EARLY UNIVERSE PHYS40772 2009 : Dr R.A. Battye

NATURAL UNITS

When using natural units one sets each of the constants $\hbar = 1.06 \times 10^{-27}\mathrm{cm}^2\,\mathrm{g}\,\mathrm{sec}^{-1}$, $k_B = 1.38 \times 10^{-16}\mathrm{erg}\mathrm{K}^{-1}$ and $c = 3.00 \times 10^{10}\mathrm{cm}\,\mathrm{sec}^{-1}$ equal to one. It is still possible to check the dimensions of a given equation, but one then has to understand that quantities such as those associated with velocity are dimensionless and that things like length and time have the same dimensions.

One can write every quantity in terms of powers of a single unit, conventionally GeV = 10^9eV. The conversion factors to the usual CGS and SI systems are:

$$
\begin{array}{rcl}
\mathrm{ENERGY} & : & 1\mathrm{GeV} = 1.60 \times 10^{-3}\mathrm{erg} = 1.6 \times 10^{-10}\mathrm{J} \\
\mathrm{TEMPERATURE} & : & 1\mathrm{GeV} = 1.16 \times 10^{13}\mathrm{K} \\
\mathrm{MASS} & : & 1\mathrm{GeV} = 1.78 \times 10^{-24}\mathrm{g} = 1.78 \times 10^{-27}\mathrm{kg} \\
\mathrm{LENGTH} & : & 1\mathrm{GeV}^{-1} = 1.97 \times 10^{-14}\mathrm{cm} = 1.97 \times 10^{-16}\mathrm{m} \\
\mathrm{TIME} & : & 1\mathrm{GeV}^{-1} = 6.58 \times 10^{-25}\mathrm{sec}
\end{array}
$$

Consider the present day critical density when the Hubble Constant is $100\mathrm{km}\,\mathrm{sec}^{-1}\mathrm{Mpc}^{-1}$ in these natural units.

$$\rho_{\mathrm{crit}} = 1.88 \times 10^{-29}\mathrm{gcm}^{-3}\,. \tag{1}$$

In natural units

$$\mathrm{gcm}^{-3} = \frac{1\mathrm{GeV}}{1.78 \times 10^{-24}}\left(\frac{1\mathrm{GeV}^{-1}}{1.97 \times 10^{-14}}\right)^{-3}\,, \tag{2}$$

and hence

$$\rho_{\mathrm{crit}} = 8.07 \times 10^{-47}\mathrm{GeV}^4\,. \tag{3}$$

Now consider the Hubble Constant, once again $100\mathrm{km}\,\mathrm{sec}^{-1}\,\mathrm{Mpc}^{-1}$. One can think of H_0^{-1} as either a length or a time (known as the Hubble radius and time respectively). Using the value of c one can show that

$$100\mathrm{km}\,\mathrm{sec}^{-1} = \frac{10^7}{3 \times 10^{10}} = \frac{1}{3000}\,, \tag{4}$$

and hence

$$H_0^{-1} = 3000\mathrm{Mpc} = 9 \times 10^{27}\left(\frac{6.58 \times 10^{-25}}{1.97 \times 10^{-14}}\right)\mathrm{sec} = 3 \times 10^{17}\mathrm{sec}\,. \tag{5}$$