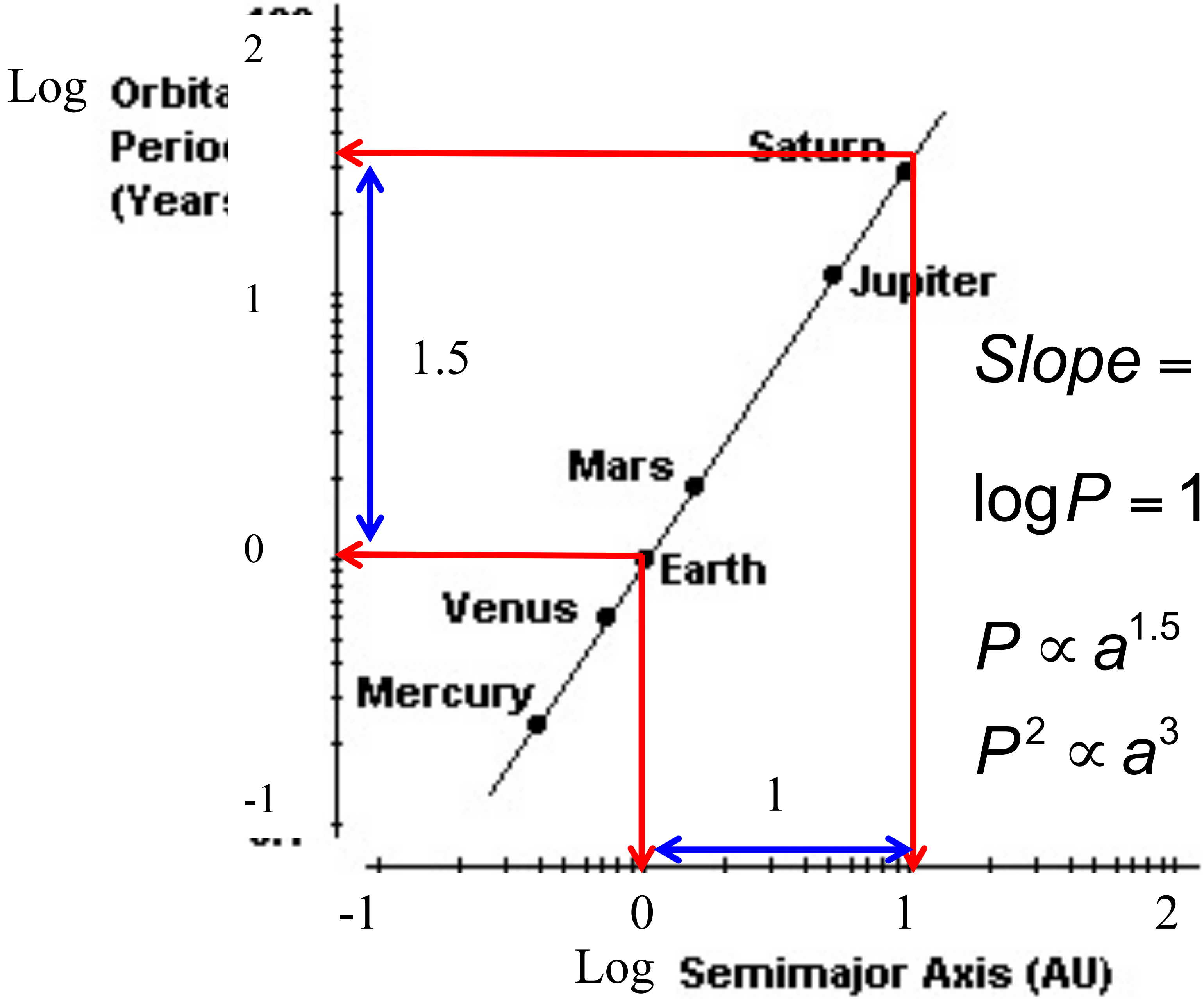


$$Slope = \frac{1.5}{1} = 1.5$$

$$\log P = 1.5 \log a$$

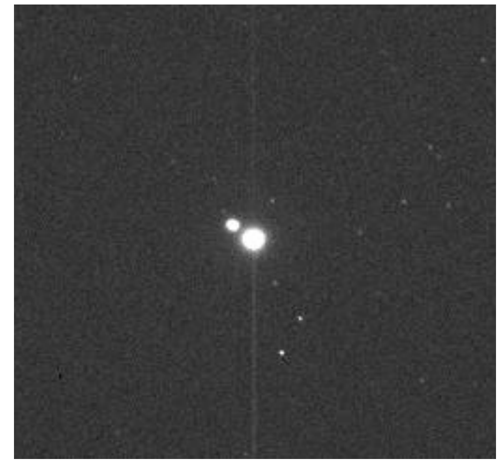
$$P \propto a^{1.5} \propto a^{\frac{3}{2}}$$

