Analytical and MHD Modelling of Merging Magnetic Flux Ropes

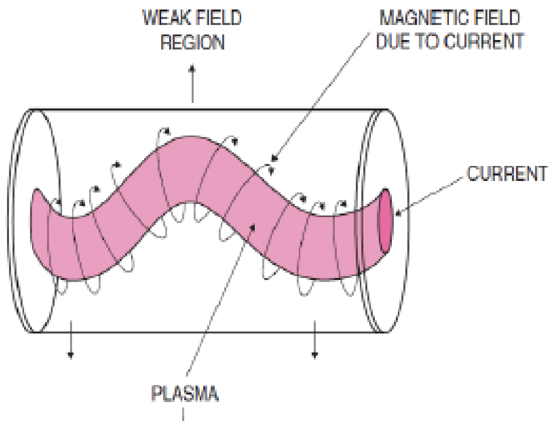
***A. S. Hussain*[1]**, P. K. Browning[2], A. W. Hood[3]

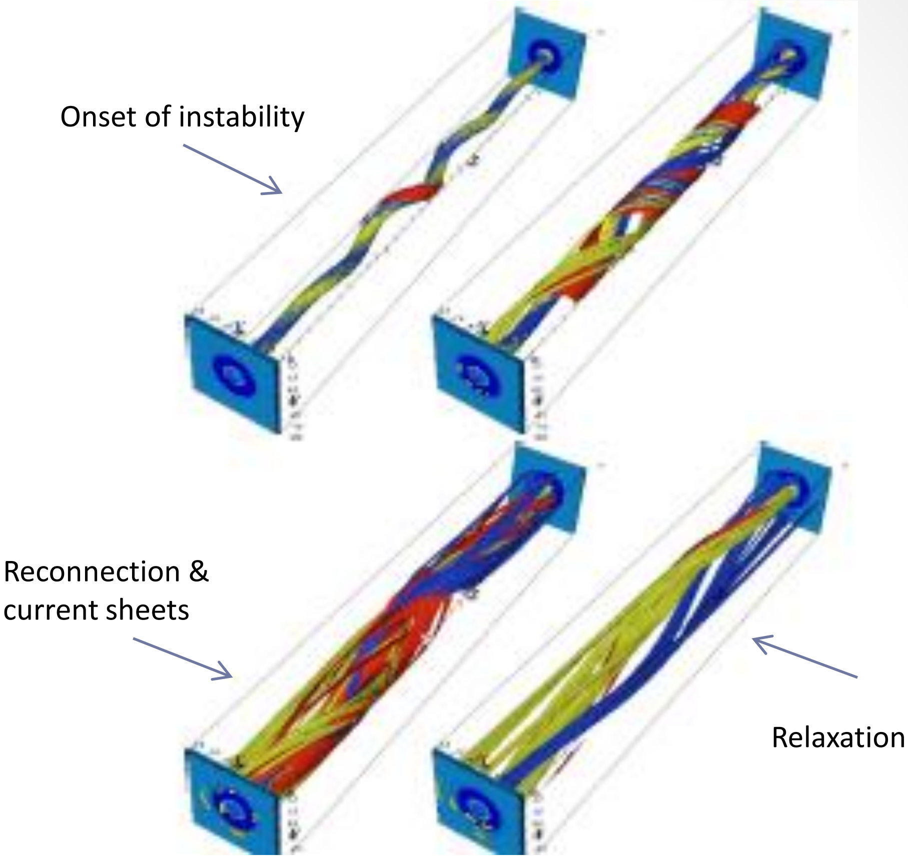
[1]School of Mechanical, Aerospace, and Civil Engineering, University of Manchester
[2]Jodrell Bank Centre for Astrophysics, School of Physics and Astronomy
[3]School of Mathematics and Statistics, University of St. Andrews

asad.hussain@postgrad.manchester.ac.uk

Alliance Meeting, Manchester 5 - 6 February 2018

Kink Instability





- Ideal plasma instability in flux rope if twist exceeds critical value in line-tied loop (Hood & Priest, 1979)
- Coronal field may be twisted by footpoint motions leading to kink instability which triggers helicity conserving relaxation (Browning and Van der Linden 2003, Browning et al 2008)
- Dissipated magnetic energy released as heat:

$$W_{heat} = W_{initial} - W_{relaxed}\ (\alpha)$$

- Coronal heating due to W_{heat} (Heyvaerts and Priest, 1983)

Kink Instability & Loop Interaction

Initial state:

Two twisted force-free cylindrical flux ropes **J** X **B** = 0

Localised field line twisting

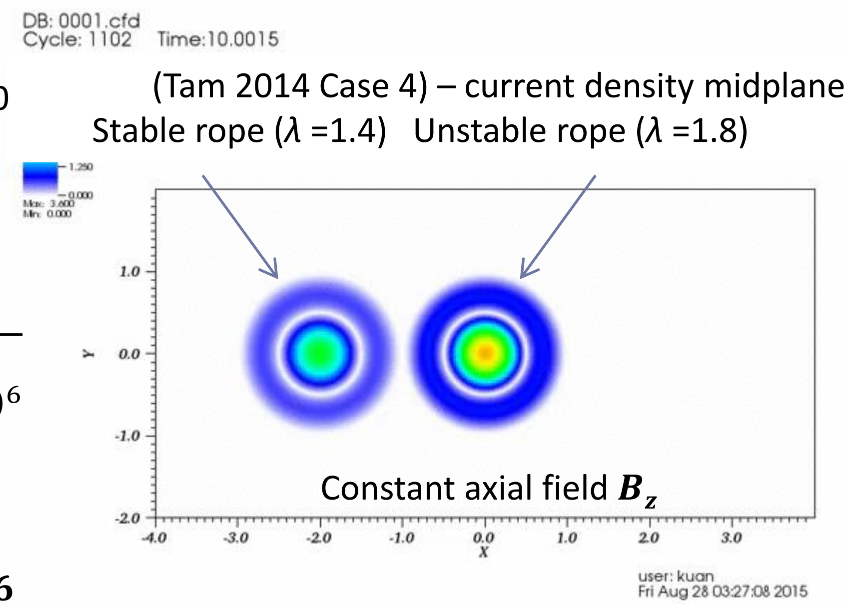
Uniform axial field between ropes

$$B_\theta = B_0 \lambda r (1 - r^2)^3$$

$$B_z = B_0 \sqrt{1 - \frac{\lambda^2}{7} + \frac{\lambda^2}{7}(1 - r^2)^7 - \lambda^2 r^2 (1 - r^2)^6}$$

$$\Phi(r = 0) = \lambda L$$

λ is a twist parameter; flux rope unstable at $\boldsymbol{\lambda = 1.6}$

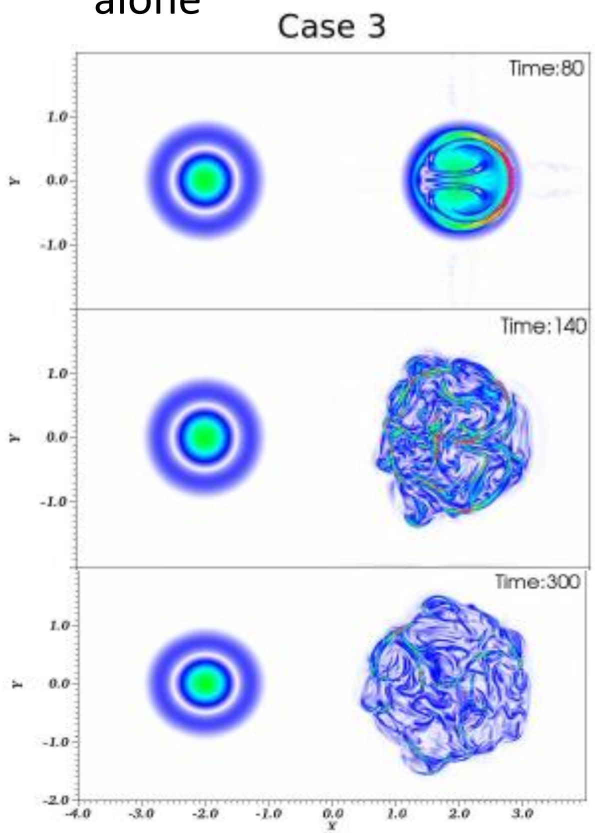


3D MHD simulations Tam et. al. (2015) :

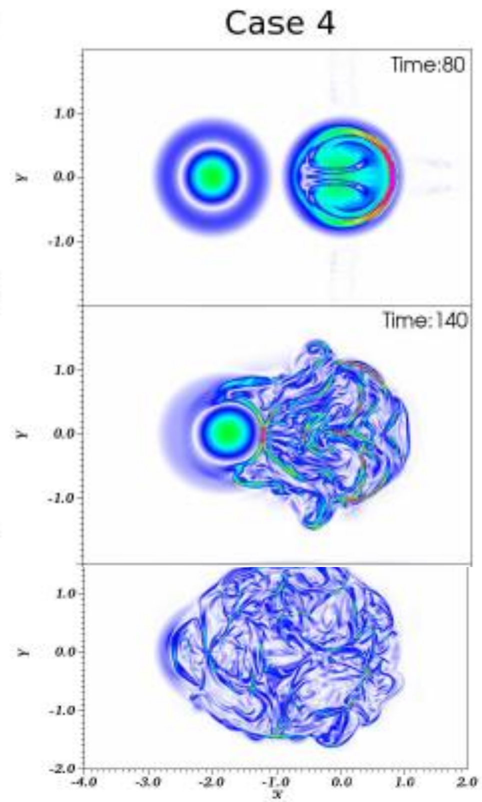
If a kink unstable rope is sufficiently close to a stable rope, it will trigger energy release in a nearby flux rope – flux ropes merge

If a kink unstable rope relaxes on its own, it yields a lower energy output than if it absorbs another flux rope.

