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PHASE TRANSITIONS IN A NUTSHELL

Discrete Goldstone model : $\phi \in R$

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi) \quad \rightarrow \quad \partial_\mu\partial^\mu\phi + \frac{dV}{d\phi} = 0 \tag{1}$$

This is Z_2 symmetric if $V(\phi) = V(-\phi)$.

Vacuum Manifold :
The vacuum manifold is defined to be

$$M = \left\{\text{Set of field values where the minimum of the potential is attained}\right\}. \tag{2}$$

Using a 'mexican hat potential', for example,

$$V(\phi) = \frac{1}{4}\lambda\left(\phi^2 - \eta^2\right)^2 \tag{3}$$

for the discrete Goldstone model, one finds that $M = \{+\eta, -\eta\}$ for Z_2 symmetry, $M = S^1$ for the $U(1)$ Goldstone model, $M = S^3$ for the $SU(2)$ model, and $M = S^{N-1}$ for the $SO(N)$ models.

Effective potential :

$$V_{\text{eff}}(\phi, T) = \frac{F(\phi, T)}{\mathcal{V}} = V(\phi) + \sum_n f_n(\phi, T) \tag{4}$$

where for bosons

$$f_n(\phi, T) = -\frac{\pi^2}{90}T^4 + \frac{m^2(\phi)}{24}T^2 \tag{5}$$

and for fermions

$$f_n(\phi, T) = -\frac{7\pi^2}{720}T^4 + \frac{m^2(\phi)}{48}T^2 \tag{6}$$

$$V_{\text{eff}}(\phi, T) = V(\phi) + \frac{1}{24}\mathcal{M}^2(\phi)T^2 - \frac{\pi^2}{90}\mathcal{N}T^4 \tag{7}$$

where

$$\mathcal{N} = \mathcal{N}_{\text{B}} + \frac{7}{8}\mathcal{N}_F \quad \mathcal{M}^2(\phi) = \sum_{n\in B} m_n^2(\phi) + \frac{1}{2}\sum_{n\in F} m_n^2(\phi) \tag{8}$$

Correlation length :
ξ is distance beyond which fields are uncorrelated. By causality $\xi < d_H(t)$.

Kibble mechanism :

$\xi < d_{\text{H}}(t) \sim t$ implies the formation of defects is inevitable. For a dimension d defect, there will be one of size ξ^d in a volume of ξ^3. ($d = 0$ for monopoles, $d = 1$ for cosmic strings and $d = 2$ for domain walls.)

Defect classification :

$\Pi_0(M) \neq I \quad \rightarrow \quad$ Domain walls
$\Pi_1(M) \neq I \quad \rightarrow \quad$ Cosmic strings
$\Pi_2(M) \neq I \quad \rightarrow \quad$ Monopoles

Classic situations for defect formation :

$Z_N \rightarrow I$ - Domain walls
$U(1) \rightarrow I$ - Cosmic strings
$G \rightarrow H \times U(1)$ - Monopoles

Derrick's Theorem :

There are no stable, time independent solutions for a global field theory which have finite energy in dimension ≥ 3.

Domain wall solution :

$$\phi(x) = \eta \tanh\left(\frac{x}{\Delta}\right) \qquad \text{where} \qquad \Delta = \left(\frac{2}{\lambda}\right)^{1/2} \eta^{-1} \equiv \text{width} \tag{9}$$

$$\sigma = \int T^0{}_0 dx = \int_{-\infty}^{\infty} \rho(x) dx = \frac{2\sqrt{2}}{3} \lambda \eta^3 \tag{10}$$

Defects evolution :

Dimensional :

$$\rho_{\rm m} \propto a^{-3}\,, \qquad \rho_{\rm str} \propto a^{-2}\,, \qquad \rho_{\rm dw} \propto a^{-1} \tag{11}$$

Self-similar :

$$\rho_{\rm str} \propto t^{-2}\,, \qquad \rho_{\rm dw} \propto t^{-1} \tag{12}$$