$$P^2 \propto a^3$$

Stellar Masses

- Visual binaries
- Spectroscopic Binaries

Types of Binary System



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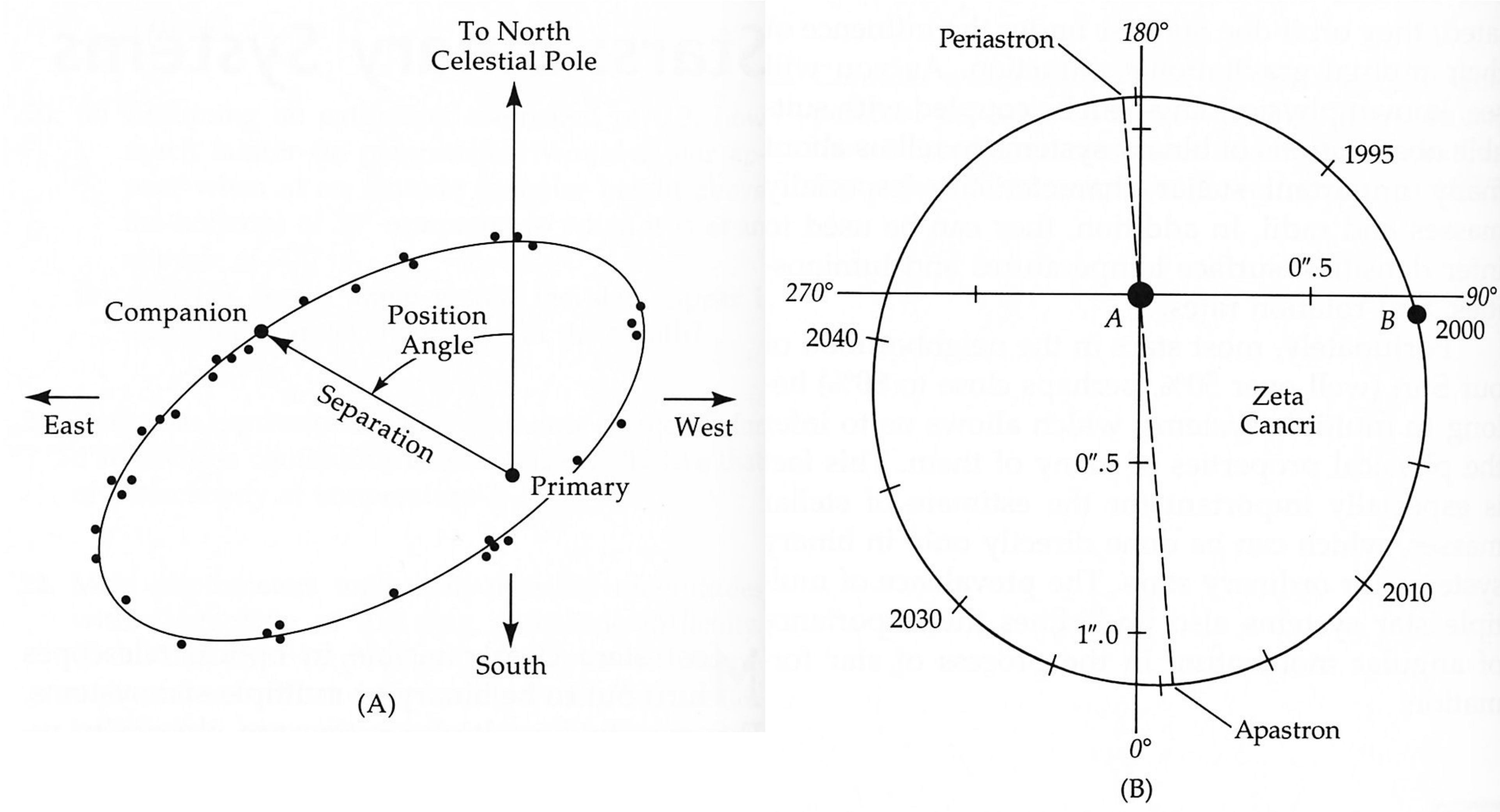
- Visual binaries
  - Two stars spatially resolved on the sky in orbit around each other
- Spectroscopic binaries
  - Two stars not spatially resolved, but orbital motion revealed through periodic Doppler shifts of their spectral lines

Masses from Visual Binaries

- measure period *P* and separation *a* (need distance *d*) $\rightarrow$ M_1+M_2 from Kepler's law
- measure angular semi-major axes of each orbit

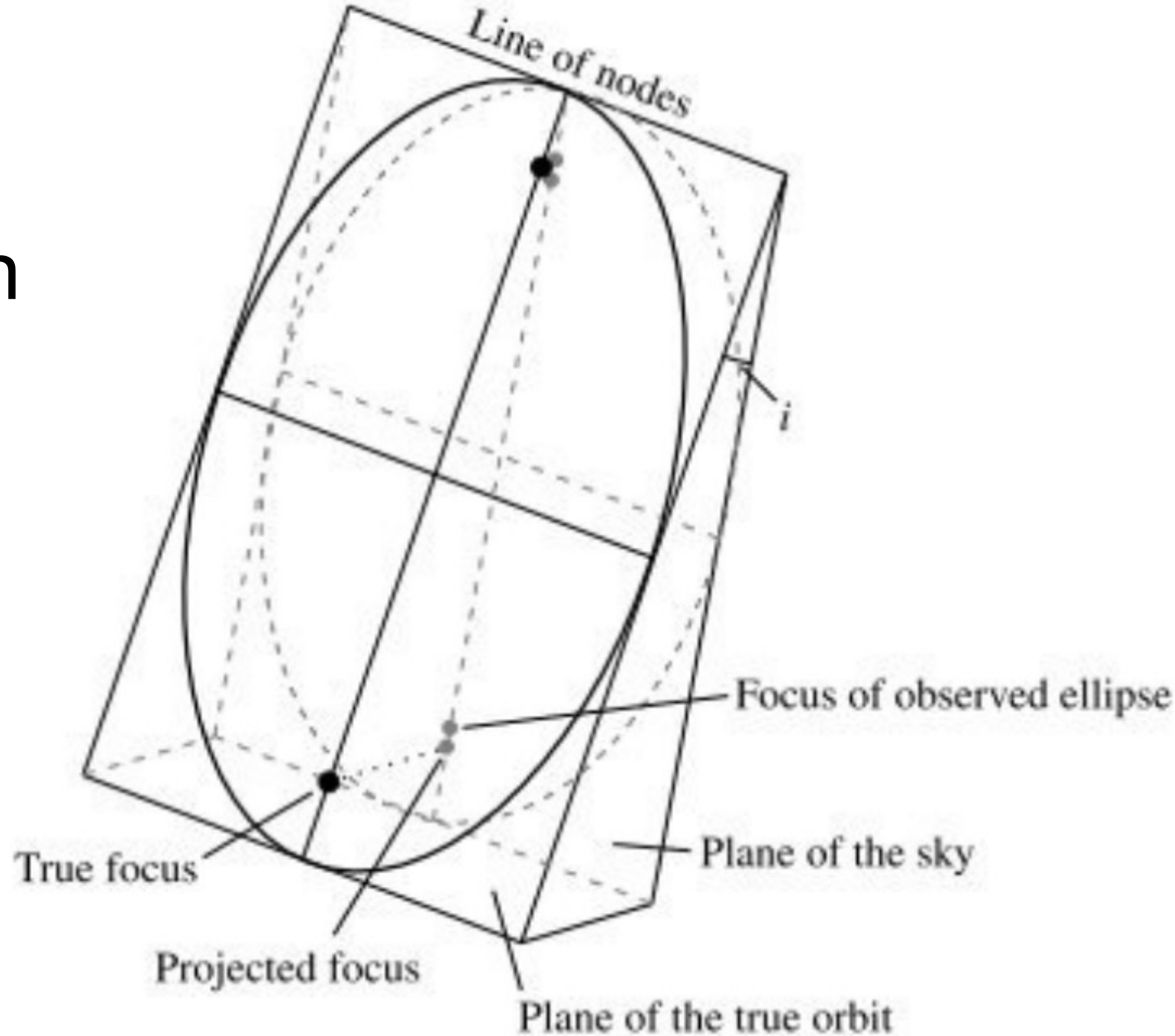
$$\frac{\theta_1}{\theta_2} = \frac{r_1}{r_2} = \frac{M_2}{M_1}$$

