Asymmetric Planetary Nebulae 5
(Ebrary, Cambridge, USA),
A.A Zijlstra, I. McDonald, eds.

The evolution of shapes in ballistic non-homologous expansion in PNe

Wolfgang Steffen

Instituto de Astronomía, UNAM, Km. 103 Carr. Tijuana-Ensenada, 22860 Ensenada, B.C. México

S. Martínez, J.A. López

We have performed analytical and semi-analytical calculations to study the changes of shape in ballistic non-homologously expanding planetary nebulae. Excluding all other dynamical effects that influence the evolving shape, we find that the changes due to deviations from homologous expansion have simple asymptotic limits, since these objects eventually always evolve towards homologous expansion. We define a parameter to describe the overal deviation from sphericity and what its final value is for a given initial value and velocity law. We show typical examples of initial shapes and their evolution for various expansion laws.

The evolution of shapes in ballistic non-homologous expansion in PNe

S. Martínez[1], W. Steffen[2], J.A. López[2]

1- Instituto Nacional de Astrofísica, Óptica y Electrónica, Puebla, México
2- Instituto de Astronomía, Universidad Nacional Autónoma de México, Ensenada, México

Abstract.
We have performed analytical and semi-analytical calculations to study the changes of shape in ballistic non-homologously expanding planetary nebulae. We find that the changes due to deviations from homologous expansion have simple asymptotic limits, since these objects eventually evolve towards homologous expansion. We define a parameter to describe the overall deviation from sphericity and what its final value is for a given initial value and velocity law. We show typical examples of initial shapes and their evolution for various expansion laws.

Analytical Calculations.

We assume the initial expansion is ballistic, but has a magnitude that changes non-linearly with distance and/or is non-radial in direction. The time evolution of the nebula outline is then given by

$$r_{(\phi,t)} = r_{(\phi,0)} + v_{r(\phi,0)}\, t$$

Where the vector quantities *r* and *v* are defined in Fig. 1.

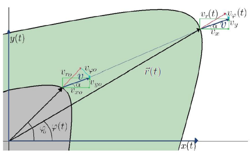

Figure 1. Fundamental quantities.

From eq. (1), normalized to the maximum distance from the center, in the limit of infinite time we get in terms of initial velocities v0 and vx:

(2) $$\lim_{t\to\infty} \frac{r(t)}{r_x(t)} = \frac{v_o}{v_x}$$

Where we have omitted the dependence on ϕ has been omitted. So the shape converges and depends only on the initial velocities (see Figure 2). The poloidal component is

(3) $$v_\varphi(t) = v_{xo} sen\varphi(t) - v_{yo}\, cos\varphi(t)$$

which quickly tends to zero for t→∞, such that the expansion soon becomes quasi-homologous (see Figure 2).

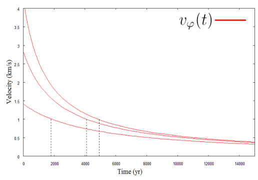

Fig. 2. The poloidal velocity components goes to zero with time, quickly reaching undetectability (limits marked) depending on its initial value if the velocity resolution of observations is low.

We define a dimensionless parameter, the relative radial extent ϵ(t), to measure the degree of "sphericity" (0≤ϵ(t)≤1) depending on the maximum and minimum distances to the center of the nebula; ϵ(t)=1 describes a perfectly spherical nebula

(4) $$\epsilon(t) = 1 - \frac{r_x(t) - r_n(t)}{r_x(t)}$$

Assuming a velocity field of the following form

(5) $$v_r(\phi) = v_0 + k\left(\frac{r(\phi,0)}{r_x}\right)^n$$

where *k* and *n* are constants, the sphericity can be expressed as

(6) $$\epsilon(t) = 1 - \frac{\epsilon_o + (1-\rho^n)(1-\gamma)\tau}{1+\tau}$$

in terms of the dimensionless parameters

(7) $$\rho = \frac{r_n}{r_x} \qquad \tau = \frac{t v_x}{r_x}$$
$$\gamma = \frac{v_0}{v_x} \qquad \kappa = \frac{k}{v_x} = 1-\gamma$$

Finally, the relative radial extent has a well defined limit

(8) $$\lim_{\tau\to\infty} \epsilon(\tau) = 1 - (1-\rho^n)(1-\gamma)$$

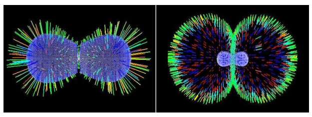

Fig. 3. Left.- Initial Shape of the planetary nebula showing non-radial velocity vectors. Right.- Final shape embedding the initial one showing quasi-radial velocity vectors; this indicates the expansion has already become homologous.

Examples

We now apply this analysis to different initial outlines of pPNs. In Figures (4) to (6) we show the expansion of different nebula structures. These plane 2D nebula outlines have been generated using the following parametric description

(9) $$\frac{r_{(\phi,0)}}{r_x} = a - b\,|\cos(h\,\phi + p)|^s\ |\cos(c\,\phi + u)|^q$$

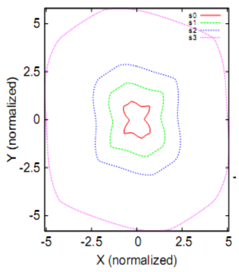

Fig. 4. Evolution of the 2D nebula structures PN1.

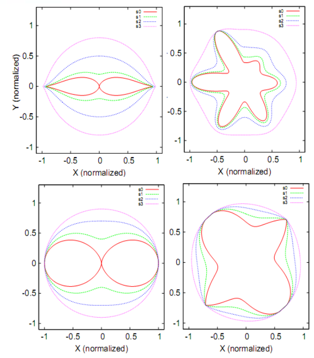

Figure 5. Evolution of different 2D nebula structures. Here the expanding shape is normalized to the maximum radius, such that the change in shape is easier to see.

"Shape" Models

We simulated the expansion of four basic 3D morpho-kinematical models for pPNs: ellipsoidal, cigar, bipolar and quadrupolar with the software Shape (Steffen et al., 2010) to investigate the final shape they reach when they start with three different velocity fields.

The velocity fields are:
Type A: radial velocity direction with magnitude gradients that increase with distance (e.g. when the PN is embedded in an environment with a strong density gradient).
Type B: radial direction with linear increase of magnitude and significant minimum extrapolated to the center.
Type C: vector directed perpendicular to the PN's surface with linear increase in magnitude.

Figure 6 shows that the sphericity ε is strongly dependent on orientation and can increase of decrease with time, depending on the initial velocity vector field.

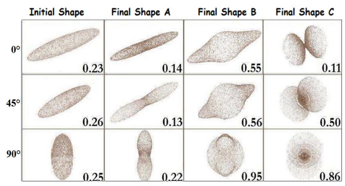

Figure 6. Initial and final shapes for our *cigar* nebula model obtained by applying the three different expansion laws. Note that type C expansion transforms the cigar nebula into a typical bipolar nebula.

References.
- Hrivnak; Smith; Su; Sahai, 2008. ApJ, 327-343
- Martínez, 2009, BSc Thesis.
- Steffen et al., 2010, IEEE Transactions of Visualization and Computer Graphics, in press.

"Get your ideas in *Shape*."
http://www.astrosen.unam.mx/shape

Funding: CONACYT grant 49447