An SPMHD Mean Field Dynamo

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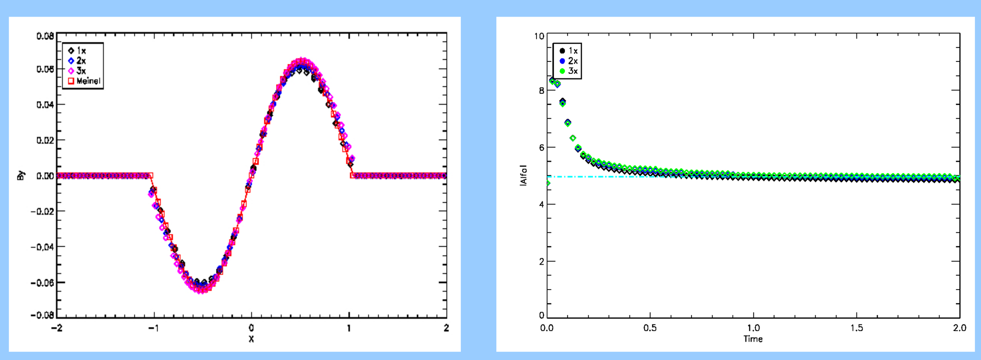
Following the developments in SPMHD, we implemented the turbulent transport terms in the induction equation for the evolution of the magnetic field in, with the aim to perform realistic modelling of dynamo action in global galaxy simulations. Besides the spatial dependent turbulent diffusion β also the α-tensor is included. For a disk setup we could verify our numerical results with a known analytical model of Meinel 1990. Further comparisons with grid based numerical simulations for disks with a galactic rotation law and an anisotropic α-effect are shown. This allow us to perform global galaxy simulations with a subgrid model for dynamo action, which can be linked to upcoming and present day radio observations

Meinel test

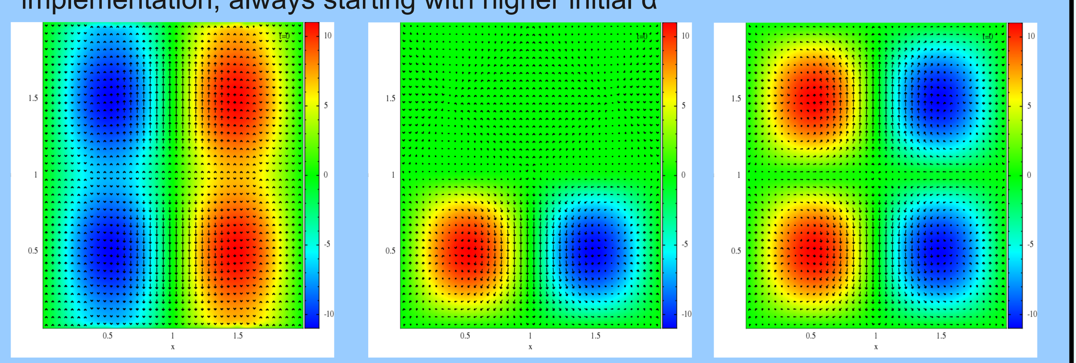
Meinel1990 show an exact solution for a pure mean field dynamo case, i.e with the velocity field equal to zero. The test setup is a cilinder with constant β and equal α in the upper and lower parts of the disk, but with different signs. Additioanally we use a height of the disk of 2 and a radius of 1, This defines our critical α which has a value of 4.98. We use a glass-like distribution for the particle positions, ensuring that particles are not particularly aligned and being a better representation of of practical cases and having a constant density field.

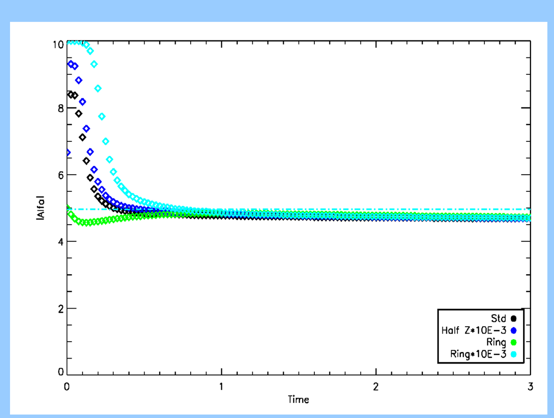
First, we set up the magnetic field in the same distribution as the solution found by Meinel1990. However we start the simulation with an α twice the critical. This will imply that at the beginning, the α terms will increase the magnetic field away from the stable solution, and then the quenching should restore the configuration and α towards the critical value. We found that we quickly restore the configurtion and find the critical α. Additionally we made a resolution study, founding that increasing the resolution we reach a little bit faster the critical values. Below we show, a cut at Y=0 and Z=0.5 of these cases for the different resolutions compared with the analitical solution, and the evolution of α value.



