Non-Gaussianity in Large Scale Structure and Minkowski Functionals: arXiv:1108.1985 (Accepted by MNRAS)

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Overview

One of the major areas of research in modern cosmology is the study of cosmological non-Gaussianities. From a fundamental perspective non-Gaussianity in the primordial perturbations that seed the CMB anisotropies and large scale structure is a direct probe of the underlying field theory that drives inflation or the conditions under which inflation occurs (e.g. modified gravity, etc). A Gaussian random field is completely characterised by its power spectrum (2-point correlation function) in the sense that all even correlators factor into products of the 2-point correlator and all odd correlation functions are identically zero. This means that the bispectrum (3-point correlator) is the lowest order correlation function for which a non-zero value denotes a deviation from Gaussianity. We present a method, applicable to galaxy clustering surveys, based on the Minkowski Functionals that retains momenta, i.e. wavenumber $k(h\,\mathrm{Mpc}^{-1})$, dependence from the underlying theory giving greater distinguishing power between different models.

Minkowski Functionals

A 3-dimensional cosmological density field can be modeled as a smoothed scalar field $\Psi(\mathbf{x})$ with zero mean $\langle\Psi(\mathbf{x})\rangle = 0$. The excursion set Q, a basic building block for Minkowski Functionals, is defined to be the set of all points above a variance, $\sigma_0^2 = \langle\Psi\rangle$, weighted threshold ν: $Q = \{\mathbf{x}|\Psi(\mathbf{x}) > \nu\sigma\}$. The analytical results for a weakly non-Gaussian random field have been derived (Matsubara 2002)[1] through a perturbative expansion in terms of the variance. The MFs are decomposed into a dimension and variance weighted amplitude and a set of normalised MFs $v_k(\nu)$ that depend on the skewness parameters S_j: $V_k(\nu) = A_k\{v_k^G(\nu) + v_k^{NG}(\nu)\}$. The non-Gaussian corrections are given by:

$$v_k^{NG}(\nu) = e^{-\nu^2/2}\left\{\left[\frac{1}{6}S_{(0)}H_{k+2}(\nu) + \frac{k}{3}S_{(1)}H_k(\nu) + \frac{k(k-1)}{6}S_{(2)}H_{k-2}(\nu)\right]\sigma_0 + \mathcal{O}(\sigma_0^2)\right\}$$

Skew-Spectra Formalism and Results

The skew-spectra are a proposed method that utilise cubic statistics constructed from the cross correlation of two fields. The generalised skewness parameters, corresponding to the skew-spectra formalism, are constructed from the cross correlations of the original random field and derivatives of the random field:

$$S_0(\mathbf{x}_1,\mathbf{x}_2) = \frac{\langle\delta^2(\mathbf{x}_1)\delta(\mathbf{x}_2)\rangle_c}{\sigma_0^4};\; S_1(\mathbf{x}_1,\mathbf{x}_2) = \frac{\langle\delta^2(\mathbf{x}_1)\nabla^2\delta(\mathbf{x}_2)\rangle_c}{\sigma_0^2\sigma_1^2};\; S_2(\mathbf{x}_1,\mathbf{x}_2) = \frac{\langle\nabla\delta(\mathbf{x}_1)\cdot\nabla\delta(\mathbf{x}_1)\nabla^2\delta(\mathbf{x}_2)\rangle_c}{\sigma_1^4};$$

Following the normal convention the bispectrum is defined as the three-point correlator of the Fourier coefficients:

