Genuine field theory is needed to support Gravitational Waves

Trevor W Marshall *Manchester*
Max K. Wallis *Cardiff* *wallismk@cf.ac.uk*

Motivation ***Our theme is that gravitational waves imply and are implied by the material status of the gravitational field.***

To properly formulate Einstein's gravitational theory as a field theory, we need a real tensor for gravitational energy-momentum, as recognised by Hilbert and accomplished by Weinberg.
The Einstein equation may be better called the Einstein-Hilbert equ'n

Gravity is to be Interpreted as a Field

The Equivalence Principle in its strongest form
- says gravity is equivalent to metric distortion -
does not fully represent the field and fluctuations

ie. Gravity is NOT ONLY Geometry

Gravity is more than Geometry

Fluctuations in the curvature of spacetime do not express transport of energy/momentum.

Poster at RAS *National Astronomy Meeting,* Manchester March 2012
Session on *Dawn of Gravitational Astronomy*

Genuine field theory to support Gravitational Waves

Gravity-field Energy is real

Spin-down of the binary pulsar proves real loss of energy

Rapid-spinning young neutron stars likewise must shed angular momentum via gravitational waves.

Electromagnetic Waves from accelerated charges relate to the inertial frame of special relativity – and the Minkowski metric. These E/m waves carry localised energy which contributes to the gravitating source of the Einstein equation.

Gravitational waves similarly relate to the Minkowski metric and contribute localised energy/mass in the Einstein equation.

Physics, Astrophysics and Cosmology with Gravitational Waves

(Sathyaprakash & Schutz*)*

Einstein uncertain over gravitational waves

Einstein himself changed his mind several times about the reality of gravitational waves, rather than a fluctuation in the metric. But when the Hulse-Taylor double pulsar[6] was found to be losing orbital energy at the rate predicted by Einstein's quadrupole formula, theorists were slow to challenge the view that 'gravity is only geometry'.

There is still a belief that *"Space tells matter how to move"* coupled to *"Matter tells space how to curve" (Wheeler).*

The material nature of gravitational energy and gravitational waves – included as a mass-energy term in the Einstein equation – show the need to state instead that *Fields tell matter how to move* (http://arxiv.org/abs/1103.6168)

Gravity Interpreted as a Field

Three requirements are frequently set on field theories of gravitation

1) Covariance
2) gauge invariance and
3) the Principle of Equivalence.

We argue for maintaining the first, abandoning the second, accepting only the weakest form of the third, namely the Eötvös Principle (EoP).

Implicit in Einstein's derivation of the quadrupole formula was that the gravitational field is carried by the Minkowski space of "Special" relativity. The notion of the gravitational field as being like the electromagnetic field of Faraday and Maxwell was taken up by Einstein's contemporaries - de Donder and Lanczos - and later by Rosen [1], Fock [3] and Weinberg [4]. This field interpretation of gravity has subsequently been developed, by Logunov and Mestvirishvili [7], and by Babak and Grishchuk, to the point where the Minkowski metric is explicitly present in the field equations.

Minkowski space is the carrier of e/m waves

Two metric tensors

Gravitational waves, like electromagnetic waves, are fields travelling through space that carry energy. The field is specified as a Riemann metric, but the space, or geometry, is not the same entity as the field; indeed it is the same space as that carrying electromagnetic waves, namely Minkowski space. So a theory of gravity requires not one but two metric tensors.

The idea was first made explicit by Nathan Rosen [1].

The Relativistic Theory of Gravitation (RTG) established a correspondence between the two metrics by going to the inertial frame of Special Relativity [7].

Inertial frame

The Minkowski (space) metric is

$$d\sigma^2 = \gamma_{\mu\nu} dx^\mu dx^\nu = dx_0^2 - \delta_{ij} dx^i dx^j$$

satisfying the Einstein-Hilbert equation and the harmonic condition[3][4]

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi T_{\mu\nu}$$

$$\partial_\mu \left[\sqrt{-g} g^{\mu\nu}\right] = 0$$

where g_{mn} is the usual field (Riemann) metric.