In the figures above, we can observe the resolution convergence, however the final critical α are slighlty lower that the predicted by Meinel. We think this is due to the inherent SPH difficulty to properly define boundaries, and therefore the volumes that will characterize this value.

Additinally, we show bellow cuts over trought thought the center of the disk of different initial configurations that we use to test the capabilities of our implementation, always starting with higher initial α





In all the cases we reach the analitical solution. Which can be seen in the right plot, where we show the evolution of the α in this cases. We can see the quenching acting different in this cases.

Finally, be test the stability of the solutions, to ensure that there are no errors that integrate and cause instabilities. Therefore we run the test 10 times longer, without any increase of the errors nor instabilities appearing.

The div(B) errors are always bellow 10E-3, which means that any kind of effects due those errors are 10E-3 times smaller than the integrations done by the induction equation.

Numerical Scheme

We make use of numerical implementationof SPMHD in **Gadget-3** (see, Dolag2009, Stasyszyn2012). Which has been already used in different astrophysical aplications from galaxies (Kotarba2009), star fromation (Bunzle 2010) and galaxy clusters (Bonafede 2011), with successful results.

In particular we implement the α term in the same way as the induction ecuation is solved, just adding the dynamo term.

We split the β term in two:

$$\nabla\times(\beta\nabla\times\vec{B})=\nabla\beta\times\nabla\times\vec{B}-\beta\Delta\vec{B}$$

We calculate the second term as a diffusion, it in the same way as we shown in Bonafede2011, which has been a succesfull tested reducing the errors by using the second derivative of the kernel.

The first term is calculated using again the same induction loop, and taking as a common factor the derivatives of the kernel, which free us to calculate the rot(B) in a previous loop and reduces any numerical noise lowering the number computations. Always, we substract the first order errors as explained in Price2011.

Introduction

The full understanding of astrophysical process, require a complete development of the astrophysical mechanisms in several scales. Some of which are well known, and others not. Such is the case of turbulence, which is a small scale process that has an intrinsic stochastic behavior,that can reach large scale effects (and vice versa).

In the case of the MHD equations and dynamo theories, to overcome the complexity inherent from this studies, the so-called *mean field theory* was developed. In this approach, one re-write the MHD equations decomposing the velocity and magnetic field into a mean field plus a *turbulent* component. Doing this decomposition we assume that the fields follow *Reynold's* rule of decomposition (see Raedler2000).

In this framework the induction equation can take the form

$$\frac{\partial\vec{B}}{\partial t}=\nabla\times(\vec{V}\times\vec{B}+\alpha\vec{B}-\eta\nabla\times\vec{B})$$

where α and β corresponds to statistical properties of the turbulent flow, being β the equivalent of a turbulent dissipation. In a general way those properties are not scalars, i.e. they are well defined as tensors to capture all posible anisotropies.

Additionally is usefull to apply a quenching mechasim, which help us to stabilze the dynamo when we reach equipartition, finding the critial α characteristic of each problem. We use a simple quenching giben by

$$\alpha=\frac{\alpha_0}{B^2/B_{eq}^2+1}$$

Where α_0 is the value at the begining of the simulation, **B** is the integrated magnetic field in our simulation and $\mathbf{B}_{eq}$ is the integrated magnetic field at equipatition.