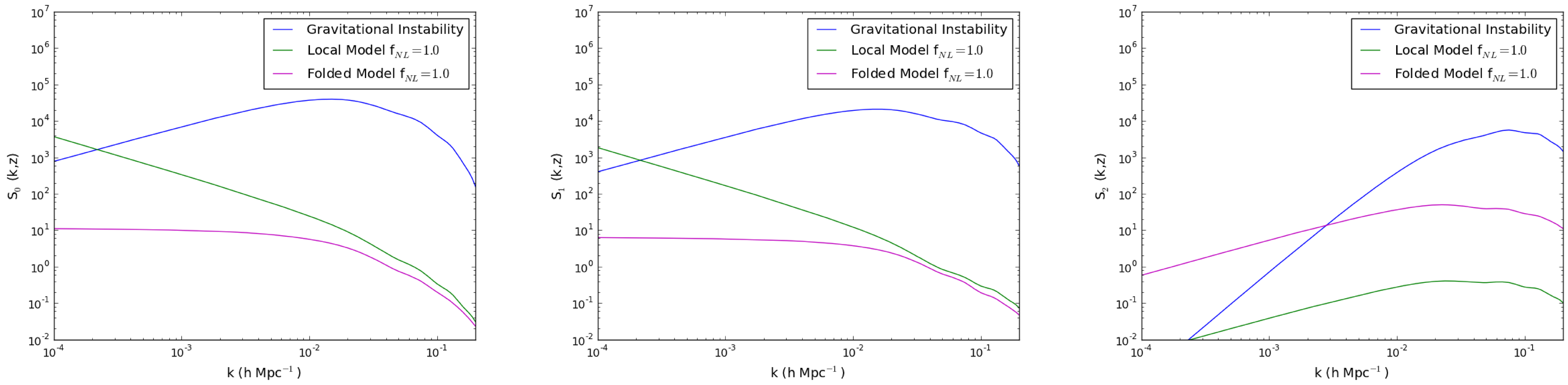
$$\langle\delta(z,\mathbf{k}_1)\delta(z,\mathbf{k}_2)\delta(z,\mathbf{k}_3)\rangle_3 = (2\pi^3)\delta_D(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)B_g(k_1,k_2,k_3,z)$$

where the Fourier coefficients are defined in terms of the density contrast of galaxies: $\delta(\mathbf{x},z) = 1/(2\pi)^3\int d^3\mathbf{k}\tilde{\delta}(\mathbf{k})e^{i\mathbf{k}\cdot\mathbf{x}}$.
From this we find that the skew-spectra reduce to integrals over the bispectrum with the appropriate weights:

$$S_0(k_2,z) = \frac{1}{\sigma_0^4}\frac{1}{4\pi^2}\int_0^\infty\frac{k_1^2dk_1}{4\pi^2}\int_{-1}^{1}d\mu\, B_\delta(k_1,k_2,|\mathbf{k}_1+\mathbf{k}_2|,z)W(k_1R)W(|\mathbf{k}_1+\mathbf{k}_2|R),$$

$$S_1(k_2,z) = \frac{3}{4\sigma_0^2\sigma_1^2}\frac{1}{4\pi^2}\int_0^\infty\frac{k_1^2dk_1}{4\pi^2}\int_{-1}^{1}d\mu\,|\mathbf{k}_1+\mathbf{k}_2|^2B_\delta(k_1,k_2,|\mathbf{k}_1+\mathbf{k}_2|,z)W(k_1R)W(|\mathbf{k}_1+\mathbf{k}_2|R),$$

$$S_2(k_2,z) = \frac{9}{4\sigma_1^4}\frac{1}{4\pi^2}\int_0^\infty\frac{k_1^2dk_1}{4\pi^2}\int_{-1}^{1}d\mu\,(\mathbf{k}_1\cdot\mathbf{k}_2)|\mathbf{k}_1+\mathbf{k}_2|^2B_\delta(k_1,k_2,|\mathbf{k}_1+\mathbf{k}_2|,z)W(k_1R)W(|\mathbf{k}_1+\mathbf{k}_2|R),$$



In the graphs above we show the skew-spectra (S_0, S_1, S_2) for three different bispectrum configurations corresponding to gravitational instability, local and folded primordial models. In this scheme we are free to specify the configuration for the bispectrum and can generate the corresponding skew spectra. This means that the momenta dependence carried by an arbitrary theory for gravitational instability (e.g. modified gravity) or the primordial density perturbations (e.g. inflationary cosmology, etc) can be probed by the generalised skew parameters and hence could be used to distinguish between different models.

Future Work

Future work includes the incorporation of real world effects such as redshift space distortions, galaxy bias, masking and survey effects into the formalism. The formalism presented here has only considered the skewness parameters up to $\mathcal{O}(\sigma_0)$ but higher order corrections can be considered and effectively provide a probe of the trispectrum. These next-to-leading order results are aptly named Kurt spectra due to their relation to the more conventional kurtosis parameters.

[1] Matsubara T., 2003, ApJ, 584, 1

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