Build-up of Coronal Magnetic Gradients from Observed Photospheric Flows

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Overview

The build-up of magnetic gradients in the Sun's atmosphere may be inferred directly from photospheric velocity data.

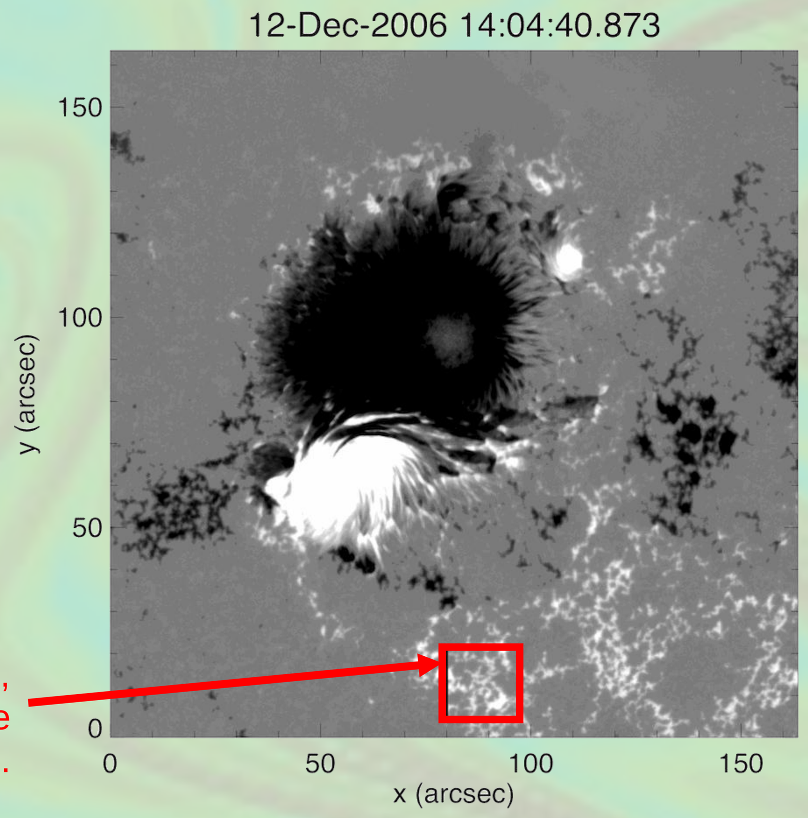
- Magnetic field connectivity measures such as the "squashing factor" [2] can be computed directly from a sequence of footpoint motions.
- Avoids the need to extrapolate a 3D magnetic field.
- Consistent with perfectly ideal evolution.
- Limitation: cannot determine initial mapping, only that resulting from subsequent footpoint motions.

**** For more details see our paper Ref [1] **.**

Observations

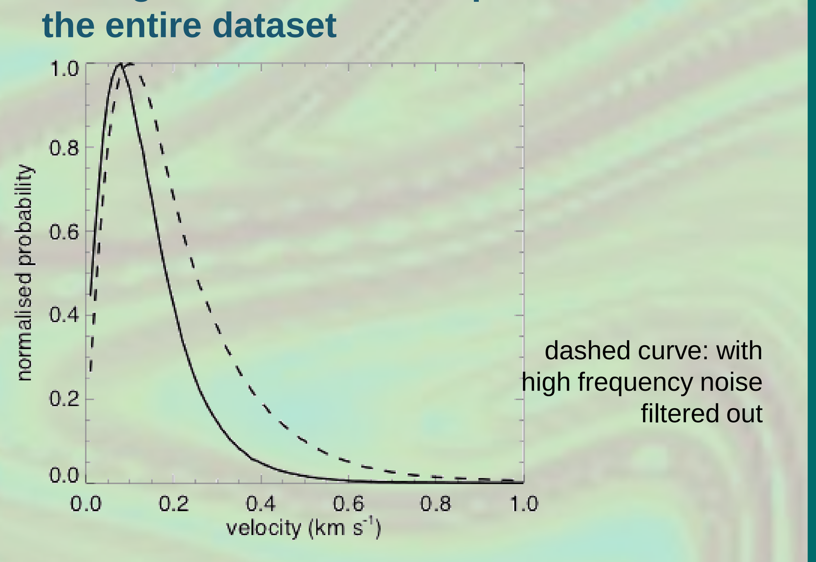
We demonstrate the method on a 12 hour sequence of (horizontal) photospheric velocities derived from *Hinode*/SOT magnetograms.

