Class Example

- How many years before planetary nebula ejecta travelling at 10 kms^{-1} reaches other stars at distances of ~1 pc?

$$t = \frac{d}{v} = \frac{3 \times 10^{16}}{10 \times 10^{3}}$$

$$= 3 \times 10^{12} \text{ s} = 10^{5} \text{ years}$$

- Short compared to MS lifetime of stars

Stellar Masses

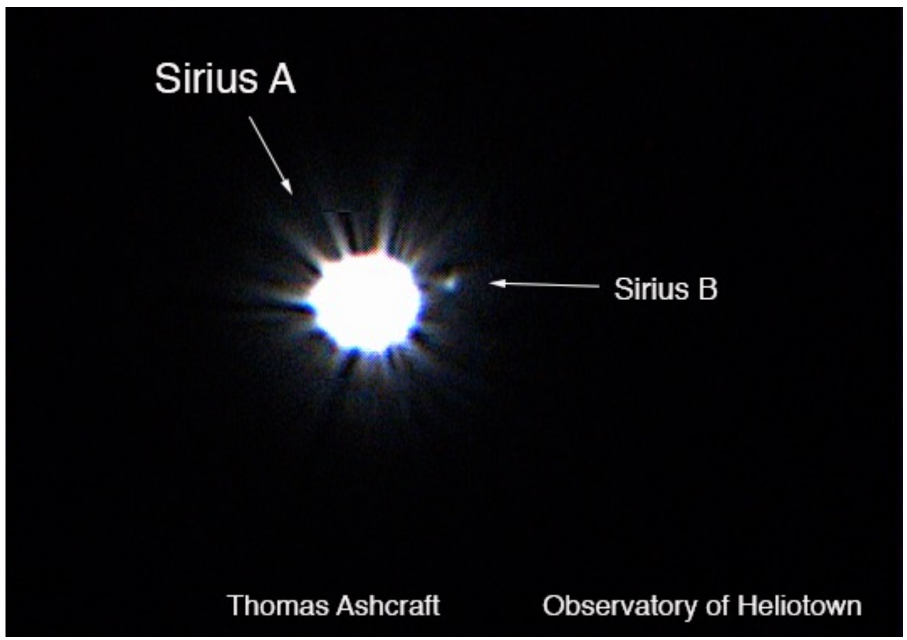
- Binary systems
- Kepler's 3rd Law
- Orbits

Mass Determination

- Mass of a star is difficult to infer from stellar spectra
- Instead use gravitational influence in a binary star system
- Most stars are in binary or multiple systems

Class Example

- How far is Sirius B from Sirius A when their separation is 11” and the distance to the system is 2.6 pc? Express answer in au.



- How far is Sirius B from Sirius A?

$$l = \theta d$$

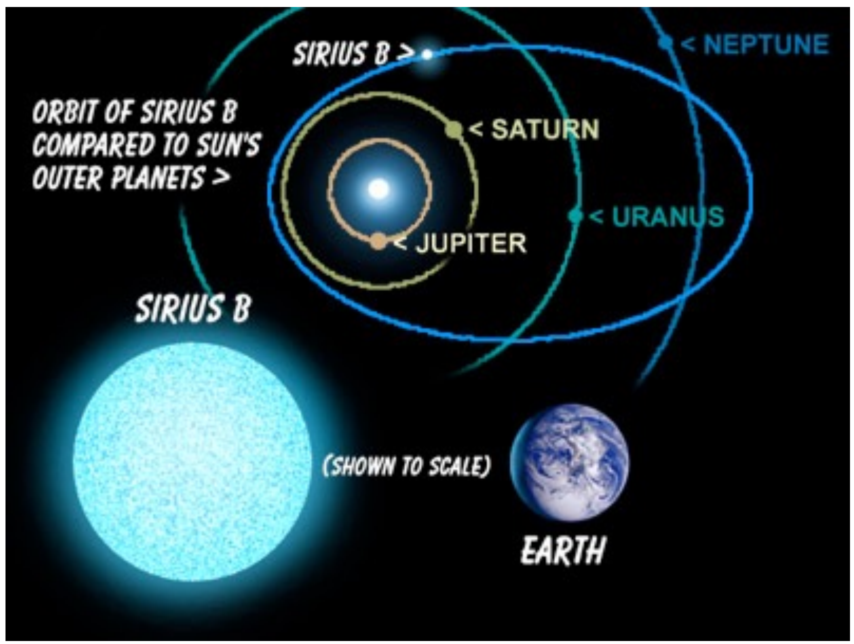
$$= \frac{11}{206265} 2.6 \times 3.1 \times 10^{16}$$