- solve for individual masses

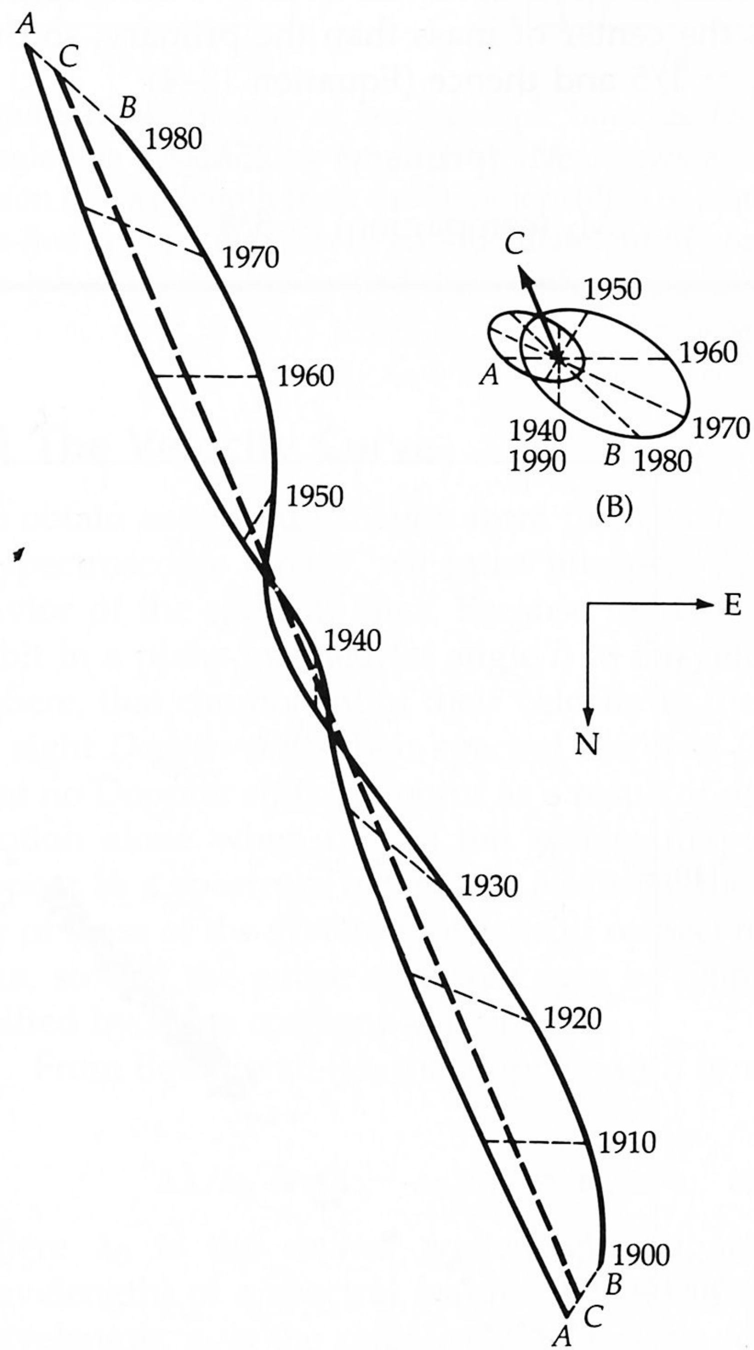


Zeilik Fig 12-1

- Unique fit of ellipse to data gives inclination



From Carroll & Ostlie



Sirius A and B

Zeilik Fig 12-2

Class Example

- The semi-major axes of the orbits of Sirius A and B are 2.5” and 5.0” respectively. With a period of 50 years and a distance of 2.6 pc what are the masses of A and B in solar masses?

Angular separation

$$\theta = \theta_1 + \theta_2 = 2.5 + 5.0 = 7.5''$$

Physical separation

$$a = \theta d = \frac{7.5}{206265} \times 2.6 \times 3.1 \times 10^{16} = 2.9 \times 10^{12} \text{ m}$$

$$M_1 + M_2 = \frac{4\pi^2 a^3}{GP^2}$$

$$= \frac{4\pi^2 (2.9 \times 10^{12})^3}{6.7 \times 10^{-11} (50 \times 3.1 \times 10^7)^2} = 6.0 \times 10^{30} \text{ kg} = 3.0 \text{ M}_{Sun}$$

$$\frac{M_2}{M_1} = \frac{a_1}{a_2} = \frac{\theta_1}{\theta_2} = \frac{2.5}{5.0} = 0.5$$