No interaction –
unstable loop relaxes
alone



Unstable loop merges
with stable loop



Onset of kink
instability

Non linear
phase

Relaxation

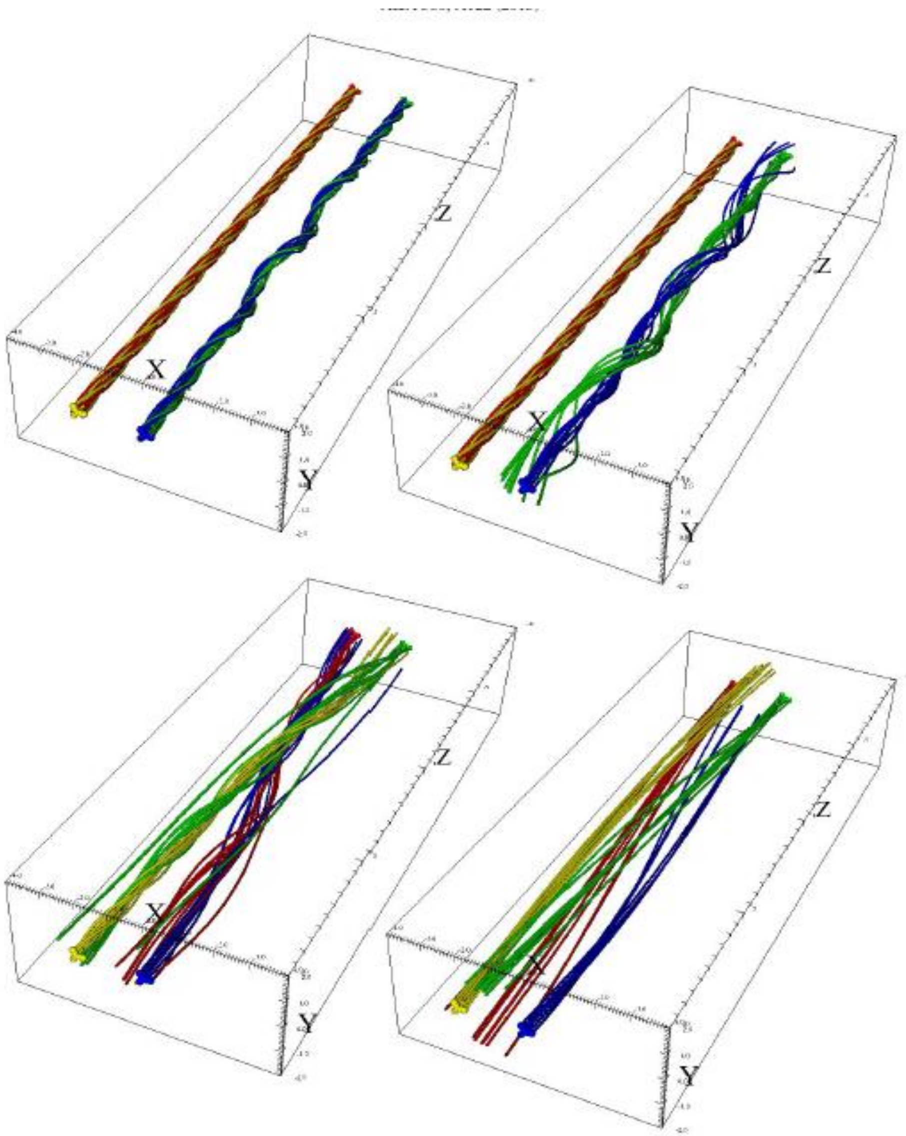
MANCHESTER 1824

The University of Manchester

MHD Simulation (Tam et. al.)

