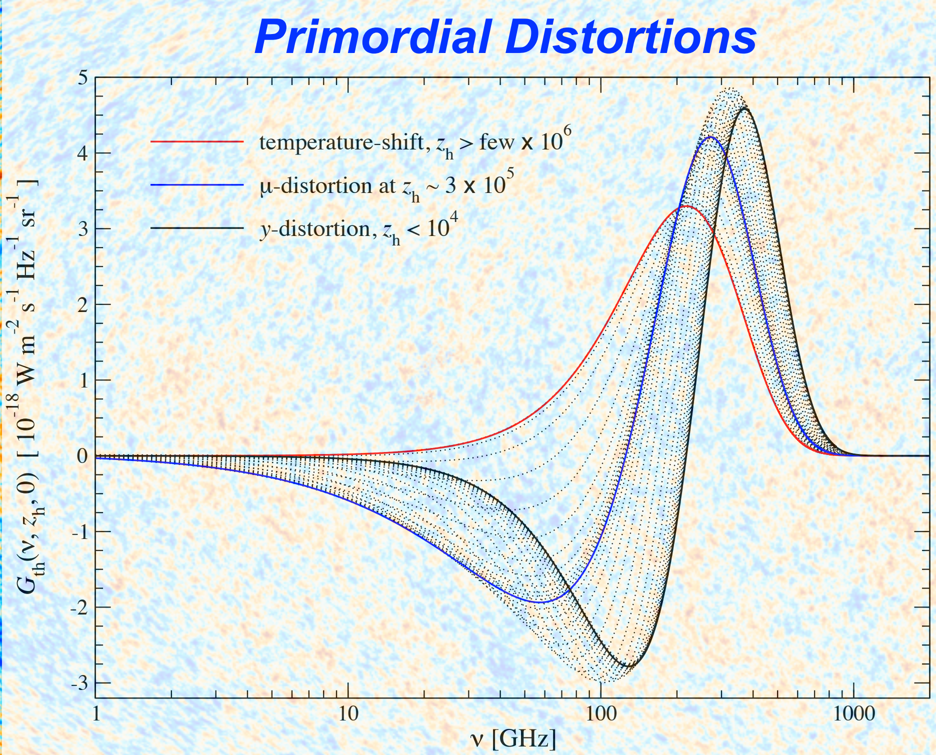
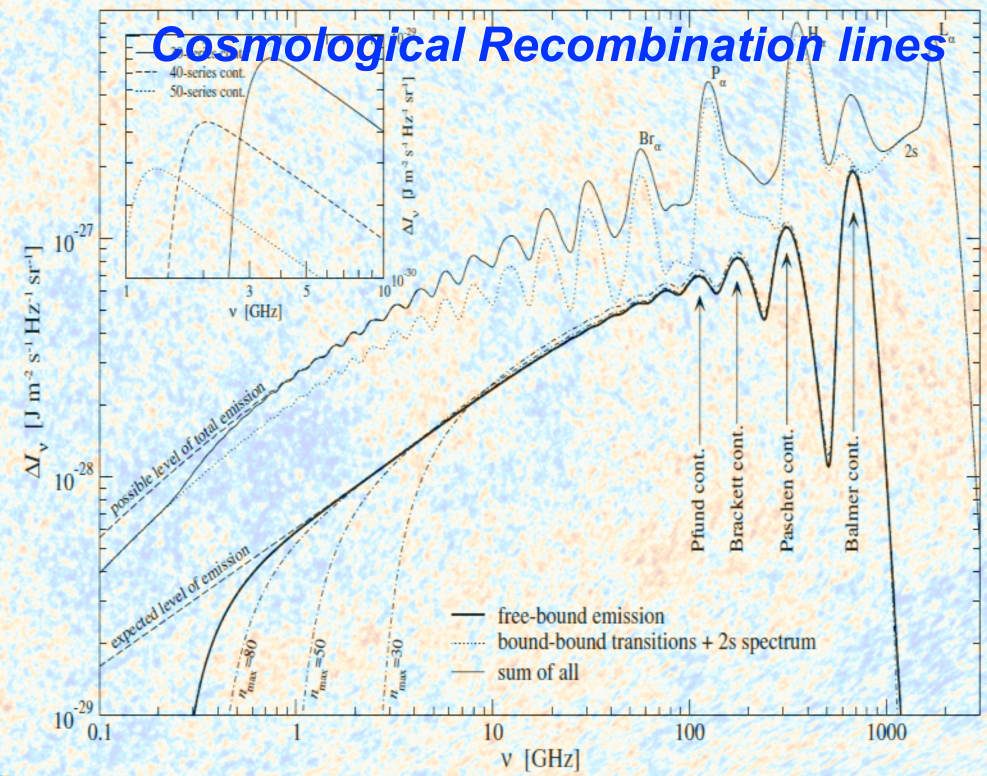
CMB Spectral Distortions





Jens Chluba

ISAPP School on Cosmology

Paris, June 25th, 2015

Main Goals of this Lecture

- Convince you that future CMB spectral distortion science will be *extremely* exciting!
- Explain how distortions evolve and thermalize
- Definition of different types of distortions
- Computations of spectral distortions
  (you should be able to do this yourself afterwards!)
- Provide an overview for different sources of primordial distortions
- Show you why the CMB spectrum provides a *complementary* probe of inflation and particle physics

References for the Theory of Spectral Distortions

- Original works
  - Zeldovich & Sunyaev, 1969, Ap&SS, 4, 301
  - Sunyaev & Zeldovich, 1970, Ap&SS, 7, 20
  - Illarionov & Sunyaev, 1975, Sov. Astr., 18, 413

Yakov Zeldovich

Rashid Sunyaev

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- Additional milestones
  - Danese & de Zotti, 1982, A&A, 107, 39
  - Burigana, Danese & de Zotti, 1991, ApJ, 379, 1
  - Hu & Silk, 1993, Phys. Rev. D, 48, 485
  - Hu, 1995, PhD thesis

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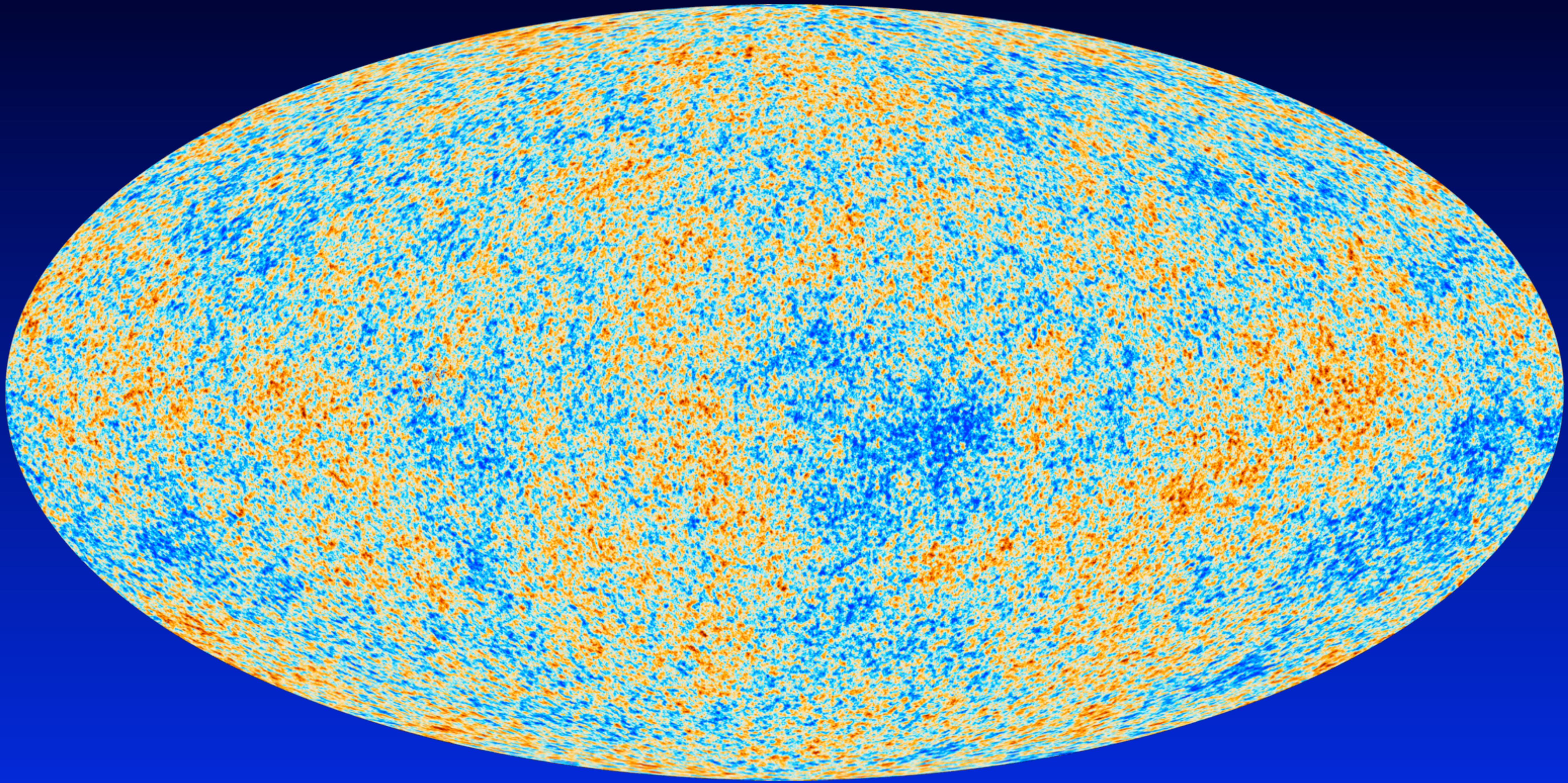
  - Hu & Silk, 1993, Phys. Rev. D, 48, 485
  - Hu, 1995, PhD thesis
- More recent overviews
  - Sunyaev & JC, 2009, AN, 330, 657
  - JC & Sunyaev, 2012, MNRAS, 419, 1294
  - JC, MNRAS, 436, 2232 & ArXiv:1405.6938
  - *CUSO-lecture notes at* www.Chluba.de/Science

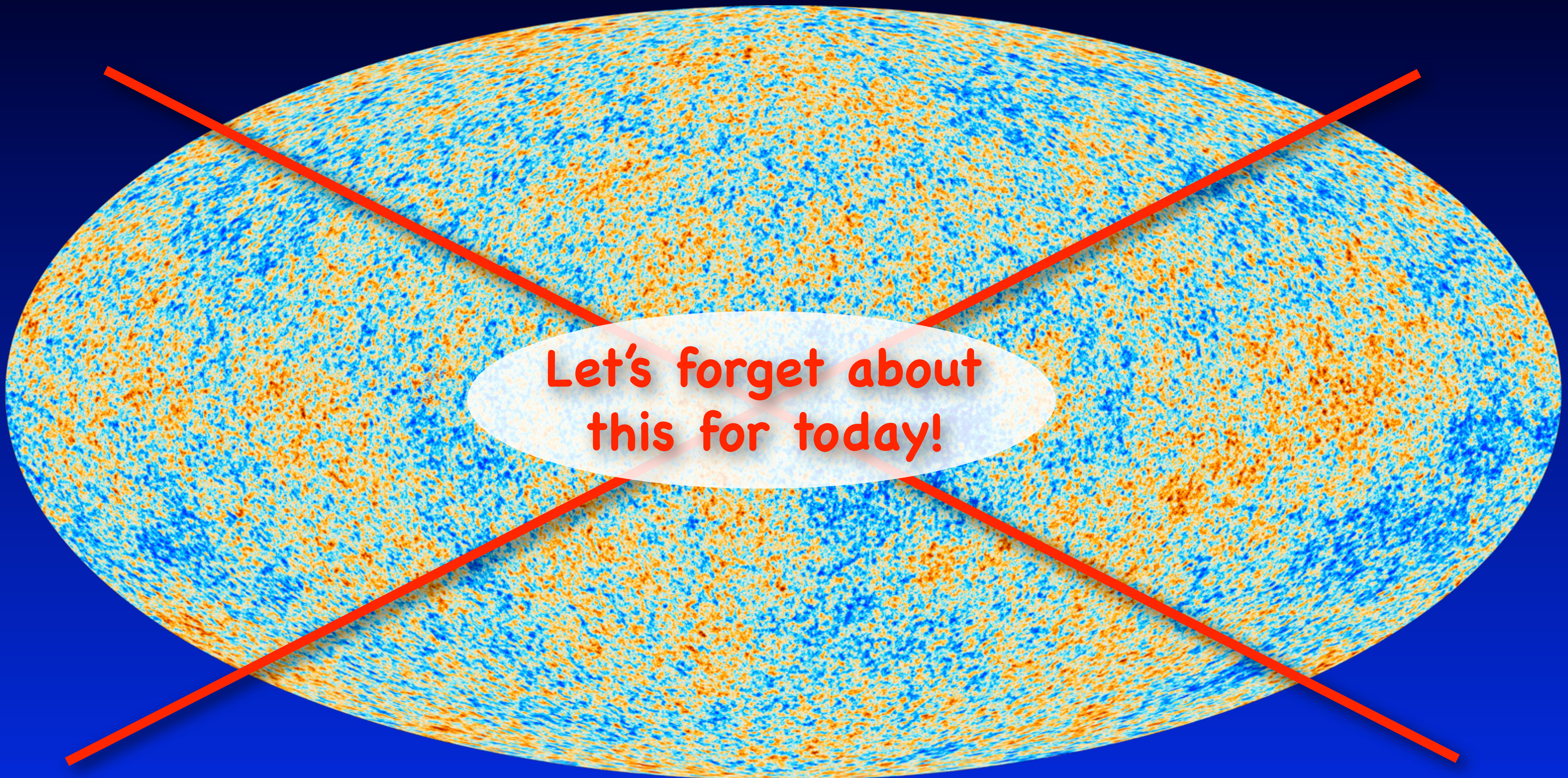
Cosmic Microwave Background Anisotropies



Planck all-sky temperature map

- CMB has a blackbody spectrum in every direction
- tiny variations of the CMB temperature $\Delta T/T \sim 10^{-5}$

Cosmic Microwave Background Anisotropies



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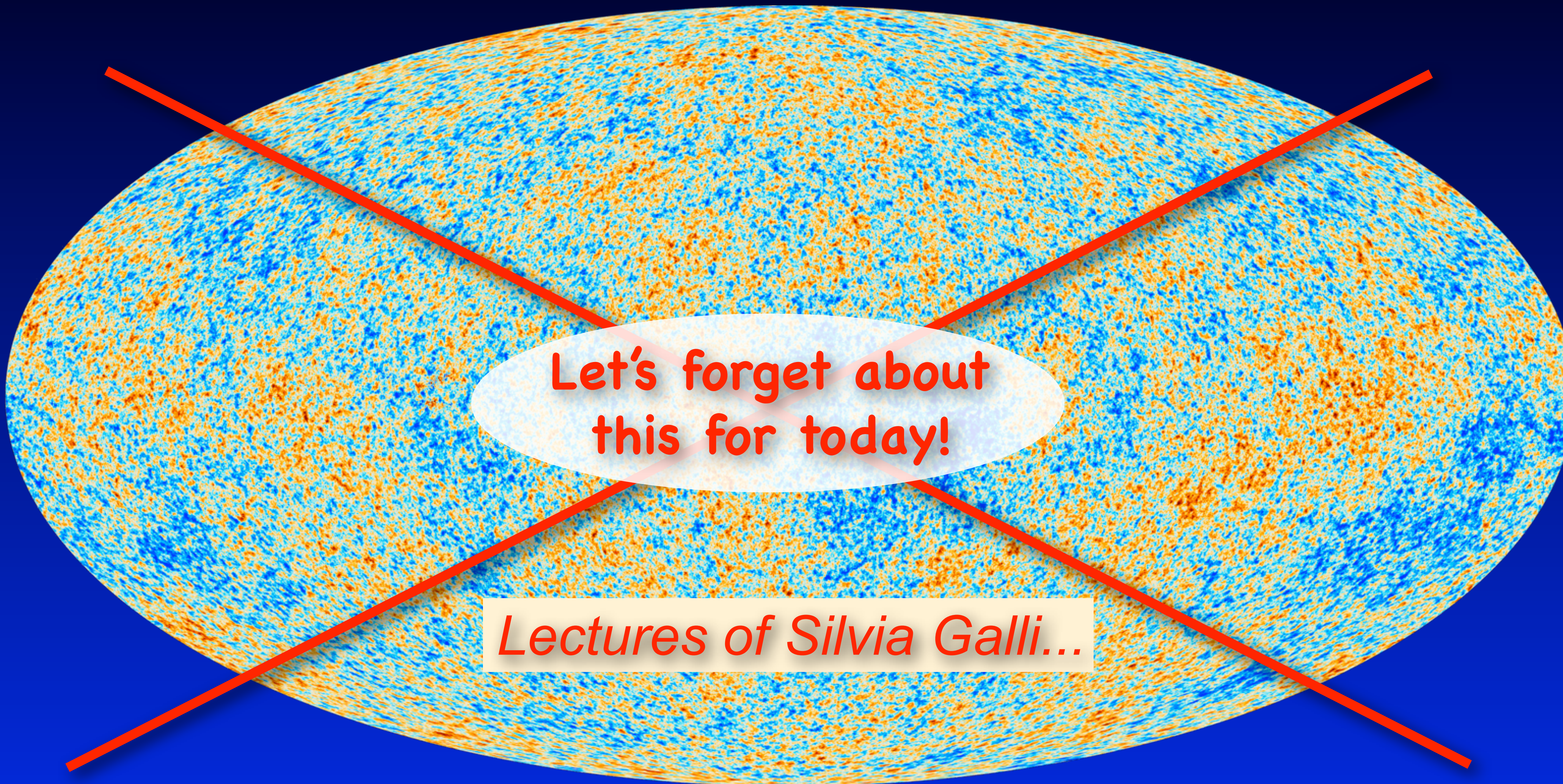


Planck all-sky
temperature map

- CMB has a blackbody spectrum in every direction
- tiny variations of the CMB temperature $\Delta T/T \sim 10^{-5}$

CMB provides another independent piece of information!

COBE/FIRAS

$$T_0 = (2.726 \pm 0.001)\,\mathrm{K}$$

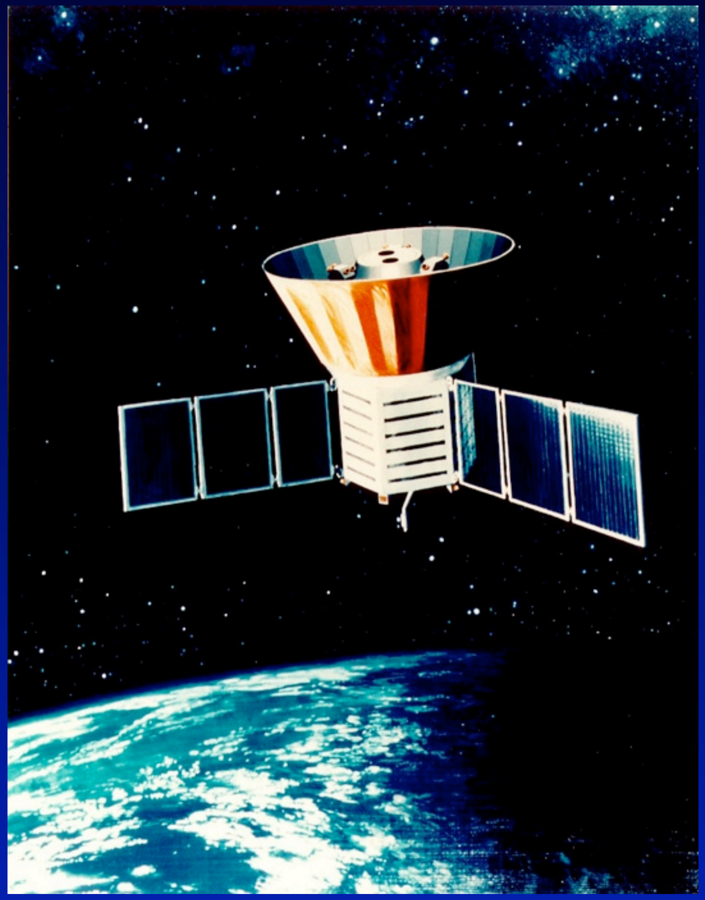
Absolute measurement required!

One has to go to space...

Mather et al., 1994, ApJ, 420, 439
Fixsen et al., 1996, ApJ, 473, 576
Fixsen, 2003, ApJ, 594, 67
Fixsen, 2009, ApJ, 707, 916

- CMB monopole is 10000 - 100000 times larger than the fluctuations

COBE / FIRAS (Far InfraRed Absolute Spectrophotometer)



$$T_0 = 2.725 \pm 0.001\,\mathrm{K}$$

$$|\,y\,| \leq 1.5 \times 10^{-5}$$

$$|\,\mu\,| \leq 9 \times 10^{-5}$$

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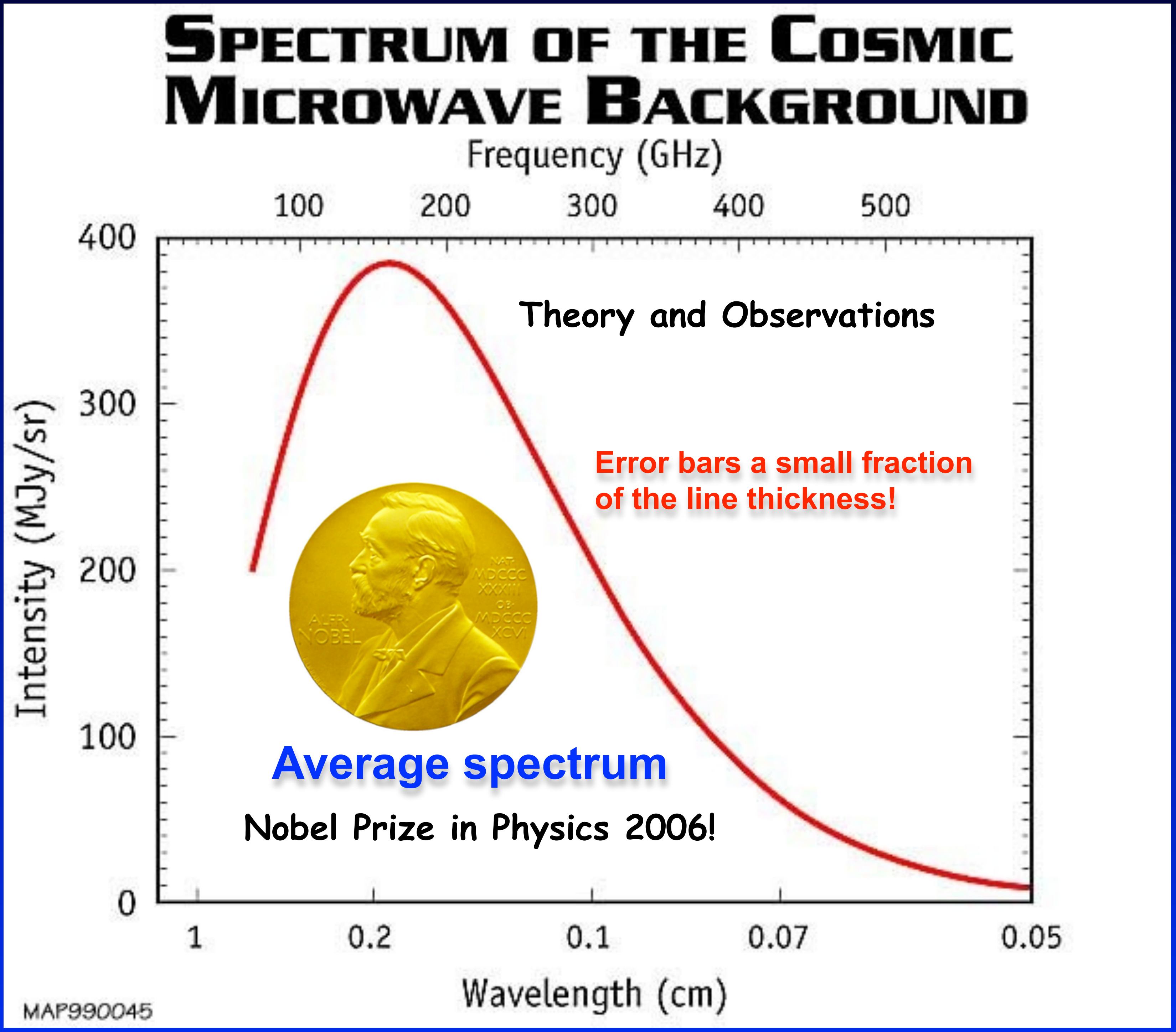

Why should one expect some spectral distortion?

Why should one expect some spectral distortion?

