A Self-Similar Expansion Model for use in Solar Wind Transient Propagation Studies

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ABSTRACT: Since the advent of wide-angle heliospheric imaging, a number of techniques have been developed to investigate the 3D structure and kinematics of solar transients from their signatures in single- and multi-spacecraft imaging observations. In the analysis of single-spacecraft imaging observations, much use has been made of the Fixed ϕ Fitting (FPF) and Harmonic Mean Fitting (HMF) techniques, in which the transient is considered a radially-propagating point source (Fixed ϕ, FP, model) and a radially-expanding circle anchored at Sun-centre (Harmonic Mean, HM, model), respectively. Initially, we compare radial speeds and propagation directions derived from applying these techniques to a large set of STEREO/Heliospheric Imager (HI) observations. As the geometries on which these two techniques are founded constitute extreme descriptions of solar transients in terms of their cross sectional extent, we describe a single-spacecraft fitting technique based on a more generalised model for which the FP and HM geometries form the limiting cases. In addition to providing transient speed and propagation direction, Self-Similar Expansion Fitting (SSEF) provides, in theory, the capability to estimate a transient's angular extent in the plane orthogonal to the field of view. Using HI observations, and a Monte-Carlo simulation, we assess the potential of the SSEF technique.

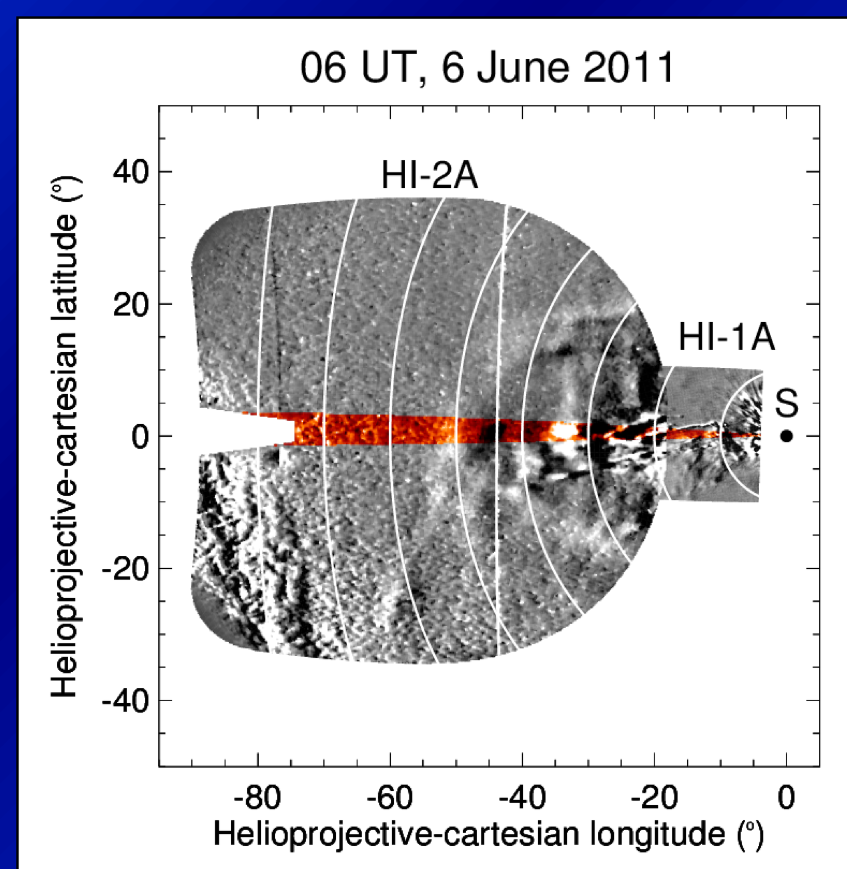


FIGURE 1

1. The Heliospheric Imager (HI) on each of the pair of NASA STEREO spacecraft comprises two wide-field, visible-light cameras, HI-1 and HI-2. FIGURE 1 presents a composite HI-1/HI-2 difference image from the STEREO-A spacecraft, showing the coronal mass ejection (CME) of 6 June 2011. In the figure, 'S' marks the location of the Sun, white curves show contours of constant elongation (the angle from Sun-centre) from 10 to 80° in steps of 10°, and the region defined by the position angle (PA) of the Earth (i.e. the ecliptic) +/- 2.5° is coloured in red.