$$= 4.3 \times 10^{12} \text{ m} = 29 \text{ au}$$

$$l(au) = \theta('')d(pc)$$

$$= 11 \times 2.6$$

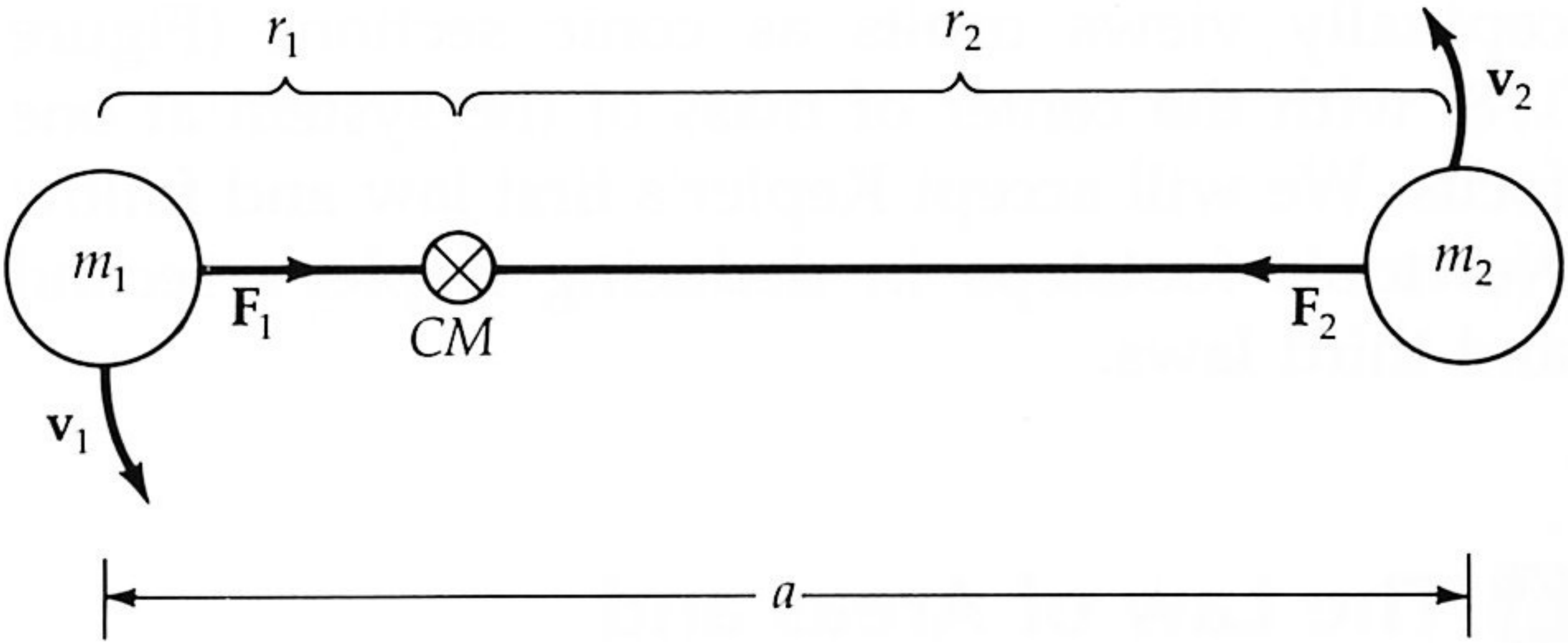
$$= 29 \text{ au}$$



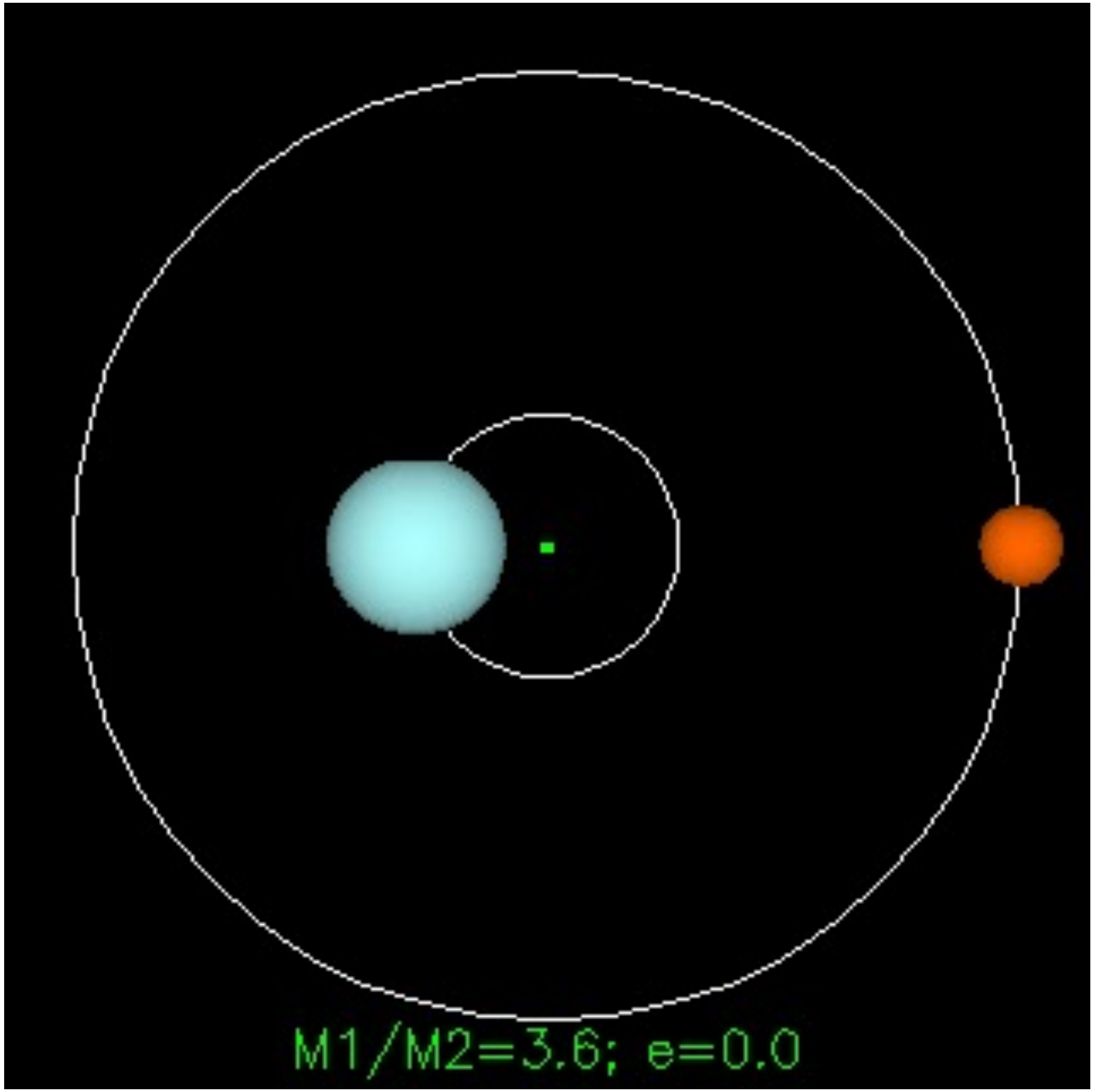
https://www.sciencecenter.net/whatsup/03/cm-stars.htm

Binary Systems

- consider two stars with masses M_1 and M_2 in circular orbits around their centre of mass (CM)
- radius of each orbit is r_1 and r_2 respectively and the total separation is a
- can use Newton's Laws and circular motion to determine masses



Zeilik Fig 1-14



http://www.astronomy.ohio-state.edu/~pogge/Ast162/Movies/visbin.html

Circular Motion

$$F_1 = \frac{M_1 v_1^2}{r_1} = \frac{4\pi^2 M_1 r_1}{P^2} \qquad v_1 = \frac{2\pi r_1}{P}$$