Full thermodynamic equilibrium (certainly valid at very high redshift)

- CMB has a blackbody spectrum at every time (not affected by expansion)
- Photon number density and energy density determined by temperature T_γ

$T_\gamma \sim 2.726\,(1+z)$ K

$N_\gamma \sim 411$ cm^{-3} $(1+z)^3 \sim 2\times10^9\, N_b$ (*entropy density dominated by photons*)

$\rho_\gamma \sim 5.1\times10^{-7}\, m_e c^2$ cm^{-3} $(1+z)^4 \sim \rho_b$ x $(1+z)$ / 925 ~ 0.26 eV cm^{-3} $(1+z)^4$

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Perturbing full equilibrium by

- Energy injection (interaction *matter* ←→ *photons*)
- Production of (energetic) photons and/or particles (i.e. change of entropy)

→ CMB spectrum deviates from a pure blackbody

→ thermalization process (partially) erases distortions

(Compton scattering, double Compton and Bremsstrahlung in the expanding Universe)

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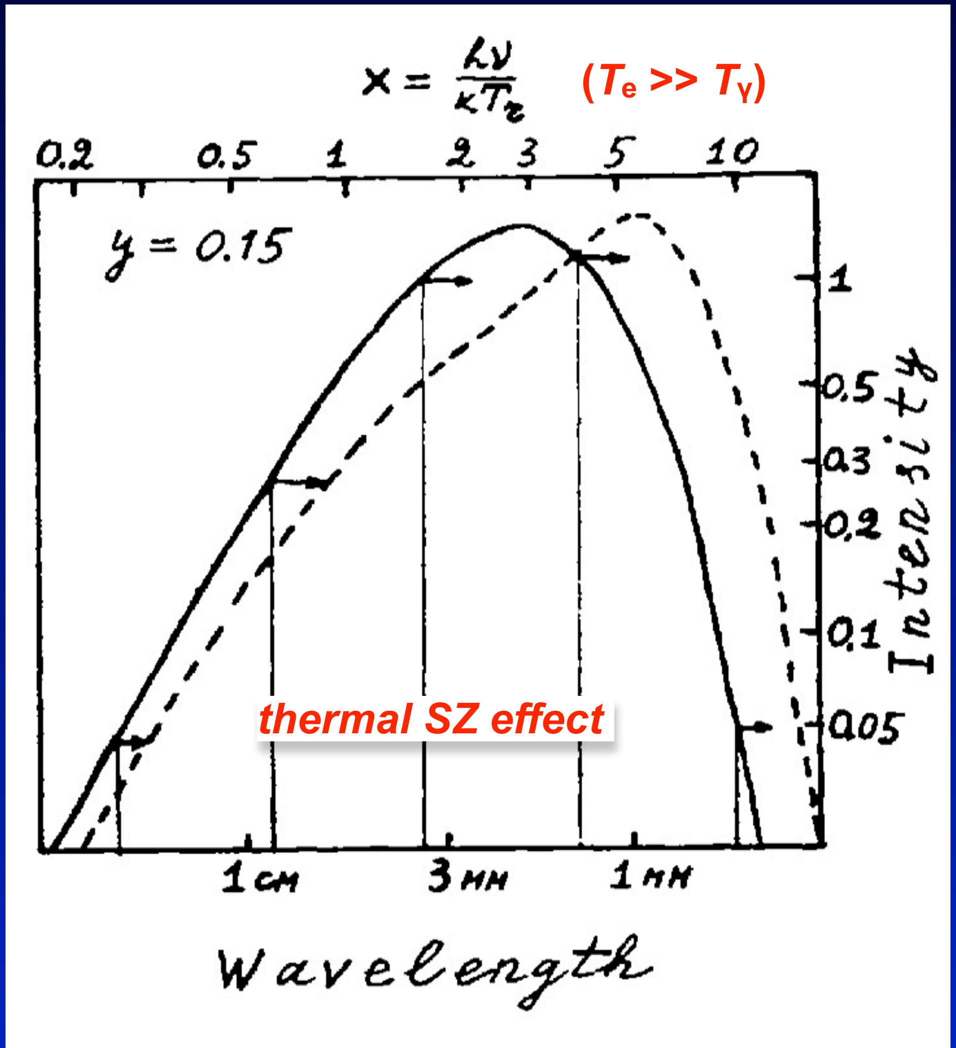
Measurements of CMB spectrum place very tight limits on the thermal history of our Universe!

Standard types of primordial CMB distortions

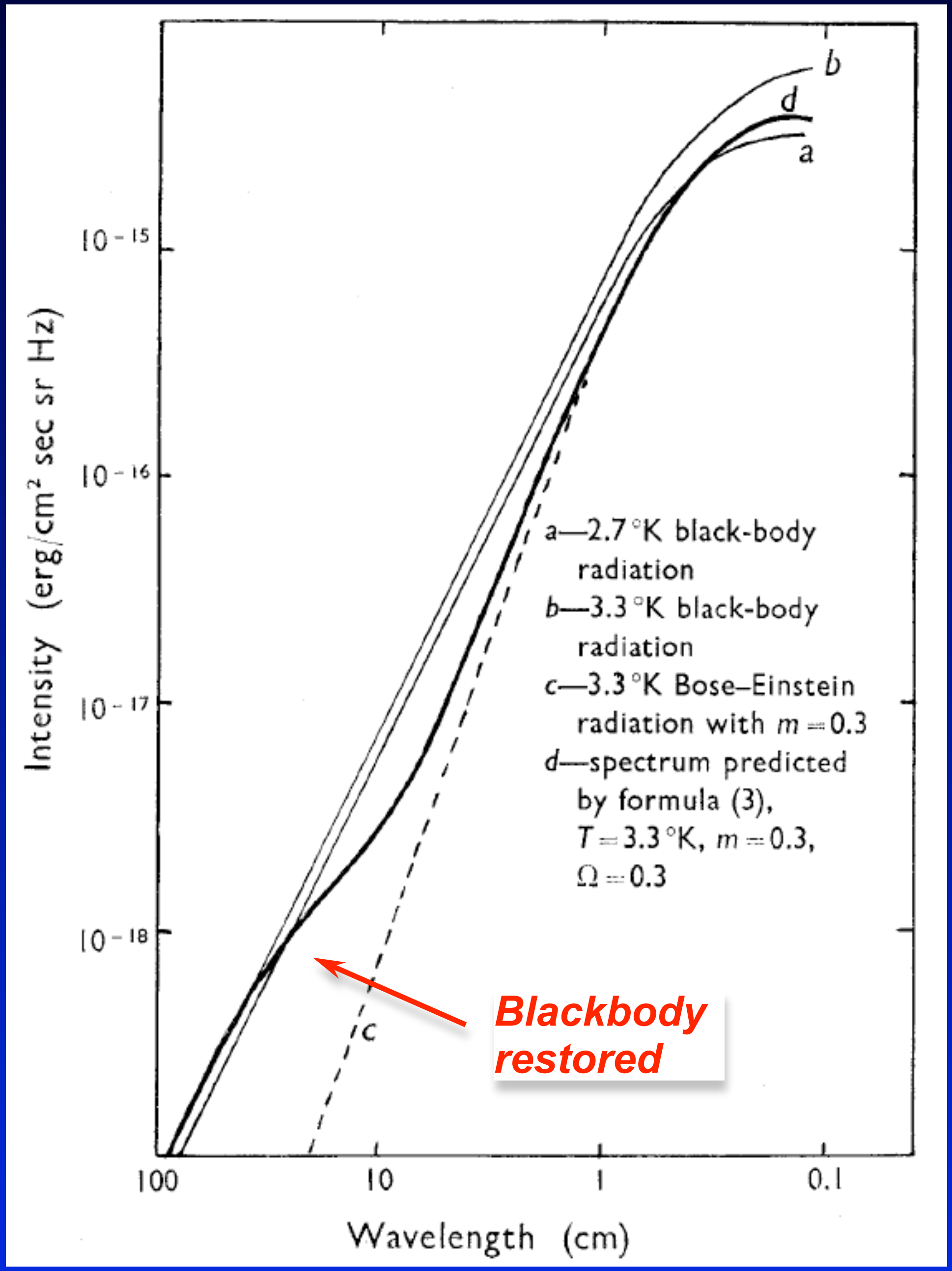
Compton y-distortion



Sunyaev & Zeldovich, 1980, ARAA, 18, 537

- also known from thSZ effect
- up-scattering of CMB photon
- important at late times (z<50000)
- scattering inefficient

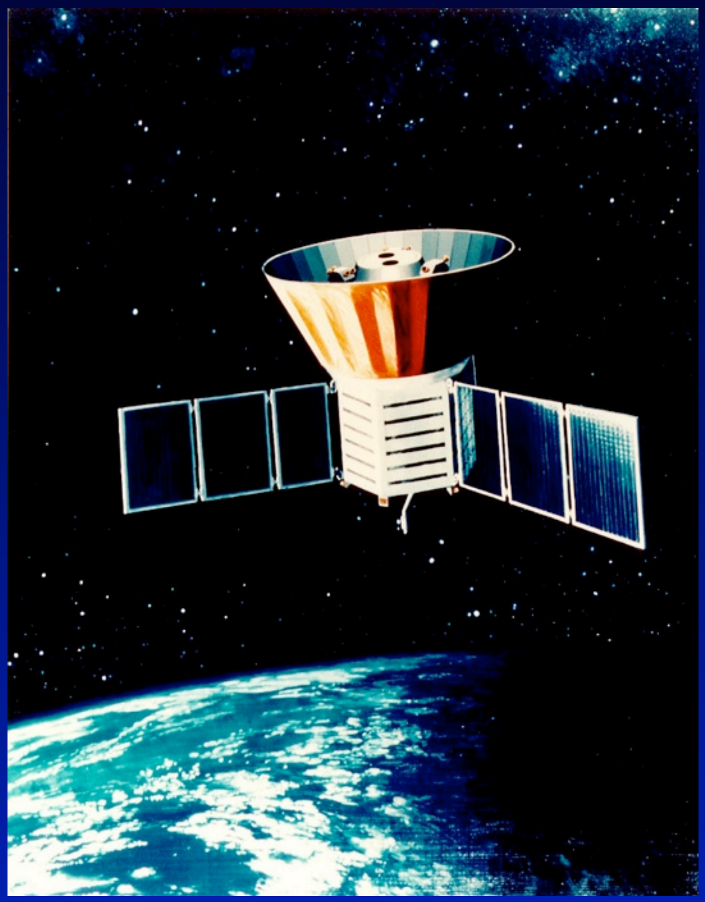
Chemical potential μ-distortion



Sunyaev & Zeldovich, 1970, ApSS, 2, 66

- important at very times (z>50000)
- scattering very efficient

COBE / FIRAS (Far InfraRed Absolute Spectrophotometer)



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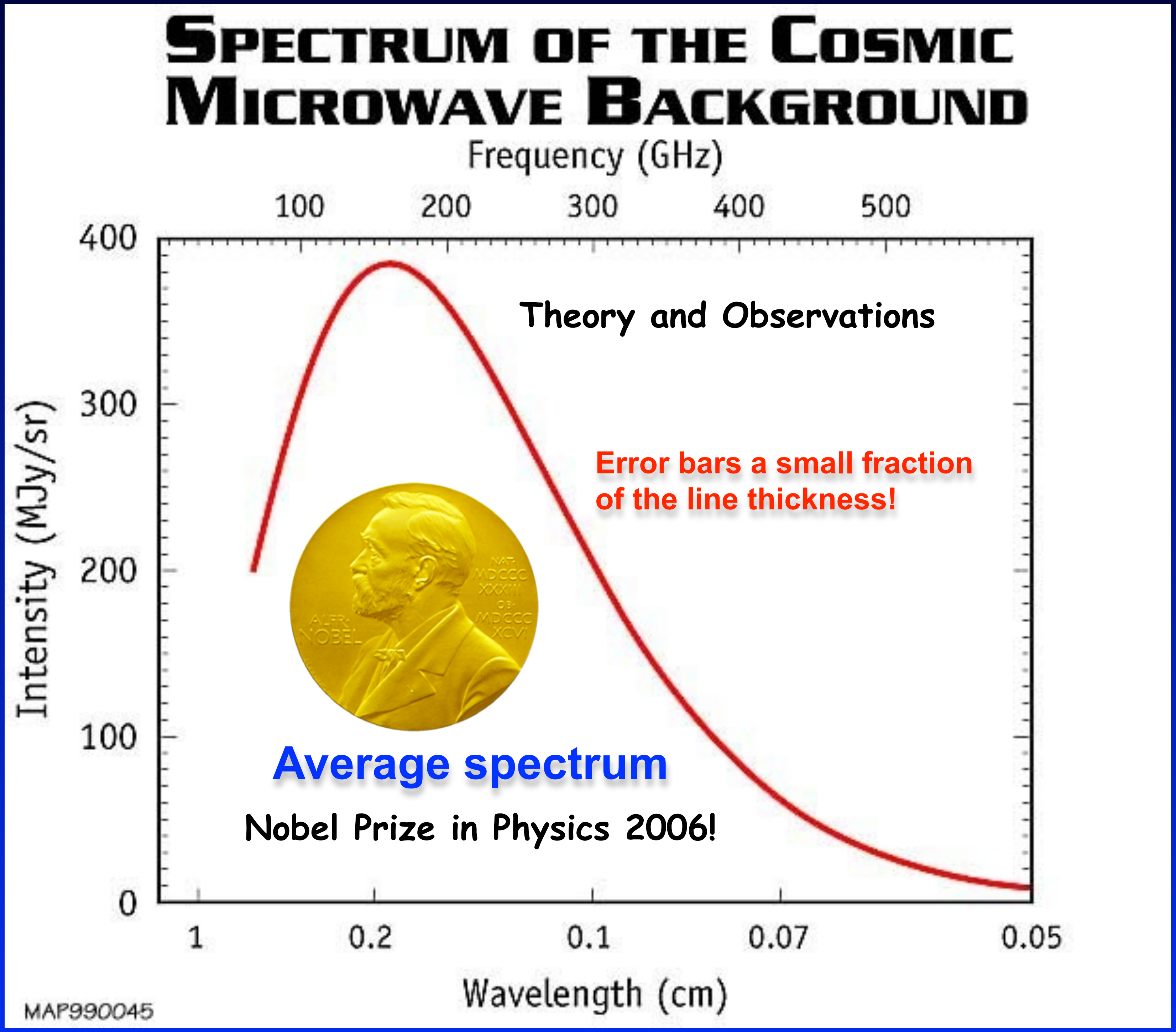

Only very small distortions of CMB spectrum are still allowed!

No primordial distortion found so far!? Why are we at all talking about this then?

Physical mechanisms that lead to spectral distortions

- *Cooling by adiabatically expanding ordinary matter*
  (JC, 2005; JC & Sunyaev 2011; Khatri, Sunyaev & JC, 2011)
- Heating by *decaying* or *annihilating* relic particles
  (Kawasaki et al., 1987; Hu & Silk, 1993; McDonald et al., 2001; JC, 2005; JC & Sunyaev, 2011; JC, 2013; JC & Jeong, 2013)
- *Evaporation of primordial black holes & superconducting strings*
  (Carr et al. 2010; Ostriker & Thompson, 1987; Tashiro et al. 2012; Pani & Loeb, 2013)
- *Dissipation of primordial acoustic modes & magnetic fields*
  (Sunyaev & Zeldovich, 1970; Daly 1991; Hu et al. 1994; JC & Sunyaev, 2011; JC et al. 2012 - Jedamzik et al. 2000; Kunze & Komatsu, 2013)
- *Cosmological recombination radiation*
  (Zeldovich et al., 1968; Peebles, 1968; Dubrovich, 1977; Rubino-Martin et al., 2006; JC & Sunyaev, 2006; Sunyaev & JC, 2009)

pre-recombination epoch

„high“ redshifts

„low“ redshifts

post-recombination

- Signatures due to first supernovae and their remnants
  (Oh, Cooray & Kamionkowski, 2003)
- Shock waves arising due to large-scale structure formation
  (Sunyaev & Zeldovich, 1972; Cen & Ostriker, 1999)
- SZ-effect from clusters; effects of reionization
  (Refregier et al., 2003; Zhang et al. 2004; Trac et al. 2008)
- more exotic processes
  (Lochan et al. 2012; Bull & Kamionkowski, 2013; Brax et al., 2013; Tashiro et al. 2013)

Physical mechanisms that lead to spectral distortions

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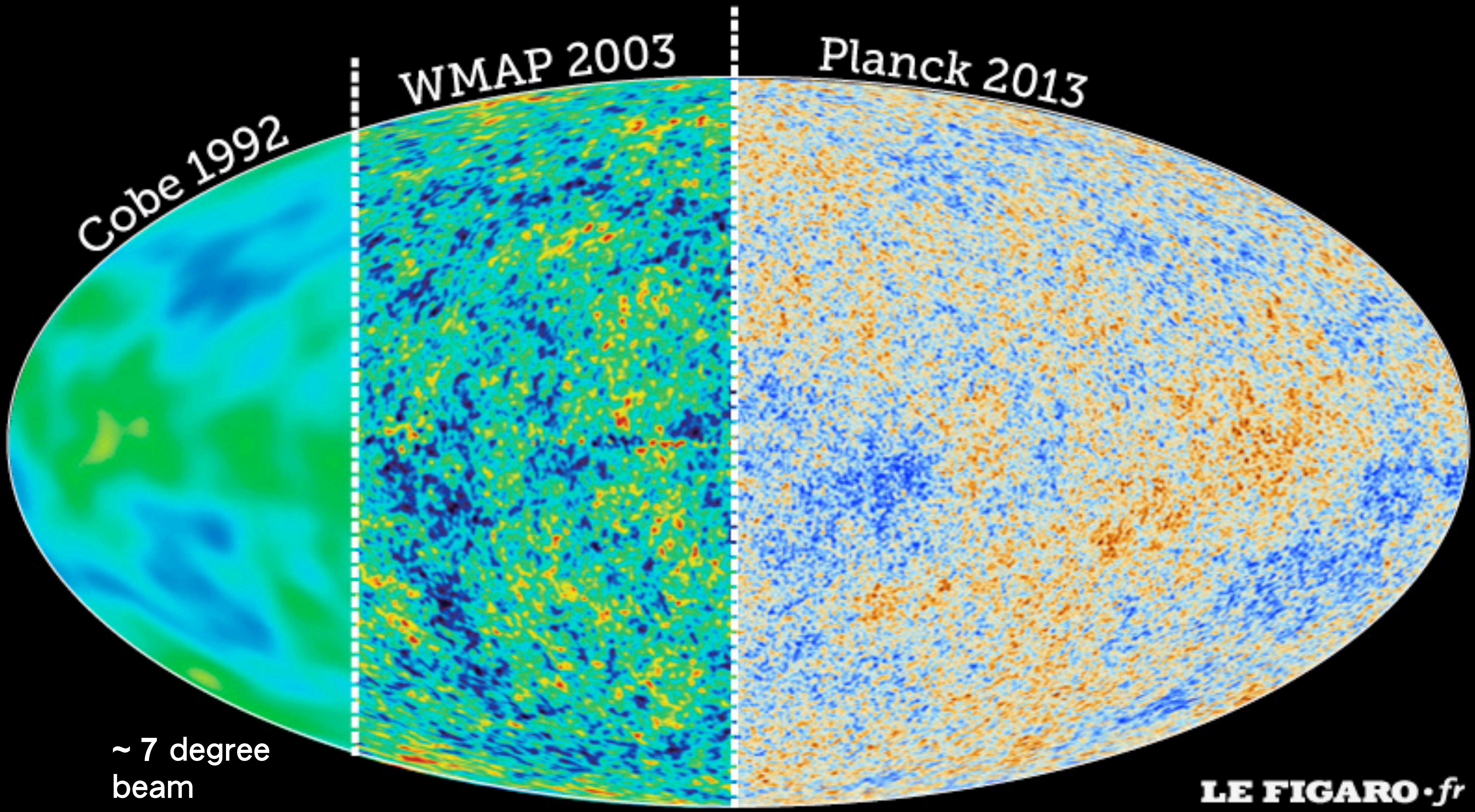
Dramatic improvements in angular resolution and sensitivity over the past decades!



~ 7 degree beam

~ 0.3 degree beam

~ 0.08 degree beam

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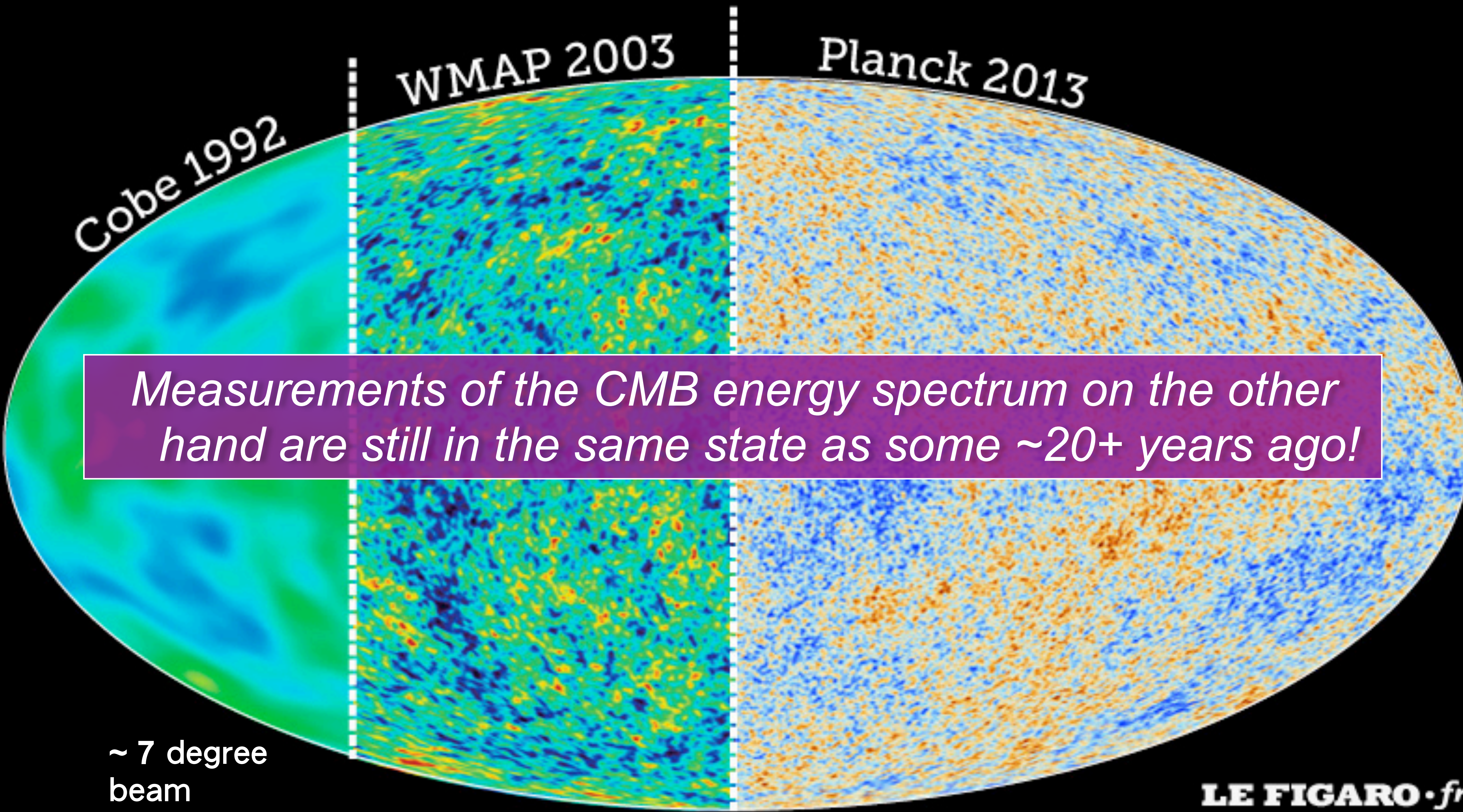


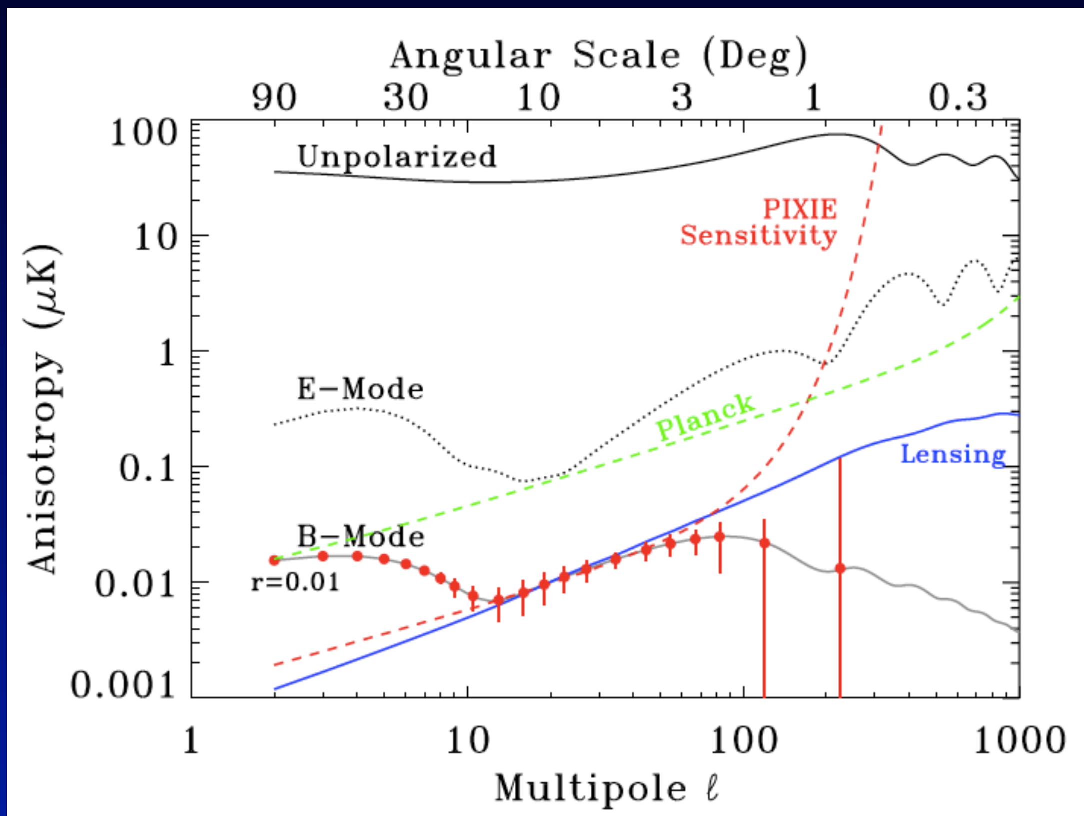
Measurements of the CMB energy spectrum on the other hand are still in the same state as some ~20+ years ago!

