Simulating the shattering of neutron star crusts

Stephanie J. Erickson

University of Southampton

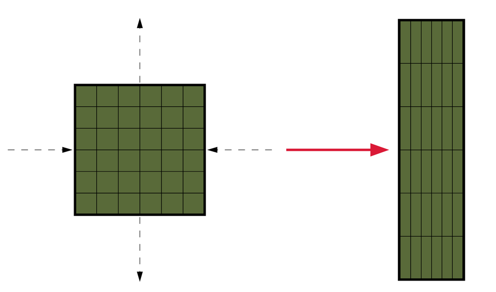
1. The physics problem

The crust of a neutron star (NS) makes up only a small portion of the mass of the star. Furthermore, the ratio between the shear modulus and compression modulus is much smaller than unity [1] in the outer crust, so the effect of shear stress will be small compared to the effect of pressure in those regions. However, the interaction between the crust and the fluid core can introduce modes that are qualitatively different from pure fluid modes. In addition, because the shear stresses are weak, the crustal and interfacial modes have lower frequencies than the purely fluid modes; this means that resonance with tidal forcing can be reached some time before merger (as in the mechanism proposed by Tsang et al. for precursors prior to short-hard gamma-ray bursts [2]). The solid crust is also important in the starquake models for pulsar glitches [3] and gamma-ray bursts [4].

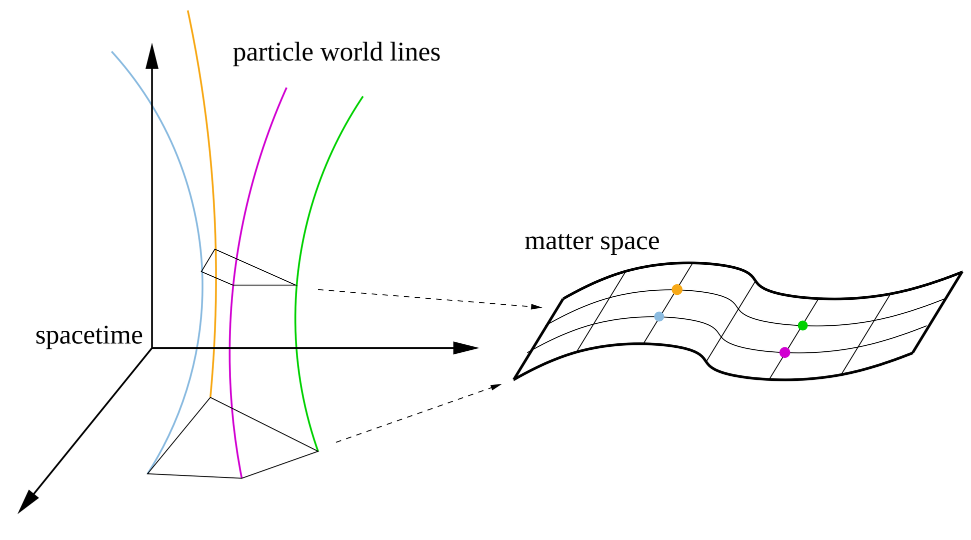
While some models appear to be based on the assumption that starquakes will occur in a manner similar to earthquakes (ie. [4]), with cracks forming and techtonic plates sliding against one another, recent molecular dynamics simulations by Horowitz and Kadau [5] and by Chugunov and Horowitz [6], suggest that the crust does not fail in this way. Because of the high pressure, any cracks that form will be immediately healed. Instead, we expect a region of the crust to *shatter*, ie. we expect that the material will instantaneously become relaxed within a certain region, and immediately refreeze to solid form with the new (relaxed) configuration. Our aim is to explore the effect that this type of crust shattering has on the dynamics of the neutron-star binary system. A major step towards this goal was presented in Gundlach et al. where a conservation-law formulation for elasticity in general relativity was demonstrated [7].