In an accelerated frame, both space and field metrics transform as second-rank covariant tensors

Gravitational Wave energy flux

The rate at which energy is transmitted across a closed surface distant from a source region, depends on the quadrupole moment D of the source's mass distribution

$$dI/dt = (G / 45c^5)\,(d^3D / dt^3)^2$$ *Einstein 1918*

This identity holds only in the inertial frame in which the field tensor satisfies the harmonic condition [L&M 1989]. Examination of Einstein's derivation reveals he obtained his result by going to this frame, but mistakenly believed that it was a frame independent result. L&M [1989] gave an explicit change of frame involving only the space coordinates, showing it to be capable of producing arbitrary changes of the right hand side.

David Hilbert in 1917 pointed out that it's not possible to define localized gravitational energy in Einstein's GR with arbitrary metric.

Goett. Nachrichten 4, 21, 1917

RTG and Gravitational collapse for the 'Dust Star' problem

Metrics for the black hole problem

The simplest black hole as described by the Schwarzschild field metric for all r > 0 has a "coordinate singularity" at r = 2m.

$$ds^2 = \frac{r-2m}{r}dt^2 - \frac{r}{r-2m}dr^2 - r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right)$$

The first correction made by RTG is simply to note that this metric fails to satisfy the harmonic condition. The field metric in the inertial frame is obtained by replacing r by r + m, that is

$$ds^2 = \frac{r-m}{r+m}dt^2 - \frac{r+m}{r-m}dr^2 - (r+m)^2\left(d\theta^2 + \sin^2\theta d\phi^2\right)$$

Apparently all this does is to shift the "coordinate singularity" to r = m:
But this is the *exterior* solution only; the *interior* region is occupied by high density matter, for which the Schwarzschild field metric is not appropriate. We obtain an *interior* field metric by going to the Tolman-Oppenheimer-Snyder (TOS) solution [8-9]

$$ds^2 = \left[(F+RF')\sqrt{RF}dR - \sqrt{y}dy\right]^2 - \left[\frac{yF'}{F^2} - (F+RF')\sqrt{\frac{R}{yF}}\right]^2 dR^2 - \left(\frac{y}{F}\right)^2\left(d\theta^2 + \sin^2\theta d\phi^2\right)$$

where the internal mass distribution $F = F(R)$ satisfies (for m=1/2):
$F = 1$ at $R=1$,
$F' > 0$ in $0= R < 1$
(F' is continuous at R=1)

The Relativistic Theory of Gravitation (RTG [7]) departs from the *Strong Equivalence Principle* of Einstein's general relativity, that:

physical processes look the same in all frames of reference.

RTG re-establishes the favoured frame of Special Relativity for properly describing gravitational waves (and *Black Holes*).

Oppenheimer-Snyder model of 1939
an idealized spherosymmetric collapsing star, a "dust ball"

used to justify the concept of a *trapped surface* in a region of space inside an event horizon – a prerequisite for the Hawking-Penrose theorem on the inevitability of black holes

The imperative of Causality

Our analysis identifies a map-reading error**, arising out of neglect for the requirement of causality.

'Hilbert causality' takes Hilbert's 1917 set of conditions on the Riemann metric of GR, forbidding a change in its signature. This excludes a metric for which a space coordinate effectively changes places with the time at the event horizon

Logunov 2001 gave a coordinate-free formulation of Hilbert causality, at the price of introducing a Minkowski metric alongside Riemann's, leading to a preferred inertial coordinate system and the Minkowski metric.

Weinberg in 1972 [4] proposed a related causality requirement - the ability to formulate the initial-value problem.

A combination of these two in 1989 [7] recognises that causality restricts the actual range of values which a set of coordinates may take – we follow this.

*** that led into a dream world, comparable to the map-reading error made by Alice, as described by Lewis Carrol.*

No Singularities in finite time

Oppenheimer & Snyder [9] argued

"… a star in its early stages of development would not possess a singular density or pressure, it is impossible for a singularity to develop in a finite time."

