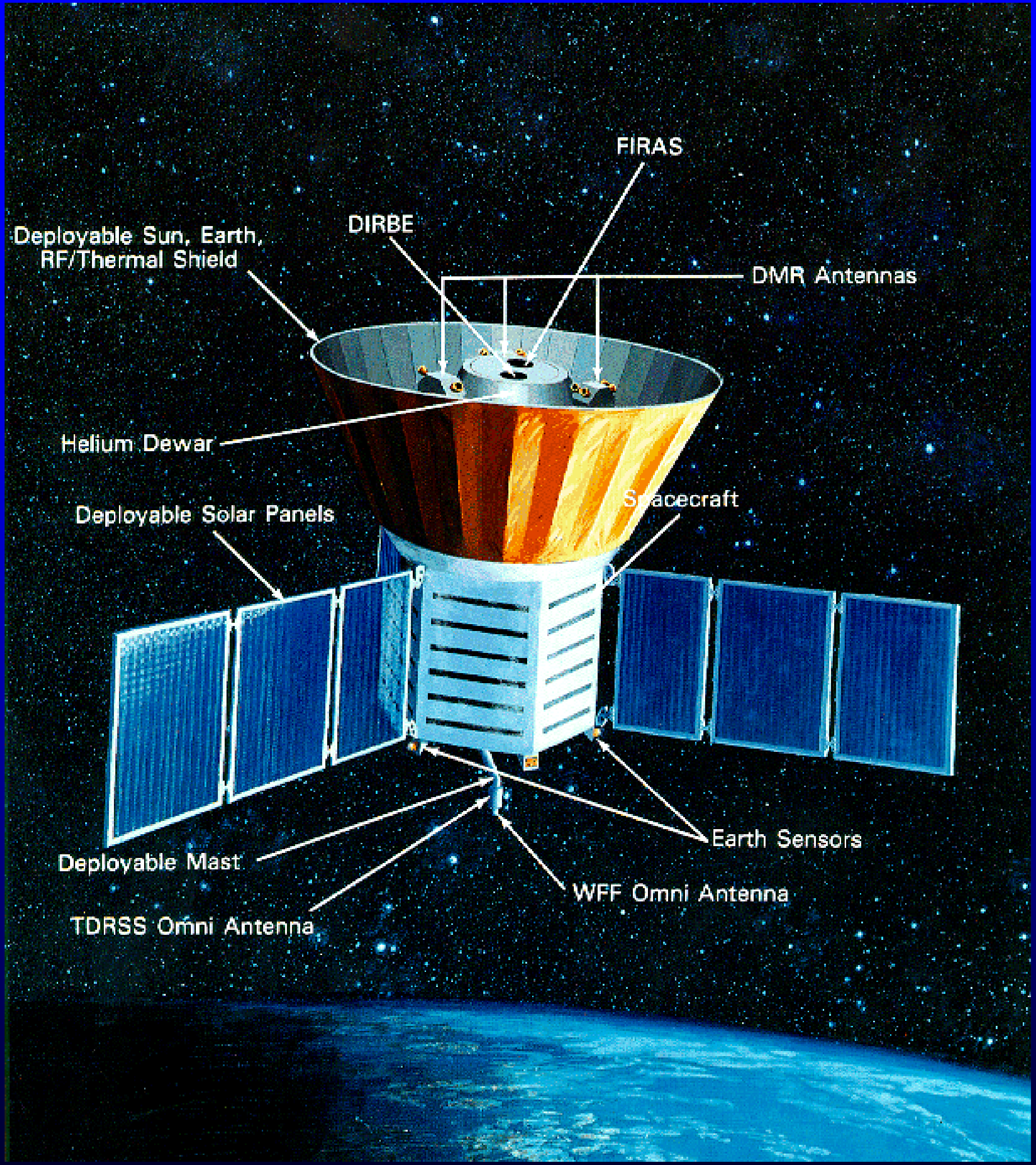
- One must either fly in very high balloons or use spacecraft