~ 7 degree beam

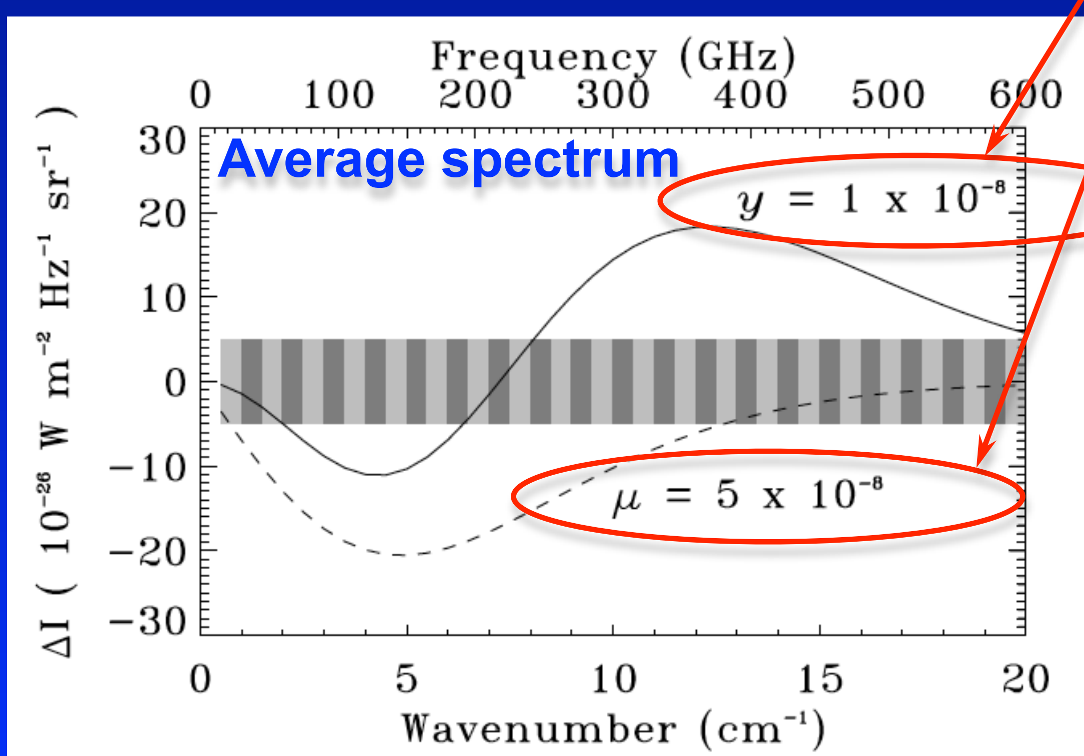
~ 0.3 degree beam

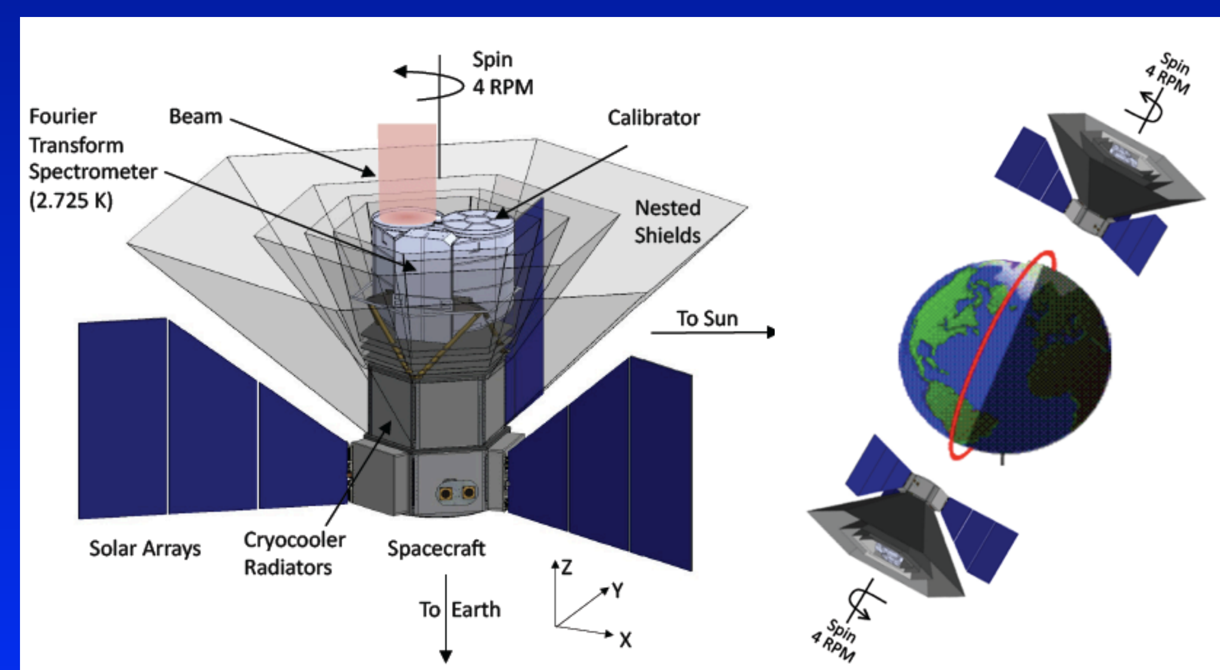
~ 0.08 degree beam

PIXIE: Primordial Inflation Explorer

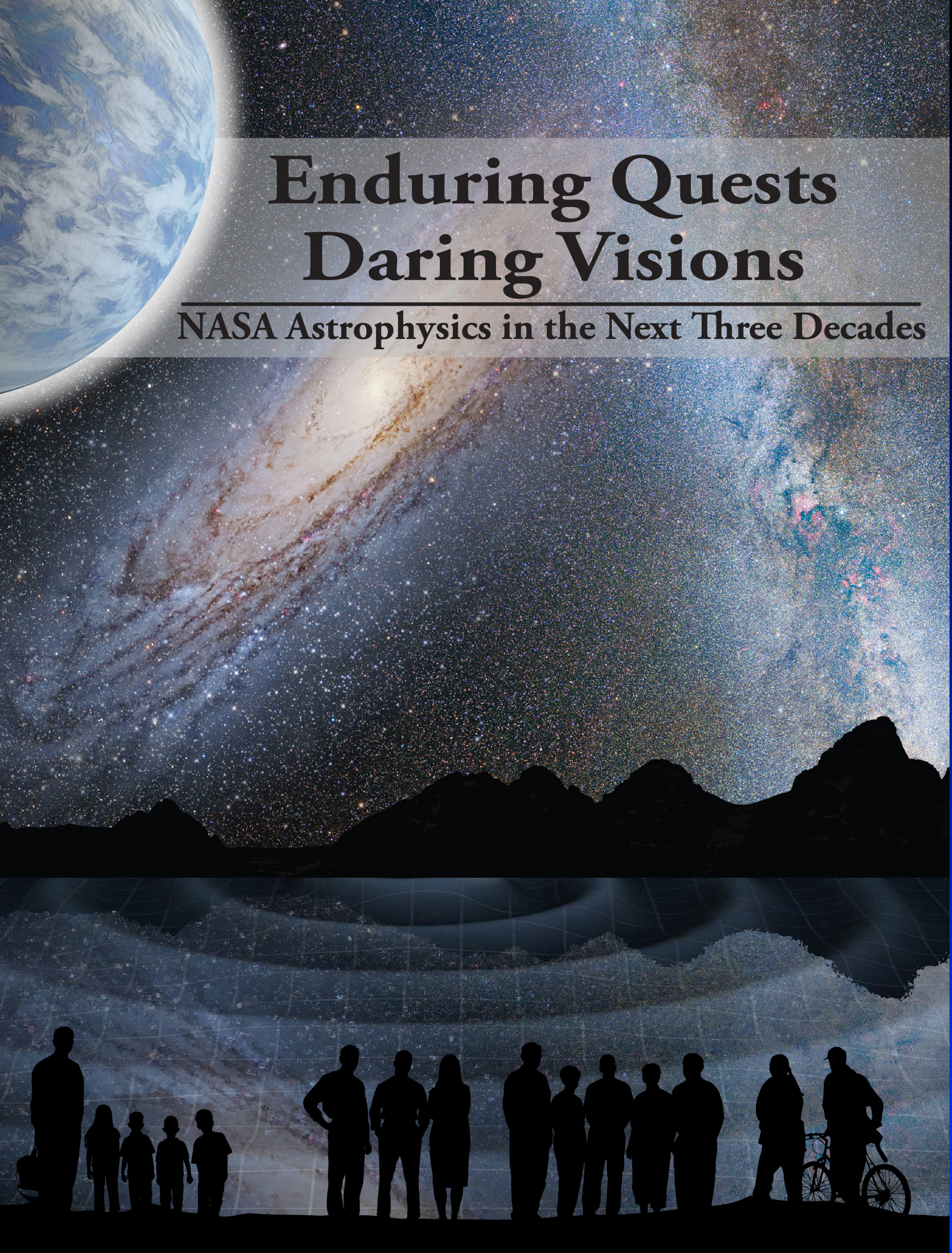


- 400 spectral channel in the frequency range 30 GHz and 6THz (Δν ~ 15GHz)
- about 1000 (!!!) times more sensitive than COBE/FIRAS
- B-mode polarization from inflation ($r \approx 10^{-3}$)
- improved limits on μ and y
- was proposed 2011 as NASA EX mission (i.e. cost ~ 200 M$)





Kogut et al, JCAP, 2011, arXiv:1105.2044



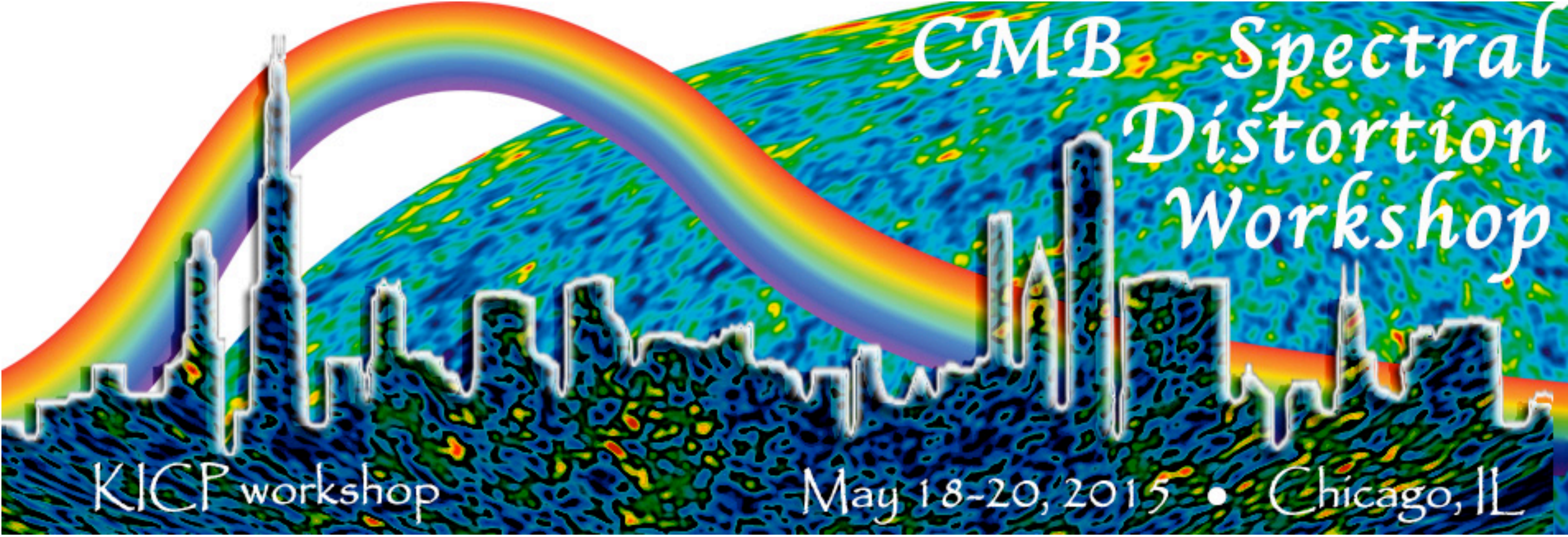
NASA 30-yr Roadmap Study

(published Dec 2013)

How does the Universe work?

"Measure the spectrum of the CMB with precision several orders of magnitude higher than COBE FIRAS, from a moderate-scale mission or an instrument on CMB Polarization Surveyor."

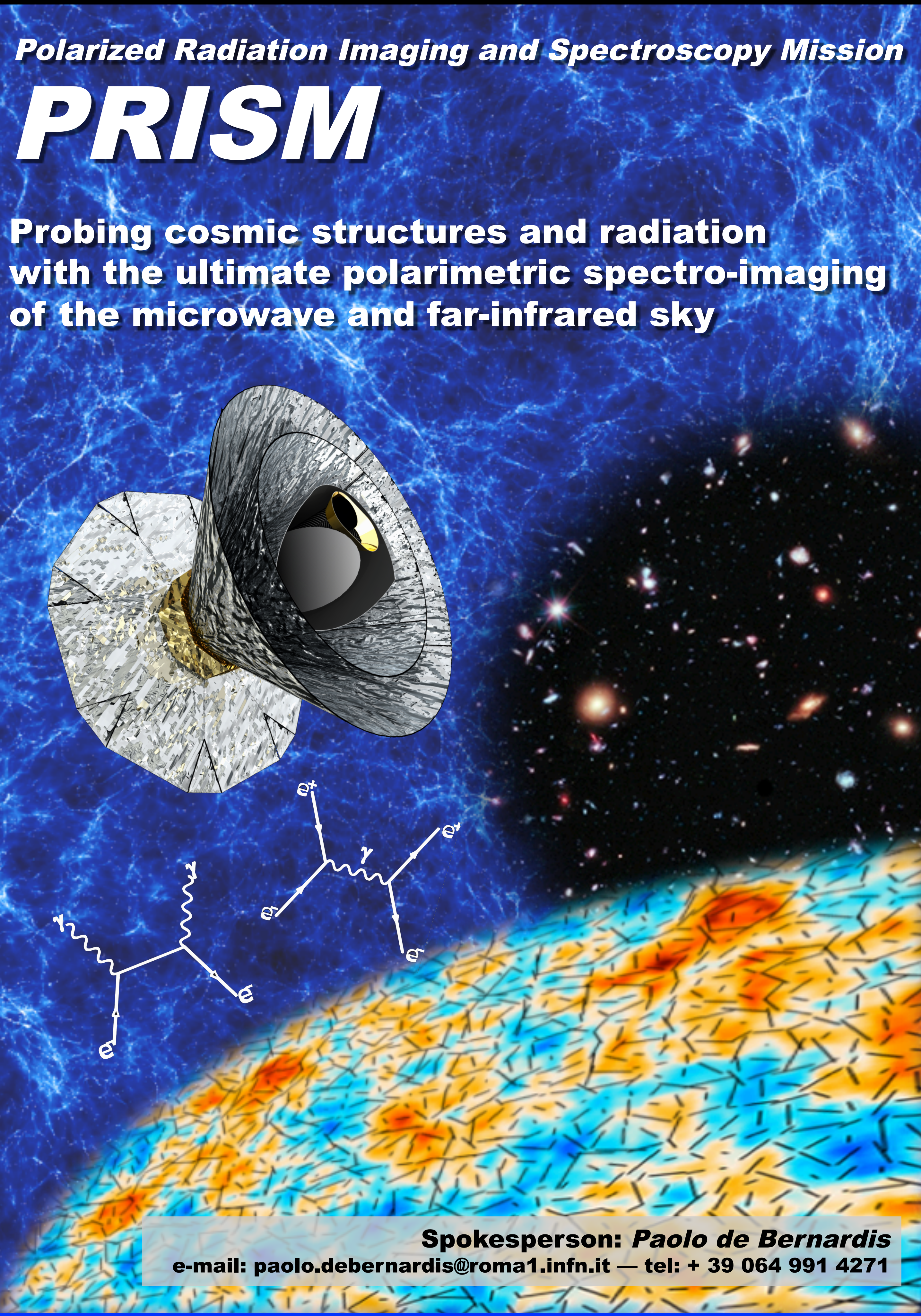
New call from NASA expected ~2-3 years from now



- Yacine Ali-Haimoud, Johns Hopkins University
- Mustafa Amin, KICC
- Nicholas Battaglia, Princeton University
- Elia S Battistelli, Sapienza, University of Rome
- Ido Ben-Dayan, DESY
- Rob Caldwell, Dartmouth
- John Carlstrom, University of Chicago
- Jens Chluba, IoA, University of Cambridge
- David Chuss, Villanova University
- Asantha Cooray, UC Irvine
- Jacques H. Delabrouille, CNRS - APC
- Scott Dodelson, Fermilab/Chicago
- Doug Finkbeiner, Harvard University
- Dale J. Fixsen, University of Maryland
- Wendy Freedman, University of Chicago
- Daniel Grin, University of Chicago
- Mark Halpern, U British Columbia
- Colin Hill, Columbia University
- Tashiro Hiroyuki, Nagoya University
- Wayne Hu, University of Chicago
- Donghui Jeong, Penn State
- Alan J. Kogut, NASA/GSFC
- Kerstin Kunze, University of Salamanca
- Alex Lazarian, University of Wisconsin
- Marilena Loverde, University of Chicago
- Alessandro Manzotti, KICP, The University of Chicago
- John C Mather, NASA/GSFC
- Stephan S. Meyer, University of Chicago
- Pavel Motloch, University of Chicago
- Lyman Page, Princeton
- Jean-Loup Puget, IAS (Institut d'astrophysique spatiale)
- Mayuri Sathyanarayana Rao, Raman Research Institute
- Christopher Sheehy, University of Chicago, KICP
- Joe Silk, Oxford
- David Spergel, Princeton
- Suzanne T Staggs, Princeton
- Albert Stebbins, Fermilab
- Ravi Subrahmanyan, Raman Research Institute, Bangalore, India
- Rashid Sunyaev, MPA Garching
- Eric R Switzer, NASA GSFC
- Andrea Tartari, Astroparticule et Cosmologie
- Miranda Vinicius, U. Chicago
- Edward J. Wollack, NASA Goddard Space Flight Center

National Science Foundation
WHERE DISCOVERIES BEGIN

THE UNIVERSITY OF CHICAGO



Instruments:

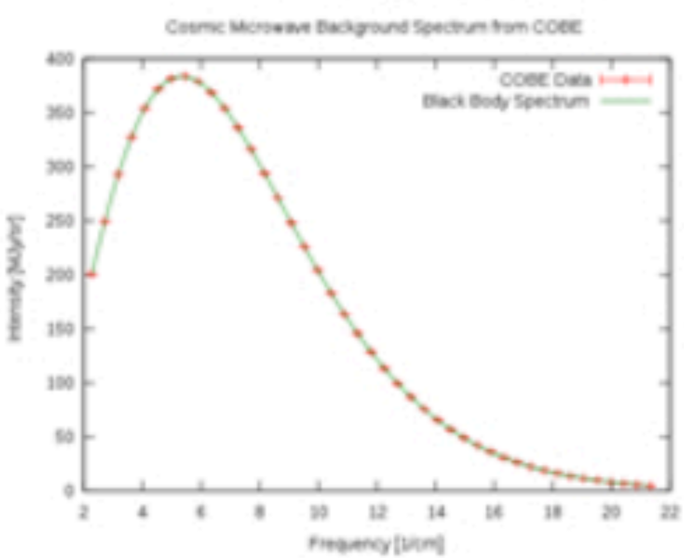
- L-class ESA mission
- White paper, May 24th, 2013
- Imager:
  - polarization sensitive
  - 3.5m telescope [arcmin resolution at highest frequencies]
  - 30GHz-6THz [30 broad ($\Delta\nu/\nu$~25%) and 300 narrow ($\Delta\nu/\nu$~2.5%) bands]
- Spectrometer:
  - FTS similar to PIXIE
  - 30GHz-6THz ($\Delta\nu$~15 & 0.5 GHz)

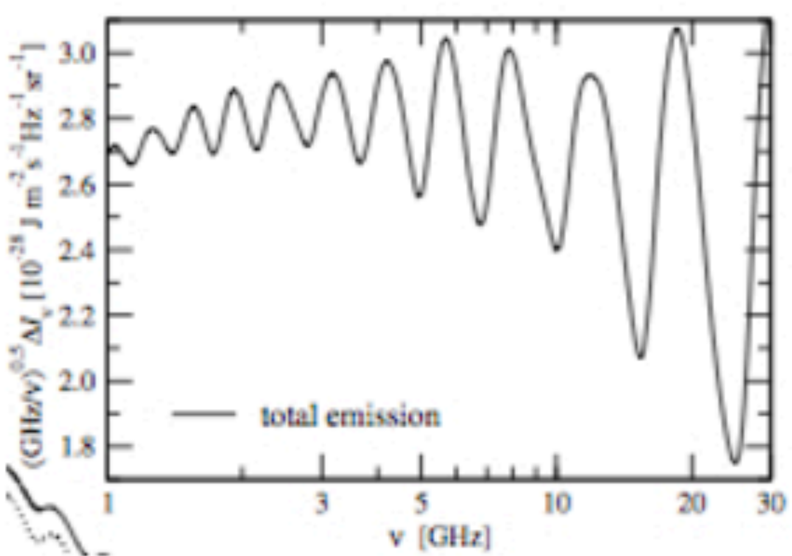
Some of the science goals:

- B-mode polarization from inflation ($r \approx 5\times10^{-4}$)
- count all SZ clusters $>10^{14}$ M_{sun}
- CIB/large scale structure
- Galactic science
- *CMB spectral distortions*

More info at: http://www.prism-mission.org/

Array of Precision Spectrometers for detecting spectral ripples from the Epoch of RecombinAtion