- Taken 12th/13th December 2006 at 2-minute cadence.
- Photospheric velocities were derived with the Fourier Local Correlation Tracking method (FLCT, [3]).
- Optimum parameters were determined by an autocorrelation analysis (for details see [4]).



We selected a unipolar plage region 12.4Mm x 12.4Mm, away from the main sunspots (to avoid large-scale rotational flow).

Histogram of horizontal speeds over the entire dataset



- Mean flow speed: ca. 0.1 km s^{-1}
- Lower than granular flows: maybe due to spatial resolution or tendency of FLCT to underestimate speeds.

Resulting mapping

The field line mapping is found by integrating trajectories/particle paths.

Observed velocity field is interpolated with a local tricubic method [5] (linear interpolation does not give smooth enough trajectories).

Two measures of the mapping gradient are the squashing factor and FTLE.

Both quantify local "stretching" through the Cauchy-Green deformation tensor J^TJ of the field line mapping, where J is the Jacobian matrix:

$$J(x,y,t) \equiv \begin{pmatrix} \frac{\partial f_x}{\partial x} & \frac{\partial f_x}{\partial y} \\ \frac{\partial f_y}{\partial x} & \frac{\partial f_y}{\partial y} \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

The **squashing factor** uses the Frobenius norm, and is used in solar physics (ridges are known as Quasi-Separatrix Layers):

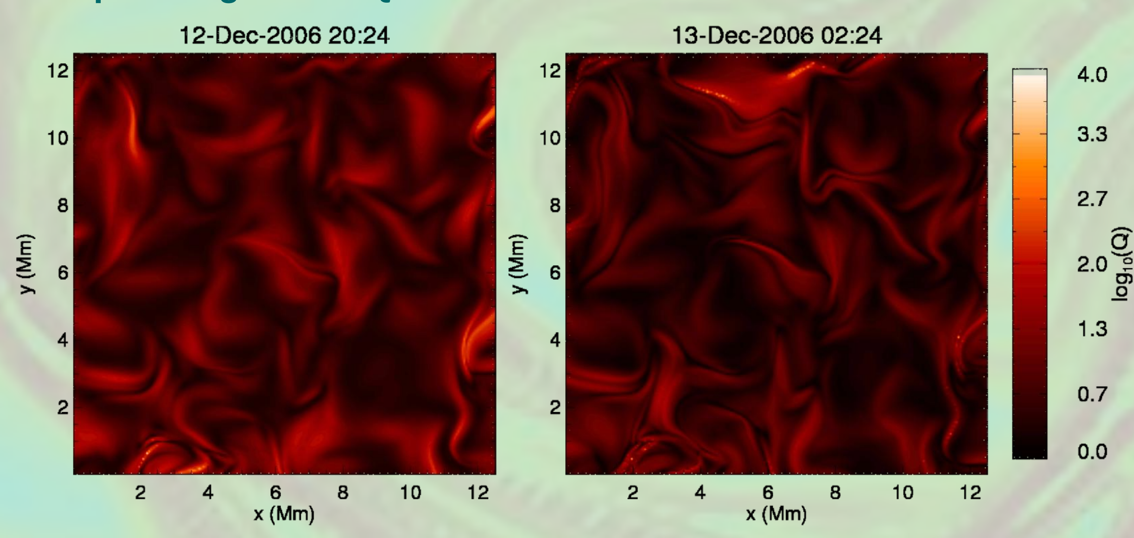
$$Q(x,y,t) = \frac{a^2+b^2+c^2+d^2}{|ad-bc|}$$

The **FTLE** [6] uses the spectral norm, has dimensions of inverse-time, and is used in fluid dynamics (ridges are known as Lagrangian Coherent Structures):

