4. Mass distributions in galaxies

Modelling gravitational lenses

Once the lens galaxies have been identified, they can be put to the use of working out the mass distributions in the lensing galaxy. This is very difficult to do using methods based on imaging the light, because one can never be sure that the dark matter distribution (which contains most of the mass) is well traced by the light distribution. For nearby galaxies, one can study the dynamics of galaxies in detail and use the gravitational effect of matter on large-scale motions in the galaxy to work out the mass of the galaxy and the distribution of the mass. In principle, gravitational lensing allows us to study masses of galaxies at large distances, up to redshifts of 0.5 or more.

The mass distributions are studied using parameterised models such as the isothermal ellipsoid model introduced in the last two sections. Radio and optical observations are first made to determine the positions and fluxes of the images, and the parameters of the model are then adjusted to give the best fit to the observations and using the additional constraint that the parity of each image must be reproduced correctly. From this we can determine the mass of the galaxy (strictly the mass within the radius of the Einstein ring) and some information about the mass distribution. Recent results have begun to give some information about mass distributions, and in particular have shown that the singular isothermal ellipsoid is a reasonable approximation although the radial distribution of mass must be varied in a minor way in some cases. Recall that the surface mass density of the isothermal model falls off as r^{-1}; typical deviations in the power-law index of 0.1-0.2 from this dependence are seen in lenses which can be well modelled.

Modelling gravitational lenses is, however, not straightforward. For a good model we must reproduce the positions and fluxes of the lensed images, together with the observed position of the lensing galaxy. To do this, we adjust parameters of the model, such as the galaxy mass, its orientation and ellipticity, and some parameters to describe how the mass falls off with radius. Table 2 shows a summary of the parameters and constraints for a four-image lens system.

Table 2. Parameters and constraints for fitting a lens mass model to a 4-image lens system, assuming the lens galaxy position is known. Each image position gives two constraints (an offset from the galaxy in the east-west direction and the north-south direction). Note that the true source position and flux is not known, and this must be fitted by the model.

Constraints	Parameters
Image positions (4x2)	Galaxy mass
Image fluxes (4)	Galaxy ellipticity
.	Galaxy orientation
.	Galaxy radial mass distribution
.	Source position (2)
.	Source flux
TOTAL: 12	**TOTAL: 7**

In any modelling procedure, the important quantity is the number of "degrees of freedom", that is, the difference between the number of constraints and the number of free parameters in the model. Obviously, the higher the number of observational constraints the better, as more parameters can then be used which may give more complete models which convey more information about the system.

> 1. How many degrees of freedom has a model of a 2-image lens where the position of the lensing galaxy is known?

With the five degrees of freedom obtainable from a 4-image lens system we might hope to obtain a good model. However, there are a number of problems:

- we would like more information about the radial mass distribution, which would cost extra free parameters
- most lens galaxies have near neighbours, which costs extra free parameters to include them in the model
- the galaxy radial mass distribution, which we would really like to know, is not very well constrained because in a 4-image lens system all of the images lie at approximately the same distance from the galaxy centre.

The ideal lens for deriving galaxy mass distributions, therefore, is rather special. One extra useful constraint is any Einstein ring present in the system. Another is more than four images. Some CLASS lensing galaxies have multiple background sources, and in one case (1933+503; see figure 3.5) the system consists of three background sources, two of them quadruply imaged and one doubly imaged.

What have we learned?

The current observational situation can be summed up as follows:

- Nearly all gravitational lens systems so far observed contain massive elliptical galaxies as lenses. A few lenses appear to be spiral galaxies. The predominance of elliptical galaxies is expected as these are usually more massive than spirals.
- The lenses so far observed have all shown normal, or nearly-normal, levels of optical light. This means that galaxies which are very dark because they never formed any stars probably do not exist in significant numbers. Like other galaxies, gravitational lens galaxies are likely to contain dark, non-luminous matter (more discussion of this point in section 6).
- The lack of odd (third or fifth) images turns out to be a useful constraint. This lack turns out to be a useful lower limit on the central surface mass density. Radio observations are ongoing to try and detect third images, and make this limit stronger.
- The mass distribution in most lenses where good constraints have been obtained appears to be close to an isothermal distribution. Some authors have experimented with more sophisticated models which can also be made to fit the data, although the number of lenses with sufficiently good constraints for this type of modelling is small.

> 2. Do you think spiral galaxy lenses are more likely to be seen with the disk face-on or edge-on, and why?

In the next section we will look at how gravitational lenses can be used as an independent measure of the value of Hubble's constant.

Answers to questions

1. How many degrees of freedom has a model of a 2-image lens where the position of the lensing galaxy is known?

Answer to question

Typically 0!

There are four positional constraints (two from each image) and two flux constraints. The free parameters are typically the source position (2) and flux, the galaxy mass, ellipticity and position angle. There are thus typically zero degrees of freedom, and the position becomes worse if further information about the galaxy mass distribution is required.

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2. Do you think spiral galaxy lenses are more likely to be seen with the disk face-on or edge-on, and why?

Answer to question

Edge-on.

The important parameter for gravitational lensing is the surface mass density (SMD), that is, the mass per unit area along the line of sight. A flattened distribution such as the disk of a spiral galaxy will be a more effective lens if it is seen edge-on, as this will give the maximum SMD for a given mass. In fact in the CLASS survey, we have discovered one edge-on and one face-on system.