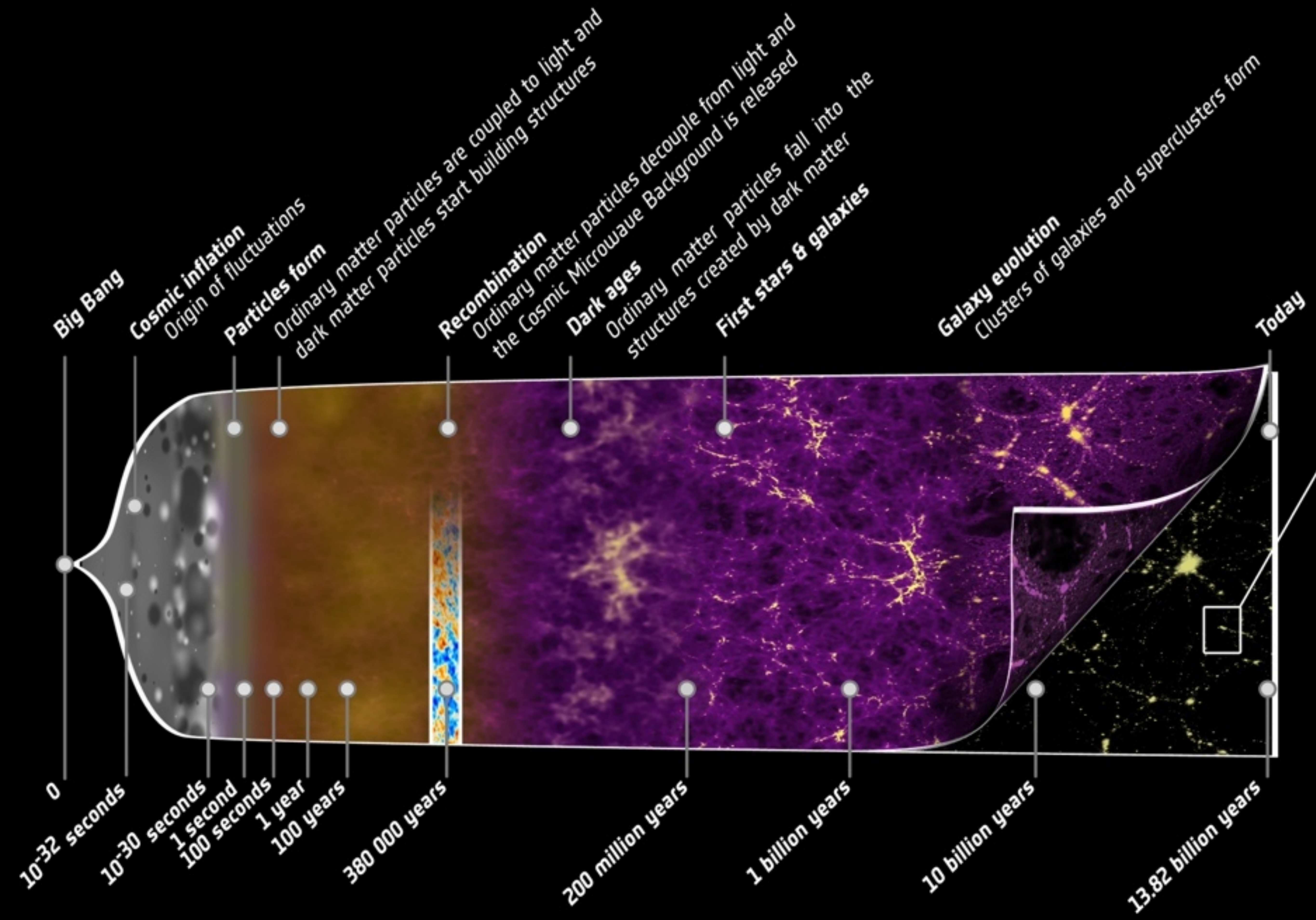




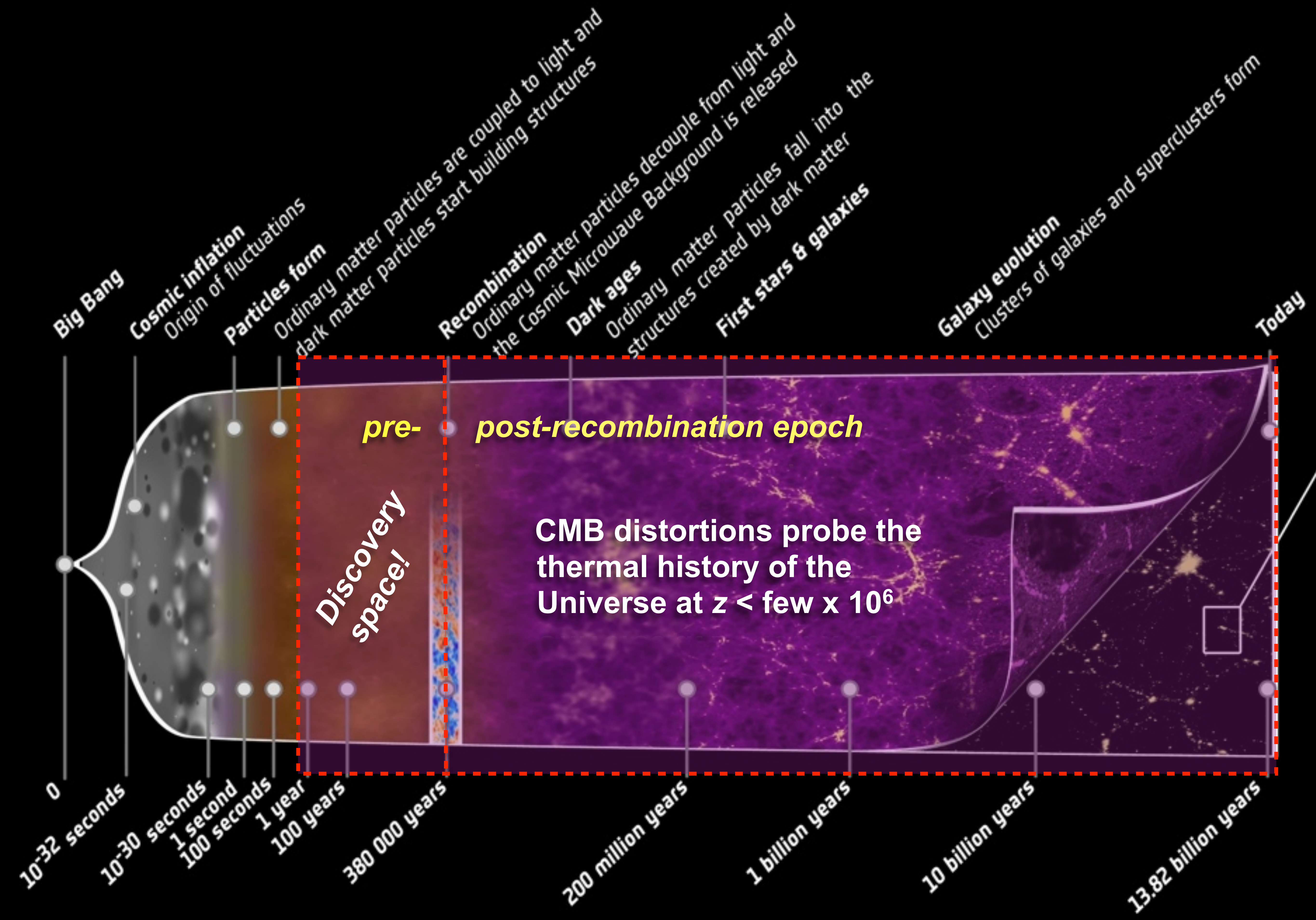
About APSERa

The Array of Precision Spectrometers for the Epoch of RecombinAtion - APSERa - is a venture to detect recombination lines from the Epoch of Cosmological Recombination. These are predicted to manifest as 'ripples' in wideband spectra of the cosmic radio background (CRB) since recombination of the primeval plasma in the early Universe adds broad spectral lines to the relic Cosmic Radiation. The lines are extremely wide because recombination is stalled and extended over redshift space. The spectral features are expected to be isotropic over the whole sky.

The project will comprise of an array of 128 small telescopes that are purpose built to detect a set of adjacent lines from cosmological recombination in the spectrum of the radio sky in the 2-6 GHz range. The radio receivers are being designed and built at the Raman Research Institute, tested in nearby radio-quiet locations and relocated to a remote site for long duration exposures to detect the subtle features in the cosmic radio background arising from recombination. The observing site would be appropriately chosen to minimize RFI from geostationary satellites and to be able to observe towards sky regions relatively low in foreground brightness.



Big Bang
Cosmic inflation
Origin of fluctuations
Particles form
Ordinary matter particles are coupled to light and
dark matter particles start building structures
Recombination
Ordinary matter particles decouple from light and
the Cosmic Microwave Background is released
Dark ages
Ordinary matter particles fall into the
structures created by dark matter
First stars & galaxies
Galaxy evolution
Clusters of galaxies and superclusters form
Today
0
10-32 seconds
10-30 seconds
1 second
100 seconds
1 year
100 years
380 000 years
200 million years
1 billion years
10 billion years
13.82 billion years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Today
pre-
post-recombination epoch
Discovery space!
CMB distortions probe the thermal history of the Universe at z < few x 10^6
0
10^-32 seconds
10^-30 seconds
1 second
100 seconds
1 year
100 years
380 000 years
200 million years
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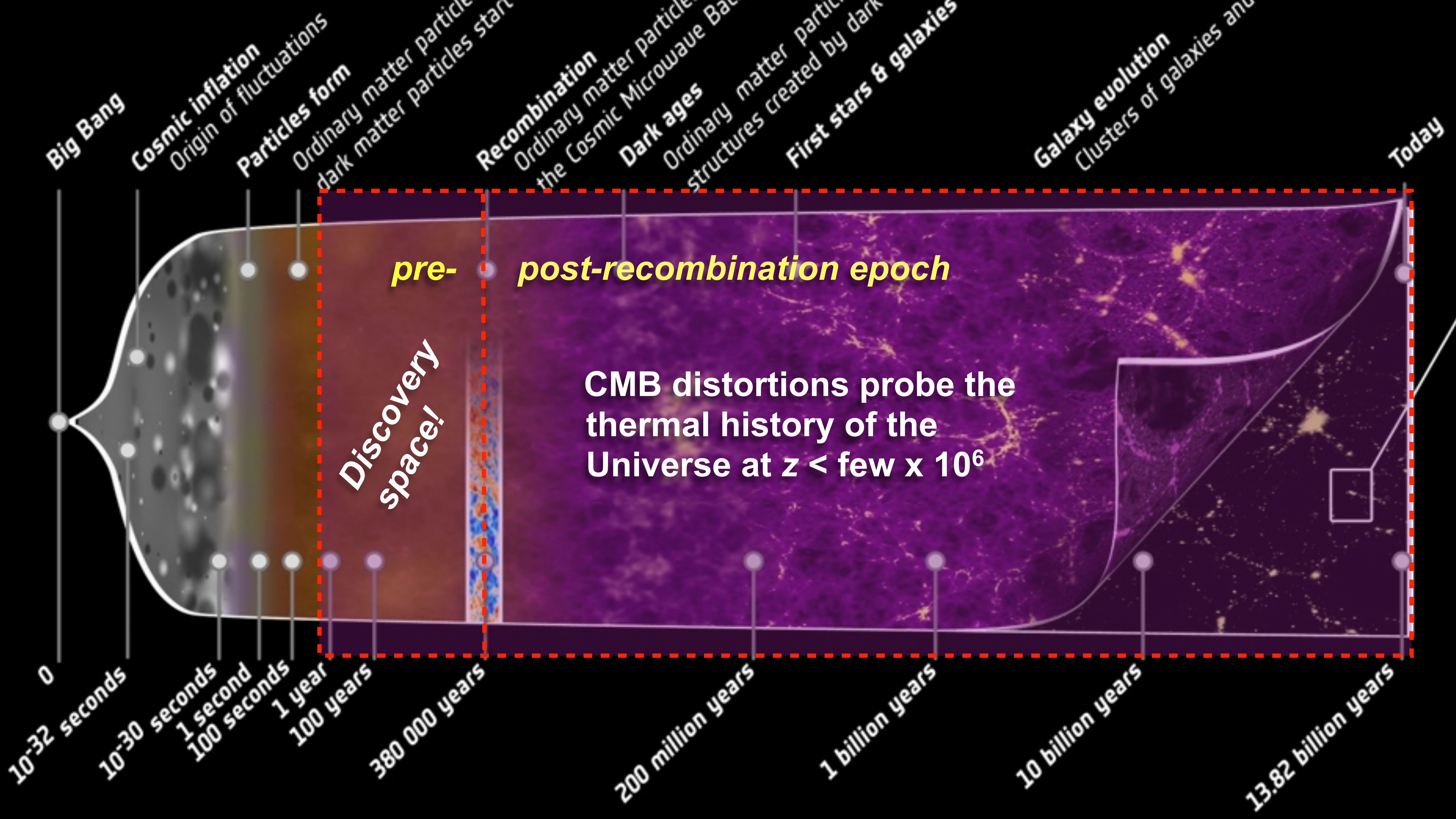
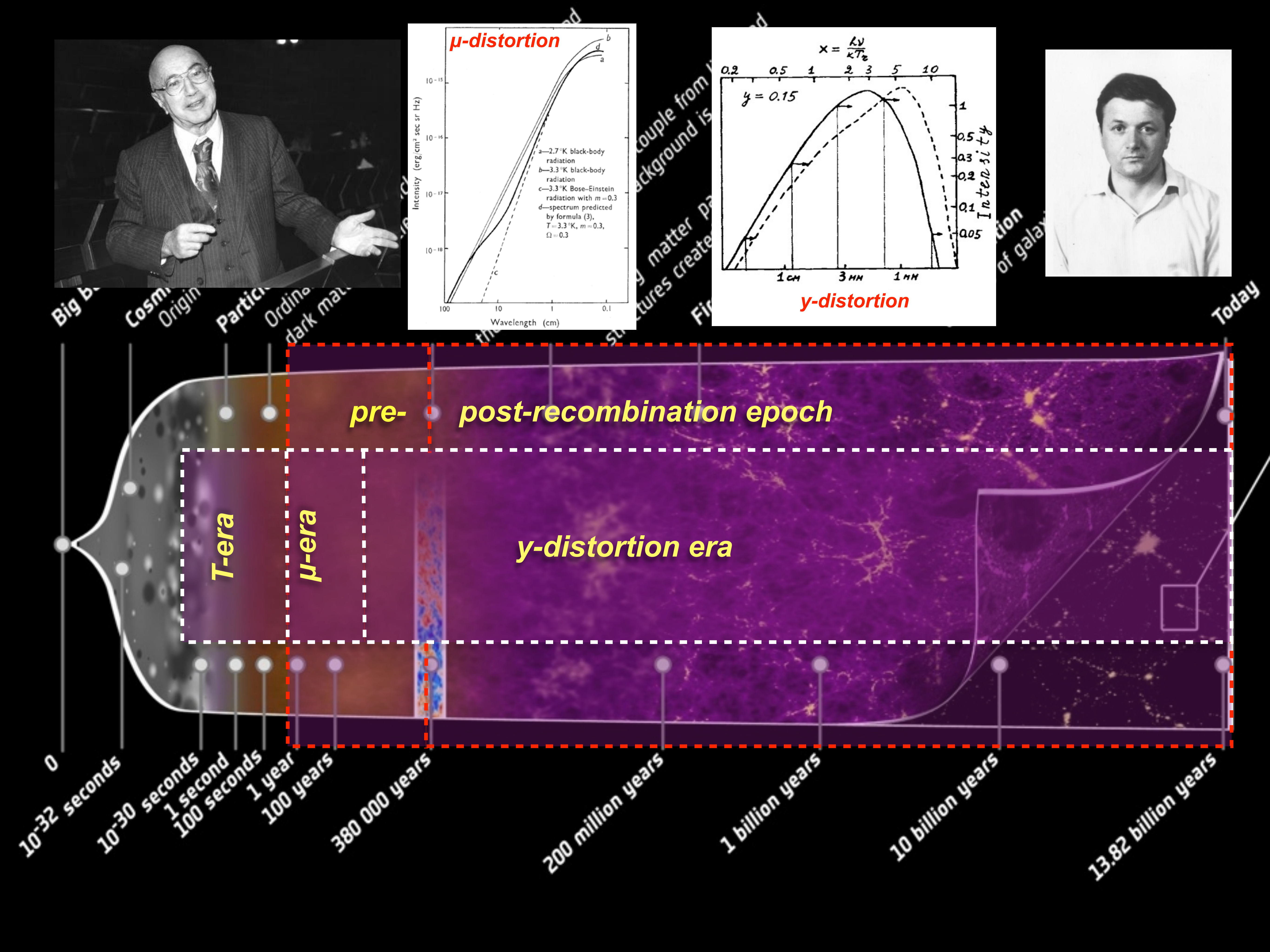
Measurements of CMB spectrum will open a new unexplored window to the early Universe!





μ-distortion
Intensity (erg/cm² sec sr Hz)
Wavelength (cm)
a—2.7°K black-body radiation
b—3.3°K black-body radiation
c—3.3°K Bose–Einstein radiation with m = 0.3
d—spectrum predicted by formula (3), T = 3.3°K, m = 0.3, Ω = 0.3
x = hν/kT_e
y = 0.15
Intensity
1 cm
3 mm
1 mm
y-distortion
pre-
post-recombination epoch
T-era
μ-era
y-distortion era
Today
0
10-32 seconds
10-30 seconds
1 second
100 seconds
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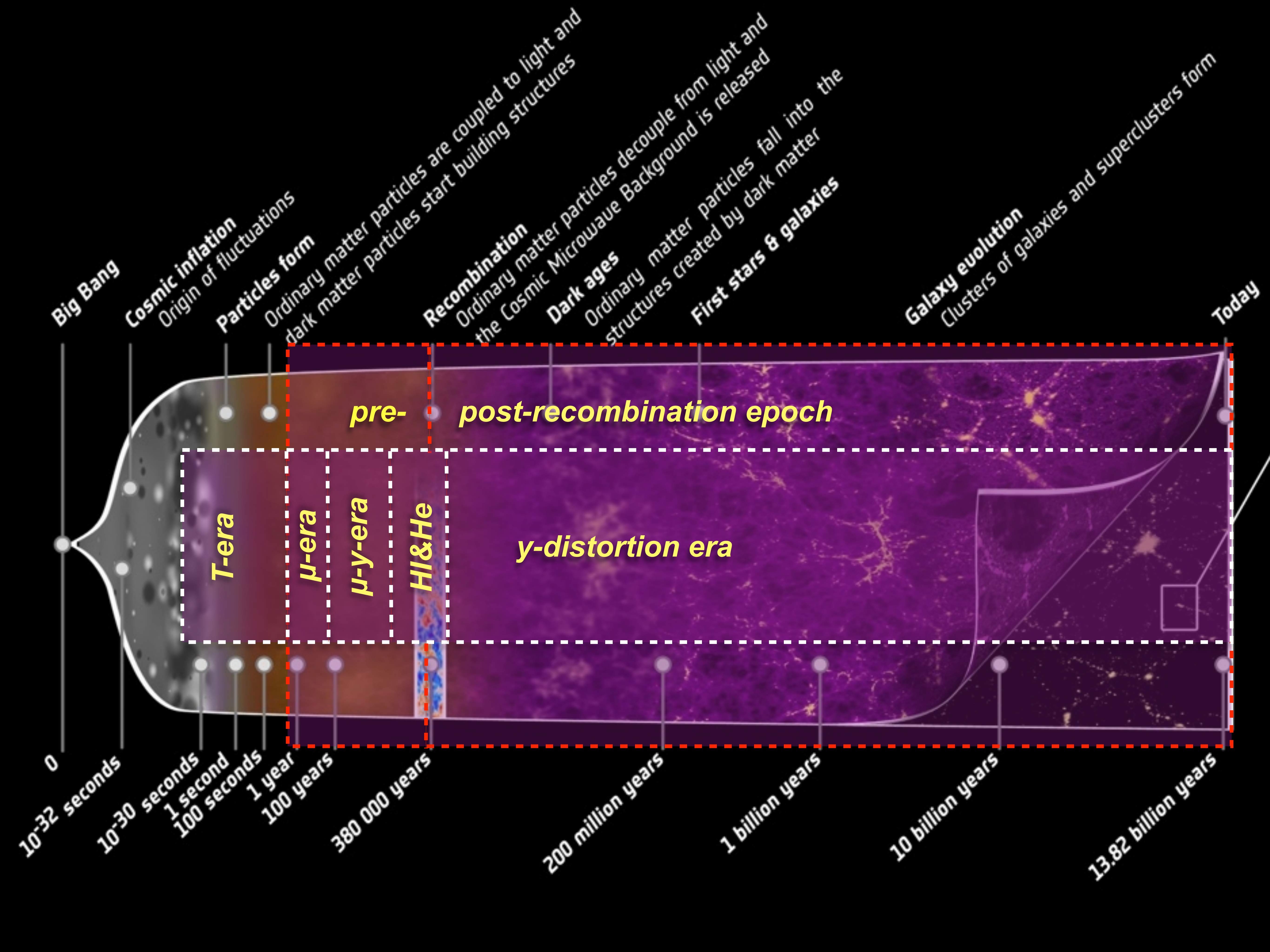

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0
10-32 seconds
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100 seconds
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200 million years
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10 billion years
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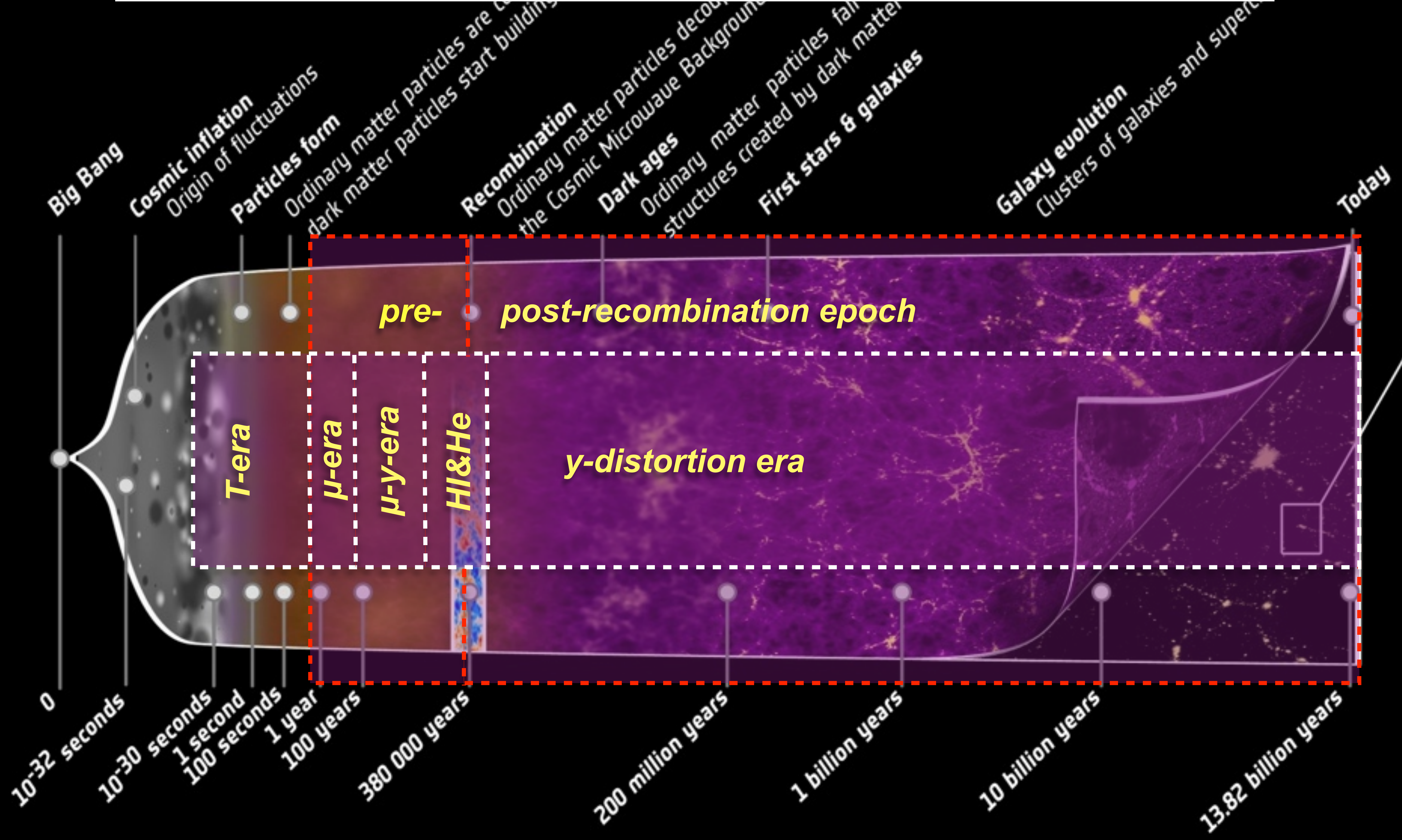
CMB spectrum adds another dimension to the problem!



How does the thermalization process work?