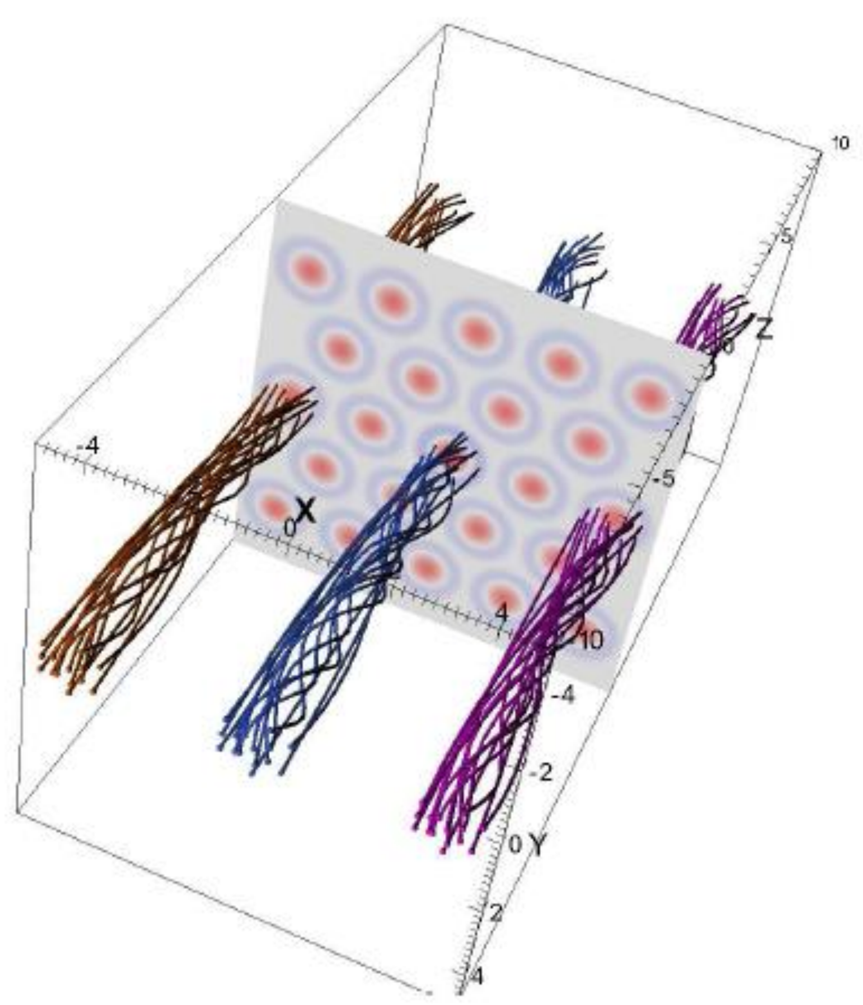
Magnetic field lines

Unstable flux rope (left)
Absorbing stable flux rope



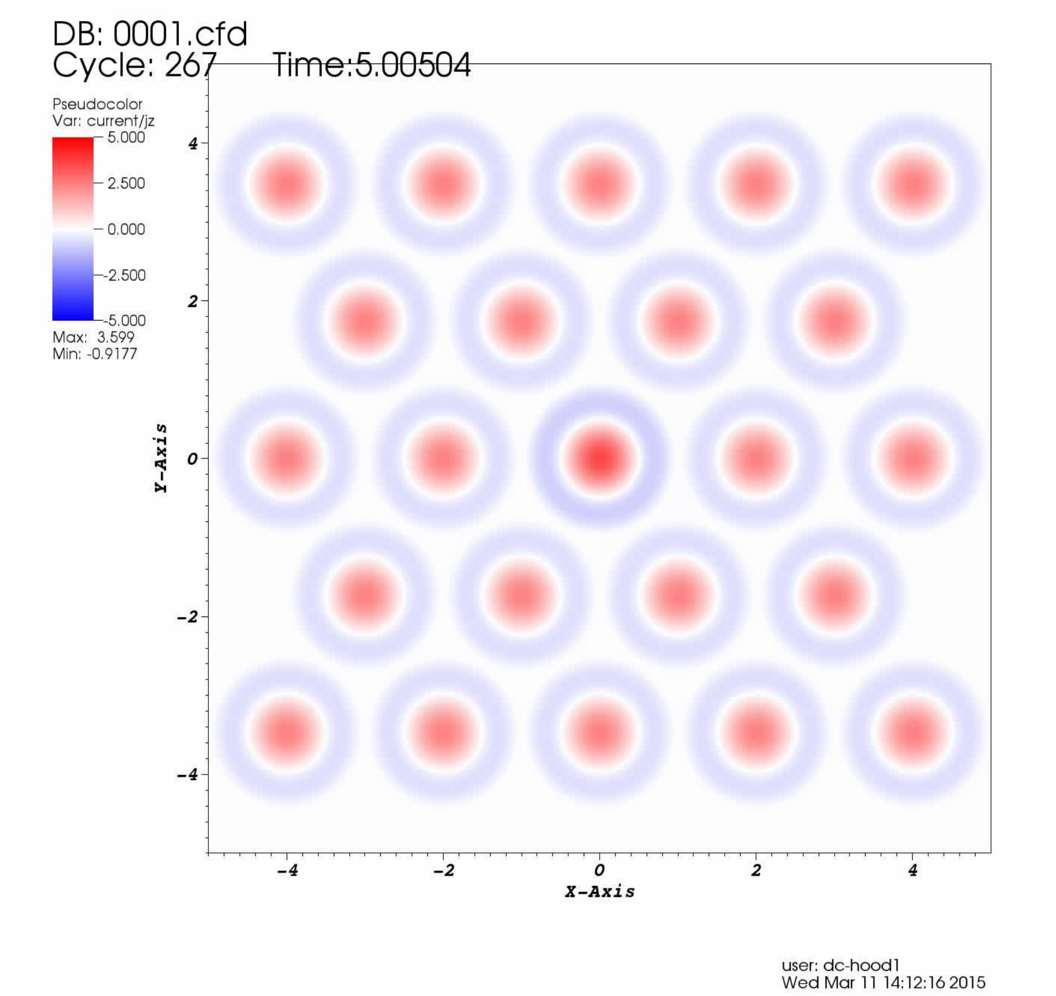
Multiple Loops (Avalanche) Sim

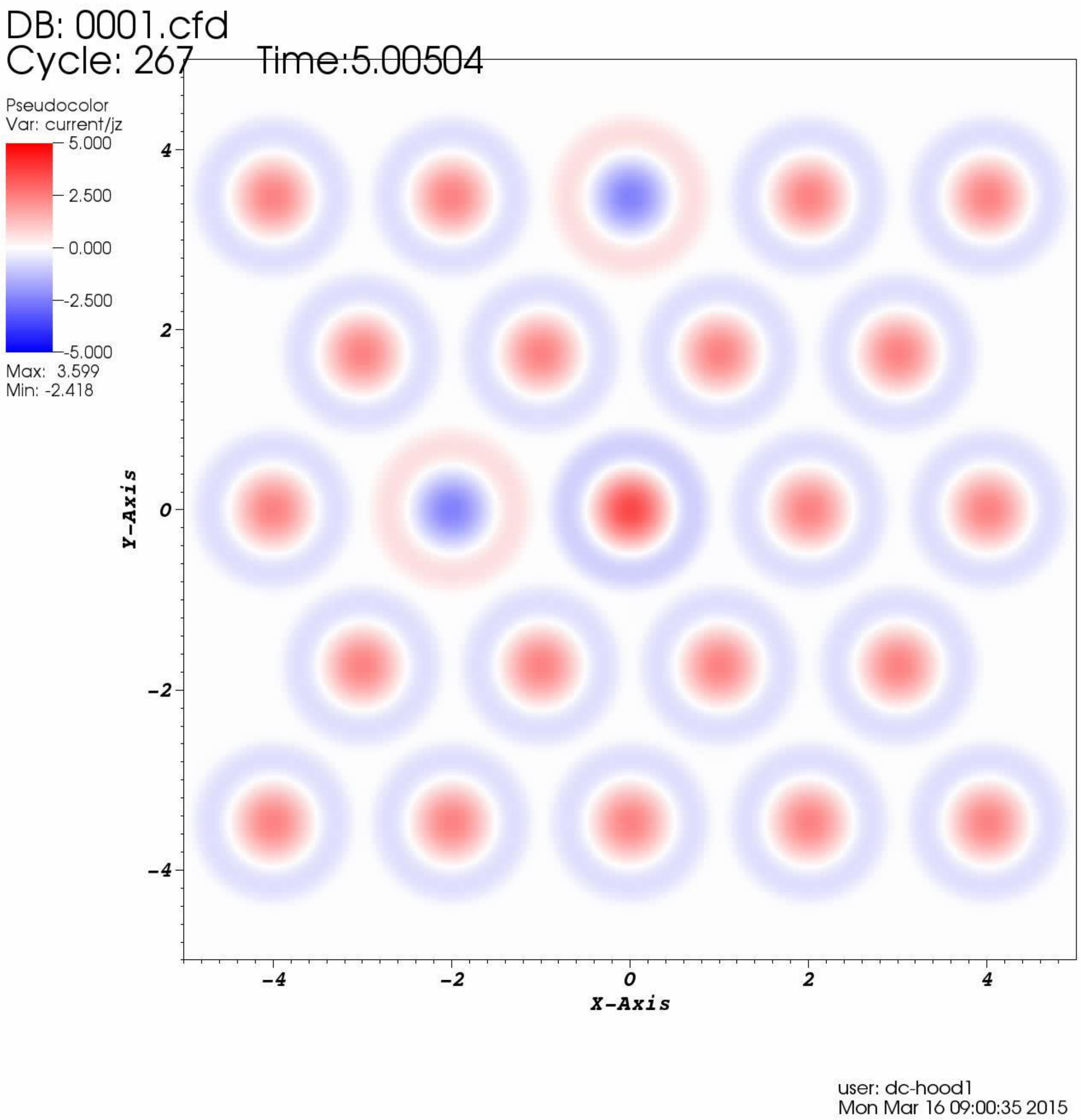
- Two flux rope model (Tam et. al, 2015) extended to consider multiple (23) flux ropes (Hood et. al, 2016).
- Simulation 1 considers all flux ropes twisted in one direction – one unstable (λ = 1.8), 22 stable (λ = 1.4)
- First demonstration using 3D MHD simulations that single kink unstable flux rope can trigger an avalanche of reconnecting flux ropes
  - (as postulated in "Cellular Automaton" Self-Organised-Criticality models e.g. Liu and Hamilton 1991)
- Simulation 2 has two oppositely-twisted (λ < 0) flux ropes
  - These partially block the avalanche, merging more slowly



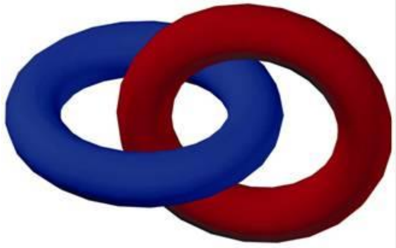
Multiple Loops (Avalanche) Simulation

Current density in midplane z = L/2




DB: 0001.cfd
Cycle: 267 Time:5.00504
Pseudocolor
Var: current/jz
5.000
2.500
0.000
-2.500
-5.000
Max: 3.599
Min: -2.418
Y-Axis
X-Axis
user: dc-hood1
Mon Mar 16 09:00:35 2015

Relaxation Theory



$$\mathbf{K} \equiv \int \mathbf{A} \cdot \mathbf{B} \, dV = 2\phi\psi$$

Minimum energy state constrained by conservation of magnetic helicity and axial flux (Taylor, 1974)

$$K = \int \boldsymbol{A}.\boldsymbol{B} dV$$

(where $\nabla \times \boldsymbol{A} = \boldsymbol{B}$)

Final state satisfies $\nabla \times \boldsymbol{B} = \alpha \boldsymbol{B}$

In cylindrical coordinates : $B_\theta = B_1 J_1(\alpha r),\ B_z = B_1 J_0(\alpha r)$

Energy release given by $W_{heat} = W_{initial}(B_0, \lambda) - W_{final}(B_1, \alpha)$

The University of Manchester

MANCHESTER 1824

Model

Initial state:

Approximate B_z to allow analytical calculation of A, K, and Φ_t

$$B_z = \sum_{n=0}^{7} C_n r^n$$

- Calculate $\boldsymbol{A}$ ($A_r = 0$)

$$A_\theta = \frac{1}{r}\int rB_z.dr$$

$$A_z = -B_0\lambda\left(\frac{r^2}{2} - \frac{3r^4}{4} + \frac{3r^6}{6} - \frac{r^8}{8}\right) + \frac{1}{8}B_0\lambda$$

- Calculate helicity and energy

$$K = 2\pi B_0\lambda L \sum_{n=0}^{7} C_n \frac{96}{(n+2)(n+4)(n+6)(n+8)(n+10)}$$

$$E = B_0^2\pi L\left(\frac{1}{2} - \frac{\lambda^2}{16}\right)$$

- Superimpose helicity and toroidal flux for multiple flux ropes

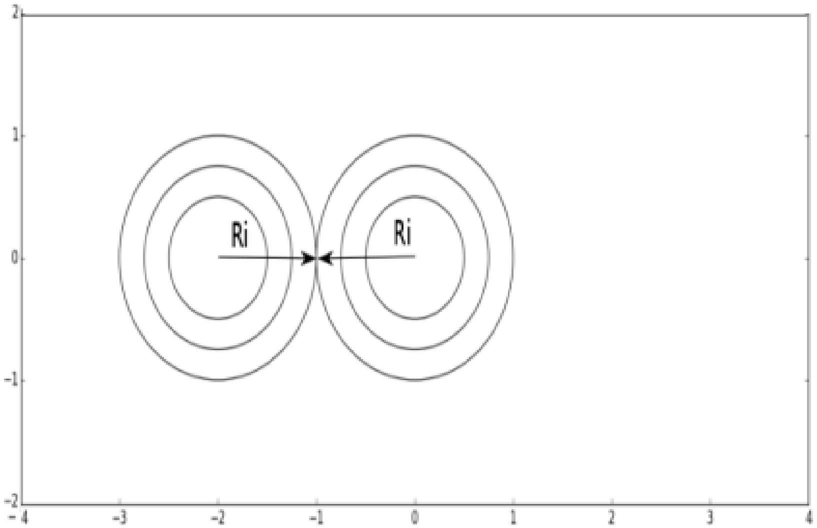
Final state:

Bessel function field with conserved helicity

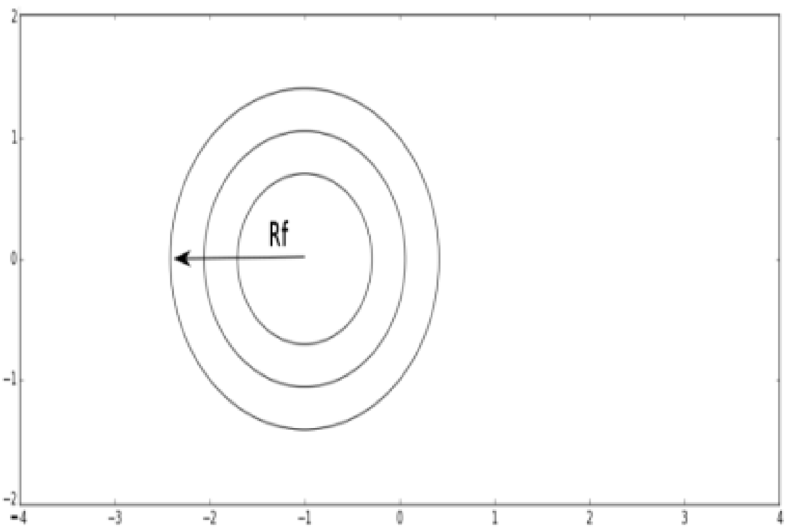
Cases Considered and Radius Restriction

Case	Conditions
Case 1	Two ropes far **apart**. Both kink **unstable**.
Case 2	Two ropes **merging**. Both kink **unstable**.
Case 3	Two ropes far **apart**. One stable, one kink **unstable**.
Case 4	Two ropes **merging**. One stable, one kink **unstable**.

