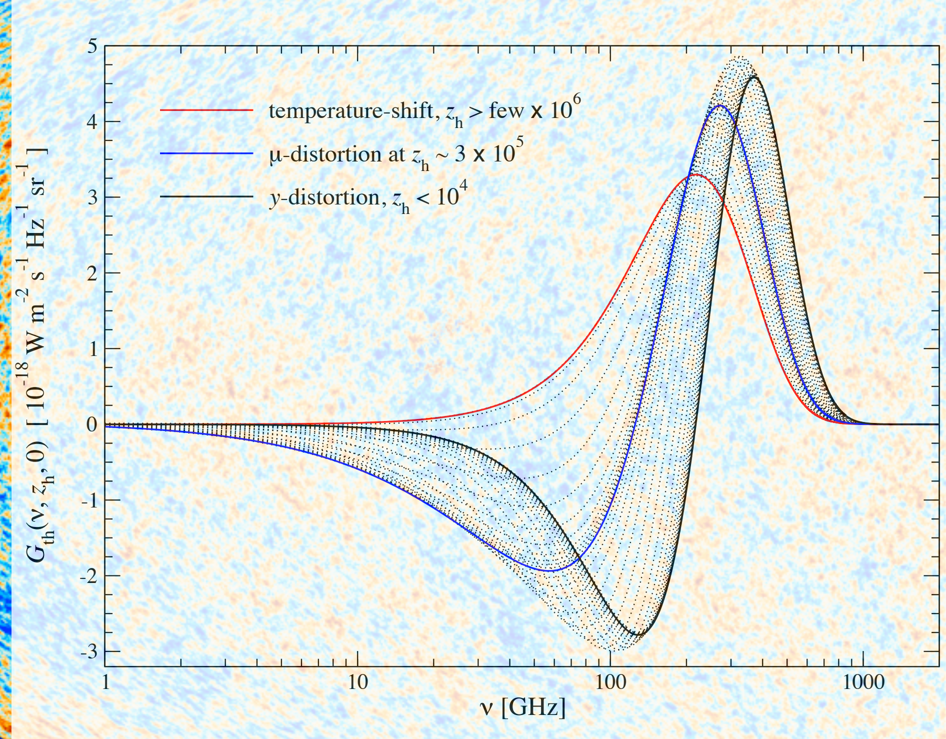
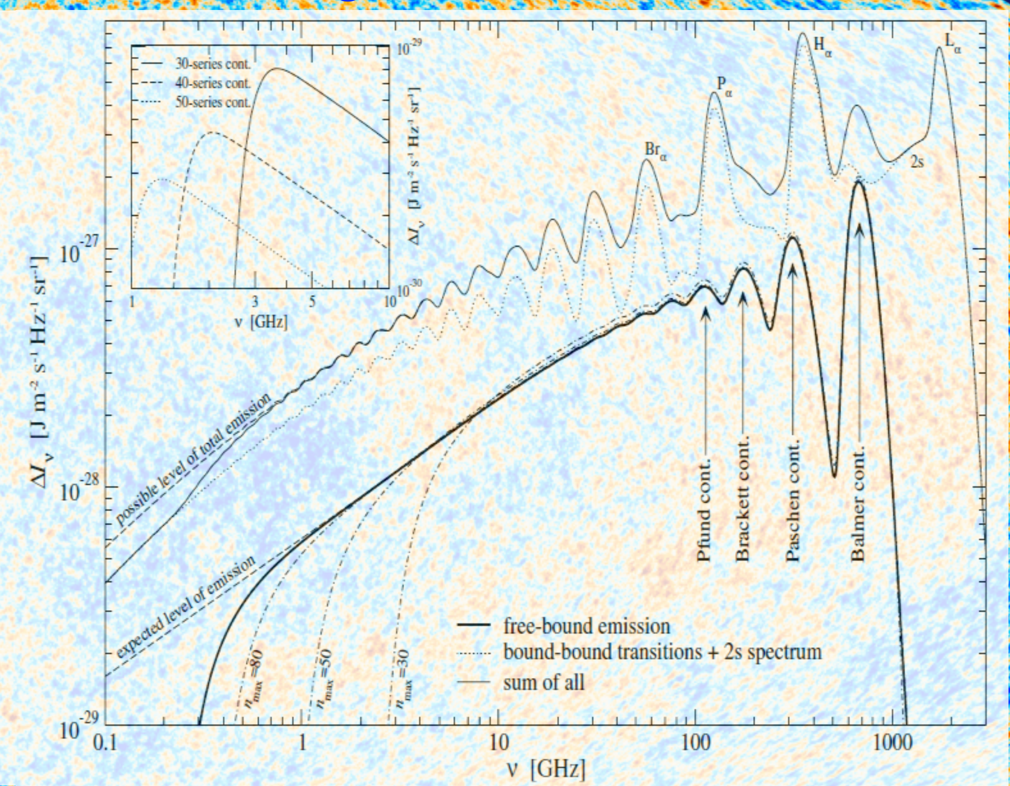
Future of Cosmology with CMB Spectral Distortions

Primordial Distortions



Cosmological Recombination lines



Jens Chluba

"Cosmology after Planck: what is next?"

Ecole de Physique, Les Houches, April 27th, 2016

MANCHESTER 1824
The University of Manchester

THE ROYAL SOCIETY

Main Goals of the lectures

- Convince you that future CMB distortions science will be *extremely* exciting and lots of fun!
- Explain in detail how distortions evolve and thermalize
- Definition of different types of distortions (μ, *y* and *r*)
- Computations of spectral distortions (*afterwards you should be able to do simple estimates yourself...*)
- Provide an overview for different sources of primordial distortions
- Show you why CMB spectral distortions provide a *complementary* probe of inflation and particle physics

References for the Theory of Spectral Distortions

- Original works
  - Zeldovich & Sunyaev, 1969, Ap&SS, 4, 301
  - Sunyaev & Zeldovich, 1970, Ap&SS, 7, 20
  - Illarionov & Sunyaev, 1975, Sov. Astr., 18, 413

Yakov Zeldovich

Rashid Sunyaev

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- Additional important milestones
  - Danese & de Zotti, 1982, A&A, 107, 39
  - Burigana, Danese & de Zotti, 1991, ApJ, 379, 1
  - Hu & Silk, 1993, Phys. Rev. D, 48, 485
  - Hu, 1995, PhD thesis

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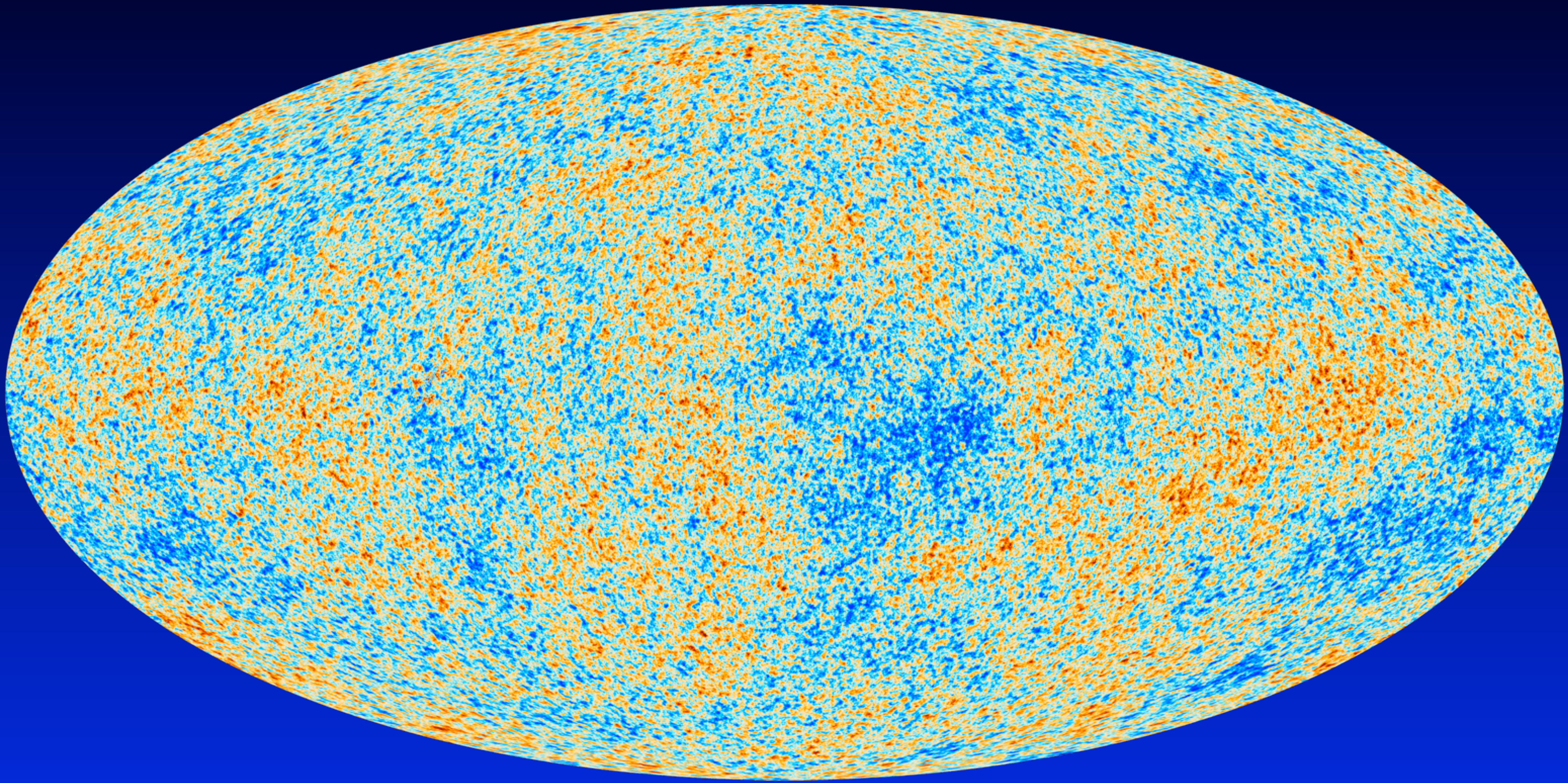
  - Burigana, Danese & de Zotti, 1991, ApJ, 379, 1
  - Hu & Silk, 1993, Phys. Rev. D, 48, 485
  - Hu, 1995, PhD thesis
- More recent overviews
  - Sunyaev & JC, 2009, AN, 330, 657
  - JC & Sunyaev, 2012, MNRAS, 419, 1294
  - JC, 2013, MNRAS, 436, 2232 & ArXiv:1405.6938

see also, *Lecture* notes at:
www.Chluba.de/Science

Part I: Theory of CMB spectral distortions

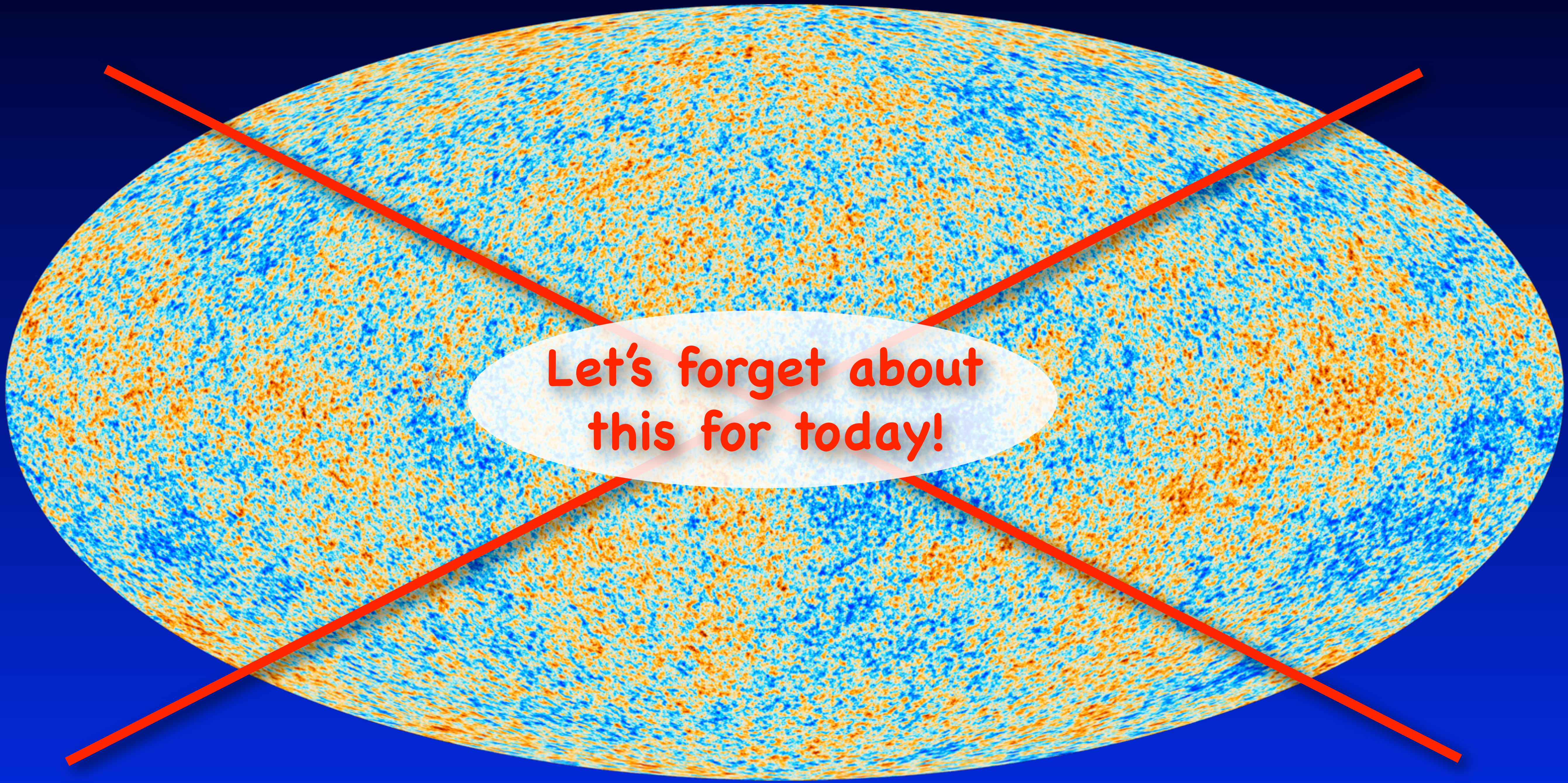
Cosmic Microwave Background Anisotropies



Planck all-sky
temperature map

- CMB has a blackbody spectrum in every direction
- tiny variations of the CMB temperature $\Delta T/T \sim 10^{-5}$

Cosmic Microwave Background Anisotropies



Planck all-sky temperature map

- CMB has a blackbody spectrum in every direction
- tiny variations of the CMB temperature $\Delta T/T \sim 10^{-5}$

CMB provides another independent piece of information!

COBE/FIRAS

$$T_0 = (2.726 \pm 0.001)\,\mathrm{K}$$

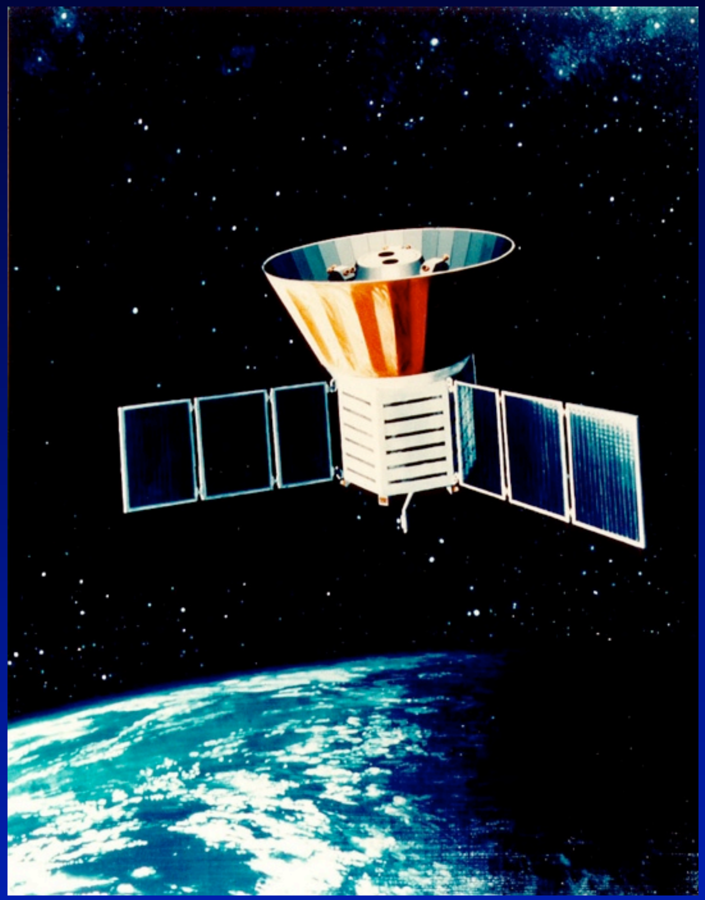
Absolute measurement required!

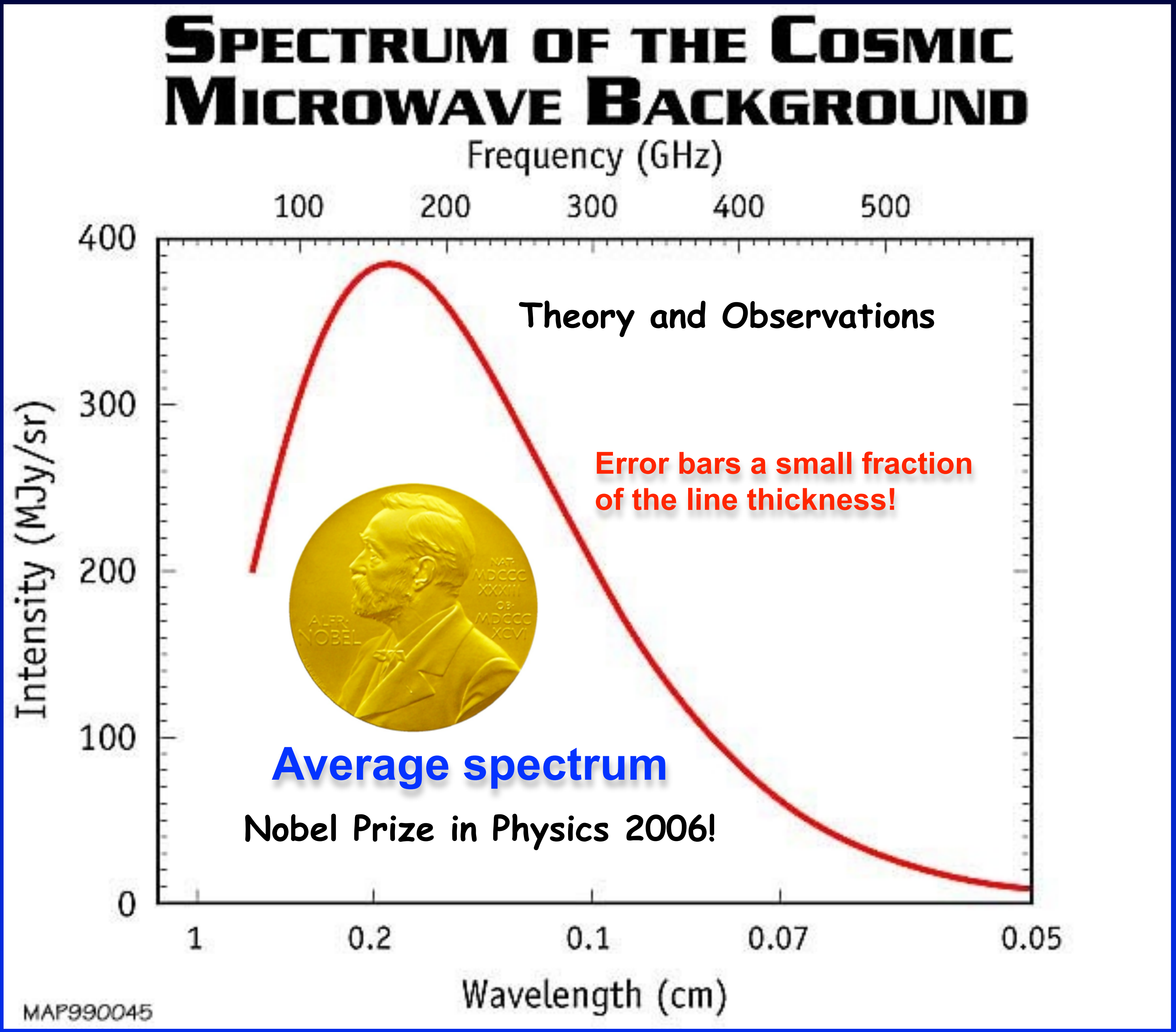
One has to go to space...

Mather et al., 1994, ApJ, 420, 439
Fixsen et al., 1996, ApJ, 473, 576
Fixsen, 2003, ApJ, 594, 67
Fixsen, 2009, ApJ, 707, 916

- CMB monopole is 10000 - 100000 times larger than the fluctuations

COBE / FIRAS (Far InfraRed Absolute Spectrophotometer)





$$T_0 = 2.725 \pm 0.001\,\mathrm{K}$$

$$|y| \leq 1.5\times10^{-5}$$

$$|\mu| \leq 9\times10^{-5}$$

Mather et al., 1994, ApJ, 420, 439
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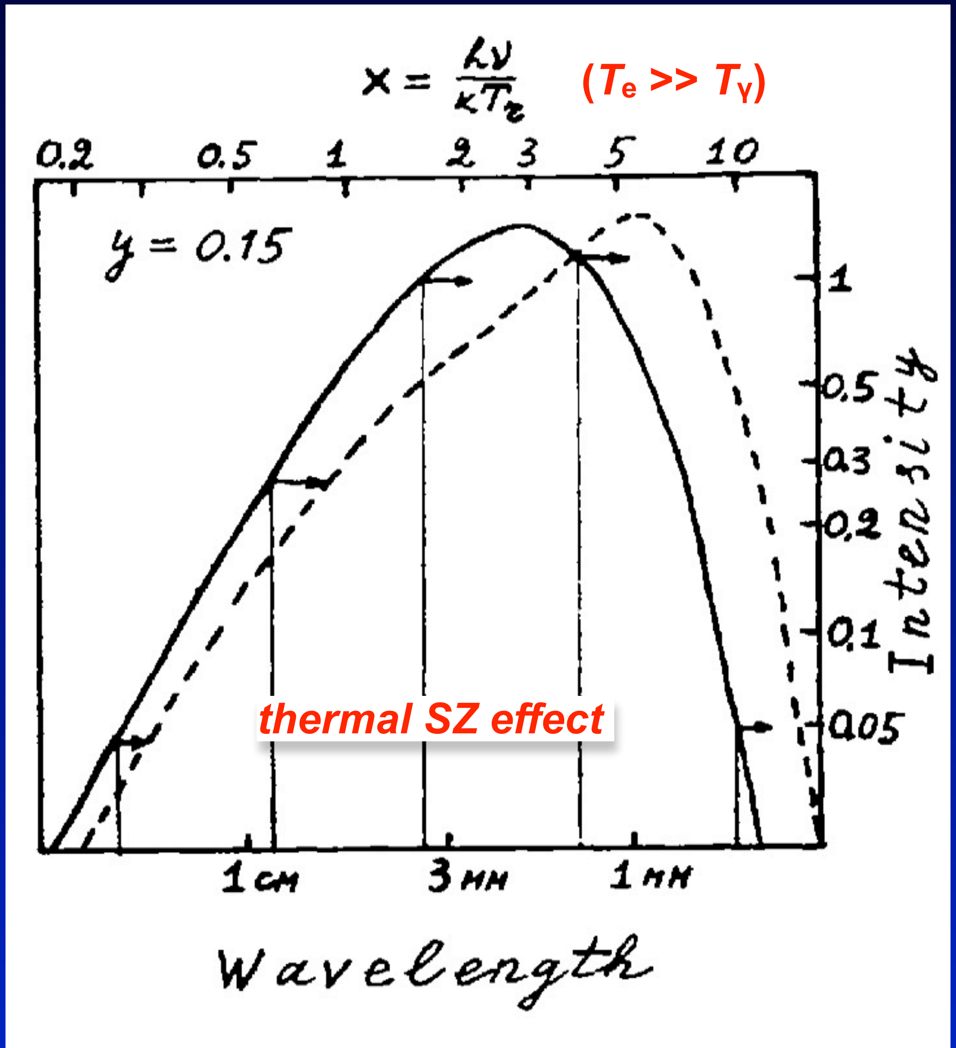
Standard types of primordial CMB distortions

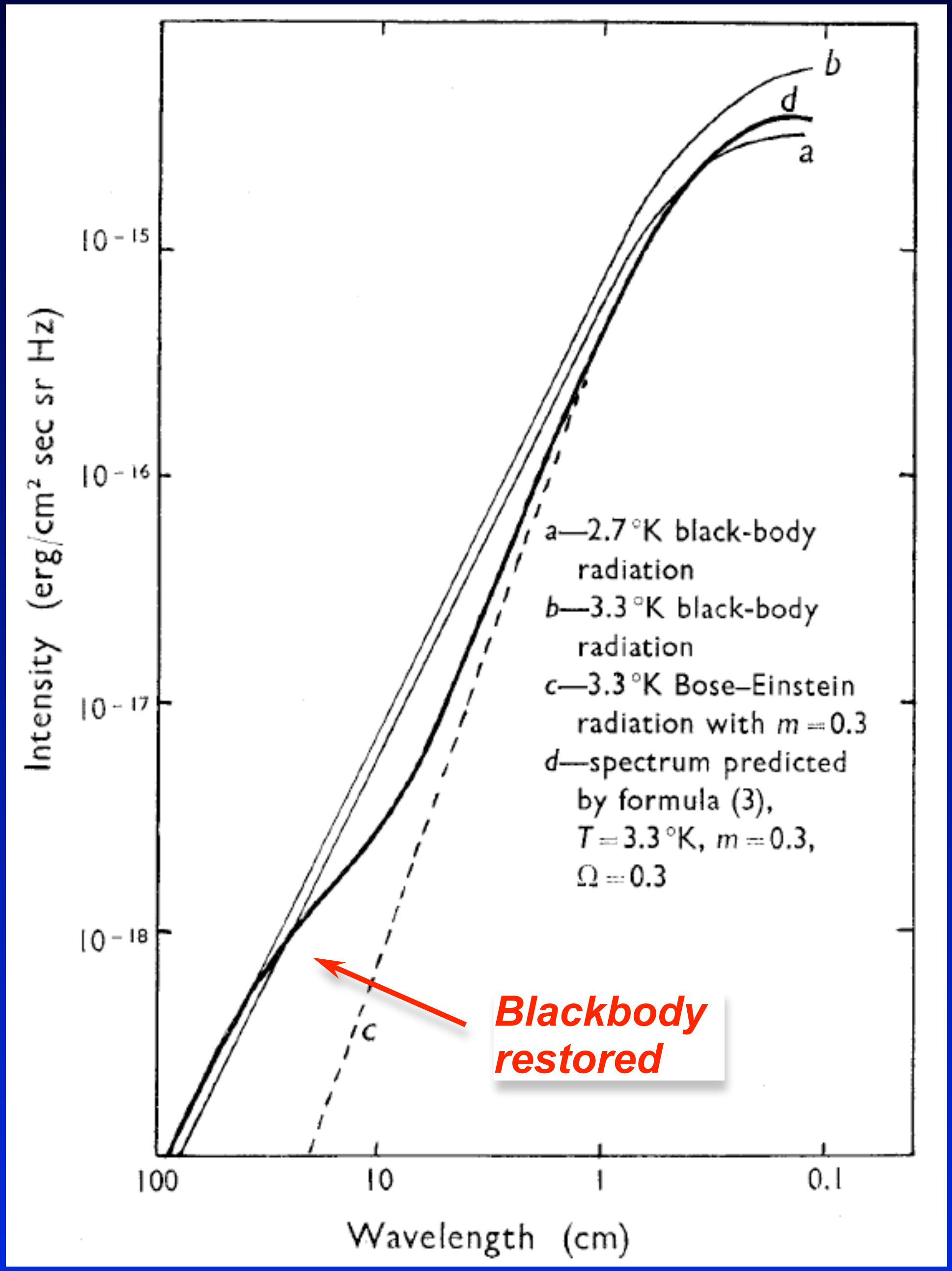
Compton y-distortion



Sunyaev & Zeldovich, 1980, ARAA, 18, 537

- also known from thSZ effect
- up-scattering of CMB photon
- important at late times (z<50000)
- scattering inefficient

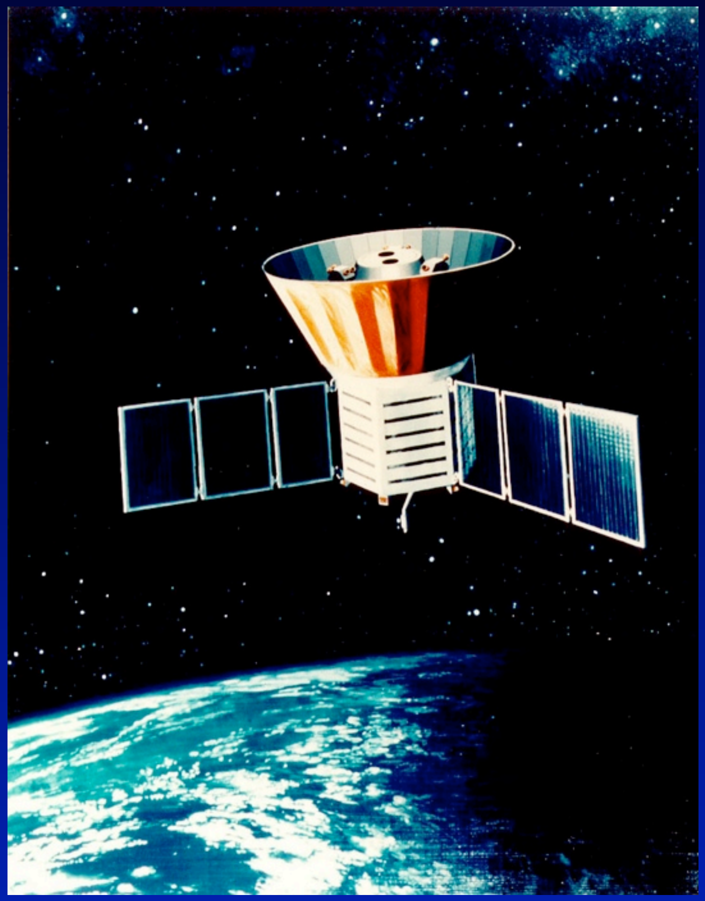
Chemical potential μ-distortion

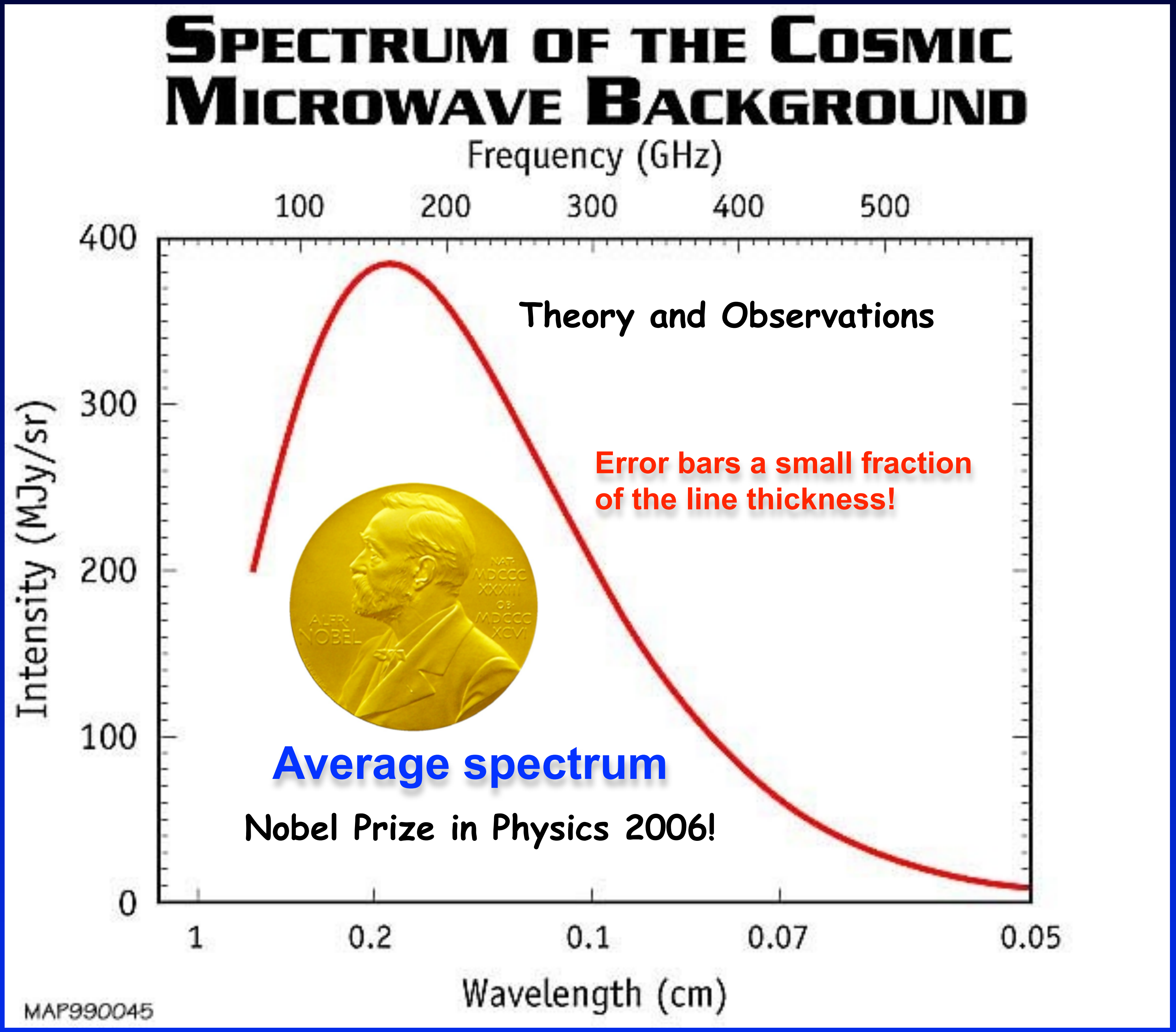


Sunyaev & Zeldovich, 1970, ApSS, 2, 66

- important at very times (z>50000)
- scattering very efficient

COBE / FIRAS (Far InfraRed Absolute Spectrophotometer)





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Only very small distortions of CMB spectrum are still allowed!

No primordial distortion found so far!? Why are we at all talking about this then?

Physical mechanisms that lead to spectral distortions

- *Cooling by adiabatically expanding ordinary matter*
  (JC, 2005; JC & Sunyaev 2011; Khatri, Sunyaev & JC, 2011)
- Heating by *decaying* or *annihilating* relic particles
  (Kawasaki et al., 1987; Hu & Silk, 1993; McDonald et al., 2001; JC, 2005; JC & Sunyaev, 2011; JC, 2013; JC & Jeong, 2013)
- *Evaporation of primordial black holes & superconducting strings*
  (Carr et al. 2010; Ostriker & Thompson, 1987; Tashiro et al. 2012; Pani & Loeb, 2013)
- *Dissipation of primordial acoustic modes & magnetic fields*
  (Sunyaev & Zeldovich, 1970; Daly 1991; Hu et al. 1994; JC & Sunyaev, 2011; JC et al. 2012 - Jedamzik et al. 2000; Kunze & Komatsu, 2013)
- *Cosmological recombination radiation*
  (Zeldovich et al., 1968; Peebles, 1968; Dubrovich, 1977; Rubino-Martin et al., 2006; JC & Sunyaev, 2006; Sunyaev & JC, 2009)

„high“ redshifts ↑ *pre-recombination epoch*

„low“ redshifts ↓ *post-recombination*

- Signatures due to first supernovae and their remnants
  (Oh, Cooray & Kamionkowski, 2003)
- Shock waves arising due to large-scale structure formation
  (Sunyaev & Zeldovich, 1972; Cen & Ostriker, 1999)
- SZ-effect from clusters; effects of reionization
  (Refregier et al., 2003; Zhang et al. 2004; Trac et al. 2008)
- more exotic processes
  (Lochan et al. 2012; Bull & Kamionkowski, 2013; Brax et al., 2013; Tashiro et al. 2013)

Physical mechanisms that lead to spectral distortions

Standard sources of distortions

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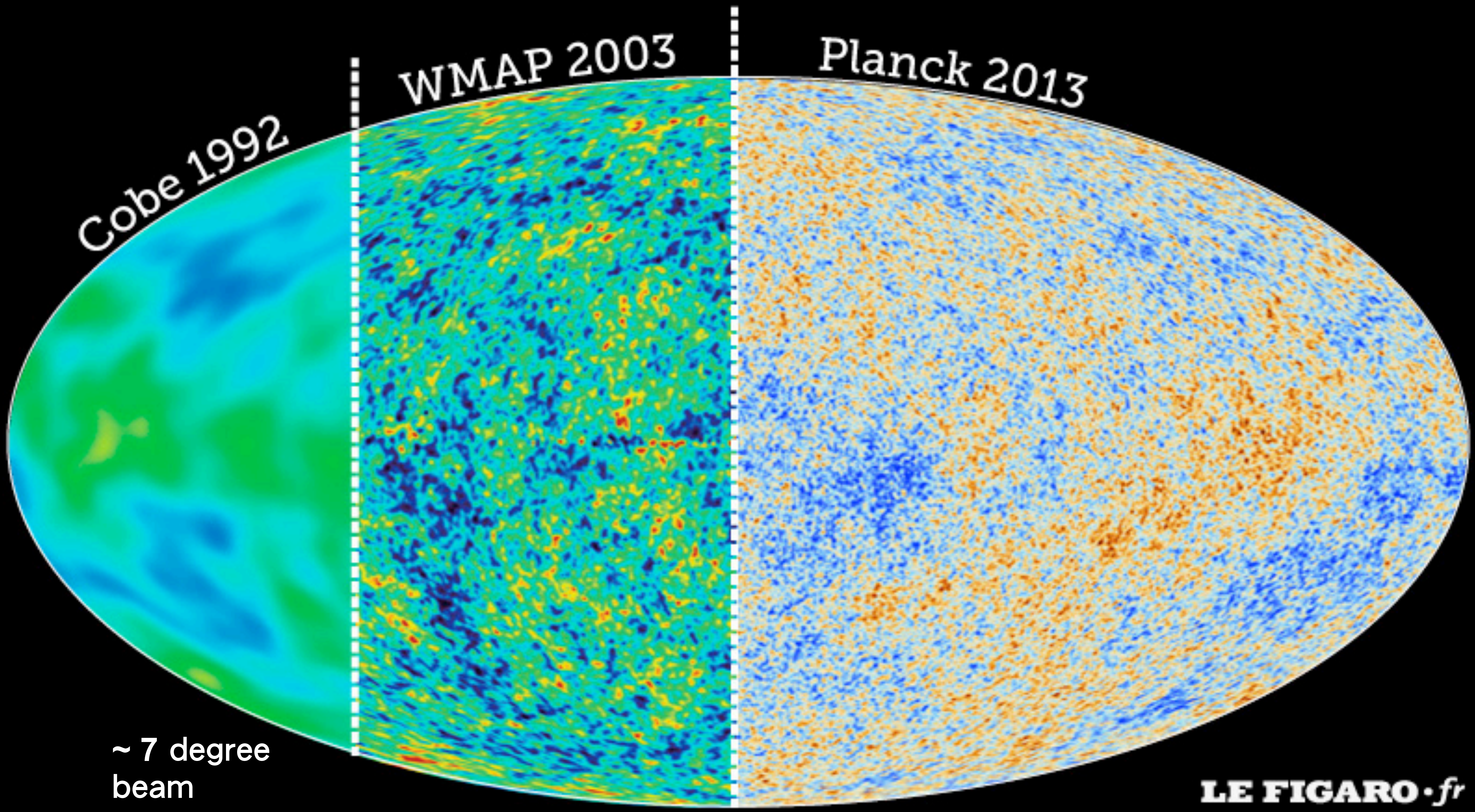
Dramatic improvements in angular resolution and sensitivity over the past decades!



~ 7 degree beam

~ 0.3 degree beam

~ 0.08 degree beam

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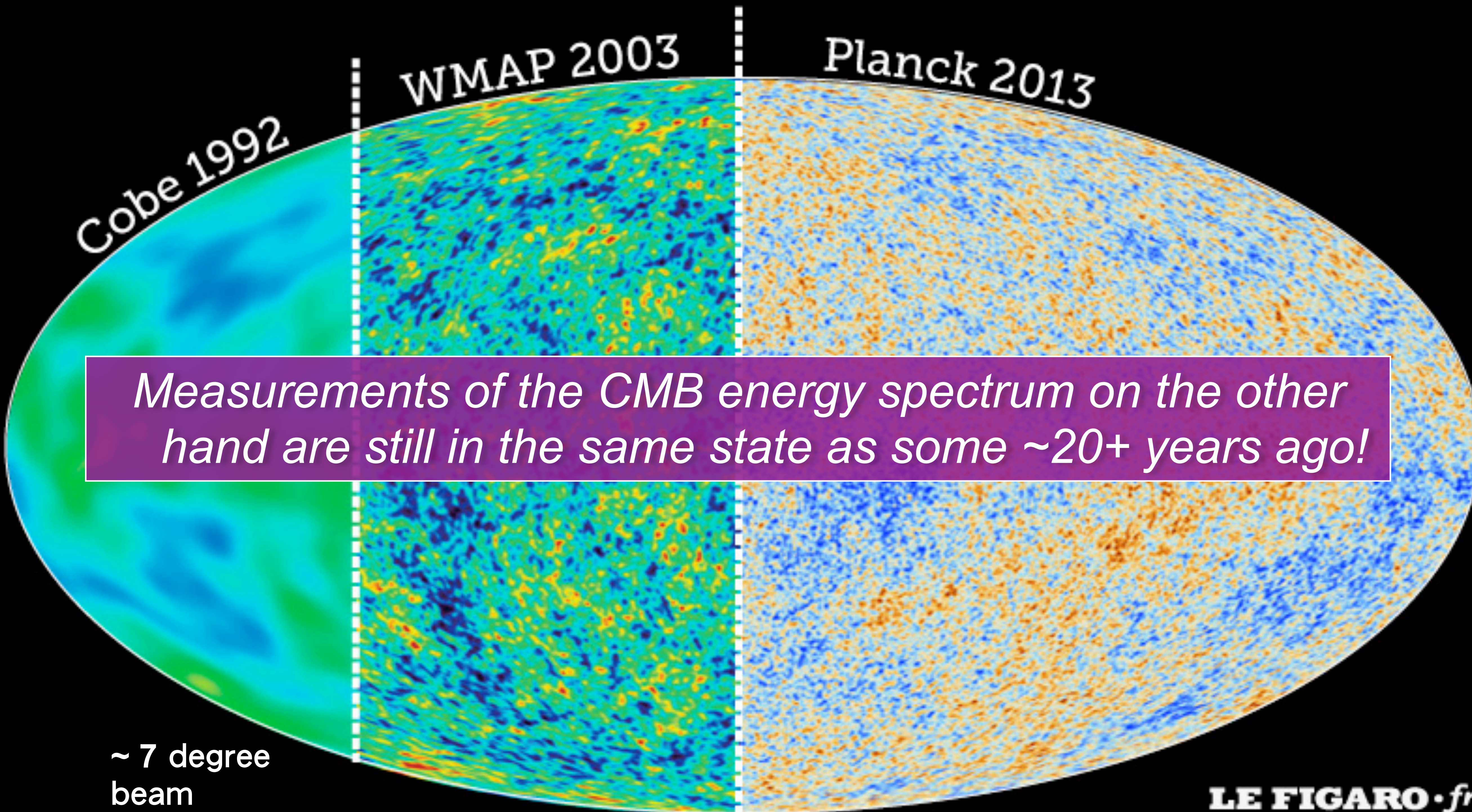


Measurements of the CMB energy spectrum on the other hand are still in the same state as some ~20+ years ago!