and

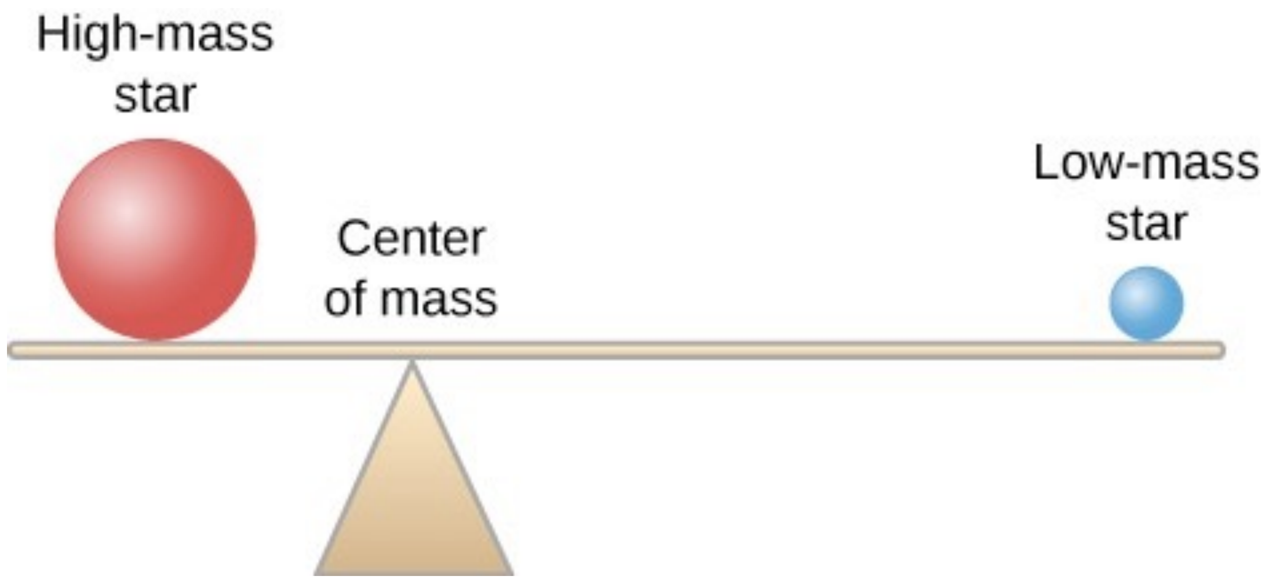
$$F_2 = \frac{M_2 v_2^2}{r_2} = \frac{4\pi^2 M_2 r_2}{P^2} \qquad v_2 = \frac{2\pi r_2}{P}$$

where P is the period which is the same for both stars

Centre of Mass

- definition of centre of mass means

$$M_1 r_1 = M_2 r_2$$



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Newton's Law of Gravity

$$F_1 = F_2 = \frac{GM_1M_2}{a^2}$$

where

$$a = r_1 + r_2$$

Newton's form of Kepler's Third Law

- combining these three equations gives

$$\frac{4\pi^2 M_1 r_1}{P^2} = \frac{G M_1 M_2}{a^2}$$

$$P^2 = \frac{4\pi^2 a^2 r_1}{G M_2}$$

Eliminate r_1 using

$$a = r_1 + r_2 = r_1 + \frac{M_1}{M_2} r_1 = \left(\frac{M_1 + M_2}{M_2} \right) r_1$$