3 separate approaches taken to restrict radius of final flux rope.



Initial (cases 2 & 4)



Final (cases 2 & 4)

Results

Constraint Magnetic Pressure

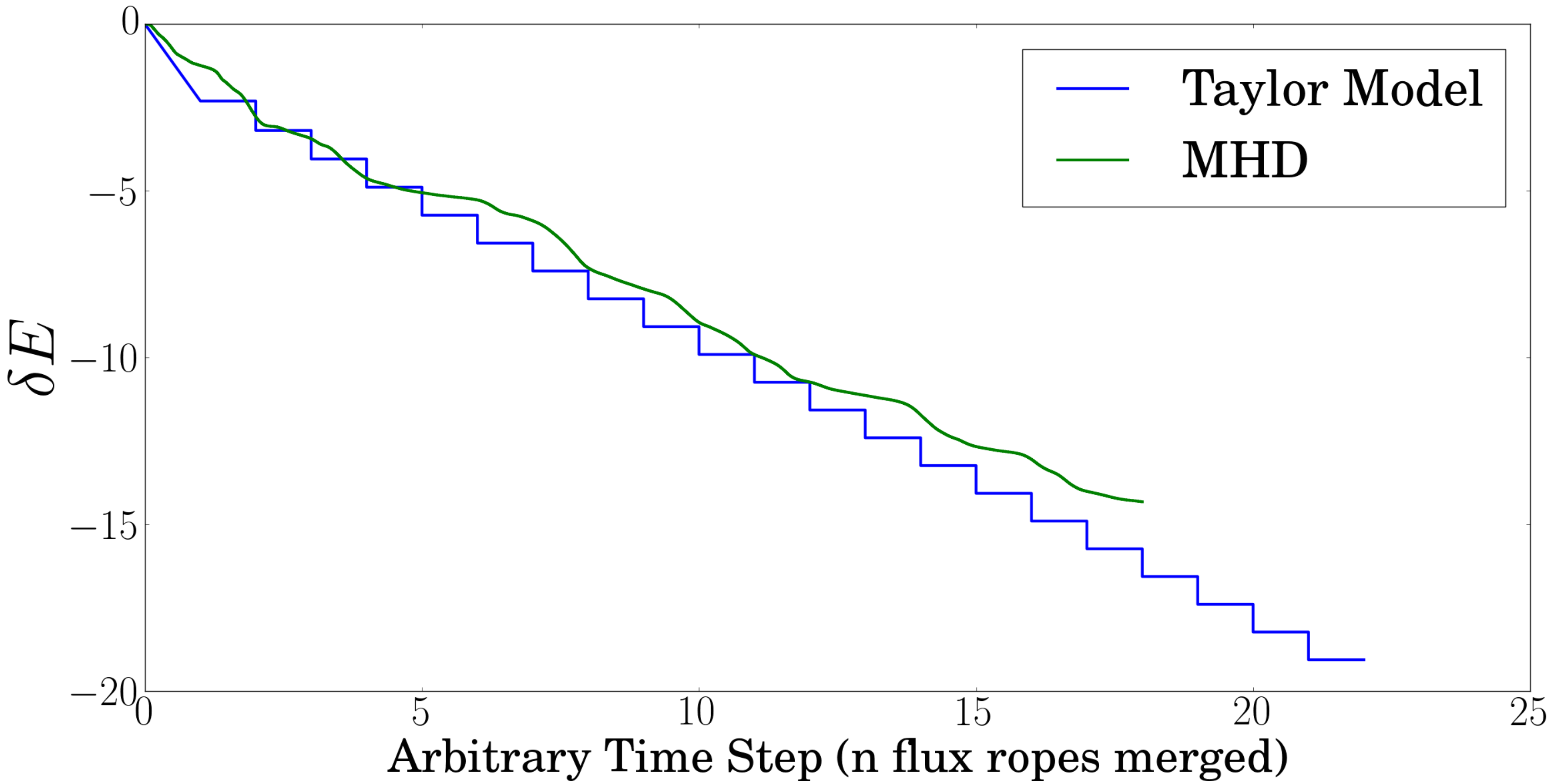
Magnetic pressure constant across flux rope boundary (Force balance)

$$B_z^2(R_f) + B_\theta^2(R_f) = B_z^2\ initial = B_0^2\left(1 - \frac{\lambda^2}{7}\right)$$

	Final Radius	ΔE_{MHD}	ΔE_{Taylor}
Case 1*	0.999 (each)	-3.031	-2.608
Case 2	1.412	-3.069	-3.164
Case 3*	0.999	-1.5	-1.304
Case 4	1.413	-2.3	-2.29

**Case 3 only considers one flux rope (in the Taylor model) resolving itself since the first rope is stable, case 1 is twice of case 3.*

Multi-loop Avalanche Modelling Results



Rules for Relaxation

- **Goal – to model energy release in loop systems with wide range of parameters (size, twist, separation etc)**
- What is the maximum distance under which an unstable flux rope will interact with a stable flux rope to result in relaxation and merger?
- Is there a minimum threshold for the twist of a stable flux rope below which it will not be triggered by a kink unstable flux rope?
- Can loops with opposite twist merge?
- At what point in an avalanche will the relaxation process no longer occur?
    - What triggers relaxation?

MHD Simulations

$$\frac{\partial \rho}{\partial t} = -\nabla.(\rho \boldsymbol{v})$$

$$\frac{\partial}{\partial t}(\rho \boldsymbol{v}) = -\nabla.(\rho \boldsymbol{v}\boldsymbol{v}) + \frac{1}{\mu_0}(\boldsymbol{\nabla} \times \boldsymbol{B}) \times \boldsymbol{B} - \nabla P + \nabla.\boldsymbol{S}$$

$$\frac{\partial \boldsymbol{B}}{\partial t} = \nabla \times (\boldsymbol{v} \times \boldsymbol{B}) - \nabla \times \left(\frac{\eta(\nabla \times \boldsymbol{B})}{\mu_0}\right)$$

$$\frac{\partial}{\partial t}(\rho) = \nabla.(\rho \boldsymbol{v}) - P\nabla.\boldsymbol{v} + \eta \boldsymbol{j}^2 + \boldsymbol{Q_{visc}}$$

Anomalous resistivity

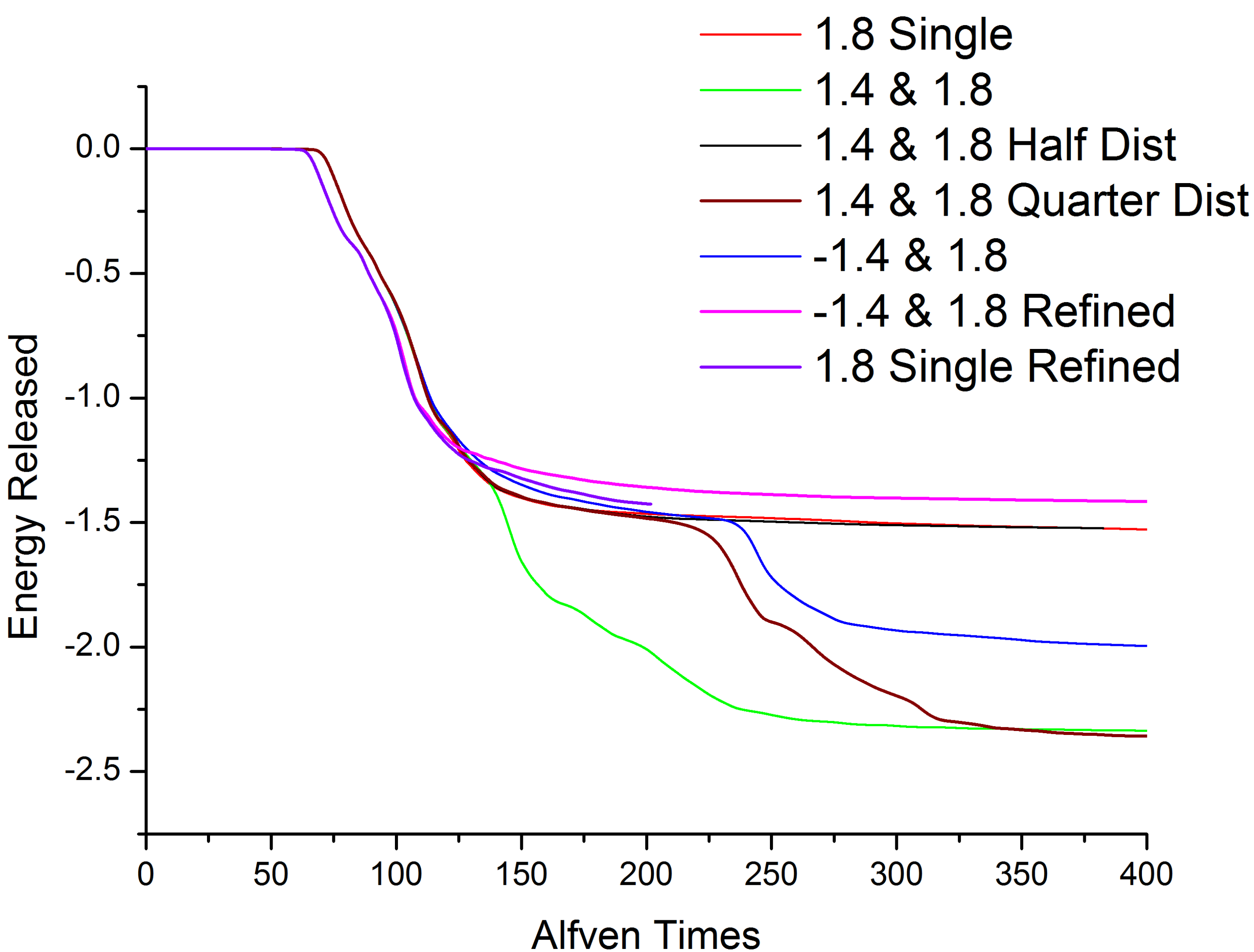
$$\eta = \begin{matrix} 0\,(j < j_{crit}) \\ \eta_0(j > j_{crit}) \end{matrix}$$

