EARLY UNIVERSE PHYS40772 2009 : Dr R.A. Battye

INFLATION IN A NUTSHELL

Definition of inflation :

$$\ddot{a} > 0 \quad \rightarrow \quad \rho + 3P < 0 \quad \rightarrow \quad 1 + 3w < 0 \quad \text{if} \quad P = w\rho \tag{1}$$

Total number of e-folds :

$$N_{\rm tot} = \log\left(\frac{a_{\rm end}}{a_{\rm start}}\right) \tag{2}$$

Potential formulation :

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi) \qquad P_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi) \tag{3}$$

$$\ddot{\phi} + 3H\dot{\phi} + \frac{dV}{d\phi} = 0 \tag{4}$$

Slow roll :

$$\dot{\phi}^2 << 1 \qquad \ddot{\phi} << 1 \quad \rightarrow \quad 3H\dot{\phi} = -\frac{dV}{d\phi} \quad \& \quad H^2 = \frac{8\pi G}{3}V \tag{5}$$

$$\dot{\phi} = -\left(\frac{1}{24\pi G}\right)^{1/2}\frac{V'}{V^{1/2}} \quad \text{and} \quad a(\phi) = a_{\rm start}\exp\left[8\pi G\int_{\phi}^{\phi_{\rm start}}\frac{V}{V'}d\phi\right] \tag{6}$$

Slow-roll parameters

$$\epsilon = \frac{m_{\rm pl}^2}{16\pi}\left(\frac{\frac{dV}{d\phi}}{V}\right)^2, \qquad \eta = \frac{m_{\rm pl}^2}{8\pi}\left[\frac{\frac{d^2V}{d\phi^2}}{V} - \frac{1}{2}\left(\frac{\frac{dV}{d\phi}}{V}\right)^2\right]. \tag{7}$$

Start and End of inflation :

$$V(\phi_{\rm start}) = M_{\rm pl}^4 \tag{8}$$

Inflation ends when either $\epsilon = 1$ or $|\eta| = 1$ whichever is earlier.

Number of e-foldings

Number of e-foldings from the end of inflation.

$$N(\phi) = 8\pi G\int_{\phi_{\rm end}}^{\phi}\frac{V}{\frac{dV}{d\phi}}d\phi \tag{9}$$

Reheating :

Instantaneous reheating implies

$$V(\phi_{\rm end}) = \frac{\pi^2}{30}gT_{\rm R}^4 \tag{10}$$

Scalar pertubations

$$\mathcal{P}_{\mathcal{R}} = \frac{2^7\pi}{3m_{\rm pl}^2}\frac{V^3}{\left(\frac{dV}{d\phi}\right)^2}\,. \tag{11}$$

Tensor (gravitational wave) perturbations

$$\mathcal{P}_{\rm G} = 128\frac{V}{M_{\rm pl}^4}\,. \tag{12}$$

Spectral indices :

$$n_{\rm S} = 1 + \frac{d(\log\mathcal{P}_{\mathcal{R}})}{d(\log k)} \qquad n_{\rm G} = \frac{d(\log\mathcal{P}_{\rm G})}{d(\log k)} \tag{13}$$

Scalar to tensor ratio:

$$r = \frac{\mathcal{P}_{\mathcal{R}}}{\mathcal{P}_{\rm G}} \tag{14}$$

Observables in terms of slow-roll

$$n_{\rm S} = 1 - 4\epsilon + 2\eta\,, \qquad r = 16\epsilon \tag{15}$$