Vorticity and surface Alfvén waves in the solar corona

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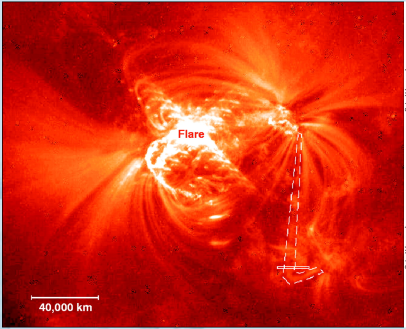
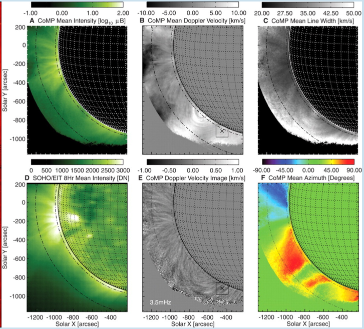
Transverse coronal waves

Transverse waves in the corona: standing (Nakariakov et al. 1999), running (Tomczyk et al. 2007)





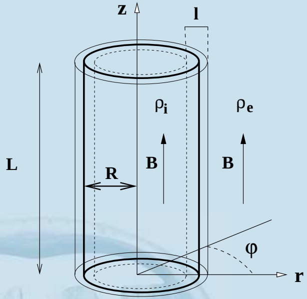
Interpretation

Running transverse waves:

- Were first interpreted as Alfvén waves by Tomczyk et al. (2007)
- Later challenged by Van Doorsselaere et al. (2008), Erdélyi & Fedun (2007): fast kink waves
- Argument by Goossens et al. (2009): ratio of magnetic pressure and magnetic tension: $\Lambda(\omega^2) = \frac{\omega^2 - \omega_\mathrm{A}^2}{\omega_\mathrm{A}^2} = \frac{\omega^2}{\omega_\mathrm{A}^2} - 1$
- $\Lambda_i(\omega_\mathrm{k}^2) = -\Lambda_\mathrm{e}(\omega_\mathrm{k}^2) = \frac{\rho_\mathrm{i} - \rho_\mathrm{e}}{\rho_\mathrm{i} + \rho_\mathrm{e}}$: tension force is more important for the kink mode than the magnetic pressure force $\rightarrow$ Alfvénic:

This work continues on this discussion! (Goossens et al. 2012, submitted to ApJ)

Classical model



(Arregui et al. 2007)

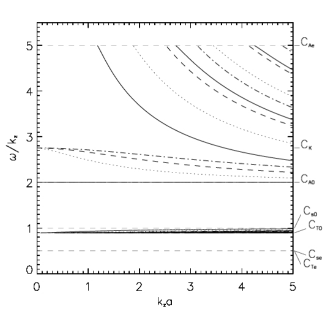
Following Edwin & Roberts (1983), description in terms of compressibility/plasma pressure:

$$\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}\left(r\frac{\mathrm{d}P'}{\mathrm{d}r}\right) = \left[\frac{m^2}{r^2} - \Gamma(\omega^2)\right] P',$$

where Γ is the "radial wave number" (m_0^2 in ER83's notation).

Kink mode found in terms of Bessel function solutions for the plasma pressure perturbations (fast/slow).

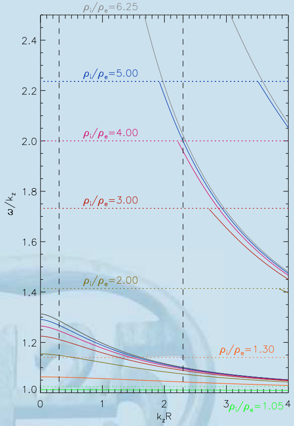
Cold plasma limit



Nakariakov & Verwichte (2005)

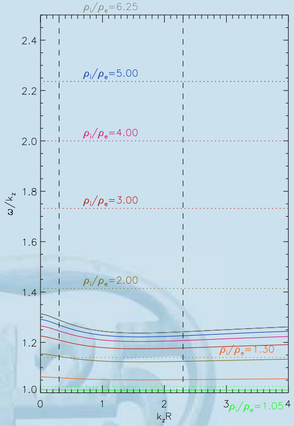
- Kink mode frequency is not in slow mode spectrum (“continuum”)
- Consider the cold plasma limit ($\beta \to 0$, $C_s \to 0$)
- Slow modes disappear, fast and Alfvén remain
- Kink mode is definitely not slow

Density contrast $\to 0$



- Consider the limit of decreasing density contrast ($\rho_\mathrm{i} - \rho_\mathrm{e} \to 0$): "no tube"
- Radial overtones disappear
- Fundamental radial overtone collapse to the Alfvén frequency
- Difference between overtones and fundamental mode!