MANCHESTER 1824
The University of Manchester

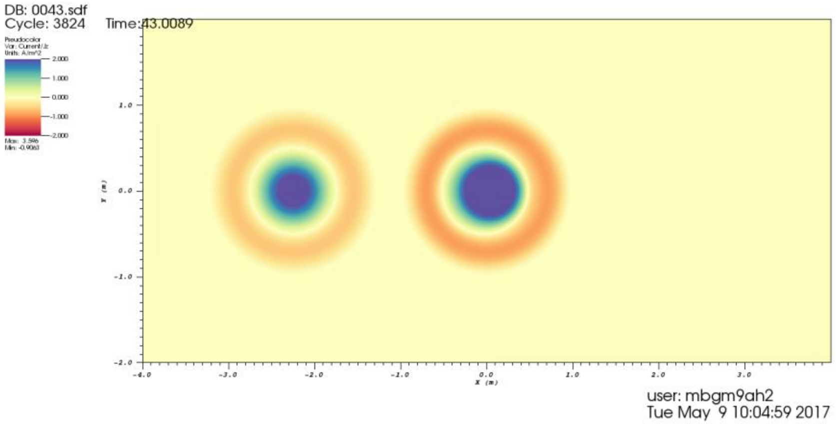
λ for various simulations

$\lambda_1 = 1.8$ unstable

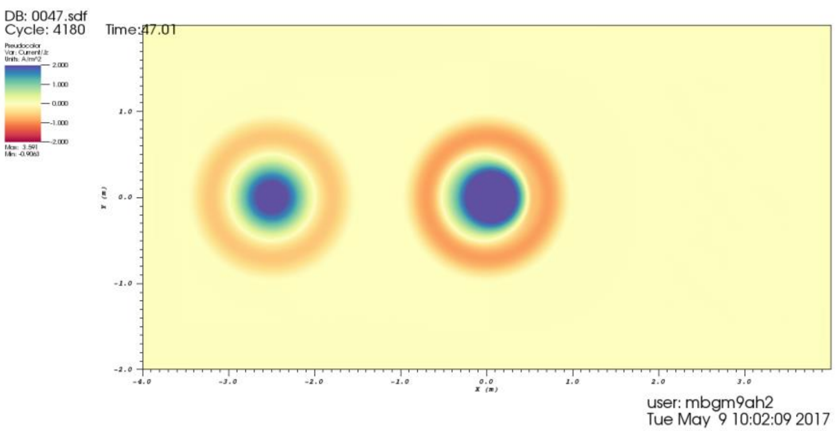
$\lambda_2 = 1.4$ stable



Distance

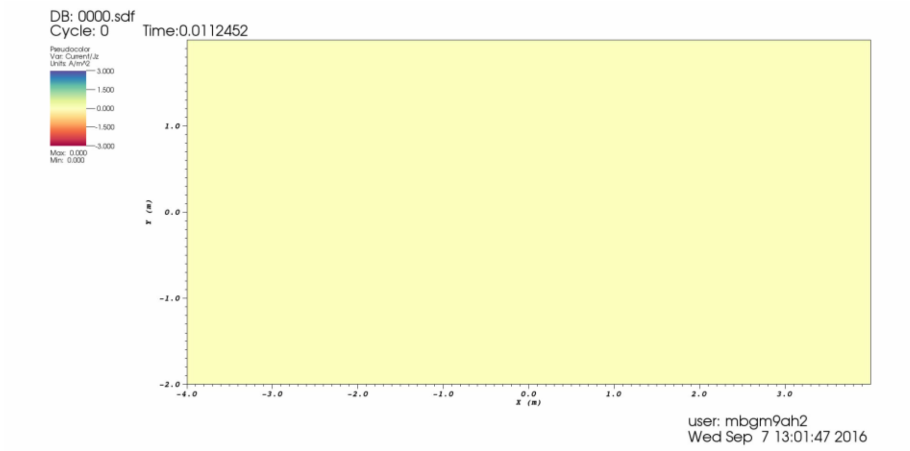


Quarter Distance

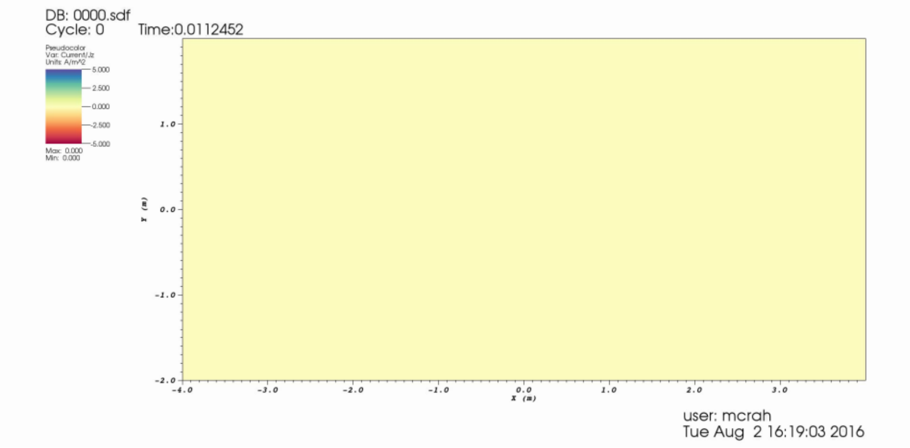


Half Distance

Distance



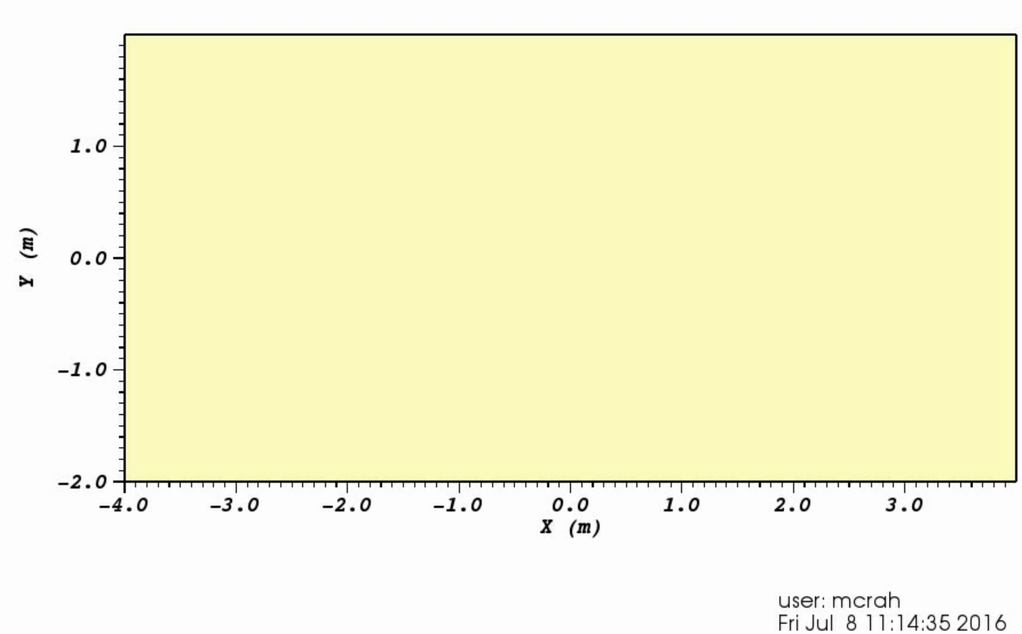
Quarter Distance

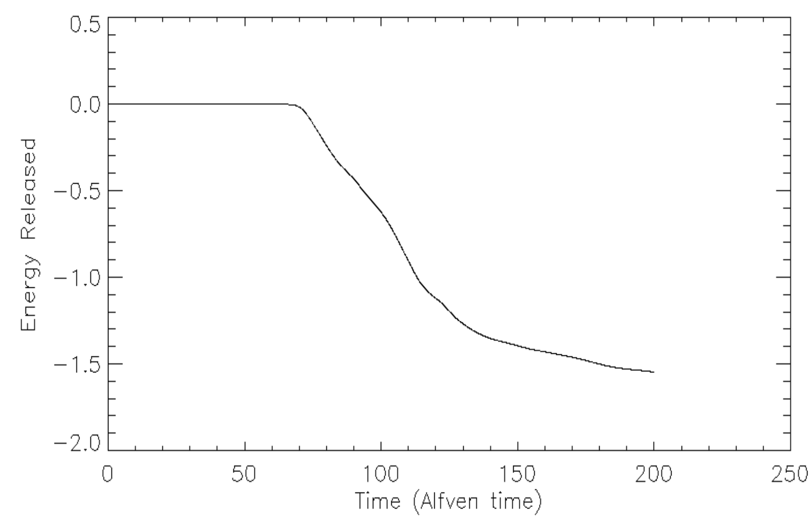


Half Distance

The University of Manchester