so

$$M_2 = 0.5M_1$$

$$M_1 + 0.5M_1 = 1.5M_1 = 3.0\ \mathrm{M}_{Sun}$$

and

$$M_1 = \frac{3.0}{1.5} = 2.0\ \mathrm{M}_{Sun}$$

$$M_2 = 0.5 \times 2.0 = 1.0\ \mathrm{M}_{Sun}$$

Masses from Spectroscopic Binaries

- Doppler shifts of their spectral lines

$$\frac{v_r}{c} = \frac{\Delta\lambda}{\lambda_0}$$

where v_r is the observed radial velocity

- for circular orbits the orbital velocities are

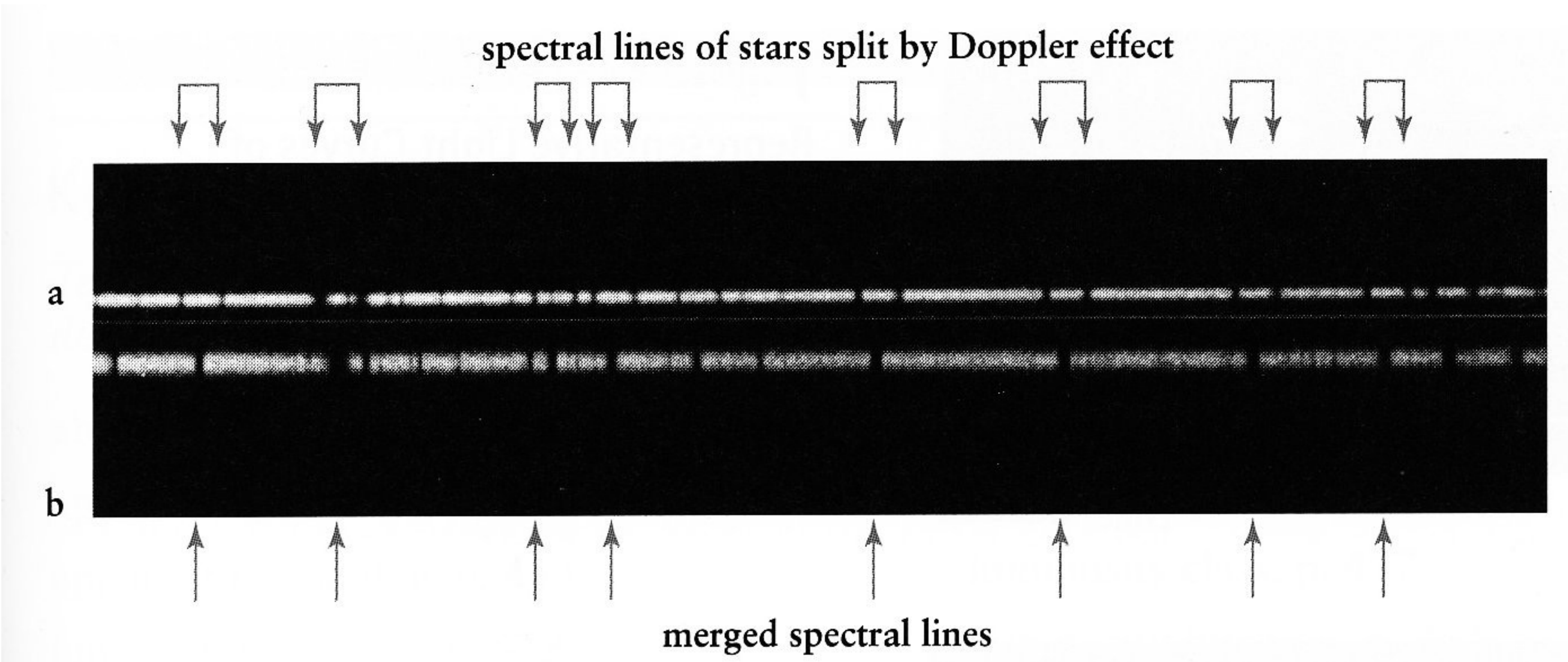
$$v_1 = \frac{2\pi r_1}{P} \quad \text{and} \quad v_2 = \frac{2\pi r_2}{P}$$