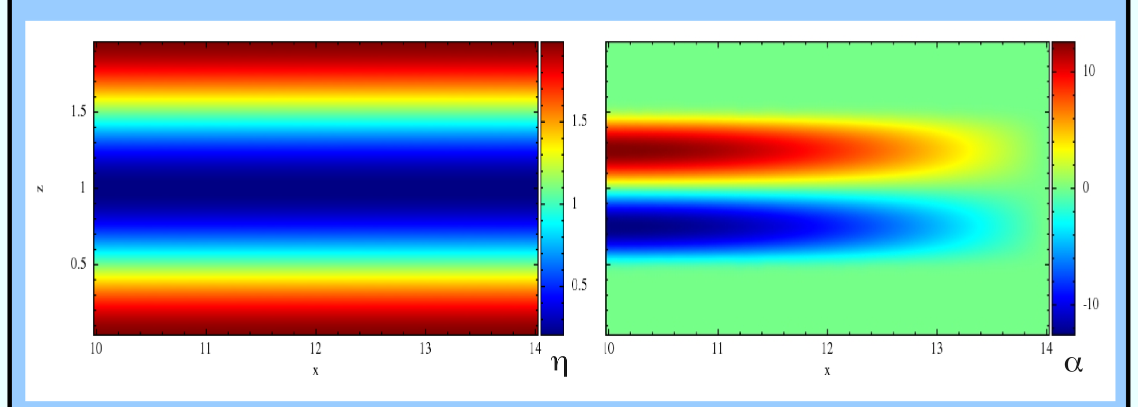
α-ω Test

We generate the initial conditions for a gravitationaly rotating disk. We define the initial velocity disttribution without any Z dependence as

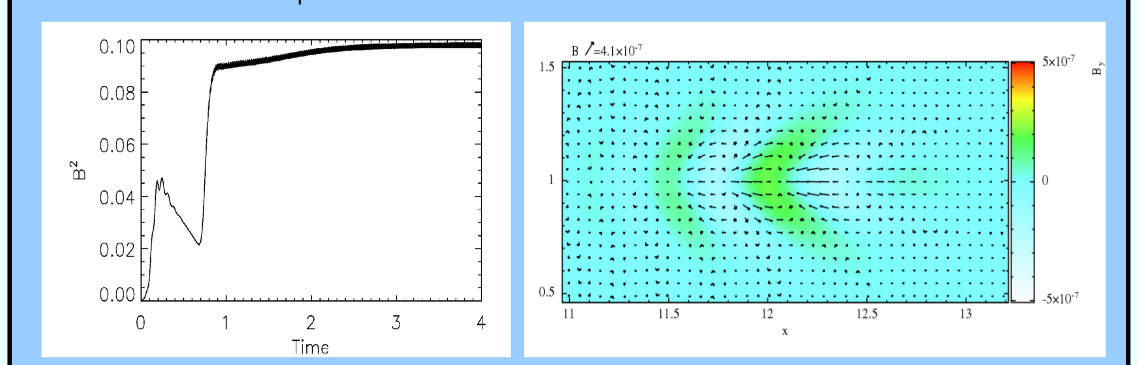
$$u_{\phi}=\frac{r\cdot\Omega}{\sqrt{1+(r/R0)^2}}$$

With Ω value of 100 [1/t] and R0 of 1.5. The disk geometry is 2 in height and 5 in radius, but we confine the α disk to be between 0.5 and 1.5 in Z direction.

For the turbulent coeficients we use sinusoidal distributions, as a step foreward real galaxy simulation. Bellow you can find the a cut trought the center of te disk showing the α and β distributions.



In particular we constrain the β to be fix between 0.2 and 2.0. In the case of α we still use the same quenching formula, conserving the signs difference and the sinusoidal shape.



Above we show the evolution magnetic field energy (left) and the magnetic field distribution (right). We begin to observe some characterisc modes from the dynamo working, however we must compare more extensively with numerical solutions, as done for the anisotropic α case.