Boomerang

Maxima

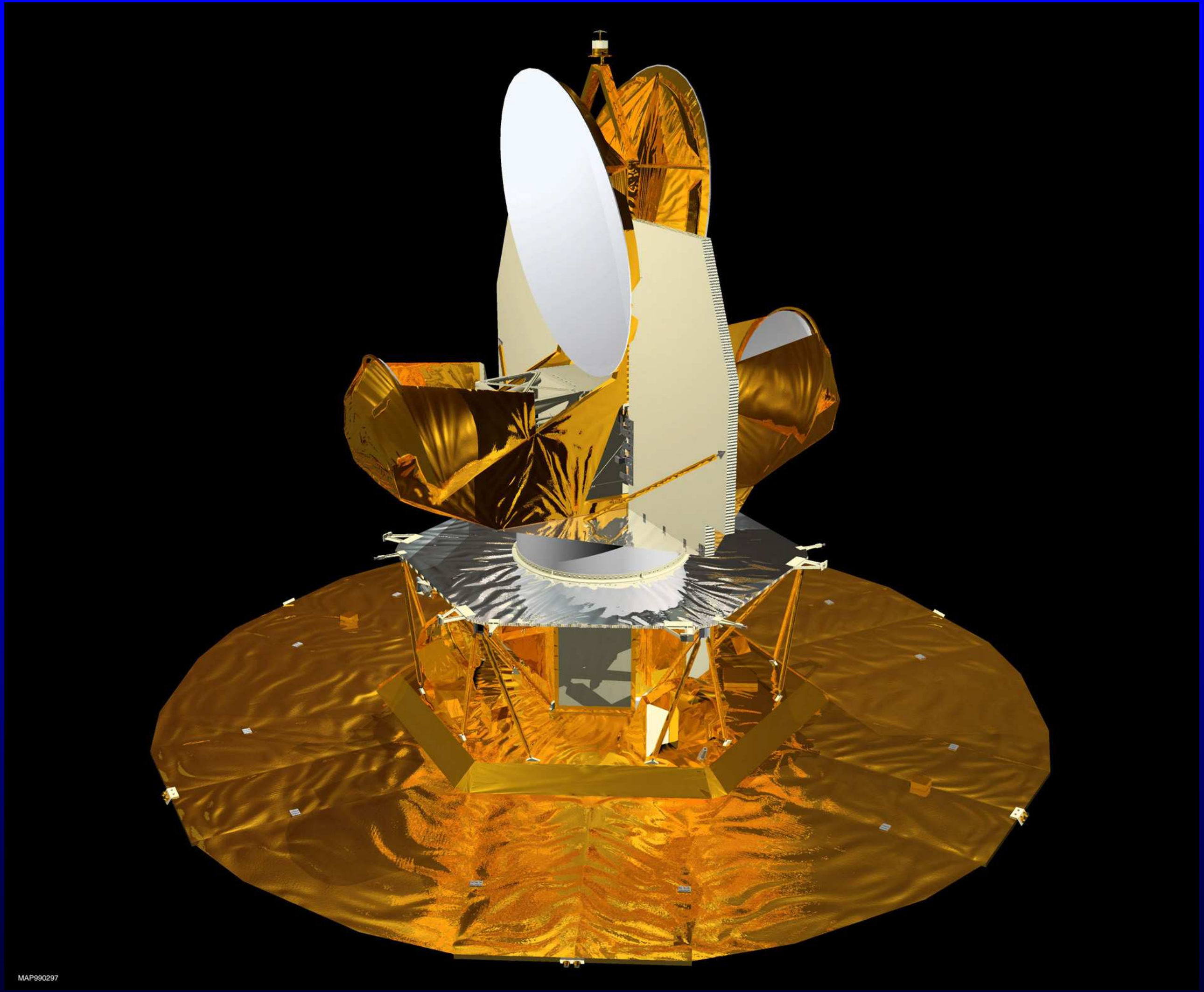
COBE

Radio and
Far Infared



WMAP

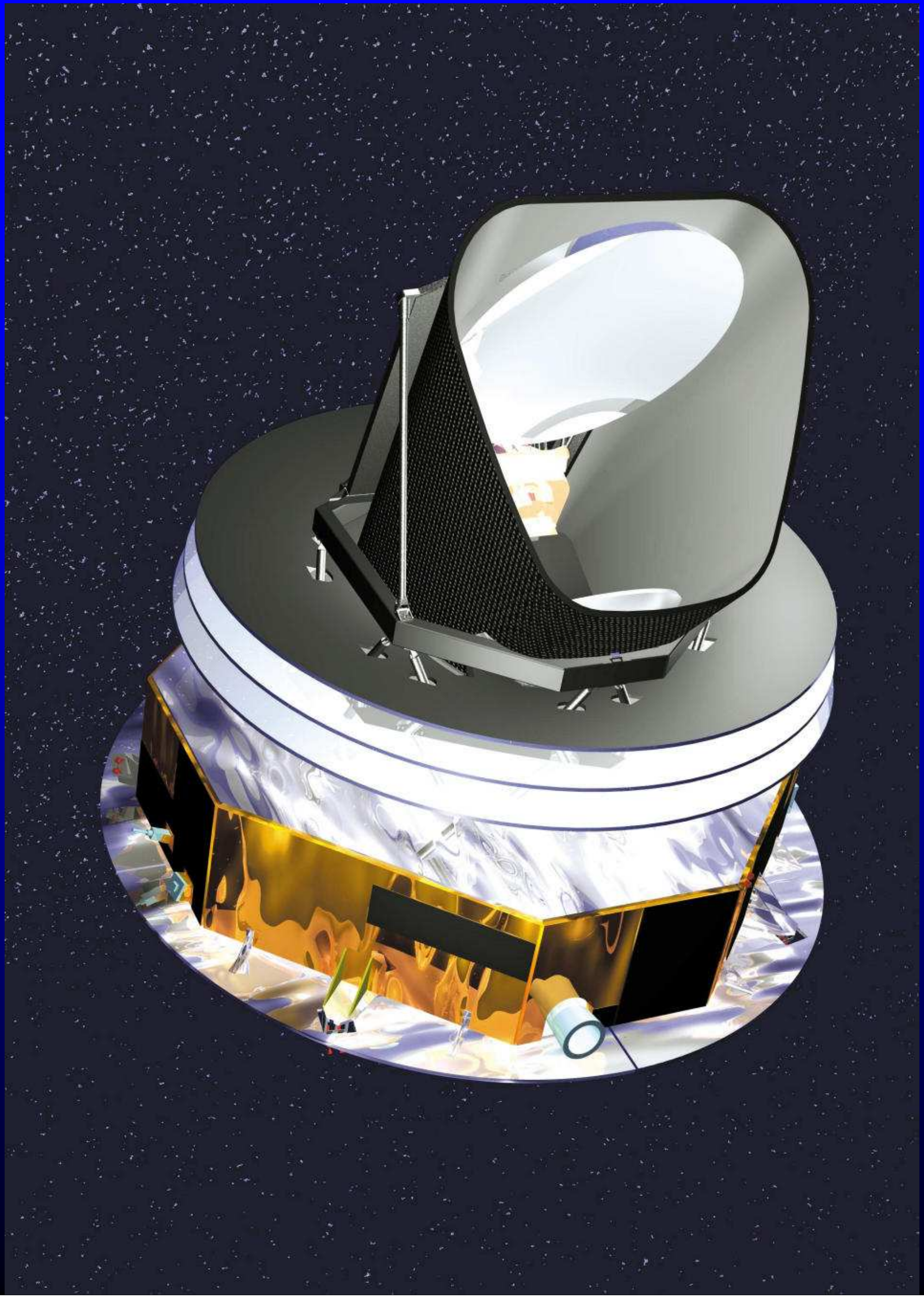
- mm wavelength radio observations



The Planck Spacecraft

mm wavelength

observations



The James Webb Space Telescope

- Infra-red observatory
- 6.5m, 18 segment mirror
- Launch 2011