so

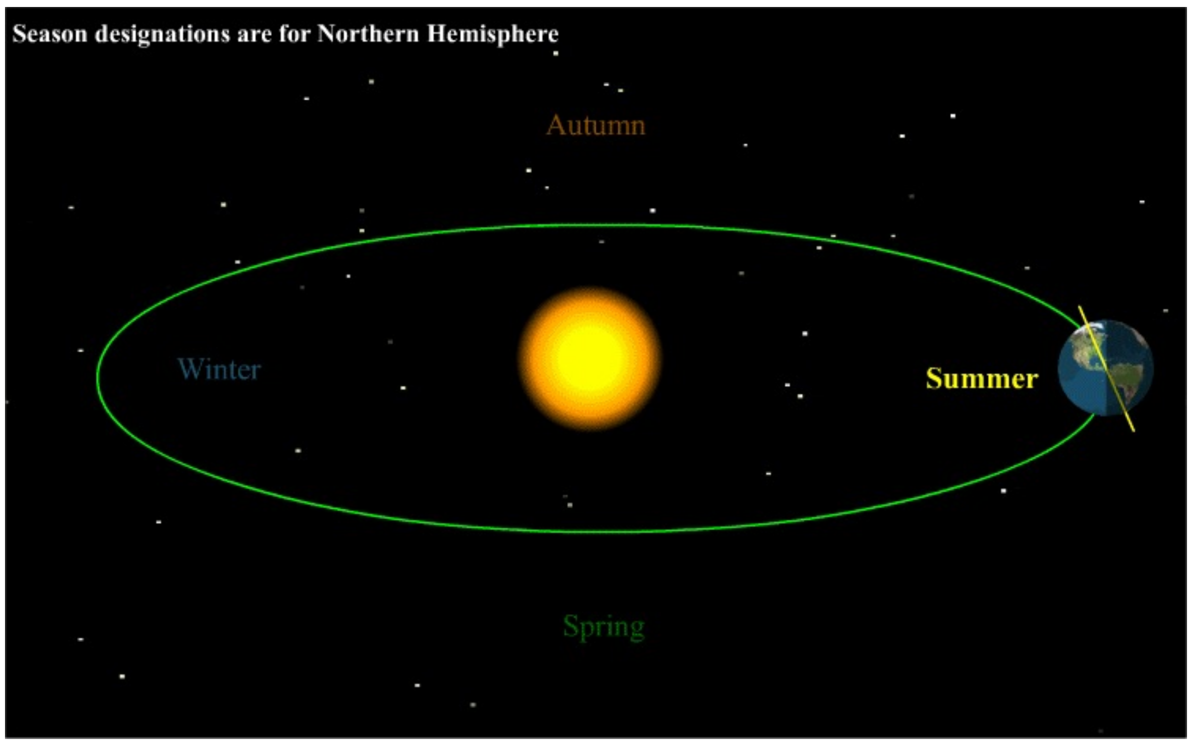
$$P^2 = \frac{4\pi^2 a^3}{G\left(M_1 + M_2\right)}$$

and

$$M_1 + M_2 = \frac{4\pi^2 a^3}{GP^2}$$

Class Example

- Use Newton's form of Kepler's Third Law to verify the mass of the Sun



- What is the mass of the Sun?

$$M_1 + M_2 = \frac{4\pi^2 a^3}{GP^2}$$

$$= \frac{4\pi^2 (1.5\times 10^{11})^3}{6.7\times 10^{-11}(3.1\times 10^{7})^2}$$