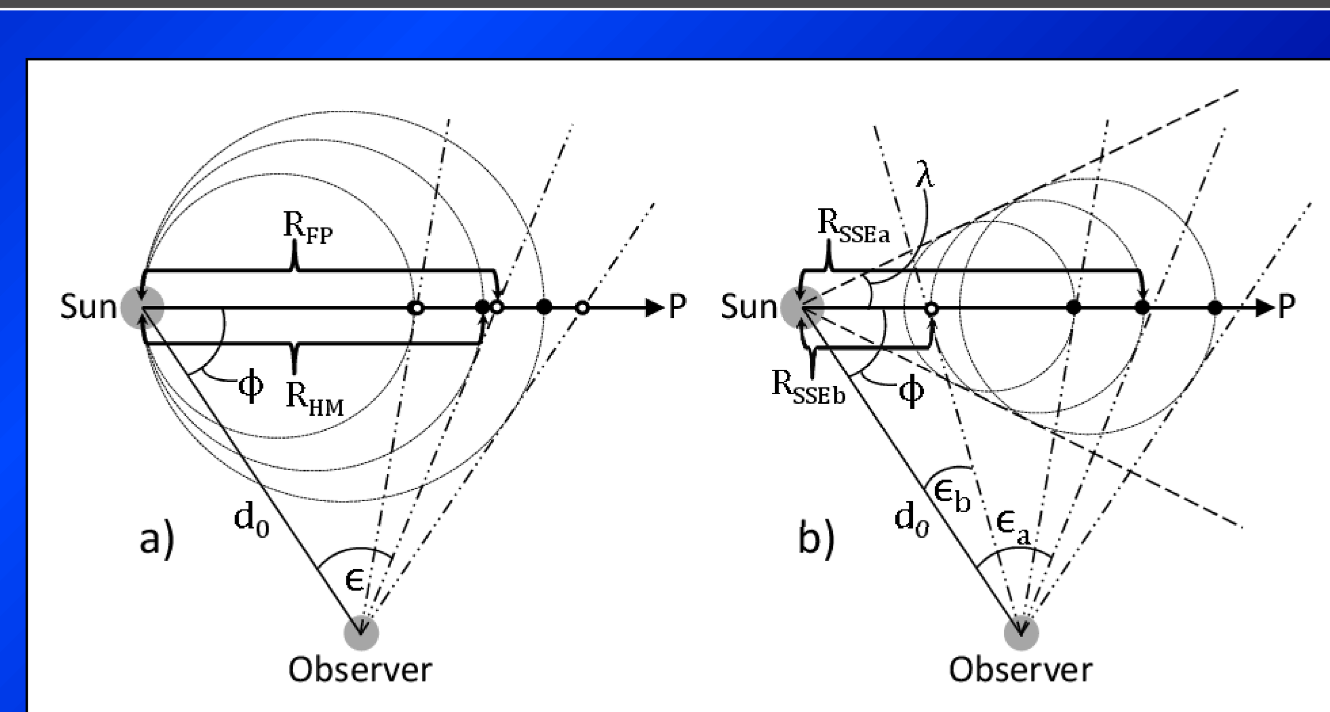


FIGURE 3

2. FIGURE 2 presents a month-long time-elongation map (J-map) constructed from STEREO-A HI-1 and HI-2 difference observations taken in the ecliptic plane (red region of FIGURE 1) during June 2011. In such a J-map, a solar transient is manifest as an inclined trace, with a positive slope, as it propagates anti-sunward through the inner heliosphere. Many such traces are evident in FIGURE 2, including the complex signature of the CME shown in FIGURE 1, as well as signatures of smaller-scale solar transients such as CIR-associated blobs (CIR: co-rotating interaction region).