- for inclination angle i the observed radial velocities are

$$v_{r1} = v_1 \sin i \quad \text{and} \quad v_{r2} = v_2 \sin i$$

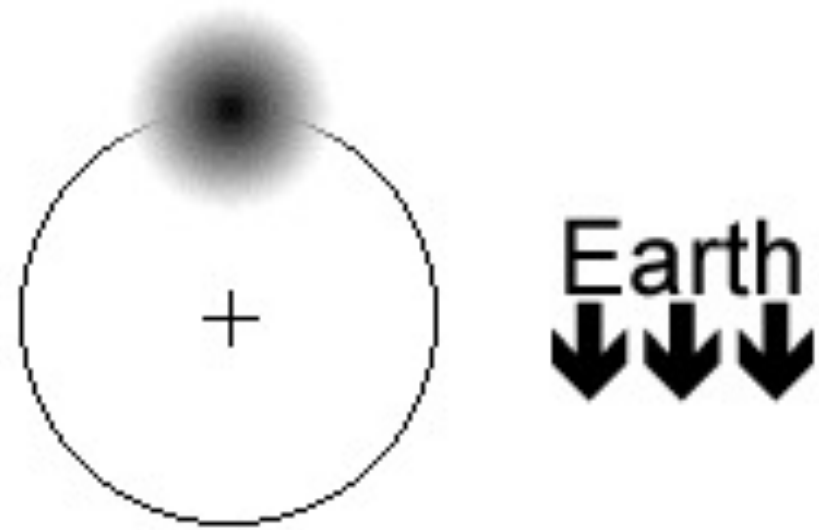
Double-lined Spectroscopic Binaries

- Spectral lines of both stars observed

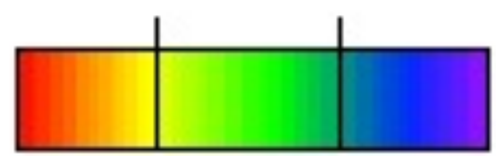


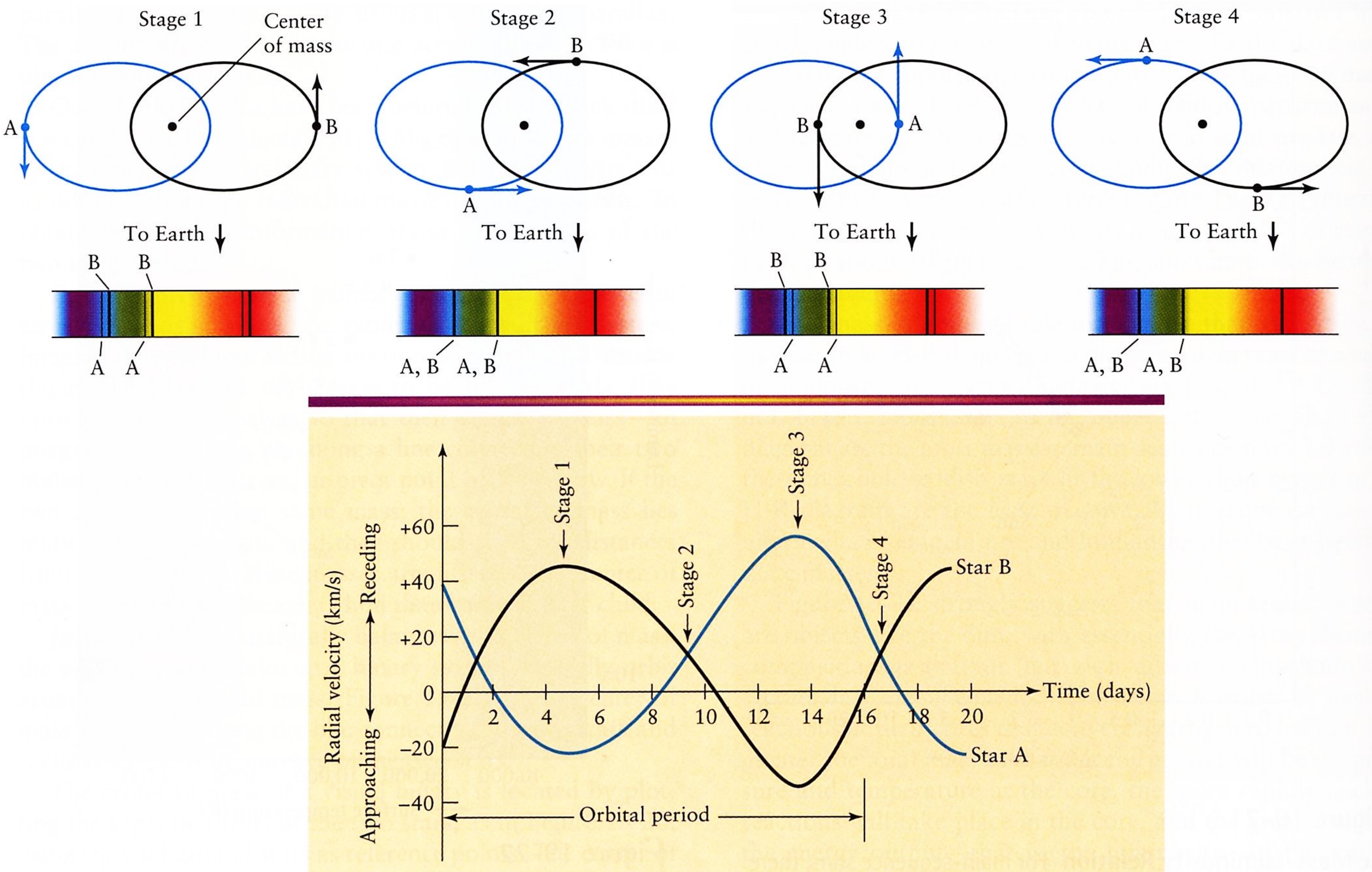
From Universe textbook

Orbit of Star Around
System's Center of Mass
(Viewed from above)



Doppler Shift
(Detects movement *along*
line of sight)





From Universe textbook

- can determine the mass ratio from

$$\frac{v_{r1}}{v_{r2}} = \frac{v_1}{v_2} = \frac{r_1}{r_2} = \frac{M_2}{M_1}$$

Also

$$a = r_1 + r_2 = \frac{P}{2\pi}(v_1 + v_2) = \frac{P}{2\pi}\left(\frac{v_{r1} + v_{r2}}{\sin i}\right)$$

so from Kepler's law

$$M_1 + M_2 = \frac{4\pi^2 a^3}{GP^2} = \frac{P}{2\pi G}\left(\frac{v_{r1} + v_{r2}}{\sin i}\right)^3$$

i.e. only a lower limit to the sum of the masses

Class Example

- The spectral lines in a double-lined spectroscopic binary exhibit sinusoidal motion with amplitudes of 150 and 350 kms^{-1} in a period of 8 hours. Assuming we view the system close to edge-on, calculate the individual masses in solar masses.

$$M_1 + M_2 = \frac{P}{2\pi G}\left(\frac{v_{r1} + v_{r2}}{\sin i}\right)^3$$

$$= \frac{8 \times 3600}{2\pi 6.7 \times 10^{-11}}\left(\frac{(150 + 350) \times 10^3}{\sin 90^o}\right)^3$$