Some important ingredients

- Plasma fully ionized before recombination (z~1000)
  - → free electrons, protons and helium nuclei
  - → photon dominated (~2 Billion photons per baryon)
- Coulomb scattering $e + p \leftrightarrow e' + p$
  - → electrons in full thermal equilibrium with baryons
  - → electrons follow thermal Maxwell-Boltzmann distribution
  - → efficient down to very low redshifts (z ~ 10-100)
- Medium homogeneous and isotropic on large scales
  - → thermalization problem rather simple!
  - → in principle *allows very precise computations*
- Hubble expansion
  - → adiabatic cooling of photons [$T_\gamma \sim (1+z)$] and ordinary matter [$T_m \sim (1+z)^2$]
  - → redshifting of photons

Blackbody Spectrum and Formula

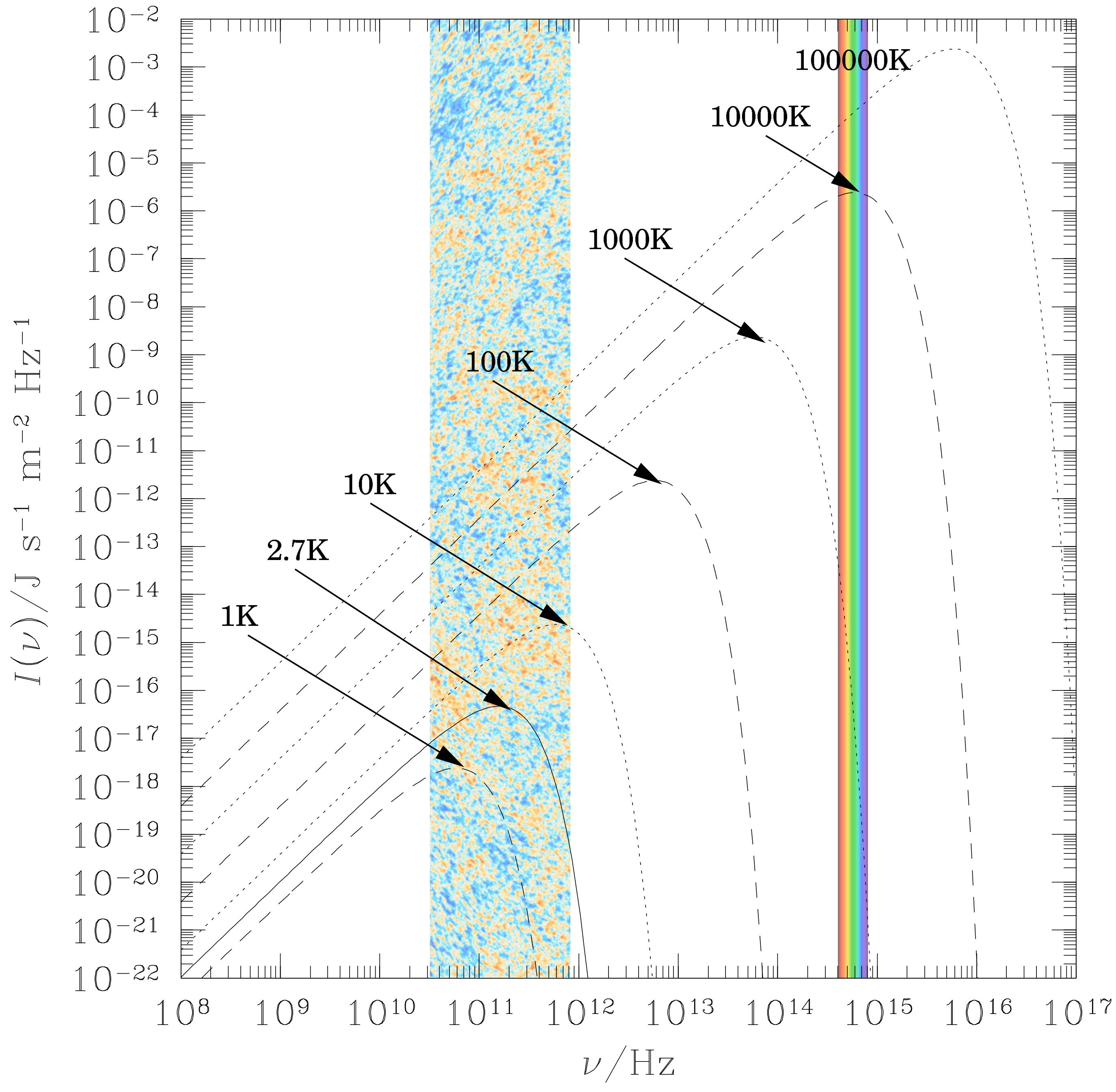


Figure 2.2: Blackbody spectrum for different temperatures. The intensity maximum is roughly at $\nu_{\max} \approx 58.8\,\mathrm{GHz\,K^{-1}}\,T$, which for the CMB blackbody today is $\nu_{\max} \simeq 160\,\mathrm{GHz}$ or at 2 mm wavelength. For $T \simeq 10^4\,\mathrm{K}$ the intensity maximum is in the visible part of the electromagnetic spectrum.

Intensity

$$B_\nu = \frac{2h\nu^3}{c^2}\frac{1}{e^{h\nu/kT}-1}$$

Occupation number

$$n_\nu^{\mathrm{bb}} = \frac{1}{e^{h\nu/kT}-1}$$

Frequency variable

$$x = \frac{h\nu}{kT}$$

Spectrum of a Temperature Shift

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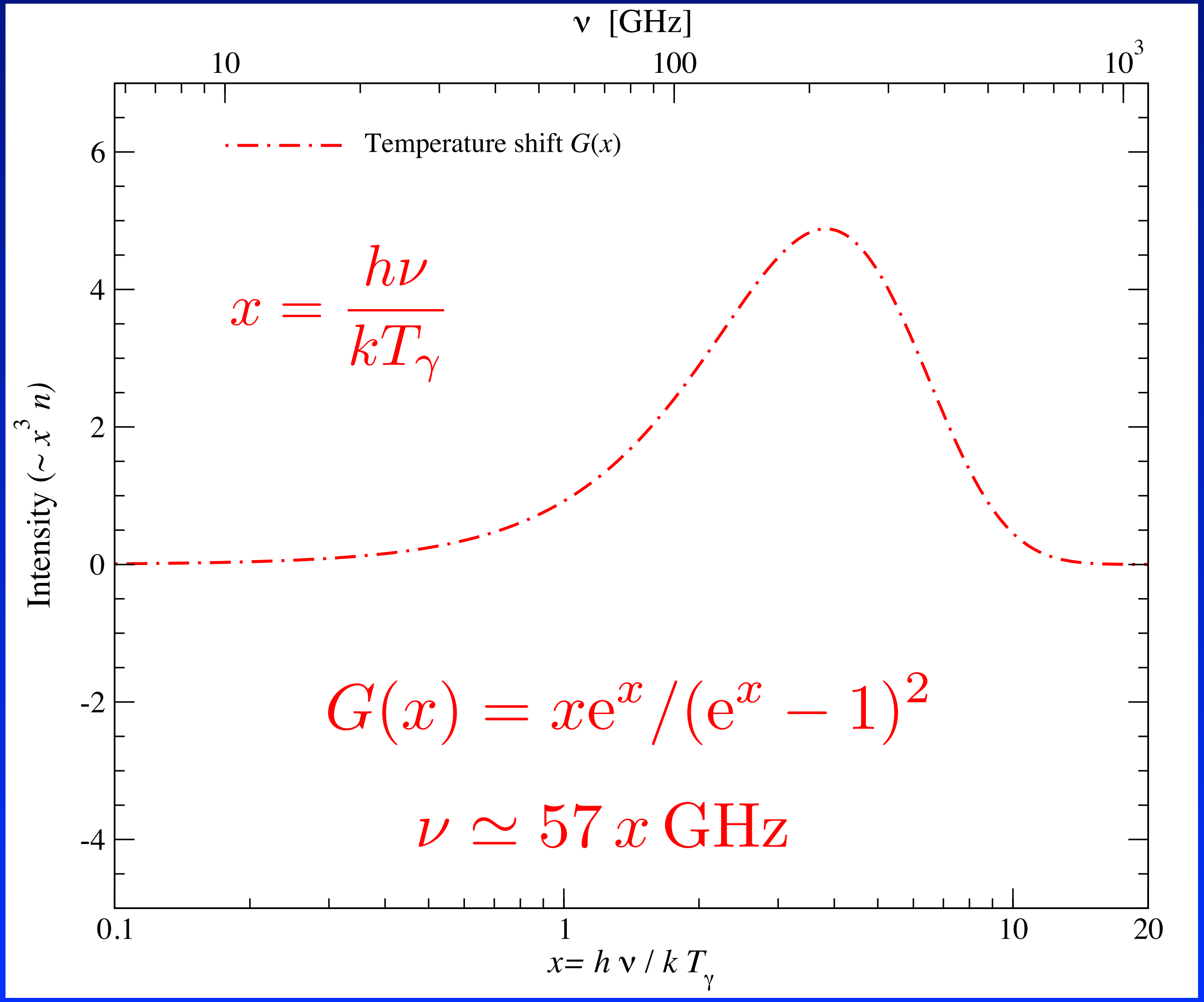

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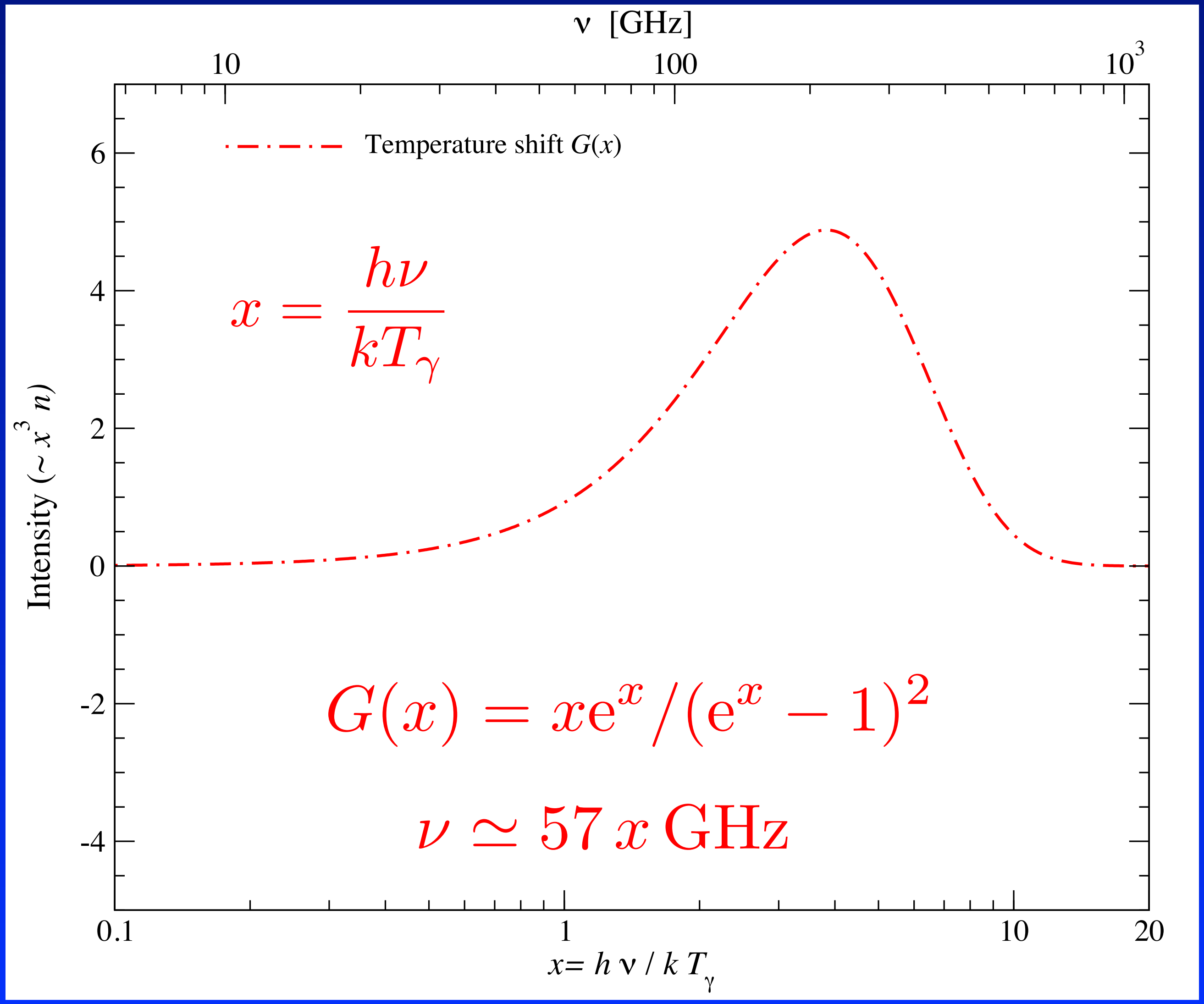

- Adiabatic condition

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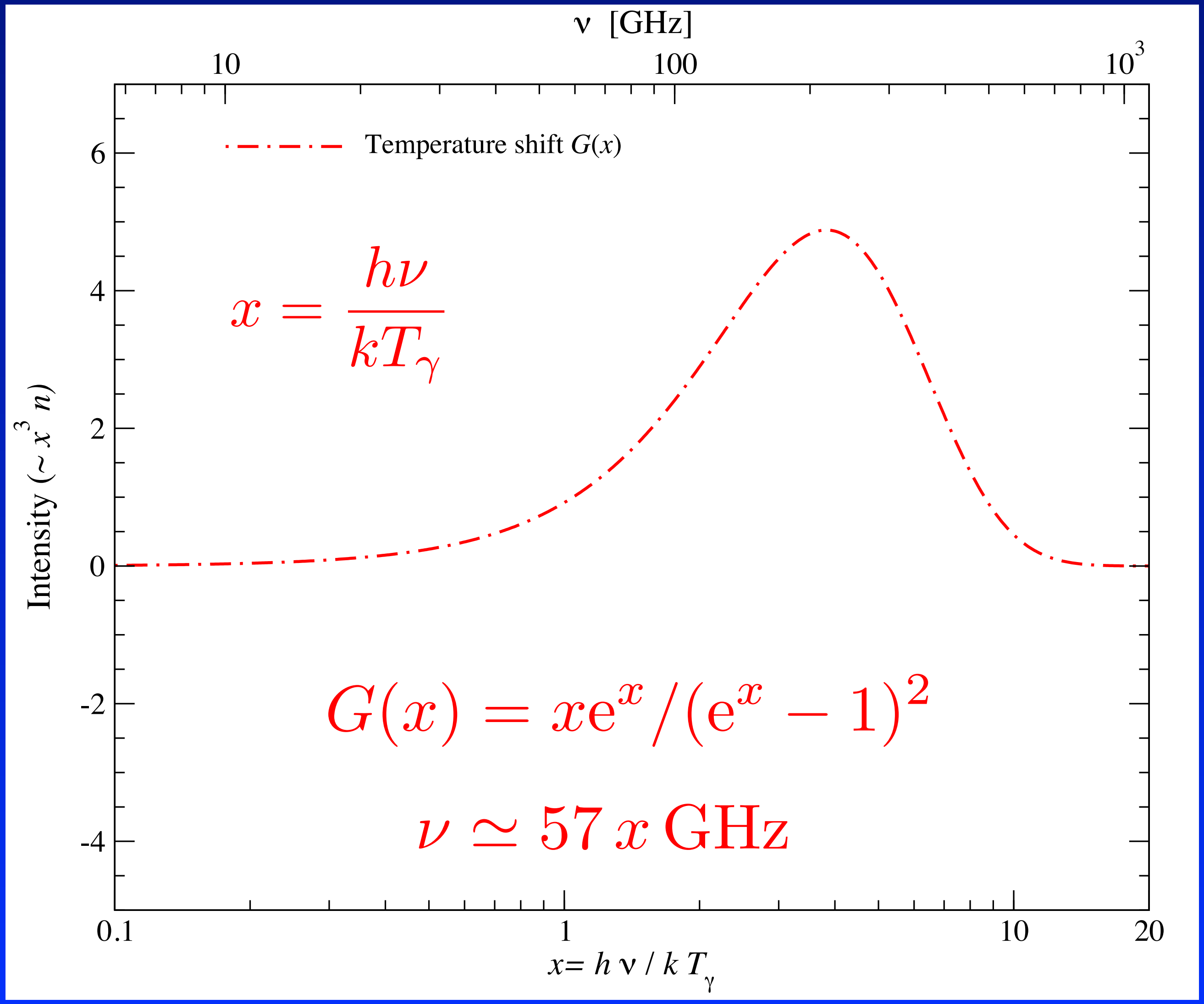

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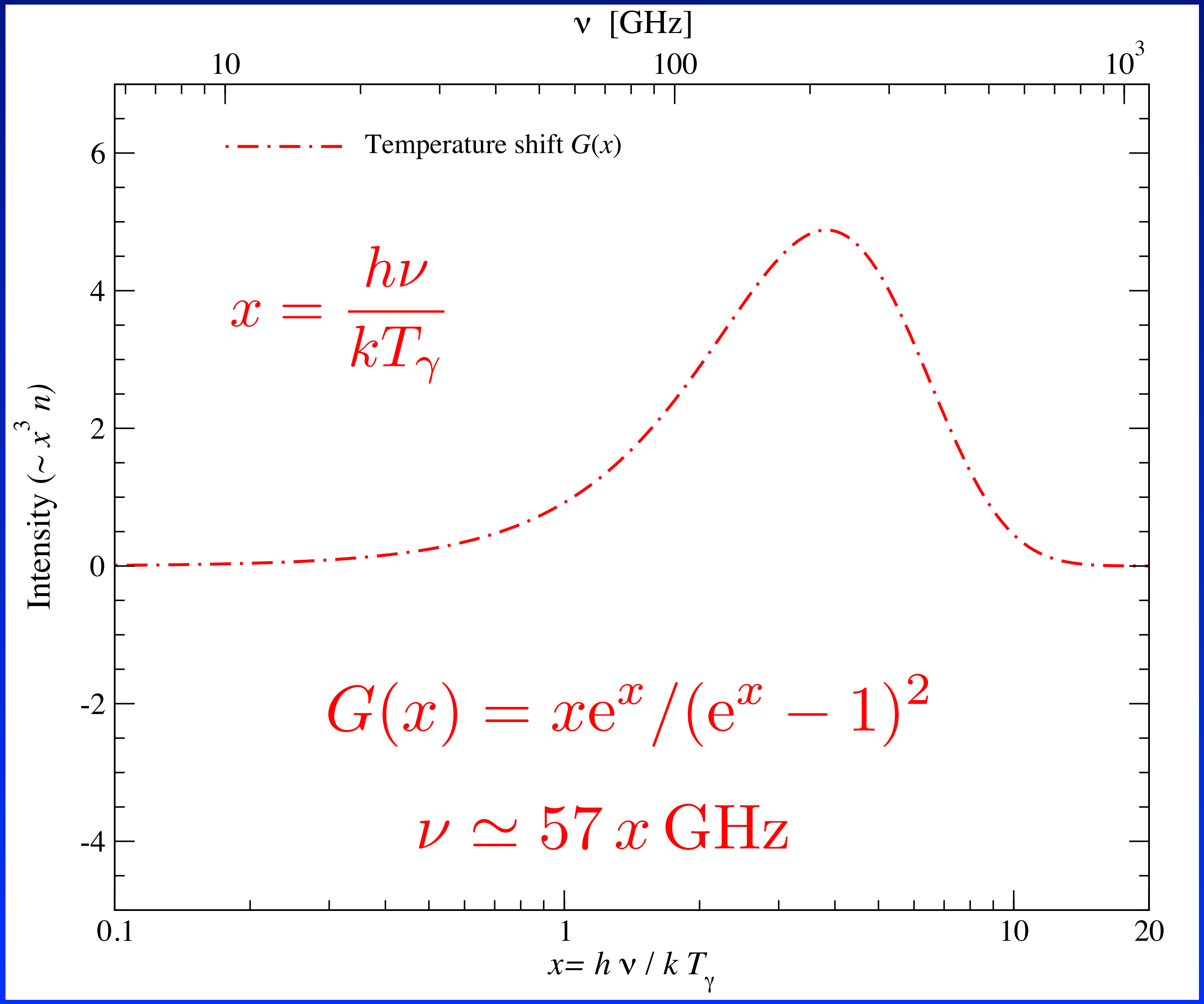

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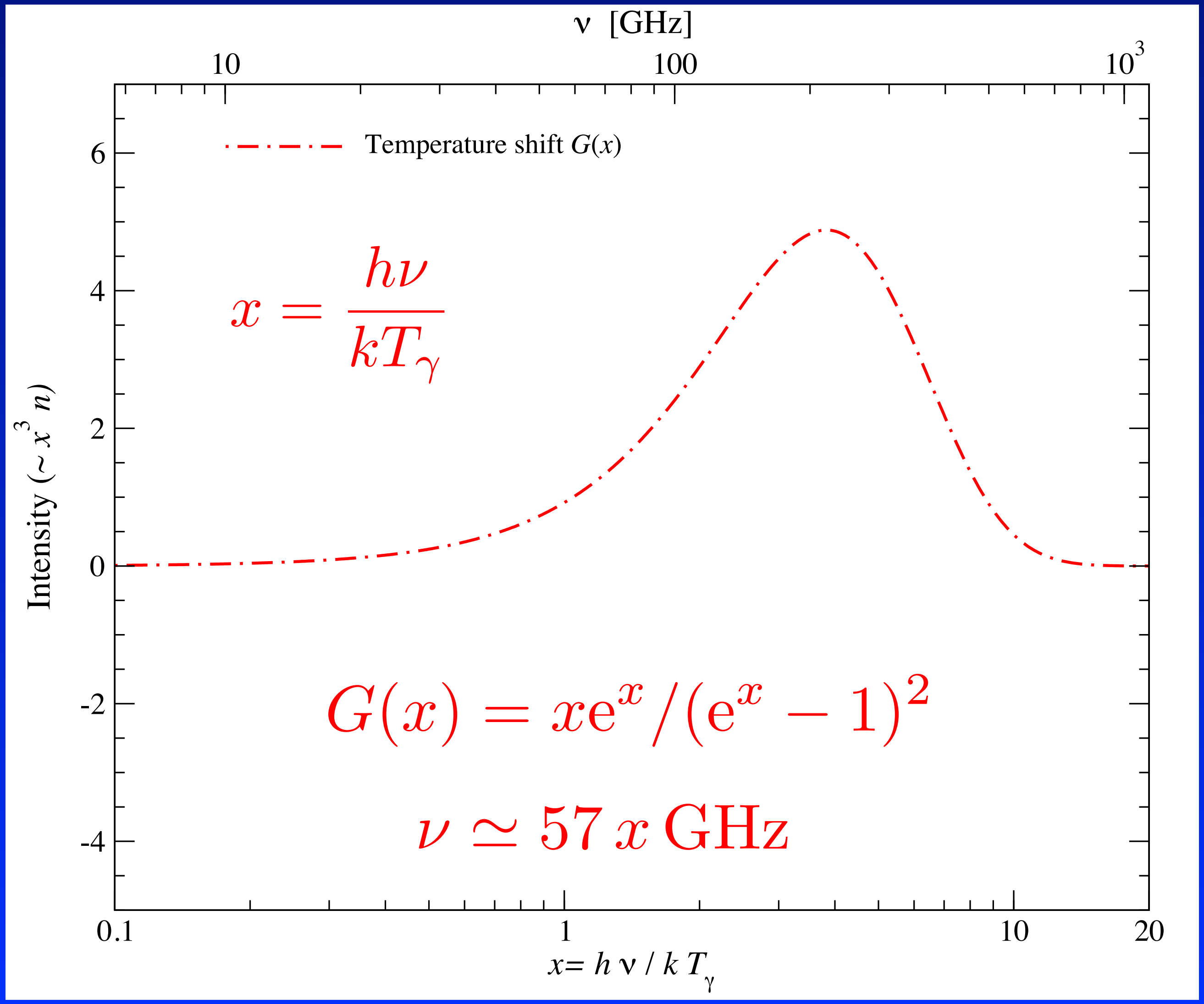

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- *But: Monopole temperature* does not really teach you anything

(• Only lowest order in Δ*T*/*T* → *superposition of blackbodies*)

Photon Boltzmann Equation for Average Spectrum

$$\frac{\mathrm{d}n_\nu}{\mathrm{d}t} = \frac{\partial n_\nu}{\partial t} + \frac{\partial n_\nu}{\partial x_i} \cdot \frac{\mathrm{d}x_i}{\mathrm{d}t} + \frac{\partial n_\nu}{\partial p} \frac{\mathrm{d}p}{\mathrm{d}t} + \frac{\partial n_\nu}{\partial \hat{p}_i} \cdot \frac{\mathrm{d}\hat{p}_i}{\mathrm{d}t} = \mathcal{C}[n]$$

Photon occupation number

Liouville operator

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redshifting term

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Compton Scattering Bremsstrahlung Double Compton

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Bremsstrahlung

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redistribution of photon over frequency

adjusting photon number

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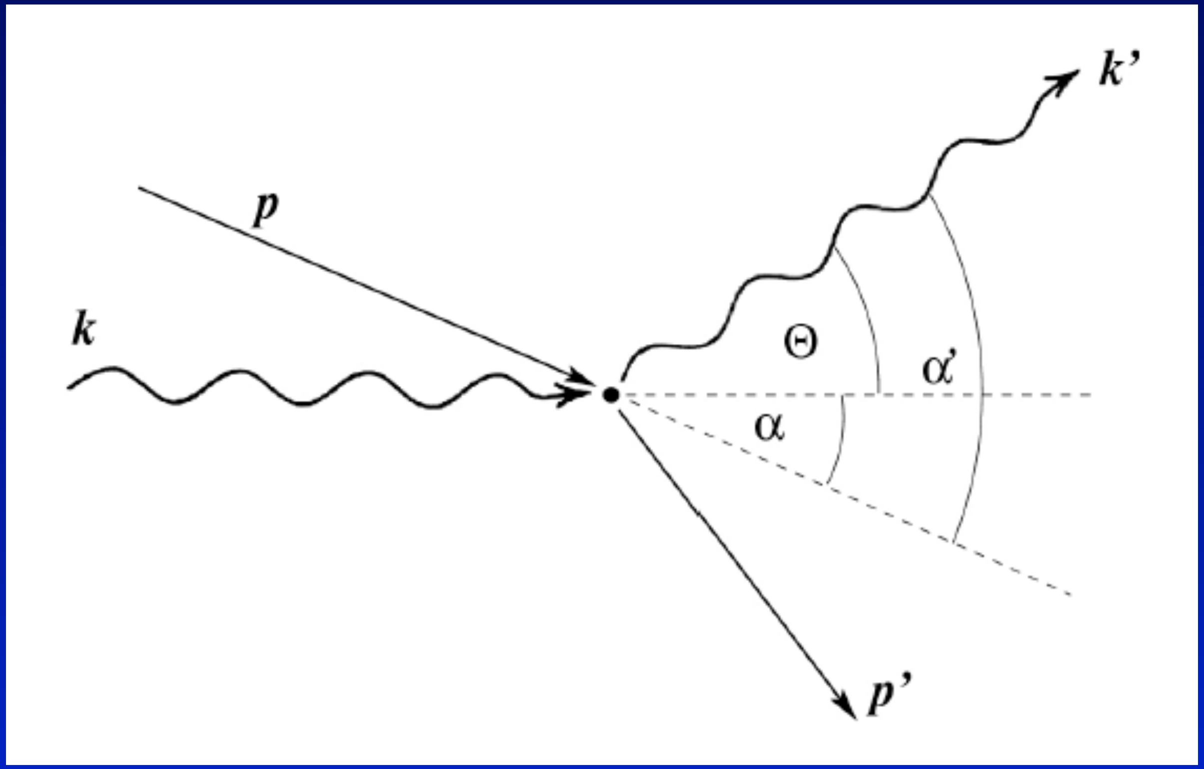
- Energy release: $\mathcal{C}[n] \neq 0 \;\Rightarrow$ *thermalization process starts*