Incompressible limit



- Consider the incompressible limit ($C_s \rightarrow \infty$)
- Fast waves have $\omega^2 = \infty$
- For any density contrast, no radial overtones are present
- Radial overtones are fast waves
- Fundamental kink mode is not fast

Say WHAAAT?

(radially) Fundamental kink mode is

- not slow (from cold plasma limit)
- not fast (from incompressible limit)
- Frequency collapses to Alfvén frequency ("no tube" limit)
- Alfvén mode
- But what about the pressure perturbations?
- And what about the vorticity?

Vorticity

ER83 again, also has an equation for vorticity:

$$\frac{\rho(\omega^2-\omega_{\mathrm{A}}^2)}{r}\frac{\mathrm{d}}{\mathrm{d}r}\left[\frac{r}{\rho(\omega^2-\omega_{\mathrm{A}}^2)}\frac{\mathrm{d}P'}{\mathrm{d}r}\right] =$$

$$\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}\left(r\frac{\mathrm{d}P'}{\mathrm{d}r}\right) = \left[\frac{m^2}{r^2}-\Gamma(\omega^2)\right]P', \quad (1)$$

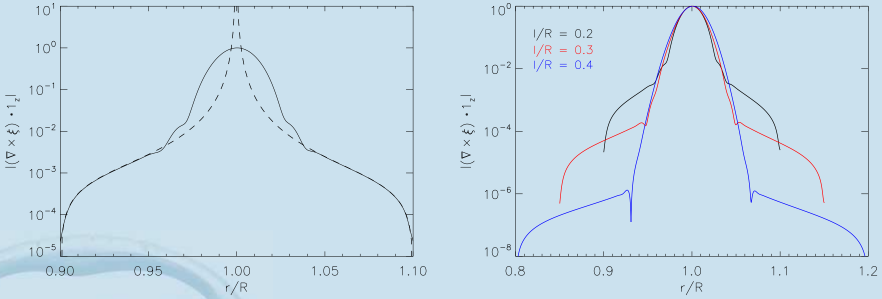
$$\rho(\omega^2-\omega_{\mathrm{A}}^2)(\nabla\times\vec{\xi})\cdot\vec{e}_z = 0 \quad (2)$$

- In either "half planes" ($r<a$ and $r>a$), $\omega\neq\omega_{\mathrm{A}}$ for the fundamental kink mode.
- Eq. 2 can only be satisfied by taking $(\nabla\times\vec{\xi})\cdot\vec{e}_z\equiv 0$ for $r<a$ and $r>a$.
- $r=a$ is special, because of density jump.

Vorticity ct'd

- In either "half planes" ($r < a$ and $r > a$), $\omega \neq \omega_\mathrm{A}$ for the fundamental kink mode.
- Eq. 2 can only be satisfied by taking $(\nabla \times \vec{\xi}) \cdot \vec{e}_z \equiv 0$ for $r < a$ and $r > a$.
- $r = a$ is special: $\frac{1}{\rho(\omega^2 - \omega_\mathrm{A}^2)} = \frac{\rho_\mathrm{i} + \rho_\mathrm{e}}{\rho_\mathrm{i}\rho_\mathrm{e}} \frac{1}{\omega_\mathrm{Ae}^2 - \omega_\mathrm{Ai}^2} [1 - 2H(r-a)]$, with a Heaviside function!
- It can be derived that the vorticity has a δ-function behaviour: $(\nabla \times \vec{\xi}) \cdot \vec{e}_z = -2i \frac{m}{R} P' \frac{\rho_\mathrm{i} + \rho_\mathrm{e}}{\rho_\mathrm{i}\rho_\mathrm{e}} \frac{1}{\omega_\mathrm{Ae}^2 - \omega_\mathrm{Ai}^2} \delta(r-a)$
- Contrary to classic Alfvén waves, the vorticity is localised in a vortex sheet on the density discontinuity.
- Surface Alfvén waves.

Smooth layers



Vorticity spread out over layer, when inhomogeneity is included!

Conclusions

- Interpretation of transverse waves in the corona
- Consider cold plasma limit and incompressible limit
- Vorticity sheet at loop boundary
- In other fields: Surface Alfvén wave
- Definitely not fast
- Use suitable model for energy considerations and seismology!