$$= 8.6 \times 10^{30}\ \text{kg}$$

$$= 4.3\ \text{M}_{Sun}$$

$$\frac{M_2}{M_1} = \frac{v_{r1}}{v_{r2}} = \frac{150}{350}$$

$$M_1 = \frac{7}{3} M_2$$

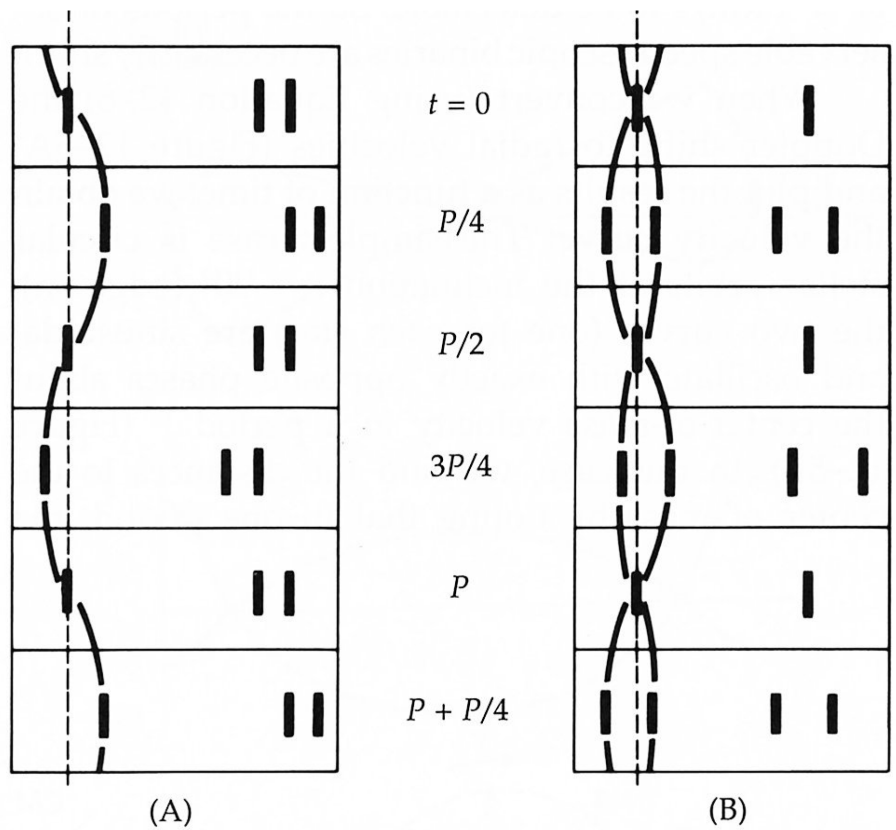
$$\frac{10}{3} M_2 = 4.3\ \text{M}_{Sun}$$

$$M_2 = 1.3\ \text{M}_{Sun}$$

$$M_1 = 3.0\ \text{M}_{Sun}$$

Single-lined Spectroscopic Binaries

- only one spectrum observed



Zeilik Fig 12-4

- Assume only v_{r1} is known so eliminate v_{r2}

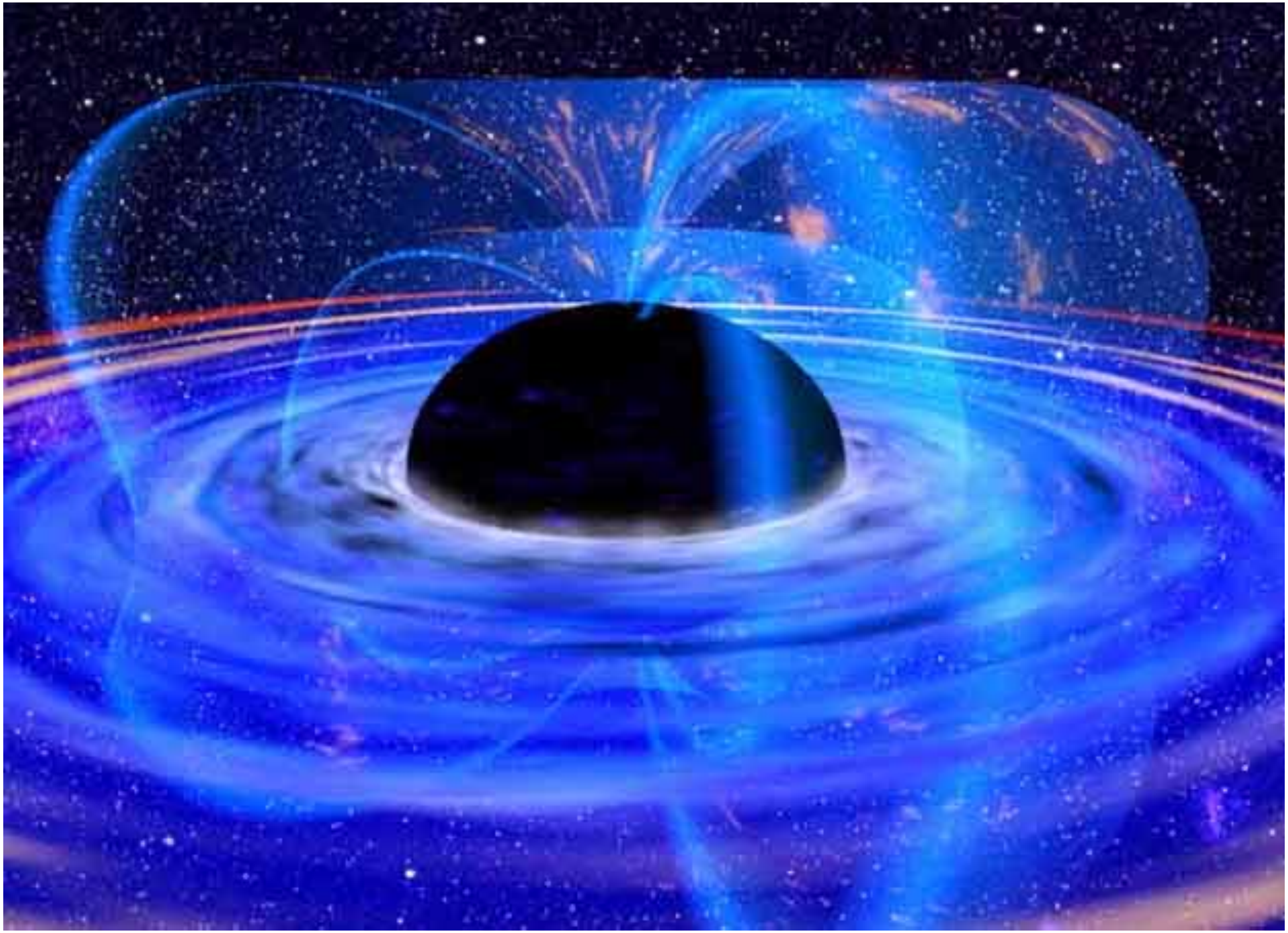
$$M_1 + M_2 = \frac{P}{2\pi G}\left(\frac{v_{r1} + \frac{M_1}{M_2} v_{r1}}{\sin i}\right)^3$$

$$M_1 + M_2 = \frac{P v_{r1}^3}{2\pi G} \left(\frac{\frac{M_1 + M_2}{M_2}}{\sin i} \right)^3$$

