Stars and Galaxies

Coursework Sheet 4 – Feedback

1. To obtain the sum of the masses of the two stars we can use Newton's form of Kepler's third law

$$M_1 + M_2 = \frac{4\pi^2 a^3}{GP^2}$$

(1 mark)

This formula contains G and all the quantities must therefore be in SI units, i.e. masses in kg, separation in m and period in seconds. To find the separation of the two stars in metres we need to convert the angular separation of the two stars which is θ=3+1=4 arcseconds into a physical separation as follows

$$a = \theta d = \frac{4}{206265}.5.3.110^{16} = 3.0.10^{12} \text{ m}$$

Hence,

$$M_1 + M_2 = \frac{4\pi^2 (3.0.10^{12})^3}{6.7.10^{-11}.(30 \times 3.15.10^7)^2} = 1.8.10^{31} \text{ kg} = 8.9\,\mathrm{M}_\odot$$

(1 mark)

Now the ratio of the masses is given by the centre of mass of the system:

$$\frac{M_2}{M_1} = \frac{a_1}{a_2} = \frac{1}{3}$$

so

$$M_1 = 3M_2$$

$$4M_2 = 8.9\,\mathrm{M}_\odot$$

and

$$M_2 = 2.2\,\mathrm{M}_\odot$$

$$M_1 = 6.7\,\mathrm{M}_\odot$$

(2 marks)

Alternatively you can use the form of Kepler's third law in solar system units:

$$M_1 + M_2 = \frac{a^3}{P^2}$$

where masses are in $\mathrm{M}_\odot$, separation is in AU and period is in years. The separation of the two stars in AU can be found using the definition of the parsec, i.e.

$$a/_{AU} = \theta/_{''}\, d/_{pc} = 4 \times 5 = 20 \text{ AU}$$

Binaries with solar-type stars will have a sum of their masses of about 2 $\mathrm{M}_\odot$ and a human lifetime is of order 100 years (let's be optimistic) or 3.10^9 s. Using Kepler's third law again this gives a separation of $4.0.10^{12}$ m = 26 AU. Hence, for typical ground-based telescope angular resolutions of about 1 arcsecond we can resolve these binaries out to distances of only 26 pc, i.e. not many can be resolved.

(1 mark)

2. For double-lined spectroscopic binaries we need the form of Kepler's third law which includes radial velocities, i.e.

$$M_1 + M_2 = \frac{P}{2\pi G}\left(\frac{v_{r1} + v_{r2}}{\sin i}\right)^3$$

$$= \frac{8.3600}{2\pi 6.7.10^{-11}}\left(\frac{(150+350).10^3}{\sin 90^{\circ}}\right)^3$$

$$= 8.6.10^{30}\ \text{kg}$$

$$= 4.3\ \mathrm{M}_{\odot}$$

(2 marks)

Again this contains *G* and so all variables have to be in SI units: masses in kg, period in seconds and velocities in ms^{-1}.

Ratio of the masses is found from the ratio of the velocity amplitudes:

$$\frac{M_2}{M_1} = \frac{v_{r1}}{v_{r2}} = \frac{150}{350}$$

$$M_1 = \frac{7}{3} M_2$$

$$\frac{10}{3} M_2 = 4.3\ \mathrm{M}_{\odot}$$

$$M_2 = 1.3\ \mathrm{M}_{\odot}$$

$$M_1 = 3.0\ \mathrm{M}_{\odot}$$

(2 marks)

To prove that we were viewing a system nearly edge-on, i.e. i~90°, we need to look for eclipses of one star by the other as they pass in front of each other thereby dimming the brightness of the system.

(1 mark)