Stars and Galaxies

Coursework Sheet 7 – Feedback

1. Formula for temperature of a dust grain at a distance d from a star of luminosity L is

   $$T = \left(\frac{L}{16\pi\sigma d^2}\right)^{\frac{1}{4}} \qquad \text{(1 mark)}$$

   For 10^{-4} pc from a 10^6 solar luminosity star then

   $$T = \left(\frac{10^6 \times 4 \times 10^{26}}{16\pi 5.8 \times 10^{-8}\left(10^{-4} \times 3.1 \times 10^{16}\right)^2}\right)^{\frac{1}{4}} = 1900 \text{ K} \qquad \text{(1 mark)}$$

   Peak wavelength is given by Wien's displacement law

   $$\lambda_{\max} = \frac{3 \times 10^{-3}}{T} = \frac{3 \times 10^{-3}}{1900} = 1.6 \times 10^{-6} \text{ m}$$

   i.e. 1.6μm or the near-infrared region of the spectrum. (1 mark)

   For 1 pc T=19 K (1 mark)
   and λ_{max}=160μm in the far-infrared. (1 mark)

   Hence, grains close to hot stars emit strongly in the near- to mid-infrared region whilst grains along way from stars have a typical temperature of a few tens of K and emit in the far-infrared. Recall that the typical distance between stars in a Galaxy is a few parcsecs. If grains get any hotter than about 2000 K then they are destroyed by sublimation into the gas phase. Hence there is no hot dust emission at wavelengths shorter than the near-infrared.

2. We need to calculate the volume that results in a mass equivalent to the 10 solar masses initially ejected. This is given by

$$V = \frac{M}{\rho} \qquad \text{(1mark)}$$

The mass density, ρ (kg m^{-3}) , is related to the number density, n (particles m^{-3}), by the mass of a particle, in this case the hydrogen atom, m_H. Hence,

$$V = \frac{M}{m_H n} = \frac{10 \times 2 \times 10^{30}}{1.7 \times 10^{-27} \times 10^{6}} = 1 \times 10^{52} \text{ m}^{-3} \qquad \text{(1mark)}$$

The radius of a sphere of this volume is

$$r = \left(\frac{3V}{4\pi}\right)^{\frac{1}{3}} = \left(\frac{3\times10^{52}}{4\pi}\right)^{\frac{1}{3}} = 1\times10^{17}\ \text{m} = 4\ \text{pc} \qquad \text{(1 mark)}$$

3. The rotation speed of the Sun around the Galaxy is 220 km s^{-1}. If the spiral pattern speed is only about half of that then the pattern speed is 110 km s^{-1}. (1 mark)

Therefore the time taken to pass through a spiral arm is

$$t = \frac{d}{v} = \frac{3.1\times10^{19}}{110\times10^{3}} = 3\times10^{14}\,s = 1\times10^{7}\,years \qquad \text{(1 mark)}$$

Note this is similar to the main sequence lifetime for massive stars and hence they only exist in spiral arms.