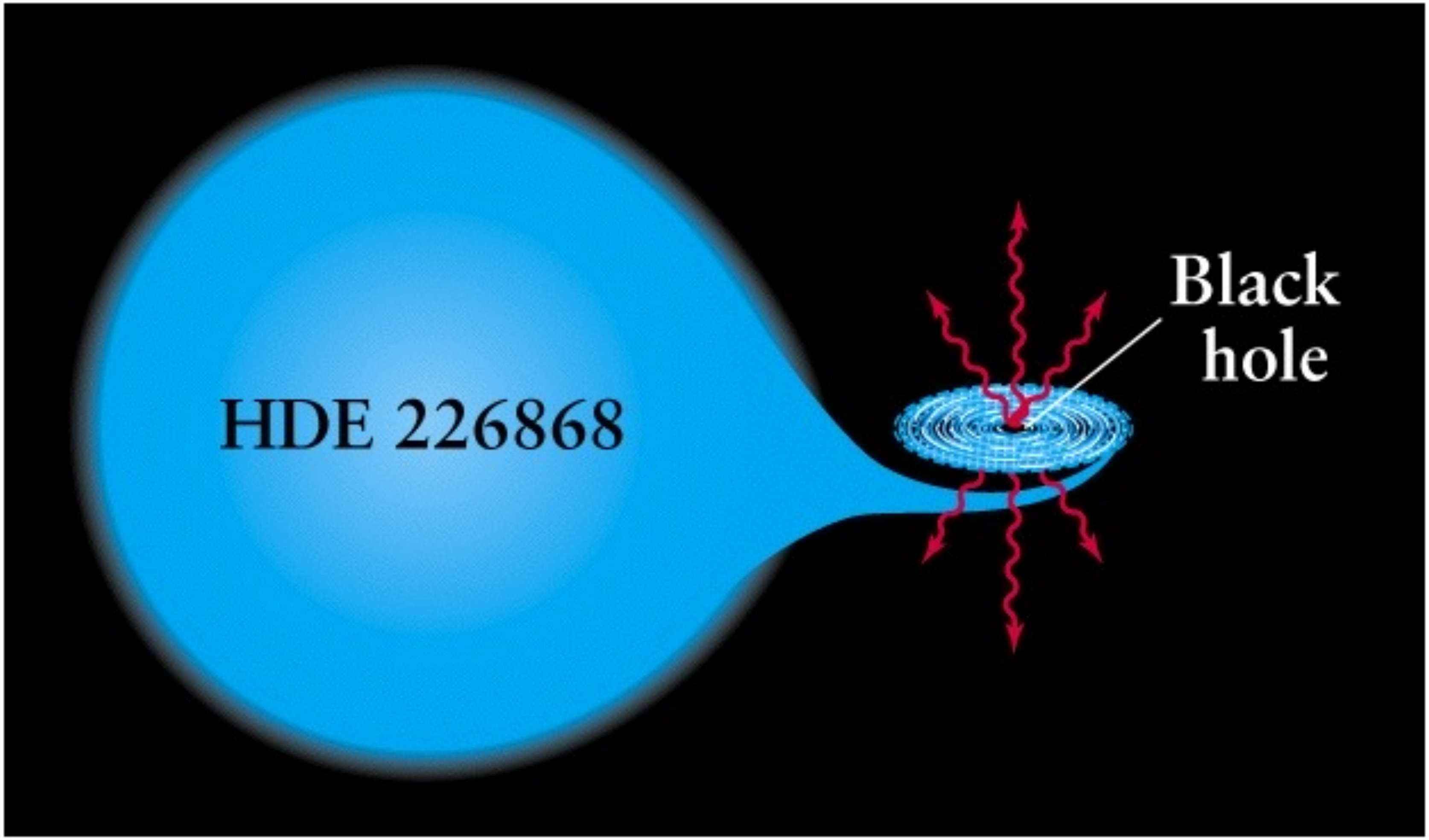
so $$\frac{M_2^3 \sin^3 i}{(M_1 + M_2)^2} = \frac{P v_{r1}^3}{2\pi G}$$