Minimum Twist



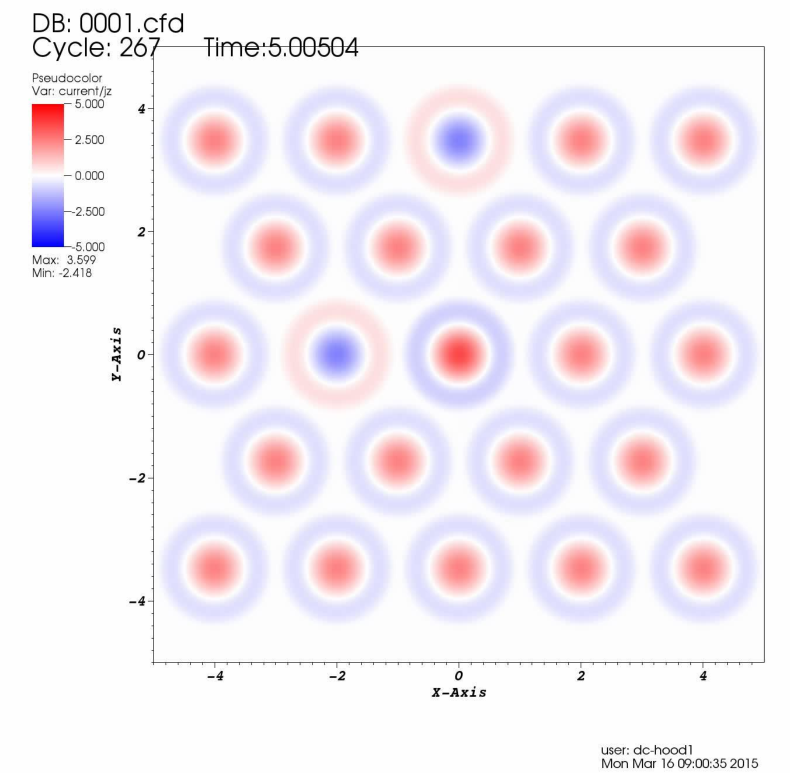


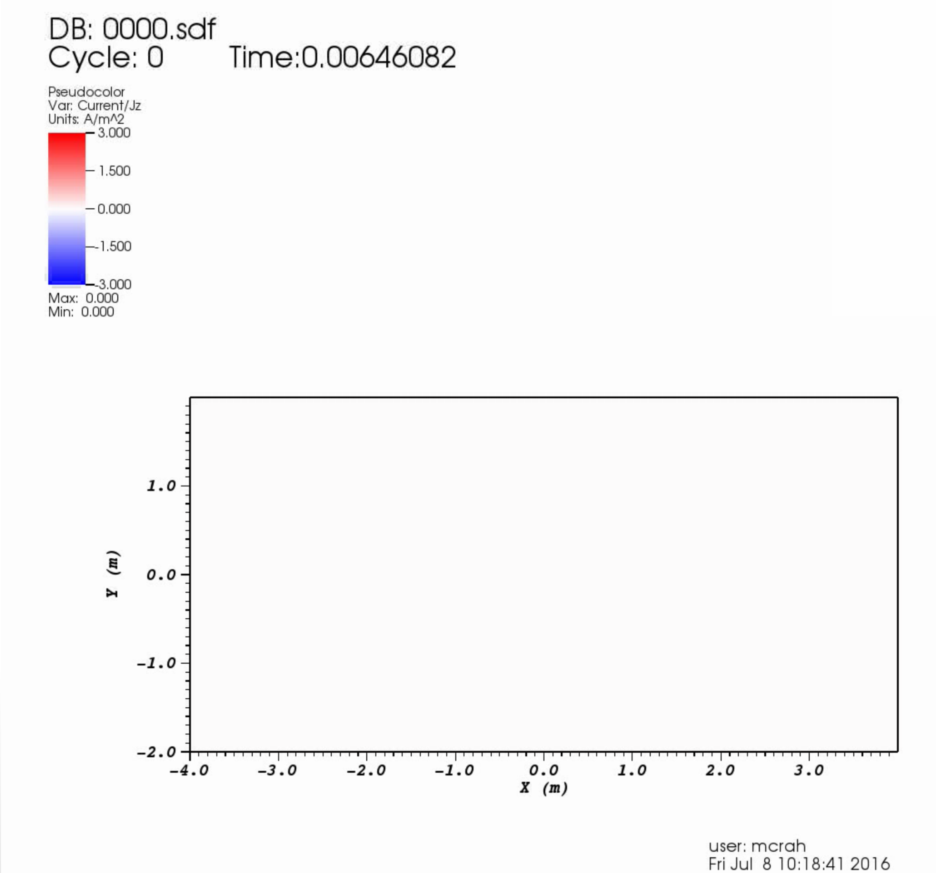
Alliance Meeting, Manchester 5 - 6 February 2018

MANCHESTER 1824

The University of Manchester

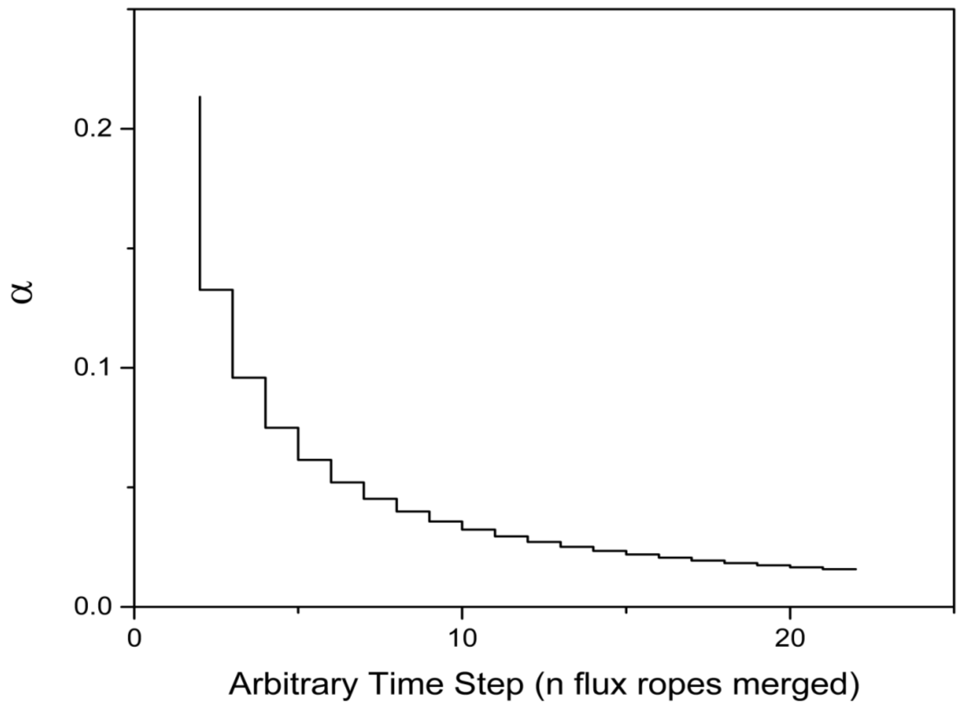
Reverse Twist



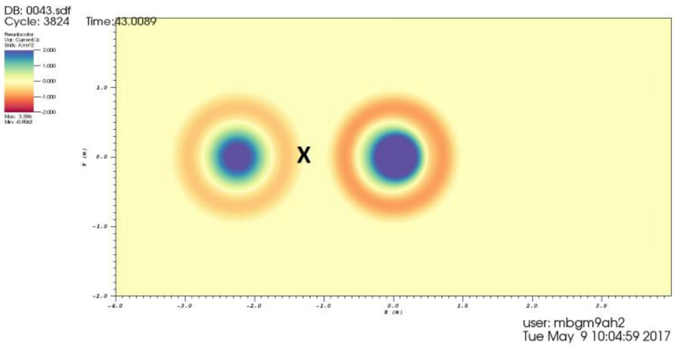


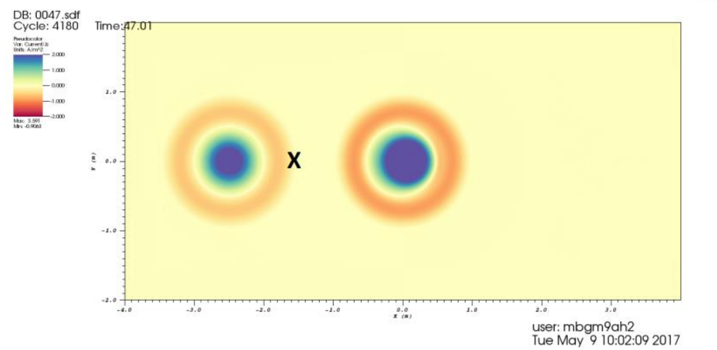
Relaxation Trigger - Current

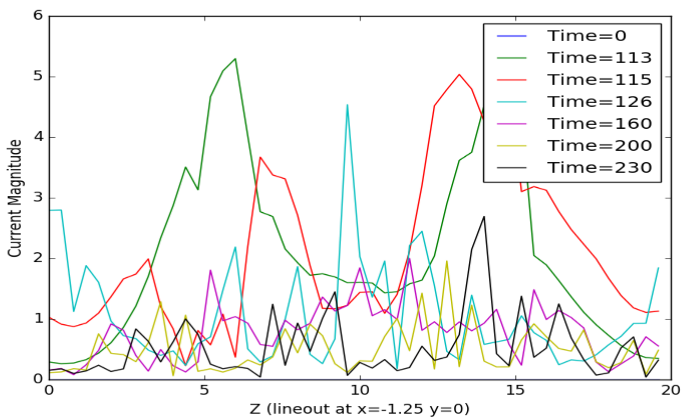
$$B_\theta = B_1 J_1(\alpha r)$$
$$B_z = B_1 J_0(\alpha r)$$
$$\nabla \times B = \alpha B$$