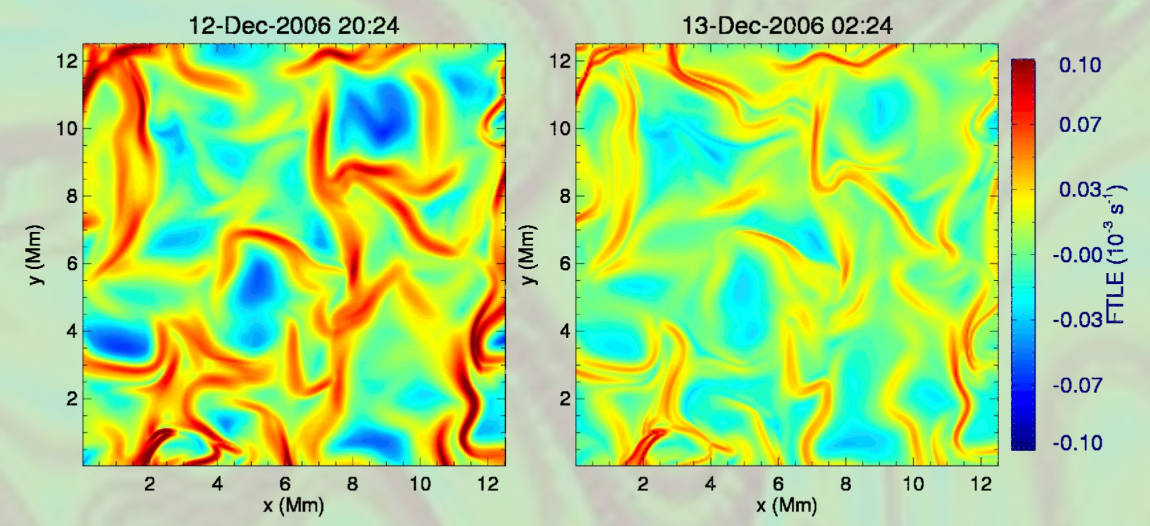
$$\sigma(x,y,t) = \frac{\ln\sqrt{\lambda_+}}{|t-t_0|}$$

(λ_+ is the largest eigenvalue of J^TJ)

