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CMB IN A NUTSHELL

Temperature fluctuations :

$$\frac{\Delta T}{T}(\theta,\phi) = \sum_{\ell=2}^{\infty}\sum_{m=-\ell}^{\ell} a_{\ell m} Y_{\ell m}(\theta,\phi) \tag{1}$$

$$C_\ell = \frac{1}{2\ell+1}\sum_{m=-\ell}^{\ell}|a_{\ell m}|^2 \tag{2}$$

$$T_{\rm CMB}^2\frac{\ell(\ell+1)C_\ell}{2\pi} = \langle T_\ell^2\rangle \approx \langle T_k^2\rangle \approx \frac{k^3}{2\pi^2}(\Delta T_k)^2 \tag{3}$$

where $\ell \approx k\eta_0$.

Dynamics of tight coupling regime :

$$\delta_{\rm r}'' + \frac{1}{3}k^2\delta_{\rm r} = \frac{4}{3}\delta_{\rm m}'' \qquad\qquad \theta_{\rm r} = \delta_{\rm m}' - \frac{3}{4}\delta_{\rm r}' \tag{4}$$

Super-horizon : $\delta_{\rm r} = 4\delta_{\rm m}/3 = 4A(k)k^2\eta^2/3$

Sub-horizon : $\delta_{\rm m} \propto \eta^2$ in mat era, $\delta_{\rm m} \propto 1$ in rad era

For radiation domination and $\eta > \eta_{\rm H} = 2\pi/k$

$$\delta_{\rm r}'' + \frac{1}{3}k^2\delta_{\rm r} = 0 \quad \rightarrow \quad \delta_{\rm r} = (16\pi^2/3)A(k)\cos[k(\eta-\eta_{\rm H})/\sqrt{3}] \tag{5}$$

$$\theta_{\rm r} = -(4\pi^2/\sqrt{3})k\sin[k(\eta-\eta_{\rm H})\sqrt{3}] \tag{6}$$

Density induced anisotropies :

$$\frac{\Delta T_k}{T} = \frac{1}{4}\delta_{\rm r}(k,\eta_{\rm dec}) \tag{7}$$

$$\frac{\langle T_k^2\rangle}{T_{\rm CMB}^2} = \frac{8\pi^2}{9}k^3|A(k)|^2\cos^2\left[\frac{2\pi}{\sqrt{3}}\left(\frac{k}{k_{\rm dec}}-1\right)\right] \tag{8}$$

for $k > k_{\rm dec}$.

Velocity induced anisotropies :

$$\frac{\Delta T_k}{T} = \hat{\mathbf{n}}\cdot\mathbf{v}(k,\eta_{\rm dec}) \tag{9}$$

$$\frac{\langle T_k^2\rangle}{T_{\rm CMB}^2} = \frac{8\pi^2}{9}k^3|A(k)|^2\sin^2\left[\frac{2\pi}{\sqrt{3}}\left(\frac{k}{k_{\rm dec}}-1\right)\right] \tag{10}$$

Acoustic peaks :
Peaks at

$$\ell = 2\pi \frac{\eta_0}{\eta_{\rm dec}} \left[1 + \frac{\sqrt{3}}{2}(n-1) \right] \tag{11}$$

for $n = 1, 2, 3, ..$ If $\Omega_{\rm m} = 1$ and $h = 0.65$ then $\eta_0/\eta_{\rm dec} \approx 45$ and hence

$$\ell \approx 280 \left(1 + \frac{\sqrt{3}}{2}(n-1) \right) \tag{12}$$

Angular diameter degeneracy :

$$\ell_{\rm peak} = \frac{\pi d_{\rm A}(z_{\rm dec})}{a_{\rm dec} r_s(z_{\rm dec})} \tag{13}$$

$$d_{\rm A}(z_{\rm dec}) = a_{\rm dec} H_0^{-1} \int_{a_{\rm dec}}^{1} \frac{da}{\sqrt{\Omega_{\rm k} a^2 + \Omega_{\rm m} a + \Omega_{\rm r}}} \tag{14}$$

Photon diffusion damping :

$$\delta_{\rm r}'' + 2k^2 \eta_\star \delta_{\rm r}' + \frac{1}{3} k^2 \delta_{\rm r} = 0 \tag{15}$$

$$\delta_{\rm r}(\eta_{\rm dec}) = e^{-k^2/k_{\rm D}^2} \delta_{\rm r}^{\rm tc}(\eta_{\rm dec}) \tag{16}$$

where $k_{\rm D}^{-2} = \eta_\star \eta_{\rm dec}$.

Integrated Sachs-Wolfe Effect :

$$\frac{\Delta T_x}{T} = \frac{1}{2} \int_{\eta_{\rm dec}}^{\eta_0} \hat{n}_i \hat{n}_j h_{ij}'(x, \eta) d\eta \tag{17}$$