$$= 2\times 10^{30}\ \text{kg}$$

- Note mass of the Earth << Sun

Kepler's Third Law

- the planets orbiting the Sun follow the relation

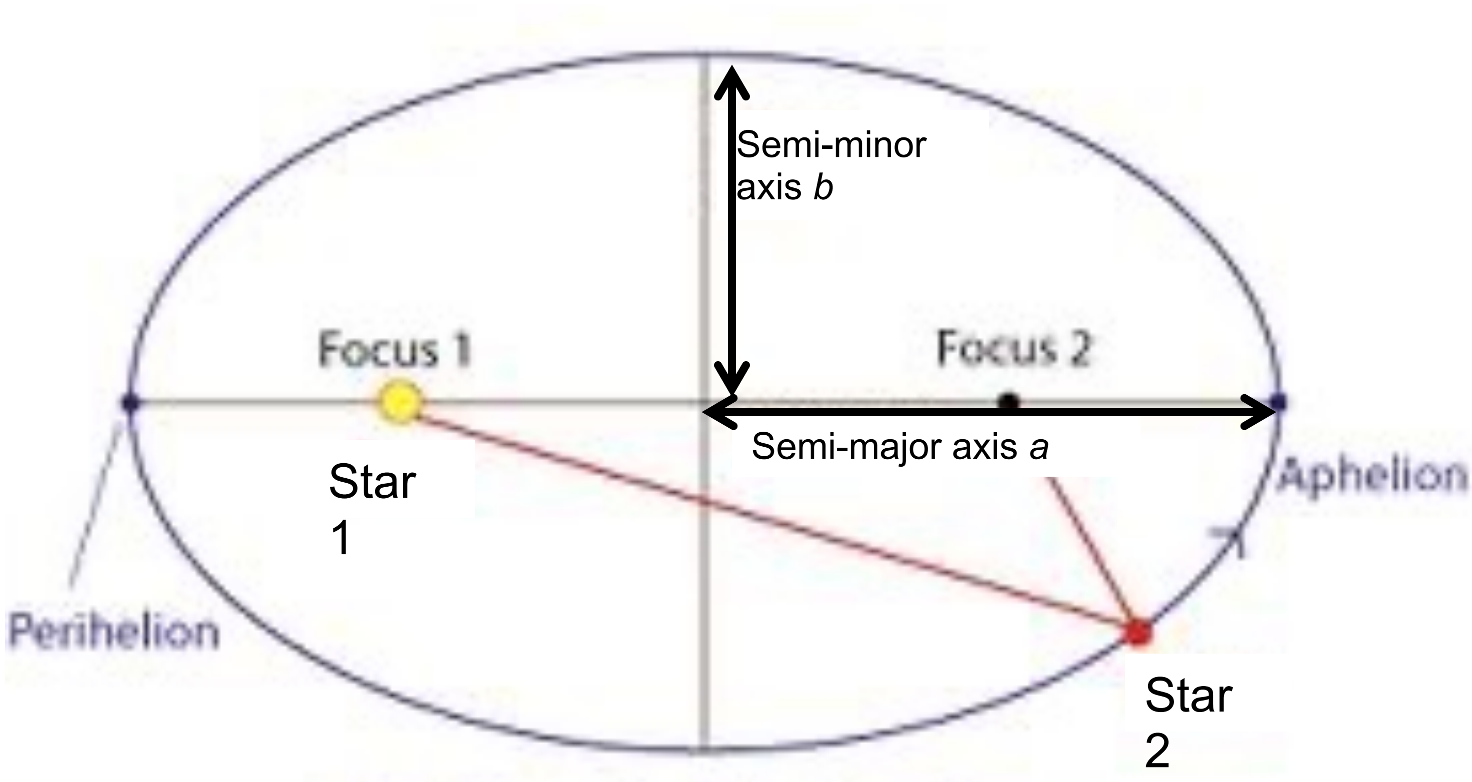
$$P^2 \propto a^3$$

- can transform into useful units:

$$\left(P/yr\right)^2 \left(M_1 + M_2 / M_{Sun}\right) = \left(a/au\right)^3$$

Real Orbits

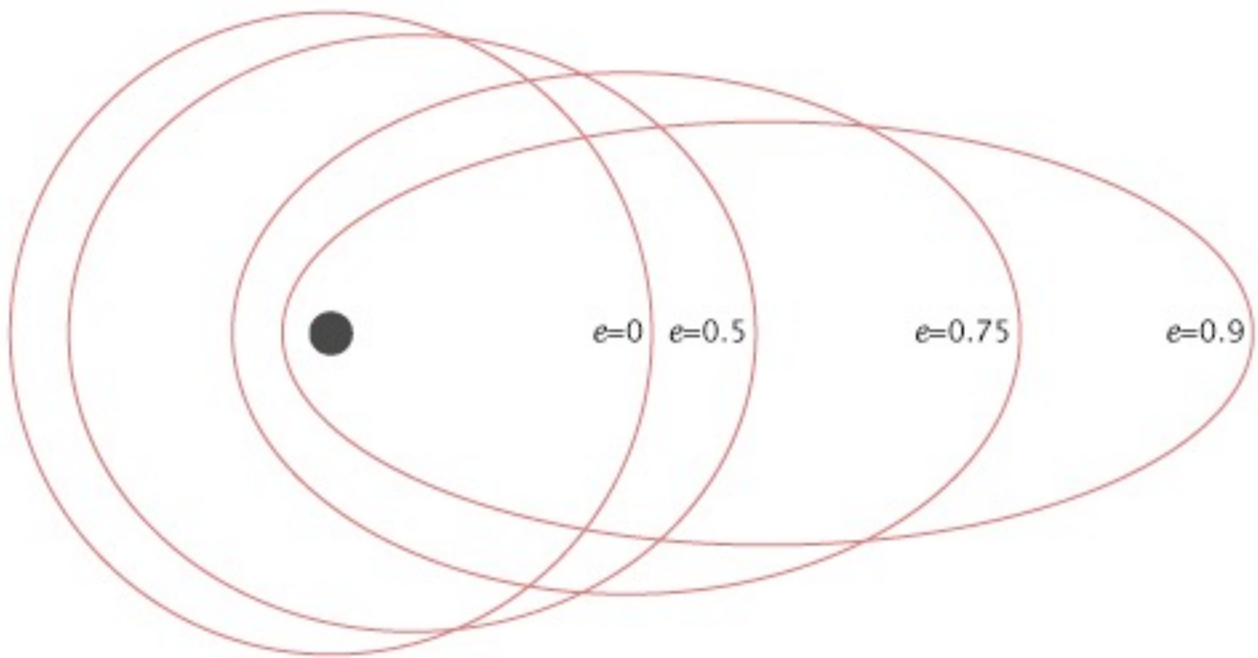
- orbits are generally elliptical described by their semi-major axis a and semi-minor axis b