Squashing Factor *Q*



Finite-Time Lyapunov Exponent σ

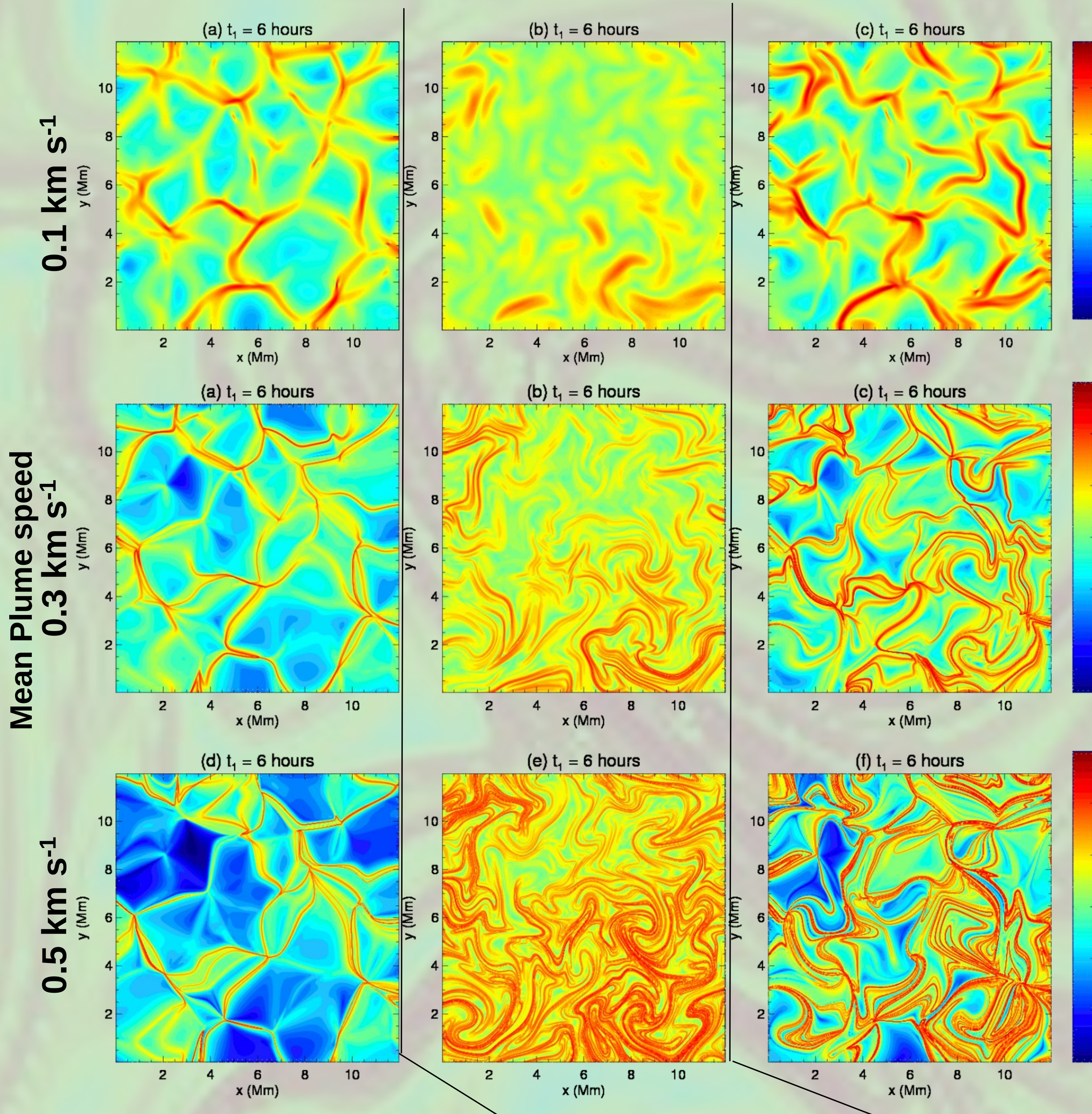


Analytical model

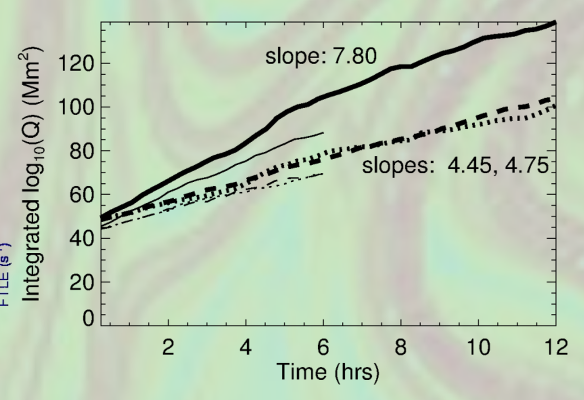
A simple analytical model of 2D convection demonstrates the origin of the observed pattern, and predicts how it would change with higher-resolution observations of faster flows.

- We model photospheric convection with a superposition of random convective "plumes" and random "vortices" (similar to [8]).
- New pattern chosen after "coherence time" of 15 mins.

FTLE field after 6 hours:



- Although visual pattern is dominated by diverging flow component, rotational component doubles rate of increase of integrated log(Q):



thick lines: observations
thin lines: analytical models

Diverging flow: network of thin LCS ridges (the usual granular network corresponds to a plot of σ in *final* frame, not initial frame as here).