3. Elasticity

Relationship with the relaxed state



If a block is stretched in one direction and squeezed in the other such that the density remains the same, then the behavior of the new configuration is exactly the same as that of the original. If the same deformation is performed on a block of elastic material, the behavior is clearly different. This is because, in elasticity, we must keep track of particle positions with respect to their relaxed state.



To do this, we follow Carter and Quintana's approach [8] of using two manifolds and a map between them. One manifold, the *matter space* (with metric, k_{AB}), keeps track of the preferred particle positions with respect to one another, while the other, the *spacetime* (with metric, g_{ab}), is the background with respect to which the material moves and deforms. The map between them, along with the metrics of both manifolds, encodes any information about deformations that occur: we compare the matter-space metric, k_{AB}, to the spacetime metric pushed forward onto the matter space, g_{AB}, with the relaxed state occurring when these two tensors are proportional to one another.

In order to write the evolution equations in first-order form, we introduce the *configuration gradient*, the spatial derivatives of the map between the two manifolds. From the commutation of partial derivatives, we can write evolution equations and constraints for the configuration gradient.

Shear Stresses

If a block of fluid is sheared so the two halves move in opposite directions, a contact discontinuity is formed (where velocity is discontinuous), but no waves propagate away from the interface; however, if this action is performed on a block of elastic material, waves *do* travel away from the interface.

Therefore, shear stresses must be included for elasticity; to do this, an anisotropic stress term is added to the usual perfect-fluid formulation of the stress-energy tensor. If examined in the rest frame of the fluid, the stress-energy tensor of a perfect fluid is diagonal; the anisotropic stress term adds off-diagonal components to the spatial part of this tensor, but not to the time-space components, which would represent heat flow.

6. Interfaces

Where is the interface?

We track the interface using a level-set function. Positive values of the function represent cells where one material is present, while negative values show where the other material is present: the interface is located where the level-set function equals zero. This function is advected over the computational grid along with the fluid, thus tracking the location of the interface.

What happens at the interface?

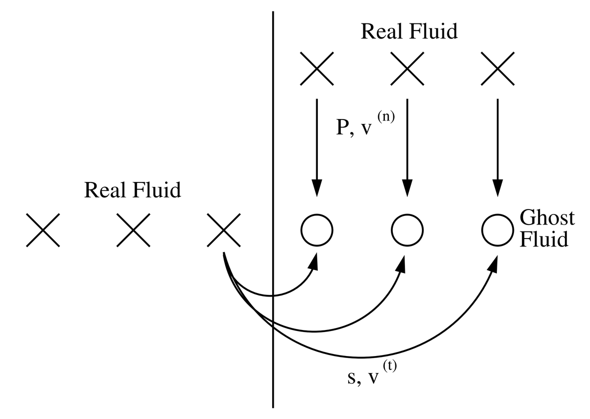
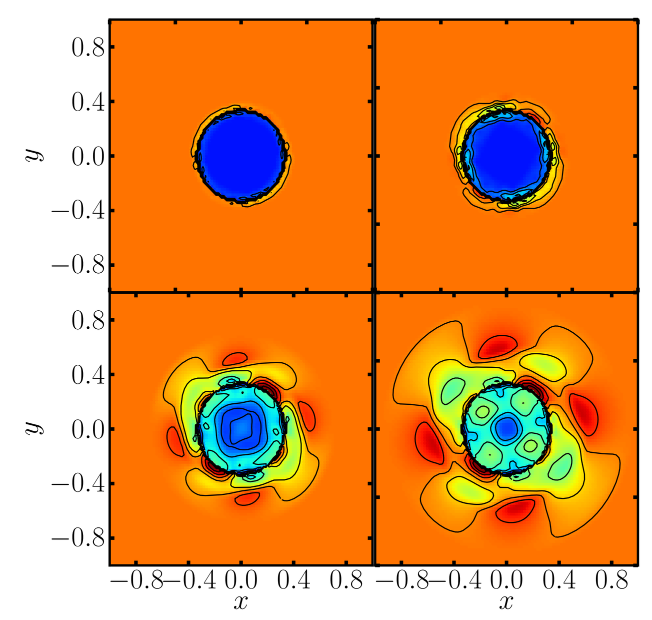


Image based on figure from [9].