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Anisotropic α

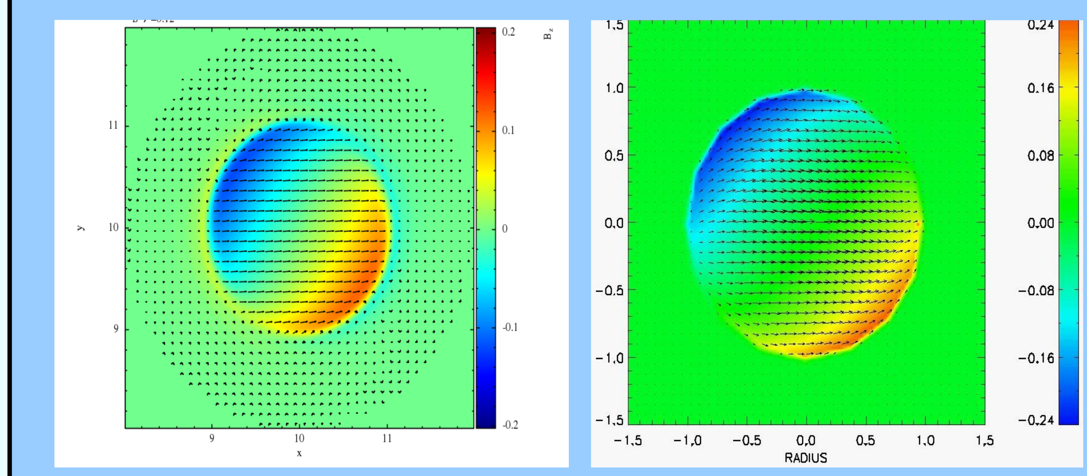
To test our implementation into a more useful and realistic case, checking the viability of our implementation in more serious environments, in which we must use a tensor version of our α (i.e Gressel2009).

We test a non isotropic α in the disk configuration similar that the used for the Meinel case, but we modify the α values to be defined in cartesian coordinates as

$$\vec{\alpha}=[\alpha/\sqrt{2},\alpha/\sqrt{2},0]$$

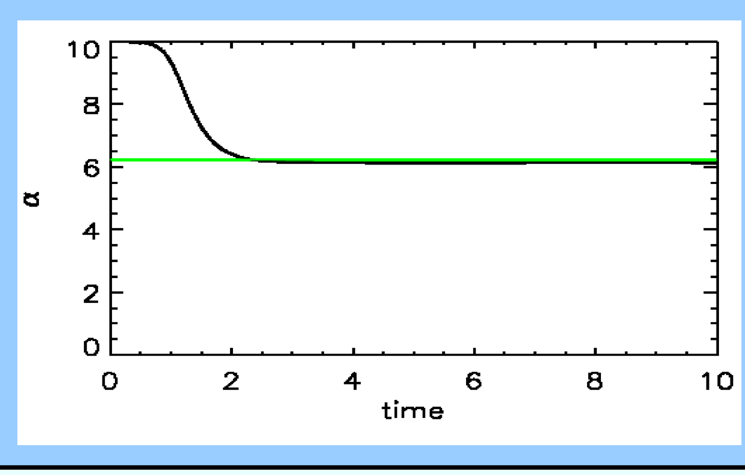
Which mean that is only afecting the horizontal plane. We also apply the same quenching mechanism as previous tests.

The solution of this problem is know but not found analitically, therefore strictly should be tested against other numerical solution. This solution involves the continuous periodic change in the m=1 mode of the disk.



Above we show two cuts of the near slightlty above the midplane, made with our method in SPH [left plot] and the solution found with an eulerial code [right plot] (Elstner2009).

Both simulations started from slighlty differenet initial condition but reach the same final configuration. The critical α for the problem is 6.24. Bellow we show the α evolution for this setup in the SPH case.



Conclusion

We sucessfully implemented the turbulent transport terms in the induction equation for the evolution of the magnetic field in SPH. Besides the spatial dependent turbulent diffusion β also we incude the α-tensor. We found as optimal implementation to use the same loop, which reduces the amount of memory and calulations.

•We verify our numerical results with a known analytical model of Meinel 1990 for a disk setup. We study the dependence of our results with resolution and various initial configurations in the distribution of the magnetic field, showing a splightly improve with resolution and almost no dependence with the initial setup at long times.

•We compare with grid based numerical simulations for disks with an anisotropic α-effect, which is common in the real cases of galactic dynamo. The results either from the critical α and the morphology of the field are comparable.

•We setup a simple galactic rotating disk to test the a complete α-ω case. The final configuration reaches a stable configuration. However more extensive tests, with other α-ω-β configurations have to be tested and the long term stability of our results.

More testing has to be done, but this are steps towards performing global galaxy simulations with a subgrid model for dynamo action, which can be linked to upcoming and present day radio observations.