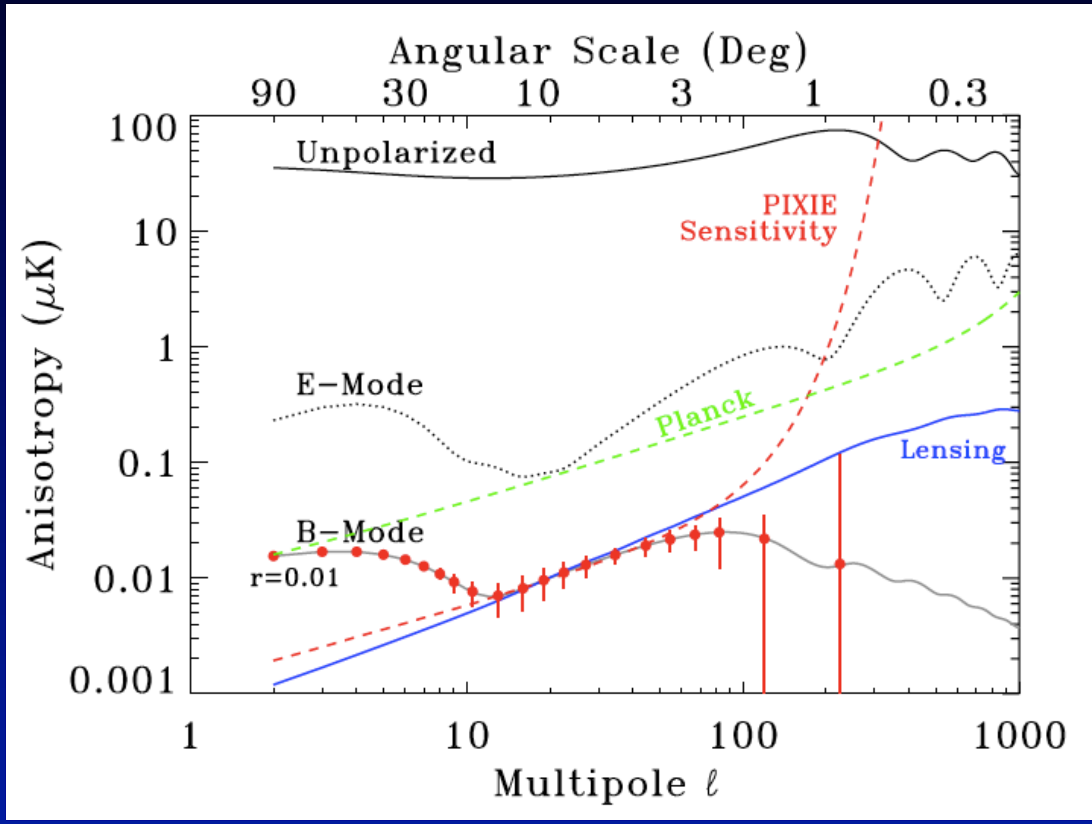
~ 7 degree beam

~ 0.3 degree beam

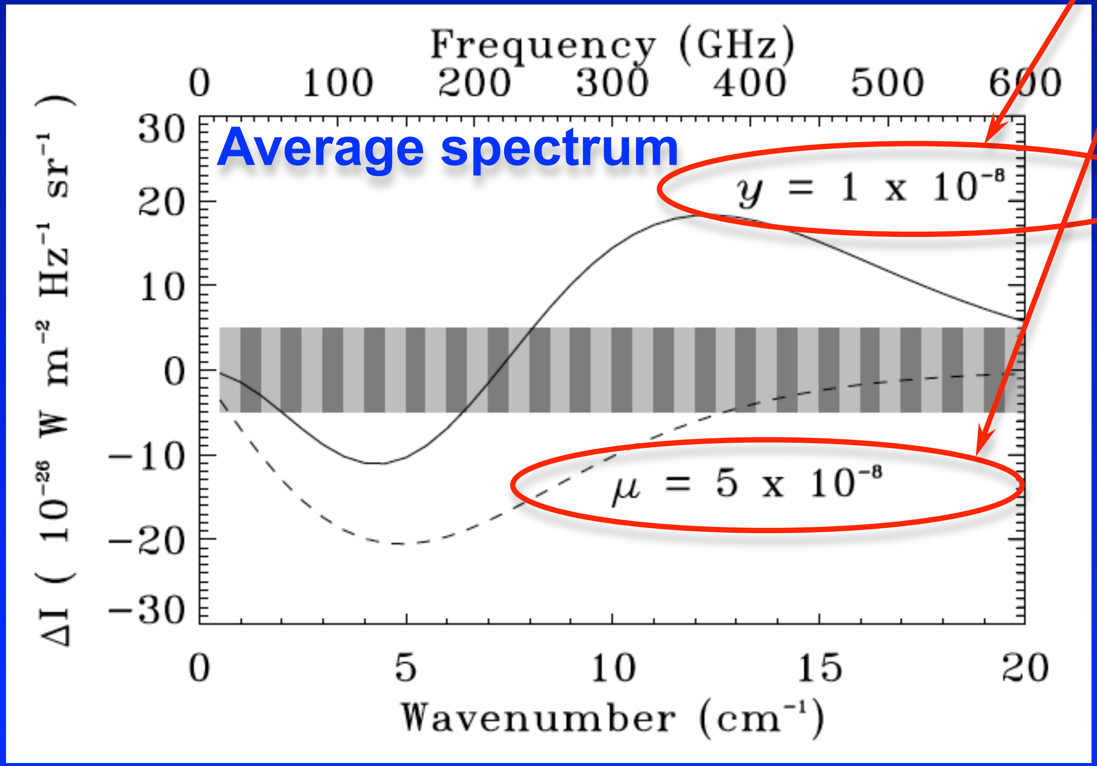
~ 0.08 degree beam

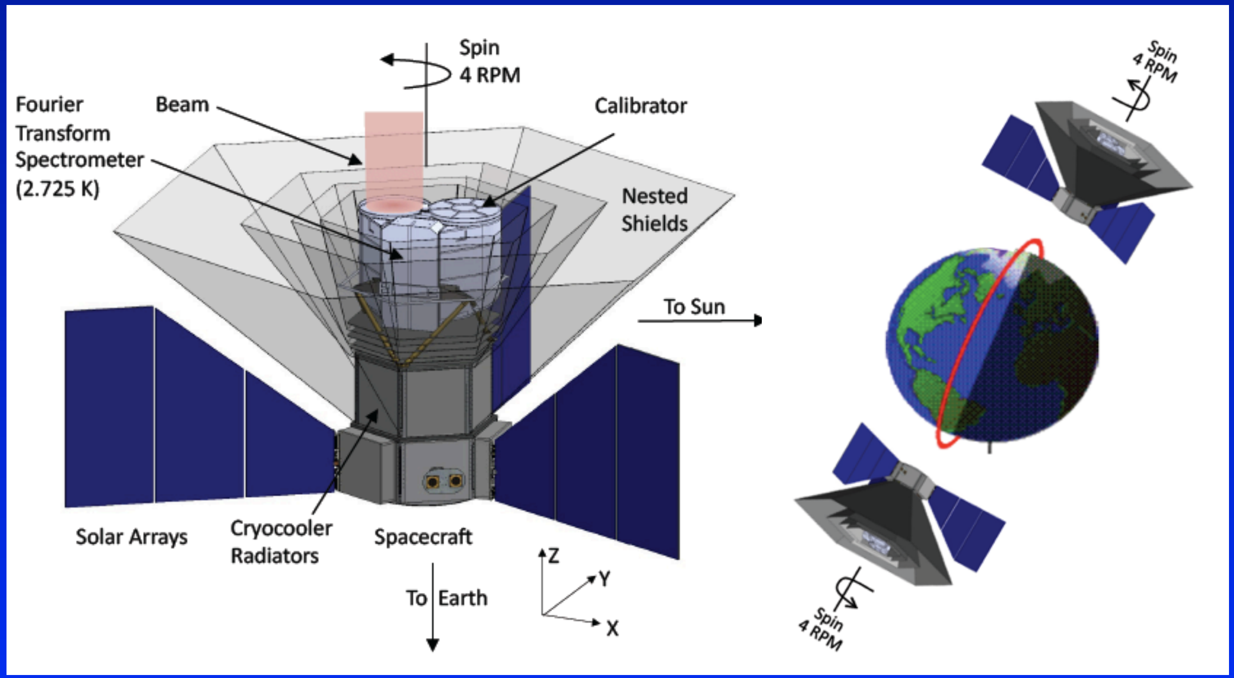
LE FIGARO·fr

PIXIE: Primordial Inflation Explorer



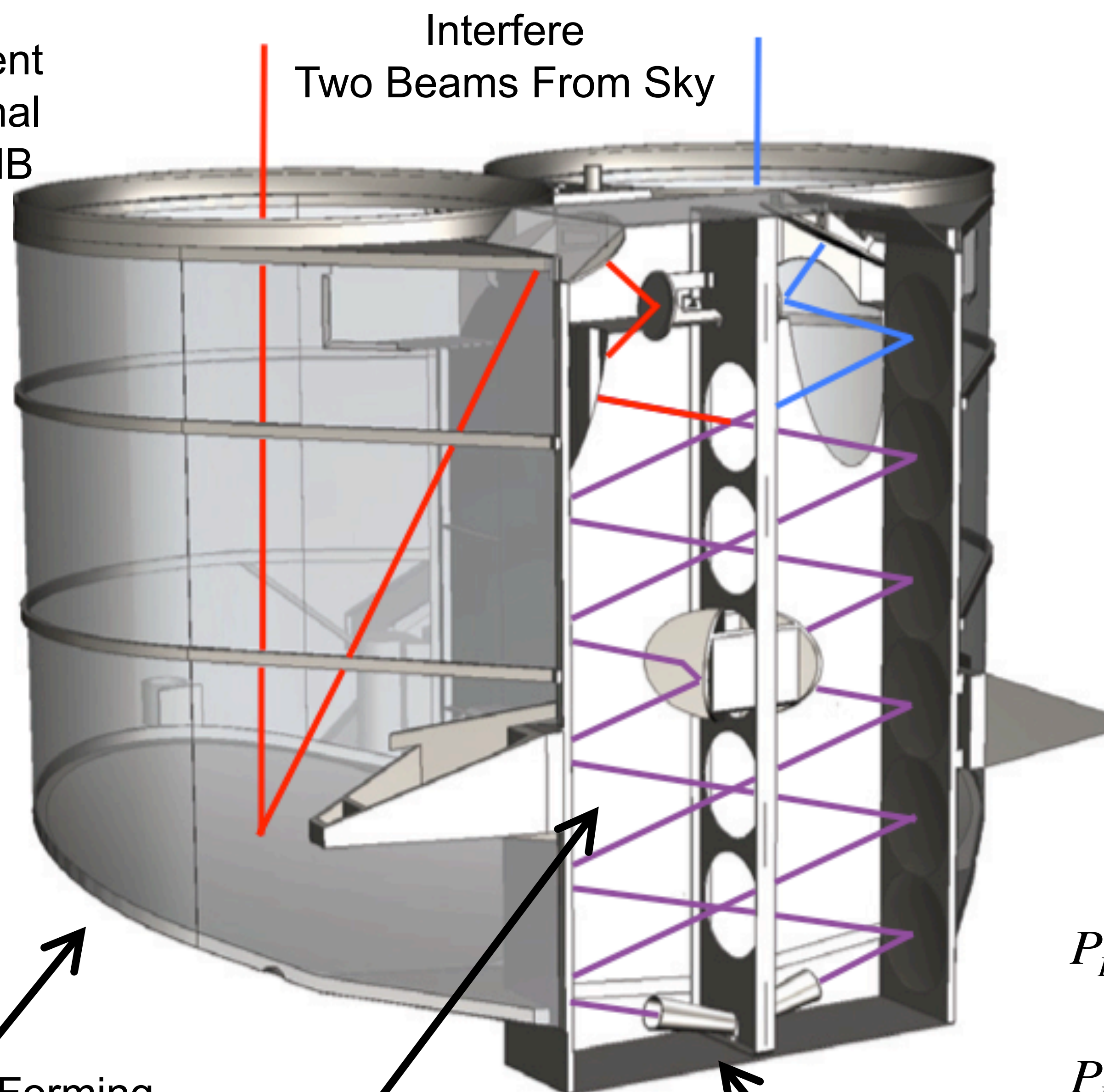
- 400 spectral channel in the frequency range 30 GHz and 6THz (Δv ~ 15GHz)
- about 1000 (!!!) times more sensitive than COBE/FIRAS
- B-mode polarization from inflation ($r \approx 10^{-3}$)
- improved limits on μ and y
- was proposed 2011 as NASA EX mission (i.e. cost ~ 200 M$)

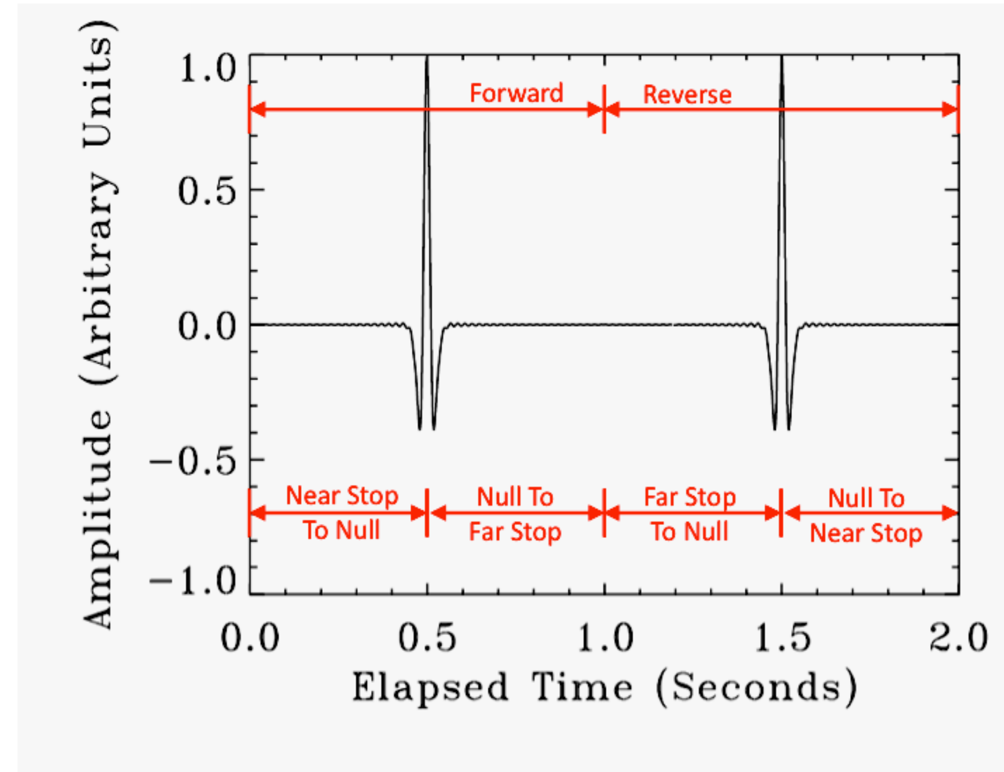




Kogut et al, JCAP, 2011, arXiv:1105.2044

PIXIE Nulling Polarimeter





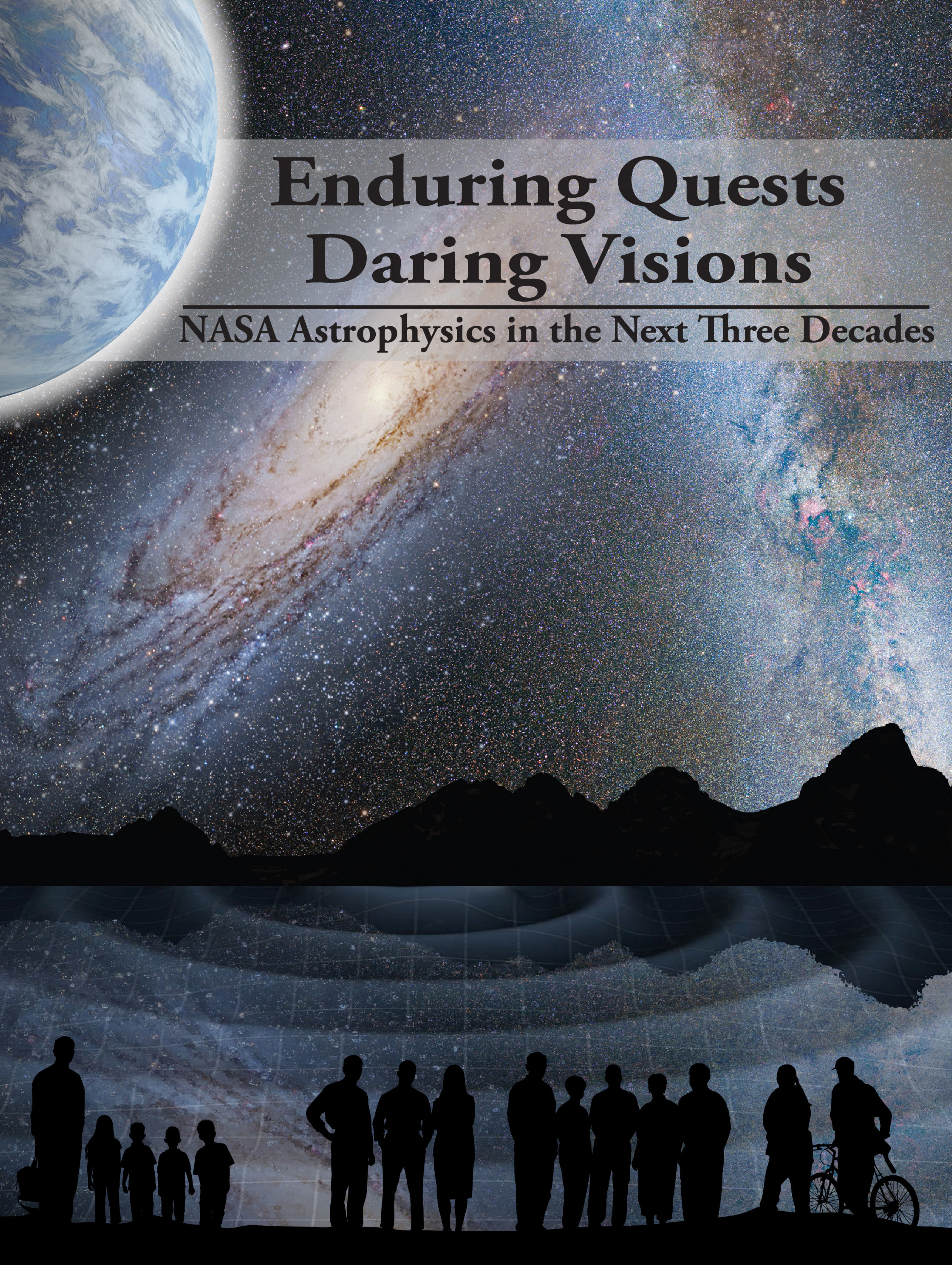
Measured Fringe Pattern Samples Frequency Spectrum of Polarized Sky Emission

$$P_{Lx} = \frac{1}{2}\int \left(E_{Ay}^2 + E_{Bx}^2\right) + \left(E_{Bx}^2 - E_{Ay}^2\right)\cos(z\omega/c)\, d\omega$$

$$P_{Ly} = \frac{1}{2}\int \left(E_{Ax}^2 + E_{By}^2\right) + \underbrace{\left(E_{By}^2 - E_{Ax}^2\right)}_{\text{Stokes Q}}\cos(z\omega/c)\, d\omega$$

FIRAS With Polarization!

courtesy Al Kogut



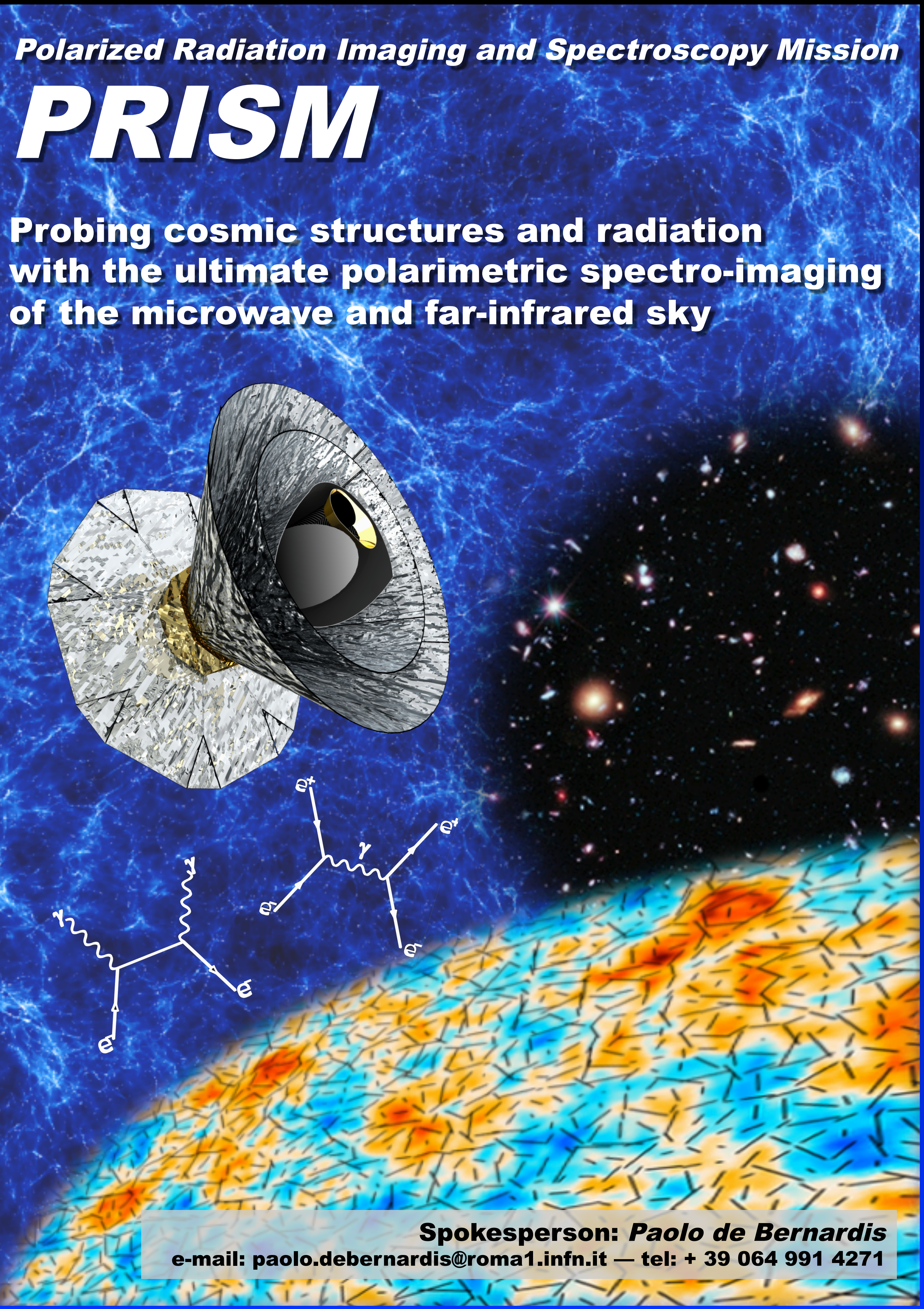
NASA 30-yr Roadmap Study

(published Dec 2013)

How does the Universe work?

"Measure the spectrum of the CMB with precision several orders of magnitude higher than COBE FIRAS, from a moderate-scale mission or an instrument on CMB Polarization Surveyor."

New call from NASA expected end 2016



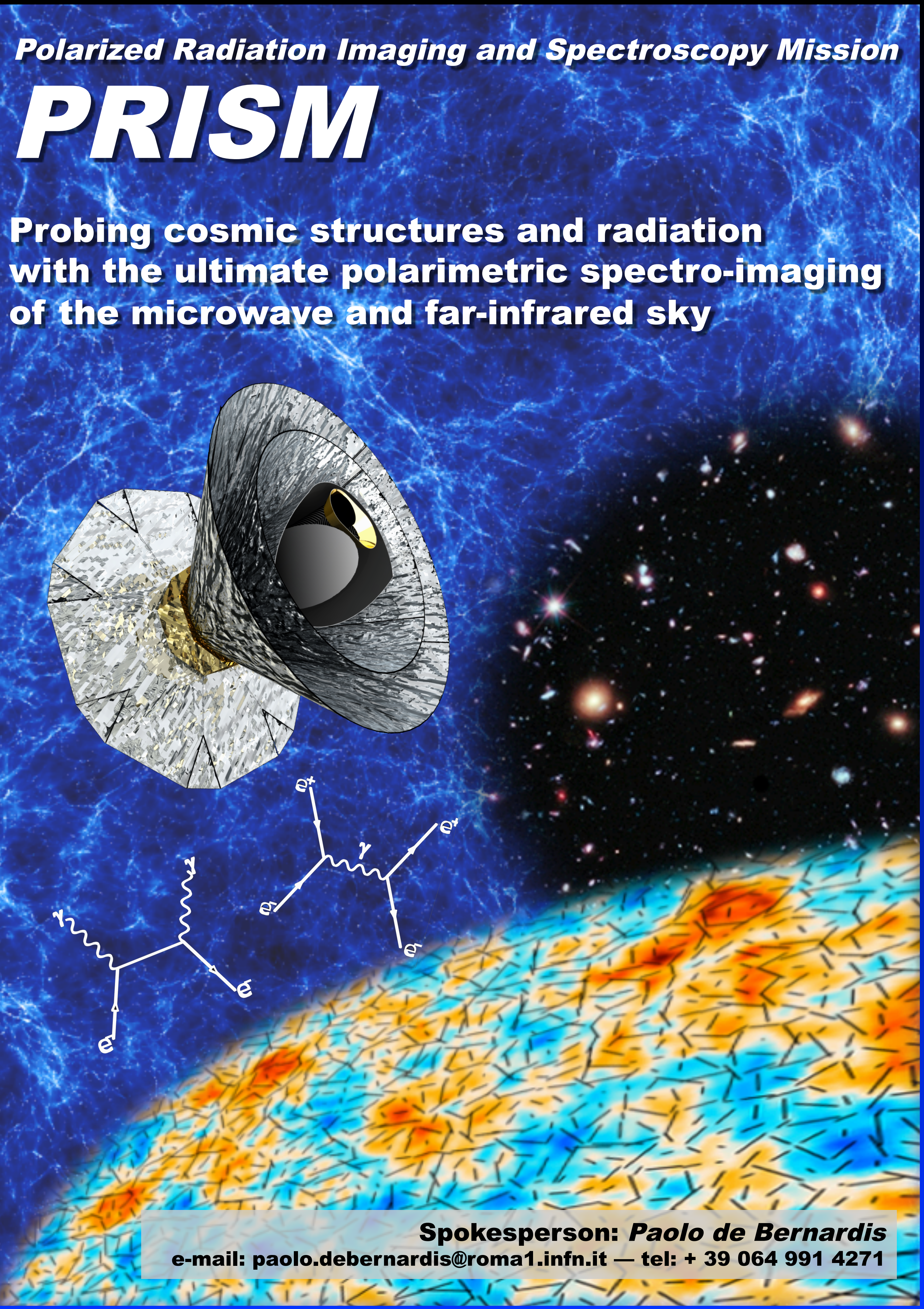
Instruments:

- L-class ESA mission
- White paper, May 24th, 2013
- Imager:
  - polarization sensitive
  - 3.5m telescope [arcmin resolution at highest frequencies]
  - 30GHz-6THz [30 broad ($\Delta\nu/\nu$~25%) and 300 narrow ($\Delta\nu/\nu$~2.5%) bands]
- Spectrometer:
  - FTS similar to PIXIE
  - 30GHz-6THz ($\Delta\nu$~15 & 0.5 GHz)

Some of the science goals:

- B-mode polarization from inflation ($r \approx 5\text{x}10^{-4}$)
- count all SZ clusters >10^{14} M_{sun}
- CIB/large scale structure
- Galactic science
- *CMB spectral distortions*

More info at:
http://www.prism-mission.org/



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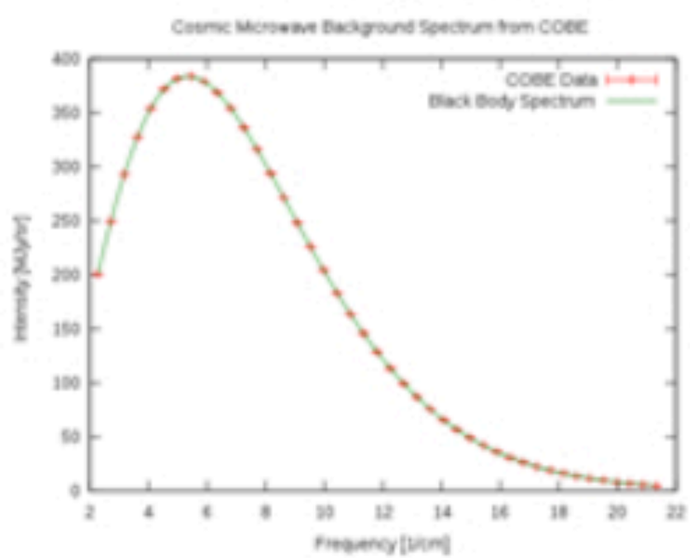
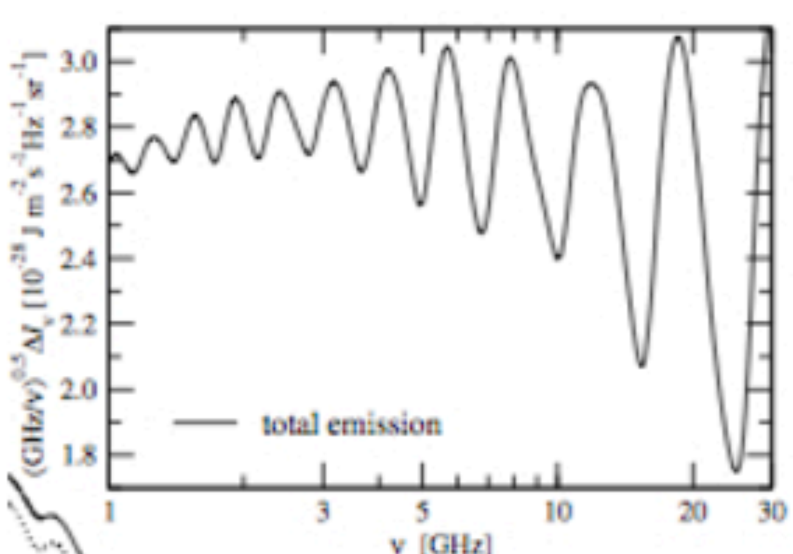
Array of Precision Spectrometers for detecting spectral ripples from the Epoch of RecombinAtion

HOME PEOPLE

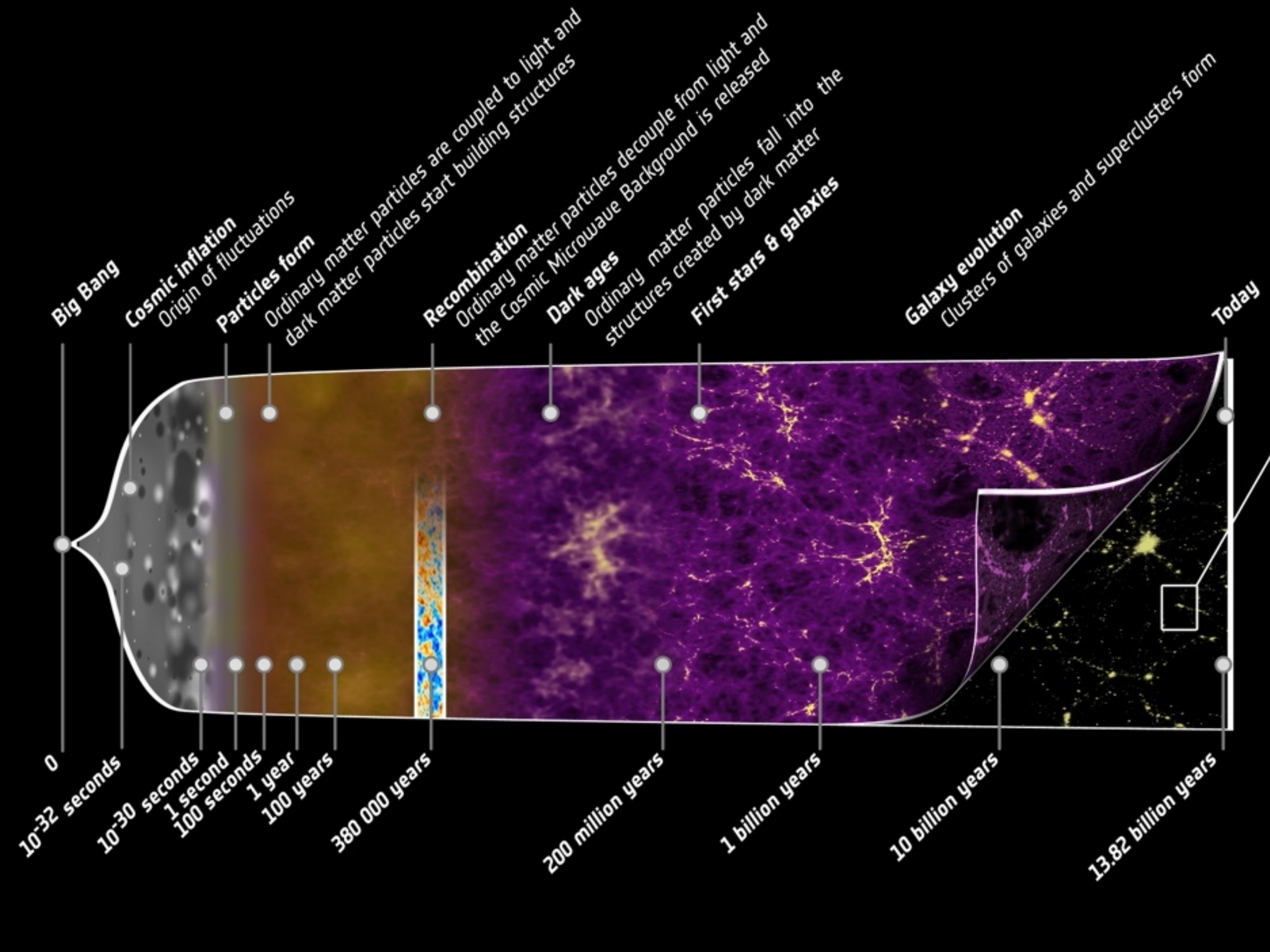




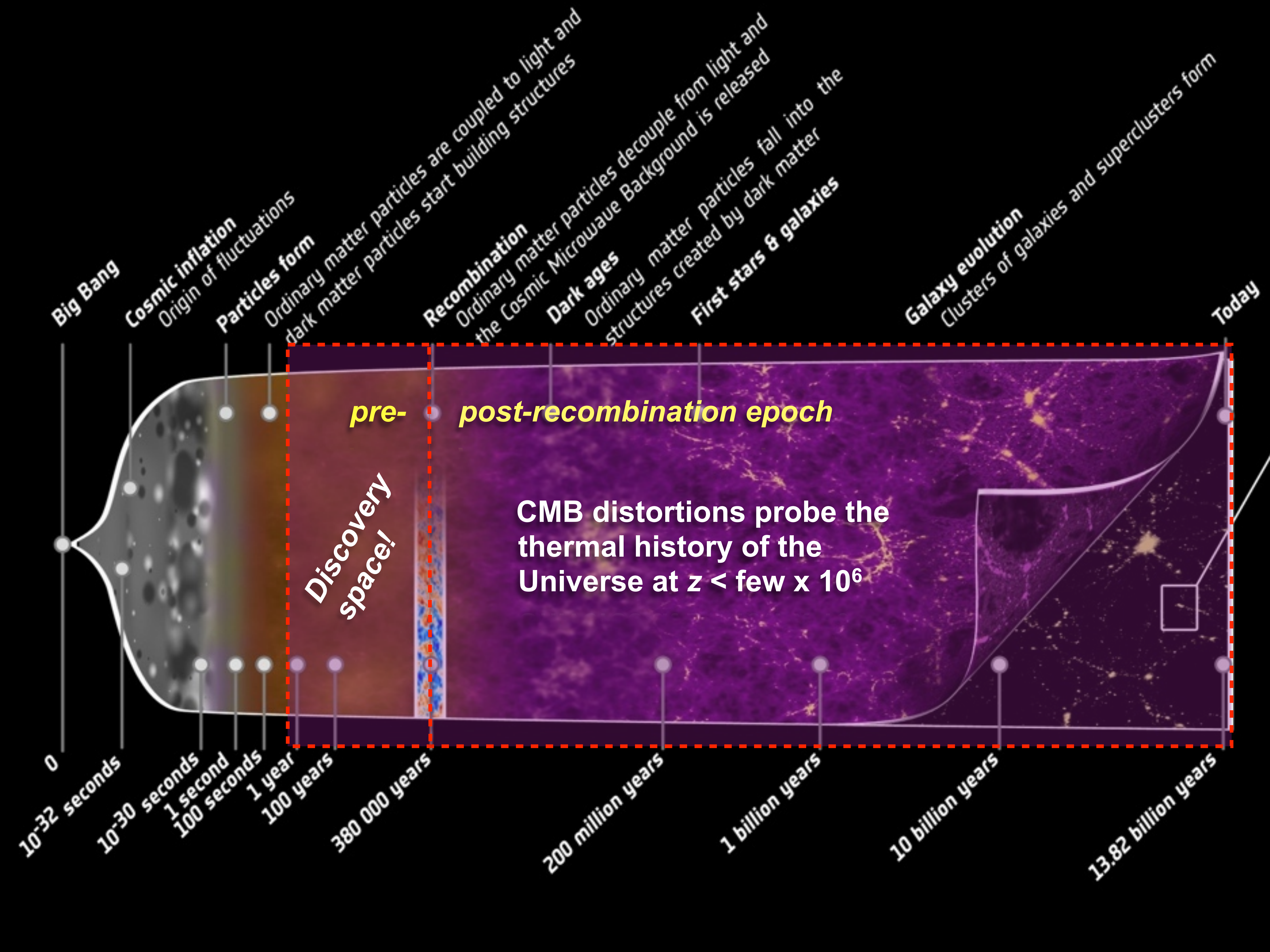
About APSERa

The Array of Precision Spectrometers for the Epoch of RecombinAtion - APSERa - is a venture to detect recombination lines from the Epoch of Cosmological Recombination. These are predicted to manifest as 'ripples' in wideband spectra of the cosmic radio background (CRB) since recombination of the primeval plasma in the early Universe adds broad spectral lines to the relic Cosmic Radiation. The lines are extremely wide because recombination is stalled and extended over redshift space. The spectral features are expected to be isotropic over the whole sky.

The project will comprise of an array of 128 small telescopes that are purpose built to detect a set of adjacent lines from cosmological recombination in the spectrum of the radio sky in the 2-6 GHz range. The radio receivers are being designed and built at the Raman Research Institute, tested in nearby radio-quiet locations and relocated to a remote site for long duration exposures to detect the subtle features in the cosmic radio background arising from recombination. The observing site would be appropriately chosen to minimize RFI from geostationary satellites and to be able to observe towards sky regions relatively low in foreground brightness.



Big Bang
Cosmic inflation
Origin of fluctuations
Particles form
Ordinary matter particles are coupled to light and dark matter particles start building structures
Recombination
Ordinary matter particles decouple from light and the Cosmic Microwave Background is released
Dark ages
Ordinary matter particles fall into the structures created by dark matter
First stars & galaxies
Galaxy evolution
Clusters of galaxies and superclusters form
Today
0
10^{-32} seconds
10^{-30} seconds
1 second
100 seconds
1 year
100 years
380 000 years
200 million years
1 billion years
10 billion years
13.82 billion years



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Clusters of galaxies and superclusters form
Today
pre-
post-recombination epoch
Discovery space!
CMB distortions probe the thermal history of the Universe at z < few x 10^6
0
10^-32 seconds
10^-30 seconds
1 second
100 seconds
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380 000 years
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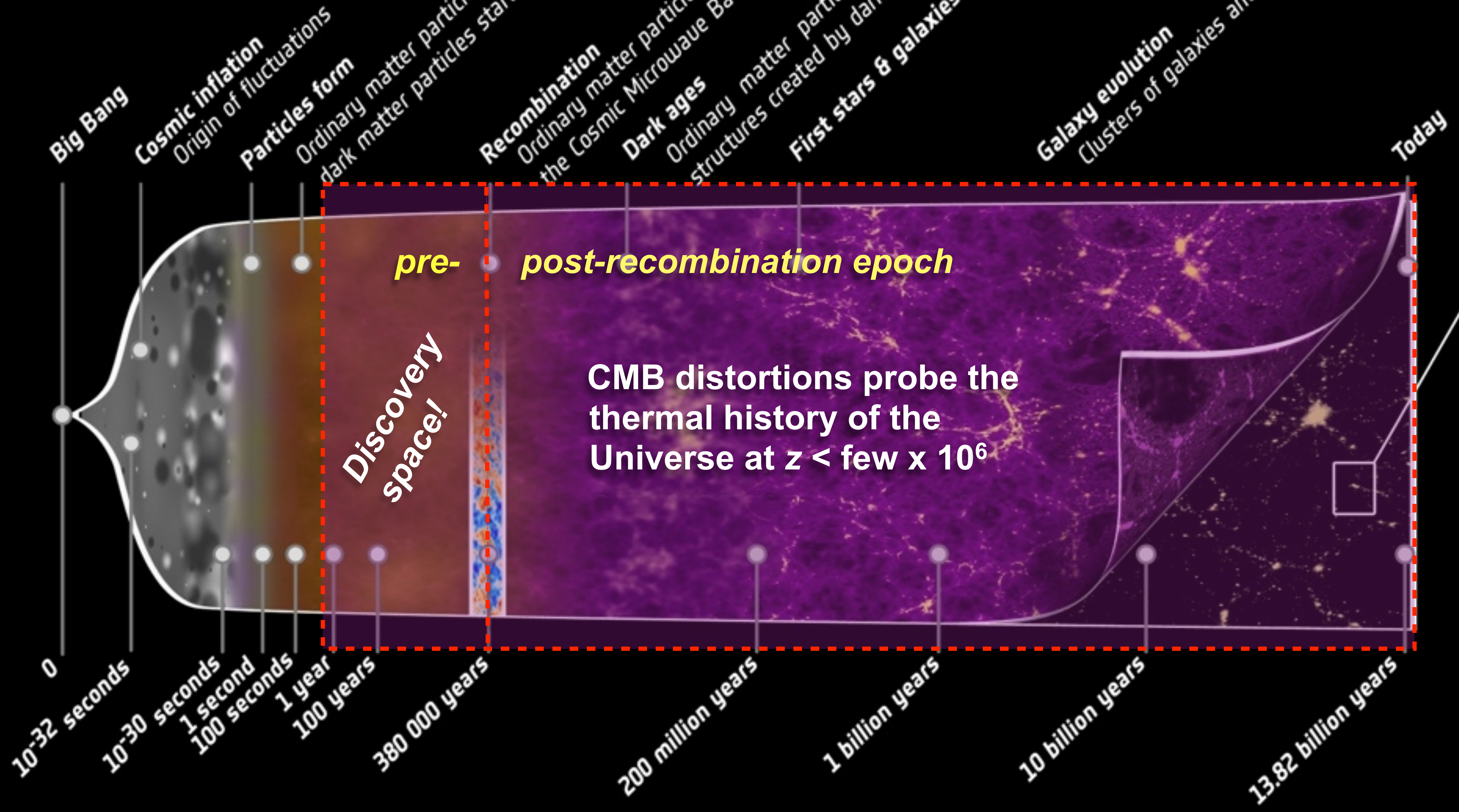
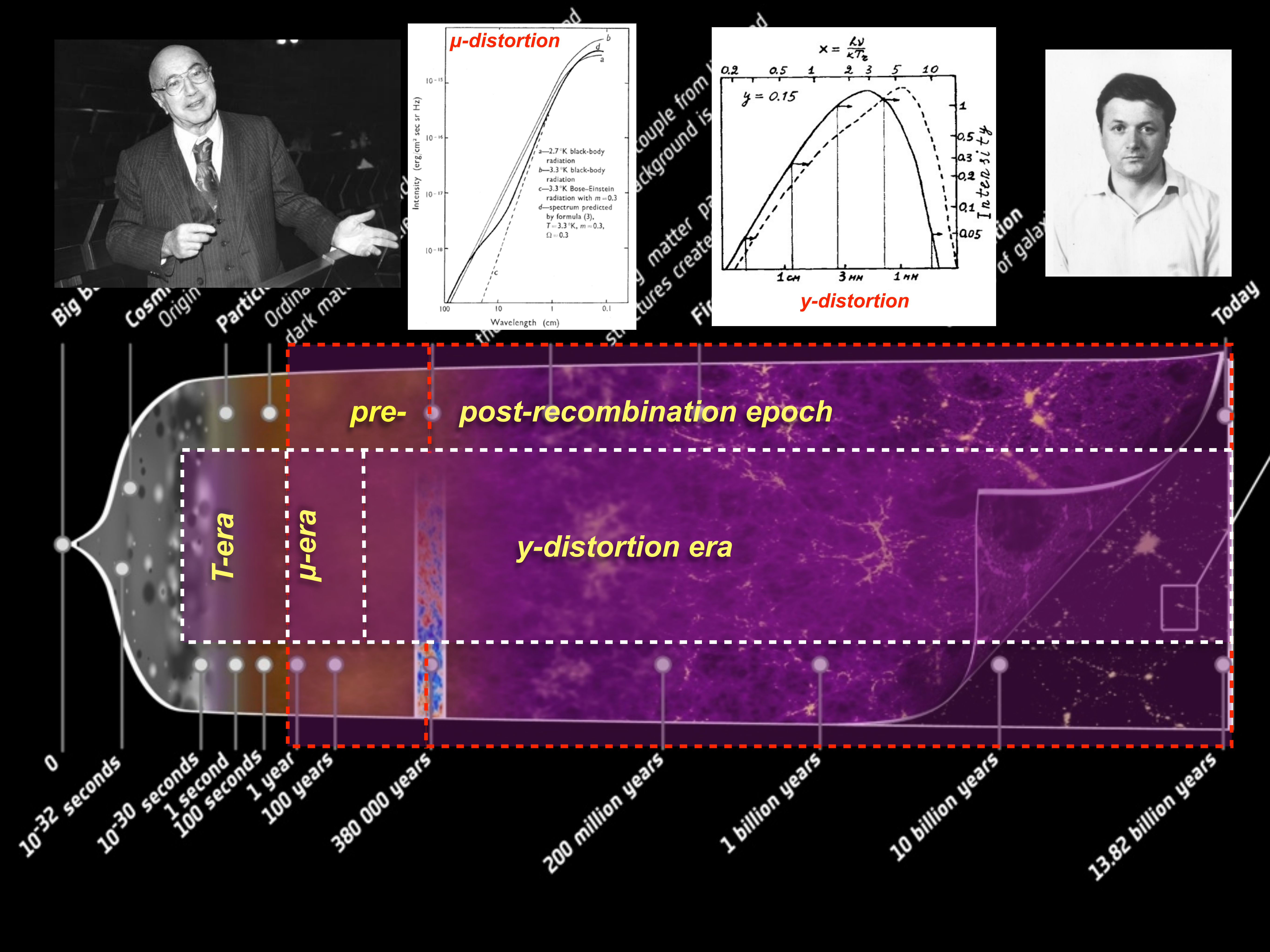
Measurements of CMB spectrum will open a new unexplored window to the early Universe!





μ-distortion
Intensity (erg/cm² sec sr Hz)
Wavelength (cm)
a—2.7 °K black-body radiation
b—3.3 °K black-body radiation
c—3.3 °K Bose–Einstein radiation with m = 0.3
d—spectrum predicted by formula (3), T = 3.3 °K, m = 0.3, Ω = 0.3
y = 0.15
Intensity
y-distortion
pre-
post-recombination epoch
T-era
μ-era
y-distortion era
Today
0
10-32 seconds
10-30 seconds
1 second
100 seconds
1 year
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380 000 years
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Why should one expect some spectral distortion?

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Full thermodynamic equilibrium (certainly valid at very high redshift)

- CMB has a blackbody spectrum at every time (not affected by expansion)
- Photon number density and energy density determined by temperature T_γ

$T_\gamma \sim 2.726\,(1+z)$ K

$N_\gamma \sim 411\ \mathrm{cm}^{-3}\,(1+z)^3 \sim 2\times10^9\,N_\mathrm{b}$ (*entropy density dominated by photons*)

$\rho_\gamma \sim 5.1\times10^{-7}\,m_\mathrm{e}c^2\ \mathrm{cm}^{-3}\,(1+z)^4 \sim \rho_\mathrm{b}\ \mathrm{x}\ (1+z)\,/\,925 \sim 0.26\ \mathrm{eV\ cm}^{-3}\,(1+z)^4$

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Perturbing full equilibrium by

- Energy injection (interaction *matter* ←→ *photons*)
- Production of (energetic) photons and/or particles (i.e. change of entropy)

→ CMB spectrum deviates from a pure blackbody

→ thermalization process (partially) erases distortions

(Compton scattering, double Compton and Bremsstrahlung in the expanding Universe)

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Measurements of CMB spectrum place very tight constraints on the thermal history of our Universe!

Some simple statements about distortions

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- Start with blackbody: T_γ, $N_\gamma^{\rm bb}(T_\gamma) \propto T_\gamma^3$, and $\rho_\gamma^{\rm bb}(T_\gamma) \propto T_\gamma^4$

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- Inject isotropic distortion: $\Delta N_\nu,\ \Delta N_\gamma = (4\pi/c) \int \Delta N_\nu \, {\rm d}\nu > 0$

$$\Delta \rho_\gamma = (4\pi/c) \int h\nu \, \Delta N_\nu \, {\rm d}\nu > 0$$

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$$\Delta\rho_\gamma = (4\pi/c)\int h\nu\,\Delta N_\nu\,{\rm d}\nu > 0$$

- Effective temperatures: $T_N^* = \left(\frac{h^3c^3N_\gamma}{16\pi k^3\zeta(3)}\right)^{1/3} \approx T_\gamma\left(1+\frac{1}{3}\frac{\Delta N_\gamma}{N_\gamma^{\rm bb}}\right) > T_\gamma$

$$N_\gamma \equiv N_\gamma^{\rm bb}(T_N^*)$$
$$\rho_\gamma \equiv \rho_\gamma^{\rm bb}(T_\rho^*)$$
$$\Longrightarrow$$
$$T_\rho^* = \left(\frac{15h^3c^3\rho_\gamma}{8\pi^5k^4}\right)^{1/4} \approx T_\gamma\left(1+\frac{1}{4}\frac{\Delta\rho_\gamma}{\rho_\gamma^{\rm bb}}\right) > T_\gamma.$$

Some simple statements about distortions

- Start with blackbody: $T_\gamma,\ N_\gamma^{\rm bb}(T_\gamma) \propto T_\gamma^3$, and $\rho_\gamma^{\rm bb}(T_\gamma) \propto T_\gamma^4$
- Inject isotropic distortion: $\Delta N_\nu,\ \Delta N_\gamma = (4\pi/c)\int \Delta N_\nu\,{\rm d}\nu > 0$

$$\Delta\rho_\gamma = (4\pi/c)\int h\nu\,\Delta N_\nu\,{\rm d}\nu > 0$$

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$$N_\gamma \equiv N_\gamma^{\rm bb}(T_N^*)$$
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- *Energy release inevitably* creates distortions (need additional photons)

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Question: Is there enough time to restore full equilibrium?

How does the thermalization process work?

Some important conditions and assumptions

- Plasma fully ionized before recombination (z~1000)
  - → free electrons, protons and helium nuclei
  - → photon dominated (~2 Billion photons per baryon)
- Coulomb scattering $e + p \leftrightarrow e' + p$
  - → electrons in full thermal equilibrium with baryons
  - → electrons follow thermal Maxwell-Boltzmann distribution
  - → efficient down to very low redshifts (z ~ 10-100)
- Medium homogeneous and isotropic on large scales
  - → thermalization problem rather simple!
  - → in principle *allows very precise computations*
- Hubble expansion
  - → adiabatic cooling of photons [$T_\gamma \sim (1+z)$] and ordinary matter [$T_m \sim (1+z)^2$]
  - → redshifting of photons

Photon Boltzmann Equation for Average Spectrum

$$\frac{\mathrm{d}n_\nu}{\mathrm{d}t} = \frac{\partial n_\nu}{\partial t} + \frac{\partial n_\nu}{\partial x_i} \cdot \frac{\mathrm{d}x_i}{\mathrm{d}t} + \frac{\partial n_\nu}{\partial p}\frac{\mathrm{d}p}{\mathrm{d}t} + \frac{\partial n_\nu}{\partial \hat{p}_i} \cdot \frac{\mathrm{d}\hat{p}_i}{\mathrm{d}t} = \mathcal{C}[n]$$

Photon occupation number

Liouville operator

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- Isotropy & Homogeneity: $\Longrightarrow$
  ($\Leftrightarrow$ *average* spectrum...)