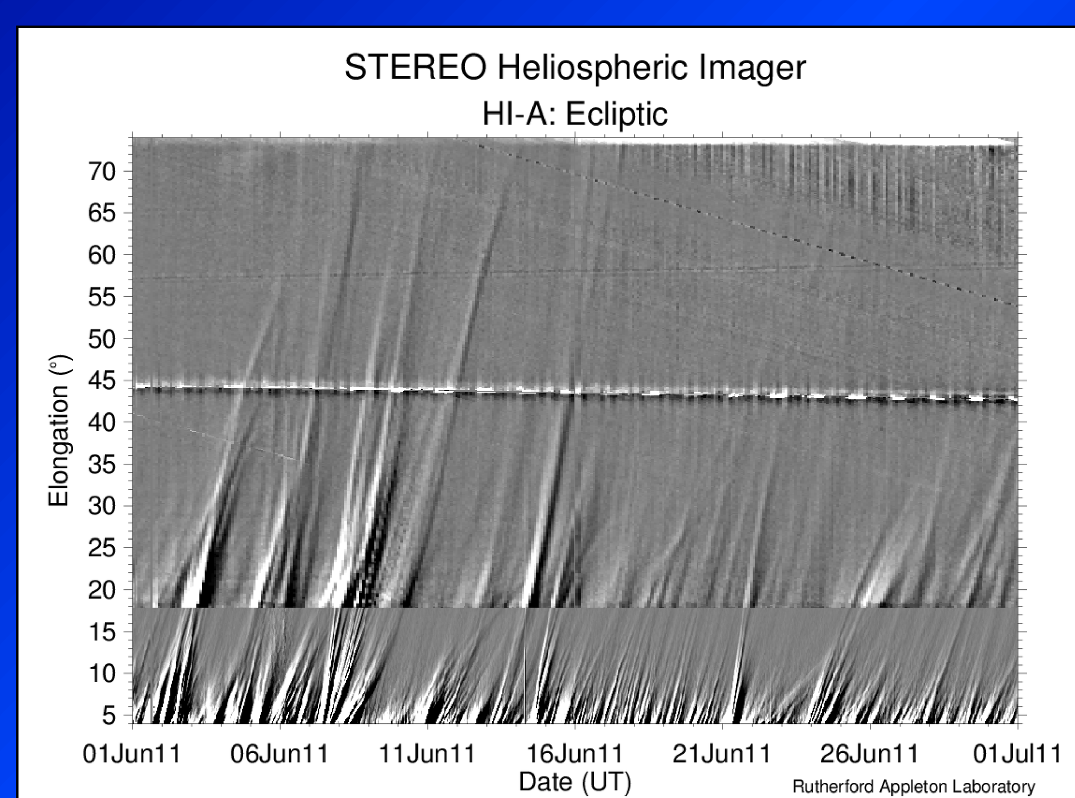


FIGURE 2

3. An observer viewing a solar transient propagating radially out to large elongations at a constant speed will perceive a non-linear time-elongation profile (see FIGURE 2), the shape depending on radial speed and propagation direction. The time-elongation profile of a solar transient viewed from a single vantage point can, thus, be analysed to retrieve best-fit estimates of these parameters. Fixed ϕ Fitting (FPF) assumes that the transient cross section in the plane described by the PA of interest (often the ecliptic) is a propagating point source (Fixed ϕ, FP, model; open dots in FIGURE 3a). Harmonic Mean Fitting (HMF), however, assumes that the transient is an expanding circle anchored to Sun-centre (Harmonic Mean, HM, model; filled dots/circles in FIGURE 3a).

4. For these two geometries, the following expressions for the elongation as a function of time, ϵ(t), can be derived:

FP:
$$\epsilon(t) = \tan^{-1}\left(\frac{V_{FP}t\sin\phi_{FP}}{d_0 - V_{FP}t\cos\phi_{FP}}\right)$$

HM:
$$\epsilon(t) = \cos^{-1}\left(\frac{-b + a\sqrt{a^2+b^2-1}}{a^2+b^2}\right)$$
$$a = \frac{2d_0}{V_{HM}t} - \cos(\phi_{HM}) \qquad b = \sin(\phi_{HM})$$

where:
V is radial speed
ϕ is propagation angle relative to observer-Sun line
d_0 is distance of the observer from the Sun.

FIGURE 4a illustrates simulated time-elongation profiles based on these two equations for four values of ϕ (offset for clarity) and a radial speed, V, of 400 km/s.

The FPF and HMF techniques are based on fitting to these equations, to yield V and ϕ.

