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STRUCTURE FORMATION IN A NUTSHELL

Density pertubation :

$$\delta(\mathbf{x},t)=\frac{\delta\rho(\mathbf{x},t)}{\rho}=\frac{\rho(\mathbf{x},t)-\bar{\rho}(t)}{\bar{\rho}(t)} \tag{1}$$

$$\delta_k=\frac{1}{V}\int d^3\mathbf{x}\delta(\mathbf{x},t)e^{i\mathbf{k}\cdot\mathbf{x}} \tag{2}$$

Correlation function :

$$\xi(\mathbf{r})=\frac{1}{V}\int d^3\mathbf{x}\delta(\mathbf{r})\delta(\mathbf{x}+\mathbf{r})=\frac{V}{(2\pi)^3}\sum_k|\delta_k|^2e^{-i\mathbf{k}\cdot\mathbf{x}} \tag{3}$$

Transfer function :

$$\delta_k(t_0)=T(k)\delta_k(t_i) \tag{4}$$

Newtonian perturbation equation :

$$\ddot{\delta}_{\rm m}+\frac{2\dot{a}}{a}\dot{\delta}_{\rm m}=\delta_{\rm m}\left[4\pi G\rho_{\rm m}-\frac{c_{\rm s}^2k^2}{a^2}\right] \tag{5}$$

Matter era solution :

$$\ddot{\delta}_{\rm m}+\frac{4}{3t}\dot{\delta}_{\rm m}-\frac{2}{3t^2}\delta_{\rm m}=0 \qquad \rightarrow \qquad \delta_{\rm m}\propto t^{2/3},t^{-1} \tag{6}$$

Mezaros equation :

$$\frac{d^2}{dy^2}\delta_{\rm m}+\frac{2+3y}{2y(1+y)}\frac{d}{dy}\delta_{\rm m}-\frac{3}{2y(1+y)}\delta_{\rm m}=0 \qquad \rightarrow \qquad \delta_{\rm m}\propto\frac{2}{3}+y \tag{7}$$

where $y=a/a_{\rm eq}$

Relativistic perturbation theory :

$$\begin{aligned} k^2H-\frac{a'}{2a}h' &= \frac{3}{2}\left[\frac{\omega_{\rm r}\delta_{\rm r}}{a^2}+\frac{\omega_{\rm m}\delta_{\rm m}}{a}\right] \\ h''+\frac{a'}{a}h' &= \frac{6\omega_{\rm r}\delta_{\rm r}}{a^2}+\frac{3\omega_{\rm m}\delta_{\rm m}}{a} \end{aligned}$$

$$
\begin{aligned}
k^2 H' &= -\frac{2\omega_{\rm r}\theta_{\rm r}}{a^2} \\
\delta'_{\rm m} &= \frac{1}{2}h' \\
\delta'_{\rm r} &= \frac{4}{3}\left[\frac{1}{2}h' - \theta_{\rm r}\right] \\
\theta'_{\rm r} &= \frac{1}{4}k^2\delta_{\rm r}
\end{aligned} \tag{8}
$$

Adiabatic pertubations :
Super-horizon modes satisfy $\delta_{\rm r} = 4\delta_{\rm m}/3$

$$\delta''_{\rm m} + \frac{2}{\eta}\delta'_{\rm m} - \frac{6}{\eta^2}\delta_{\rm m} = 0 \quad \text{(mat)} \qquad \delta'_{\rm m} + \frac{1}{\eta}\delta'_{\rm m} - \frac{4}{\eta^2}\delta_{\rm m} = 0 \quad \text{(rad)} \tag{9}$$

Sub-horizon modes satisfy $\langle\delta_{\rm r}\rangle \approx 0$

$$\delta''_{\rm m} + \frac{2}{\eta}\delta'_{\rm m} - \frac{6}{\eta^2}\delta_{\rm m} = 0 \quad \text{(mat)} \qquad \delta''_{\rm m} + \frac{1}{\eta}\delta'_{\rm m} = 0 \quad \text{(rad)} \tag{10}$$

Conformal Newtonian Gauge

$$h_{00} = 2\psi\,, \qquad h_{ij} = -2\phi\delta_{ij}\,. \tag{11}$$

$$\psi = \alpha' + \frac{a'}{a}\alpha\,, \qquad \phi = H - \frac{a'}{a}\alpha\,, \qquad \alpha = \frac{1}{2k^2}(h' + 6H')\,. \tag{12}$$

$$\delta^{(\rm con)} = \delta^{(\rm syn)} - 3H(1+w)\alpha\,, \quad \theta^{(\rm con)} = \theta^{(\rm syn)} + k^2\alpha\,, \quad \delta P^{(\rm con)} = \delta P^{(\rm syn)} + \alpha\bar{P} \tag{13}$$

CDM transfer function :

$$T(k) \propto 1 \text{ for } k << k_{\rm eq} \qquad \text{and} \qquad T(k) \propto \left(\frac{k}{k_{\rm eq}}\right)^{-2} \text{ for } k >> k_{\rm eq} \tag{14}$$

HDM and Baryons :

$$T(k) \approx 0 \text{ for } k >> k_{\rm FS} \qquad \text{and} \qquad T(k) \approx 0 \text{ for } k >> k_{\rm J} \tag{15}$$