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redshifting term

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Compton Scattering

Bremsstrahlung

Double Compton

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redistribution of photon over frequency

adjusting photon number

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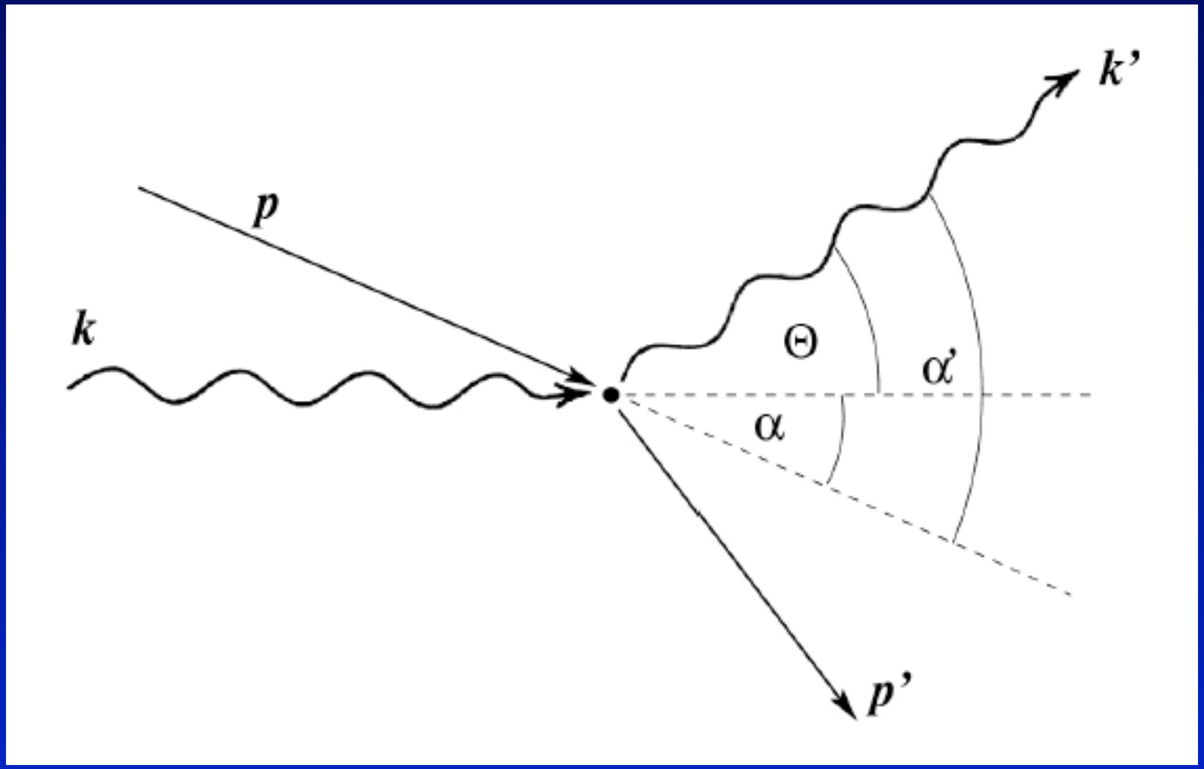
- Energy release: $\mathcal{C}[n] \neq 0 \;\Rightarrow$ *thermalization process starts*

Compton scattering

Redistribution of photons by Compton scattering

- Reaction: $\gamma + e \longleftrightarrow \gamma' + e'$



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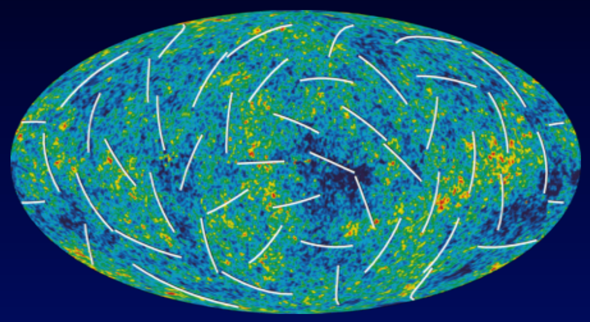

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→ *no energy exchange* ⇒ *Thomson limit*

⇒ *important for anisotropies*

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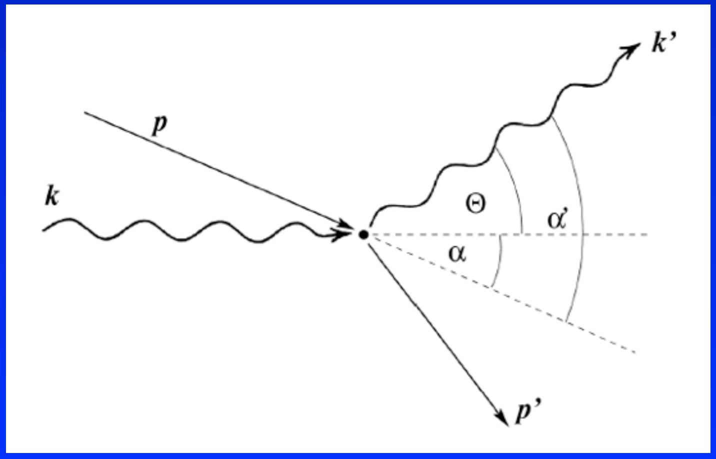

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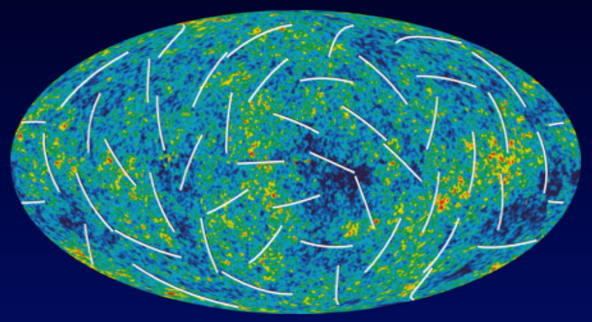

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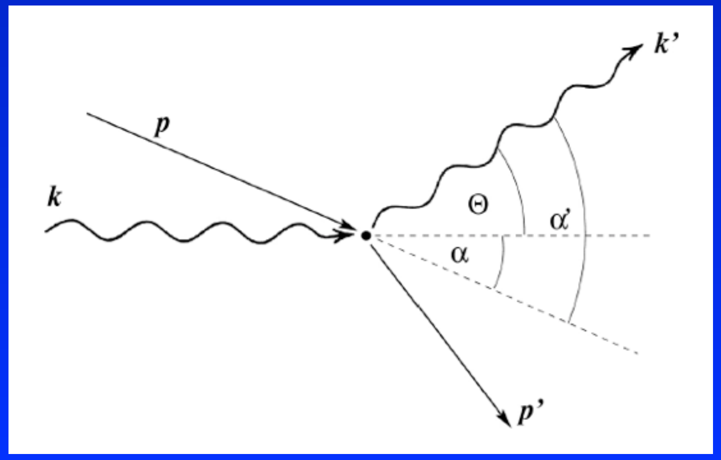
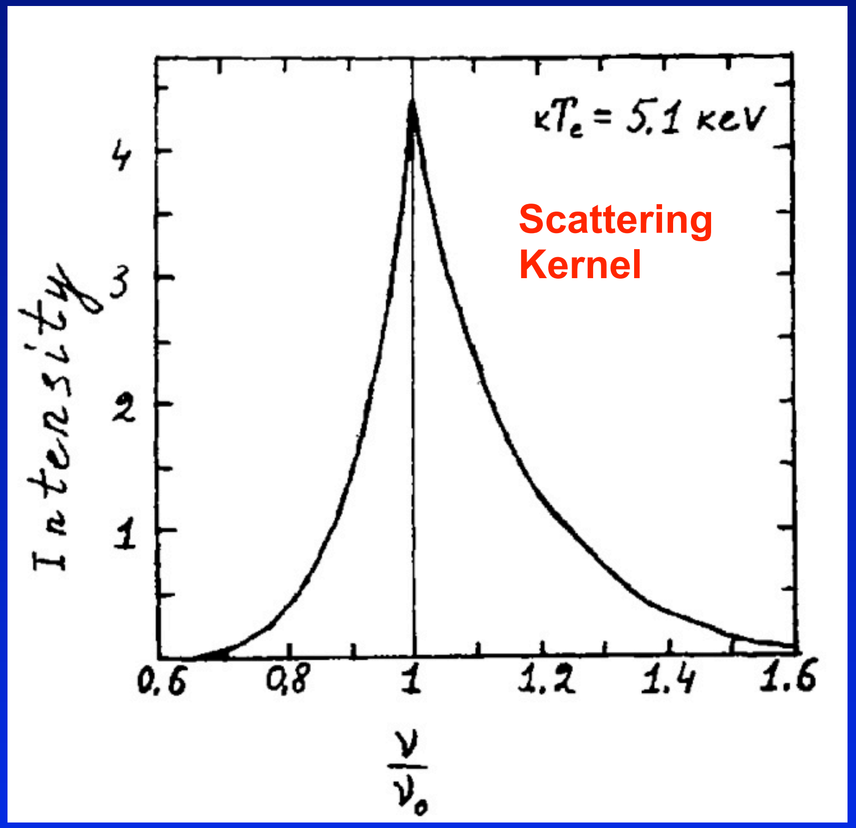
→ *energy exchange included*

- up-scattering due to the ***Doppler*** effect for $h\nu < 4kT_{\mathrm{e}}$
- down-scattering because of ***recoil*** (and stimulated recoil) for $h\nu > 4kT_{\mathrm{e}}$
- ***Doppler*** broadening $\frac{\Delta\nu}{\nu} \simeq \sqrt{\frac{2kT_{\mathrm{e}}}{m_{\mathrm{e}}c^2}}$





Sunyaev & Zeldovich, 1980, ARAA, 18, 537

Compton Collision Term / Kompaneets Equation

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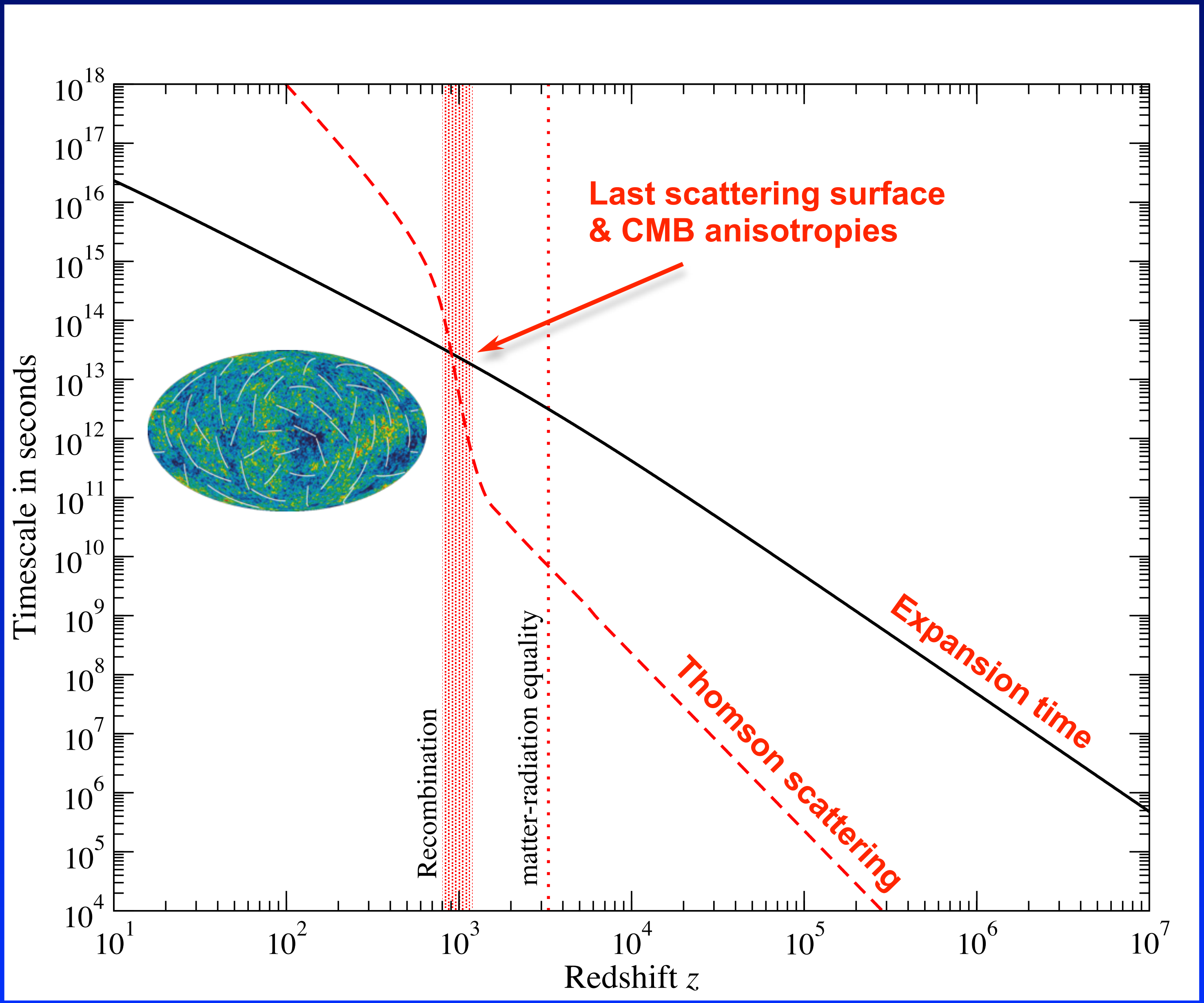
$$\mathrm{d}\tau = \sigma_{\mathrm{T}} N_{\mathrm{e}} c\, \mathrm{d}t$$

Thomson optical depth

Important Timescales for Compton Process

- *Thomson scattering* $t_{\mathrm{C}} = (\sigma_{\mathrm{T}} N_{\mathrm{e}} c)^{-1} \approx 2.3 \times 10^{20}\, \chi_{\mathrm{e}}^{-1} (1+z)^{-3}\, \mathrm{sec}$



Radiation dominated

$$t_{\exp} = H^{-1} \simeq 4.8 \times 10^{19} (1+z)^{-2}\, \mathrm{sec}$$
$$\simeq 8.4 \times 10^{17} (1+z)^{-3/2}\, \mathrm{sec}$$

Matter dominated

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Scattering Kernel

Stimulated scattering

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Doppler broadening & boosting

recoil and stimulated recoil

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also allows absorbing redshifting term!

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Kompaneets equation (A.S. Kompaneetz, 1956, JETP, 47, p. 1939)

Compton Collision Term / Kompaneets Equation

$$\left.\frac{\mathrm{d}n_\nu}{\mathrm{d}\tau}\right|_{\mathrm{C}} = \int \left[\left(\frac{\nu'}{\nu}\right)^2 P(T_{\mathrm{e}}, \nu' \to \nu)\, n_{\nu'}(1+n_\nu) - P(T_{\mathrm{e}}, \nu \to \nu')\, n_\nu(1+n_{\nu'}) \right] \mathrm{d}\nu'$$

- *Fokker-Planck expansion:* $\quad \dfrac{kT_{\mathrm{e}}}{m_{\mathrm{e}}c^2} \ll 1 \quad \text{and} \quad \dfrac{\Delta\nu}{\nu} \ll 1$

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- *Initially developed to describe repeated scattering of thermal neutrons*

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While Compton cooling is fast:

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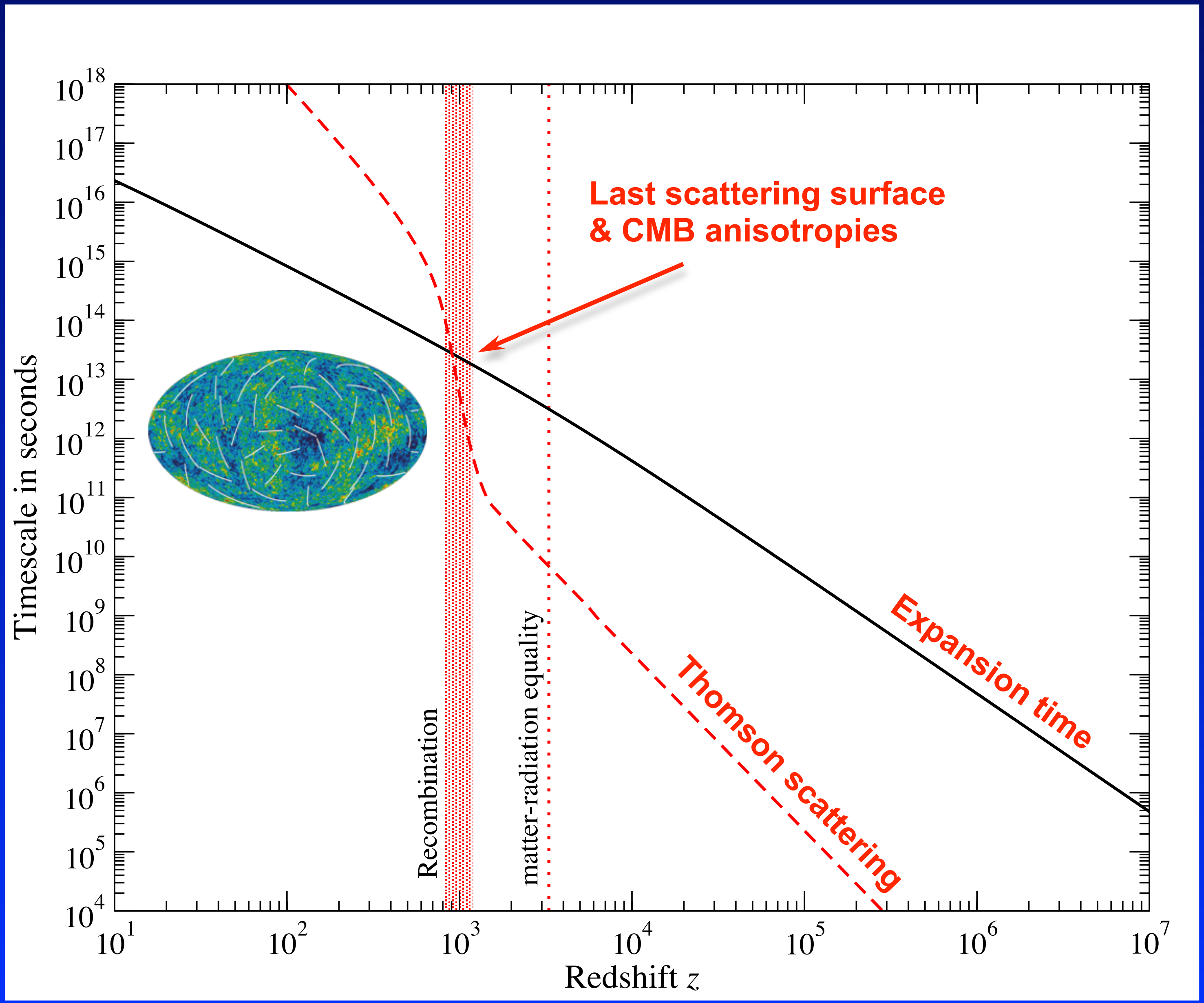

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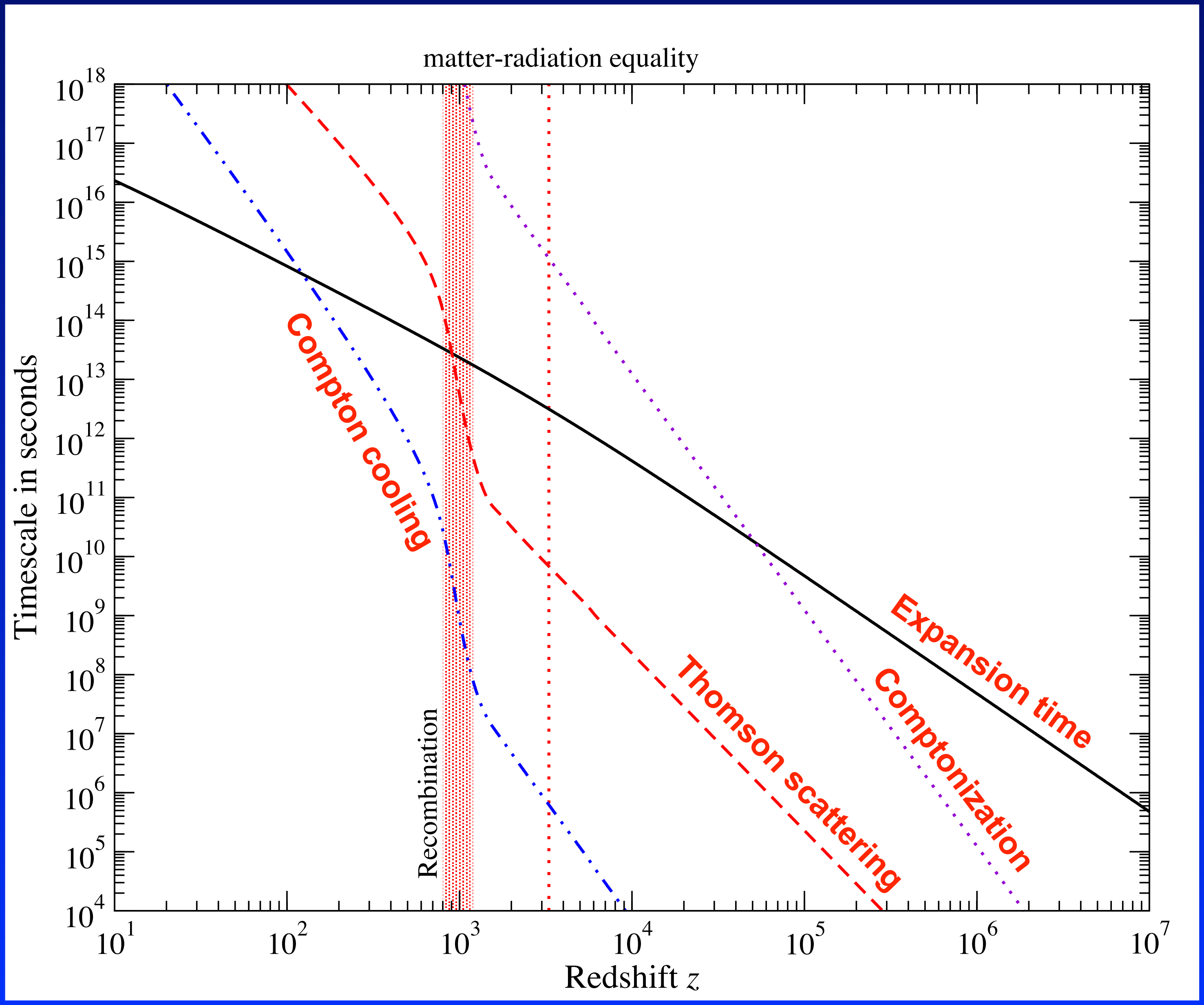

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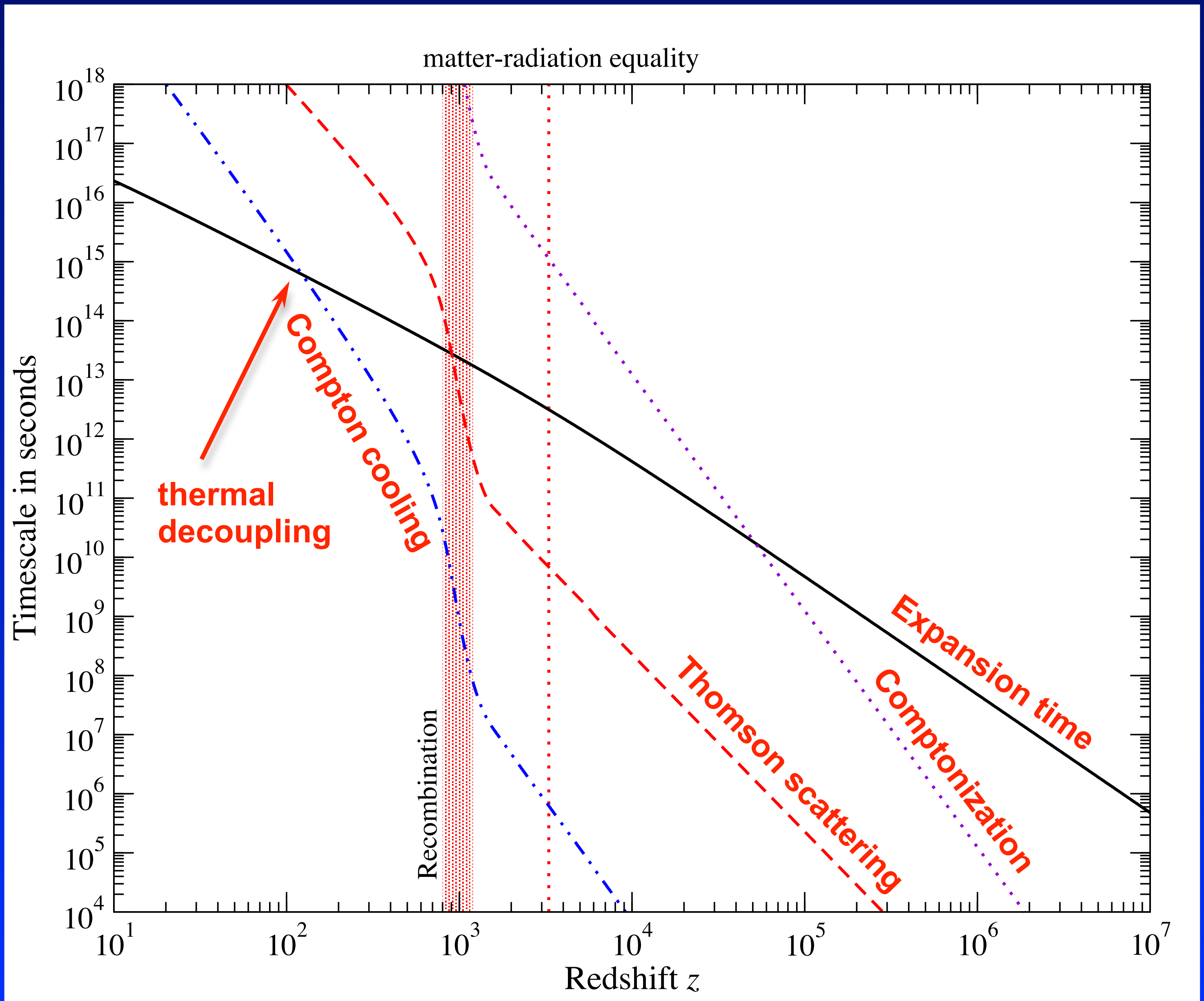

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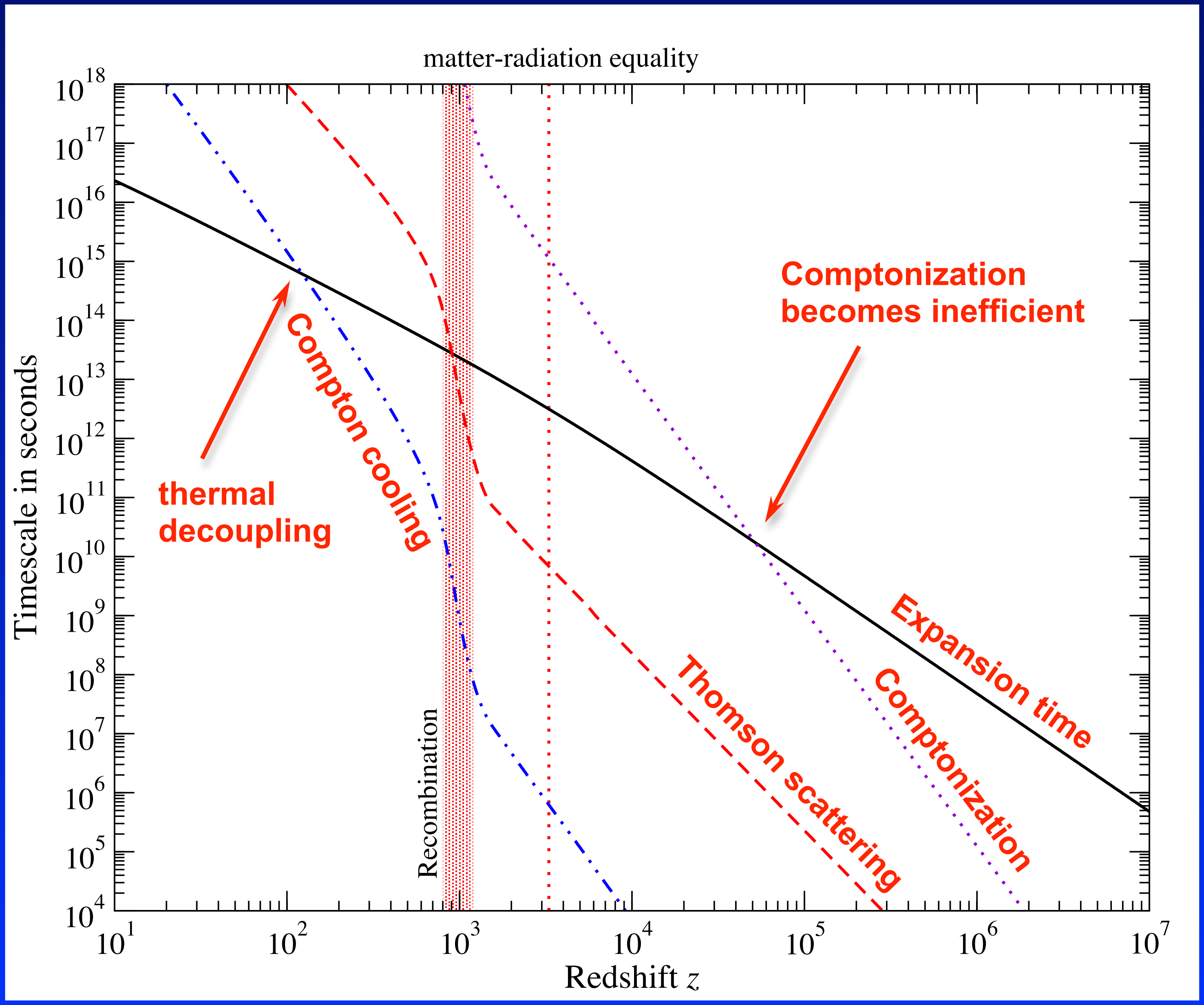

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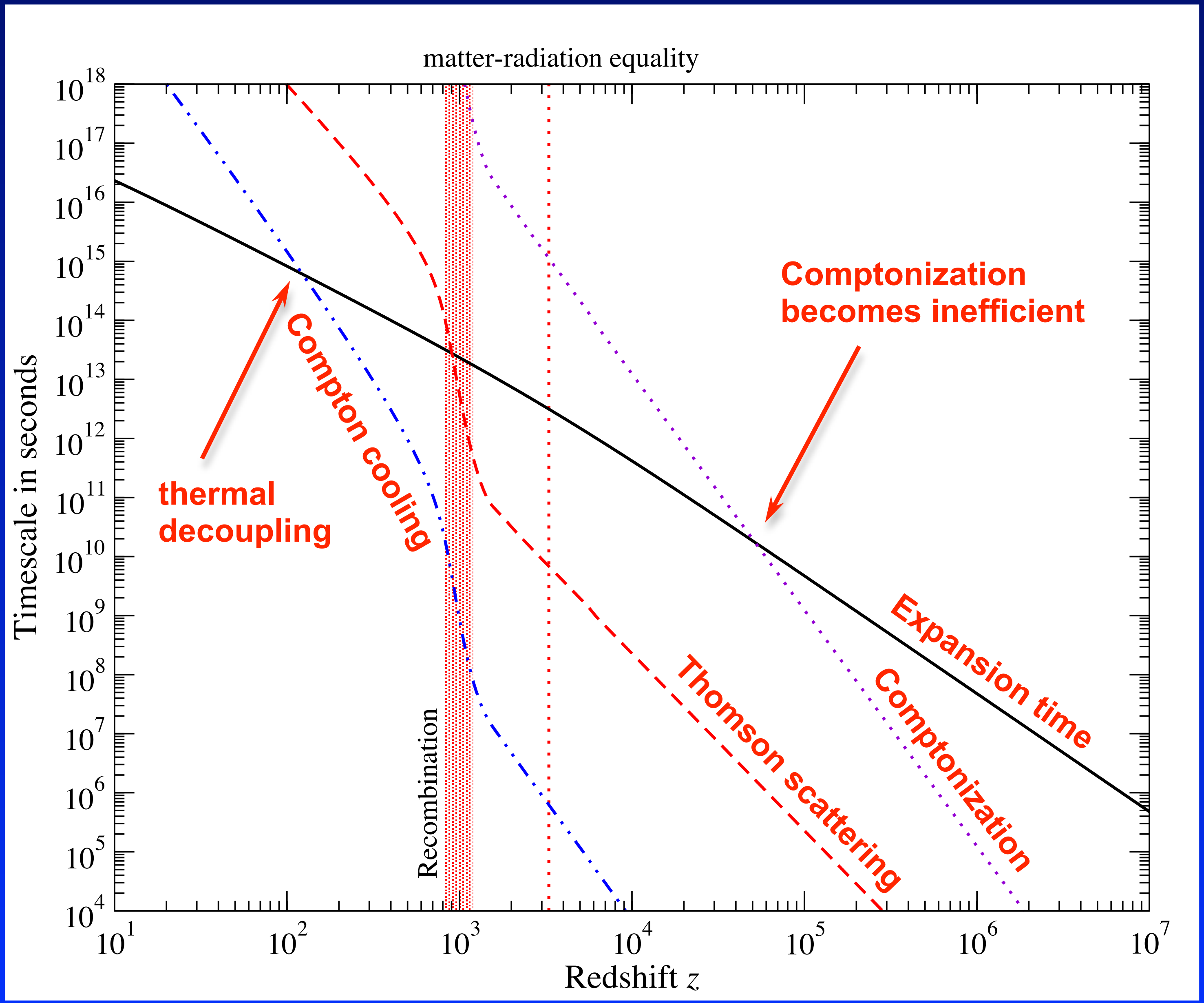

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⇒ character of distortion changes at z_K ! μ ⇔ y

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What are y- and μ-distortions?

Compton y-distortion / thermal SZ effect

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spectrum of y-distortion

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- *if* $T_{\mathrm{e}} = T_\gamma \quad \Longrightarrow \quad \left.\frac{\mathrm{d}n}{\mathrm{d}\tau}\right|_{\mathrm{C}} = 0$ *(thermal equilibrium with electrons)*
- *if* $T_{\mathrm{e}} < T_\gamma \quad \Longrightarrow$ *down-scattering of photons / heating of electrons*
- *if* $T_{\mathrm{e}} > T_\gamma \quad \Longrightarrow$ *up-scattering of photons / cooling of electrons*
- *for* $T_{\mathrm{e}} \gg T_\gamma \quad \Longrightarrow$ *thermal Sunyaev-Zeldovich effect (up-scattering)*

Simplest spectral shapes

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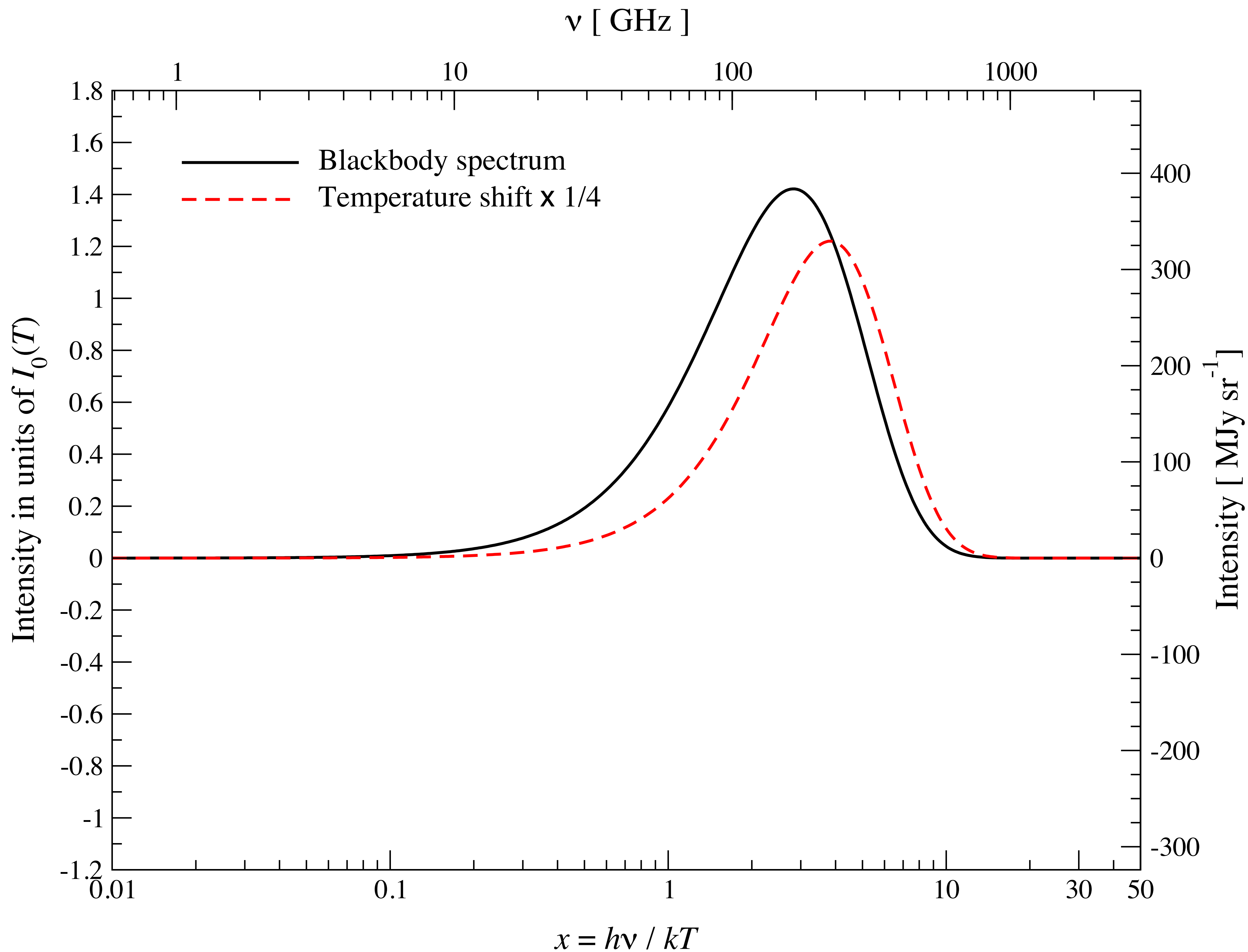

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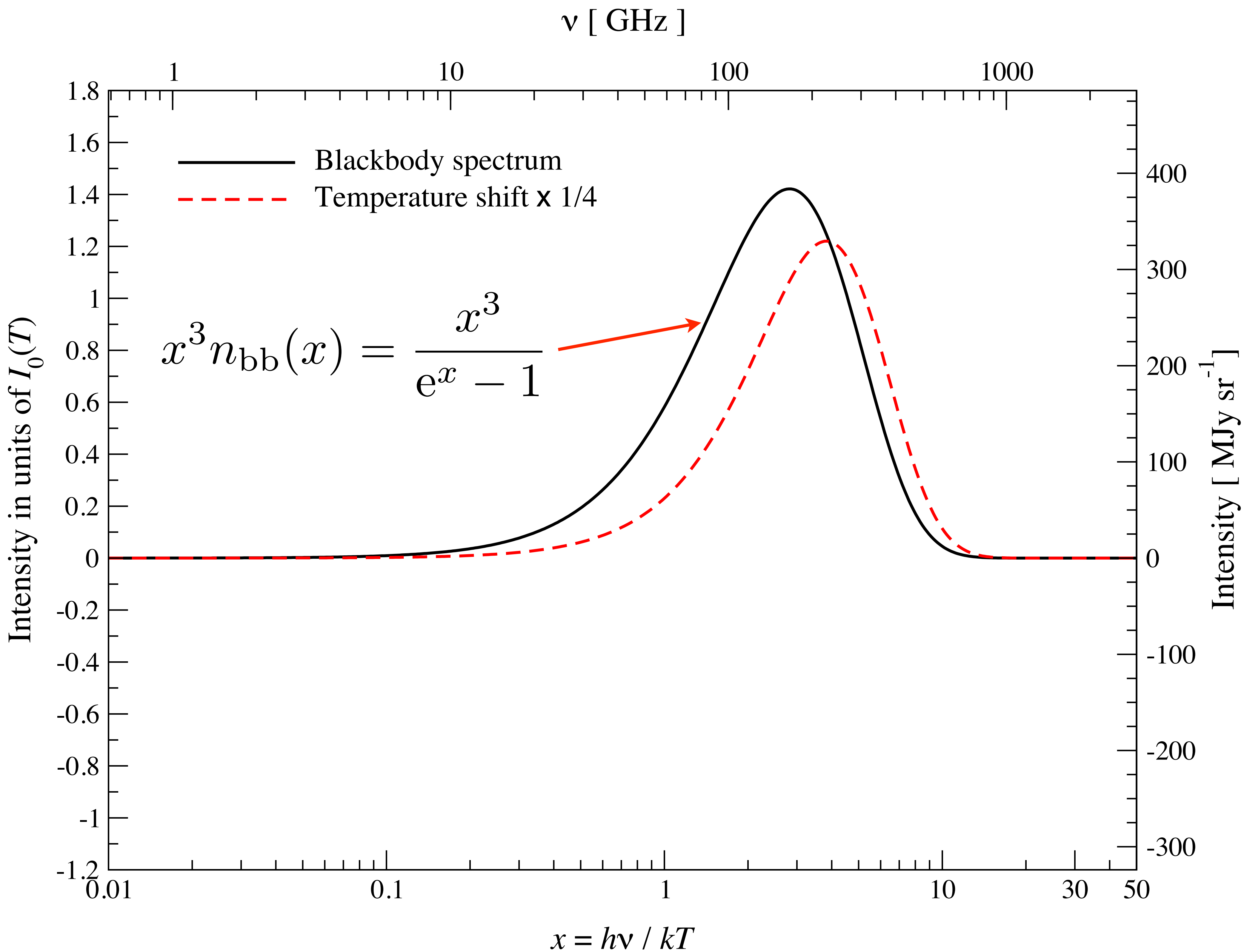

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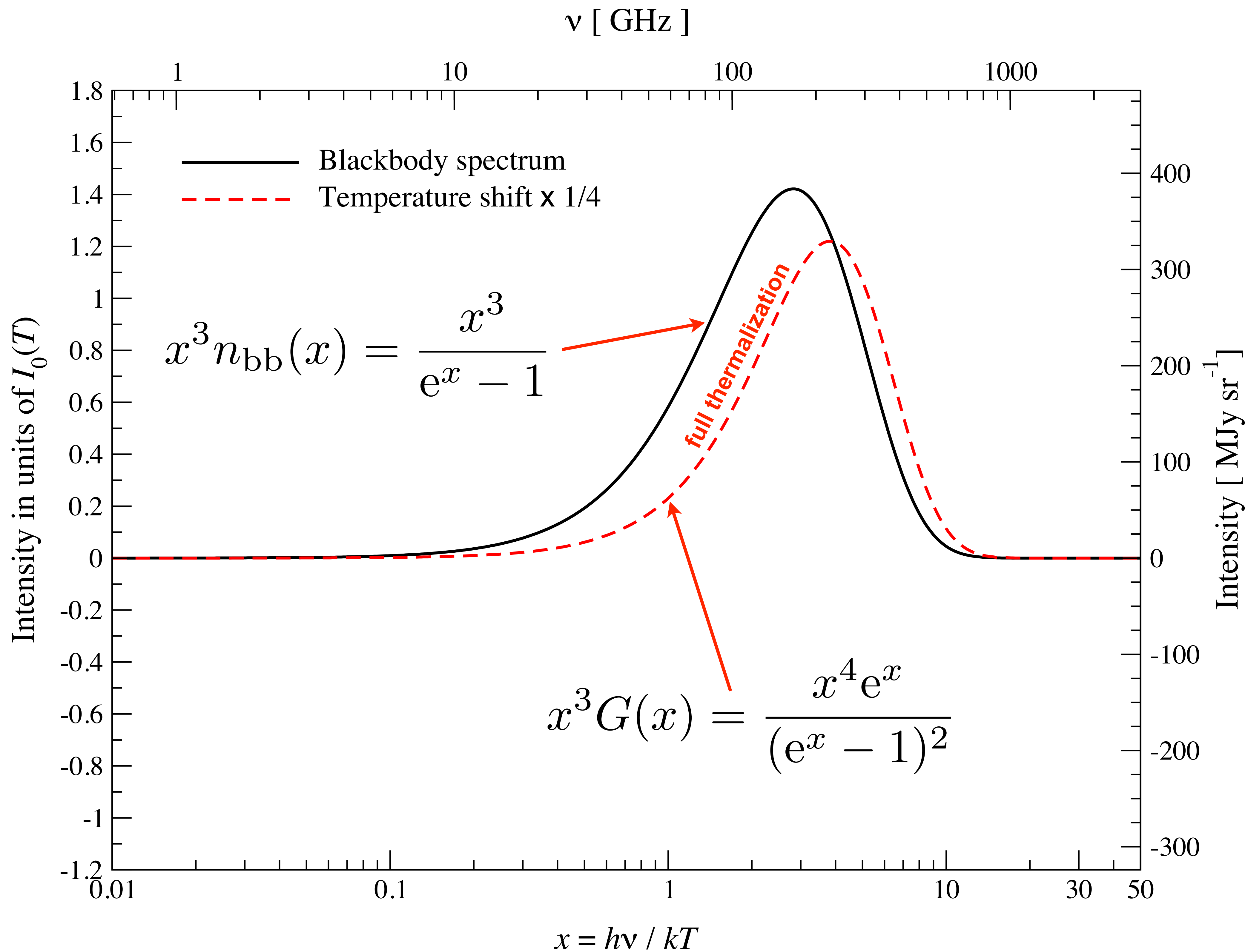

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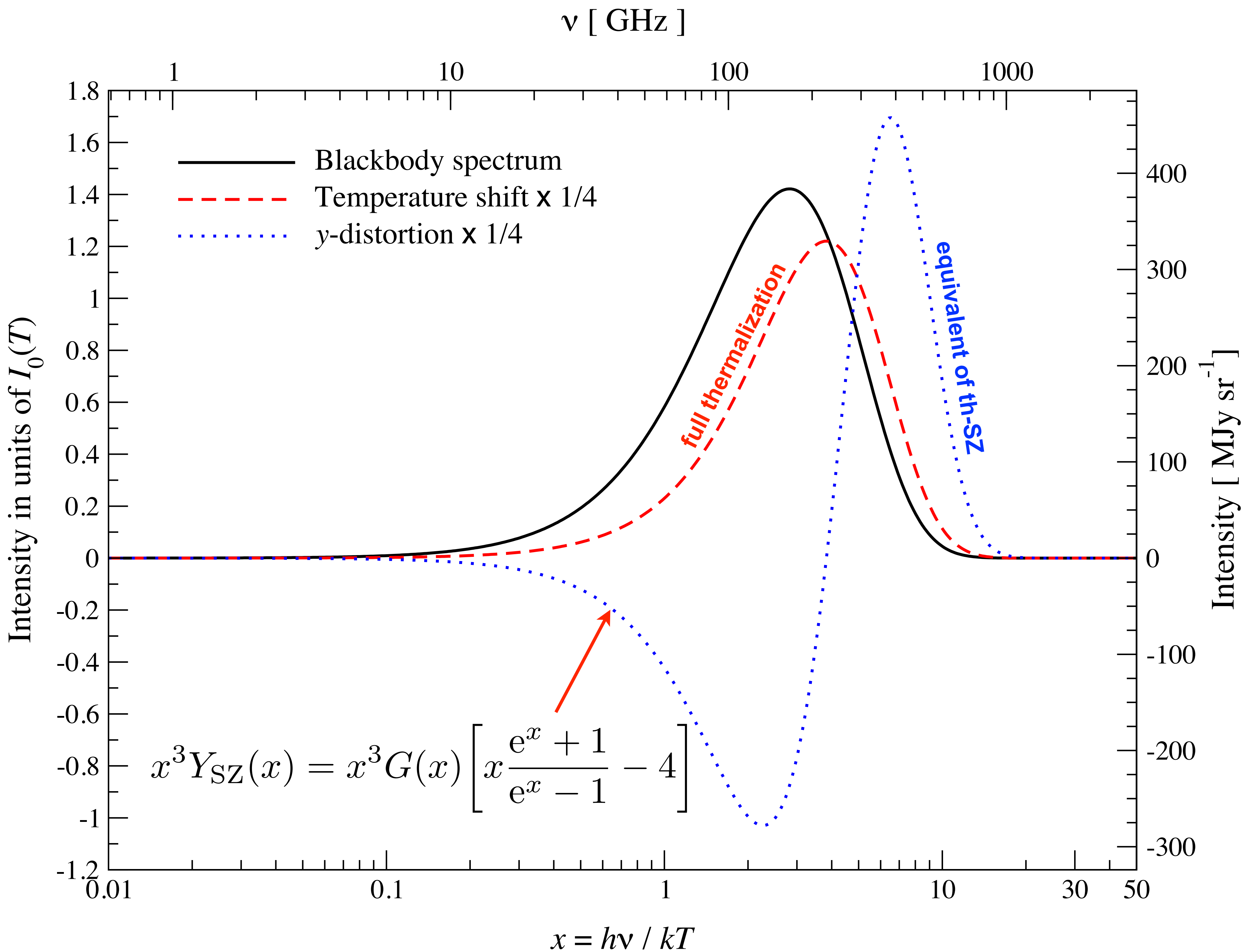

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- *General equilibrium solution: Bose-Einstein spectrum with*

$T_\gamma = T_{\mathrm{e}} \equiv T_{\mathrm{eq}}$ and $\mu_0 = \mathrm{const}$ ($\equiv 0$ for blackbody)

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$$\Longrightarrow (1+\partial_x \mu) = -\frac{T_\gamma}{T_{\mathrm{e}}} \Longrightarrow x + \mu = x\frac{T_\gamma}{T_{\mathrm{e}}} + \mu_0$$

constant

- *General equilibrium solution:* *Bose-Einstein spectrum with*

$$T_\gamma = T_{\mathrm{e}} \equiv T_{\mathrm{eq}} \quad \text{and} \quad \mu_0 = \text{const} \ (\equiv 0 \text{ for blackbody})$$

Something is missing? How do you fix T_{e} and μ_0?