Rotational flow: LCS ridges more diffuse and space-filling (efficient mixing and "infilling" of LCS)

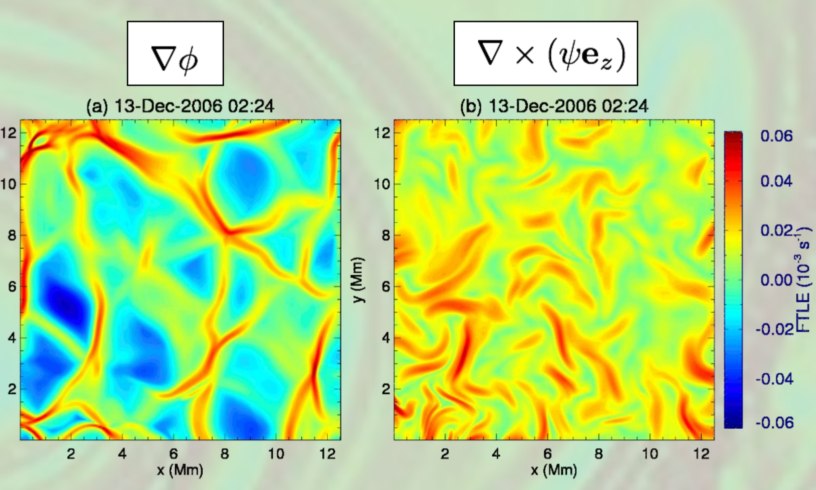
Combined flow: matches observations

Prediction:
higher resolution observed flows will produce more space-filling QSLs with steeper gradients.

Helmholtz decomposition

To understand the origin of the FTLE/*Q* pattern we decompose *v* into irrotational and solenoidal components.

$$\mathbf{v} = \nabla\phi + \nabla\times(\psi\mathbf{e}_z)$$

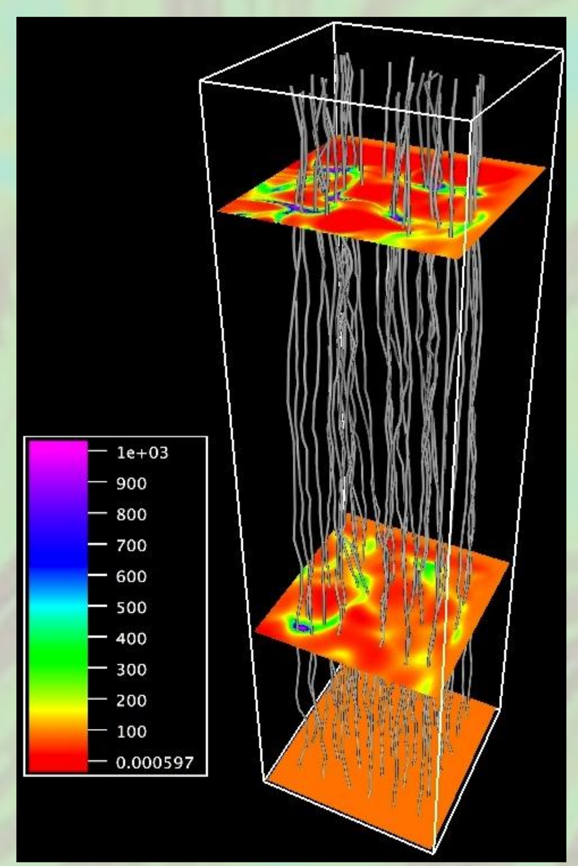


(fix unique solution by setting $\mathbf{n}\cdot\mathbf{v} = \frac{\partial\phi}{\partial n}$ on the boundary)

- Components are determined by solving the Poisson equation $\nabla^2\phi = \nabla\cdot\mathbf{v}$ with a fast-Poisson solver [7].

Reconstructed magnetic field

For future investigation, we construct a 3D magnetic field with the observed mapping (only defined up to an ideal deformation and an arbitrary initial B_z distribution).



(1) Set field lines of **B** to the trajectories, with z corresponding to time.

$$\mathbf{B}(x,y,z) = B(x,y,z)\left(v_x\mathbf{e}_x + v_y\mathbf{e}_y + \mathbf{e}_z\right)$$

(2) Adjust amplitude $B(x, y, z)$ to
(a) make **B** divergence-free
(b) match B_z on lower boundary

This determines **B** uniquely.

Contour slices show resulting B_z at different z (initial distribution is a uniform field of strength 88G).

References

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Acknowledgements

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