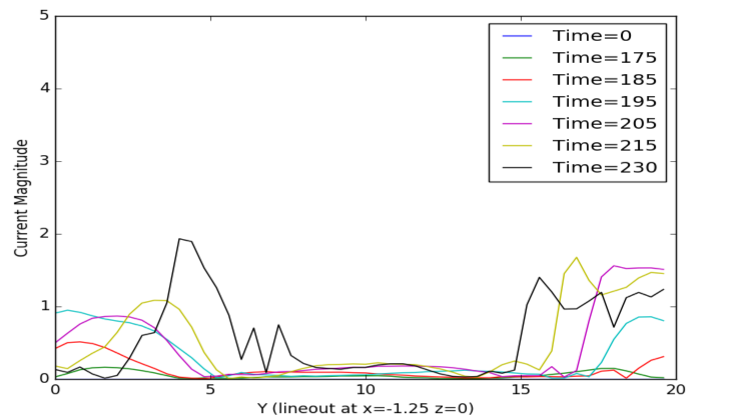


Relaxation Trigger - Current



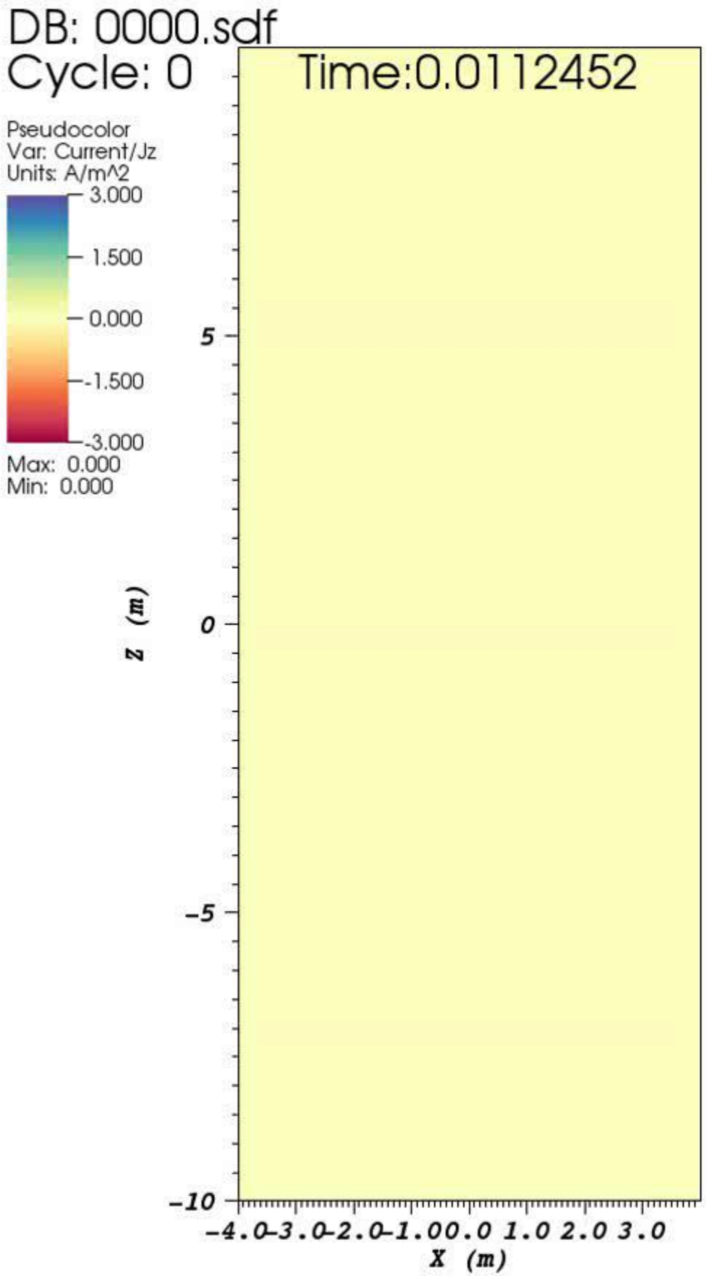




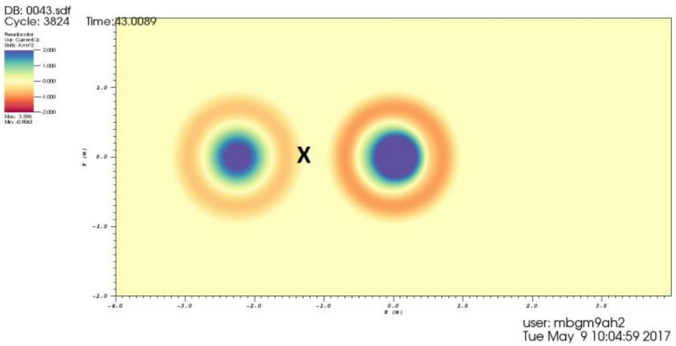


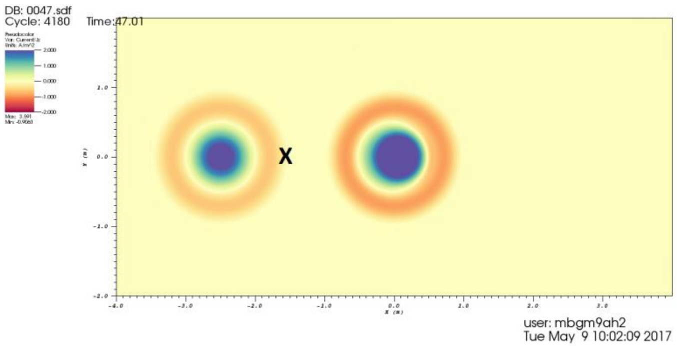
MANCHESTER 1824

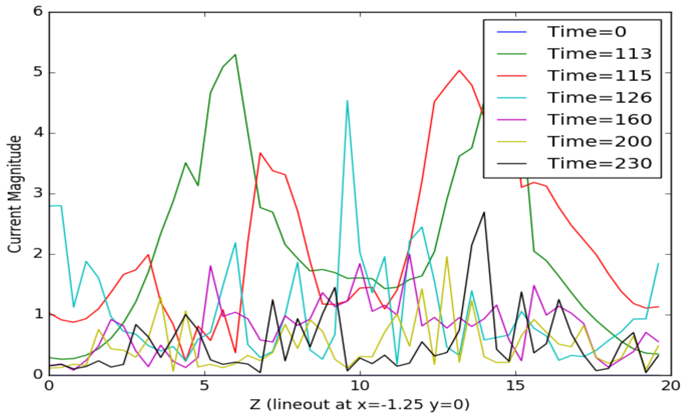
The University of Manchester

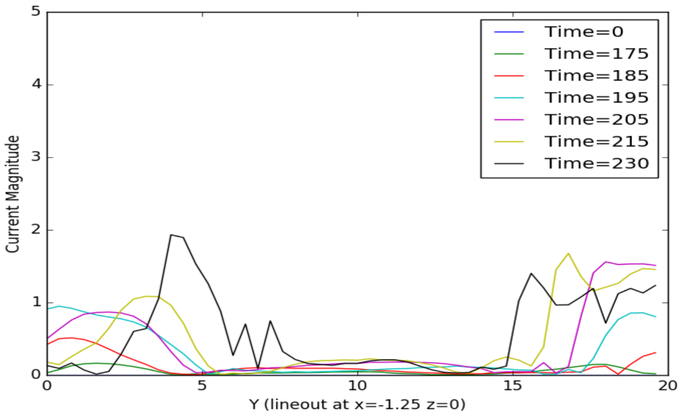


The University of Manchester

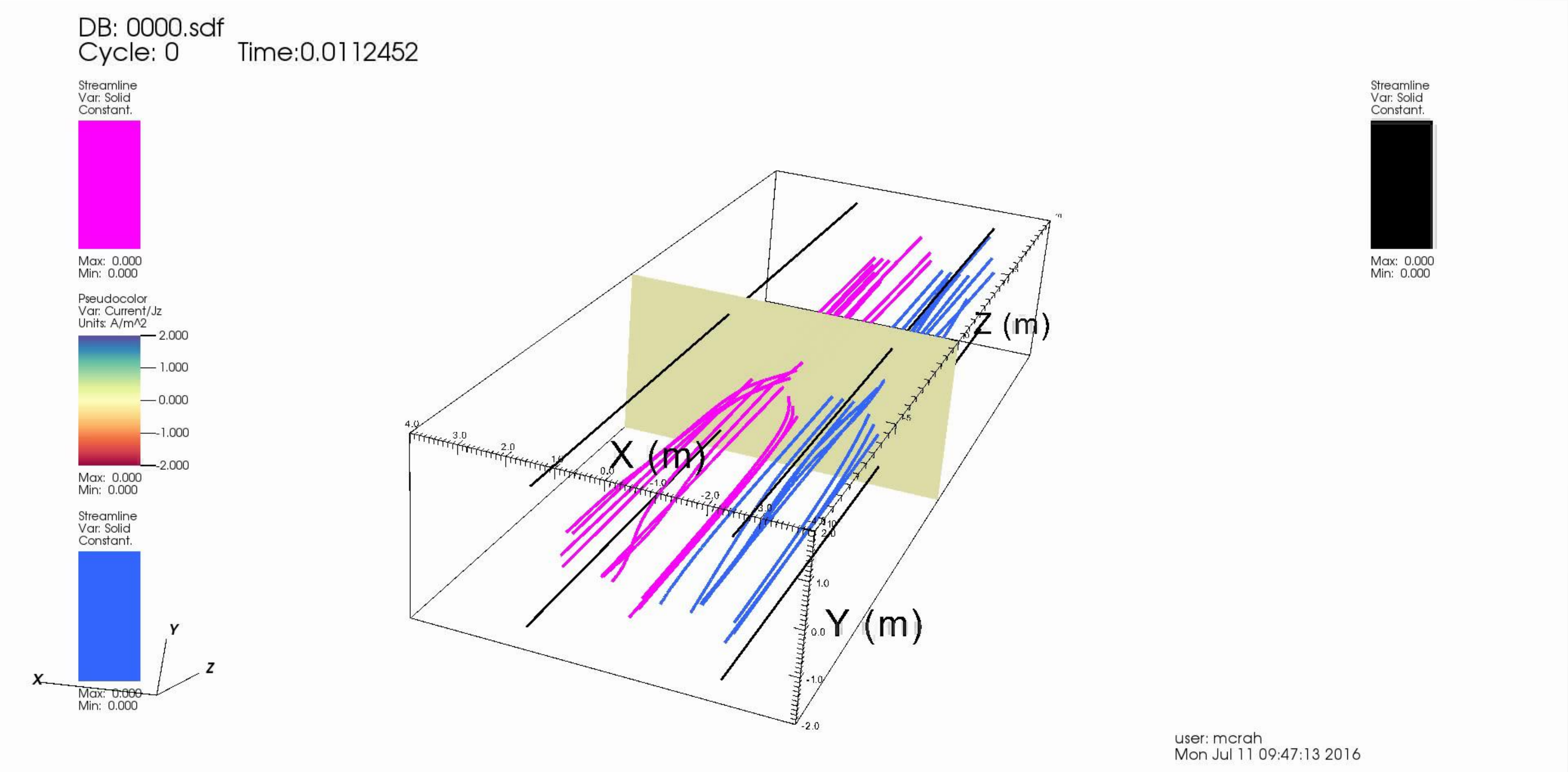




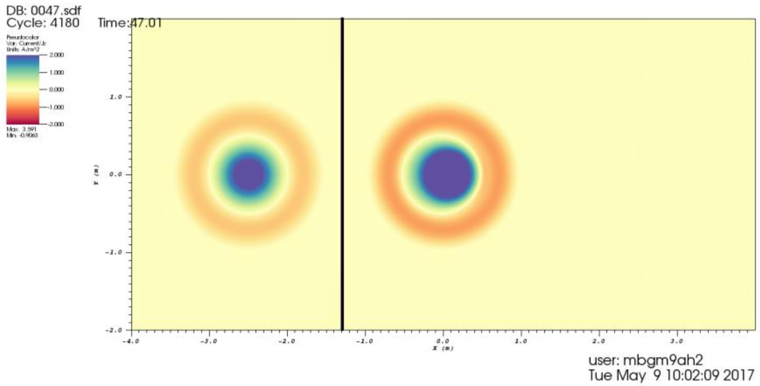


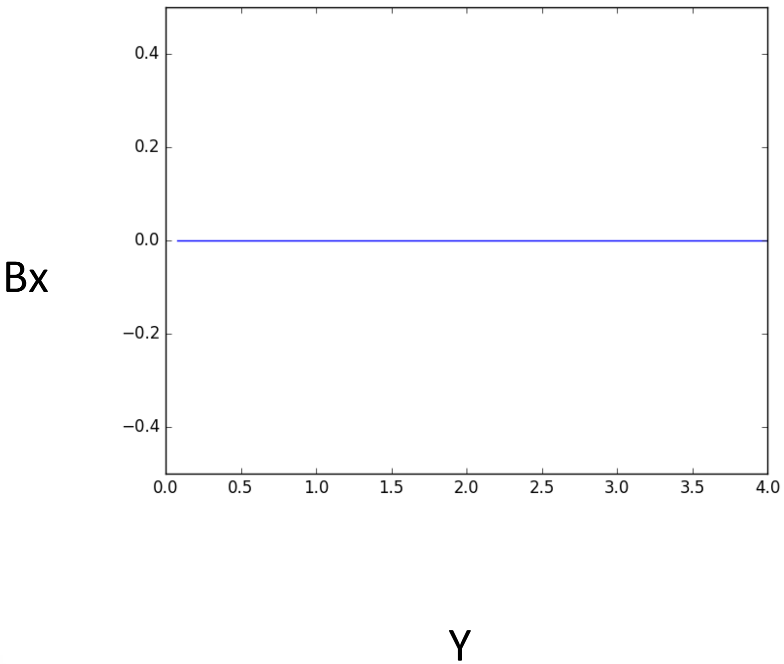


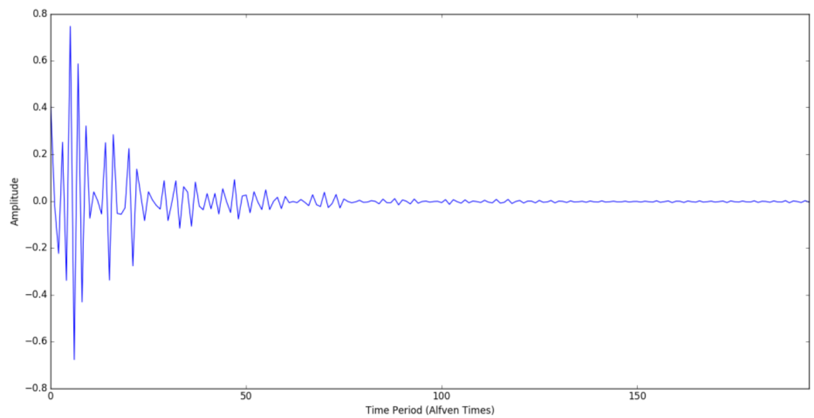
Relaxation Trigger - Wave



Relaxation Trigger - Wave



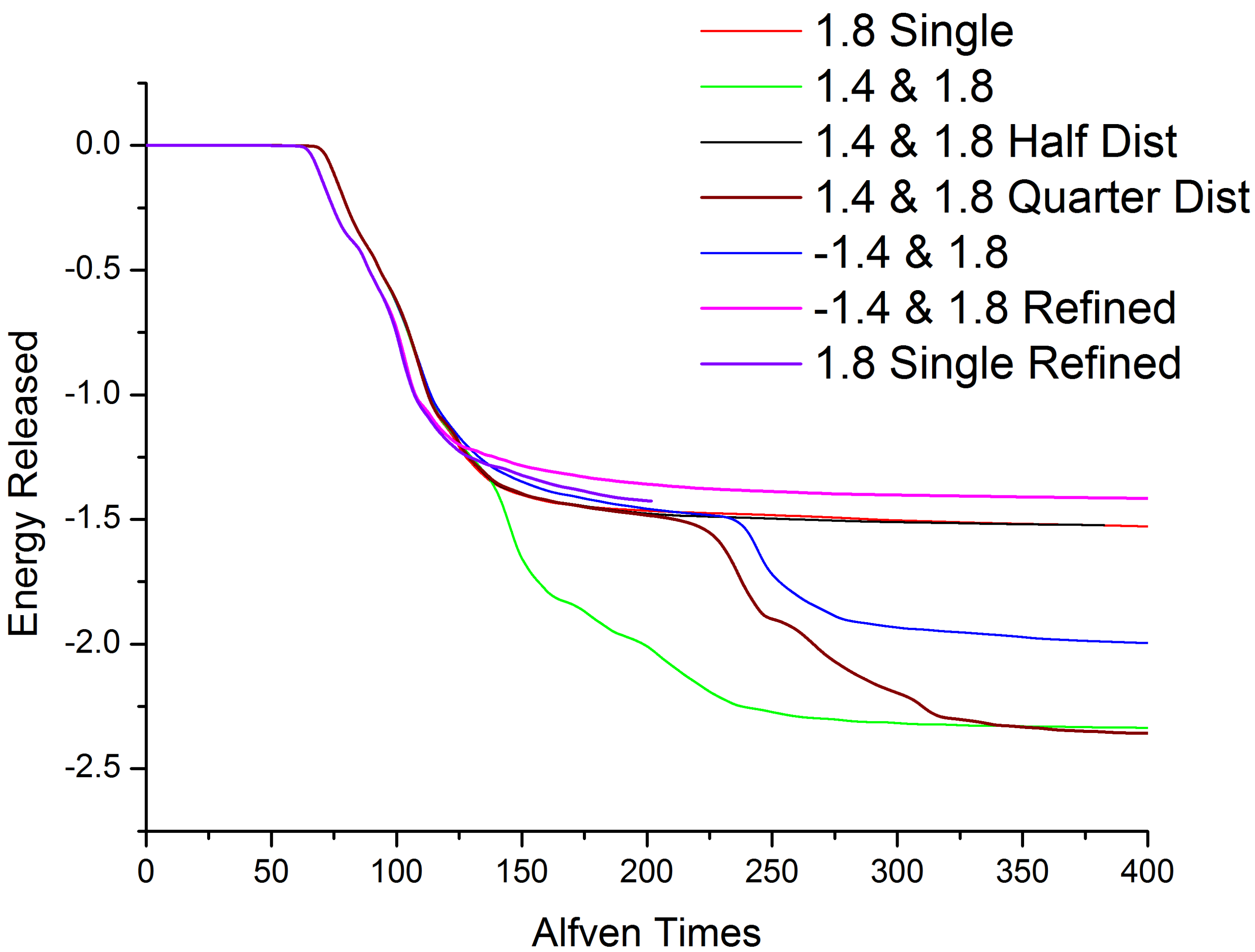




λ for various simulations

$\lambda_1 = 1.8$ *unstable*

$\lambda_2 = 1.4$ *stable*



Conclusions

- Twisted flux ropes in the solar corona may merge through magnetic reconnection, releasing stored magnetic energy and heating the solar corona
- The relaxation model appears to have good agreement with MHD simulations.
  - Kink unstable and merging flux ropes are well described by relaxation theory
- The model is not computationally intensive, allowing possibilities for modelling wider parameter spaces, larger regions of the solar corona, and varied twist profiles due to photospheric footpoint motion.
- We have established conditions under which flux ropes merge.