Final definition of μ-type distortion

- *initial condition:* $N_\gamma = N_\gamma^{\rm bb}(T_\gamma)$ and $\rho_\gamma = \rho_\gamma^{\rm bb}(T_\gamma)$

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- *after energy release* $\Rightarrow$

$$\approx 1.368$$

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- *Solution:* $\dfrac{\Delta T}{T_\gamma} \approx \dfrac{\pi^2}{18\zeta(3)}\mu_0 \approx 0.456\,\mu_0$ *and* $\boxed{\mu_0 \approx 1.401\,\dfrac{\Delta\rho_\gamma}{\rho_\gamma}}$

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$$\Longrightarrow \boxed{M(x) = G(x)\left[\frac{\pi^2}{18\zeta(3)} - \frac{1}{x}\right]}$$

μ-distortion spectrum
(photon number conserved)

Final definition of μ-type distortion

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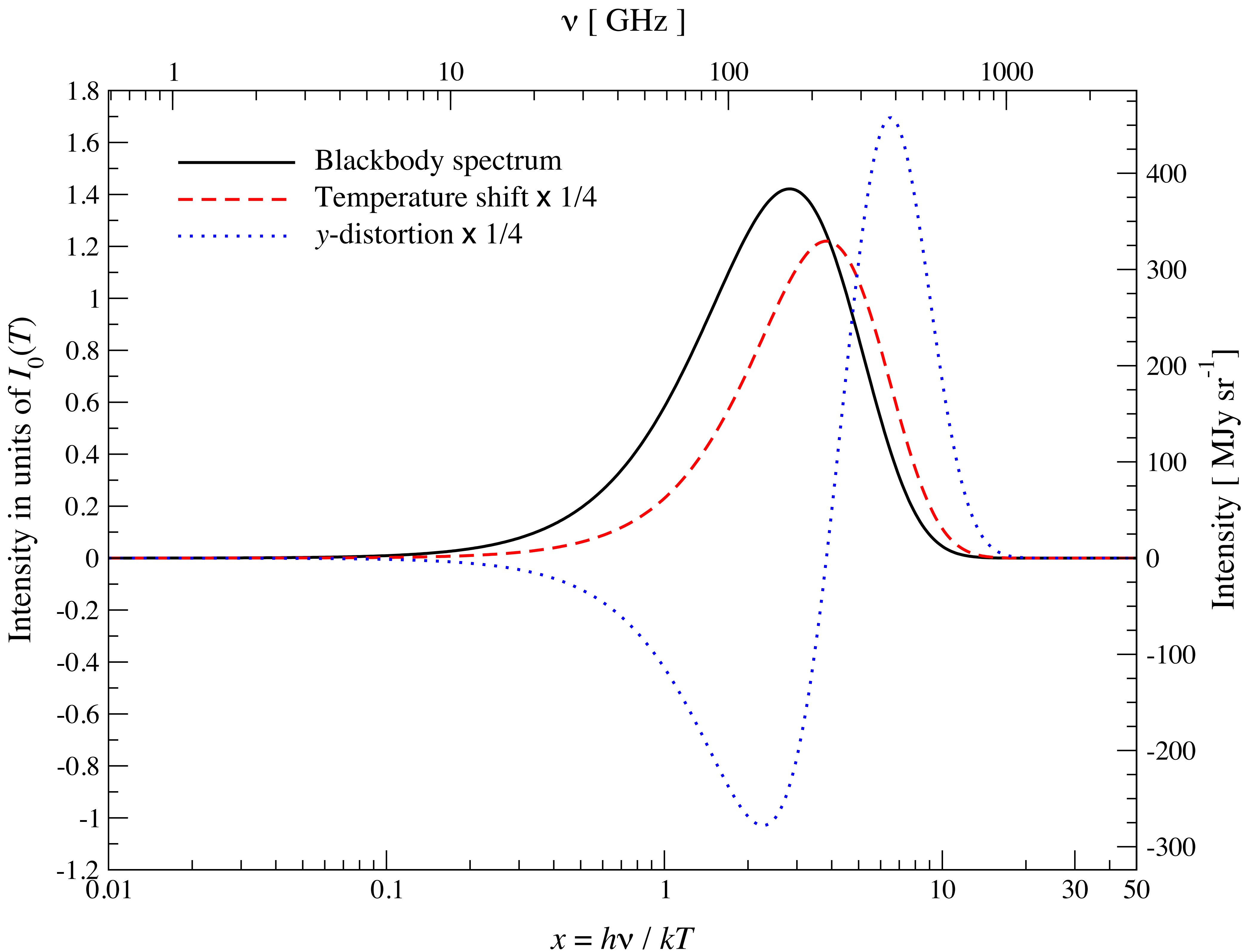
μ-distortion spectrum
(photon number conserved)

- $\mu_0 > 0$ ⇒ *too few photons / too much energy*
- $\mu_0 < 0$ ⇒ *too many photons / too little energy*

$$\frac{\Delta\rho_\gamma}{\rho_\gamma^{\rm bb}} \approx \frac{4}{3}\frac{\Delta N_\gamma}{N_\gamma^{\rm bb}}$$

Simplest spectral shapes



$$I_0 = (2h/c^2)(kT_0/h)^3 \approx 270\,\mathrm{MJy\,sr}^{-1}$$

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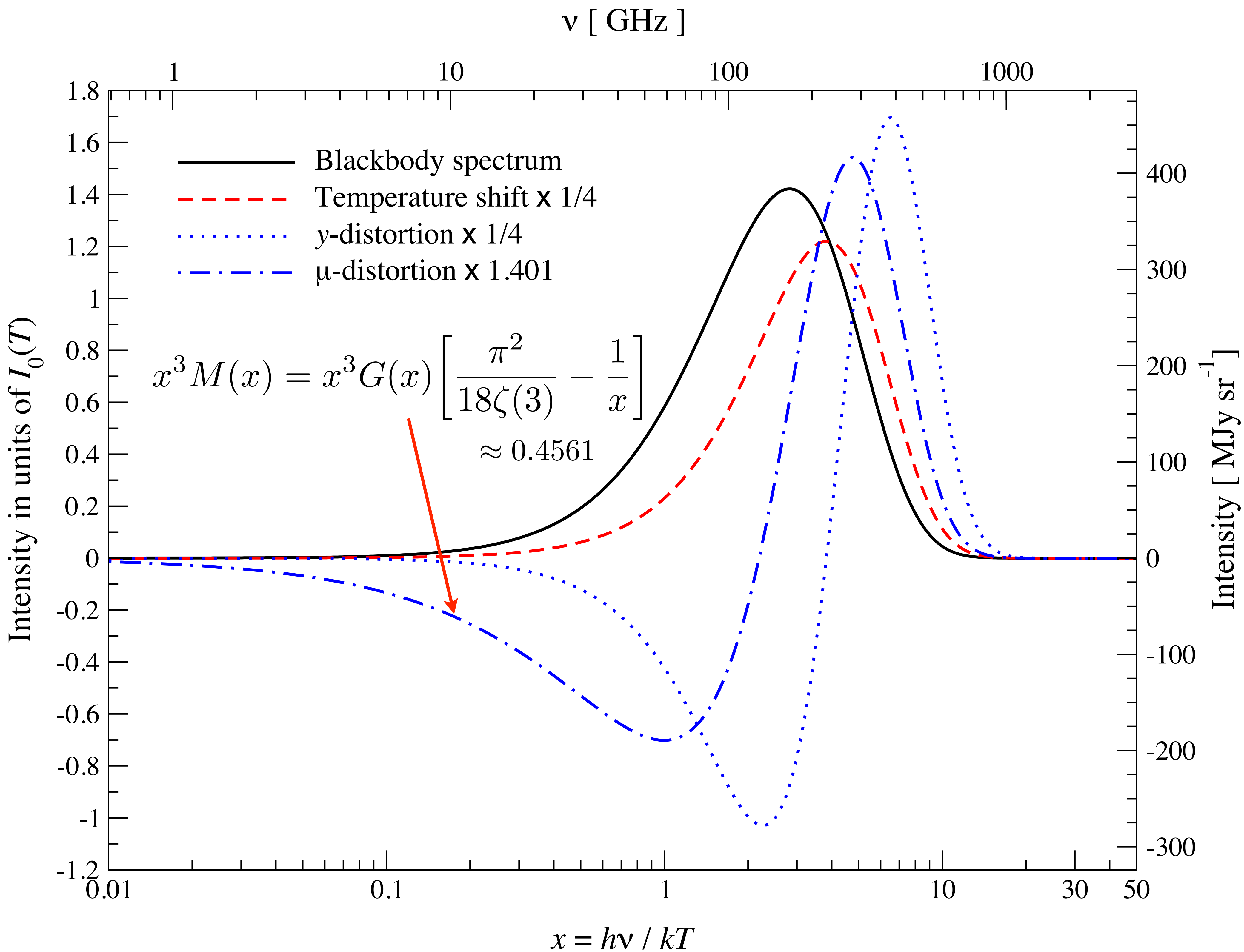

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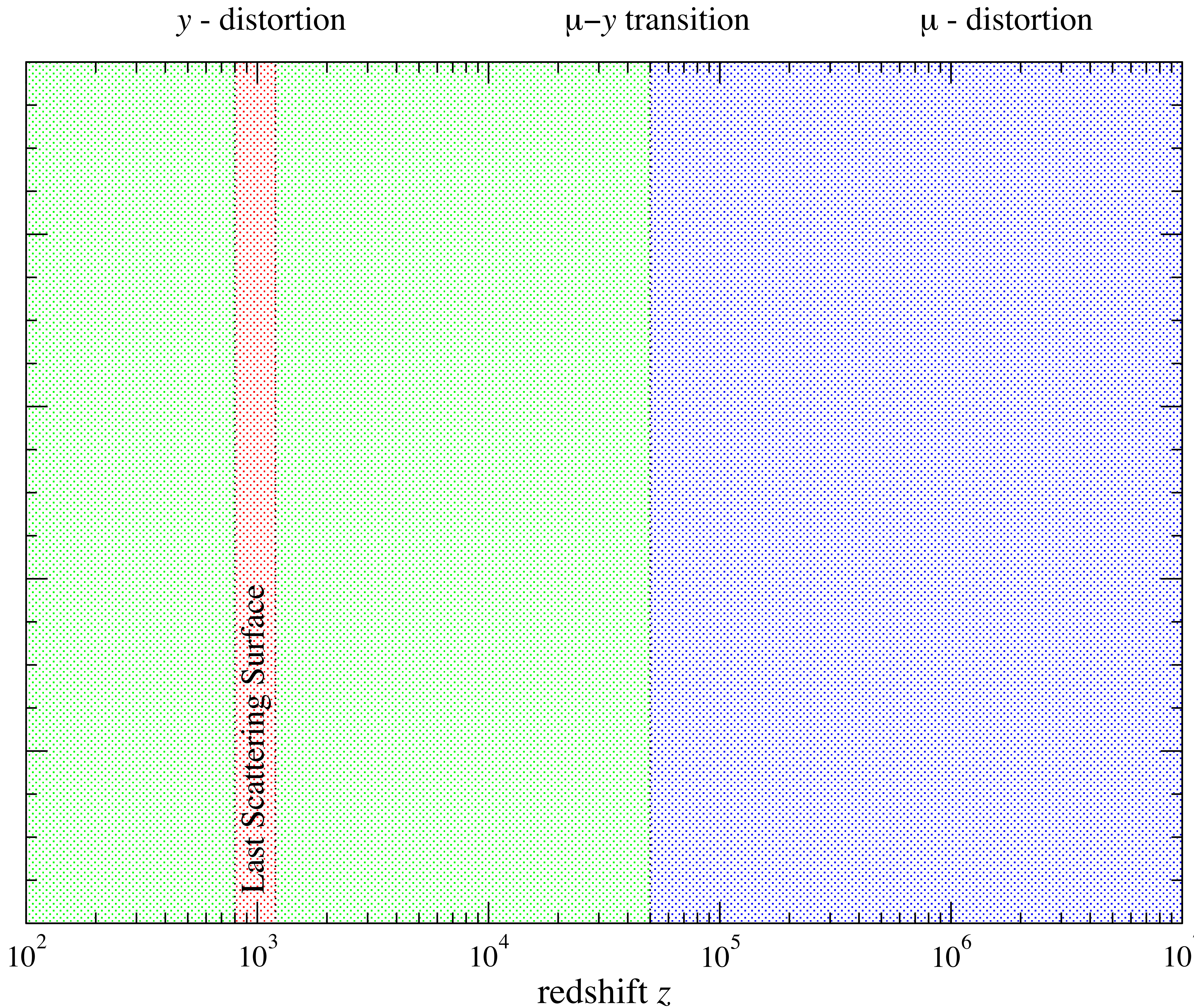
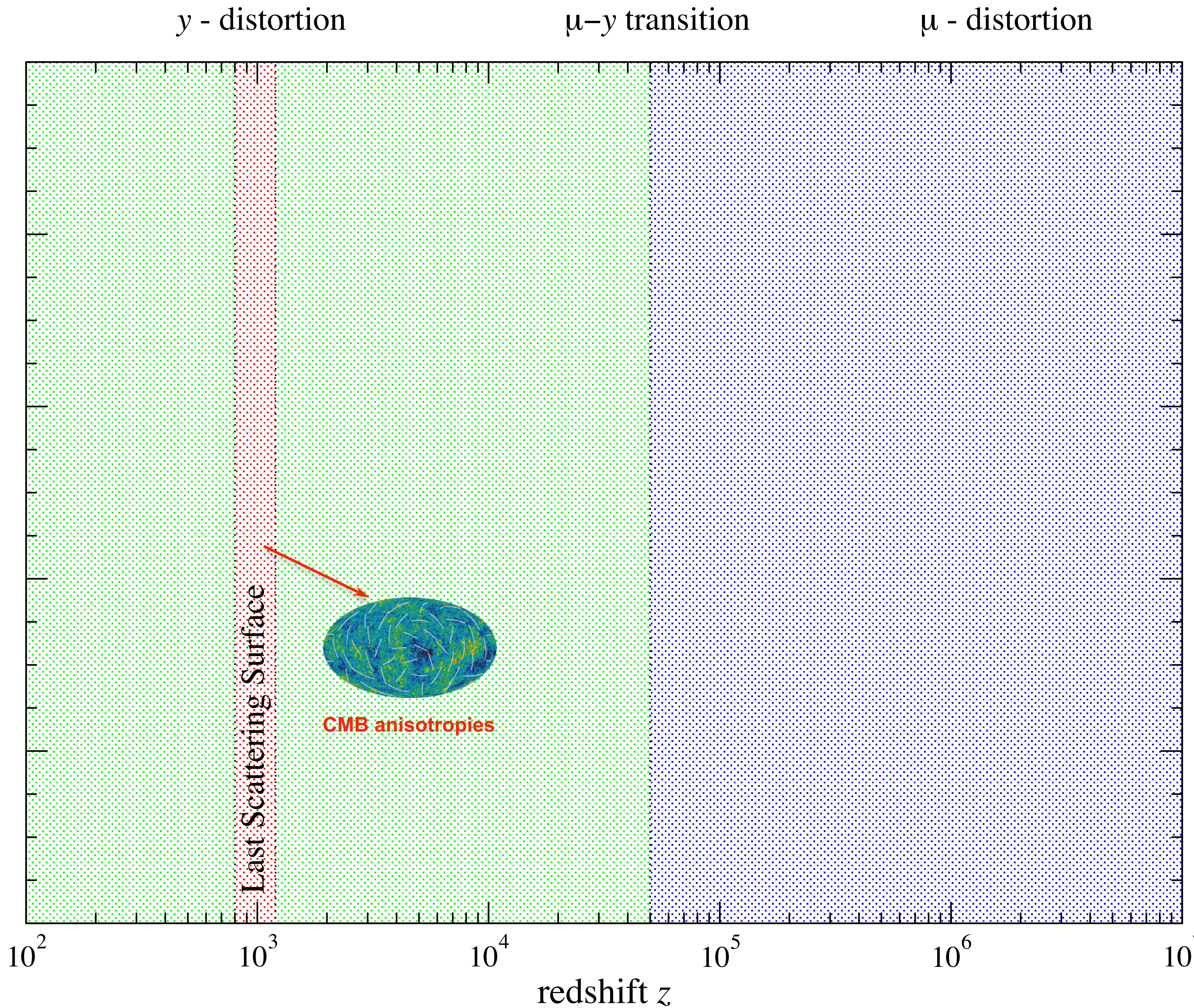


y - distortion
μ−y transition
μ - distortion
Last Scattering Surface
10^2
10^3
10^4
10^5
10^6
10^7
redshift z



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Last Scattering Surface
CMB anisotropies
10^2
10^3
10^4
10^5
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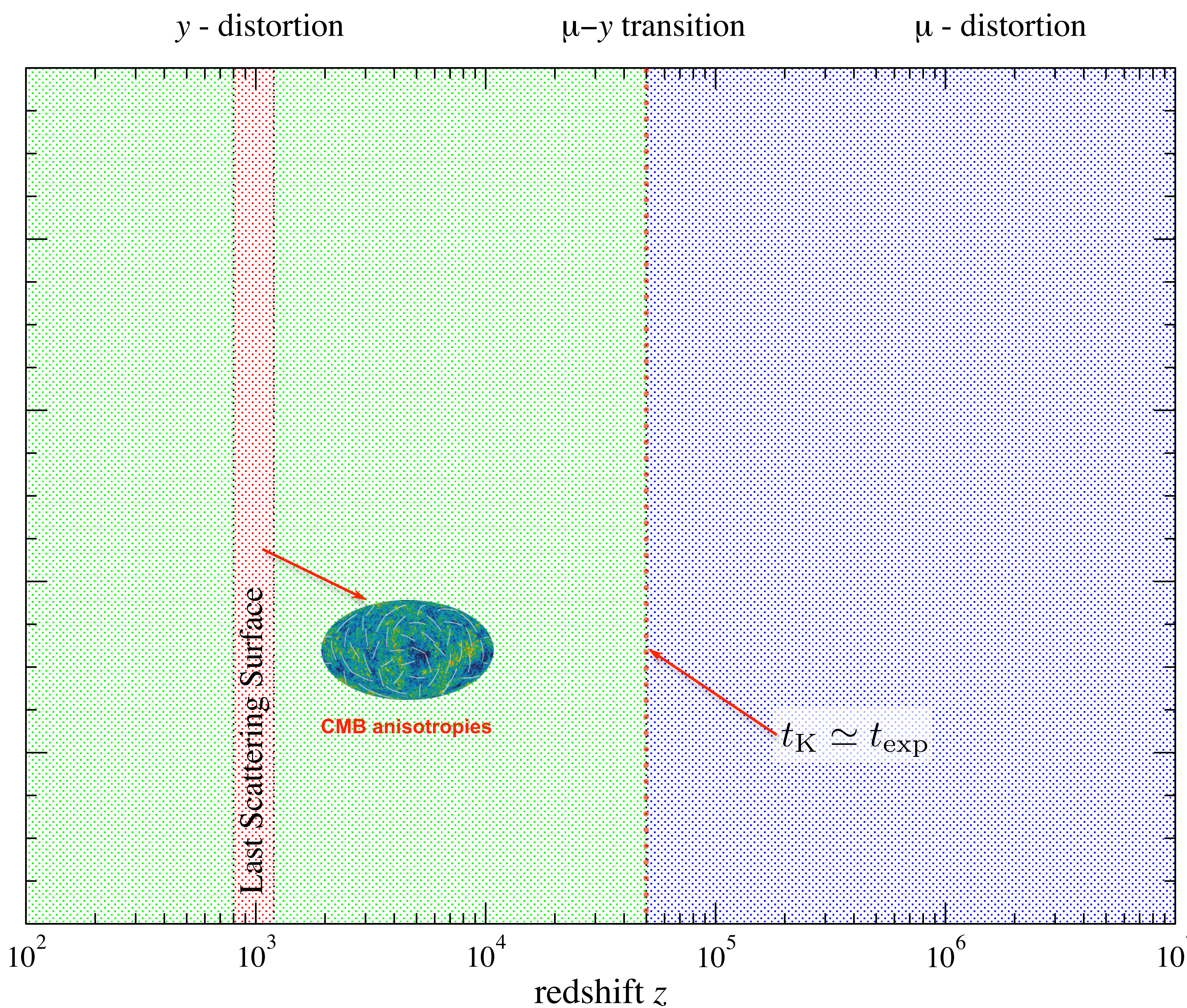
redshift z



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CMB anisotropies
$t_{\mathrm{K}} \simeq t_{\mathrm{exp}}$
10^2
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10^4
10^5
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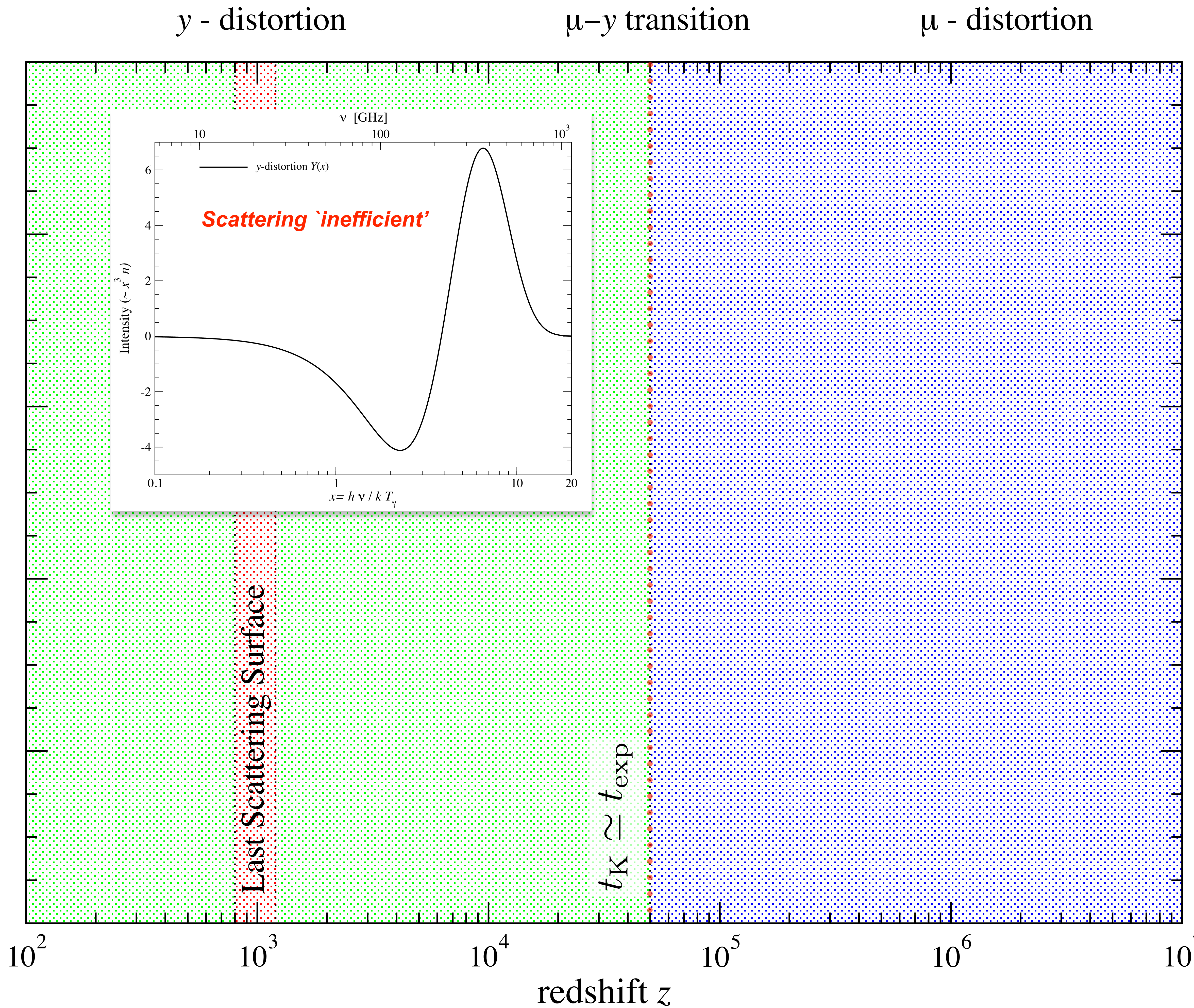
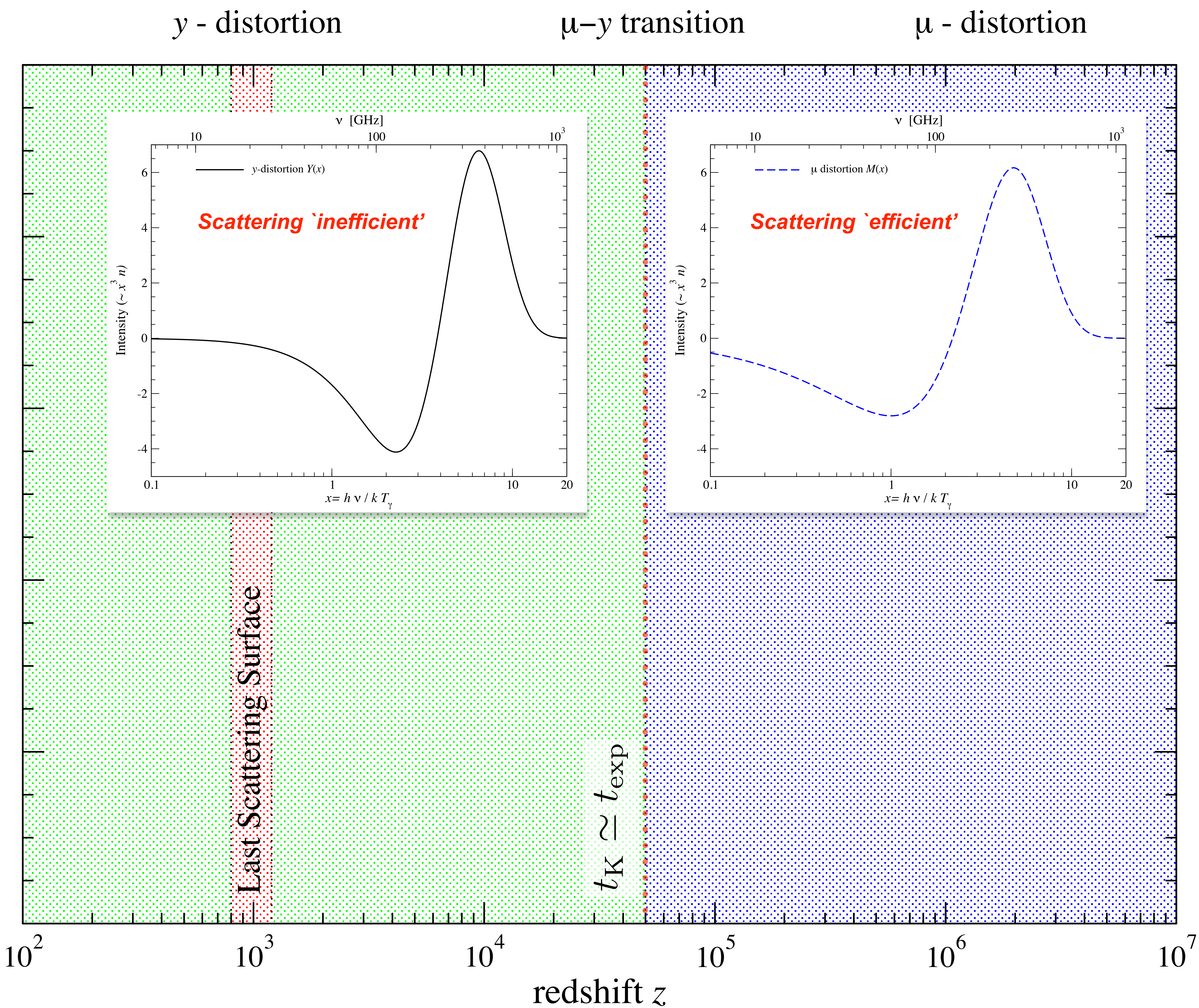


y - distortion
μ−y transition
μ - distortion
ν [GHz]
10
100
10^3
y-distortion Y(x)
Scattering `inefficient'
Intensity (~ x^3 n)
6
4
2
0
-2
-4
0.1
1
10
20
x= h ν / k T_γ
Last Scattering Surface
t_K ≃ t_exp
10^2
10^3
10^4
10^5
10^6
10^7
redshift z



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What about photon production processes?

Adjusting the photon number

- Bremsstrahlung $e + p \leftrightarrow e' + p + \gamma$

  → 1. order α correction to *Coulomb* scattering

  → production of low frequency photons

  → important for the evolution of the distortion at low frequencies and late times ($z < 2 \times 10^5$)

- Double Compton scattering
  (Lightman 1981; Thorne, 1981)

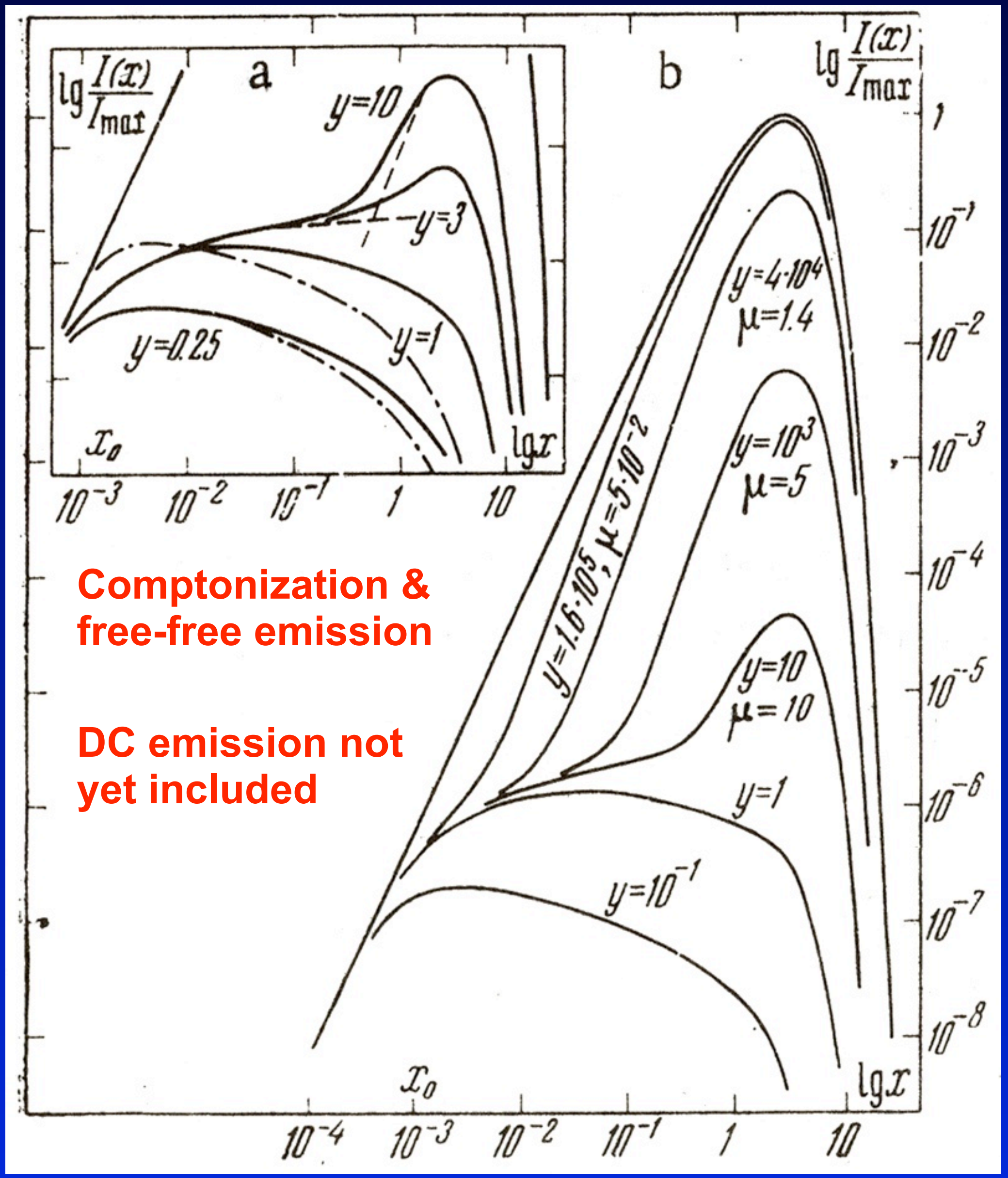
$$e + \gamma \leftrightarrow e' + \gamma' + \gamma_2$$

  → 1. order α correction to *Compton* scattering

  → was only included later **(Danese & De Zotti, 1982)**

  → production of low frequency photons

  → very important at high redshifts ($z > 2 \times 10^5$)



Illarionov & Sunyaev, 1975, Sov. Astr, 18, pp.413

Example: *Energy release by decaying relict particle*

redshift

difference between electron and photon temperature

- initial condition: *full equilibrium*
- total energy release:

  $\Delta\rho/\rho \sim 1.3 \times 10^{-6}$
- most of energy release around:

  $z_X \sim 2 \times 10^6$
- positive μ-distortion
- high frequency distortion frozen around $z \simeq 5 \times 10^5$
- late ($z < 10^3$) free-free absorption at very low frequencies ($T_e < T_\gamma$)

today $x = 2 \times 10^{-2}$ means $\nu \sim 1$GHz

Computation carried out with *CosmoTherm*
(JC & Sunyaev 2012)

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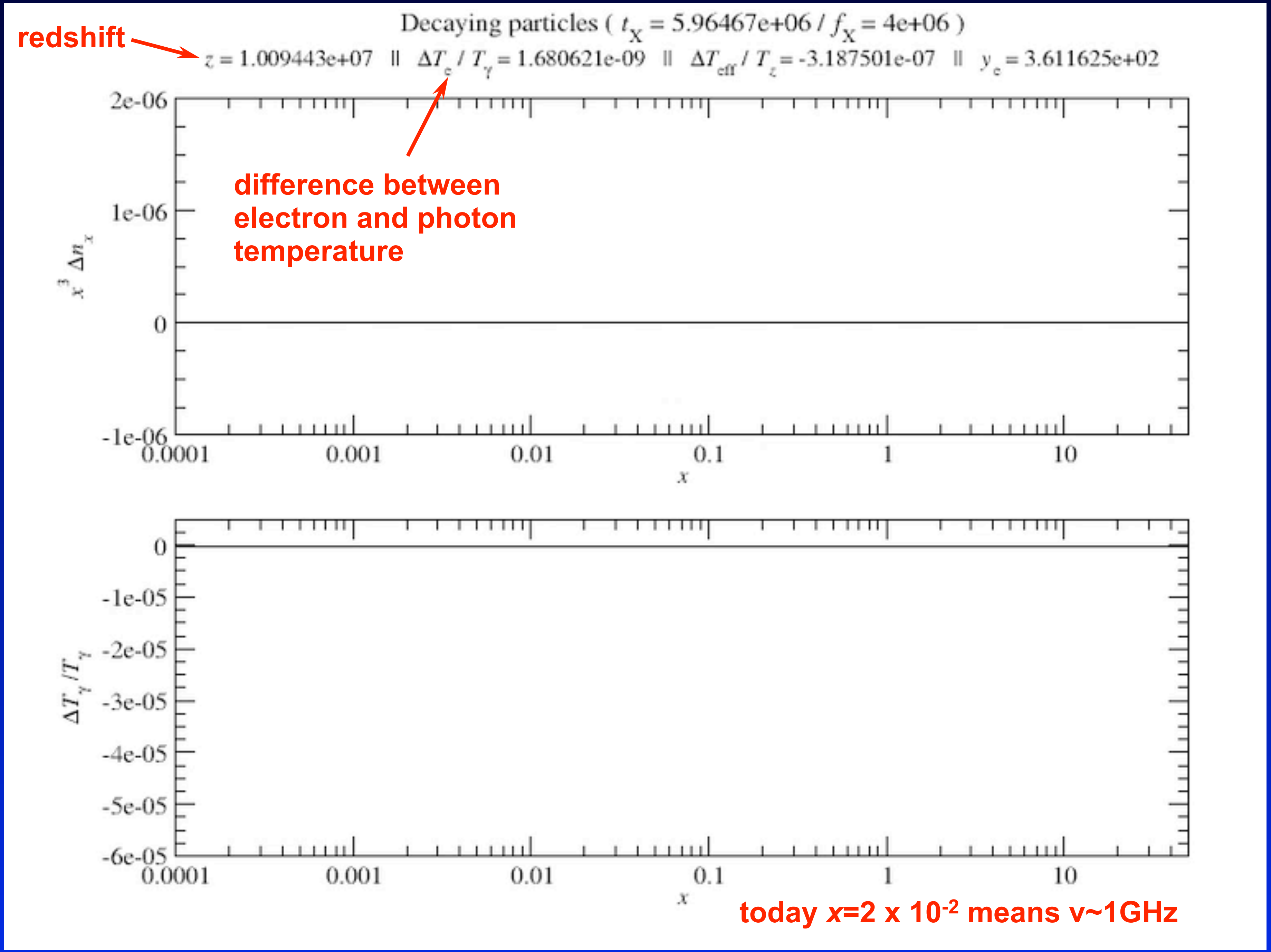

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Is there a simple way to include the effect of photon production at low frequencies?

Analytic Approximation for μ-distortion

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- *Comptonization efficient!* $\Longrightarrow$ $\left.\frac{\mathrm{d}n}{\mathrm{d}\tau}\right|_{\mathrm{C}} + \left.\frac{\mathrm{d}n}{\mathrm{d}\tau}\right|_{\mathrm{em/abs}} \approx 0$

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critical frequency

- *low frequency limit & small distortion* $\Longrightarrow$ $$\mu(x, z) \approx \mu_0(z)\,\mathrm{e}^{-x_{\mathrm{c}}(z)/x}$$

(**e.g., see Sunyaev & Zeldovich, 1970, ApSS, 7, 20; Hu 1995,** PhD Thesis)

chemical potential
at high frequencies

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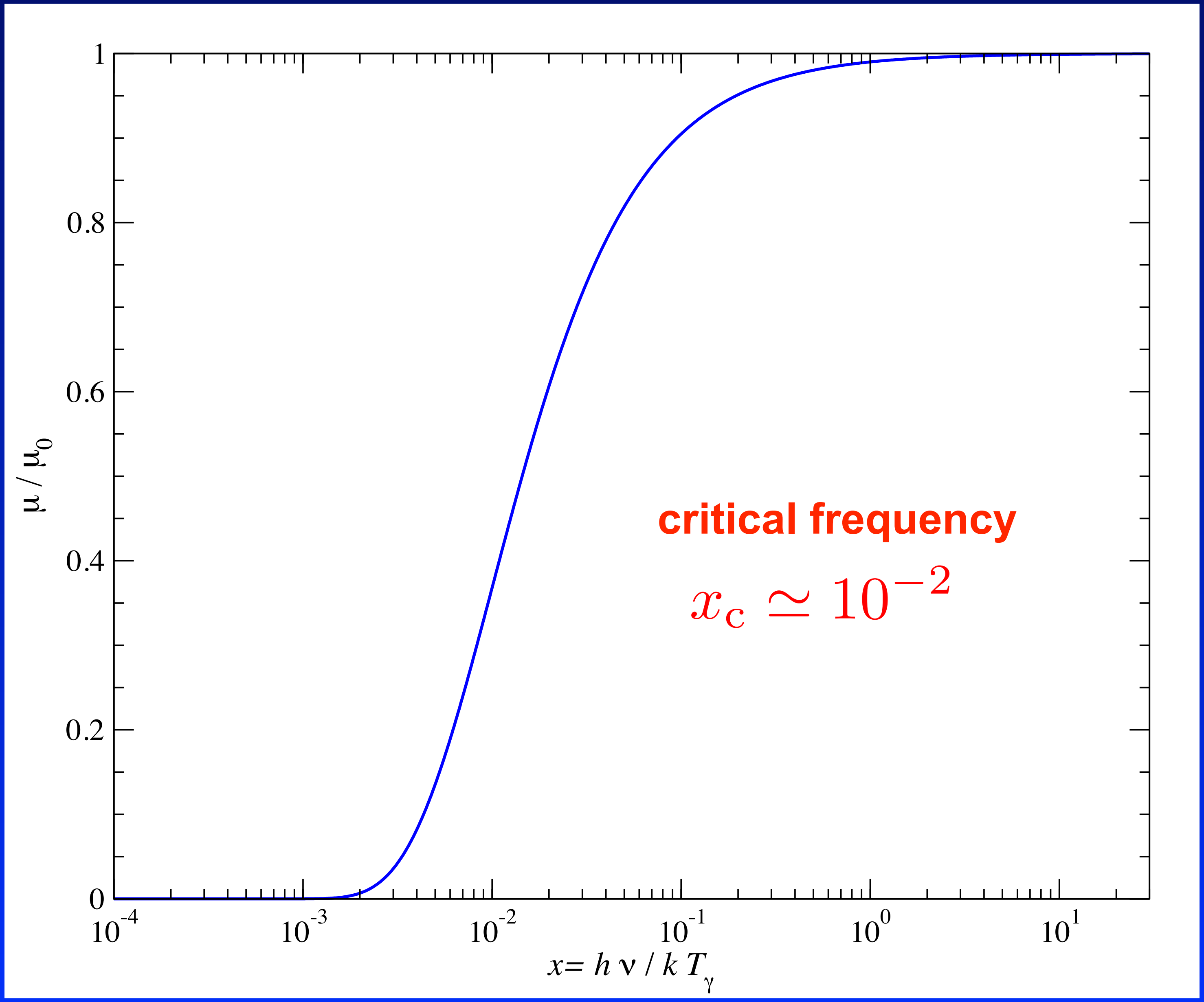

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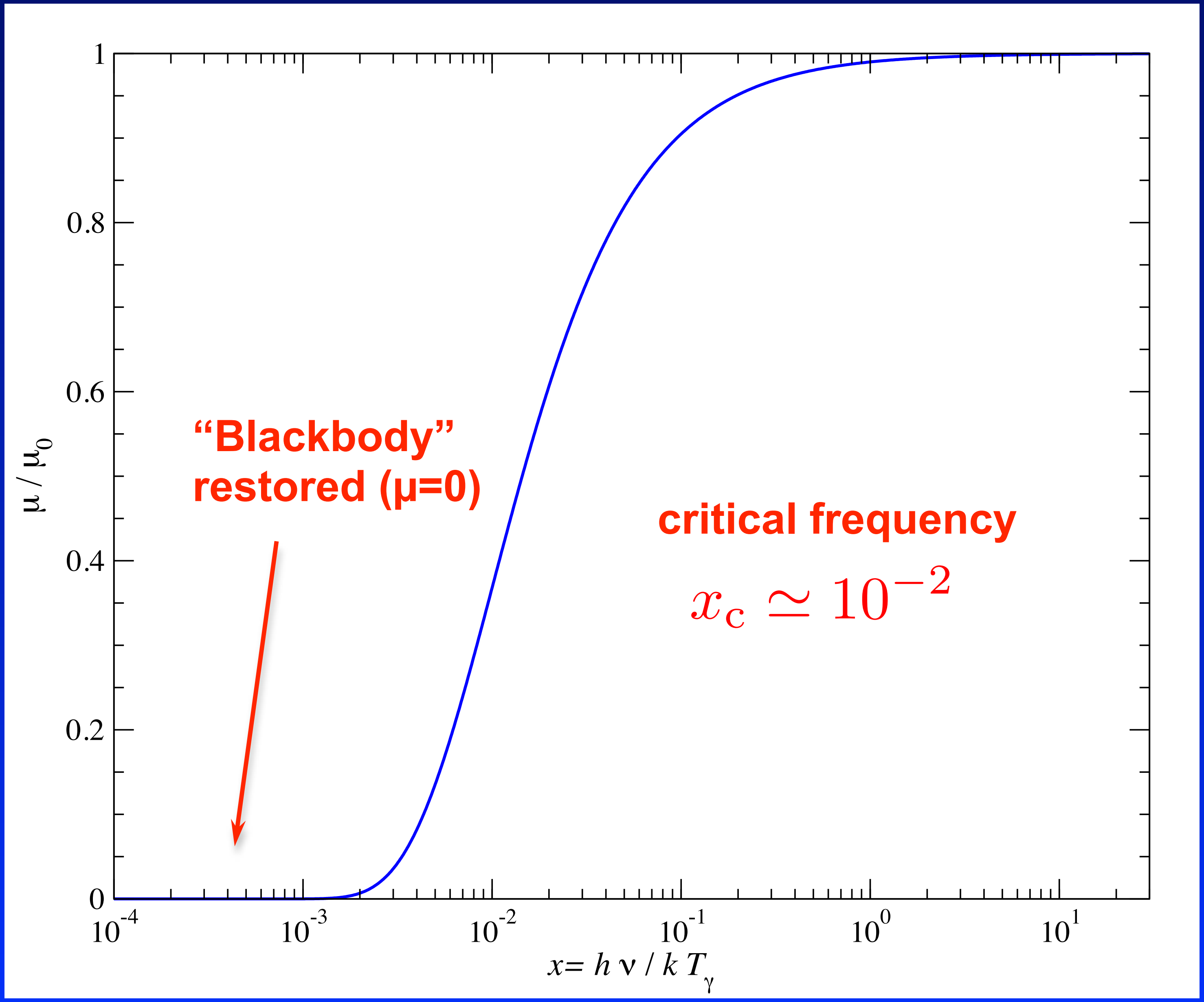

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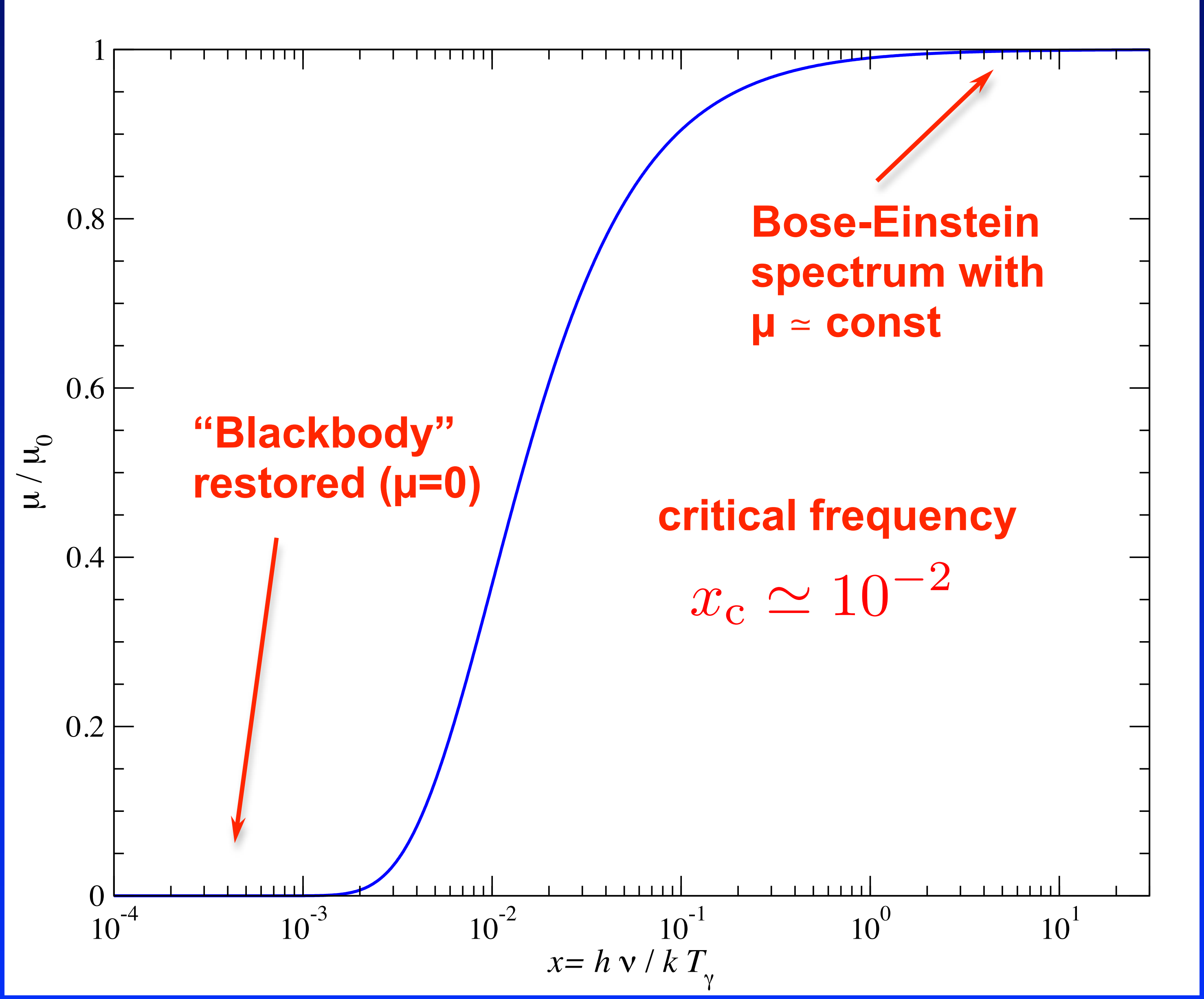

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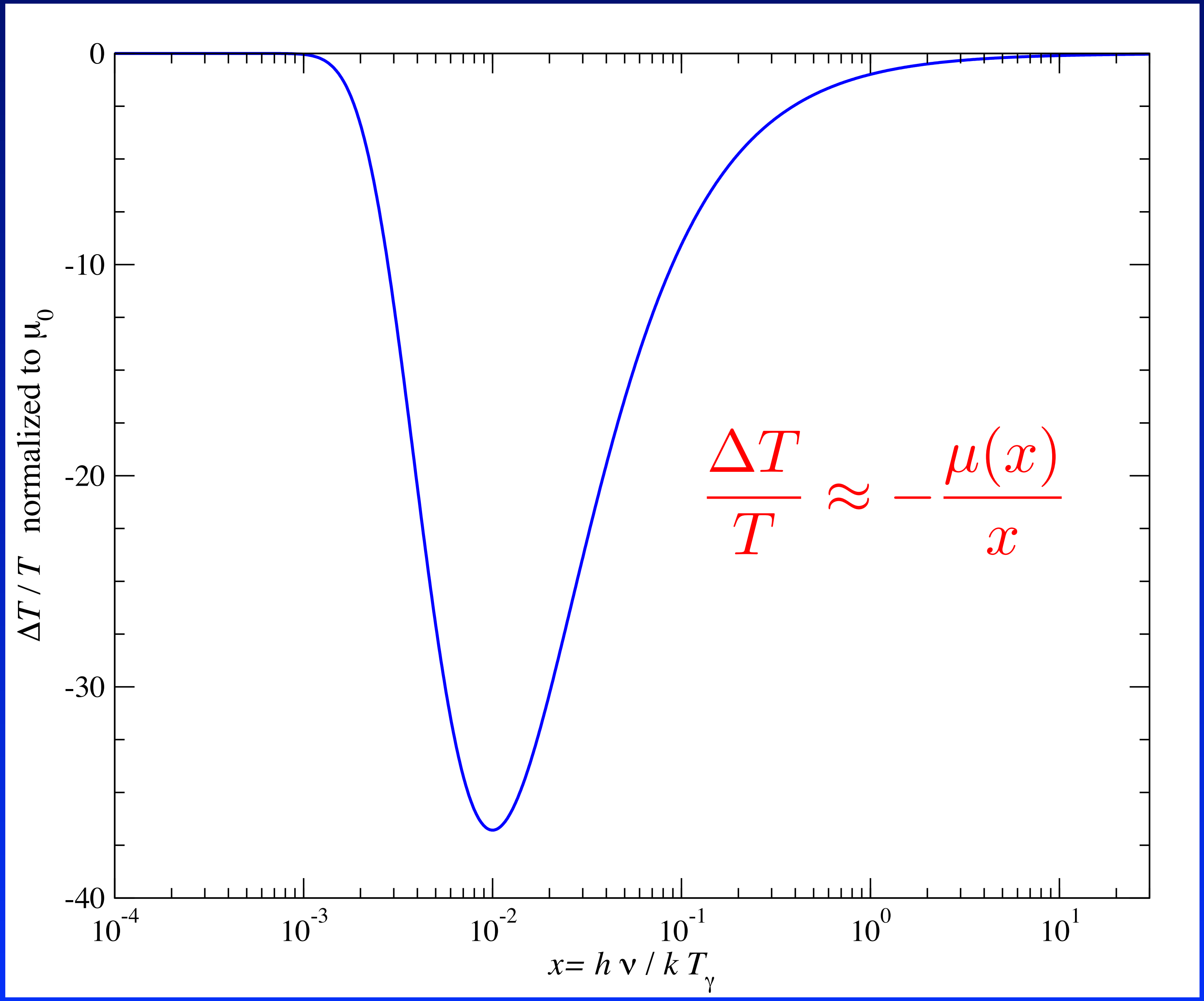

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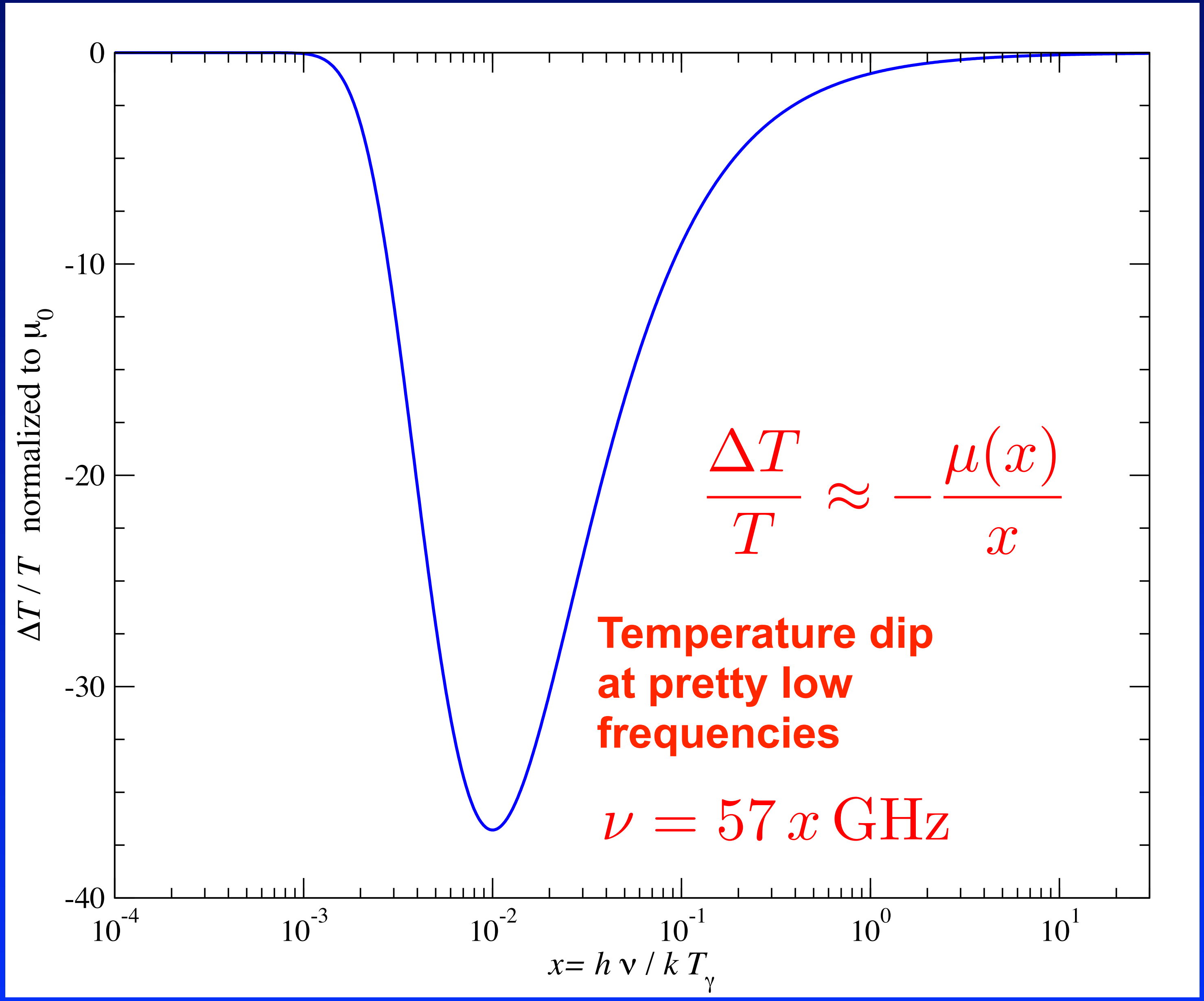

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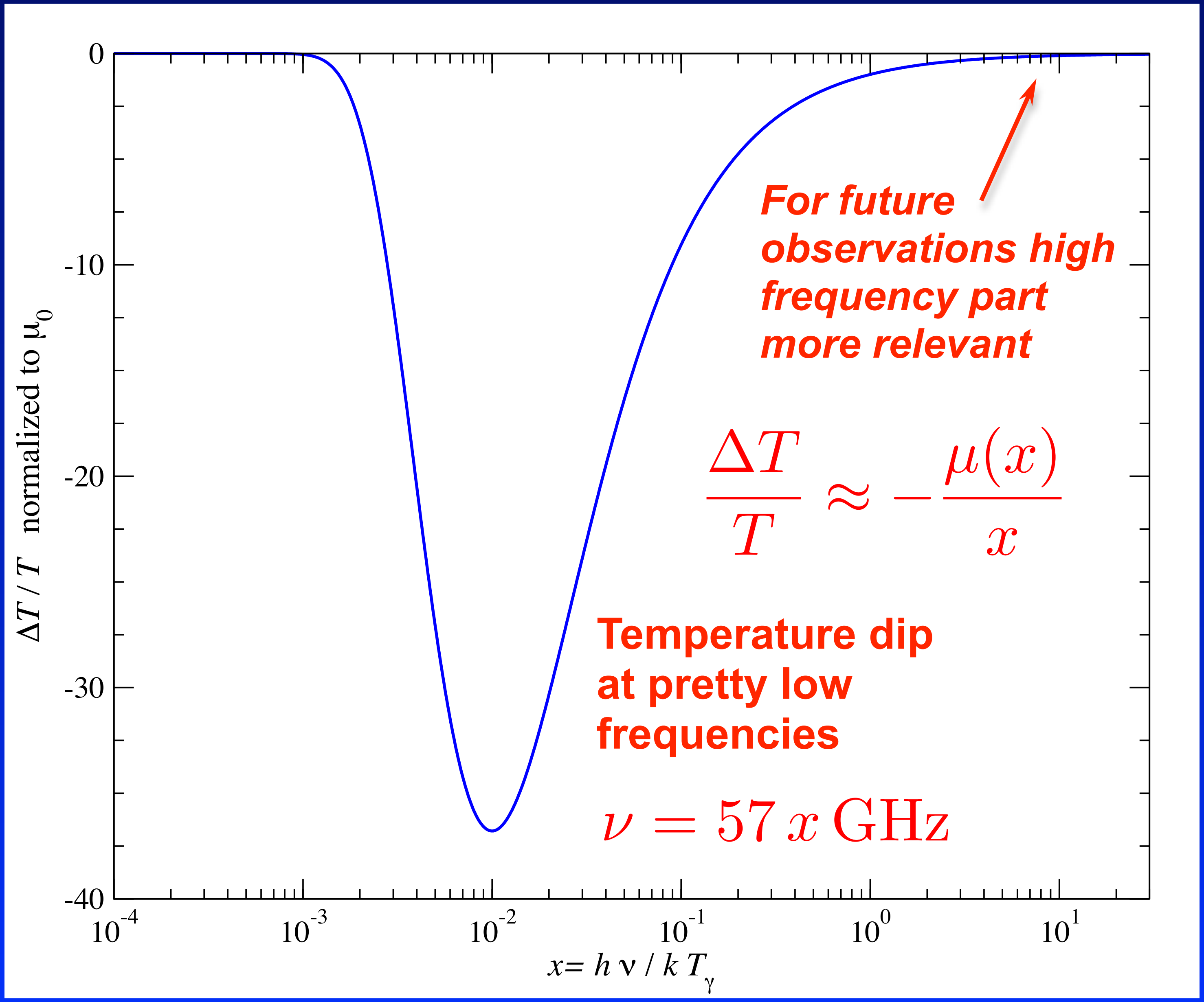

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Last step: How does $\mu_0(z)$ depend on z?