outreach.atnf. csiro.edu

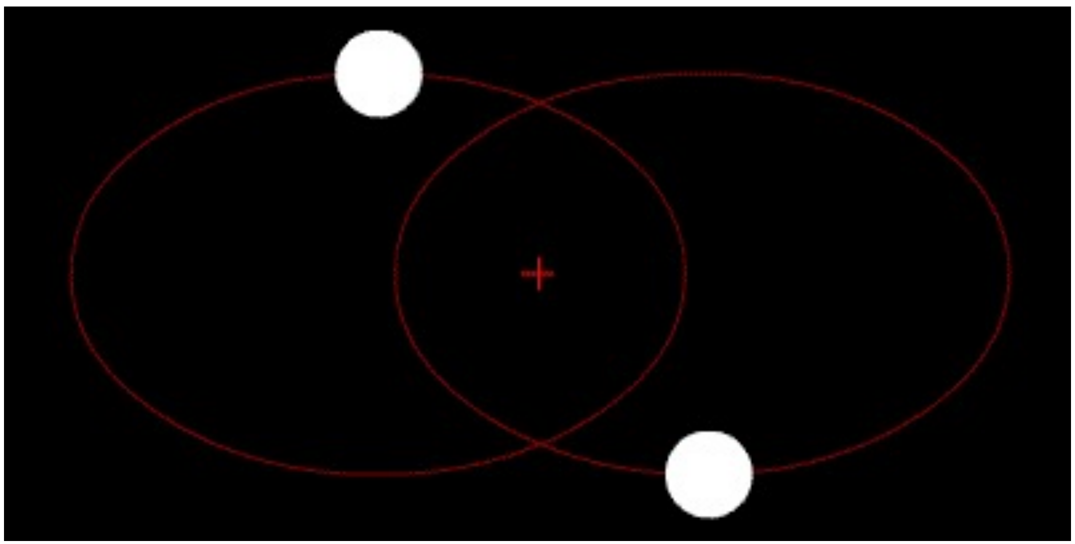
- eccentricity of elliptical orbit is defined by

$$e = \frac{\sqrt{a^2 - b^2}}{a} \qquad e = 0 \Rightarrow \text{circular orbit}$$



https://www.tutorialspoint.com/satellite_communication/s
al_mechanics.htm

- Newton's form of Kepler's third law also applies to elliptical orbits with *a* the sum of the semi-major axes ($a = a_1 + a_2$)



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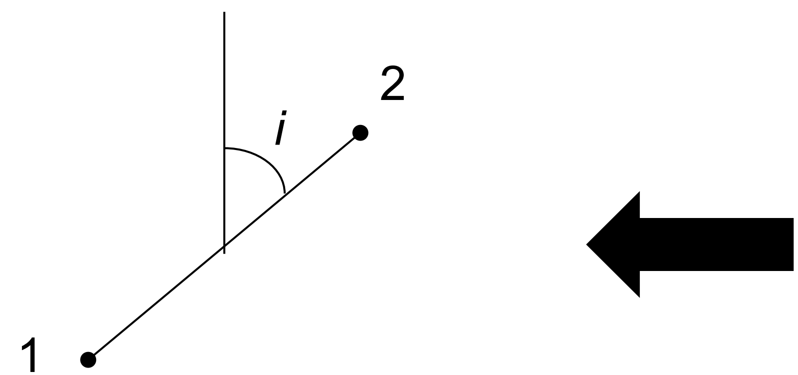
Orbital Inclination

- in general the orbital plane of a binary system will be inclined by some angle *i* to the plane of the sky:

$$i = 0^o \quad \Rightarrow \text{ face on}$$

$$i = 90^o \quad \Rightarrow \text{ edge on}$$



Summary

- Binaries are the only direct way of measuring stellar masses
- Newton's form of Kepler's 3rd law is the starting point for measuring stellar masses

Class Example

- Confirm Kepler's Third law by measuring the slope of this graph