i.e. if we can estimate M_1

we can constrain M_2

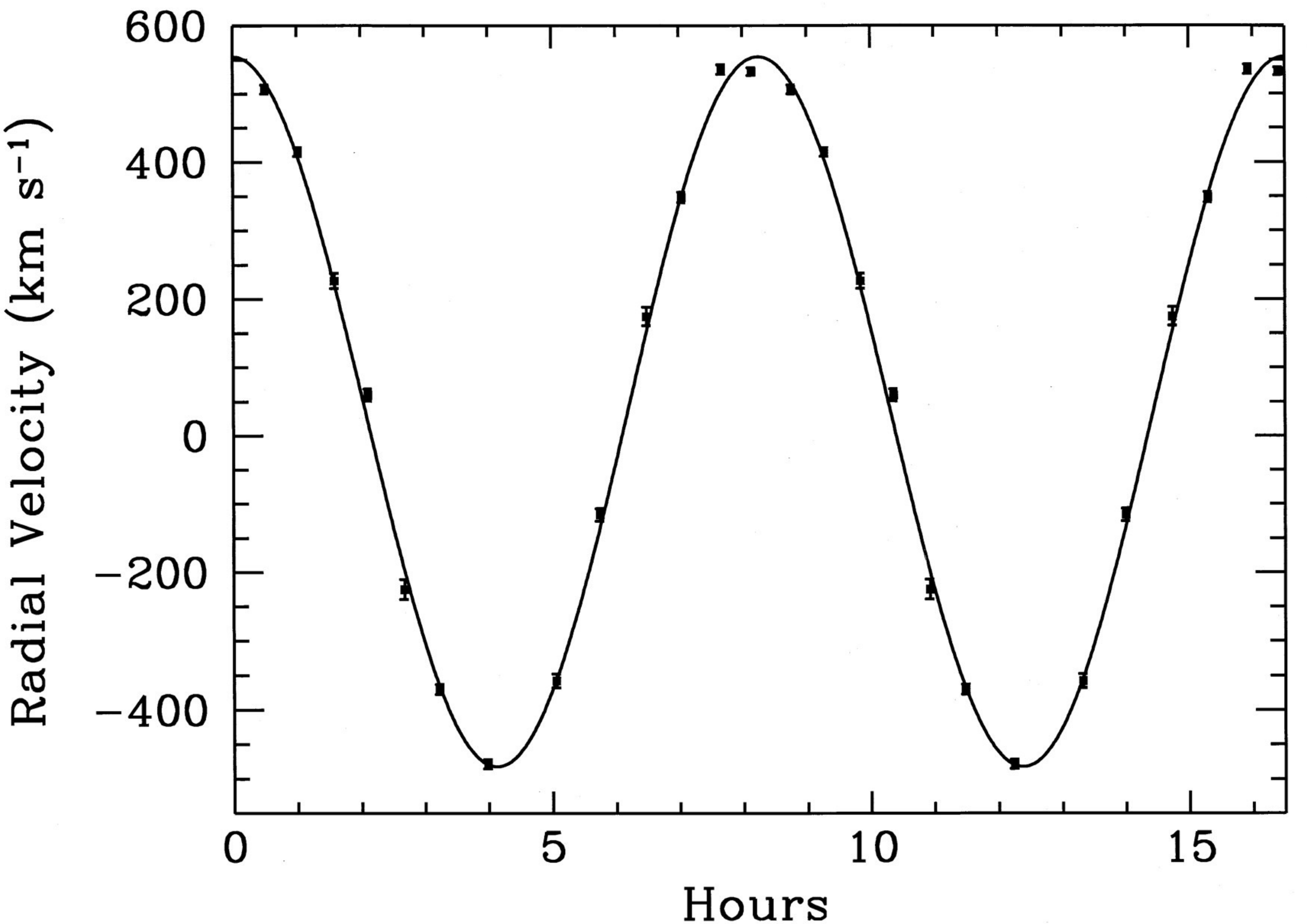


From Universe textbook



a

From Universe textbook



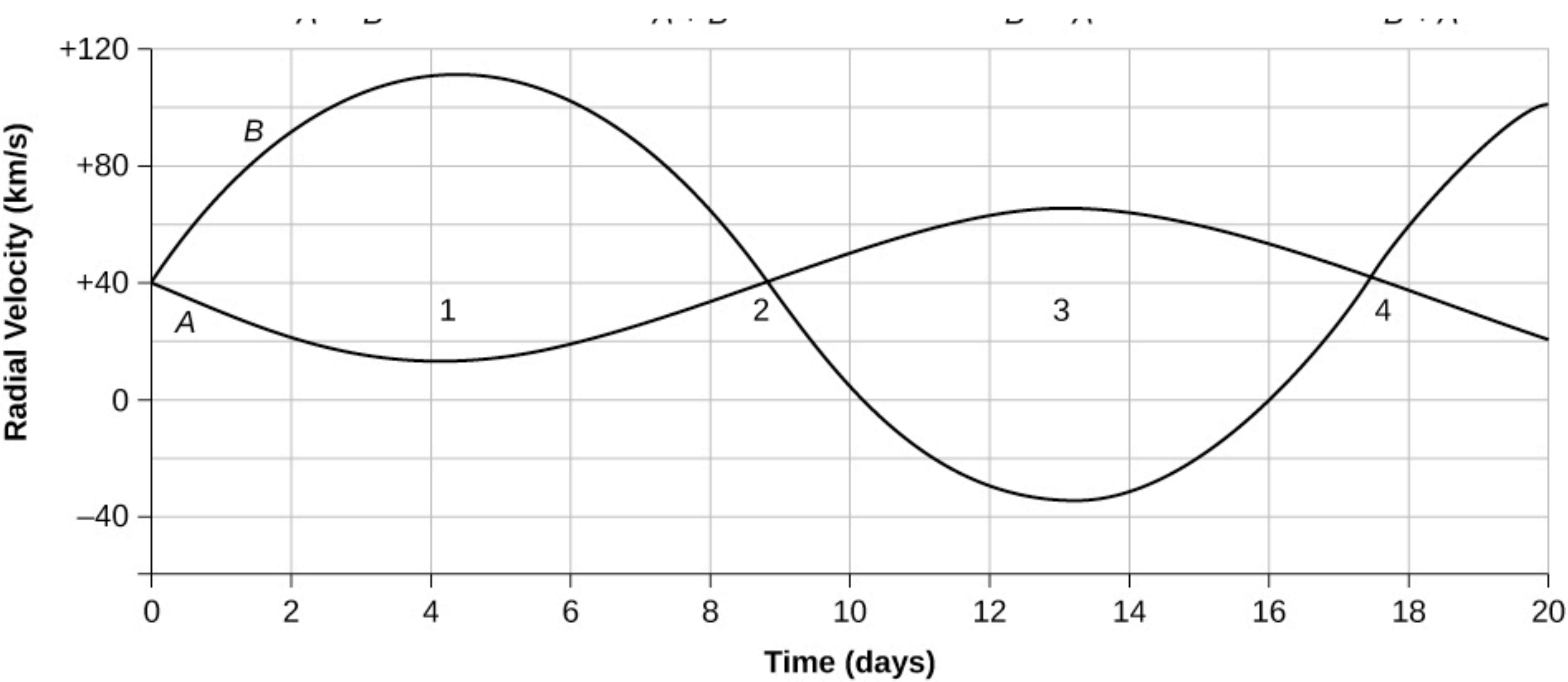
Radial-velocity curve of the visible star in the X-ray binary GS 2000 + 25
Fillipenko et al. (1999) www.pnas.org/content/96/18/9993.full
Shows that invisible compact companion star is a 5 solar mass black hole

Summary

- visual binaries provide accurate masses, but not many known
- spectroscopic binaries only usually constrain the masses with inclination the greatest uncertainty unless the system is eclipsing
- spectroscopic binaries used to find black holes and planets orbiting other stars

Class Example

- The graph overleaf shows the radial velocity curves of two stars in a spectroscopic binary system. What are the amplitudes of the radial velocities of the two stars? What is the radial velocity of the centre of mass of the binary system?


Radial Velocity (km/s)
+120
+80
+40
0
–40
Time (days)
0
2
4
6
8
10
12
14
16
18
20
A
B
1
2
3
4