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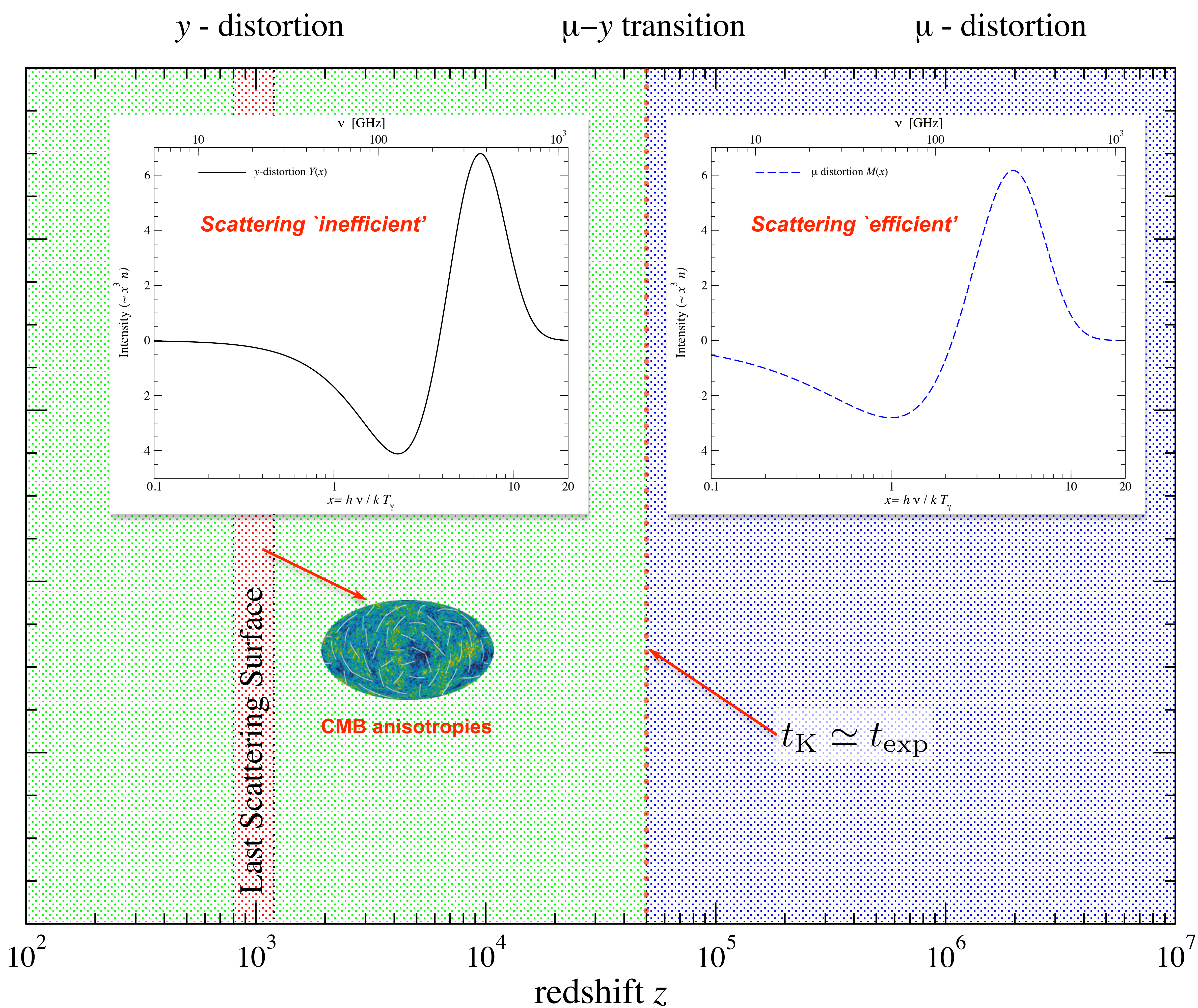
- *Transition between μ and y modeled as simple step function*

Classical approximations for μ and y



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μ−y transition
μ - distortion
ν [GHz]
y-distortion Y(x)
Scattering `inefficient'
Intensity (~ x^3 n)
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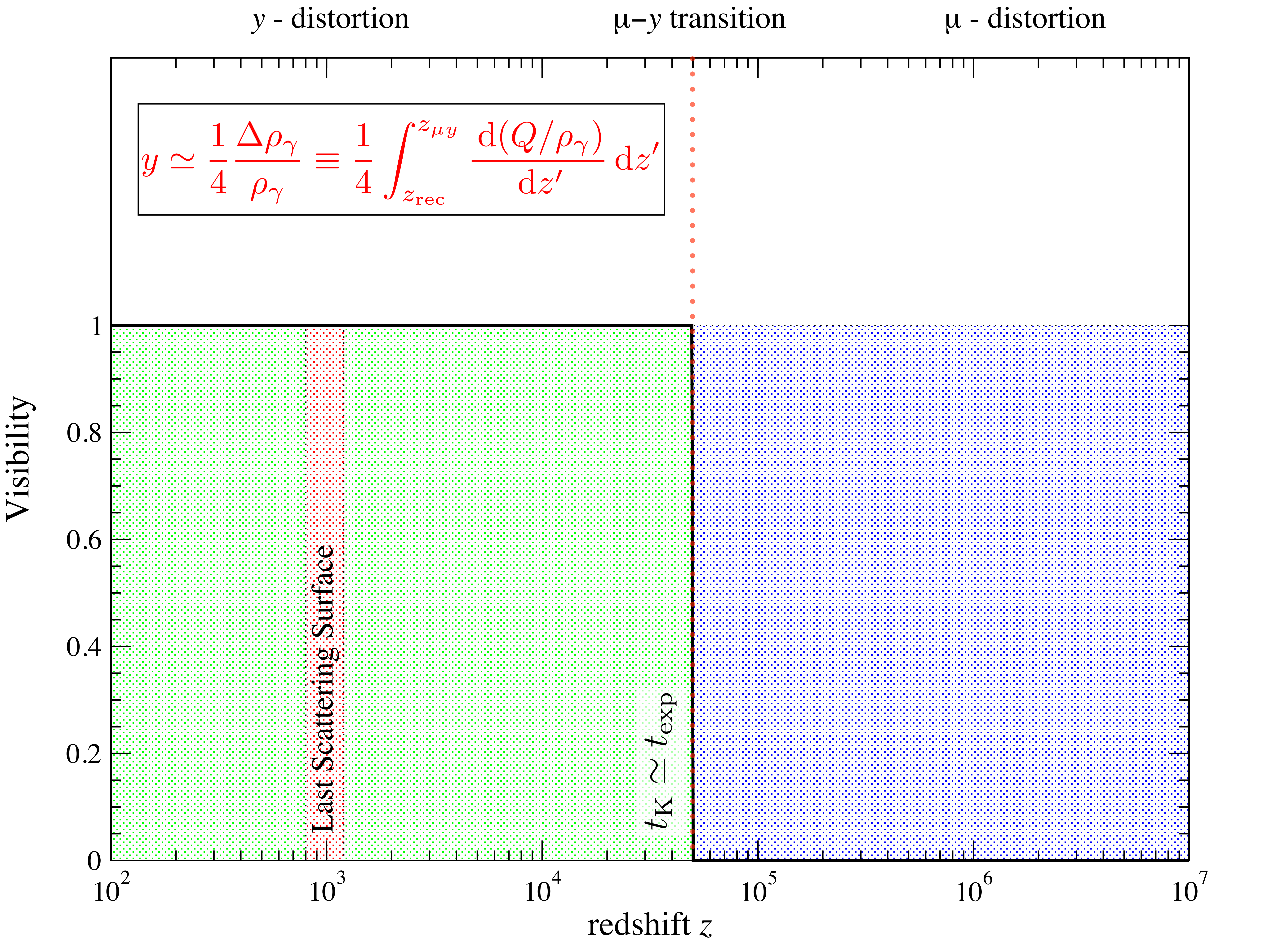

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μ–y transition
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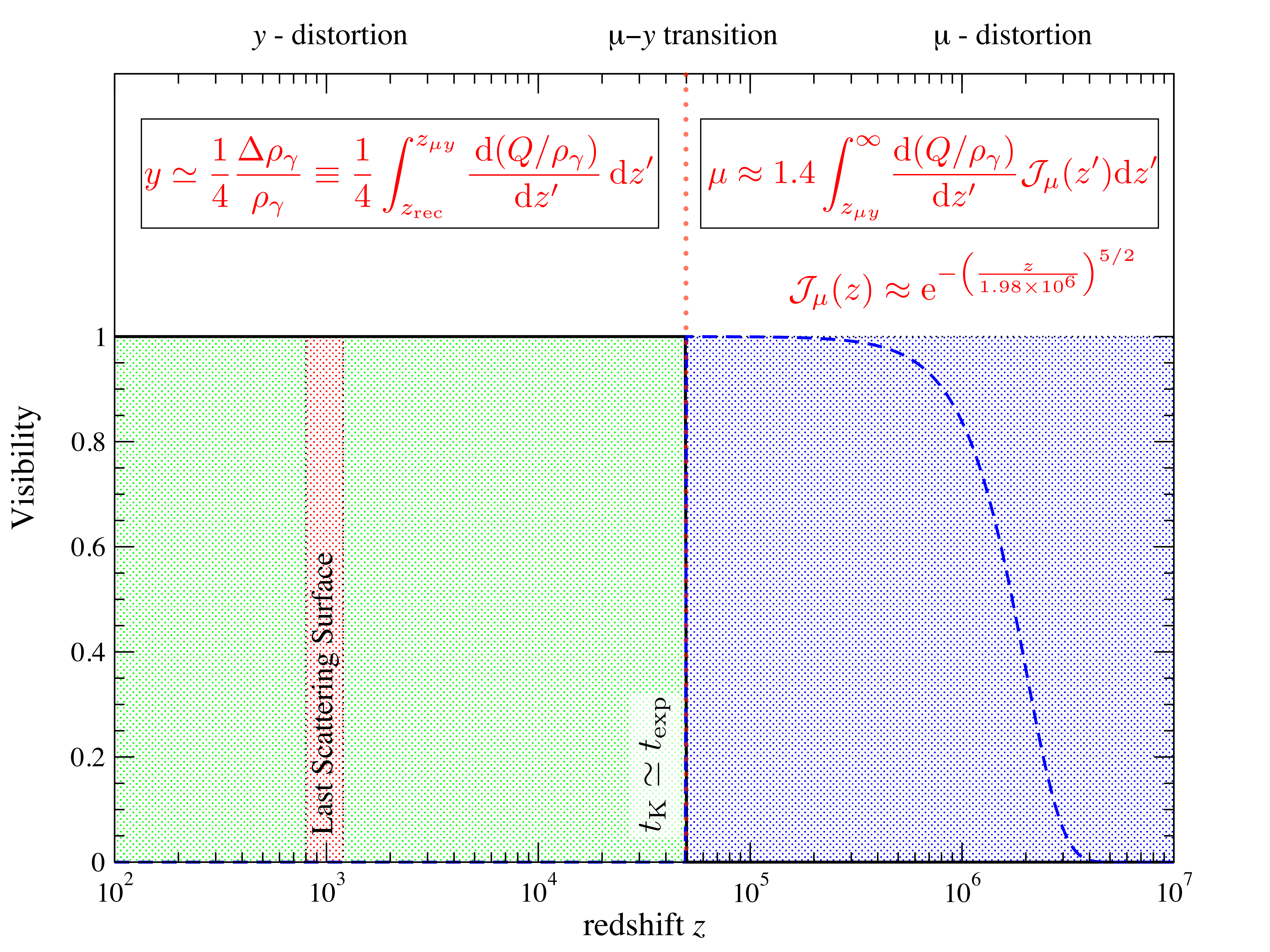
Visibility
Last Scattering Surface
$t_{\rm K} \simeq t_{\rm exp}$
redshift z



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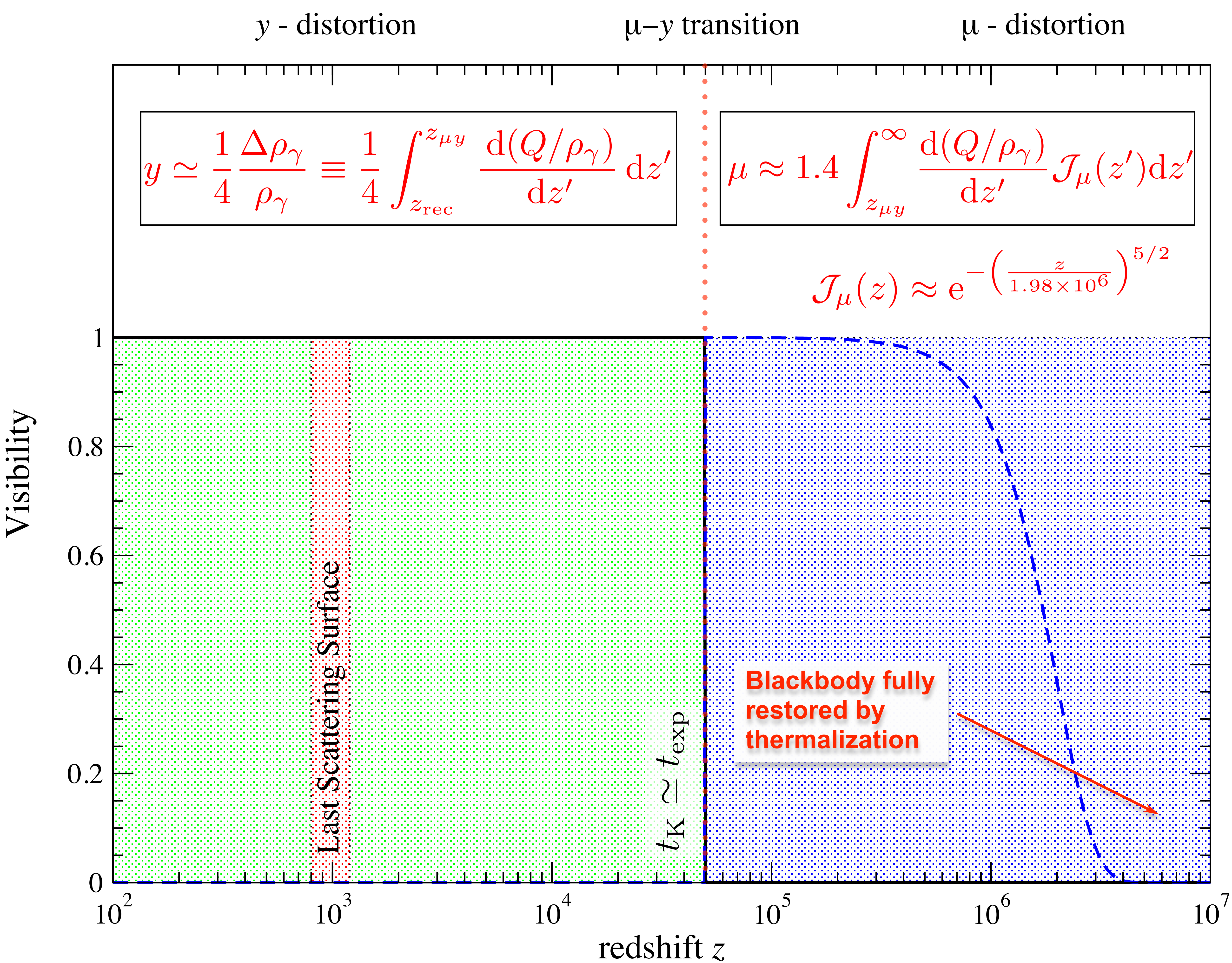
$\mathcal{J}_\mu(z) \approx {\rm e}^{-\left(\frac{z}{1.98\times10^6}\right)^{5/2}}$
Visibility
Last Scattering Surface
$t_{\rm K} \simeq t_{\rm exp}$
redshift z



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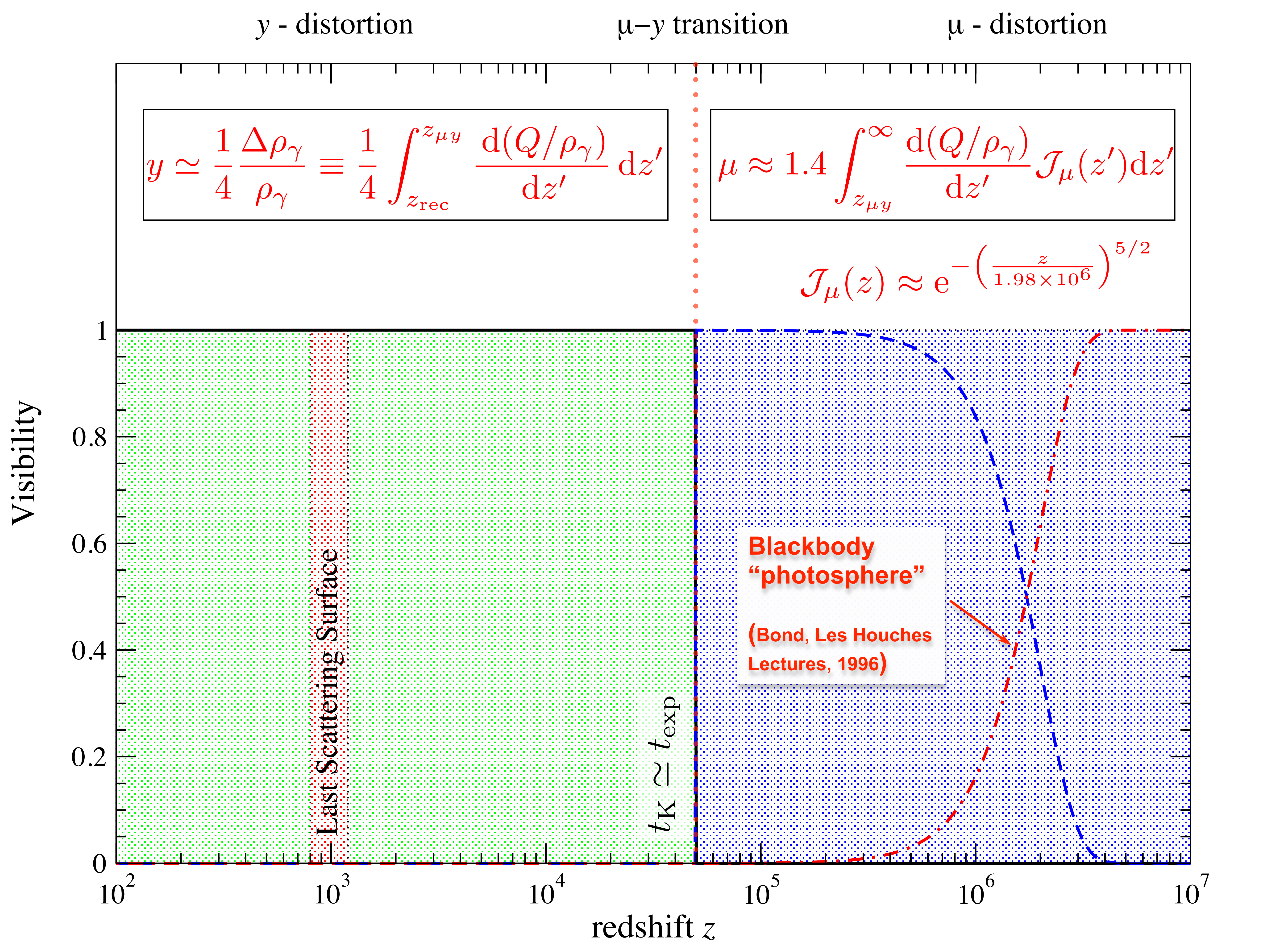
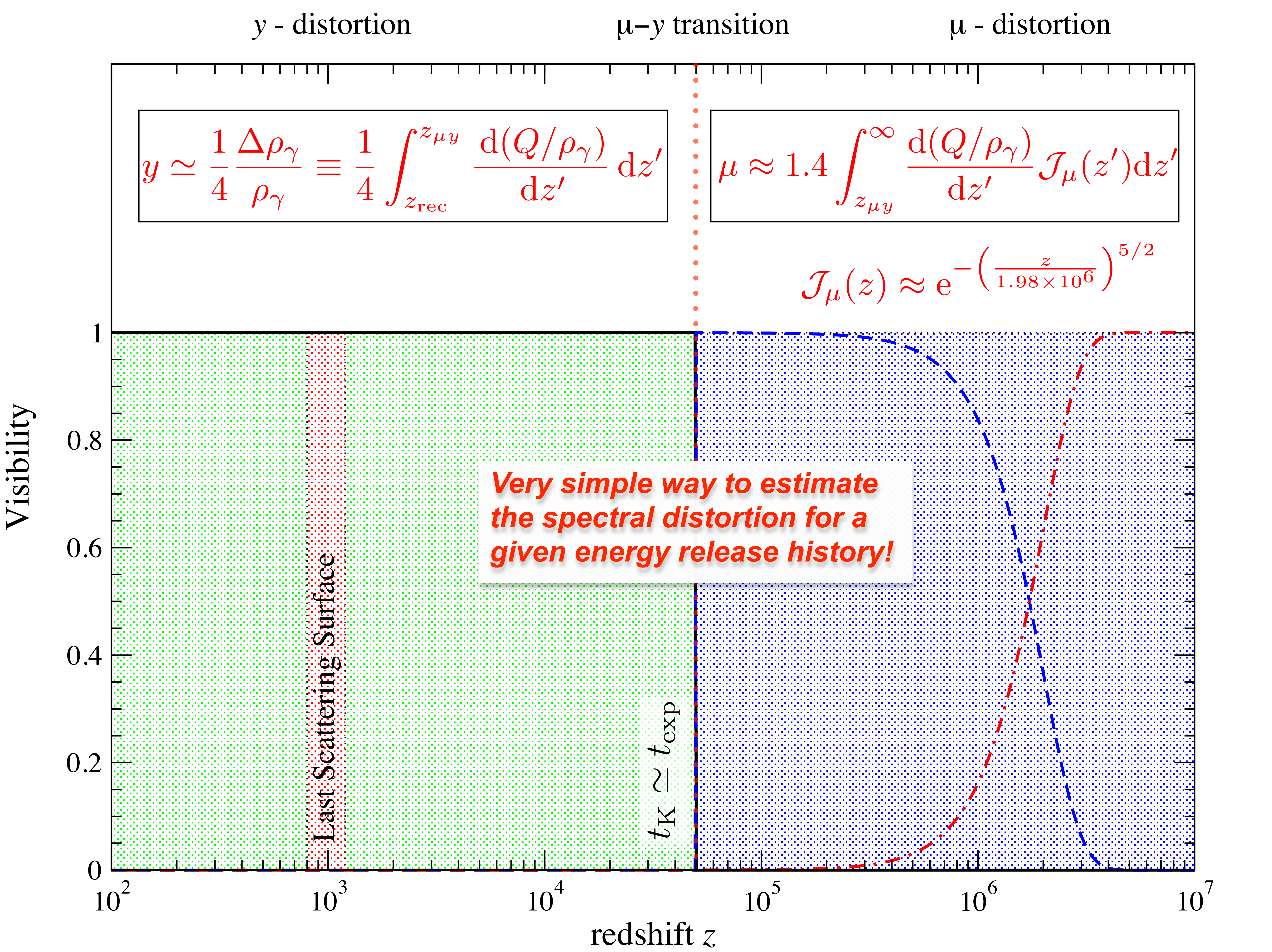
Visibility
Last Scattering Surface
$t_{\rm K} \simeq t_{\rm exp}$
Blackbody fully restored by thermalization
redshift z


y - distortion
μ–y transition
μ - distortion
$y \simeq \frac{1}{4}\frac{\Delta\rho_\gamma}{\rho_\gamma} \equiv \frac{1}{4}\int_{z_{\rm rec}}^{z_{\mu y}} \frac{{\rm d}(Q/\rho_\gamma)}{{\rm d}z'}\,{\rm d}z'$
$\mu \approx 1.4 \int_{z_{\mu y}}^{\infty} \frac{{\rm d}(Q/\rho_\gamma)}{{\rm d}z'}\mathcal{J}_\mu(z'){\rm d}z'$
$\mathcal{J}_\mu(z) \approx {\rm e}^{-\left(\frac{z}{1.98\times10^6}\right)^{5/2}}$
Visibility
Last Scattering Surface
$t_{\rm K} \simeq t_{\rm exp}$
Blackbody "photosphere"
(Bond, Les Houches Lectures, 1996)
redshift z



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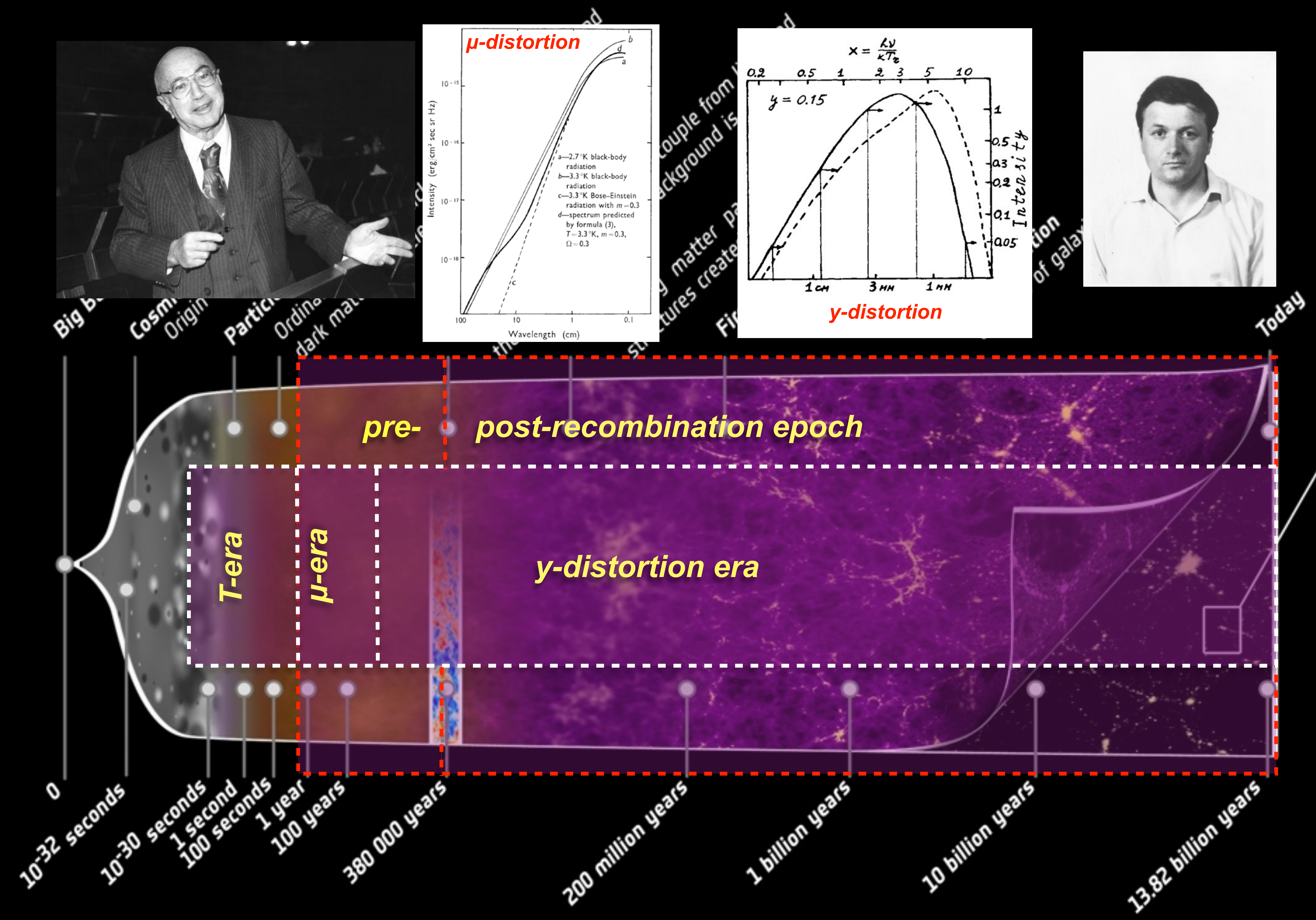
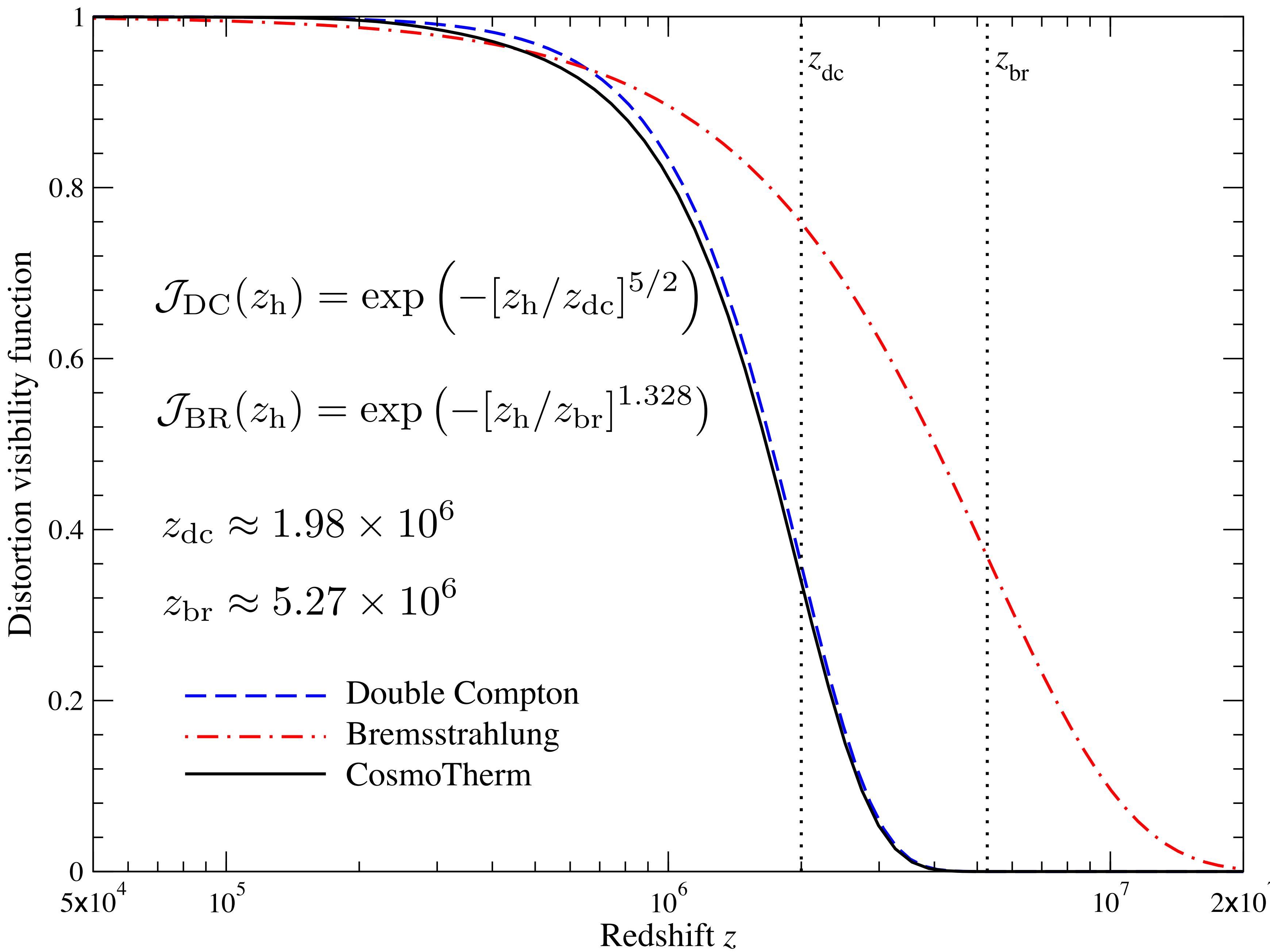
Very simple way to estimate the spectral distortion for a given energy release history!
Last Scattering Surface
$t_{\rm K} \simeq t_{\rm exp}$
Visibility
redshift z



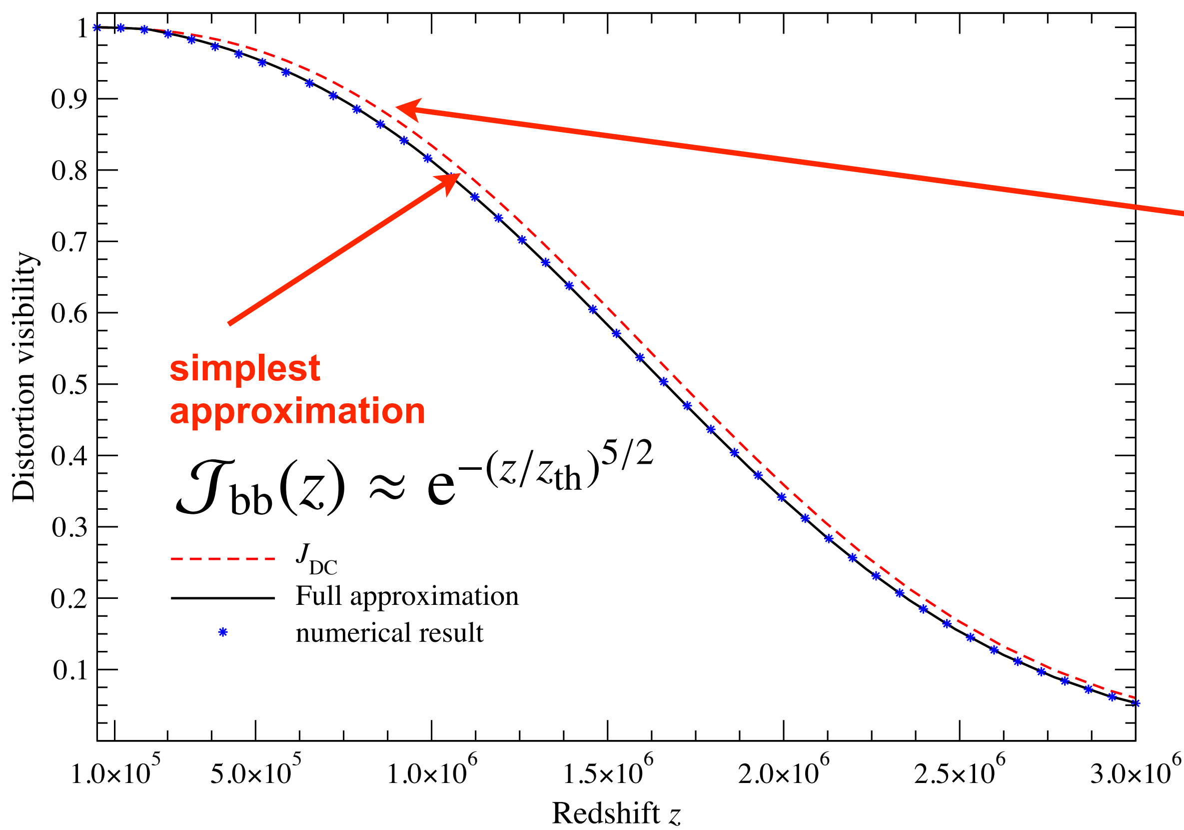
μ-distortion
Intensity (erg/cm² sec sr Hz)
Wavelength (cm)
a—2.7 °K black-body radiation
b—3.3 °K black-body radiation
c—3.3 °K Bose–Einstein radiation with m = 0.3
d—spectrum predicted by formula (3), T = 3.3 °K, m = 0.3, Ω = 0.3
x = hν/kT_e
y = 0.15
Intensity
1 cm
3 mm
1 mm
y-distortion
Today
pre-
post-recombination epoch
T-era
μ-era
y-distortion era
0
10-32 seconds
10-30 seconds
1 second
100 seconds
1 year
100 years
380 000 years
200 million years
1 billion years
10 billion years
13.82 billion years

Distortion visibility for BR and DC



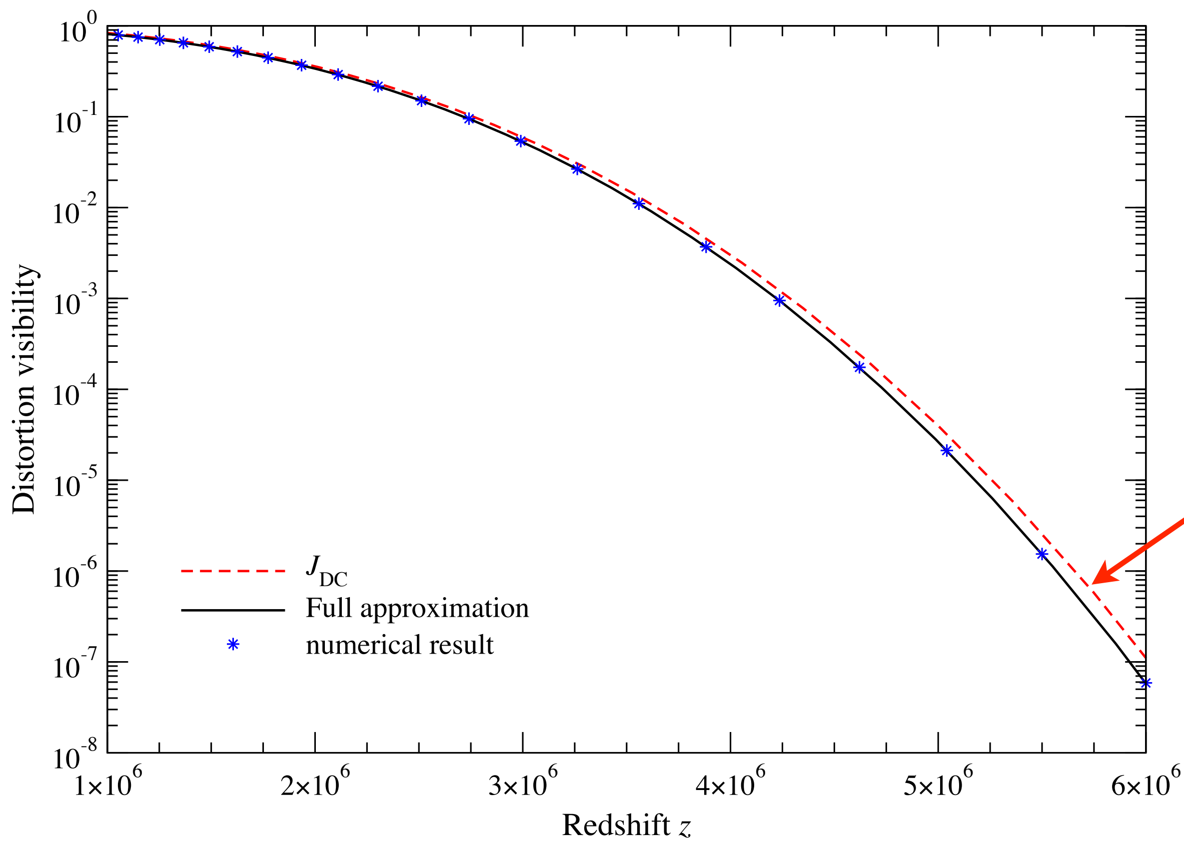
- Original estimates only included the effect of BR
- Double Compton emission was first included by Danese & de Zotti, 1982
- DC changes the distortion visibility quite strongly

Improved shape for the distortion visibility function



→ correction from photons being stuck at low frequencies important here

- Approximation given in terms of simple integrals
- Evaluation very fast and precise
- time-dependent and relativistic corrections can be included
- will be available as part of the *CosmoTherm* package



visibility is off by a factor of ~2 with simplest approximation

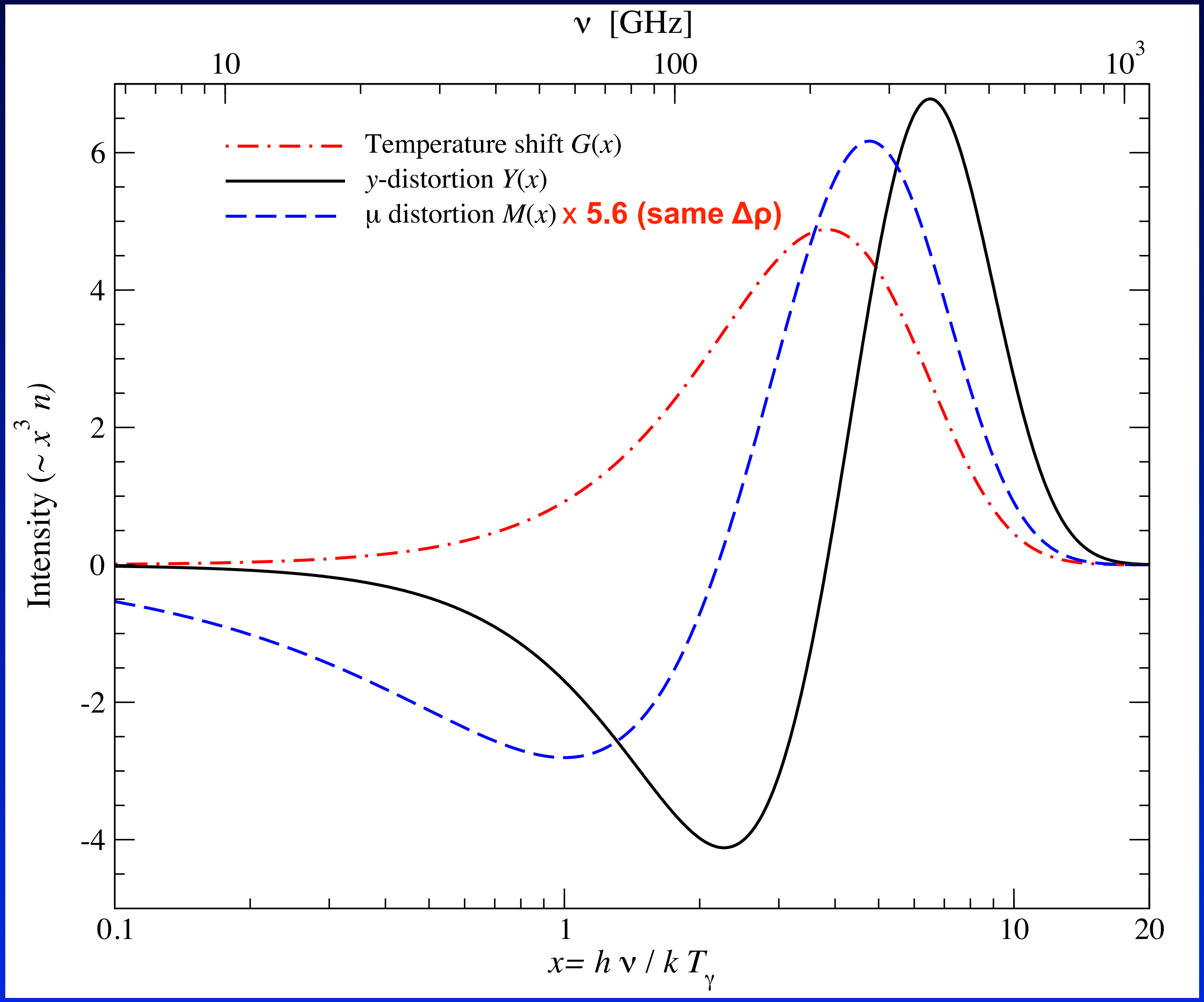
Khatri & Sunyaev, 2012

JC 2014, ArXiv:1312.6030

What about the μ-y transition regime?

Is the transition really as abrupt?