Compton scattering

Redistribution of photons by Compton scattering

- Reaction: $\gamma + e \longleftrightarrow \gamma' + e'$



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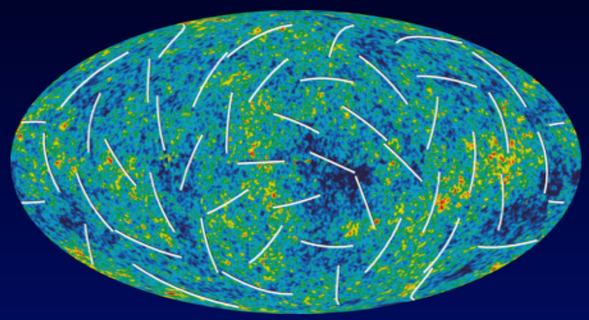

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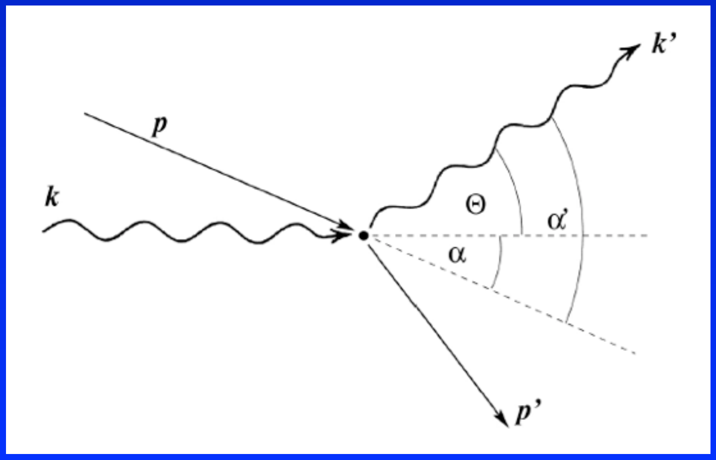
→ *no energy exchange ⇒ Thomson limit*

⇒ important for anisotropies

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \frac{3\sigma_{\mathrm{T}}}{16\pi}\left[1 + (\hat{\gamma}\cdot\hat{\gamma}')^2\right]$$



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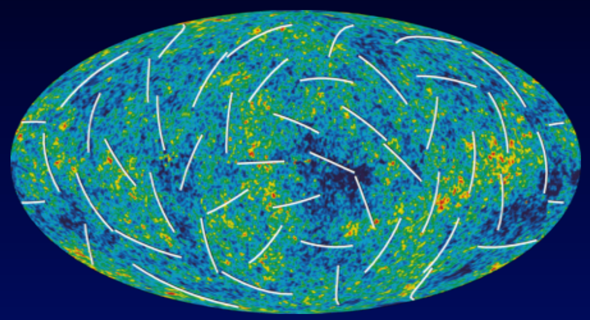

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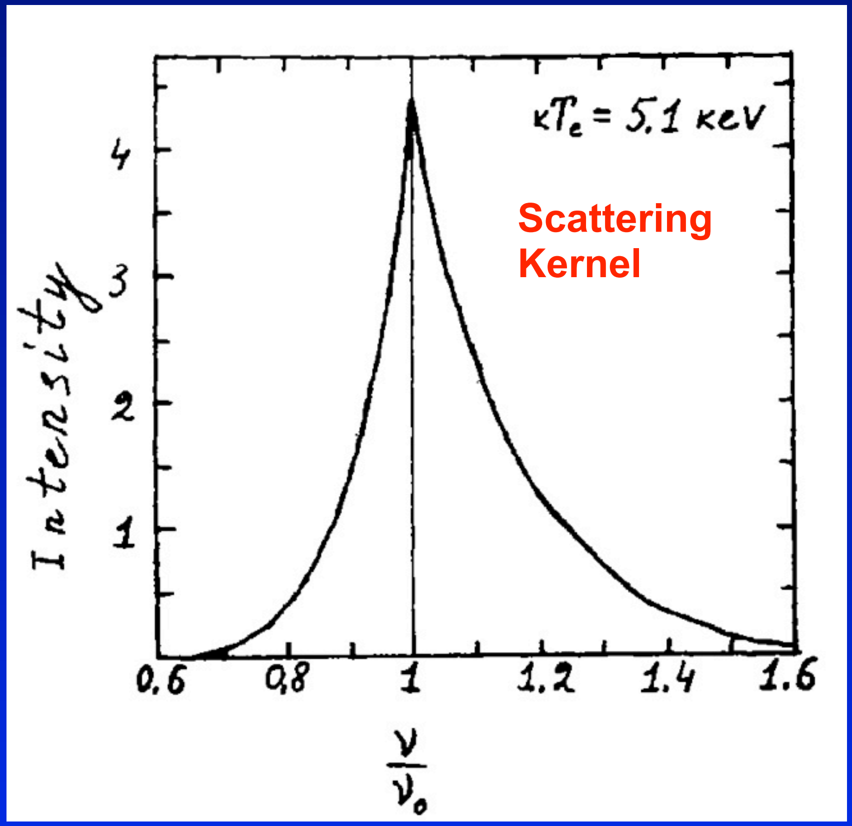
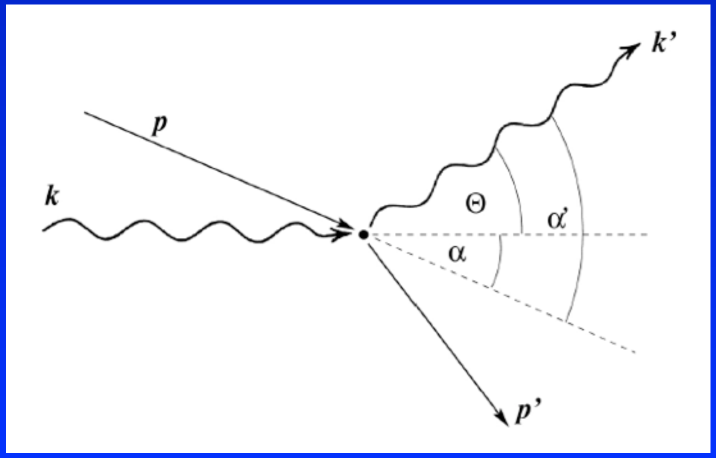
→ *energy exchange included*

- up-scattering due to the ***Doppler*** effect for $h\nu < 4kT_{\mathrm{e}}$
- down-scattering because of ***recoil*** (and stimulated recoil) for $h\nu > 4kT_{\mathrm{e}}$
- ***Doppler*** broadening $\frac{\Delta\nu}{\nu} \simeq \sqrt{\frac{2kT_{\mathrm{e}}}{m_{\mathrm{e}}c^2}}$

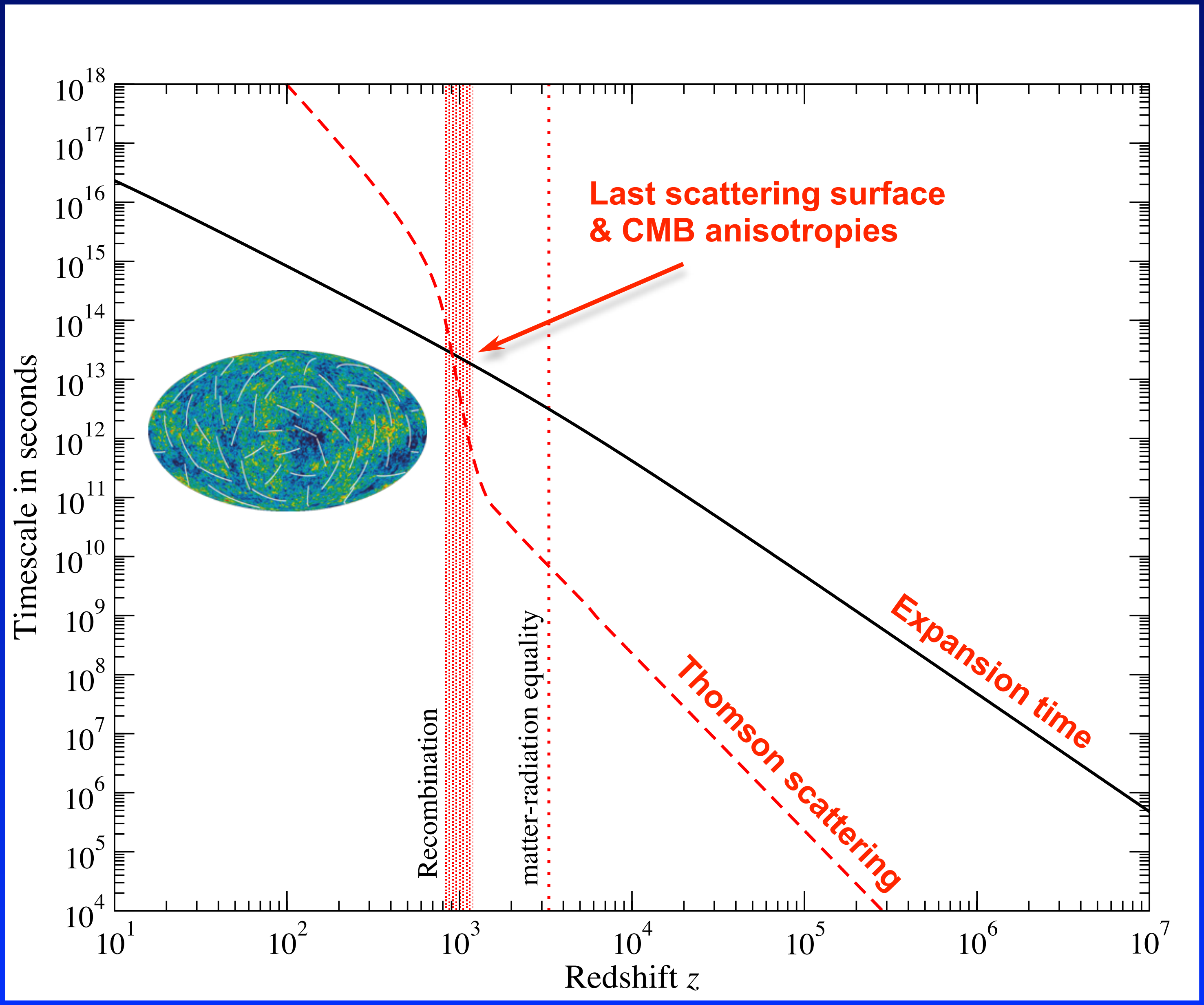


Sunyaev & Zeldovich, 1980, ARAA, 18, 537



Important Time-scales for Compton Process

- *Thomson scattering* $t_{\rm C} = (\sigma_{\rm T} N_{\rm e} c)^{-1} \approx 2.3 \times 10^{20}\, \chi_{\rm e}^{-1} (1+z)^{-3}\,\rm sec$



Radiation dominated

$$t_{\rm exp} = H^{-1} \simeq 4.8 \times 10^{19} (1+z)^{-2}\,\rm sec$$
$$\simeq 8.4 \times 10^{17} (1+z)^{-3/2}\,\rm sec$$

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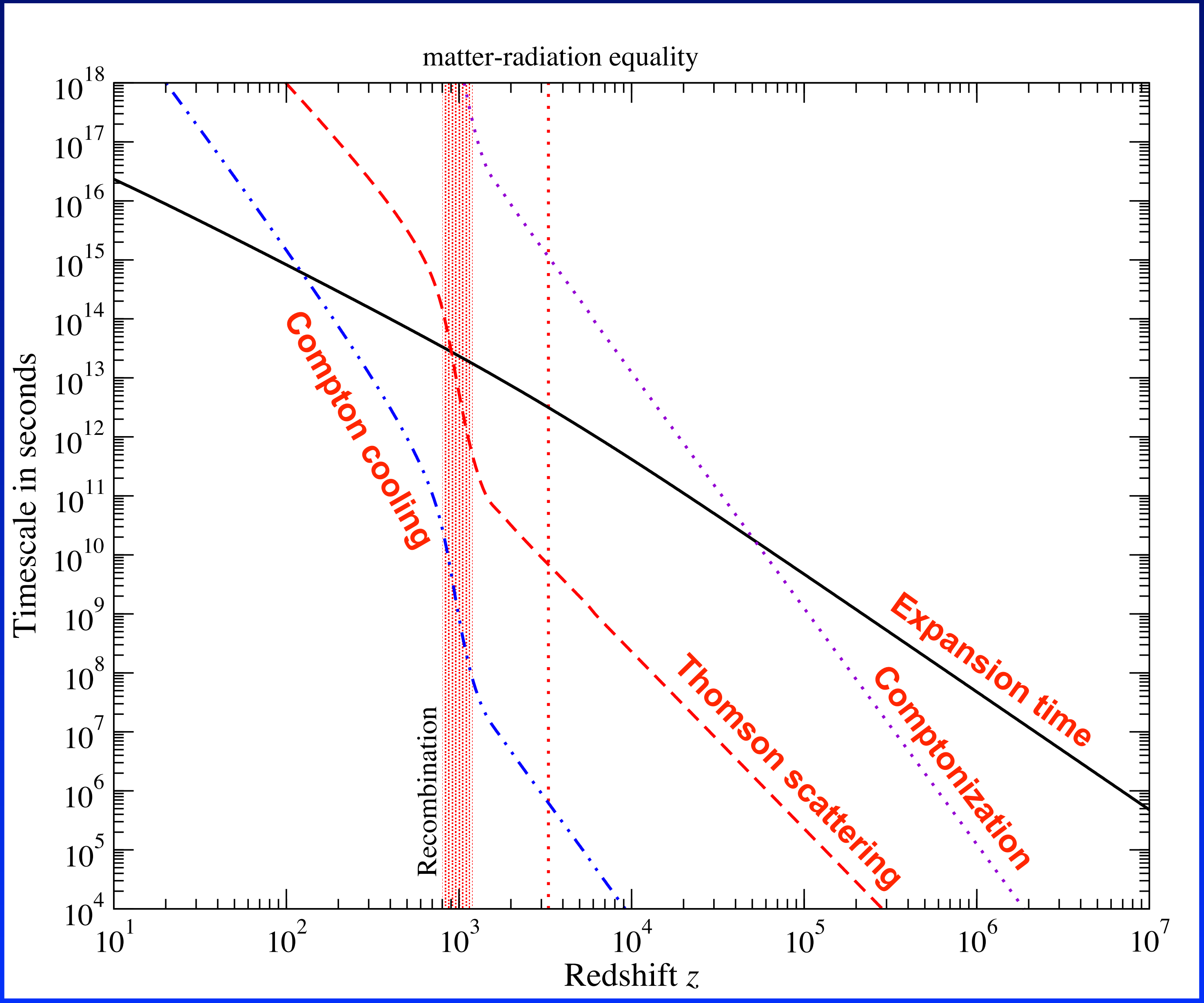

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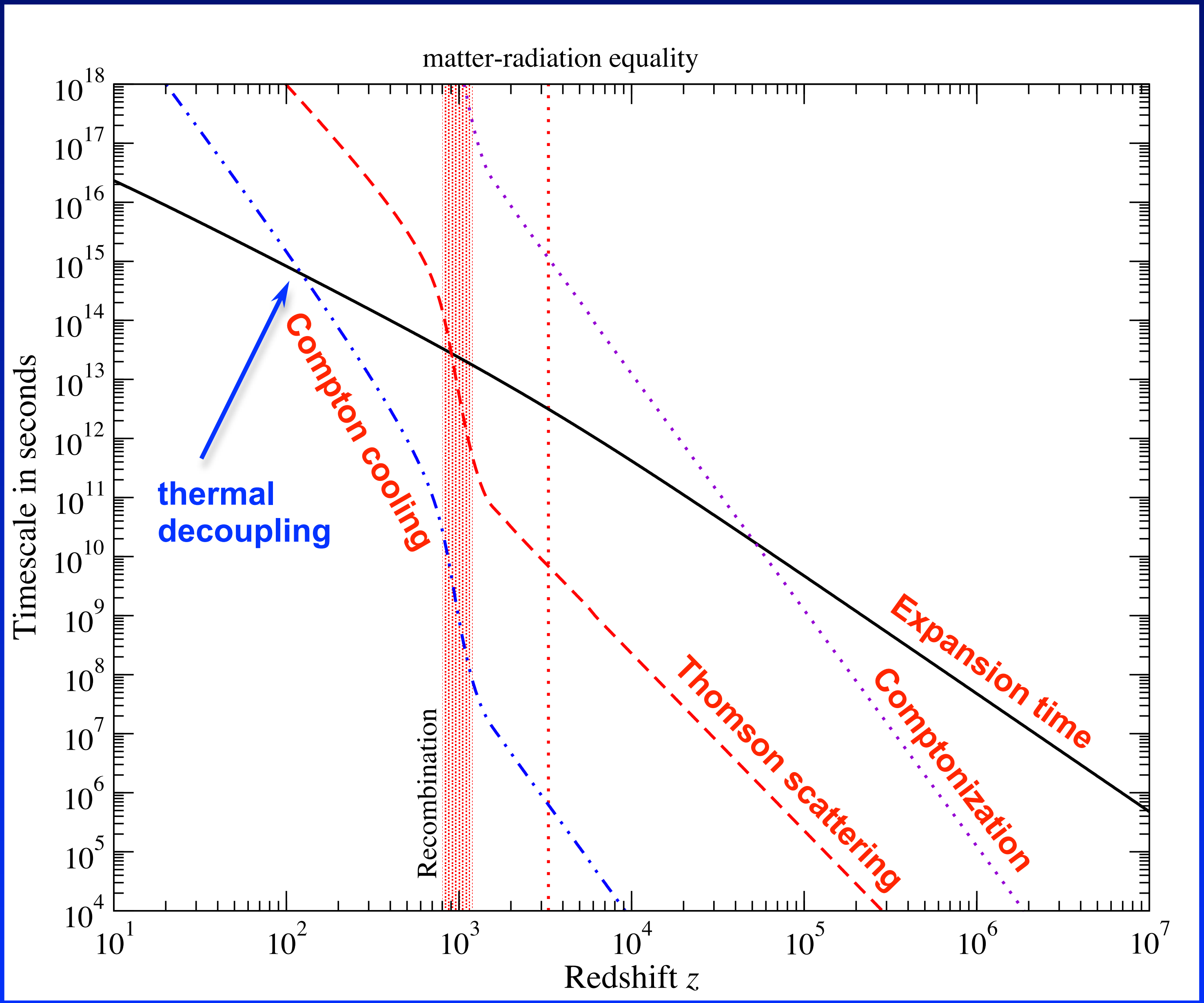

- *matter temperature starts deviating from Compton equilibrium temperature at z ≲ 100-200*

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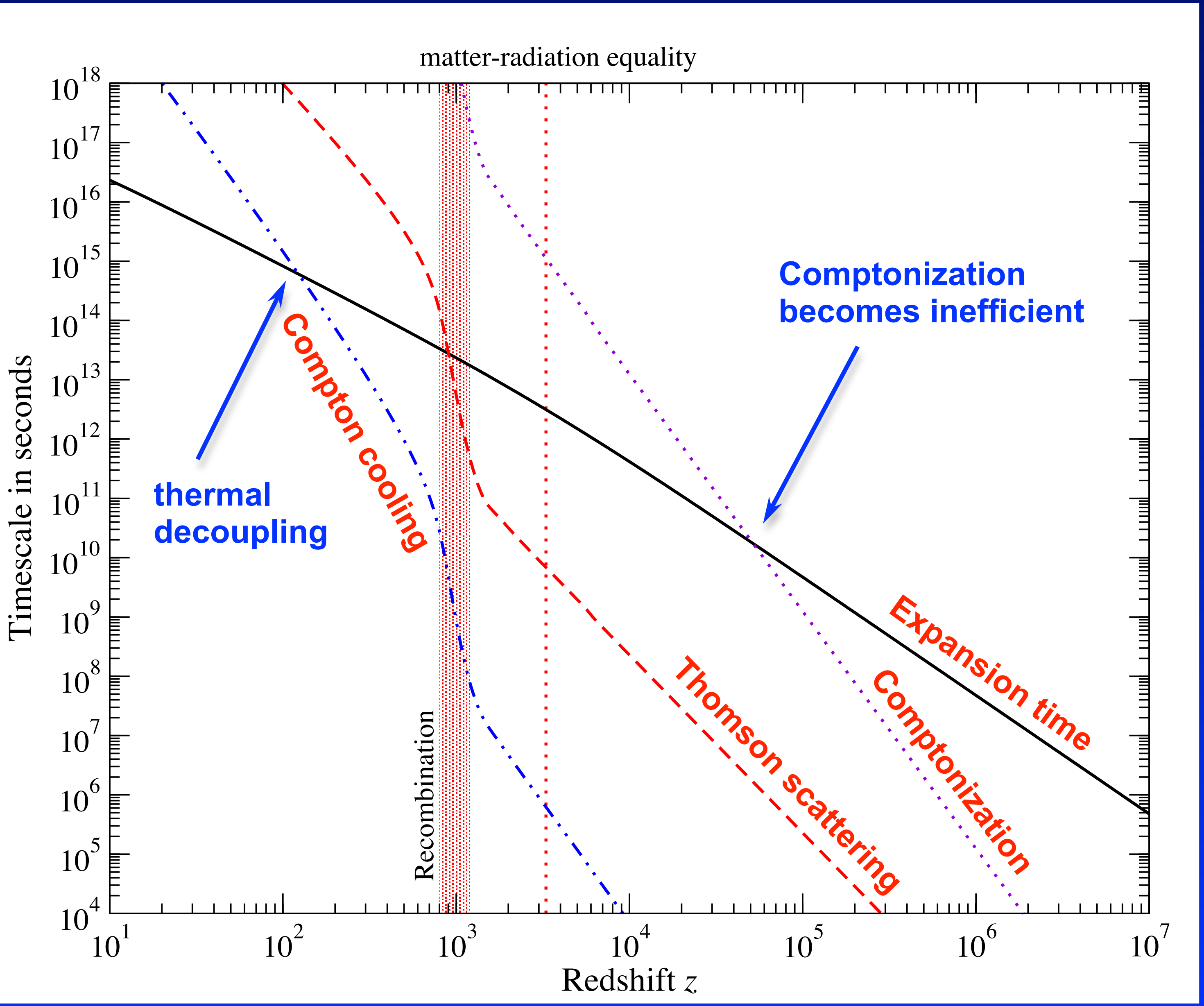

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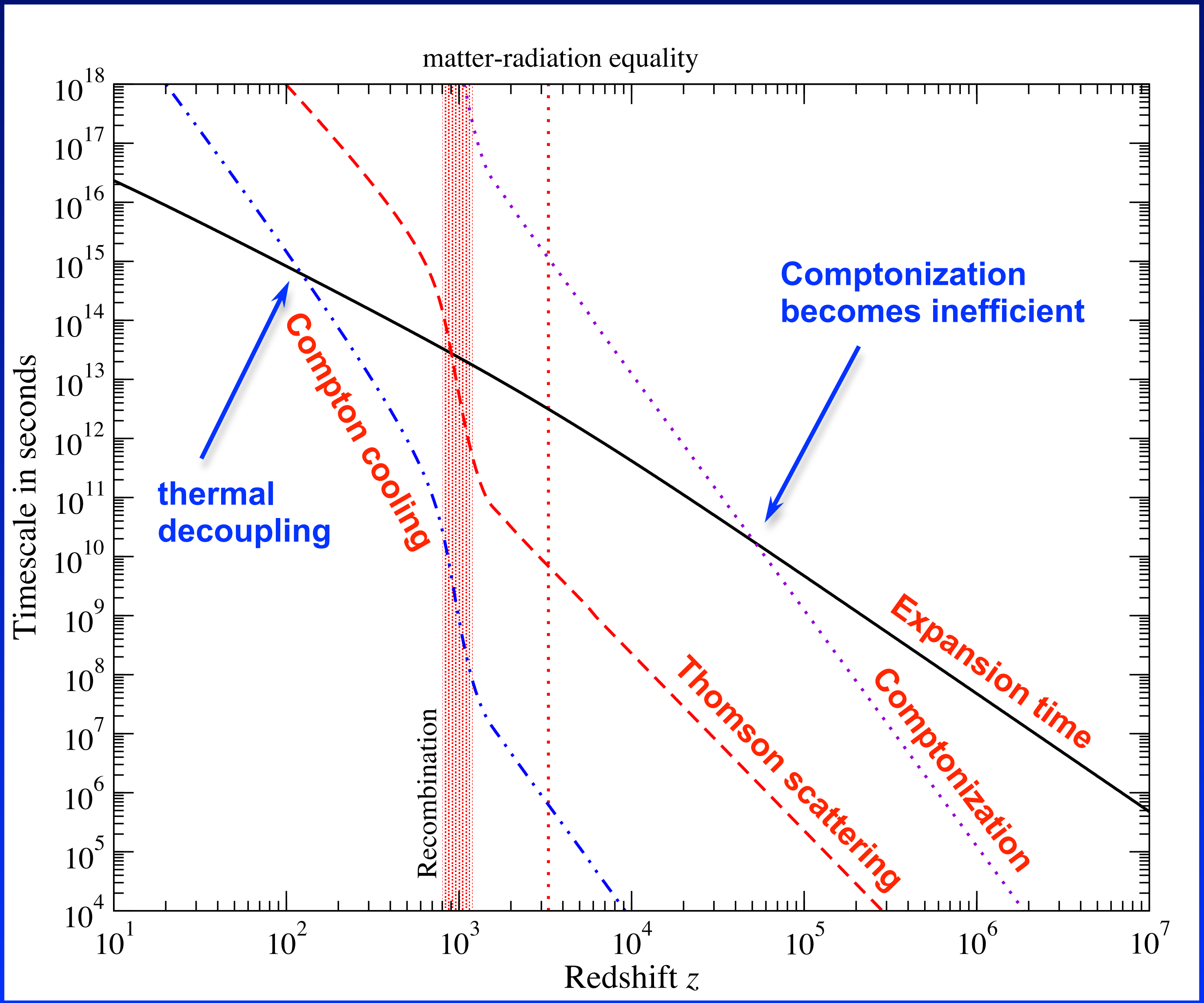

- *matter temperature starts deviating from Compton equilibrium temperature at $z \lesssim$ 100-200*
- *Comptonization becomes inefficient at $z_{\rm K} \simeq 50000$*

⇒ character of distortion should change around $z_{\rm K}$!

$$t_{\rm exp} = H^{-1} \simeq 4.8 \times 10^{19} (1+z)^{-2}\,{\rm sec} \quad \text{Radiation dominated}$$

$$\simeq 8.4 \times 10^{17} (1+z)^{-3/2}\,{\rm sec} \quad \text{Matter dominated}$$

What are y- and μ-distortions?

