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COSMOLOGICAL DISTANCE MEASURES

Luminosity Distance

The luminosity distance, $d_{\rm L}(z)$, by

$$F_{\rm obs} = \frac{L_{\rm emit}}{4\pi d_L^2}\,,$$

where $F_{\rm obs}$ is the observed flux and $L_{\rm emit}$ is the intrinsic luminosity.

In order to derive an expression for $d_{\rm L}$ consider the definition of the observed flux as

$$F_{\rm obs} = \frac{1}{A}\frac{dE_{\rm obs}}{dt_{\rm obs}}\,,$$

where A is the area over which the radiation spreads out. Within the FRW metric this is given by $A = 4\pi[r(z)]^2$ where $r(z)$ is the coordinate distance to redshift z.

Redshifting relates the observered and emitted energies and timescales, so that $E_{\rm obs} = E_{\rm emit}(1+z)^{-1}$ and $dt_{\rm obs} = (1+z)dt_{\rm emit}$.

Putting all this together gives

$$F_{\rm obs} = \frac{L_{\rm emit}}{4\pi[r(z)]^2(1+z)^2}\,,$$

and hence one can deduce that $d_{\rm L}(z) = (1+z)r(z)$.

Angular Diameter Distance

We define the angular diameter distance, $d_{\rm A}$, by

$$R = d_{\rm A}\theta\,,$$

where R is the comoving size of the object and θ is the observed angular size.

The physical size as observed today is scaled by the expansion, that is, $a(t_0)R/a(t)$. Using the convention that $a(t_0) = 1$, this is given by $R(1+z)$ and can be equated with $r(z)\theta$.

Hence, one can deduce that

$$d_{\rm A} = \frac{r(z)}{(1+z)}\,.$$