Corda & Mosquera Cuesta 2011 remove black hole singularities via a particular non-linear electrodynamics Lagrangian, finding an exact solution of Einstein's field equations.

No trapped surfaces

Mitra (2000, 2010) found for arbitrary EOS and radiation transport properties, spherical gravitational collapse does not lead to the formation of trapped surfaces. He argues for *eternally collapsing objects* as alternative interpretation of stellar collapse.

the Black Hole problem in Inertial Coordinates

'Dust Star' model of cold contracting particles

In the comoving frame, a freely falling particle inside the mass distribution satisfies the simple equation of motion

R = constant.

The inertial coordinates, satisfied by the inertial coordinates are related to the co-moving ones by the harmonic conditions [4] written with ÿ the d'Alembertian operator as

$$\Box t = 0, \quad \Box r = -\frac{2rF^2}{y^2}$$

$$\Box = g^{\mu\nu}\partial_\mu\partial_\nu + \Phi^\mu\partial_\mu, \quad \Phi^\mu = \frac{1}{\sqrt{g}}\partial_\nu\left(g^{\mu\nu}\sqrt{g}\right)$$

The equations are integrated [10] in the region $R < 1$ with boundary conditions of continuity at $R = 1$ (linkage to exterior solution) and asymptotic conditions at $y \to +\infty$ (Correspondence Principle limit).

a new solution – collapsing cold-dust "star"

The integration is performed by Marshall (2007,2009) [10] giving the inertial coordinates (t,r) as functions of (y, R), from which it is possible to obtain r (R; t).

Since R is a co-moving coordinate, we obtain the trajectories of individual particles inside the collapsing "star"; a given particle being indexed by R which is its asymptotic position ($t \to -\infty$).

The interval of t is the entire physical range $-\infty < t < +\infty$; there is no singularity, ie. no black hole.

We thereby confirmed the judgments of both Eddington[11] and Einstein[12] on Black Holes:

their existence would violate the *Principle of Locality* - the basis of Special Relativity (SR) – which does not allow any material particle to cross the superluminal speed barrier.

- It is natural and appropriate to reach this conclusion by reinstating the inertial frame to the privileged position it has in SR
- such a point of view requires dropping the strong form of the Equivalence Principle, restricting it to the weak Eötvös Principle (EoP), originally conceived by Einstein in 1911.

The map of space-time in Oppenheimer-Snyder coordinates R, t (Figure 1)
with particle geodesics depicted by continuous and light geodesics by broken lines.