Compton y-distortion / thermal SZ effect

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- *Kompaneets equation:* $$\left.\frac{\mathrm{d}n}{\mathrm{d}\tau}\right|_{\mathrm{C}} \approx \frac{\theta_{\mathrm{e}}}{x^2}\frac{\partial}{\partial x} x^4 \left[\frac{\partial n}{\partial x} + \frac{T_\gamma}{T_{\mathrm{e}}}\, n(1+n)\right]$$

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$$y = \int \frac{k[T_{\mathrm{e}} - T_\gamma]}{m_{\mathrm{e}}c^2}\,\sigma_{\mathrm{T}} N_{\mathrm{e}} c\,\mathrm{d}t$$

Compton y-parameter

Compton y-distortion / thermal SZ effect

- *Kompaneets equation:* $$\left.\frac{\mathrm{d}n}{\mathrm{d}\tau}\right|_{\mathrm{C}} \approx \frac{\theta_{\mathrm{e}}}{x^2}\frac{\partial}{\partial x}x^4\left[\frac{\partial n}{\partial x}+\frac{T_\gamma}{T_{\mathrm{e}}}n(1+n)\right]$$
- *insert:* $n \approx n^{\mathrm{bb}} = 1/(\mathrm{e}^x-1) \implies \Delta n \approx y\,Y(x)$ *for* $y \ll 1$

$$y = \int \frac{k[T_{\mathrm{e}}-T_\gamma]}{m_{\mathrm{e}}c^2}\sigma_{\mathrm{T}}N_{\mathrm{e}}c\,\mathrm{d}t \qquad Y(x) = \frac{x\mathrm{e}^x}{(\mathrm{e}^x-1)^2}\left[x\frac{\mathrm{e}^x+1}{\mathrm{e}^x-1}-4\right]$$

Compton y-parameter *spectrum of y-distortion*

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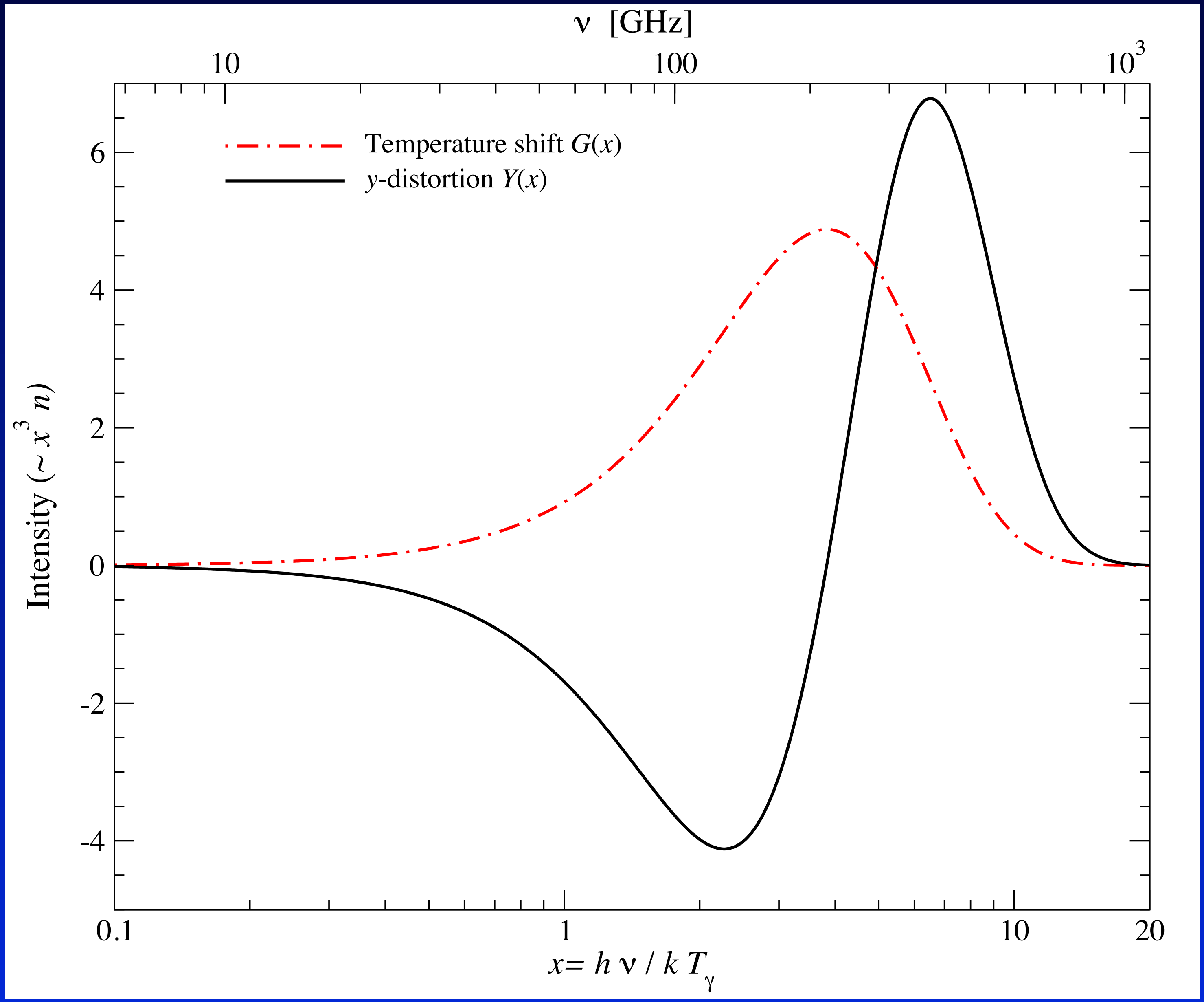
- *if* $T_{\mathrm{e}} > T_\gamma \quad \Longrightarrow$ *up-scattering of photons / cooling of electrons*
- *for* $T_{\mathrm{e}} \gg T_\gamma \quad \Longrightarrow$ *thermal Sunyaev-Zeldovich effect (up-scattering)*

Temperature shift ↔ y-distortion



$$Y(x) = \frac{x e^x}{(e^x - 1)^2}\left[x\frac{e^x + 1}{e^x - 1} - 4\right]$$

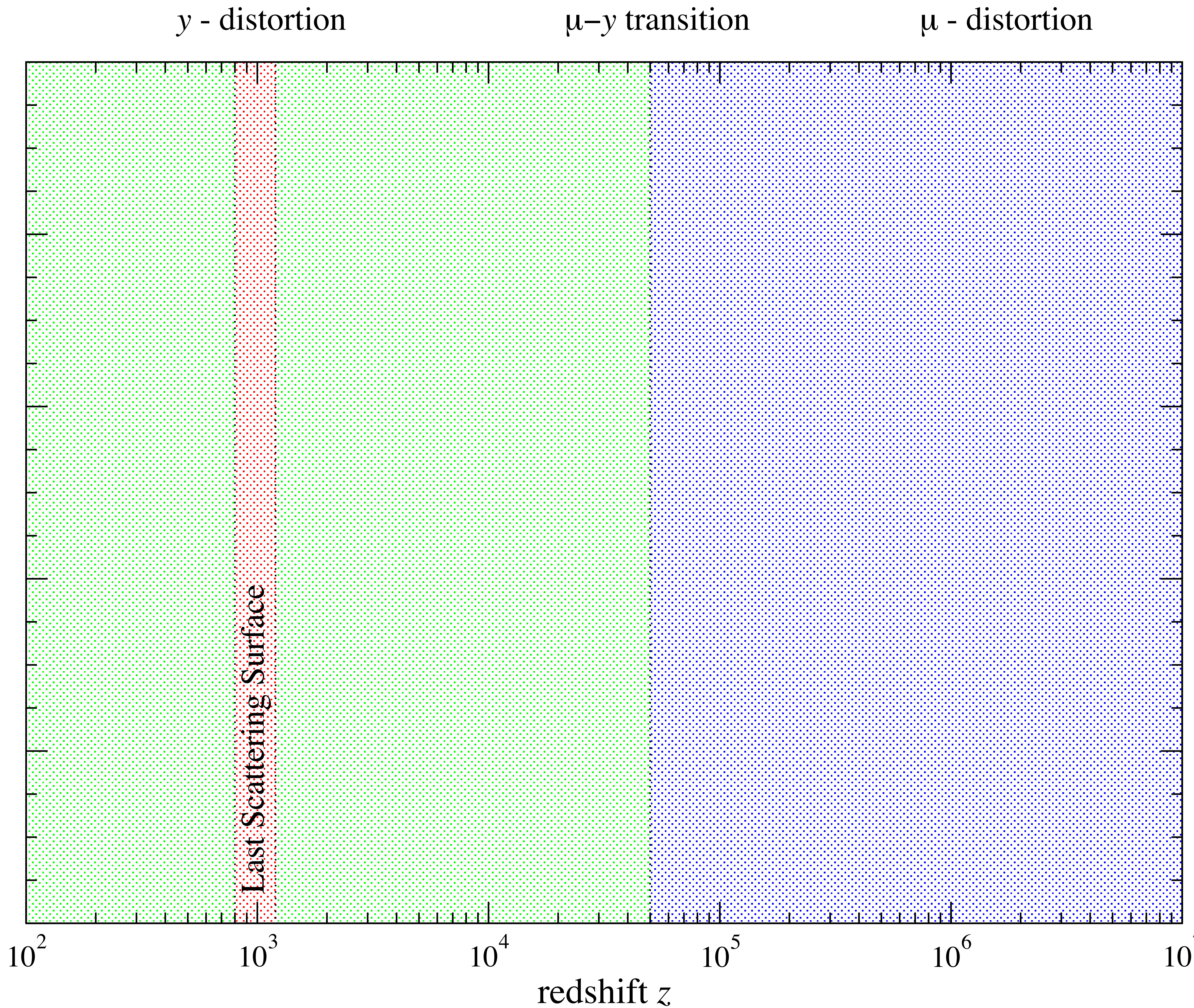
- thermal SZ spectrum (non-relativistic)
- *important for* $y \ll 1$
- null at ν ~ 217 GHz (x~3.83)
- photon number conserved

$$\int x^2 Y(x)\,\mathrm{d}x = 0$$

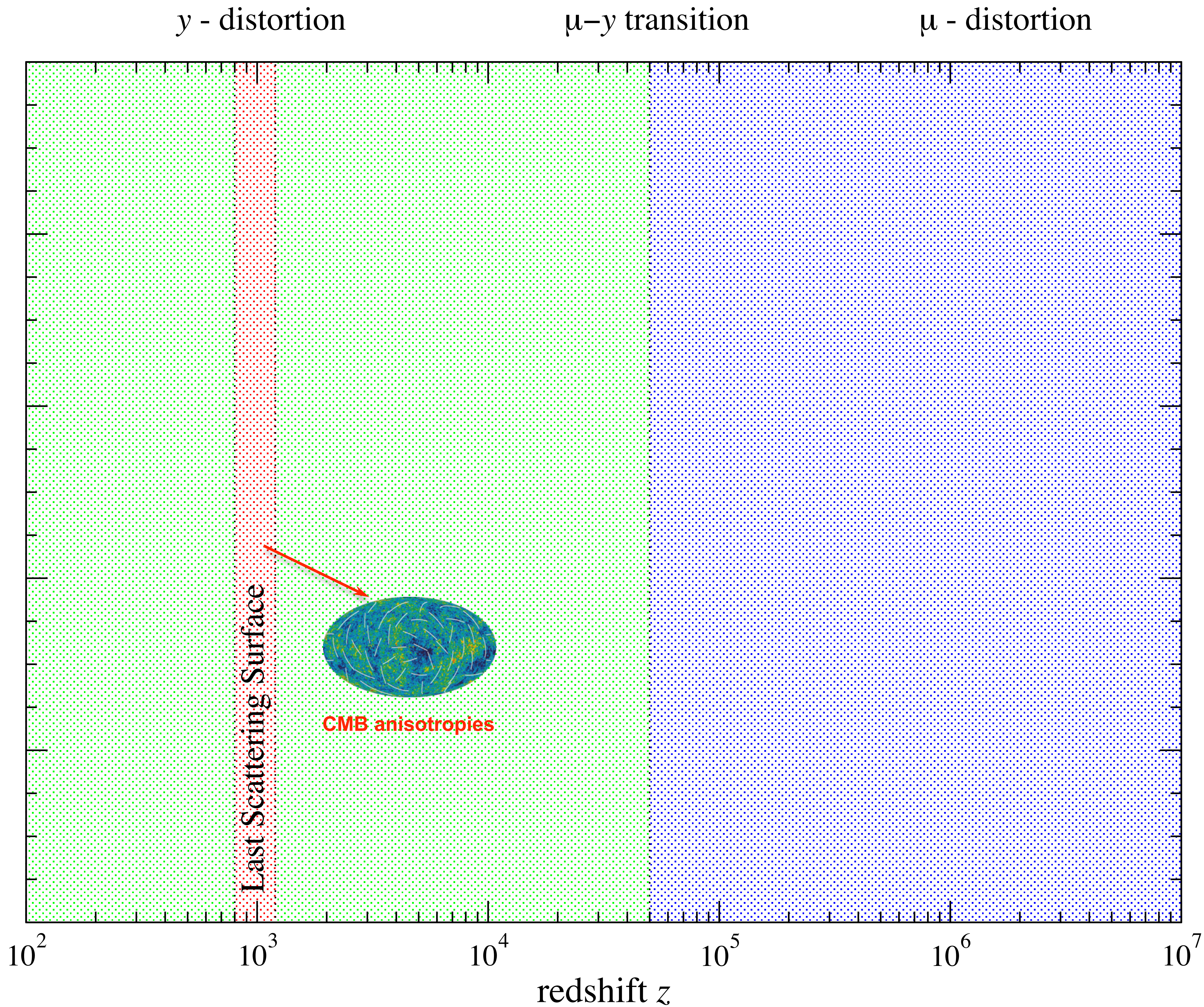
- energy exchange

$$\int x^3 Y(x)\,\mathrm{d}x = \frac{4\pi^4}{15} \leftrightarrow 4\rho_\gamma$$

- *direction of energy flow* depends on difference between T_e and T_γ



y - distortion
μ−y transition
μ - distortion
Last Scattering Surface
10^2
10^3
10^4
10^5
10^6
10^7
redshift z



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Last Scattering Surface
CMB anisotropies
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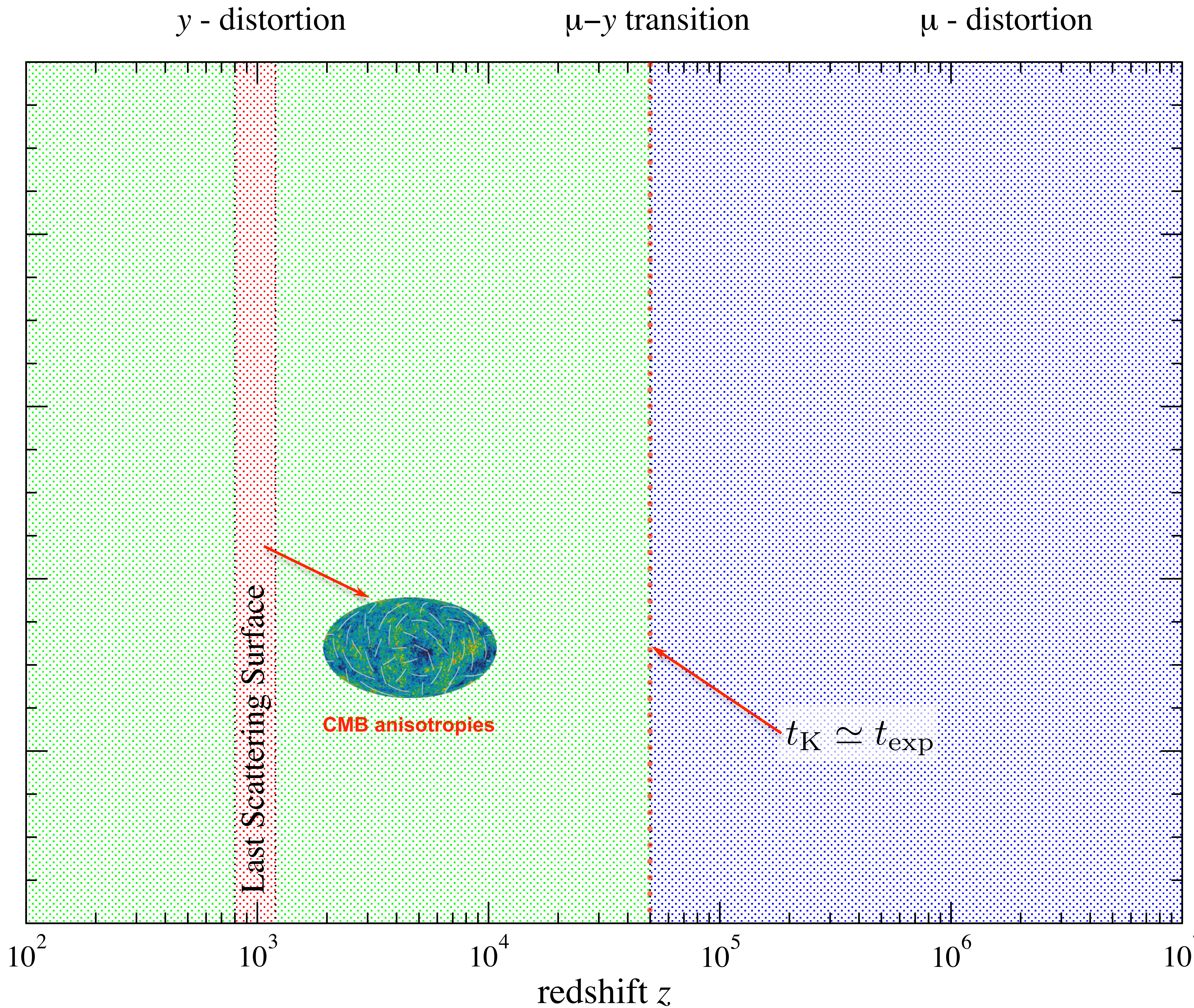

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$t_{\rm K} \simeq t_{\rm exp}$
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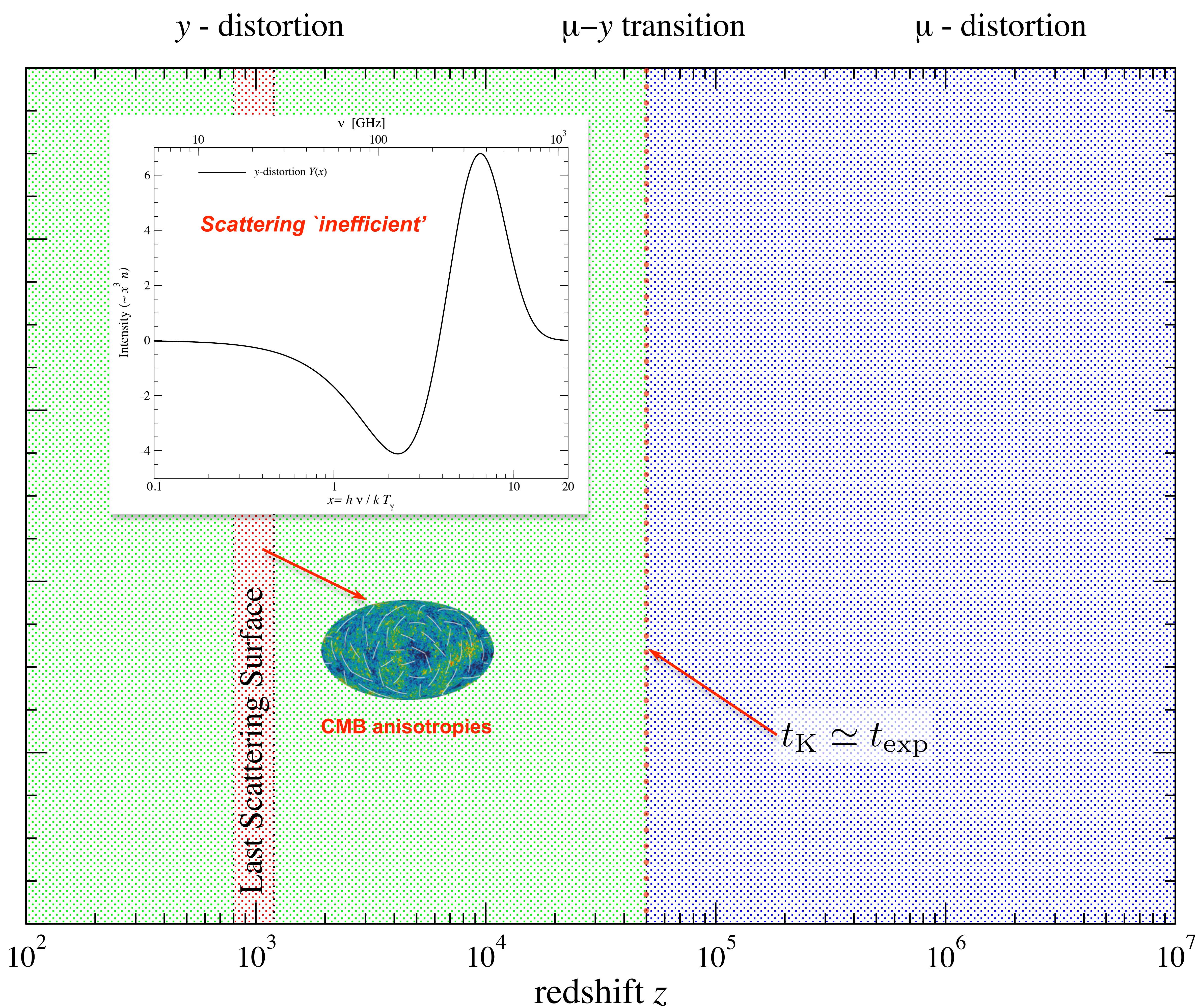
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Chemical Potential / μ-parameter

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chemical potential parameter (*"wrong" sign*)

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constant

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Something is missing? How do you fix T_{e} and μ_0?