Temperature shift ↔ y-distortion ↔ μ-distortion



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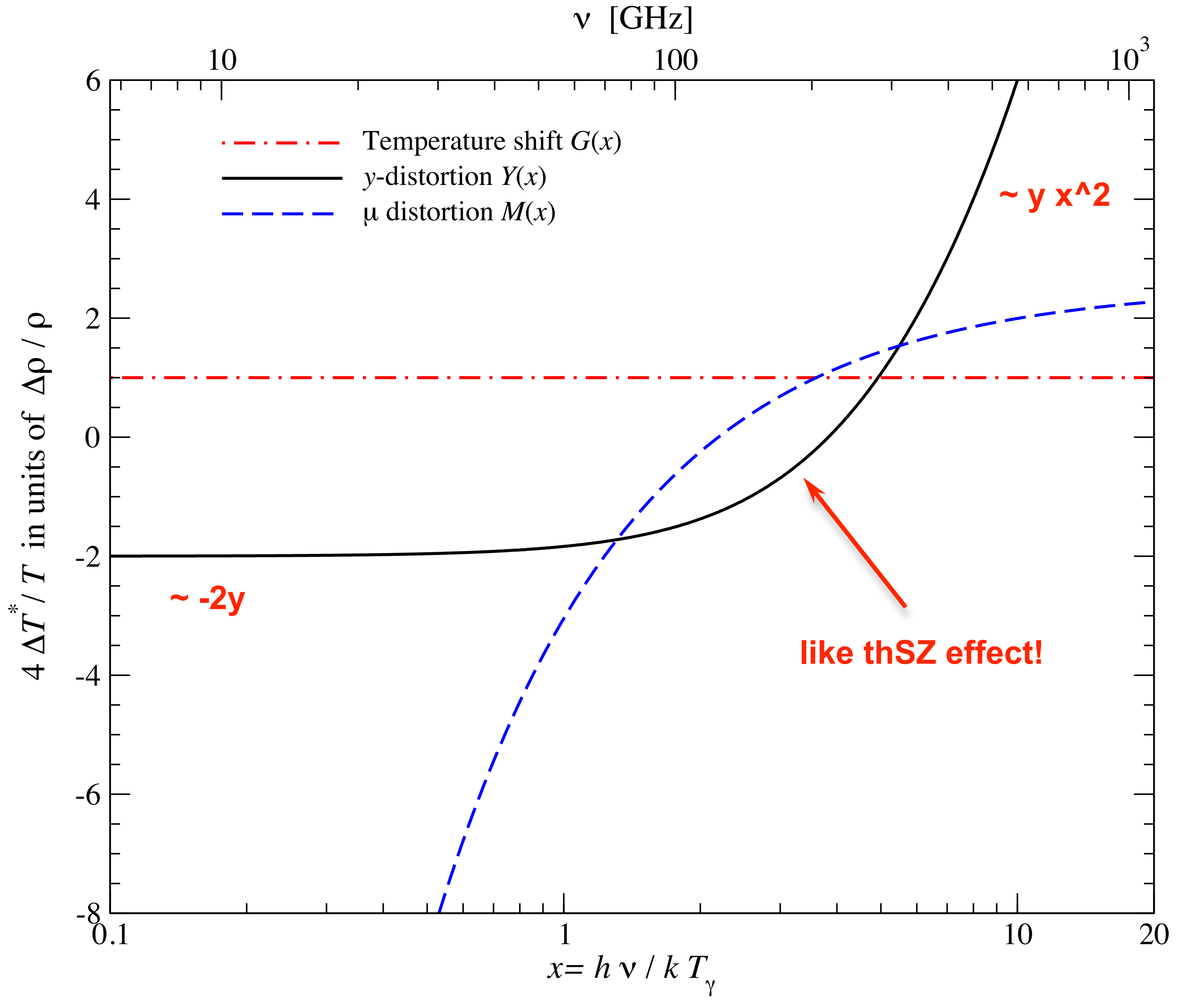


Same as before but as effective temperature

$$\frac{\Delta T^*}{T} \approx \frac{\Delta n(x)}{G(x)}$$

Transition from y-distortion → μ-distortion

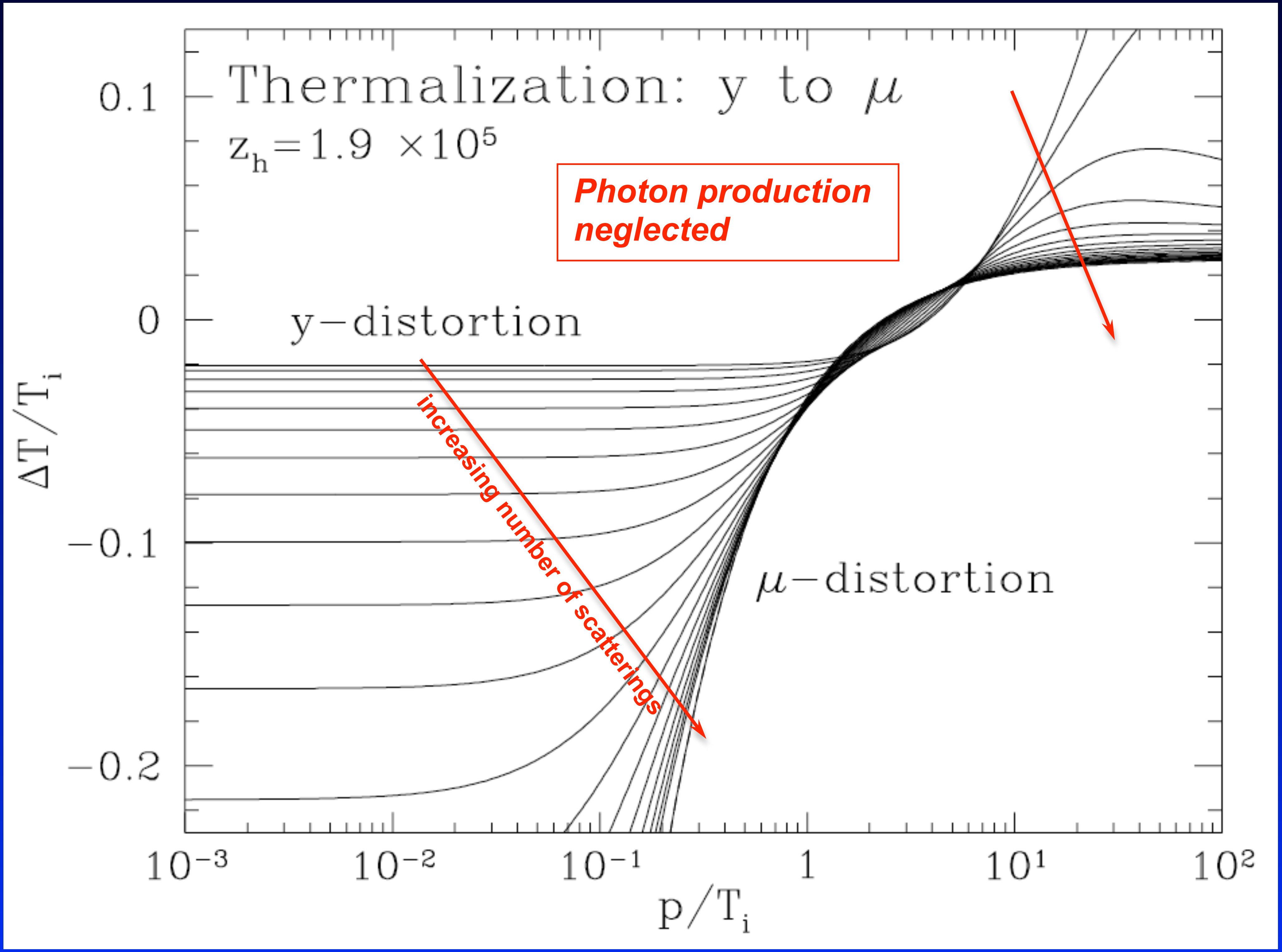
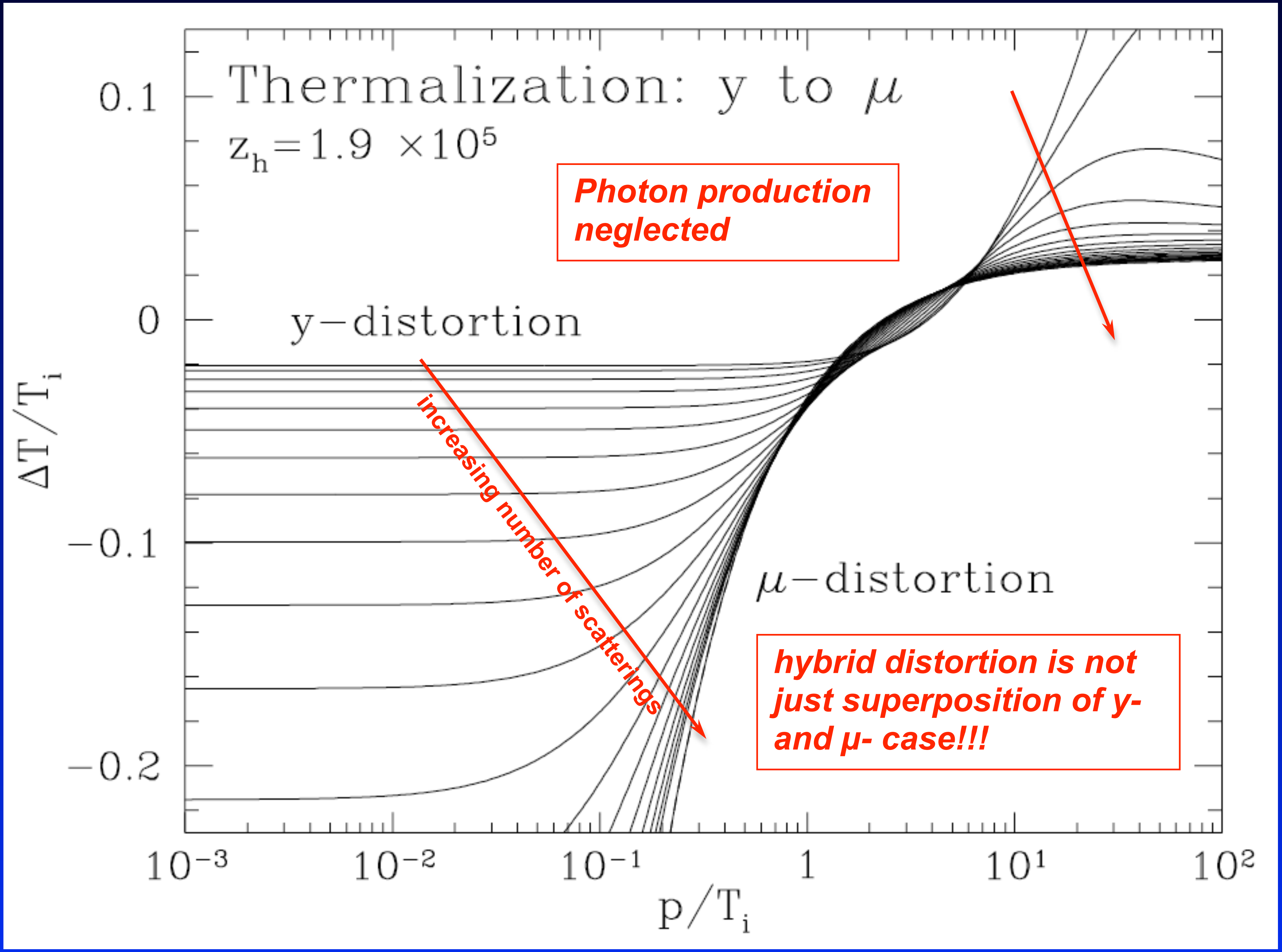


Figure from Wayne Hu's PhD thesis, 1995, but see also discussion in Burigana, 1991

Transition from y-distortion → μ-distortion



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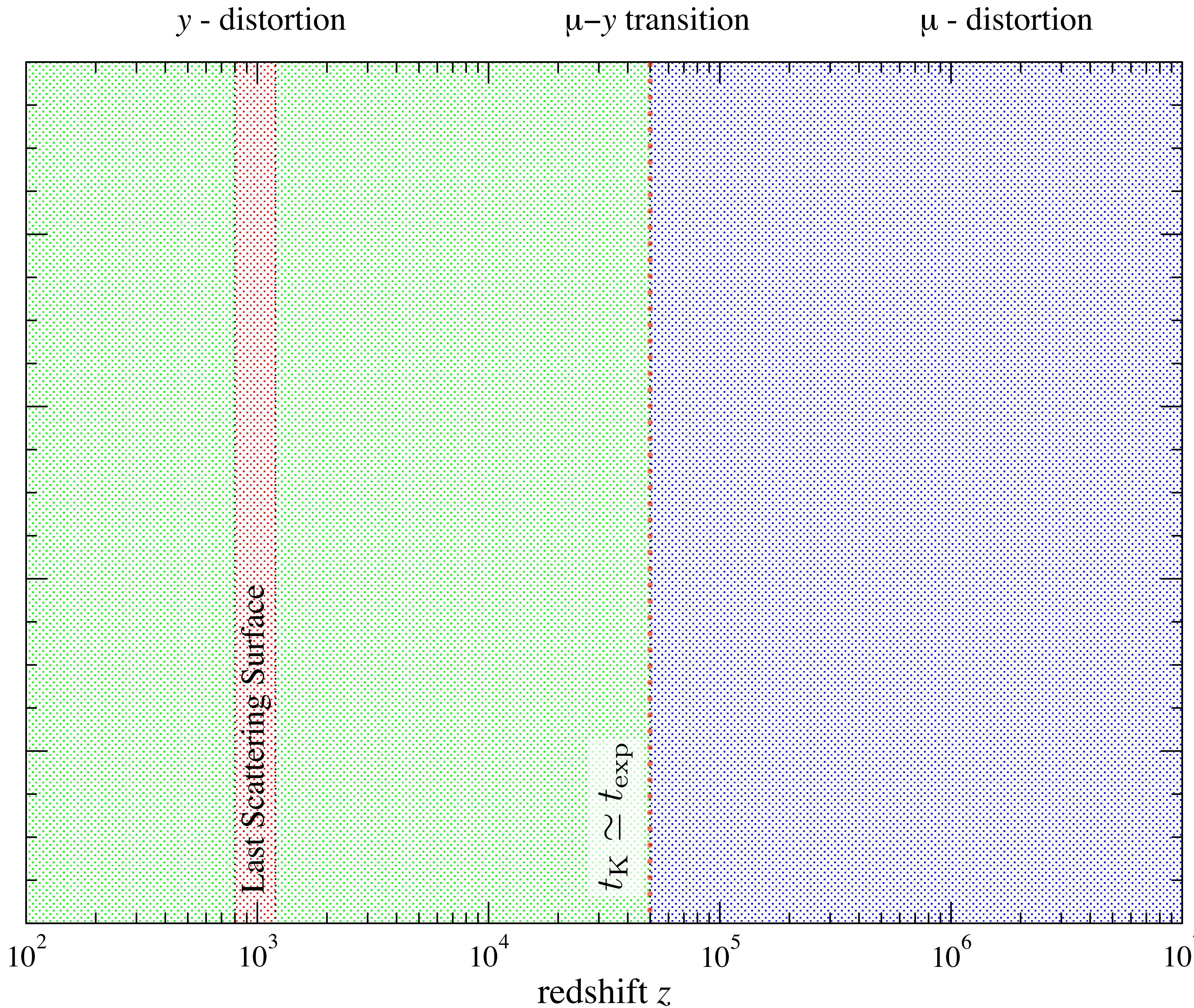
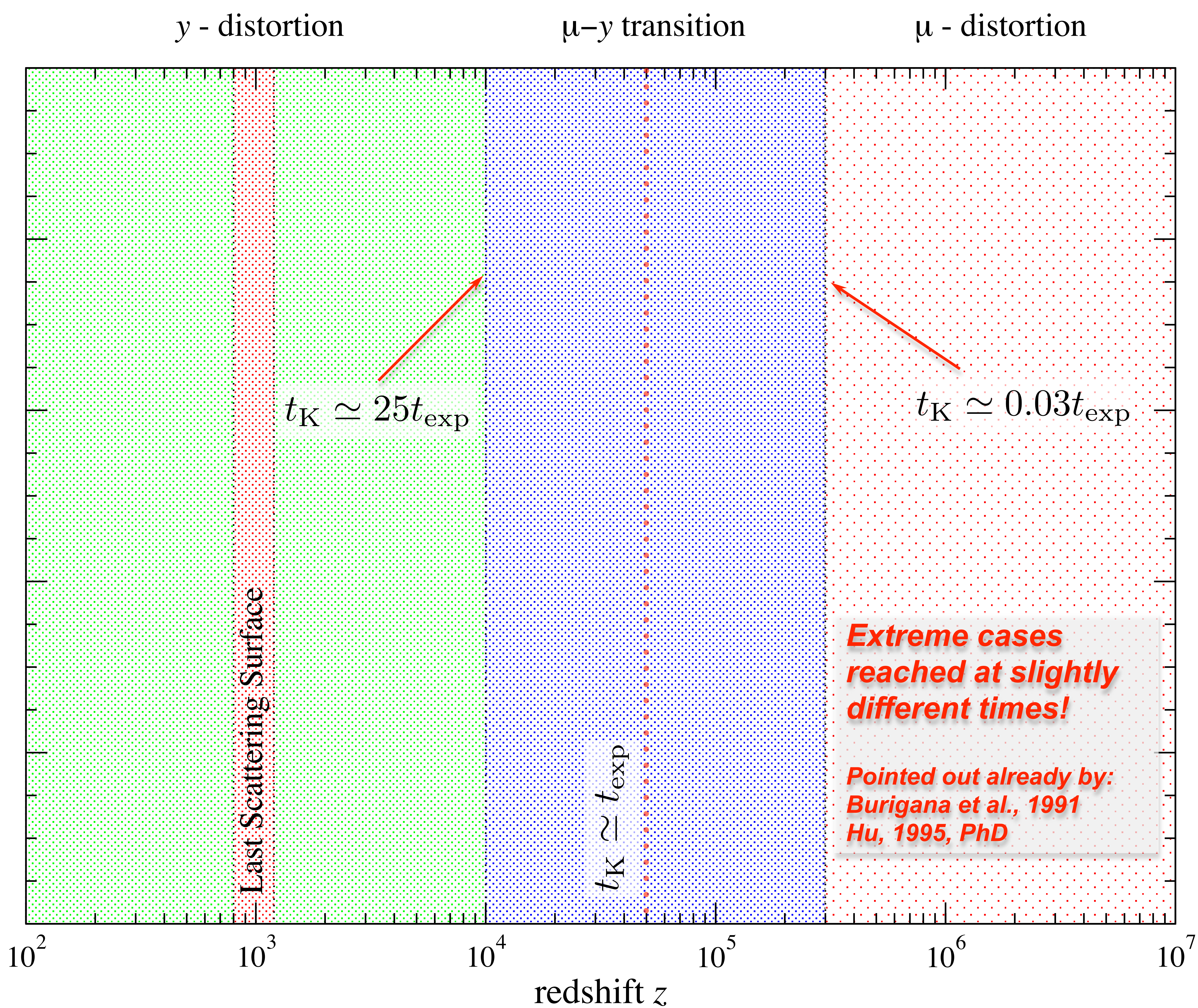
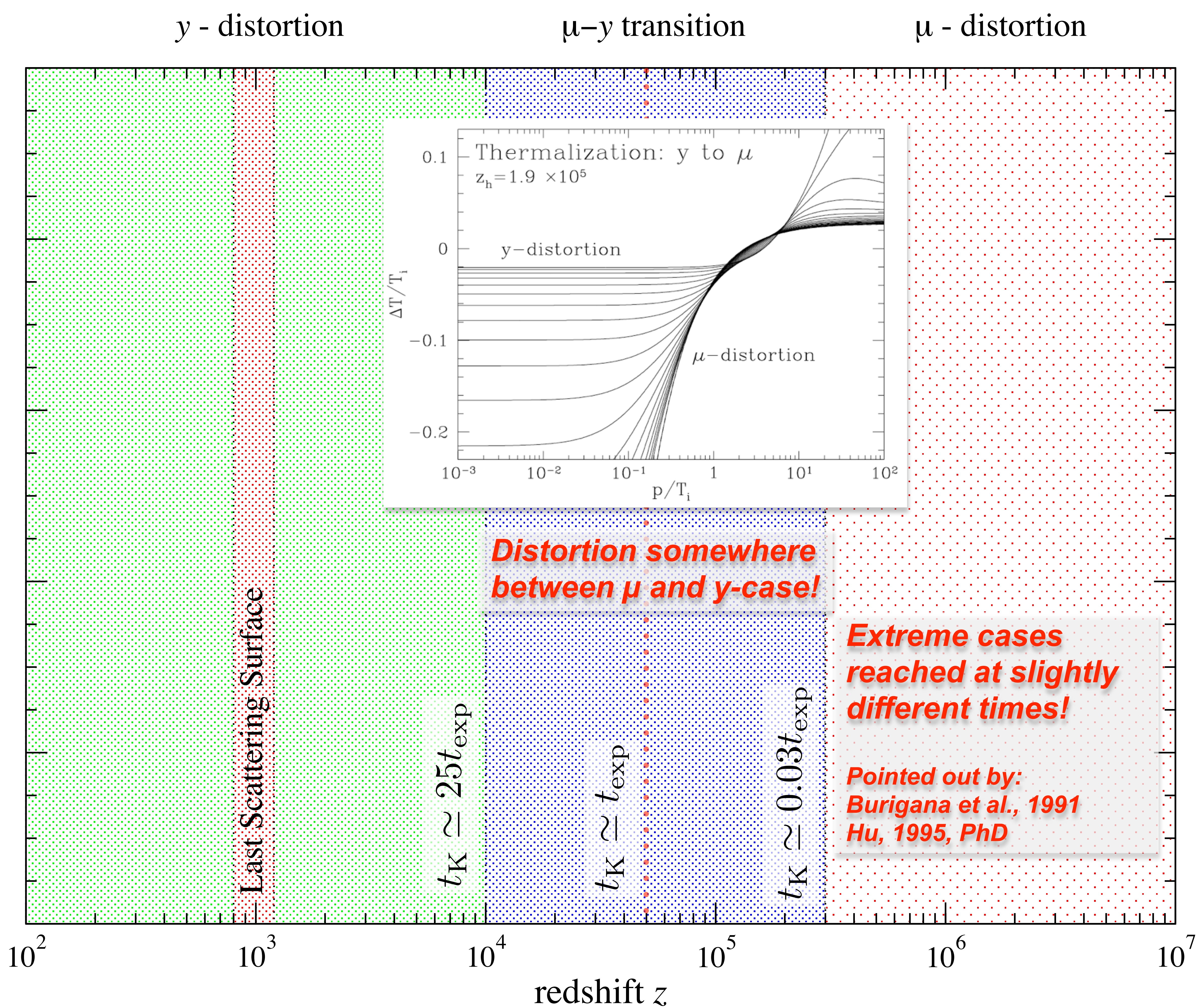


y - distortion
μ–y transition
μ - distortion
Last Scattering Surface
$t_{\mathrm{K}} \simeq t_{\mathrm{exp}}$
10^2
10^3
10^4
10^5
10^6
10^7
redshift z

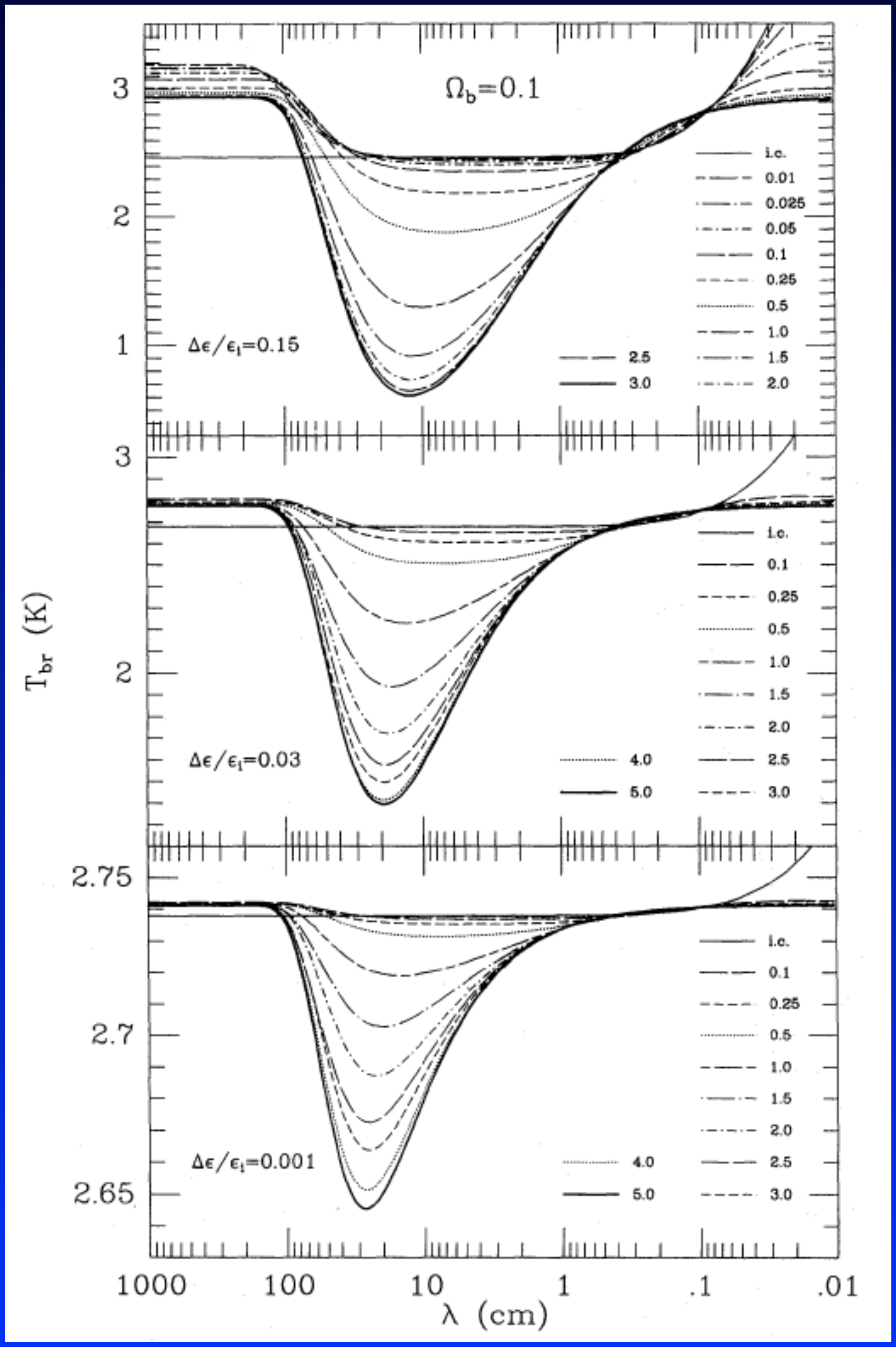


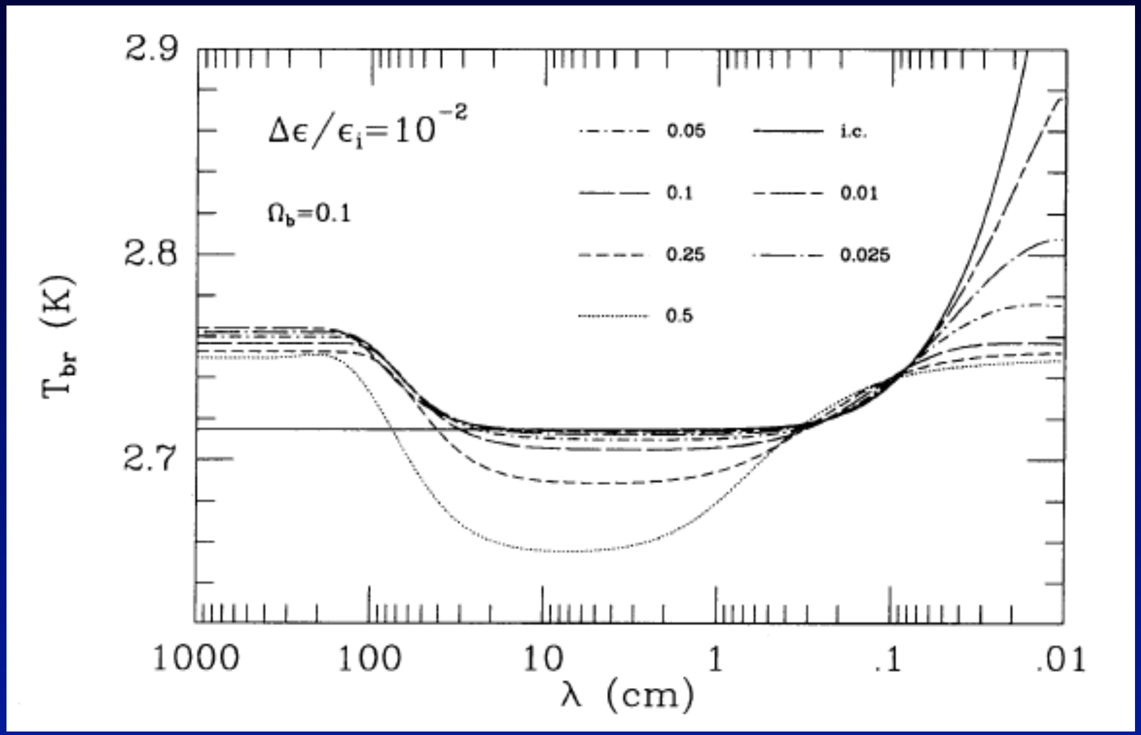
y - distortion
μ–y transition
μ - distortion
Last Scattering Surface
$t_{\rm K} \simeq 25 t_{\rm exp}$
$t_{\rm K} \simeq t_{\rm exp}$
$t_{\rm K} \simeq 0.03 t_{\rm exp}$
Extreme cases reached at slightly different times!
Pointed out already by:
Burigana et al., 1991
Hu, 1995, PhD
10^2
10^3
10^4
10^5
10^6
10^7
redshift z



y - distortion
μ–y transition
μ - distortion
Thermalization: y to μ
z_h=1.9 ×10^5
y−distortion
μ−distortion
ΔT/T_i
p/T_i
Distortion somewhere between μ and y-case!
Extreme cases reached at slightly different times!
Pointed out by:
Burigana et al., 1991
Hu, 1995, PhD
Last Scattering Surface
t_K ≃ 25t_exp
t_K ≃ t_exp
t_K ≃ 0.03t_exp
redshift z

Thermalization from *y* → *μ at low frequencies*





- amount of energy

  ↔ amplitude of distortion

  ↔ position of 'dip'

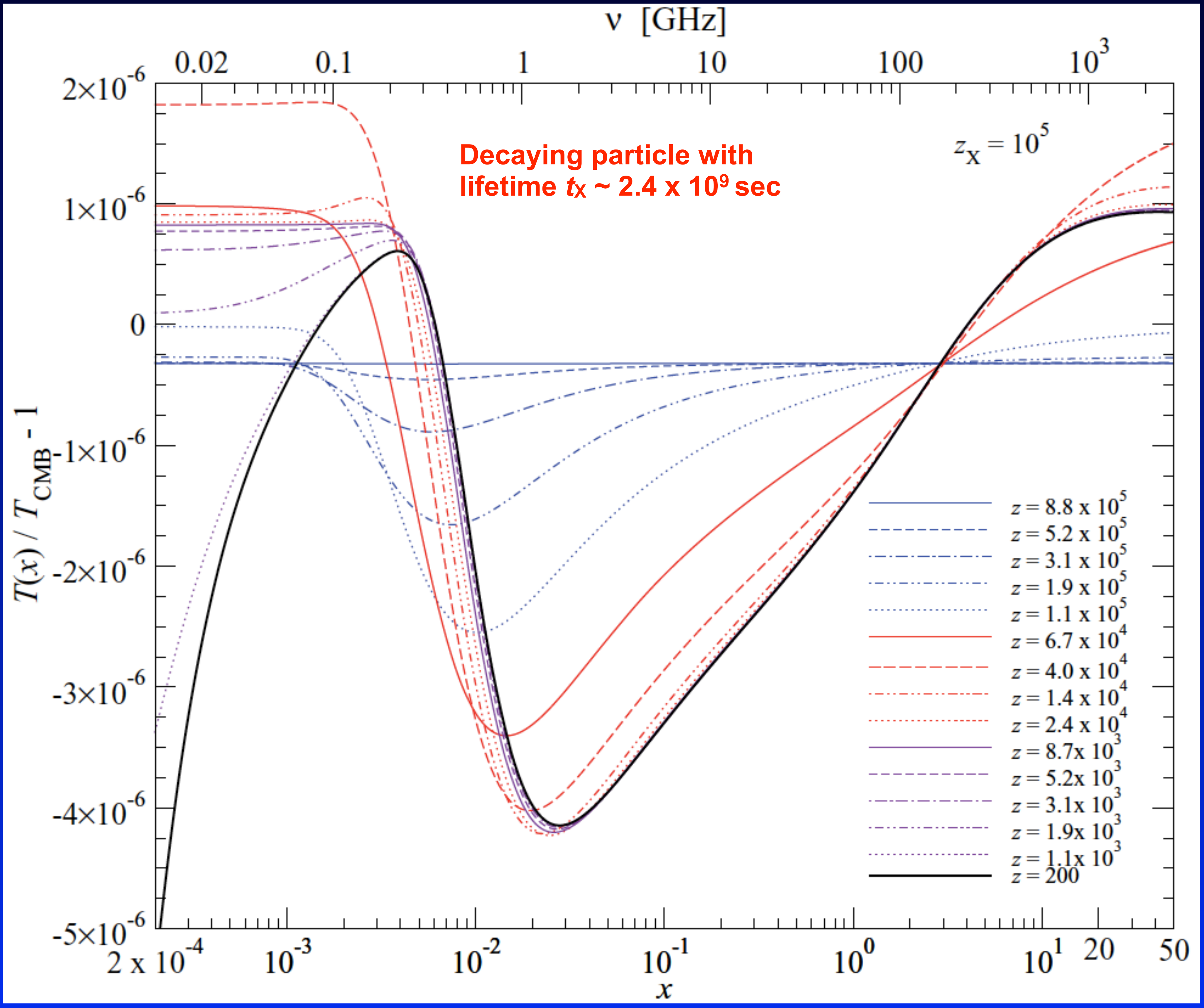
- Intermediate case ($3 \times 10^5 \geq z \geq 10000$)

  ⇒ mixture between *μ* & *y* + *residual*

- details at very low frequencies change

Burigana, De Zotti & Danese, 1991, ApJ
Burigana, Danese & De Zotti, 1991, A&A

Distortion *not* just superposition of μ and y-distortion!



Computation carried out with *CosmoTherm*
(JC & Sunyaev 2011)

First explicit calculation that showed that there is more!

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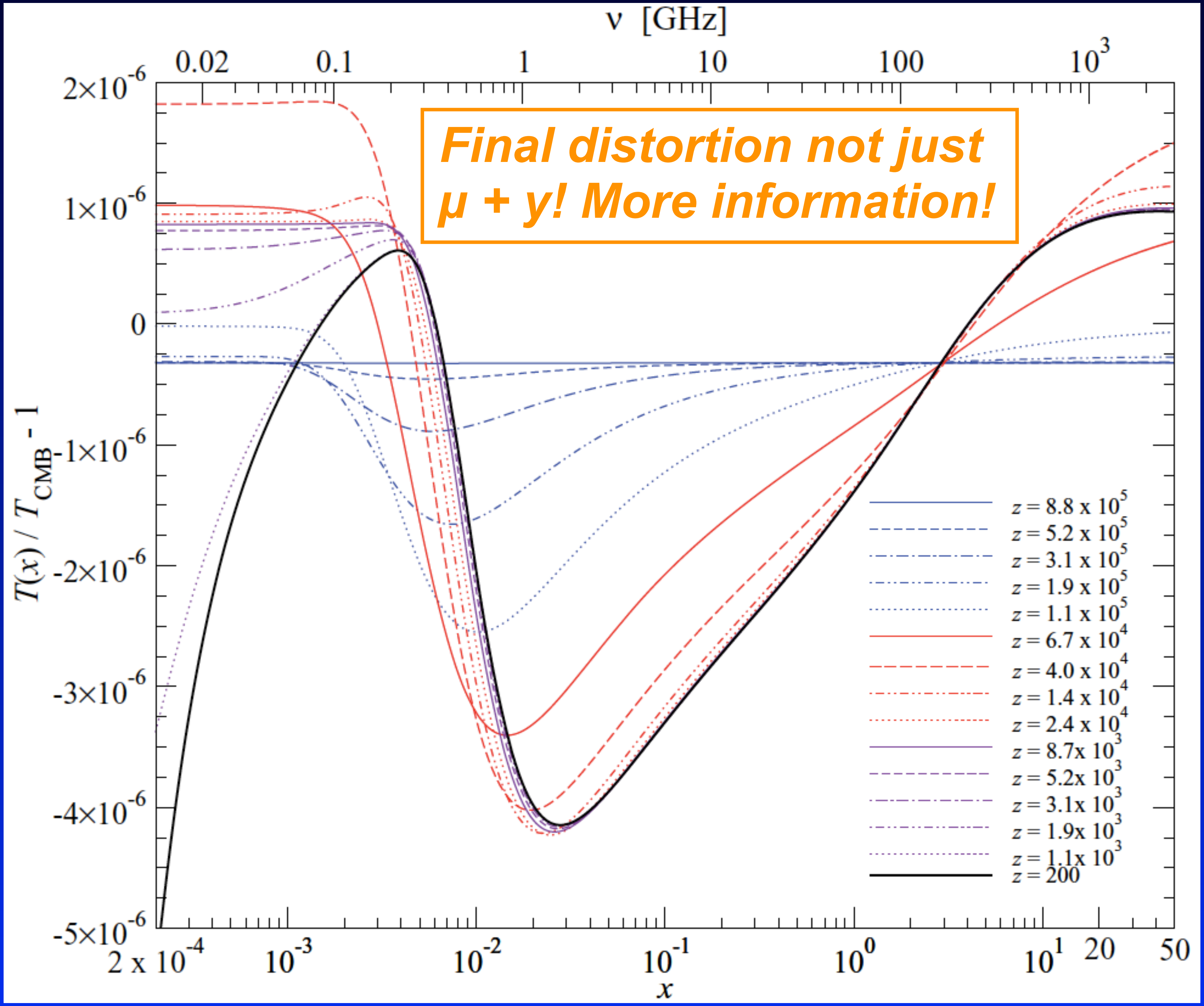

Computation carried out with *CosmoTherm*
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First explicit calculation that showed that there is more!

Quasi-Exact Treatment of the Thermalization Problem

- *For real forecasts of future prospects a precise & fast method for computing the spectral distortion is needed!*
- *Case-by-case computation of the distortion (e.g., with CosmoTherm, JC & Sunyaev, 2012, ArXiv:1109.6552) still rather time-consuming*
- *But: distortions are small ⇒ thermalization problem becomes linear!*
- *Simple solution: compute "response function" of the thermalization problem ⇒ Green's function approach (JC, 2013, ArXiv:1304.6120)*
- *Final distortion for fixed energy-release history given by*

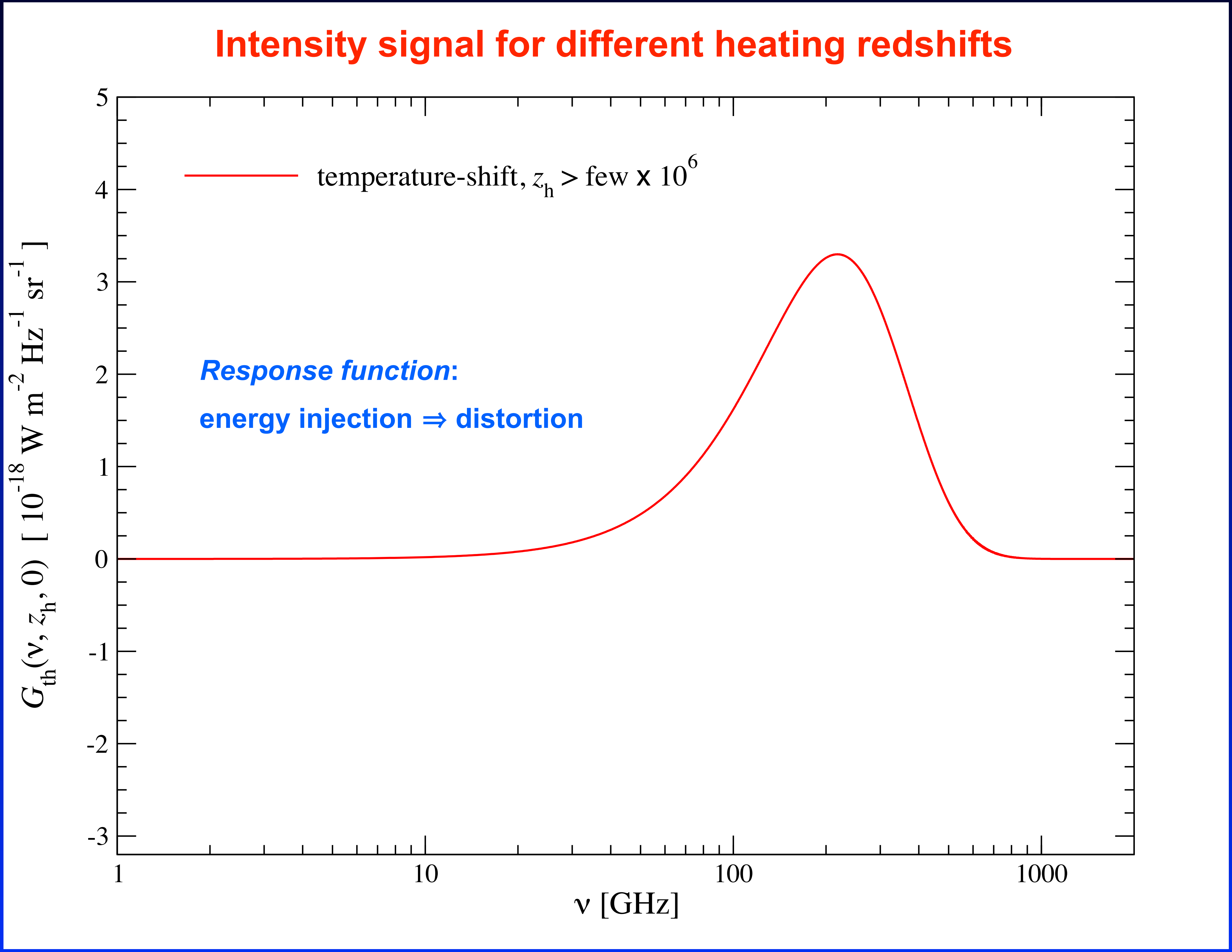
$$\Delta I_\nu \approx \int_0^\infty G_{\rm th}(\nu, z') \frac{\mathrm{d}(Q/\rho_\gamma)}{\mathrm{d}z'} \mathrm{d}z'$$

Thermalization Green's function

- *Fast and quasi-exact! No additional approximations!*

CosmoTherm available at: www.Chluba.de/CosmoTherm

What does the spectrum look like after energy injection?



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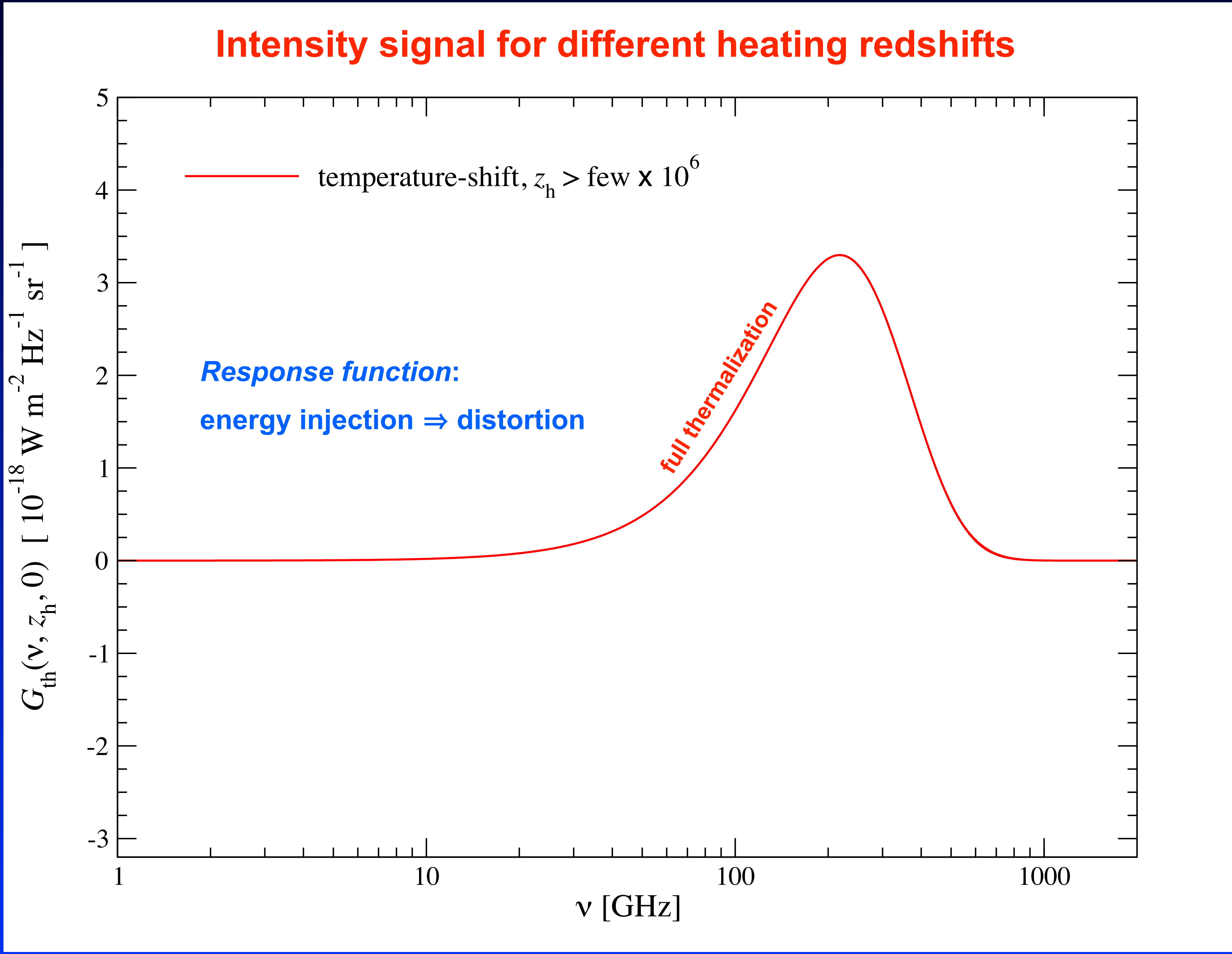

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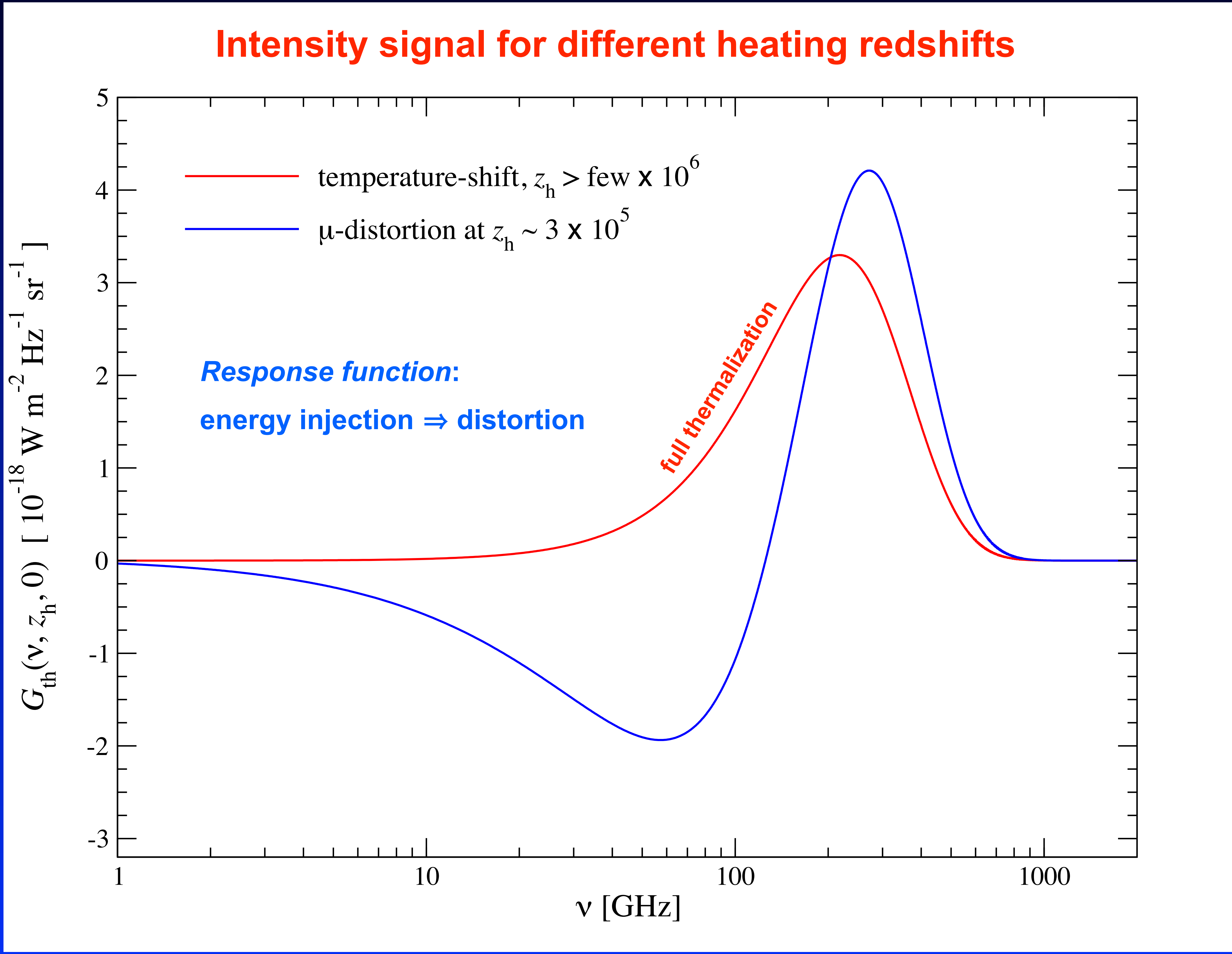

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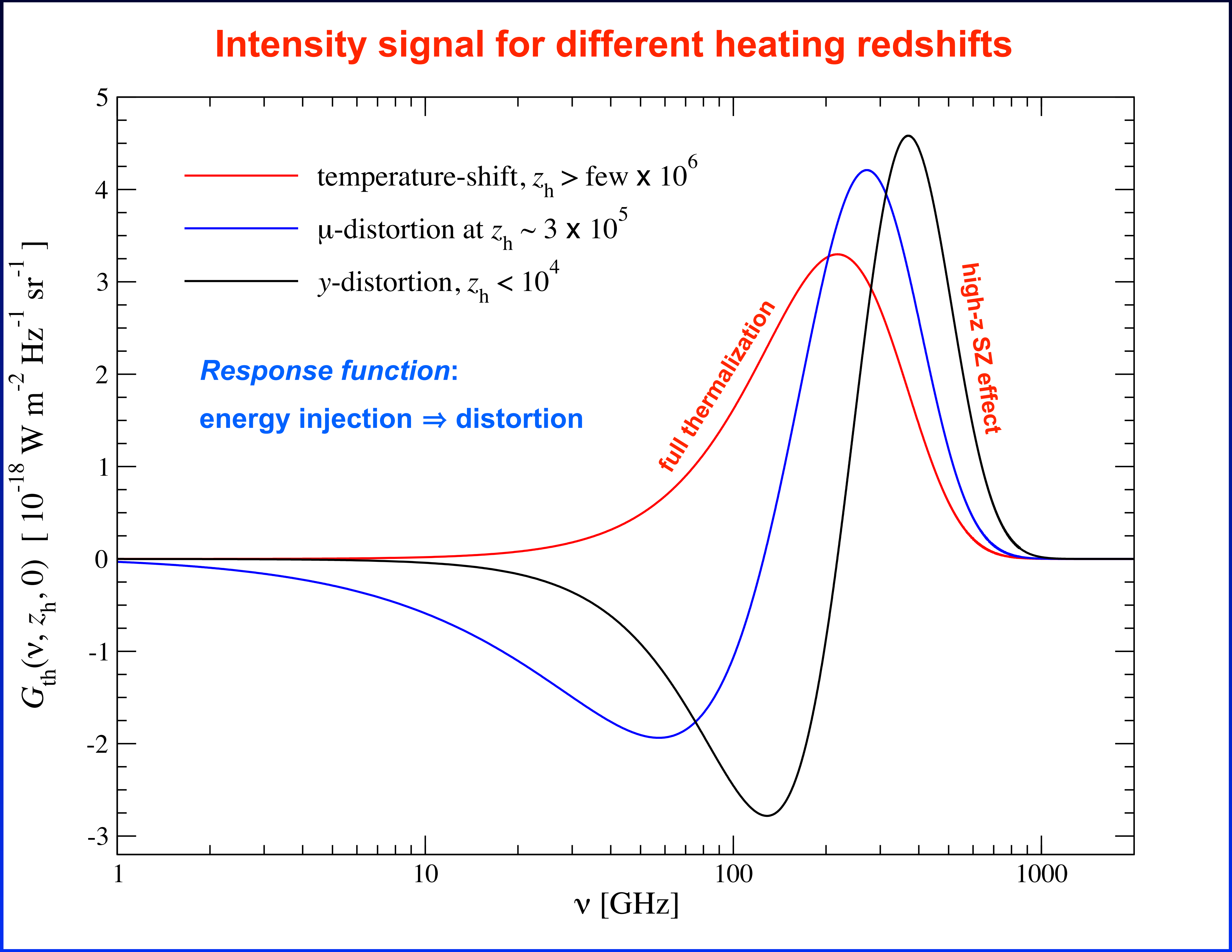