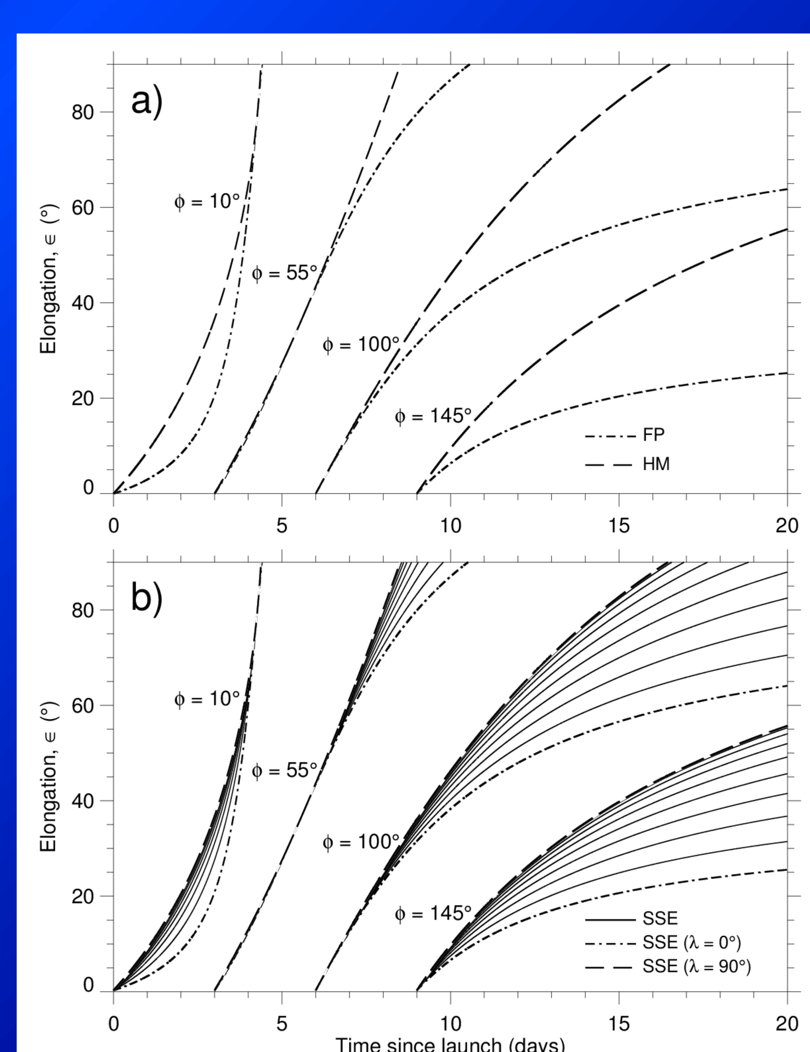
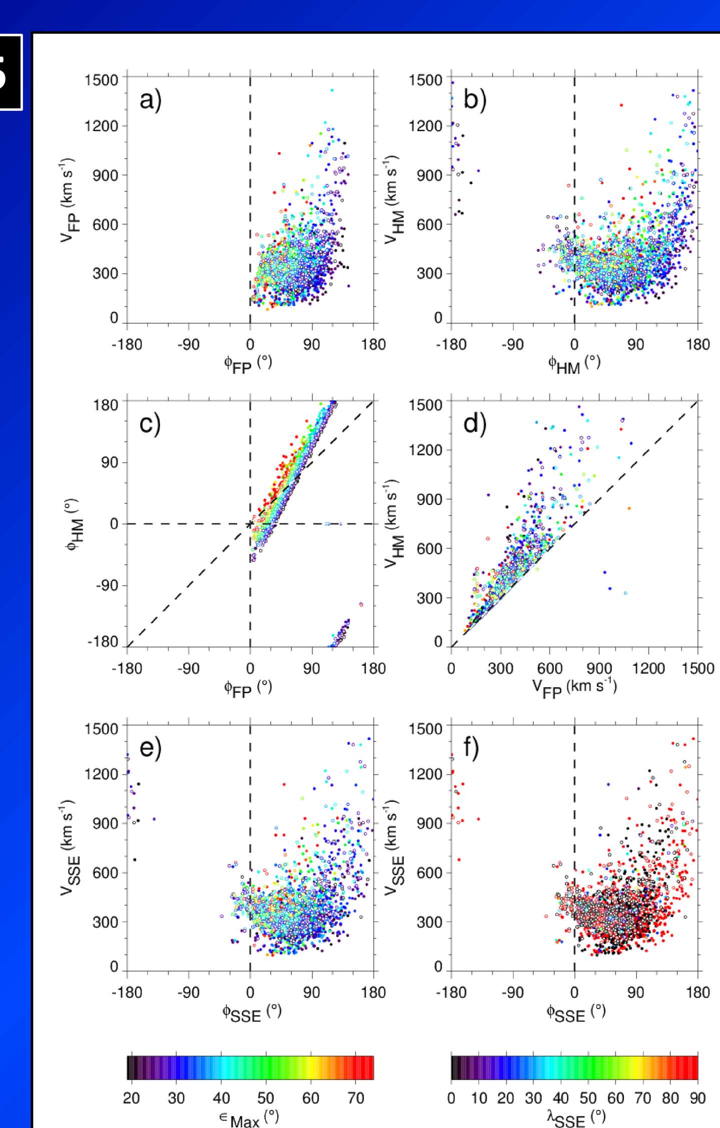