To treat the boundary, we evolve each material separately, assigning ghost-fluid cells at the edges of the regions occupied by another material. In the original ghost-fluid method (GFM) [9], ghost cells are assigned in a relatively intuitive way: since pressure and velocity normal to the interface should be continuous, these are copied from the real fluid occupying the same cells as the ghost fluid. Tangential velocity and entropy can be discontinuous, so we copy these from the nearest real-fluid cell that is of the material associated with the ghost-fluid cell we are considering. Density is then calculated from entropy and pressure.

While this is the most intuitive approach, other methods can also be used determine the behavior at the material interface. For example, Barton and Drikakis [10] have developed a modified GFM where a linearized interface Riemann solution is used to determine the physically correct behavior at the boundary; the ghost-fluid cells are then assigned to reproduce the correct behavior to the real-fluid side of the interface.

5. Shattering



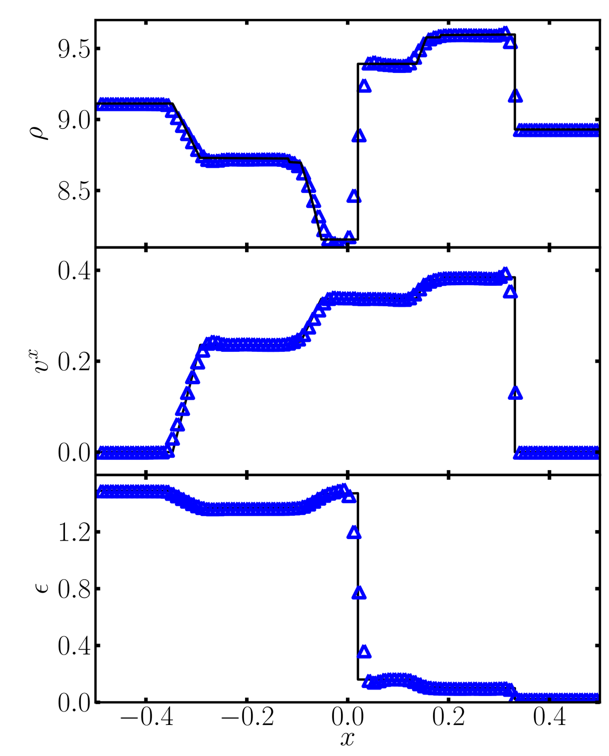
Shattering is the instantaneous relaxation of the material in some region. Numerically, this is achieved by resetting the matter-space metric such that it is proportional to the spacetime metric pushed forward onto matter space, where the constant factor is the density.

The figure to the left shows a shear-stress component of the stress tensor in the time evolution following a shattering event. Here we took homogeneous initial data and shattered a circular region in the center, setting the shear stress in that region to zero.

5. Results

Riemann problems are standard tests for high-resolution shock-capturing numerical codes: we have used several Riemann problems to test our code [7].

- ▶ The code can reproduce published Newtonian exact Riemann solutions in elasticity.
- ▶ The results of the relativistic code approach the results of the Newtonian code and the Newtonian exact solution in the Newtonian limit ($v << c$).
- ▶ The relativistic results match the relativistic exact solution.
- ▶ The elasticity code functions in 2D, and the 2D results are consistent with 1D results.
- ▶ A Riemann test in 2D cylindrical coordinates demonstrates that the formalism works for curved coordinates.
- ▶ Initial tests using the GFM for 1D fluid interfaces agree with published Newtonian and special relativistic tests.
- ▶ Initial artificial "shattering" simulations in 1D and 2D have encountered no numerical problems.



The results of a Newtonian elastic Riemann problem match the exact solution published in [11].

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S.J.Erickson@soton.ac.uk

University of Southampton, Highfield, Southampton SO17 1BJ