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CURVILINEAR COORDINATE SYSTEMS

Plane polar coordinates (r, θ)

$$x = r\cos\theta \quad y = r\sin\theta \quad 0 \le r < \infty,\ 0 \le \theta < 2\pi$$

Spherical polar coordinates (r, θ, ϕ)

$$x = r\sin\theta\cos\phi \quad y = r\sin\theta\sin\phi \quad z = r\cos\theta \quad 0 \le r < \infty,\ 0 \le \theta < \pi,\ 0 \le \phi < 2\pi$$

Cylindrical polar coordinates (r, θ, z)

$$x = r\cos\theta \quad y = r\sin\theta \quad z = z \quad 0 \le r < \infty,\ 0 \le z < \infty,\ 0 \le \theta < 2\pi$$

Toroidal coodinates (u, v, ϕ)

$$x = a\frac{\sinh v\cos\phi}{\cosh v - \cos u} \quad y = a\frac{\sinh v\sin\phi}{\cosh v - \cos u} \quad z = a\frac{\sin u}{\cosh v - \cos u}$$

$$0 \le u < \infty,\ 0 \le v < \infty,\ 0 \le \phi < 2\pi$$

SOME USEFUL SETS

Definition : an n-tuple is an ordered array of numbers $(x^1, .., x^n)$

$$R^n = \left\{(x^1, .., x^n)\ :\ x^i \in R,\ i = 1, .., n\right\}$$

$$S^n = \left\{(x^1, .., x^{n+1})\ :\ (x^1)^2 + .. + (x^{n+1})^2 = 1,\ x^i \in R,\ i = 1, .., n+1\right\}$$

NB: S^1 is a circle and S^2 is a sphere. S^n is called the n-sphere and is a sphere in R^{n+1}.

BALLS

An open ball in R^n of radius ϵ centred at $\mathbf{a} \in R^n$ is defined to be

$$B_{\mathbf{a}}(\epsilon) = \{\mathbf{x} \in R^n : |\mathbf{x} - \mathbf{a}| < \epsilon\}\ ,$$

and the corresponding closed ball is

$$\bar{B}_{\mathbf{a}}(\epsilon) = \{\mathbf{x} \in R^n : |\mathbf{x} - \mathbf{a}| \le \epsilon\}\ .$$