FIGURE 4

5. The top four panels of FIGURE 5 show the results of FPF and HMF analyses of nearly 3000 time-elongation profiles that correspond to the solar transients in the RAL HI Event List. The profiles have been manually extracted from ecliptic HI-1/HI-2 J-maps, derived from the STEREO-A (filled dots) and STEREO-B observations (open dots). In these top four panels, colour coding indicates the maximum elongation of the profile.

The results of the FPF and HMF analysis can be very different, particularly over certain regimes, which is not surprising given the extreme nature of the underlying model geometries on which these two techniques are based.

FIGURE 5



6. Hence, we have developed a more generalised model, for which the FP and HM models form limiting cases. In the Self-Similar Expansion (SSE) model, the transient - although still circular - is not anchored to the Sun, but has a radius that increases as the transient propagates anti-sunward such that it subtends a fixed angle with respect to Sun-centre (see FIGURE 3b). The time-elongation profile for the SSE geometry can be expressed

SSE:
$$\epsilon(t) = \cos^{-1}\left(\frac{-bc + a\sqrt{a^2+b^2-c^2}}{a^2+b^2}\right)$$
$$a = \frac{d_0(1+c)}{V_{SSE}t} - \cos(\phi_{SSE}) \quad b = \sin(\phi_{SSE}) \quad c = \pm\sin(\lambda_{SSE})$$

where λ is the transient's angular half-width in the plane described by the PA of interest.

FIGURE 4b illustrates simulated time-elongation profiles for the SSE geometry. For each value of ϕ, a set of 10 curves correspond to λ ranging from 10 to 90°, in steps of 10°.

The Self-Similar Expansion Fitting (SSEF) technique is based on fitting to this equation for V, ϕ and λ.

The lower two panels of FIGURE 5 show results of SSE analysis of the transients in the RAL HI event List, colour coded by maximum elongation (left) and best-fit λ (right). It appears that SSE analysis tends to mainly return values of λ near 0° or 90° (i.e. either FP or HM solutions).