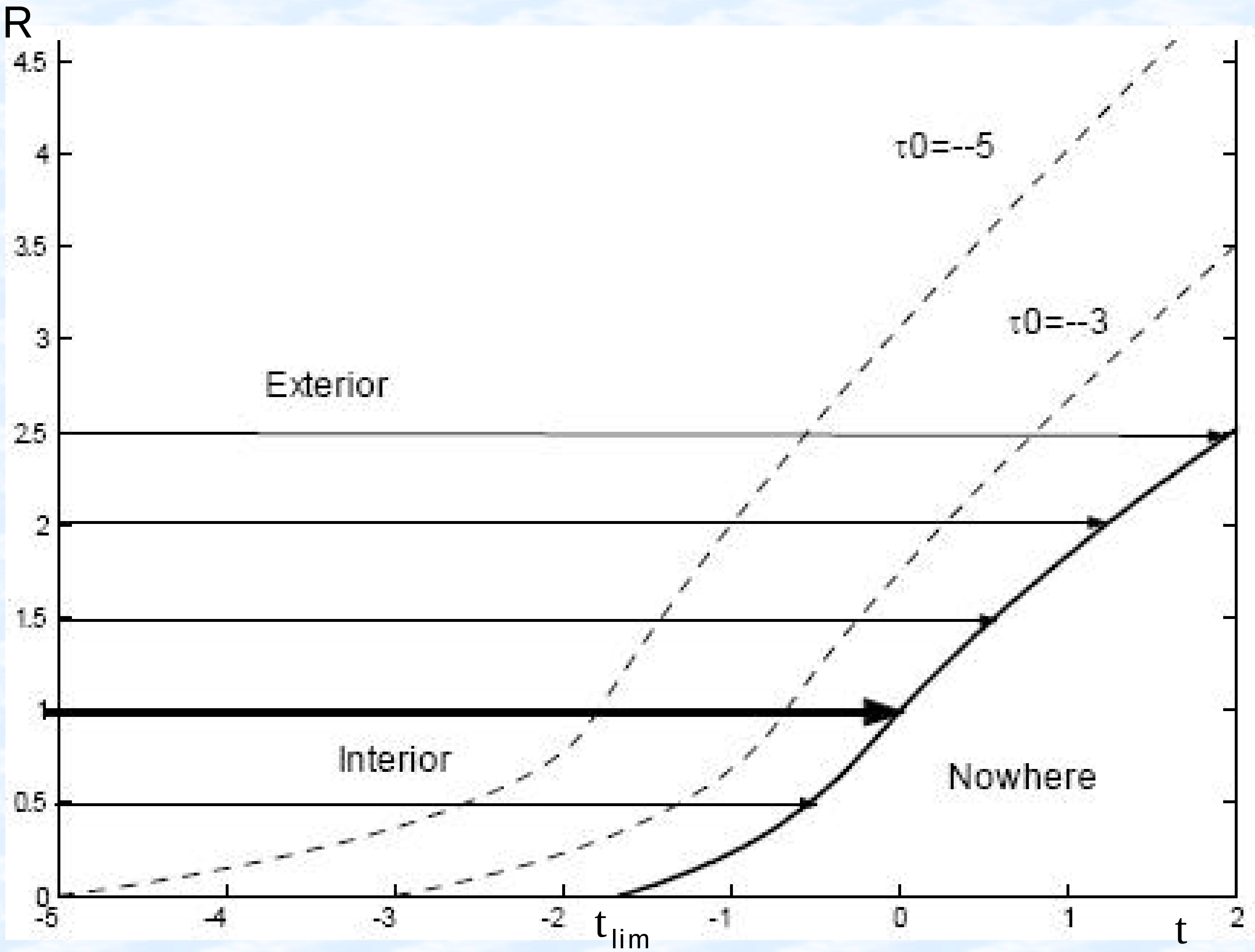
The geodesic R = 1 is the boundary between OS exterior and interior regions

OS used co-moving coord's, ‘exterior’ for empty space and the ‘interior’ co-moving with the contracting dust.

We discover the ***limit curve*** (bold) defining a region to the right - labelled ***Nowhere*** - here Hawking & Penrose discovered trapped surfaces.

Physical space and time are confined to the region to the left of the limit curve.

Light paths (broken lines) cross the particle paths R=const., but none after t_{lim} - the physical time $t = +\infty$