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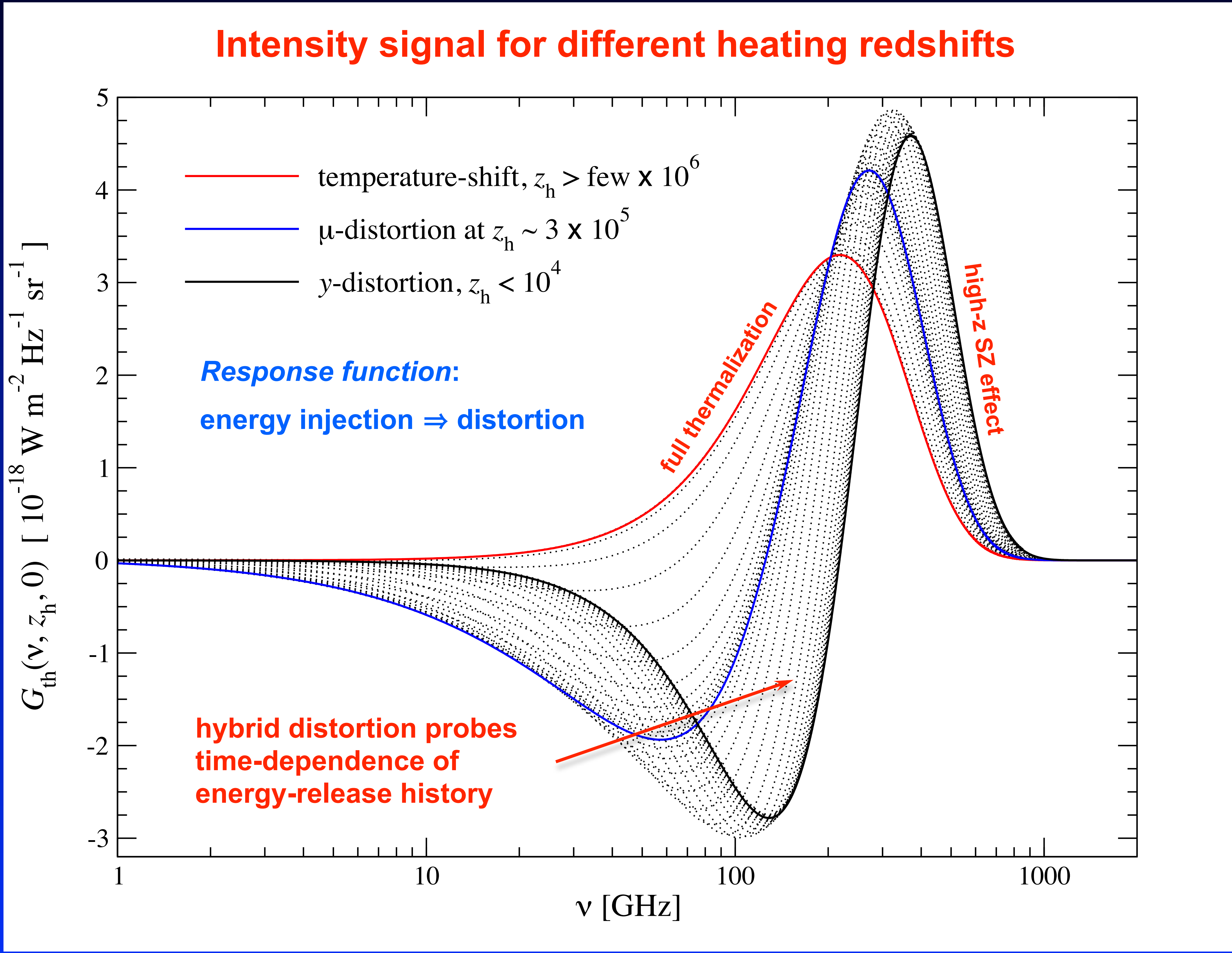

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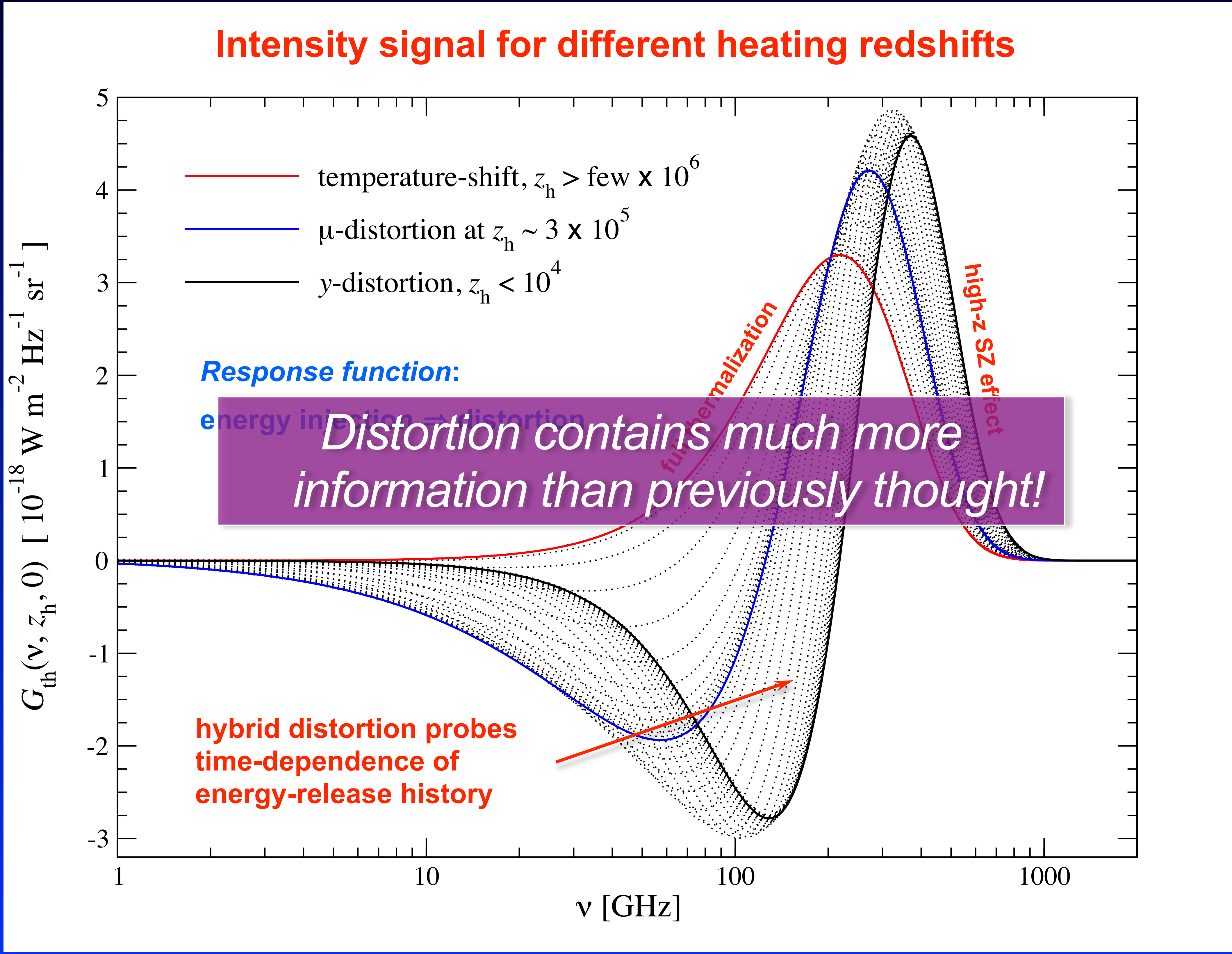
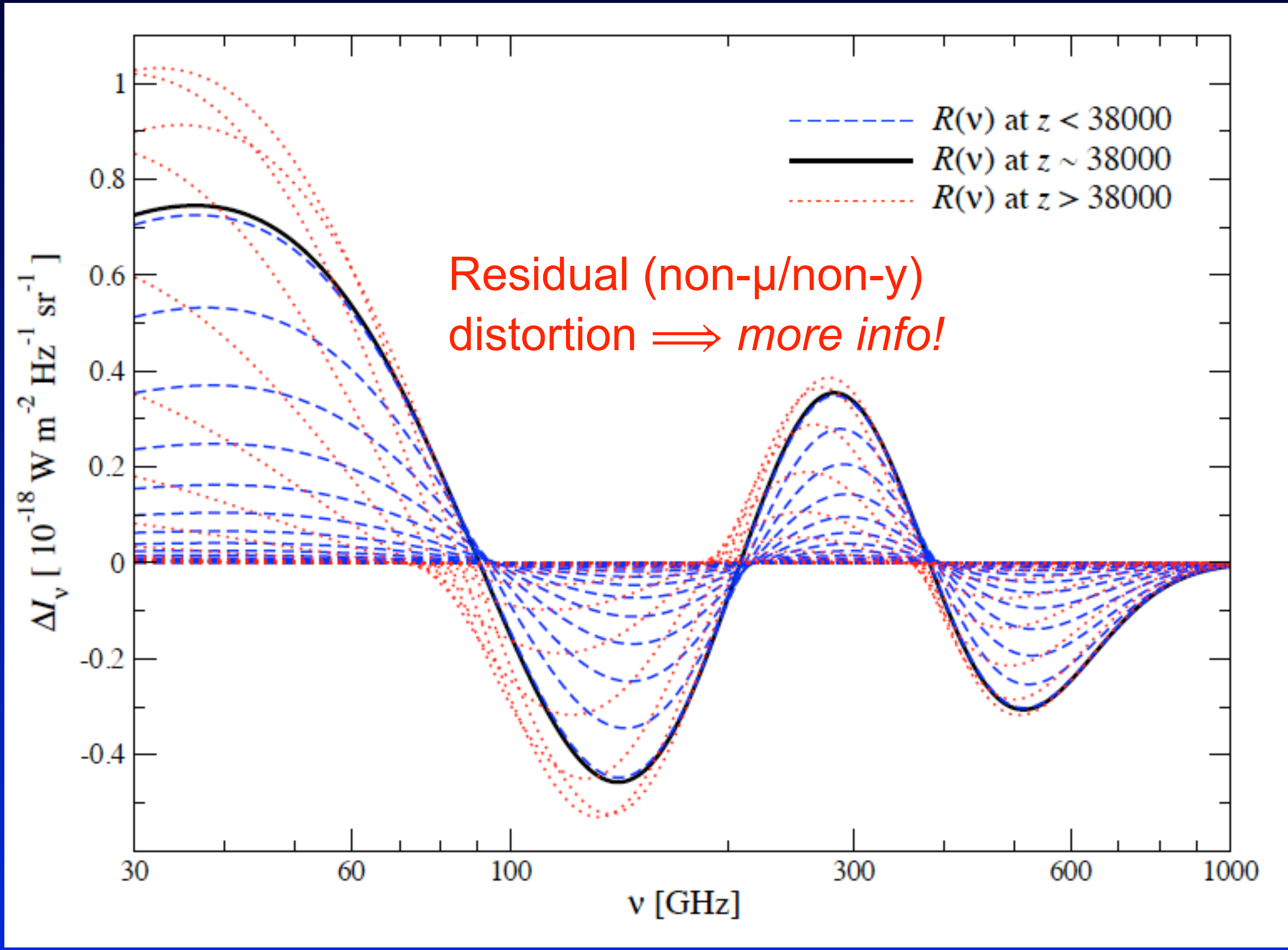


Distortion contains much more information than previously thought!

JC & Sunyaev, 2012, ArXiv:1109.6552
JC, 2013, ArXiv:1304.6120

Explicitly taking out the superposition of *T*, μ & y distortion



- *Allows us to distinguish different energy release scenarios!*

JC & Sunyaev, 2012, ArXiv:1109.6552
JC, 2013, ArXiv:1304.6120; JC, 2013, ArXiv:1304.6121; JC & Jeong, 2013

Is there a simple way to model μ and y during transition regime?

Simple estimates for the distortion μ- and y-parameters caused by energy release

- Generalization of classical approximations:

$$y = \frac{1}{4} \left.\frac{\Delta\rho_\gamma}{\rho_\gamma}\right|_y = \frac{1}{4}\int_0^\infty \mathcal{J}_y(z')\,\frac{\mathrm{d}(Q/\rho_\gamma)}{\mathrm{d}z'}\,\mathrm{d}z'$$

$$\mu = 1.401 \left.\frac{\Delta\rho_\gamma}{\rho_\gamma}\right|_\mu = 1.401\int_0^\infty \mathcal{J}_\mu(z')\,\frac{\mathrm{d}(Q/\rho_\gamma)}{\mathrm{d}z'}\,\mathrm{d}z'$$

Energy release history

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Energy release history

- Differences in the approximations are due to *visibility functions*
- An overview can be found in ArXiv:1603.02496

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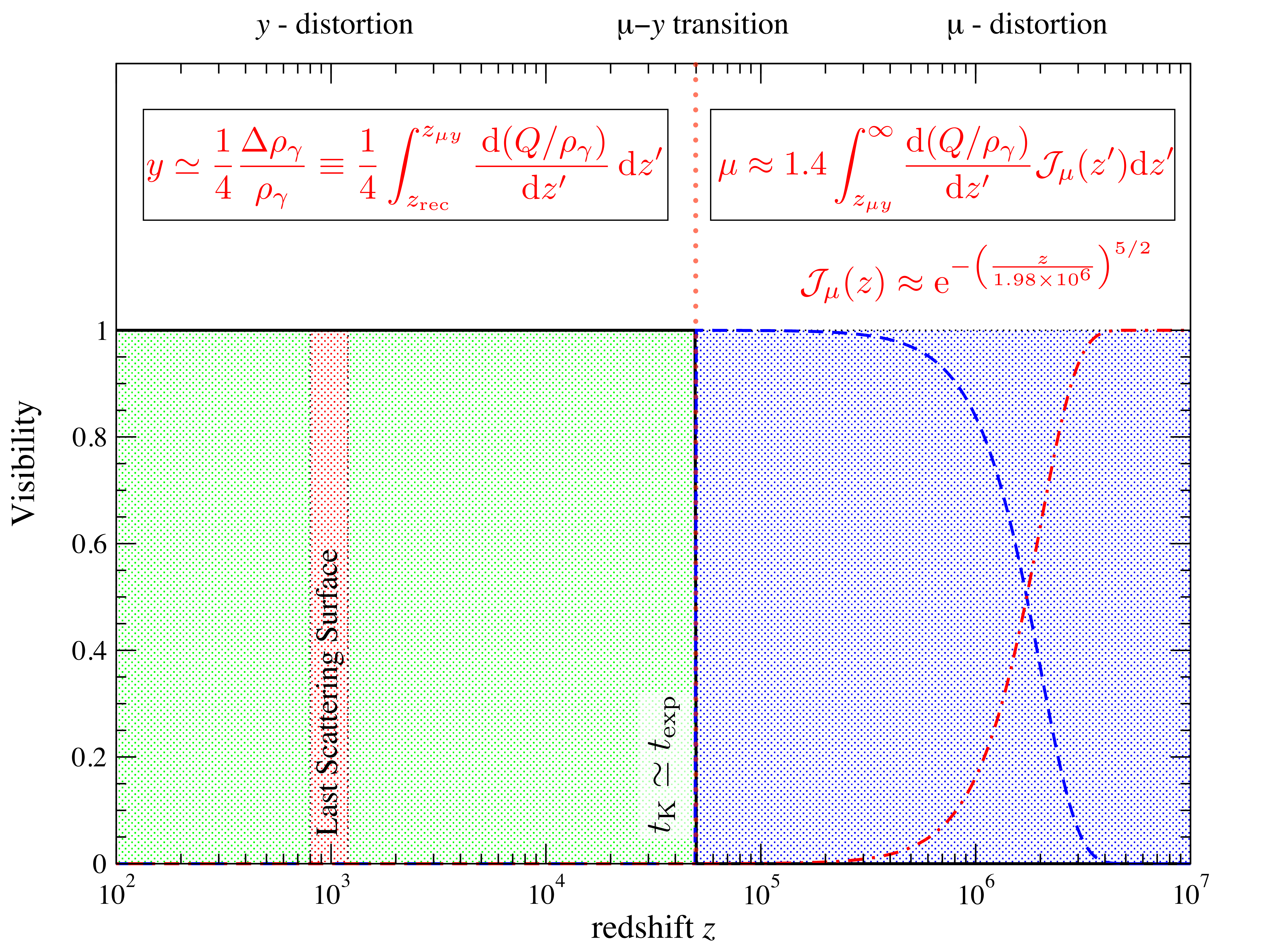
- Differences in the approximations are due to *visibility functions*
- An overview can be found in ArXiv:1603.02496
- One *commonly* used approximation (e.g., see Hu&Silk, 1993):

$$\mathcal{J}_y(z) = \begin{cases} 1 & \text{for } z_{\rm rec} \le z \le z_{\mu y} \\ 0 & \text{otherwise} \end{cases}$$

$$\mathcal{J}_\mu(z) = \begin{cases} \mathcal{J}_{\rm bb}(z) & \text{for } z_{\mu y} \le z \\ 0 & \text{otherwise.} \end{cases}$$

- step-function transition between μ and *y* around $z_{\mu y} \simeq 5\times 10^4$
- accounts for thermalization efficiency with *distortion visibility* $\mathcal{J}_{\rm bb}(z) \approx \mathrm{e}^{-(z/z_{\rm th})^{5/2}}$ $z_{\rm th} \approx 1.98\times 10^6$



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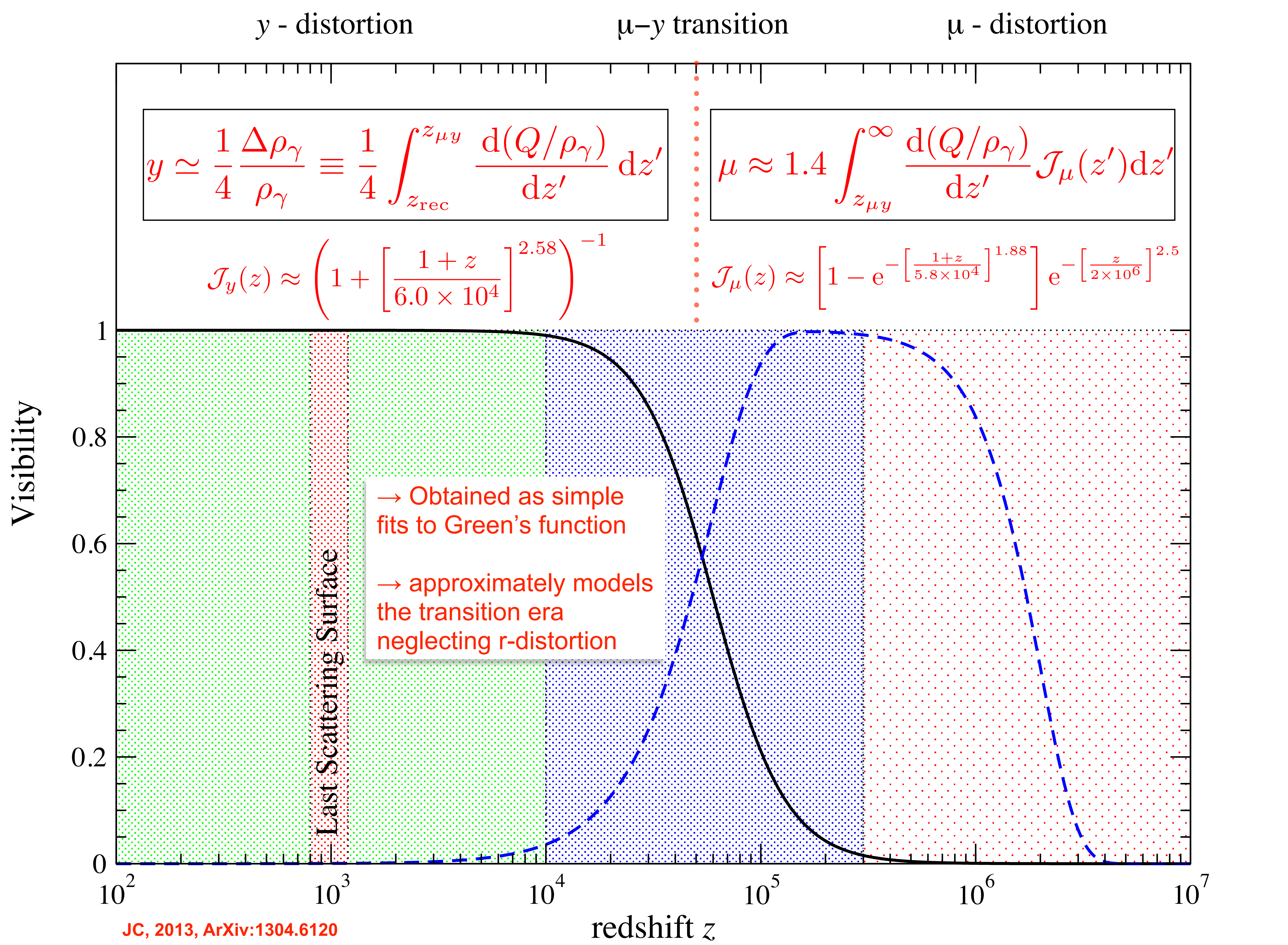
Last Scattering Surface
$t_{\rm K} \simeq t_{\rm exp}$
Visibility
1
0.8
0.6
0.4
0.2
0
10^2
10^3
10^4
10^5
10^6
10^7
redshift z



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$\mathcal{J}_y(z) \approx \left(1+\left[\frac{1+z}{6.0\times10^4}\right]^{2.58}\right)^{-1}$
$\mathcal{J}_\mu(z) \approx \left[1-{\rm e}^{-\left[\frac{1+z}{5.8\times10^4}\right]^{1.88}}\right]{\rm e}^{-\left[\frac{z}{2\times10^6}\right]^{2.5}}$
Visibility
1
0.8
0.6
0.4
0.2
0
Last Scattering Surface
→ Obtained as simple fits to Green's function
→ approximately models the transition era neglecting r-distortion
10^2
10^3
10^4
10^5
10^6
10^7
redshift z
JC, 2013, ArXiv:1304.6120

Distortions created by photon injection

Physical mechanisms that lead to spectral distortions

Standard sources of distortions

- *Cooling by adiabatically expanding ordinary matter*
  (JC, 2005; JC & Sunyaev 2011; Khatri, Sunyaev & JC, 2011)
- Heating by *decaying* or *annihilating* relic particles
  (Kawasaki et al., 1987; Hu & Silk, 1993; McDonald et al., 2001; JC, 2005; JC & Sunyaev, 2011; JC, 2013; JC & Jeong, 2013)
- *Evaporation of primordial black holes & superconducting strings*
  (Carr et al. 2010; Ostriker & Thompson, 1987; Tashiro et al. 2012; Pani & Loeb, 2013)
- *Dissipation of primordial acoustic modes & magnetic fields*
  (Sunyaev & Zeldovich, 1970; Daly 1991; Hu et al. 1994; JC & Sunyaev, 2011; JC et al. 2012 - Jedamzik et al. 2000; Kunze & Komatsu, 2013)
- *Cosmological recombination radiation*
  (Zeldovich et al., 1968; Peebles, 1968; Dubrovich, 1977; Rubino-Martin et al., 2006; JC & Sunyaev, 2006; Sunyaev & JC, 2009)

pre-recombination epoch

„high“ redshifts

„low“ redshifts

post-recombination

- Signatures due to first supernovae and their remnants
  (Oh, Cooray & Kamionkowski, 2003)
- Shock waves arising due to large-scale structure formation
  (Sunyaev & Zeldovich, 1972; Cen & Ostriker, 1999)
- SZ-effect from clusters; effects of reionization
  (Refregier et al., 2003; Zhang et al. 2004; Trac et al. 2008)
- other exotic processes
  (Lochan et al. 2012; Bull & Kamionkowski, 2013; Brax et al., 2013; Tashiro et al. 2013)

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Comparison of energy release and photon injection

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Energy release scenario:

- *Medium* is heated (or cooled)
- *Energy transfer* to photons via Compton scattering
- *Photon number* remains unchanged by the energy release process

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$$y \approx \frac{1}{4} \frac{\Delta\rho_\gamma}{\rho_\gamma}$$

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Photon injection scenario:

- *Photons* directly added to photon field
- Means both *energy injection* and *addition of photons*
- *Medium* is affected by Compton scattering (heating/cooling)

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rich spectral shape

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- *Medium* is affected by Compton scattering (heating/cooling)

Low redshift:
(inefficient Comptonization)

rich spectral shape

High redshift:
(efficient Comptonization)

$$\mu \approx 1.4\left[\frac{\Delta\rho_\gamma}{\rho_\gamma} - \frac{4}{3}\frac{\Delta N_\gamma}{N_\gamma}\right]$$

Comparison of energy release and photon injection

Energy release scenario:

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Narrow line:

$$\frac{\Delta\rho_\gamma}{\rho_\gamma} \approx 0.37\, x_{\rm i}\, \frac{\Delta N_\gamma}{N_\gamma}$$

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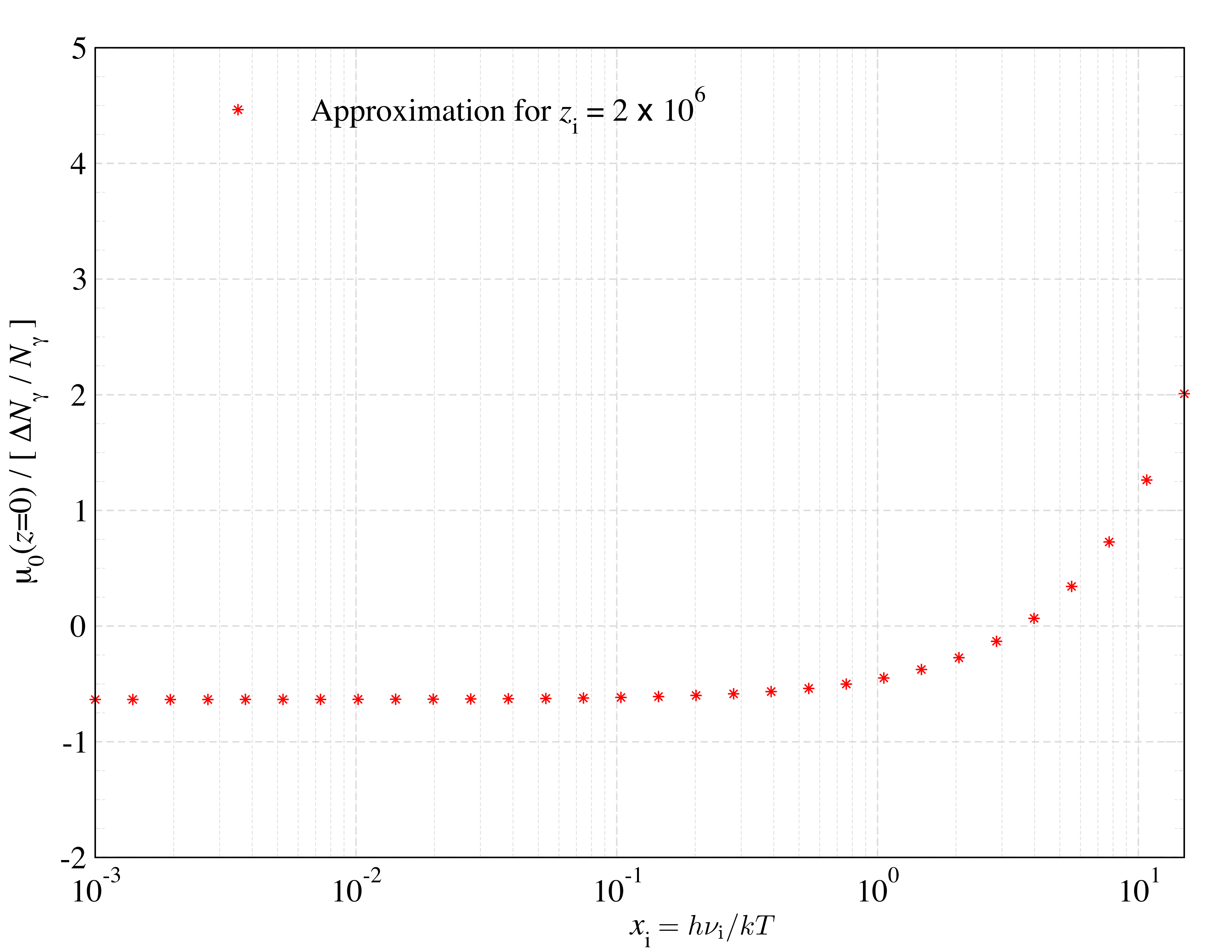
Net μ-distortion can vanish or be negative!

Photon injection in the μ-era



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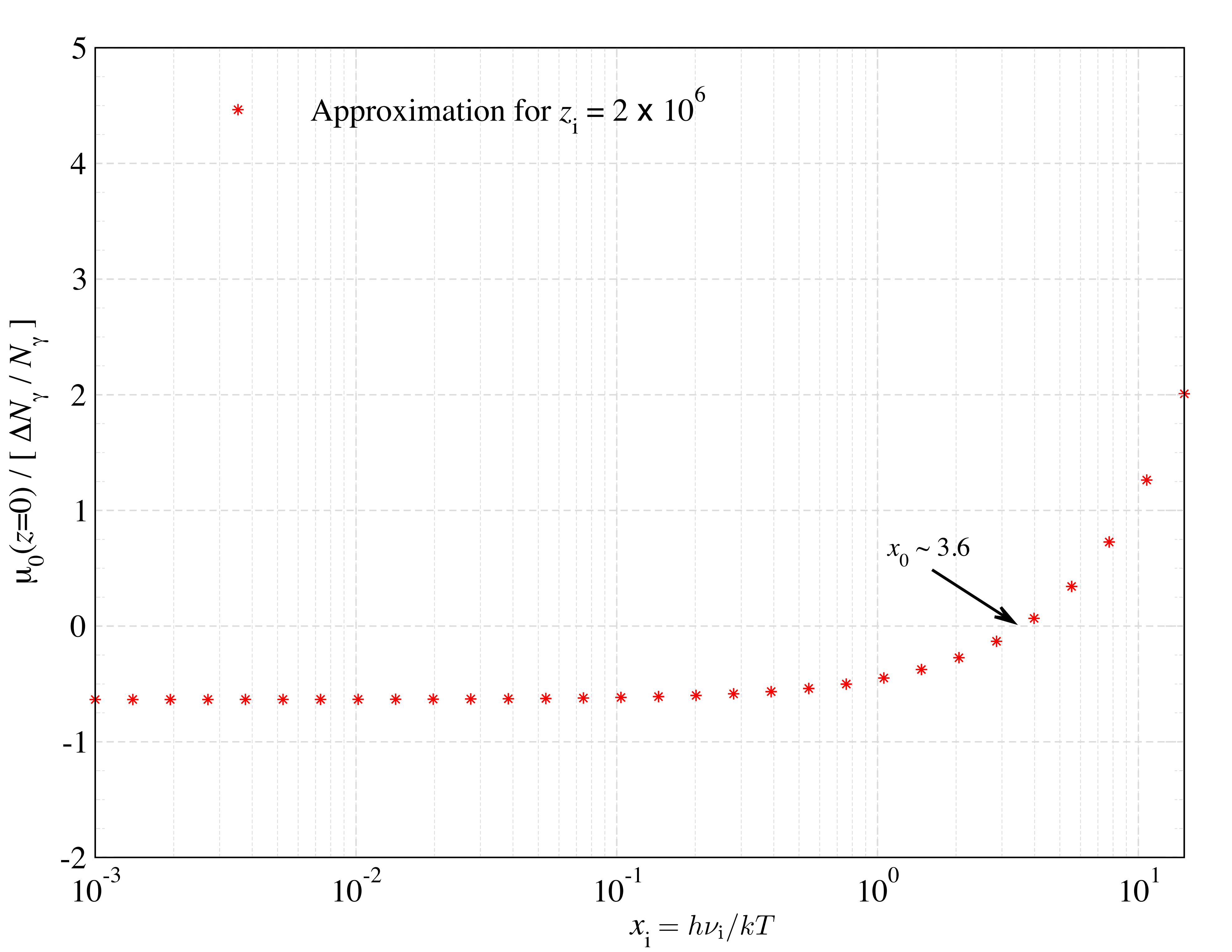
JC 2015, ArXiv:1506.06582

Photon injection in the μ-era



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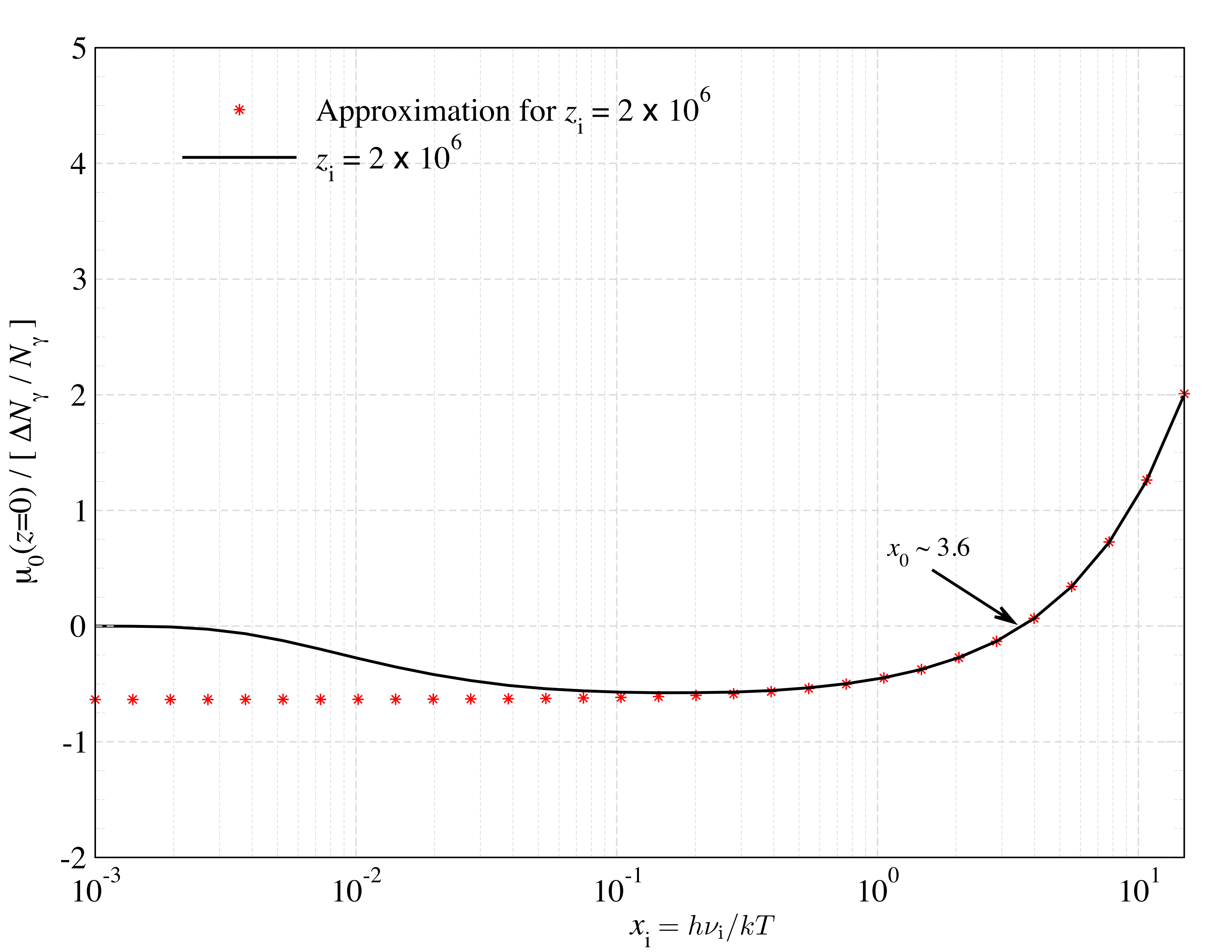

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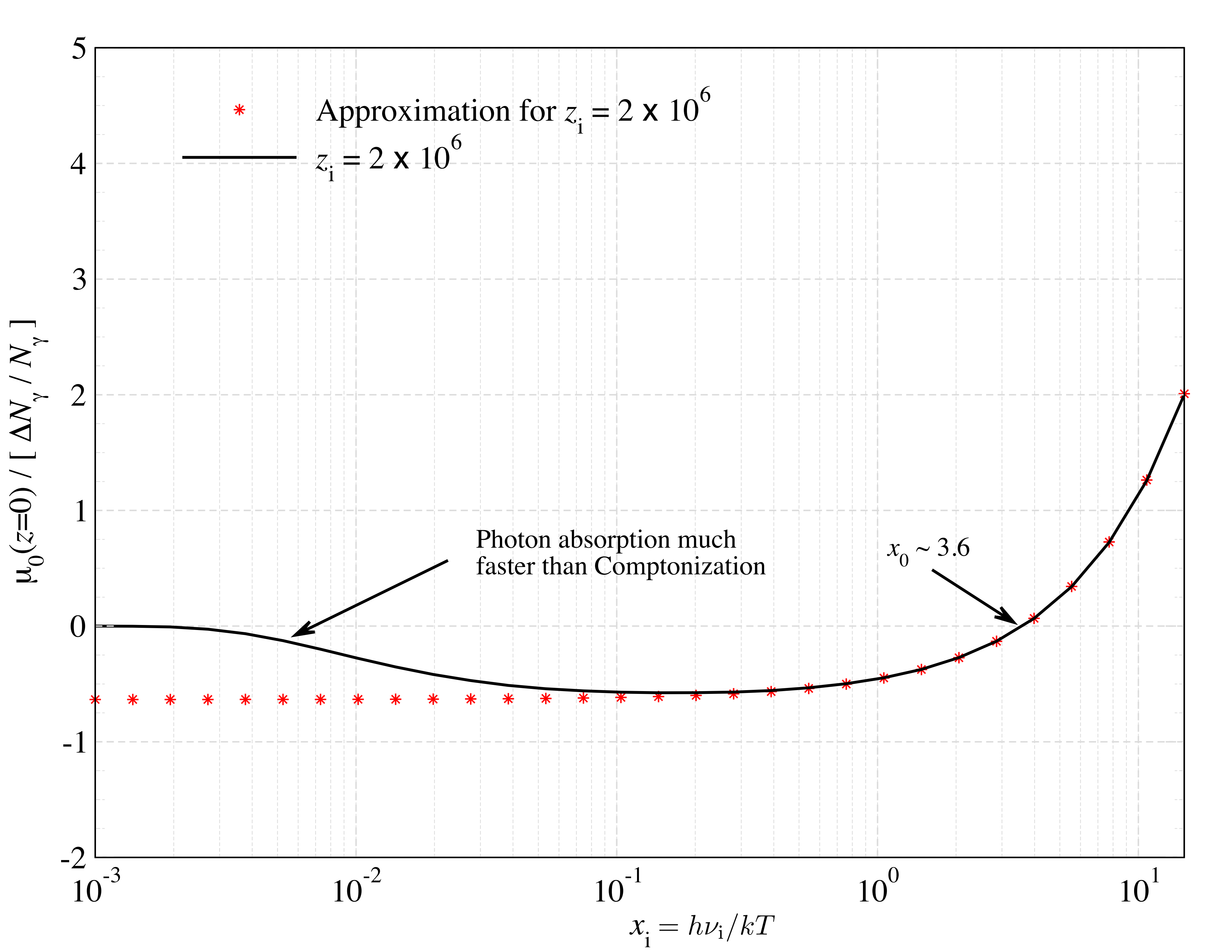

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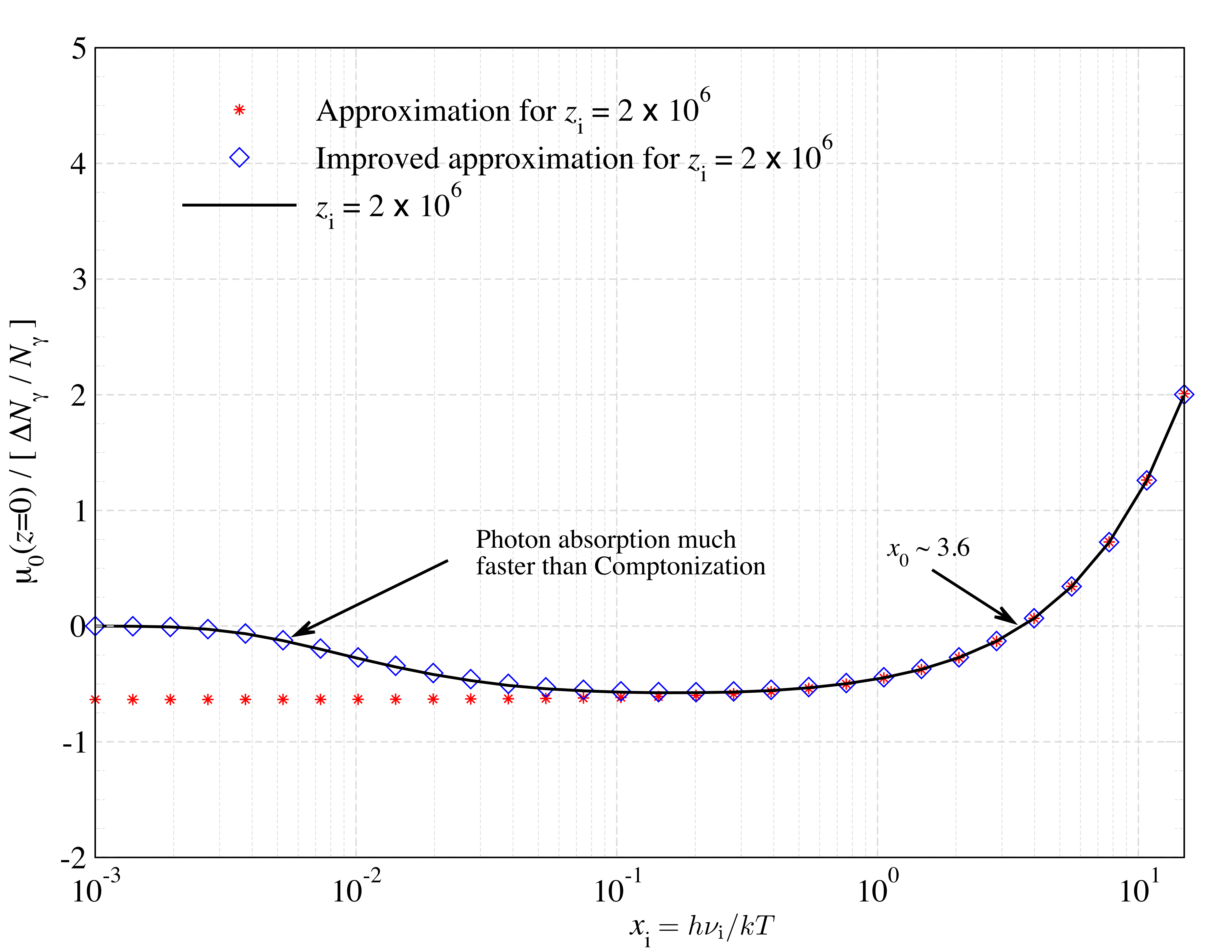

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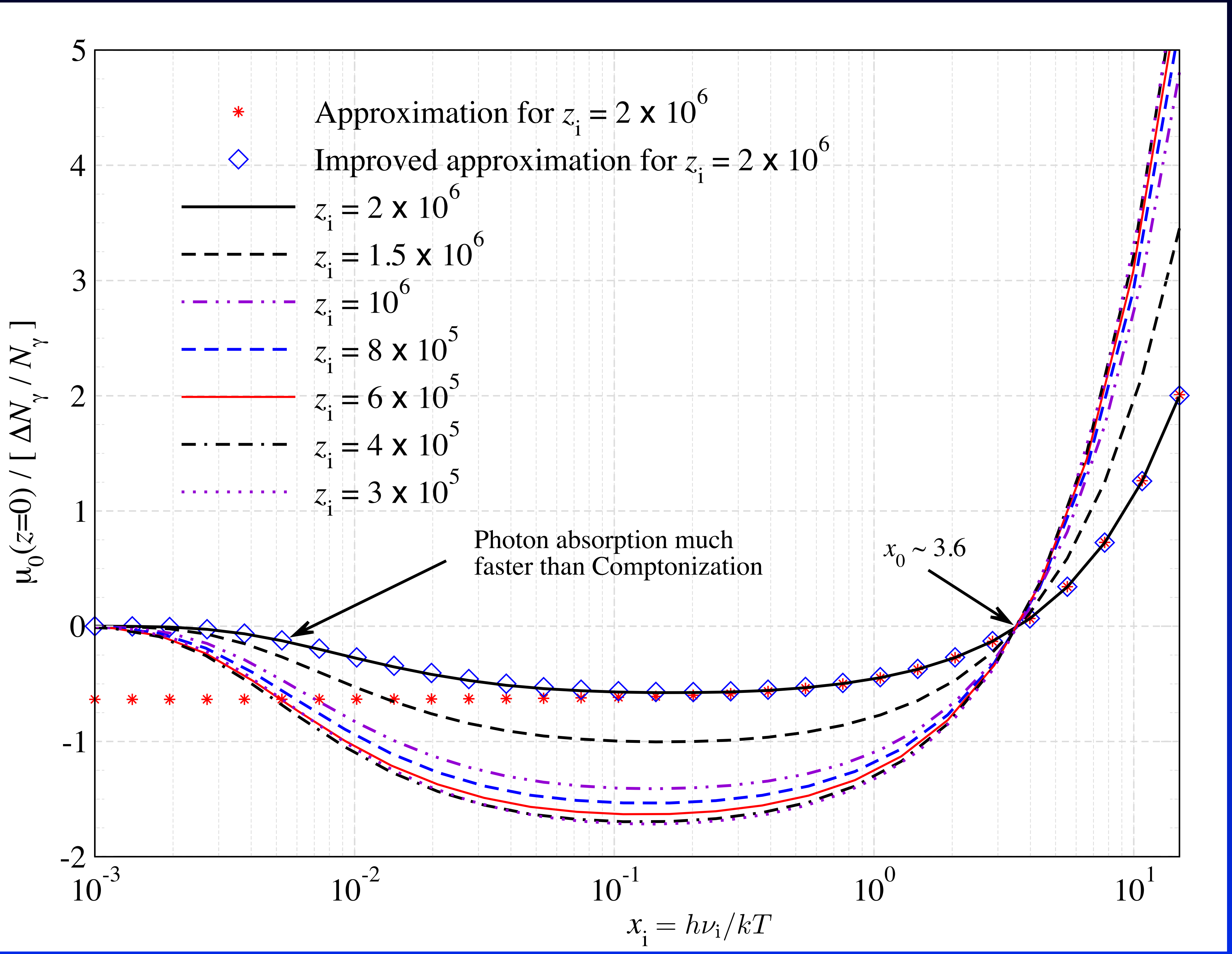

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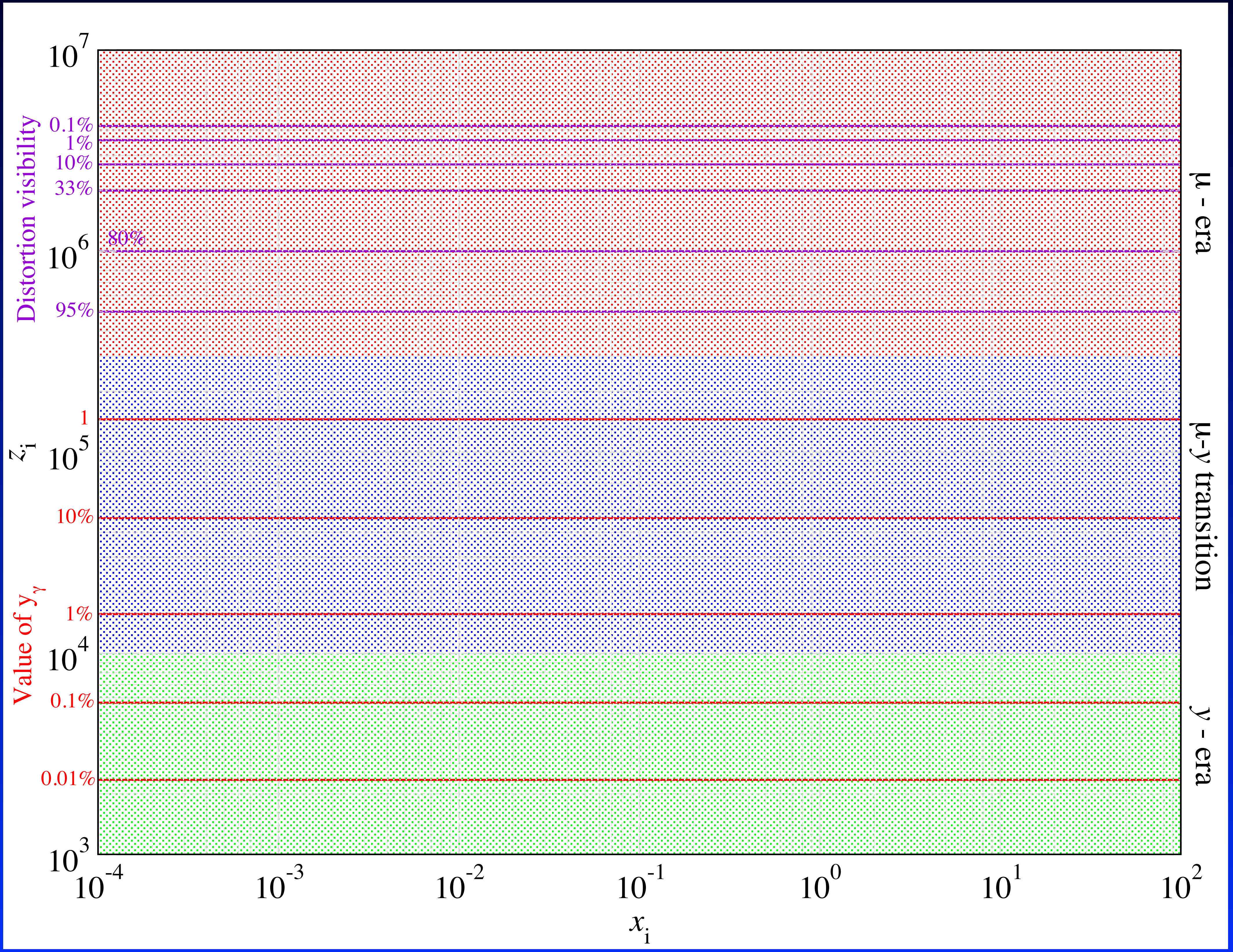
Different regimes for photon injection



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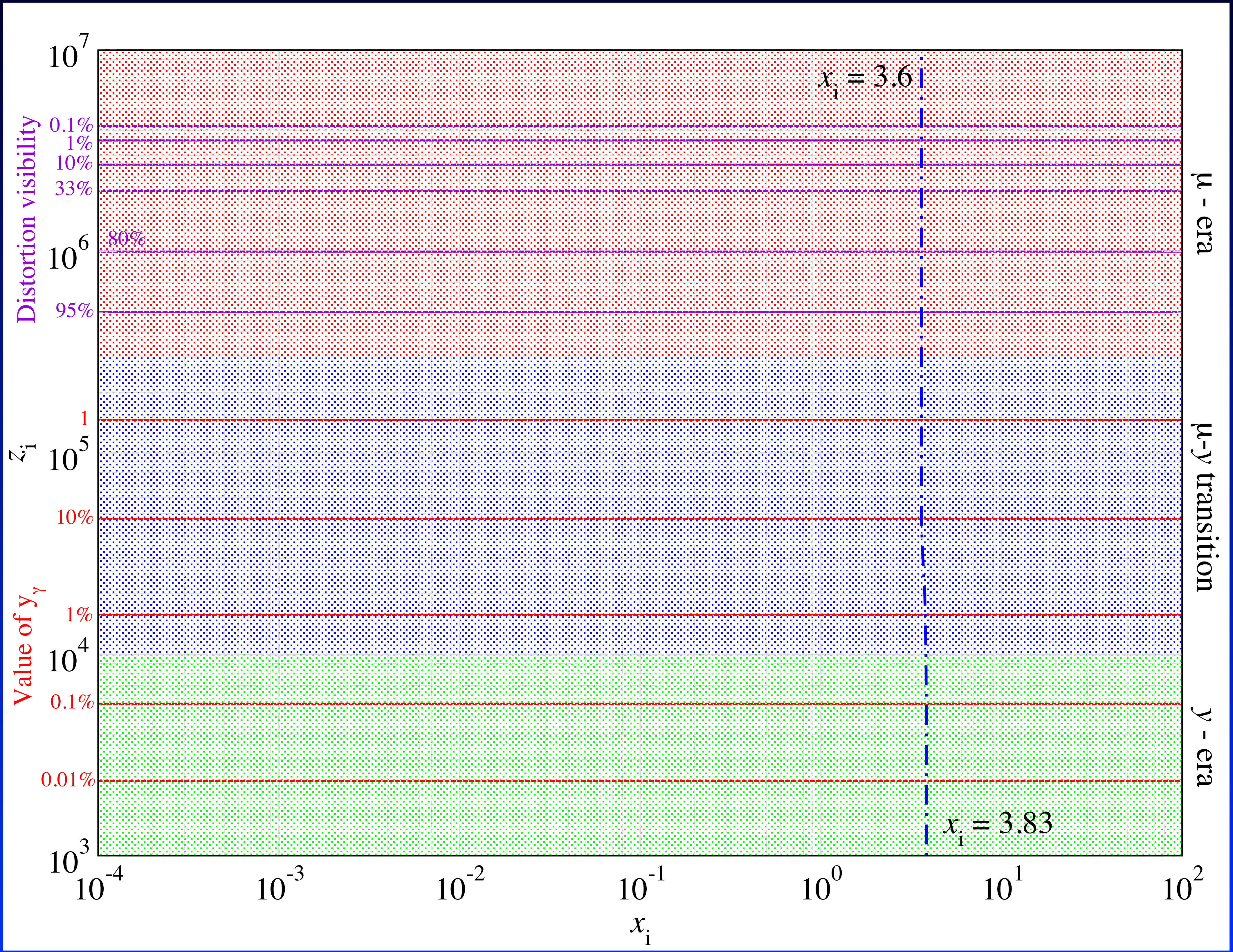