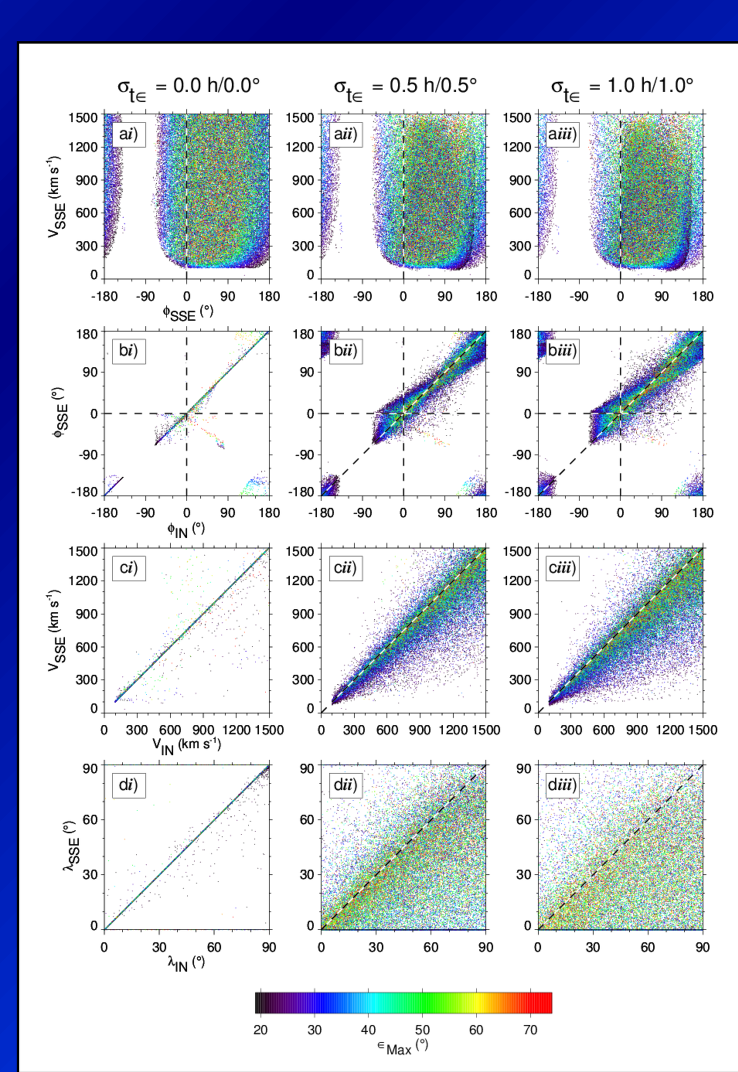


FIGURE 6

7. In an attempt to validate the SSE technique, we performed a Monte-Carlo simulation of 100,000 time-elongation profiles, based on uniformly distributed values of V, ϕ and λ.

FIGURE 6 presents the results of the SSE analysis of these profiles, for three levels of simulated error (increasing left to right). Even for realistic errors, it is difficult to retrieve accurate values of λ for all but the most extended profiles (panels di to diii). As for the HI observations, values of 0° or 90° are returned for λ in many cases.

There is some evidence that more accurate estimates of λ appear to be retrieved in the case of smaller speeds and larger angles relative to the spacecraft (see FIGURE 7).

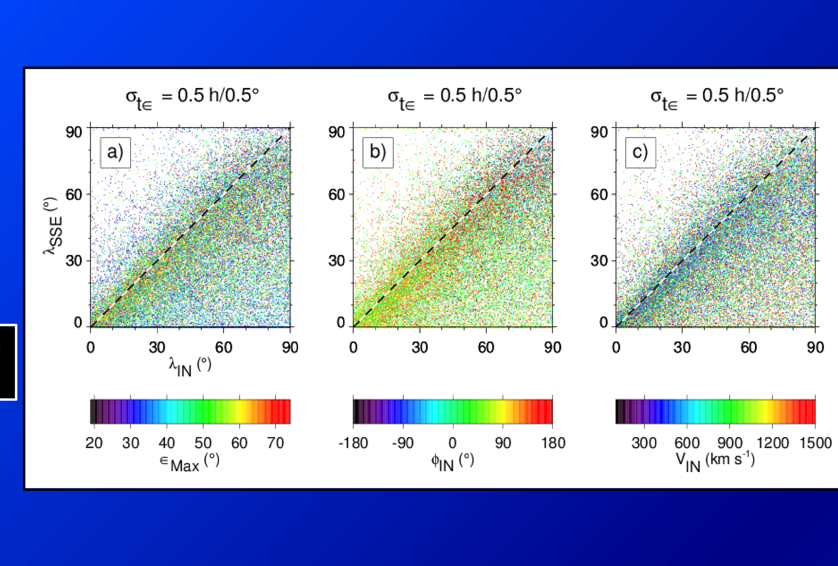


FIGURE 7

CONCLUSIONS:

- FPF and HMF are often-used techniques for retrieving solar transient speed and propagation direction based on single-spacecraft heliospheric imaging observations.
- However, these techniques are based on extreme solar transient geometries, and can produce very different results.
- We present a more generalised method (SSEF) that can, in theory, yield simultaneous estimates of the speed, direction and half-width.
- Although simultaneous fitting for all three parameters is met with limited success, the SSEF technique has much potential based on modification of its underlying equations.
- Not least, it provides the basis for use of a more physically realistic value of half-width than is assumed in either the FPF and HMF techniques.
- This, and other possible adaptations, should make the SSEF technique at least as successful as the FPF and HMF techniques have proved to be.

For full details see Davies et al. ApJ, in press, 2012 (and references therein).