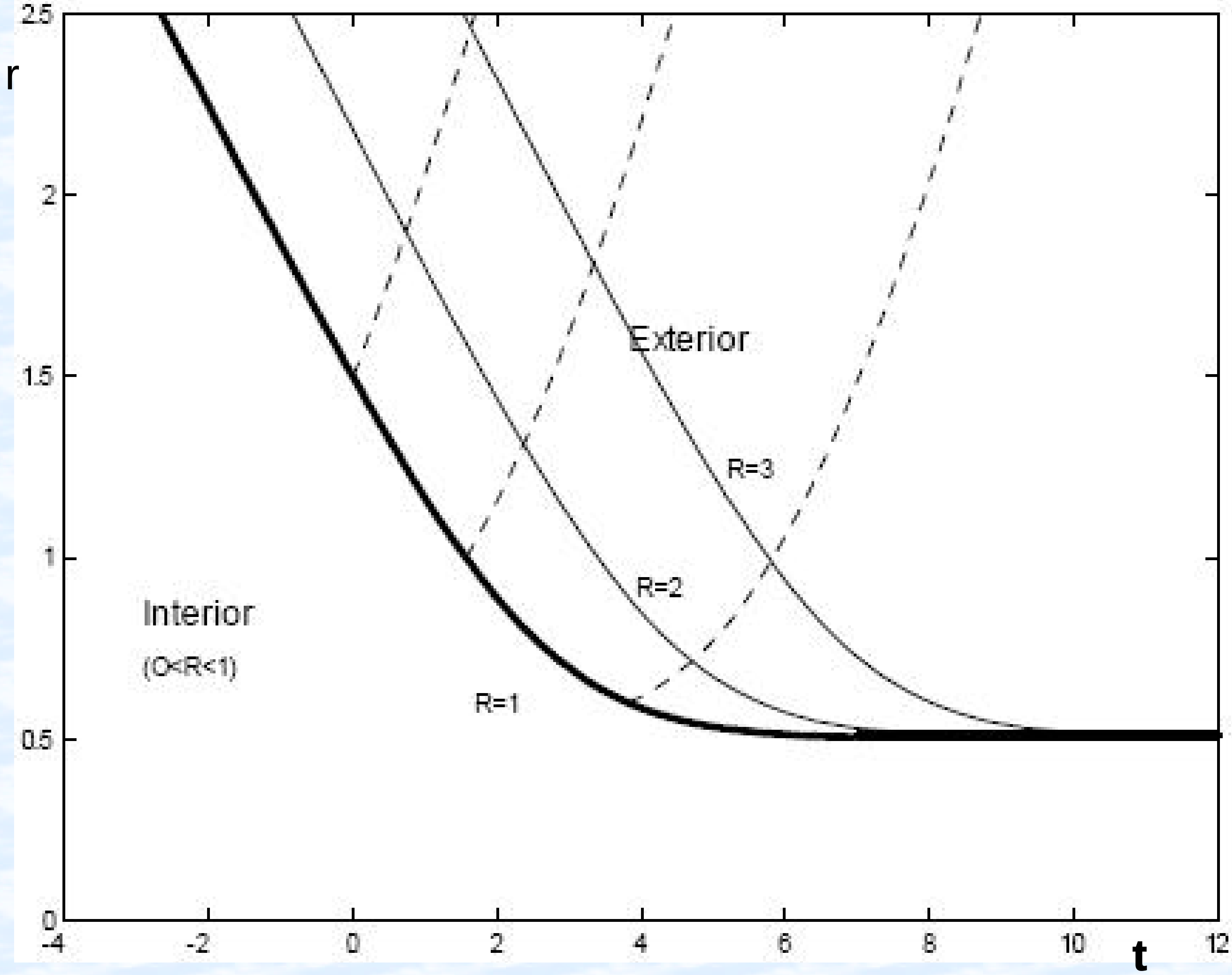
The space-time of Figure 1 mapped onto r, t coordinates (Figure 2)
shows all points of the physical space to be *causally connected* at all times,

Any light ray originating in the interior region eventually escapes (broken trajectories) far into the exterior region.

r is in units of 2m and **t** is in units of 2m/c, with $m = 2M/c^2$.

Dust particle R=const. trajectories all converge on the limit surface R=1 at large t.

We still have to map trajectories in the inner region, where the increasingly high gravity field tends to force them to also converge in a shell at R=1



Gravitational attraction and repulsion

We have established with consistent field and space metrics of [7]:

- the collapse can go no further than the Schwarzschild radius
- the collapse to this state takes an infinite time
- the density of particles in the limit becomes infinite at the surface of the cloud.

This confirms the findings of Logunov [2], that gravity changes from being attractive to repulsive for certain high-density conditions.

The analysis shows that "continued gravitational contraction" is incorrect – this idea was not fully justified by Oppenheimer & Snyder [9] though their work is considered to be the basis for the contemporary “Black Hole".