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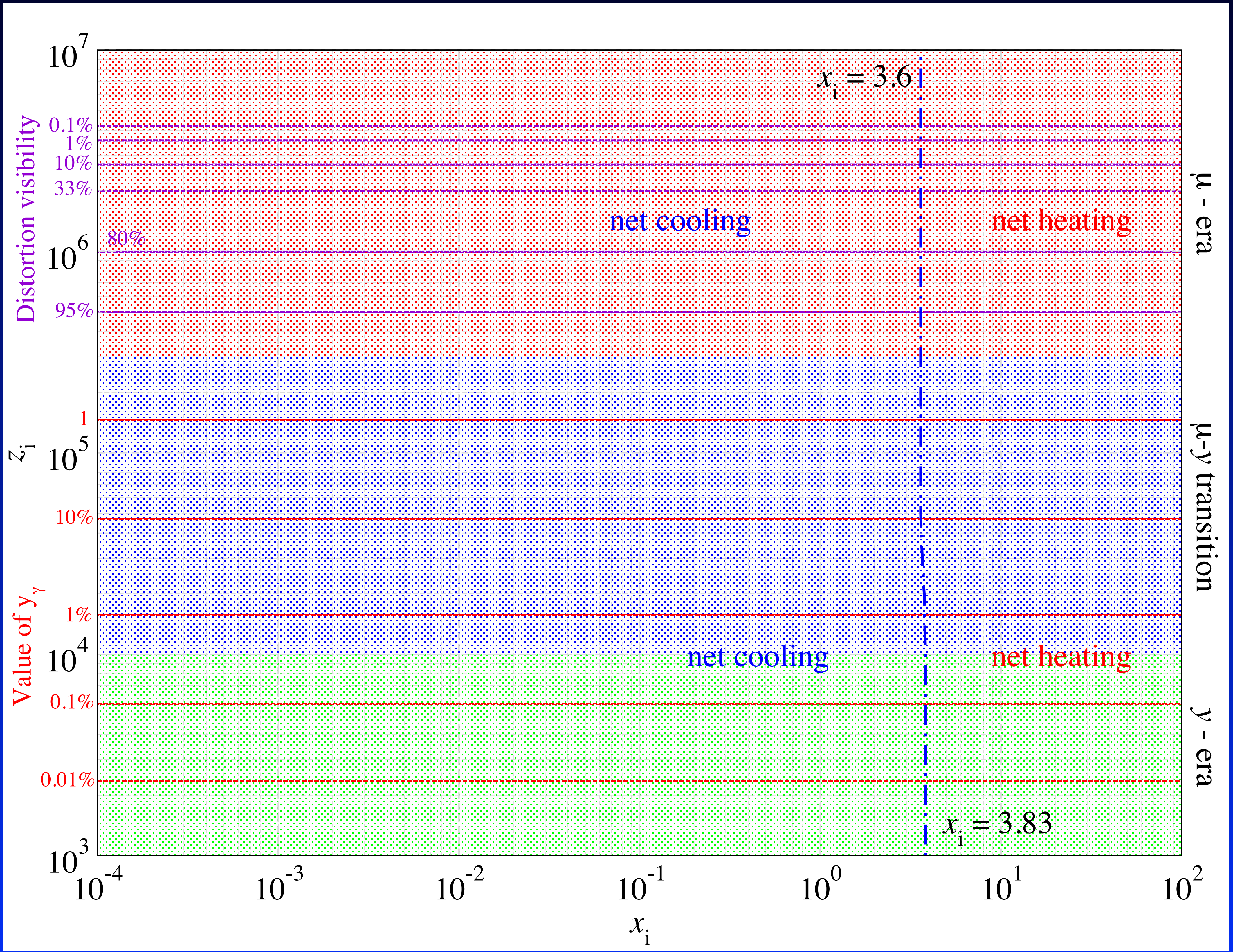

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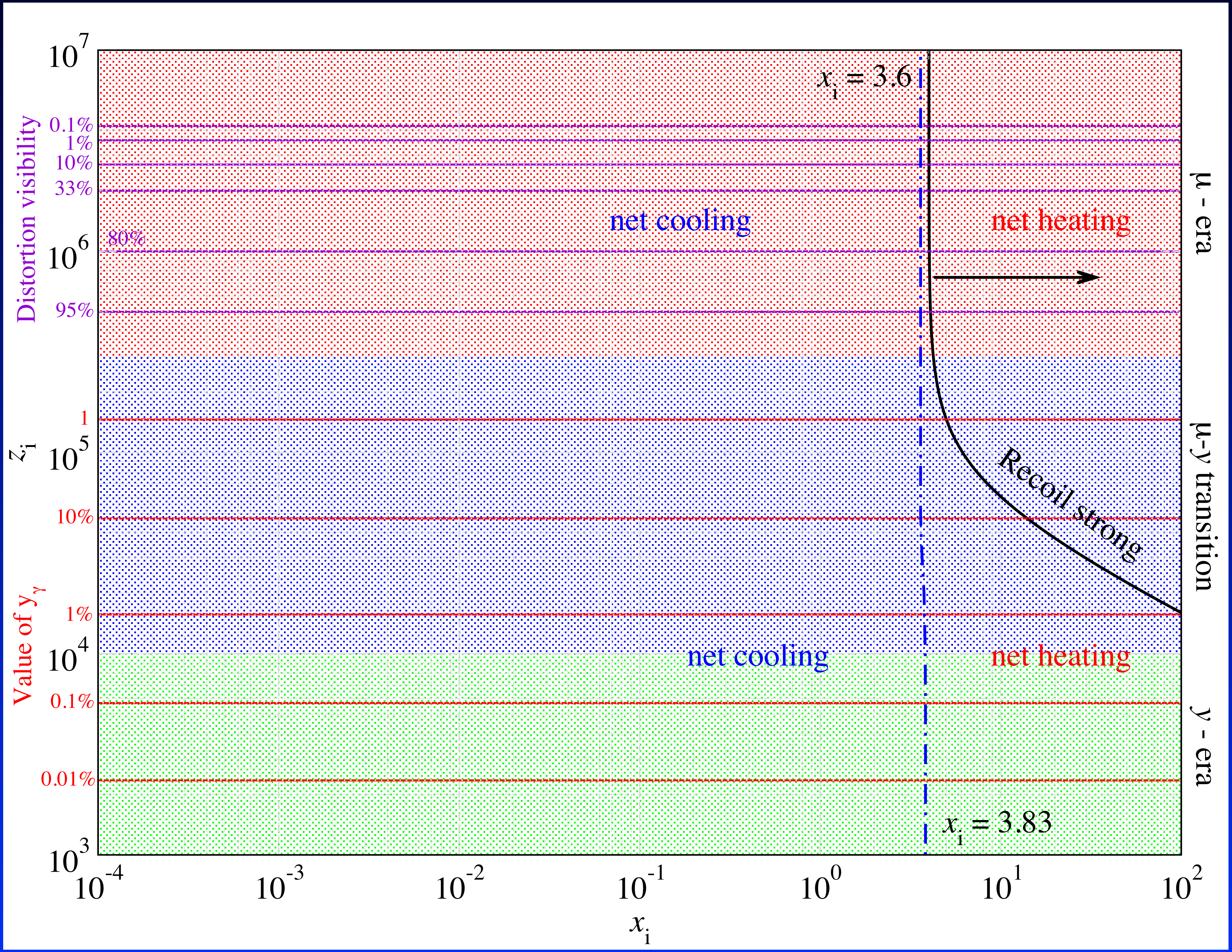

JC 2015, ArXiv:1506.06582

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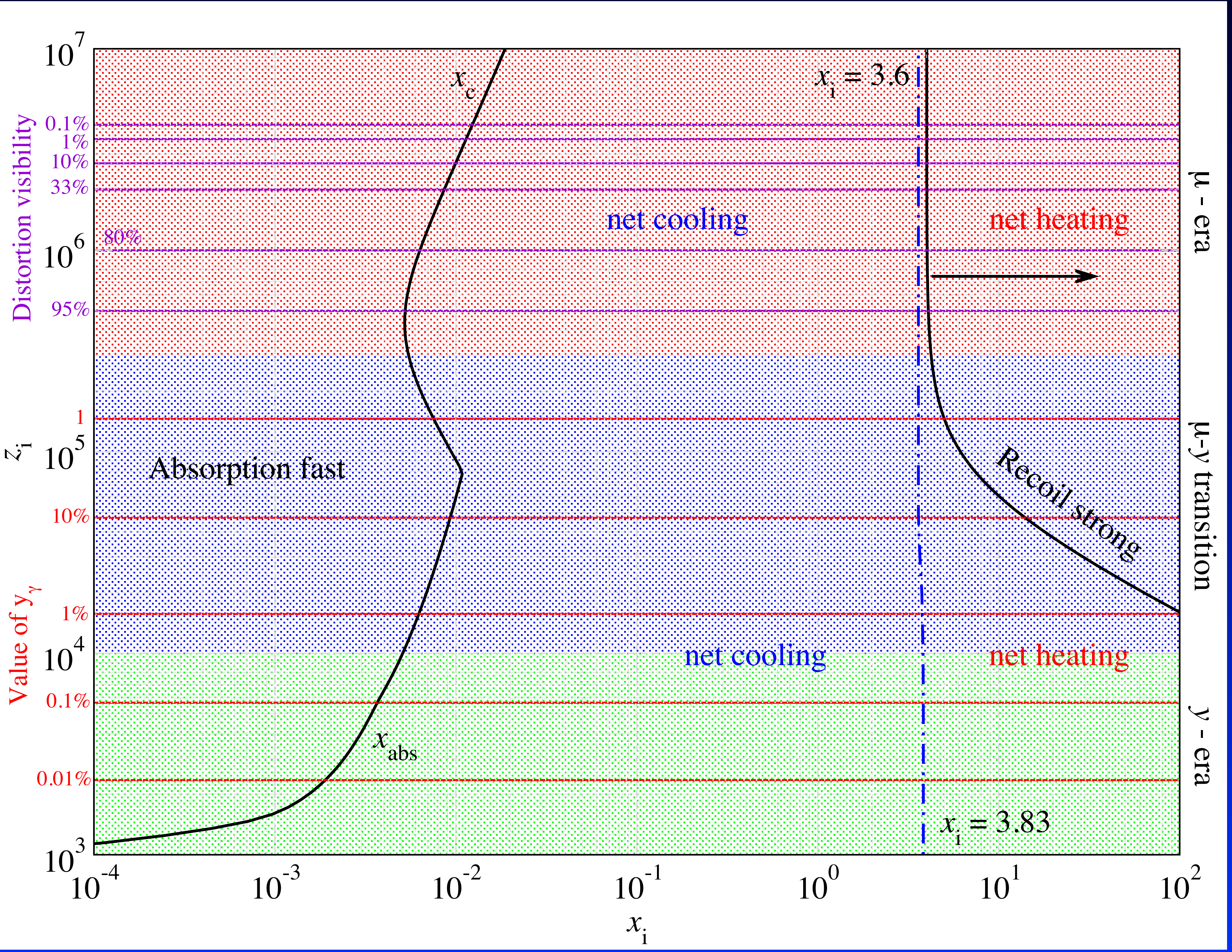

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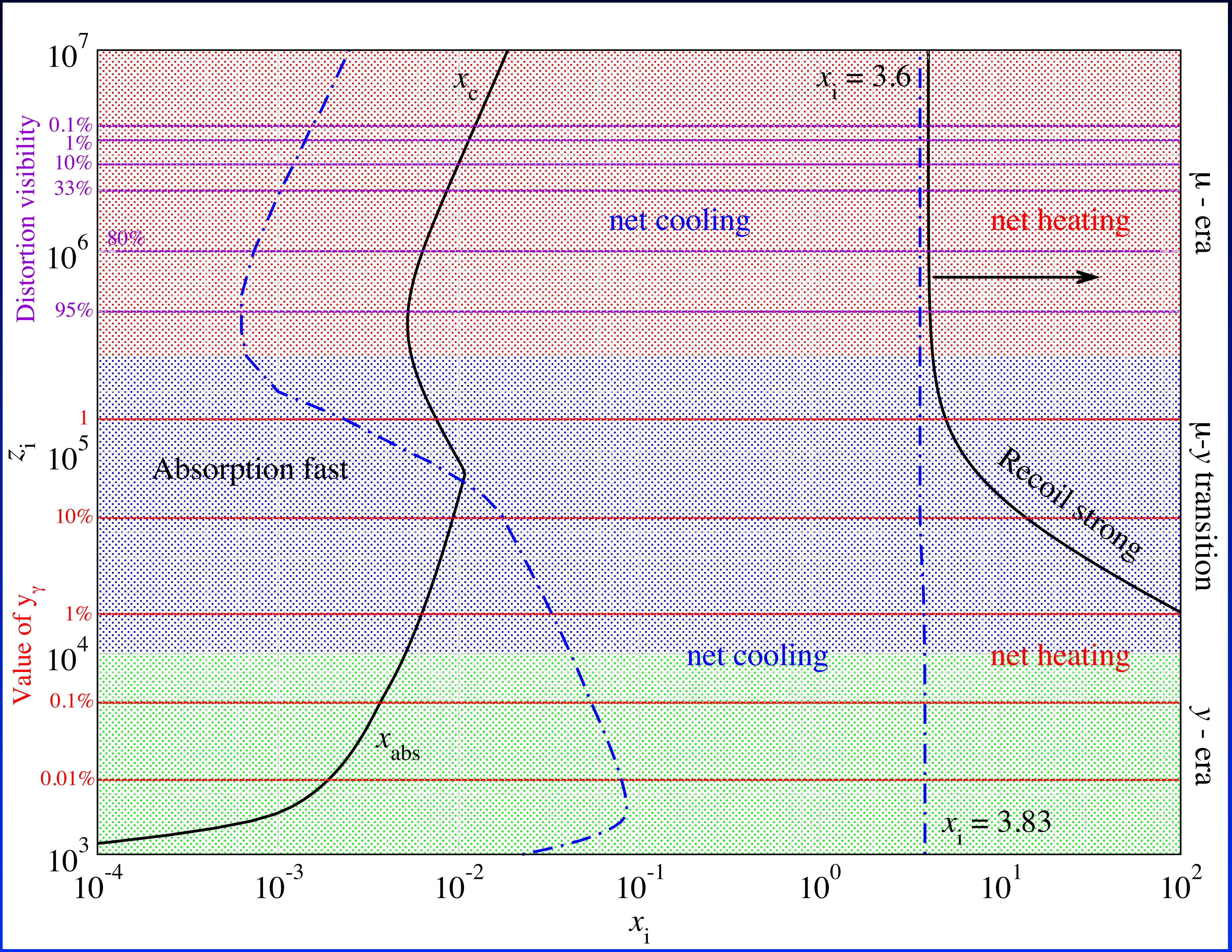
Different regimes for photon injection



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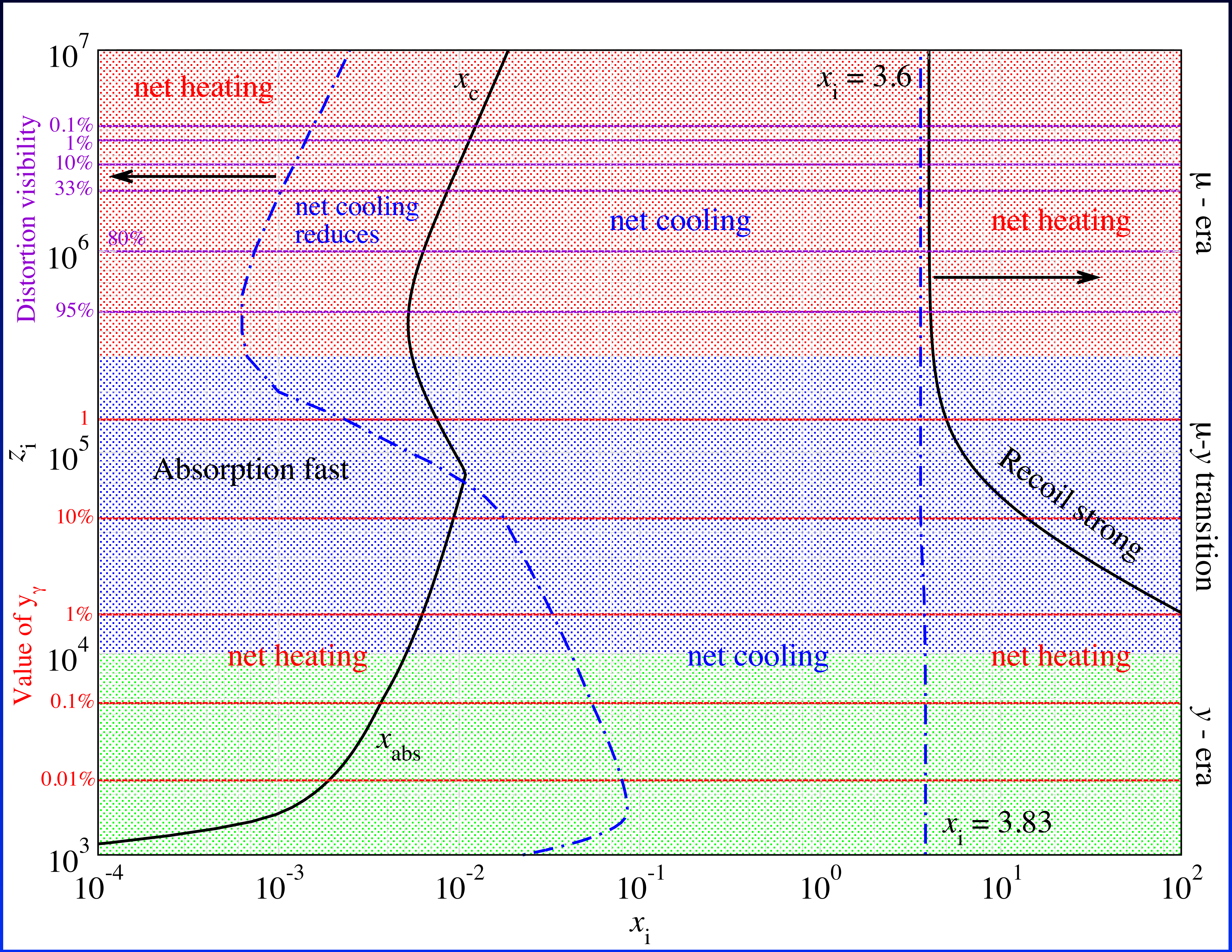
Different regimes for photon injection



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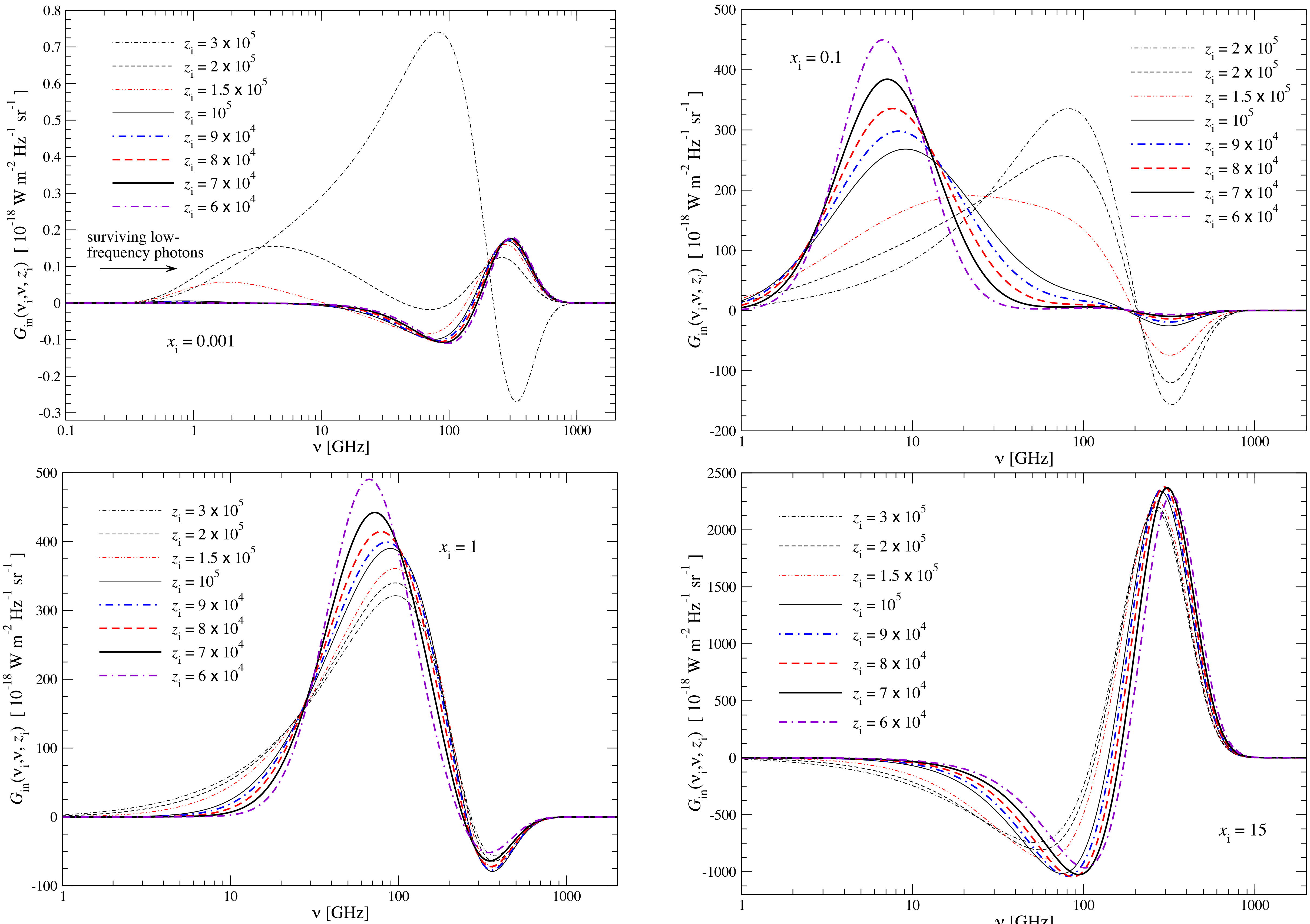
Photon injection at later times



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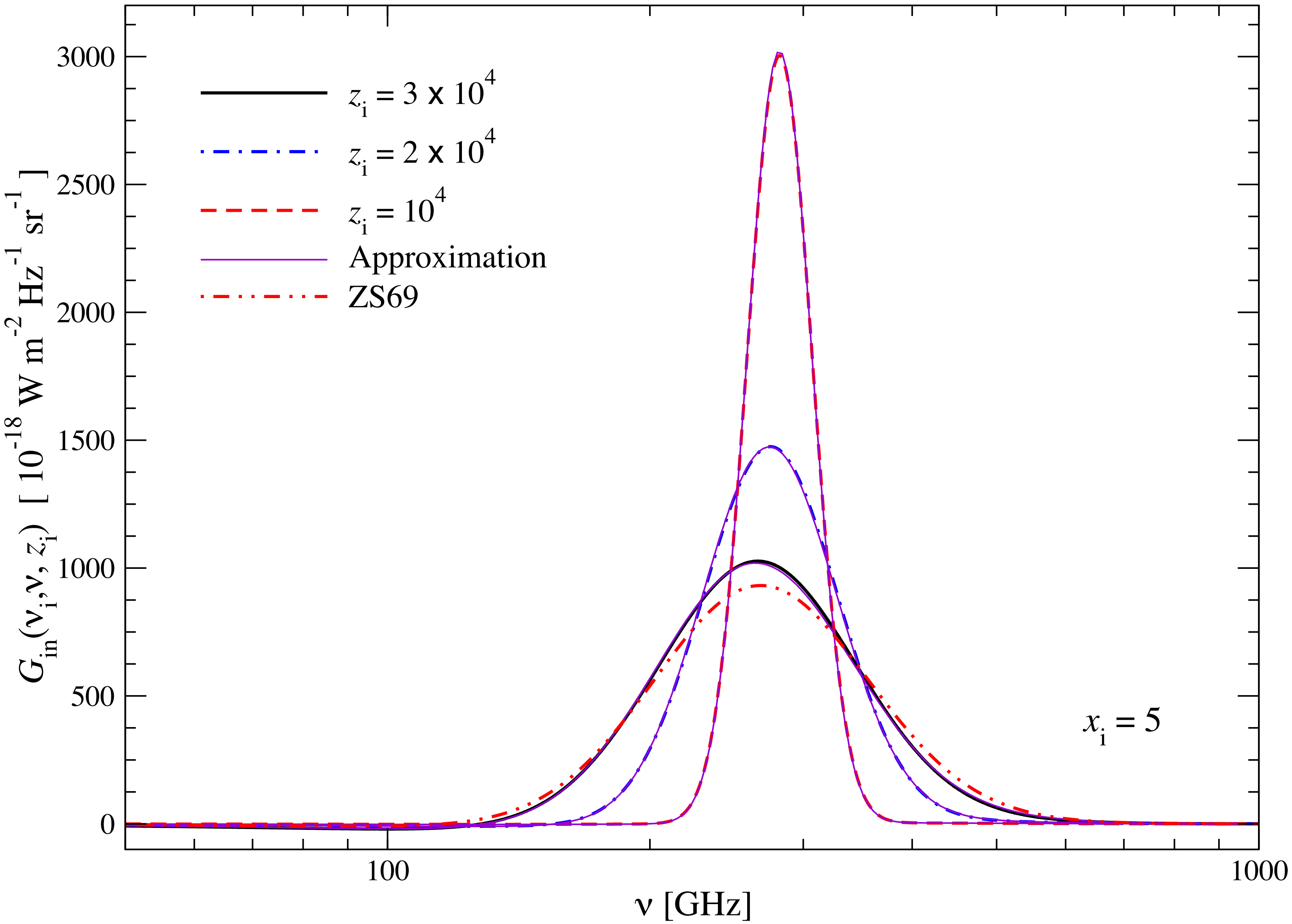
Regime with very few scatterings



Classical solution by Zeldovich & Sunyaev, 1969

$$\Delta n(x, y) = \frac{A}{\sqrt{4\pi y}} \frac{e^{-[\ln(x/x_i)-3y]^2/4y}}{x^3}$$

Improved solution capturing recoil and stimulated scattering

$$\Delta n^*(x, y) = \frac{A}{\sqrt{4\pi y\, \beta(x_i, y)}} \frac{e^{-\left[\ln(x/x_i)-\alpha(x_i,y)\,y+\ln(1+x_i y)\right]^2/4y\,\beta(x_i,y)}}{x^3}$$

$$\alpha = [3 - 2f(x_i)]/\sqrt{1 + x_i y} \qquad f(x_i) = e^{-x_i}(1 + x_i^2/2)$$

$$\beta = 1/[1 + x_i y(1 - f(x_i))]$$

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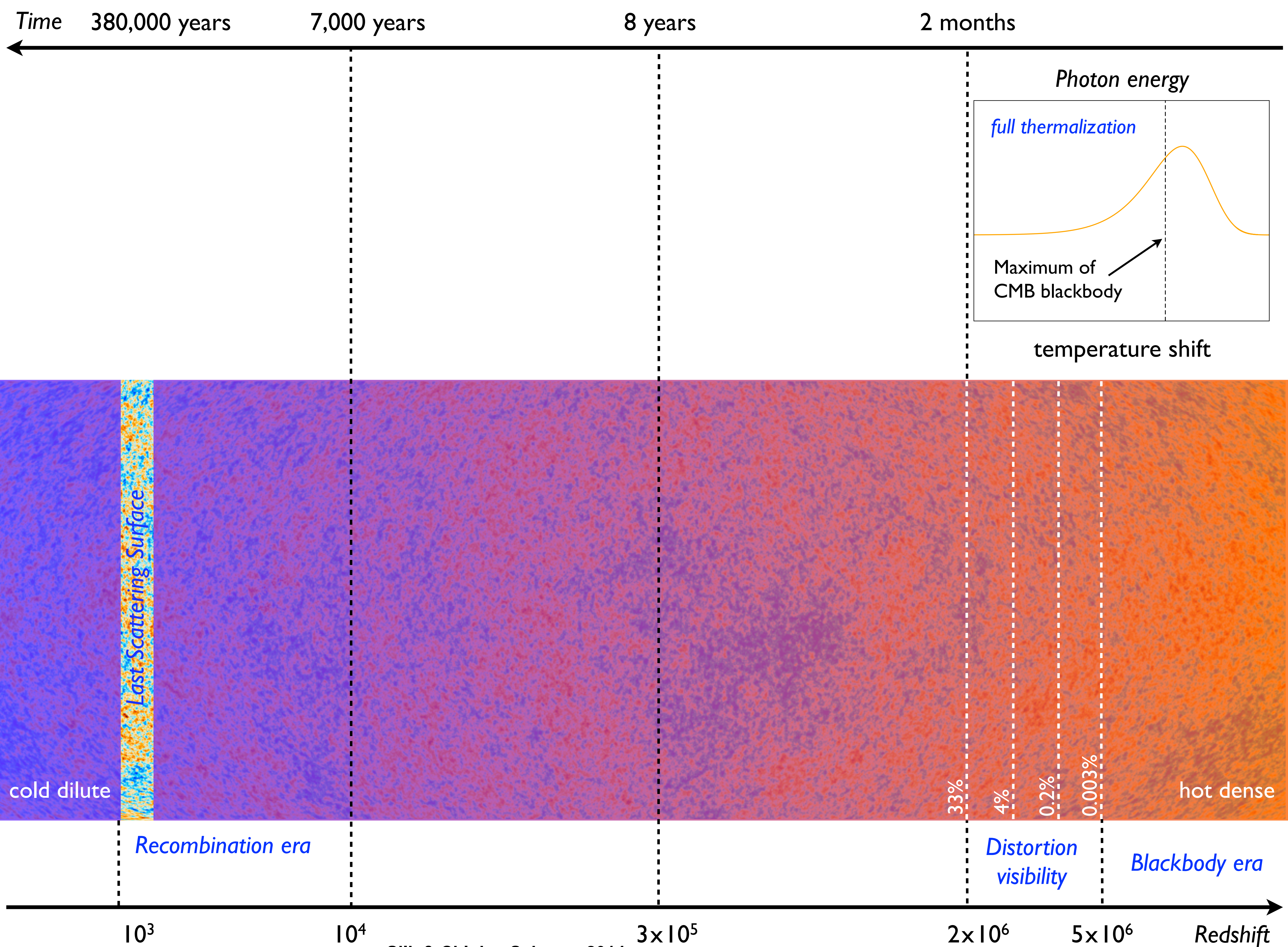
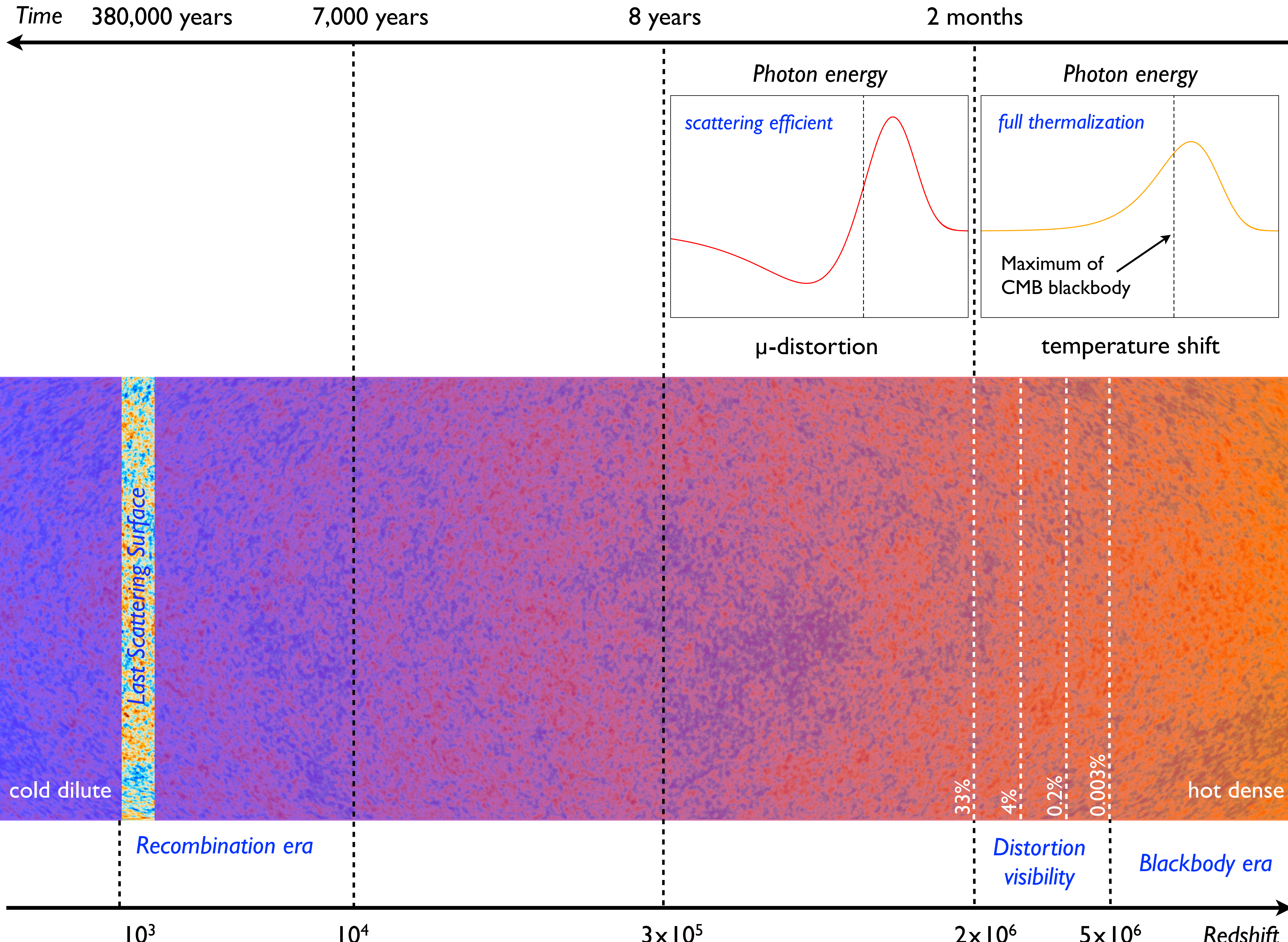


Silk & Chluba, Science, 2014



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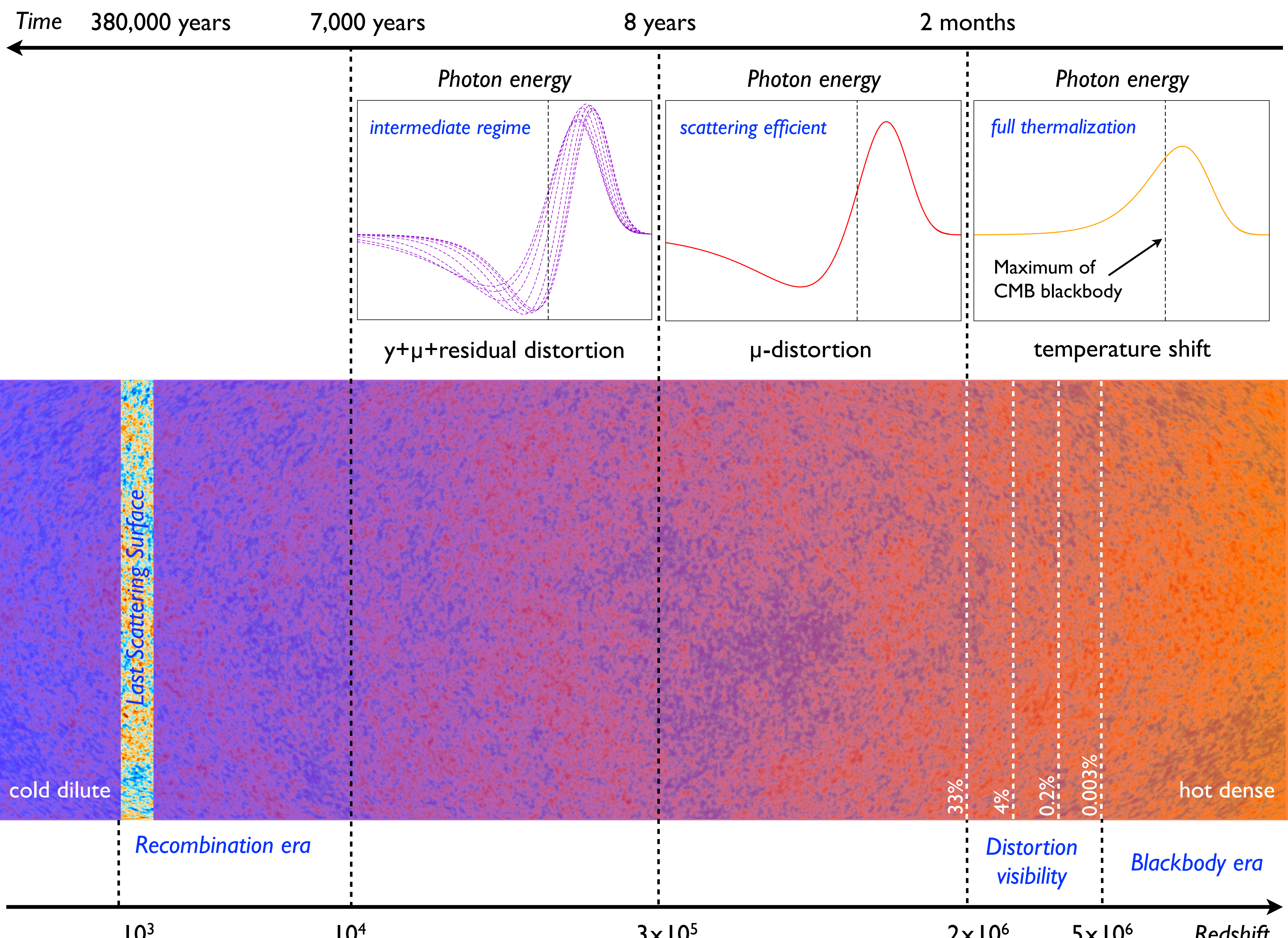

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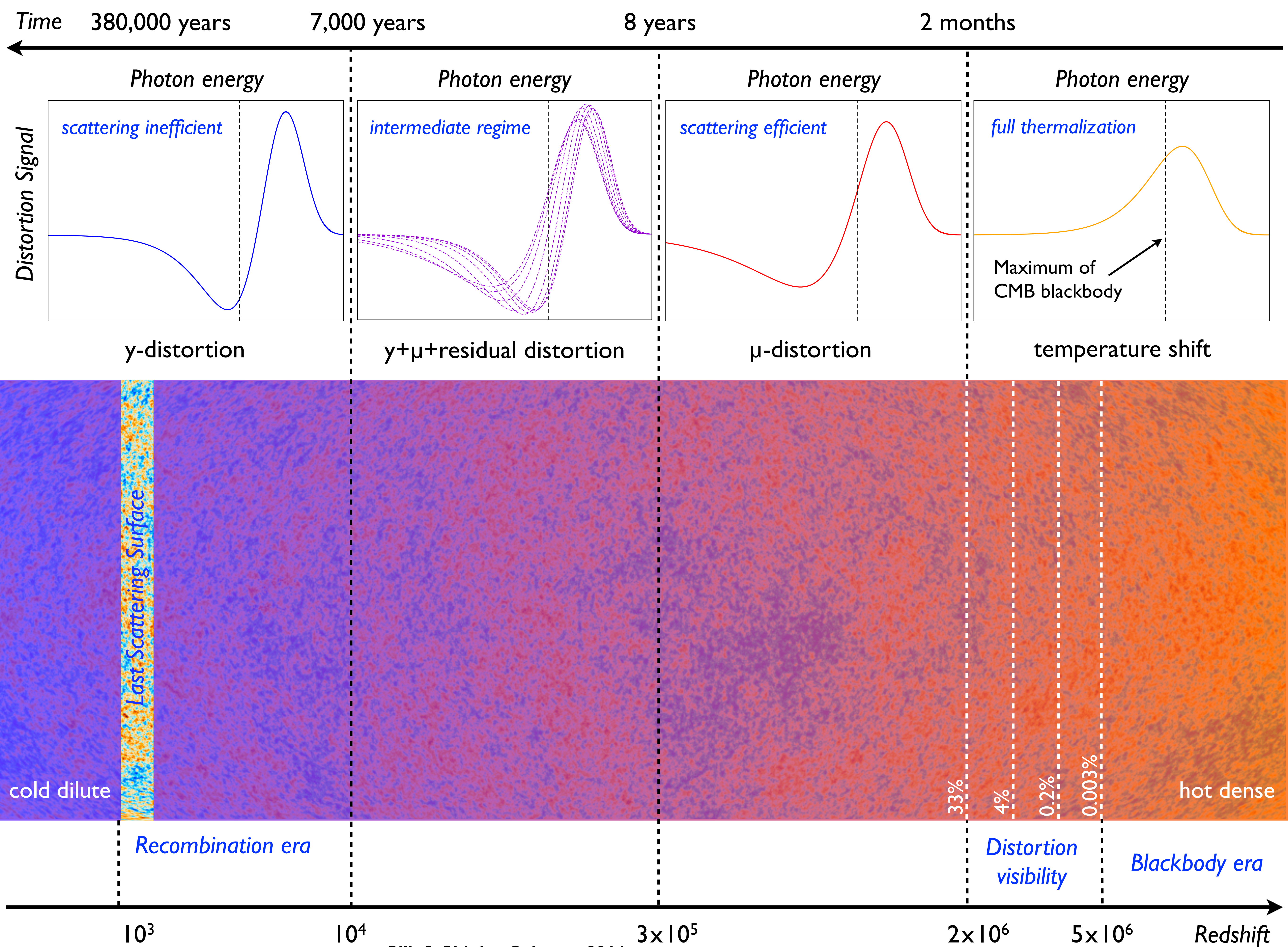

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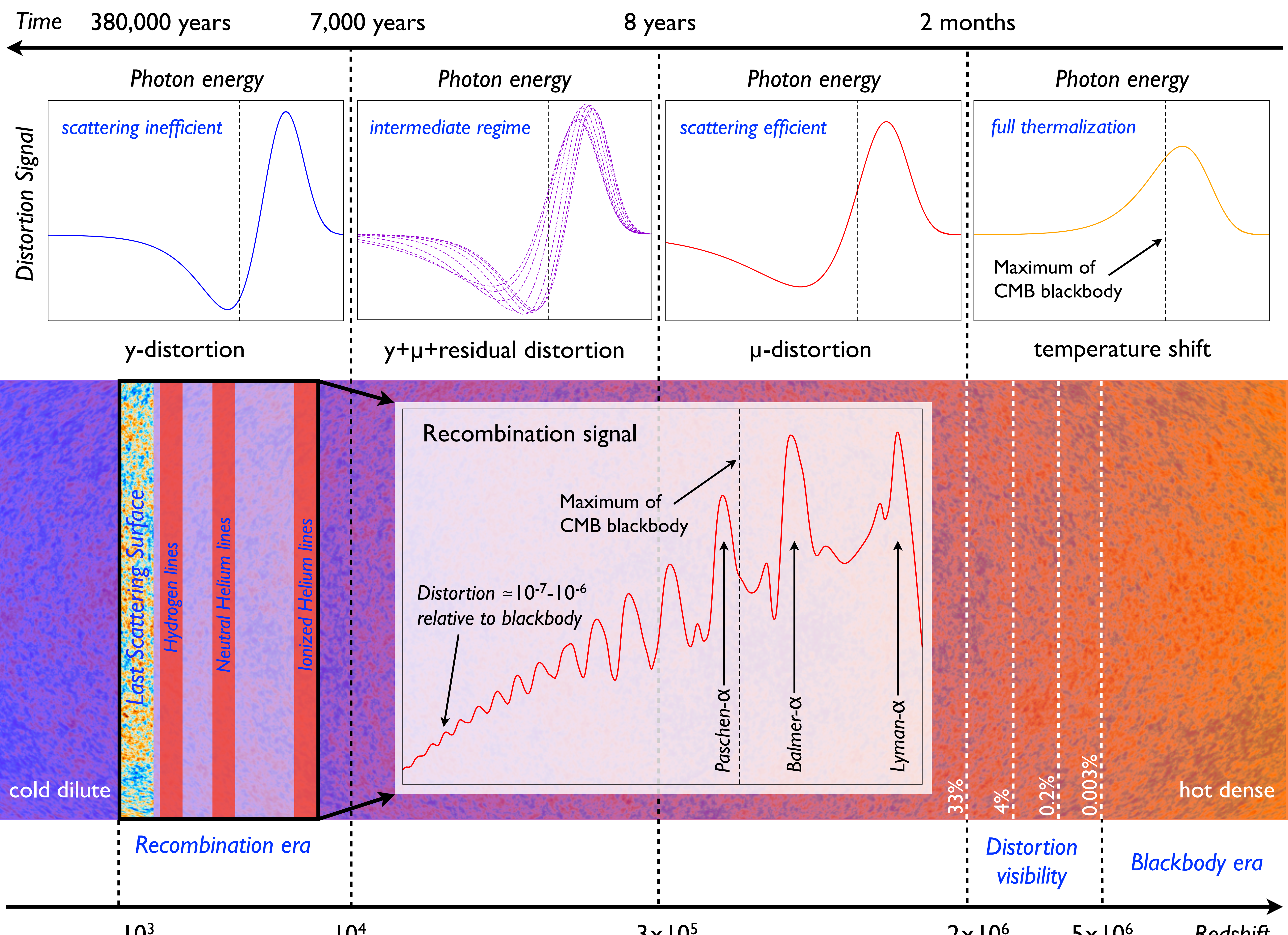

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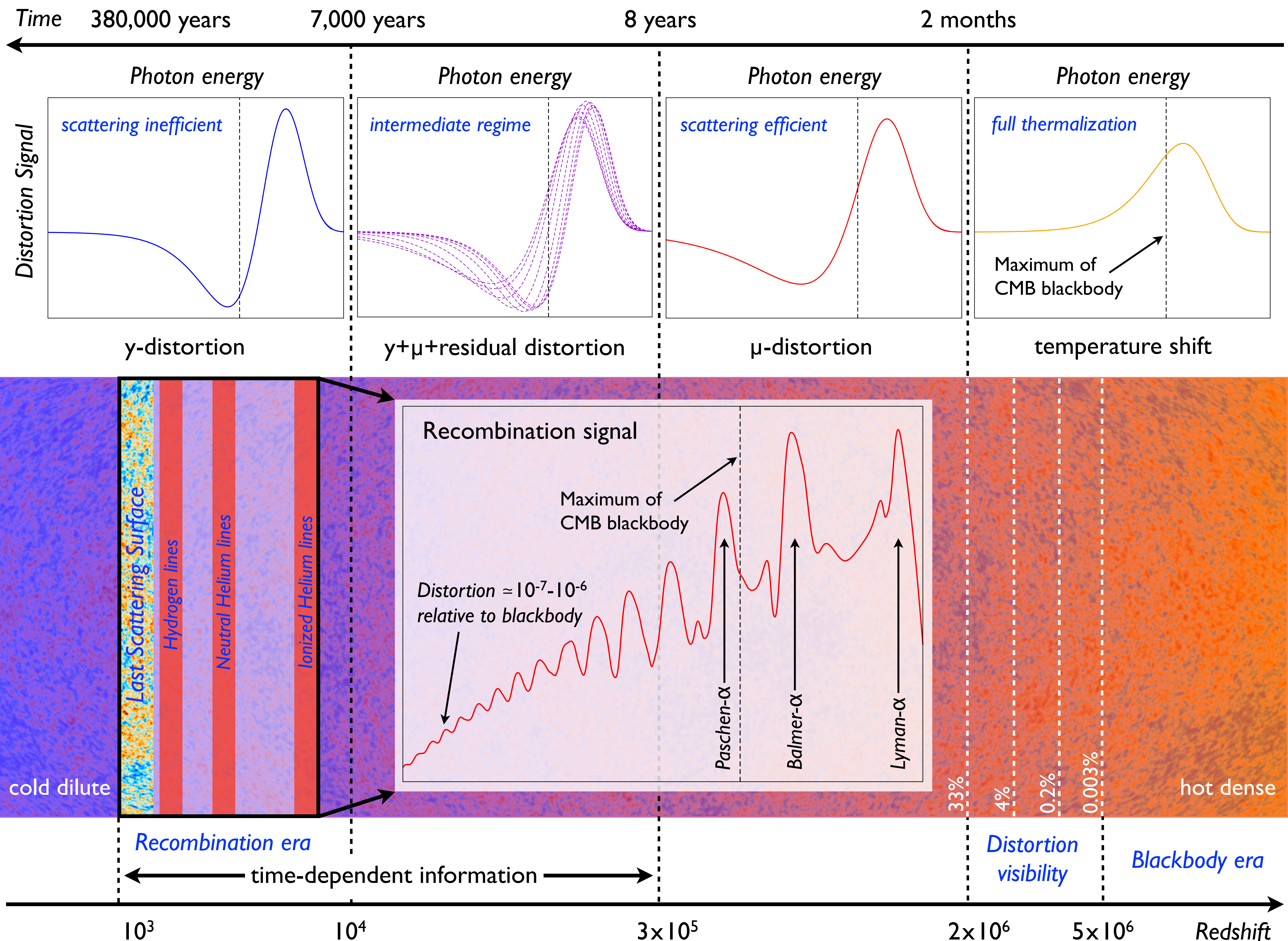

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