It is more appropriate to call the process one of *gravitational compression*; the combination of an overall attraction of the surface particles with a repulsion of the particles beneath the surface produces an infinite density at the surface; presumably this balance of gravitational forces will be modified by the action of nuclear forces in an actual neutron star, so the state of infinite density is never reached. But in any case the contraction does not go beyond the Schwarzschild radius.

Case of a nonzero cosmological constant.

The modified Einstein-Hilbert equation incorporating a cosmo-logical constant is a natural and indeed necessary feature of RTG.

Gerstein et al. [13] suggest this will stop the compression process after a certain time, and convert it into a cyclical, or pulsating, process. However, it is not clear what will happen if, as plausible, the nuclear forces intervene before these extra gravitational terms can take effect.

[1] N. Rosen, Phys. Rev. 57, 147-153 (1940) and Ann. Phys. (New York) 22,1 (1963)

[2] A. A. Logunov, The Theory of Gravity, Nauka, Moscow (2001)

[3] V. A. Fock, The Theory of Space, Time and Gravitation, Pergamon, New York (1959)

[4] S. Weinberg, Gravitation and Cosmology (John Wiley, New York, 1972) pp. 161-163

[5] A. Einstein, Sitzungsber. preuss. Akad. Wiss., 1, 154 (1918)

[6] A. S. Eddington, The Mathematical Theory of Relativity, University Press, Cambridge, pp128-131 (1924)

[7] A. Logunov and M Mestvirishvili, Relativistic Theory of Gravitation, Moscow (1989)

[8] R. C. Tolman, Proc. Nat. Acad. Sci USA, 20, 169-176 (1934)

[9] J. R. Oppenheimer and H. Snyder, Phys. Rev., 56, 455-459 (1939)

[10] T. W. Marshall, http://arxiv.org/abs/0707.0201; abs/0907.2339

[11] A. S. Eddington, The Observatory, 58, 373 (1935)

[12] A. Einstein, Ann. Math. 40, 922 (1939)

[13] S. S. Gerstein, A. A. Logunov and M. A. Mestvirishvili, Theor. Math. Phys. 155, 715 (2008)

also commentaries on Blog:
http://crisisinphysics.wordpress.com

Gravitational waves are propagating oscillations of the gravitational field, just as light and radio waves are propagating oscillations of the electromagnetic field **

Energy in the static field in the Newtonian approx.

the field $GM\underline{\mathbf{r}}/r^3$ has energy density $G^2M^2/8pe_or^4$

This integrates outside a star of radius r_o in mass units as

$$G^2M^2/2e_or_oc^2 = (r_{schwar}/r_o)ßM/2 \quad \text{where the permittivity } e_o=G/ß$$

? the energy inside depends on ?(r) and just adds a fraction.

? the factor ß = 7 for RTG but differs for other metrics.

? for the Sun $r_{schwar}/r_o \sim 2x10^{-6}$ => field energy ~ 1 Earth mass.

The field energy of a binary system is more strongly dependent on M and r_o, and still small until $r_o \sim r_{schwar}$.
The compact object at the galactic centre $M=4.3x10^6$ solar masses could be a few times its r_{schwar}; limits on the size of the SgrA* radio source are about 6 r_{schwar}, so the field energy could exceed 0.6M.

Robertson & Leiter (2004) interpreted galactic black hole candidates and active galactic nuclei as having intrinsic magnetic moments, so no event horizon, but magnetic centrally compact objects.

Mitra (2010) argues condensed objects, not black holes, explain the central engine of quasars, micro-quasars, and energetic Gamma Ray Bursts.

Changes in the mass distribution put a part of this energy into gravitational waves. Both the static energy and changes in it are real energies – so are independent of the GR metric.

For a collapsar, immense energy goes into the field (*not* lost in a Black Hole)

=> *compact object disruptions can be astrophysically mega-luminous*

** Sathyaprakash & Schutz "*Physics, Astrophysics and Cosmology with Gravitational Waves*"
http://relativity.livingreviews.org/Articles/lrr-2009-2/