Final definition of μ-type distortion

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$$\approx 1.368$$

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$$\approx 1.111$$

$$\rho_\gamma^{\rm bb}(T_\gamma) + \Delta\rho_\gamma = \rho_\gamma^{\rm BE}(T_{\rm e}, \mu_0) \approx \rho_\gamma^{\rm bb}(T_\gamma)\left(1 + 4\frac{\Delta T}{T_\gamma} - \frac{90\zeta(3)}{\pi^4}\mu_0\right)$$

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- *Solution:* $\frac{\Delta T}{T_\gamma} \approx \frac{\pi^2}{18\zeta(3)}\mu_0 \approx 0.456\,\mu_0$ *and* $\mu_0 \approx 1.401\,\frac{\Delta\rho_\gamma}{\rho_\gamma}$

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$$\Longrightarrow \quad M(x) = G(x)\left[\frac{\pi^2}{18\zeta(3)} - \frac{1}{x}\right]$$

μ-distortion spectrum
(photon number conserved)

Final definition of μ-type distortion

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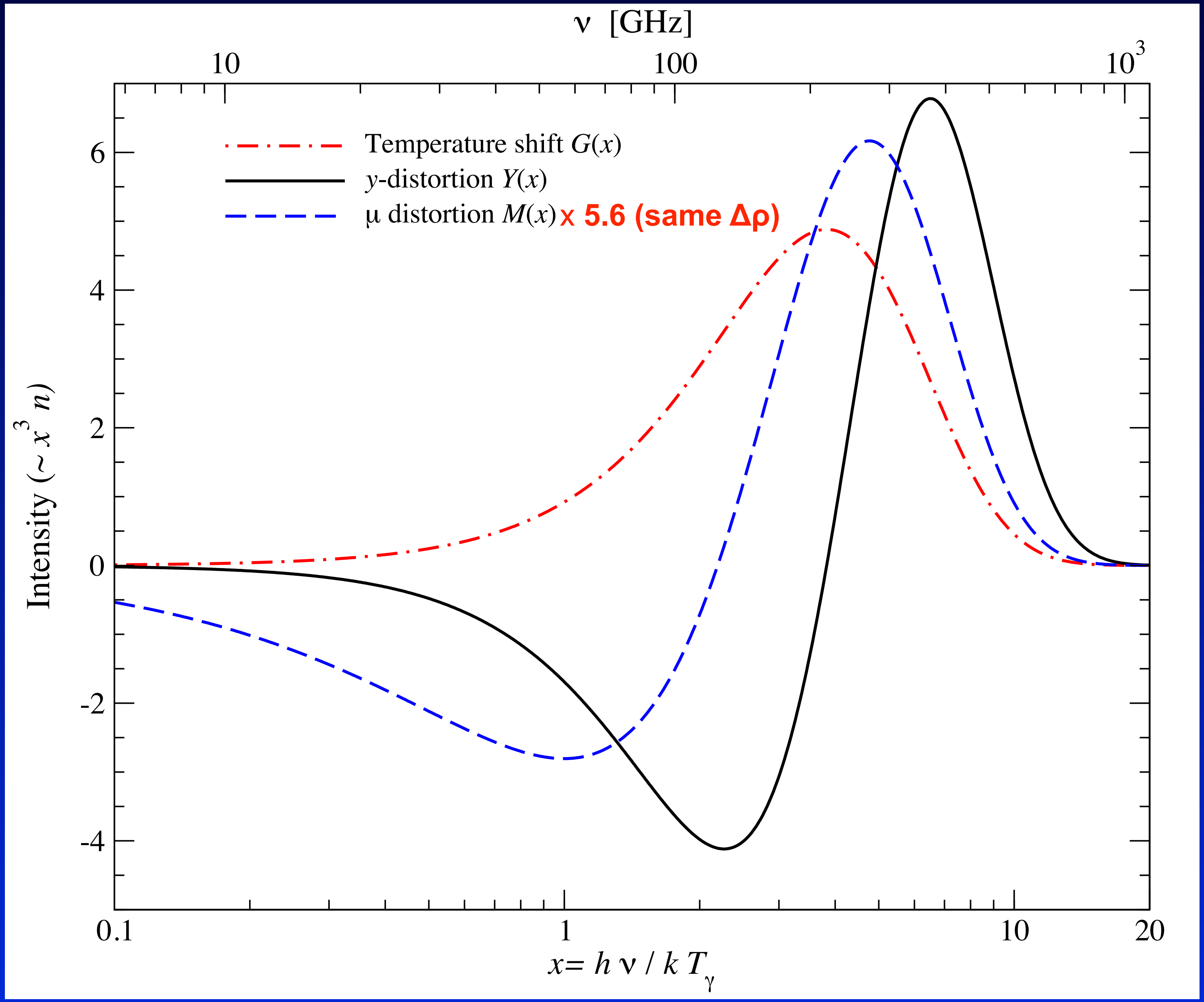
μ-distortion spectrum
(photon number conserved)

- $\mu_0 > 0 \Rightarrow$ *too few photons / too much energy*
- $\mu_0 < 0 \Rightarrow$ *too many photons / too little energy*

$$\frac{\Delta\rho_\gamma}{\rho_\gamma^{\rm bb}} \approx \frac{4}{3}\,\frac{\Delta N_\gamma}{N_\gamma^{\rm bb}}$$

Temperature shift ↔ y-distortion ↔ μ-distortion



$$M(x) = G(x)\left[\frac{\pi^2}{18\zeta(3)} - \frac{1}{x}\right]$$

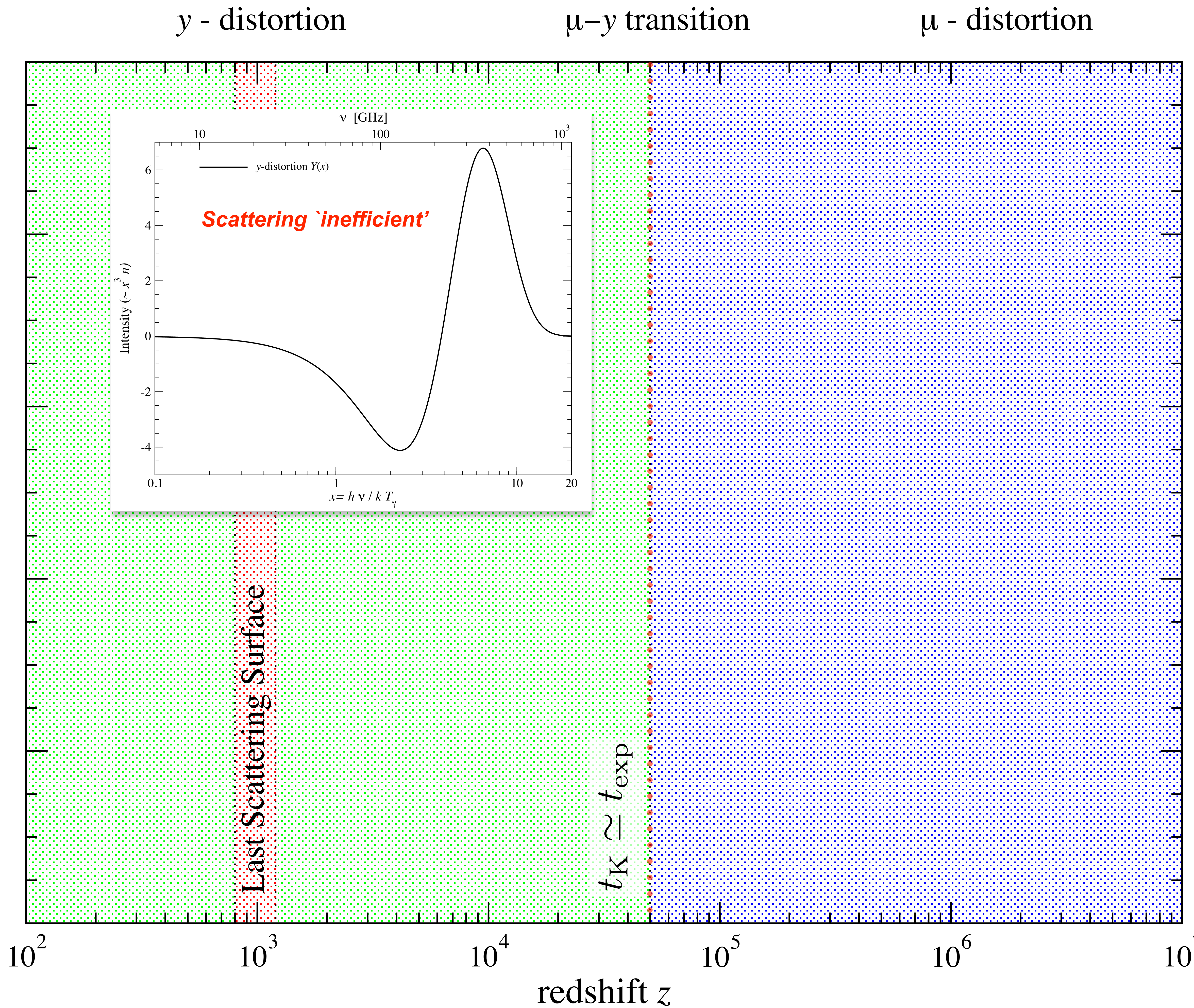
- *important for in the limit of many scatterings*
- null at ν ~ 125 GHz (x~2.19)
- photon number conserved

$$\int x^2 M(x)\mathrm{d}x = 0$$

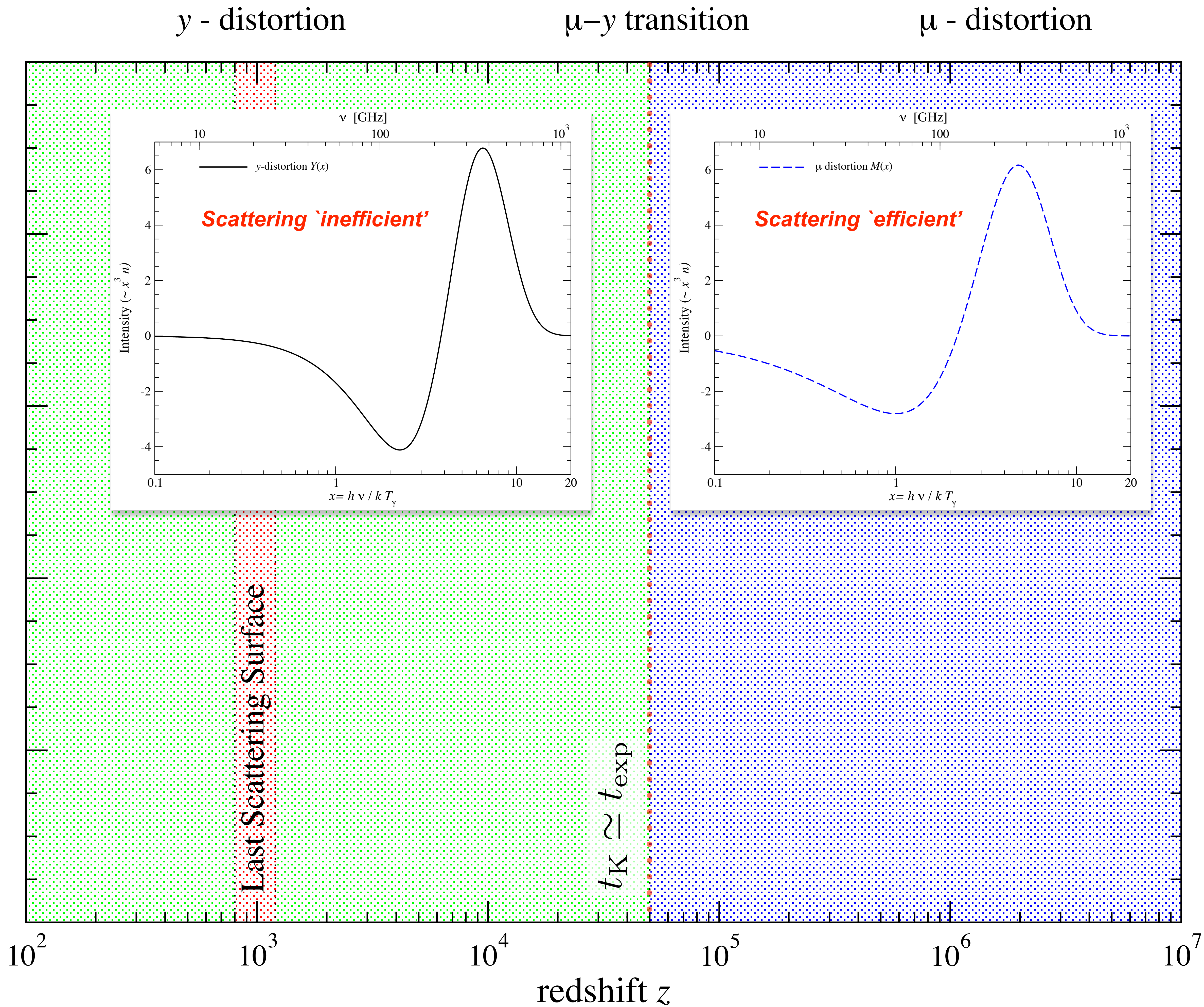
- energy exchange

$$\int x^3 M(x)\mathrm{d}x \approx \frac{\pi^4/15}{1.401} \leftrightarrow \frac{\rho_\gamma}{1.401}$$

- *can only be created at very early times*



y - distortion
μ−y transition
μ - distortion
ν [GHz]
10
100
10^3
y-distortion Y(x)
Scattering `inefficient'
Intensity (~ x^3 n)
6
4
2
0
-2
-4
0.1
1
10
20
x= h ν / k T_γ
Last Scattering Surface
t_K ≃ t_exp
10^2
10^3
10^4
10^5
10^6
10^7
redshift z



y - distortion
μ–y transition
μ - distortion
ν [GHz]
y-distortion Y(x)
Scattering `inefficient'
Intensity (~ x^3 n)
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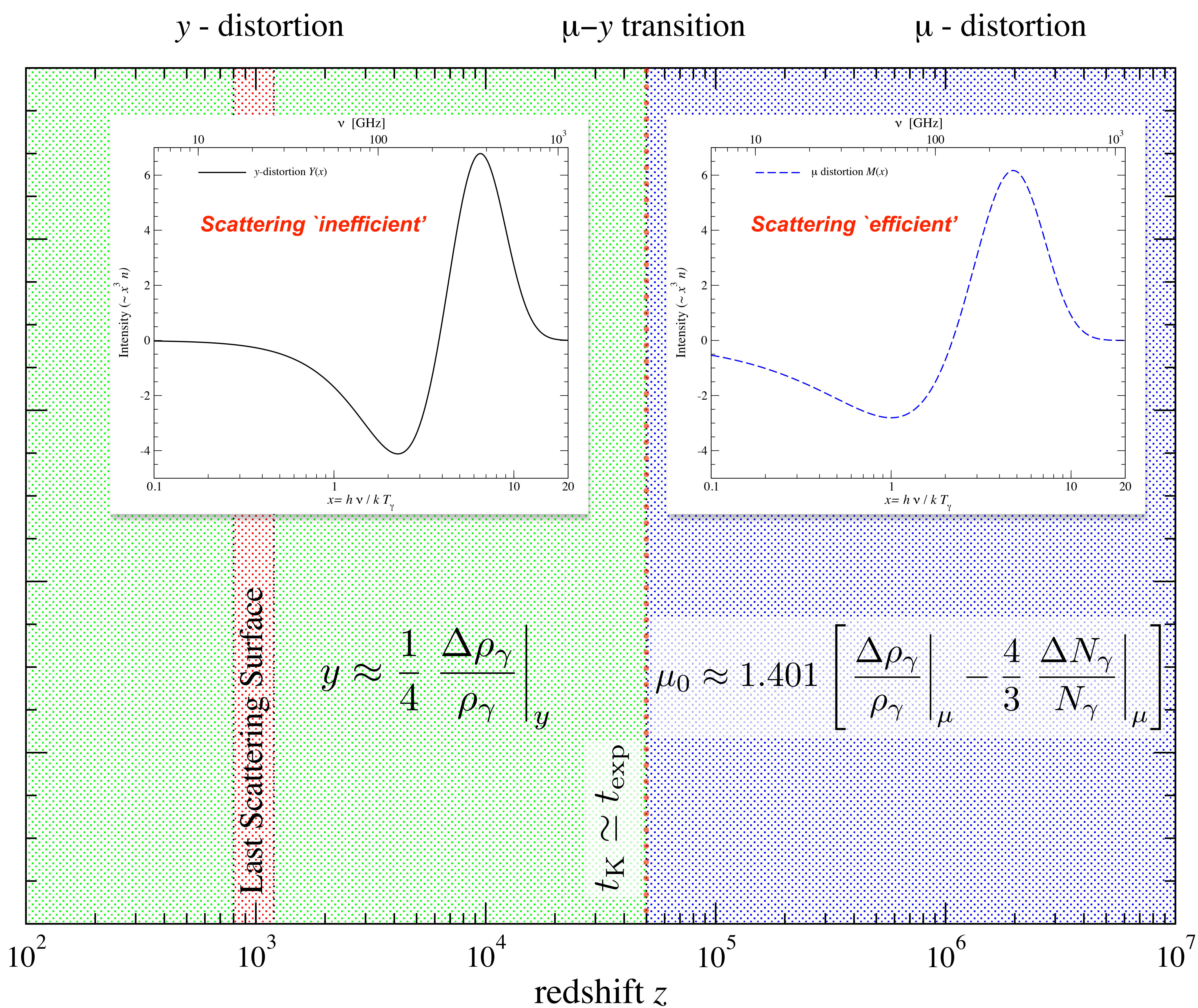
μ distortion M(x)
Scattering `efficient'
Last Scattering Surface
$t_K \simeq t_{exp}$
redshift z



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Scattering `inefficient'
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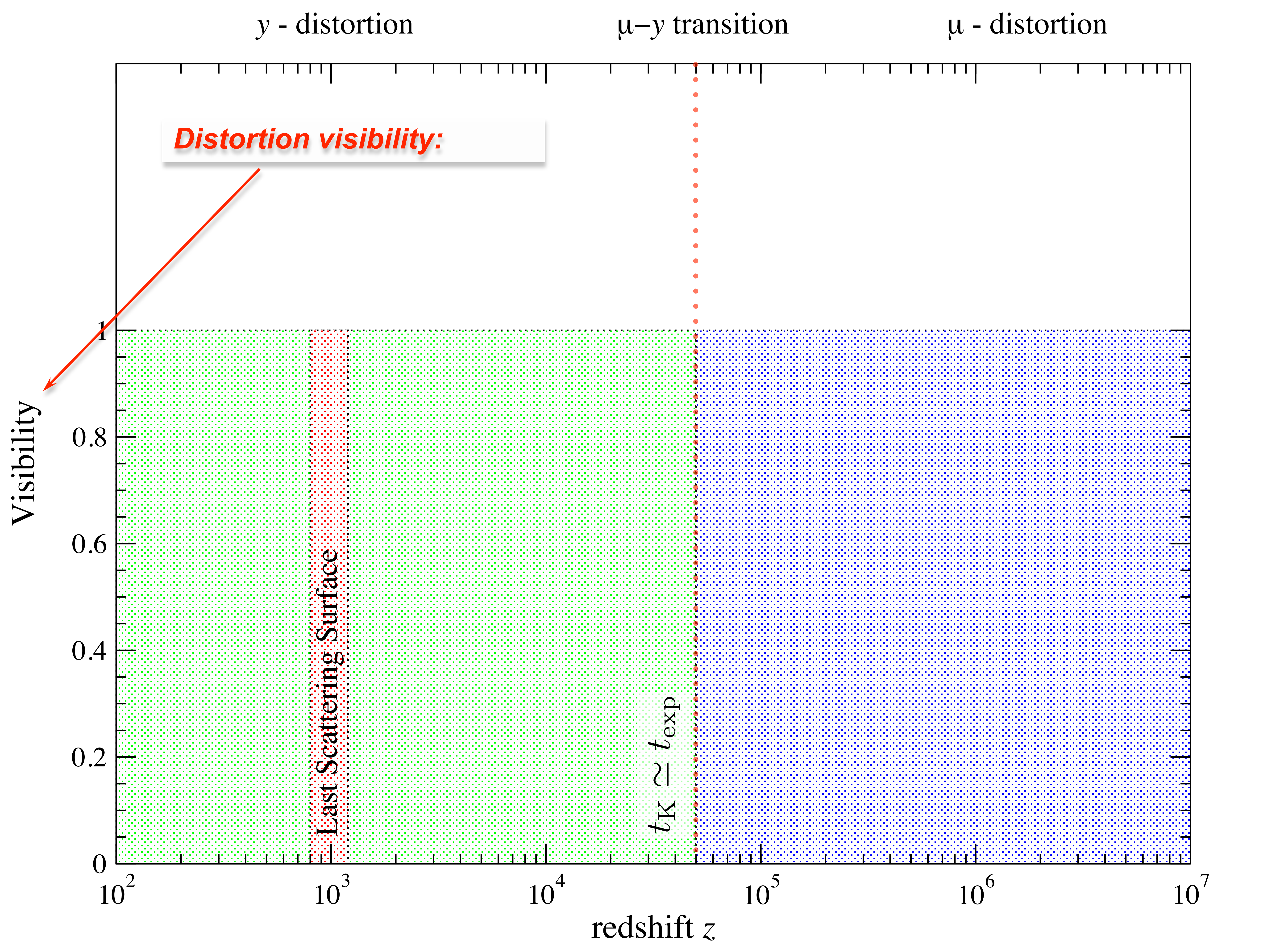
Last Scattering Surface
$y \approx \frac{1}{4} \frac{\Delta\rho_\gamma}{\rho_\gamma}\Big|_y$
$\mu_0 \approx 1.401 \left[\frac{\Delta\rho_\gamma}{\rho_\gamma}\Big|_\mu - \frac{4}{3} \frac{\Delta N_\gamma}{N_\gamma}\Big|_\mu \right]$
$t_K \simeq t_{exp}$
redshift z

But how do we thermalize?



y - distortion
μ–y transition
μ - distortion
Distortion visibility:
Visibility
1
0.8
0.6
0.4
0.2
0
Last Scattering Surface
$t_K \simeq t_{exp}$
10^2
10^3
10^4
10^5
10^6
10^7
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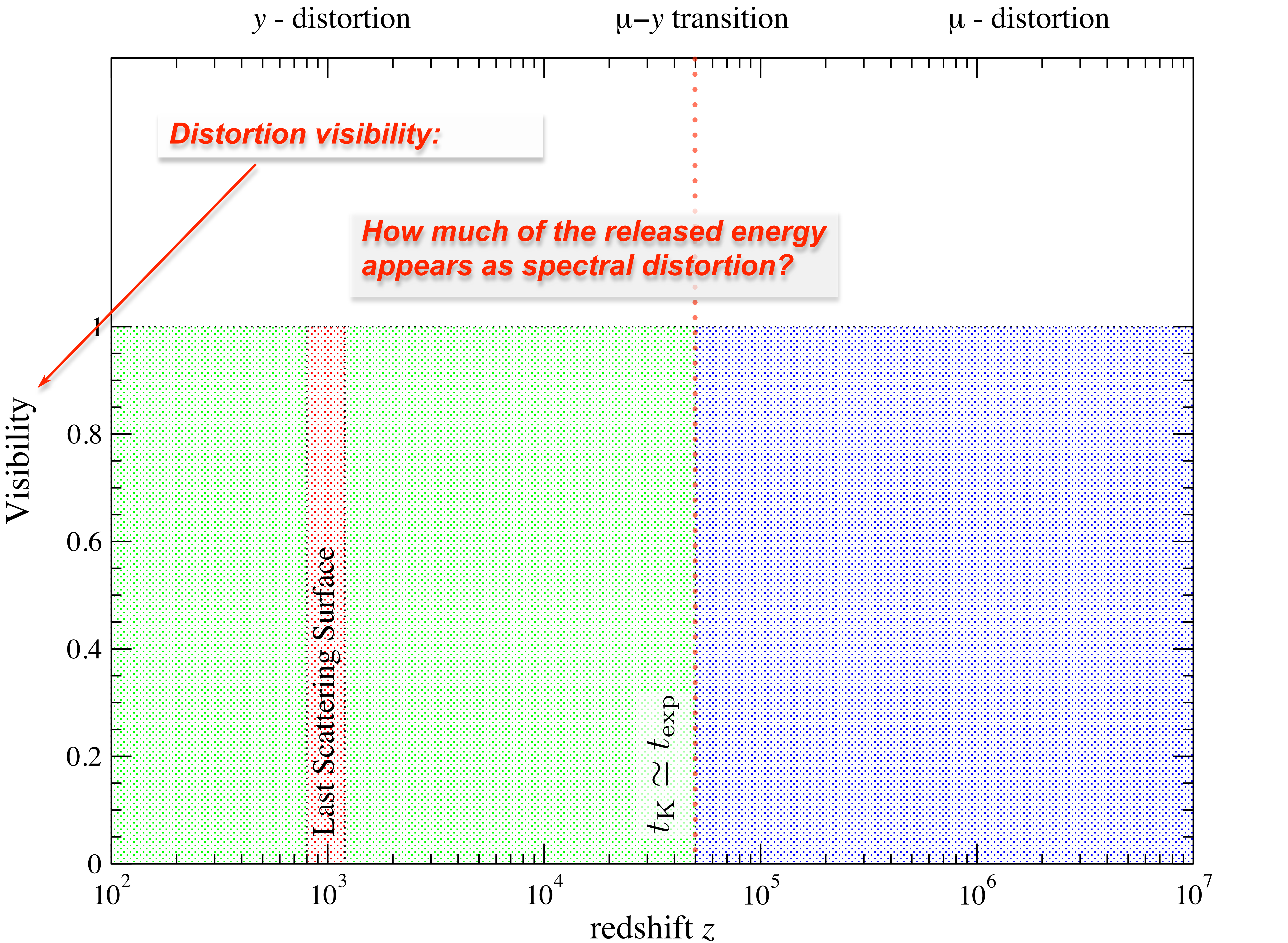

y - distortion
μ–y transition
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Distortion visibility:
How much of the released energy appears as spectral distortion?
Visibility
Last Scattering Surface
$t_K \simeq t_{exp}$
1
0.8
0.6
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0
10^2
10^3
10^4
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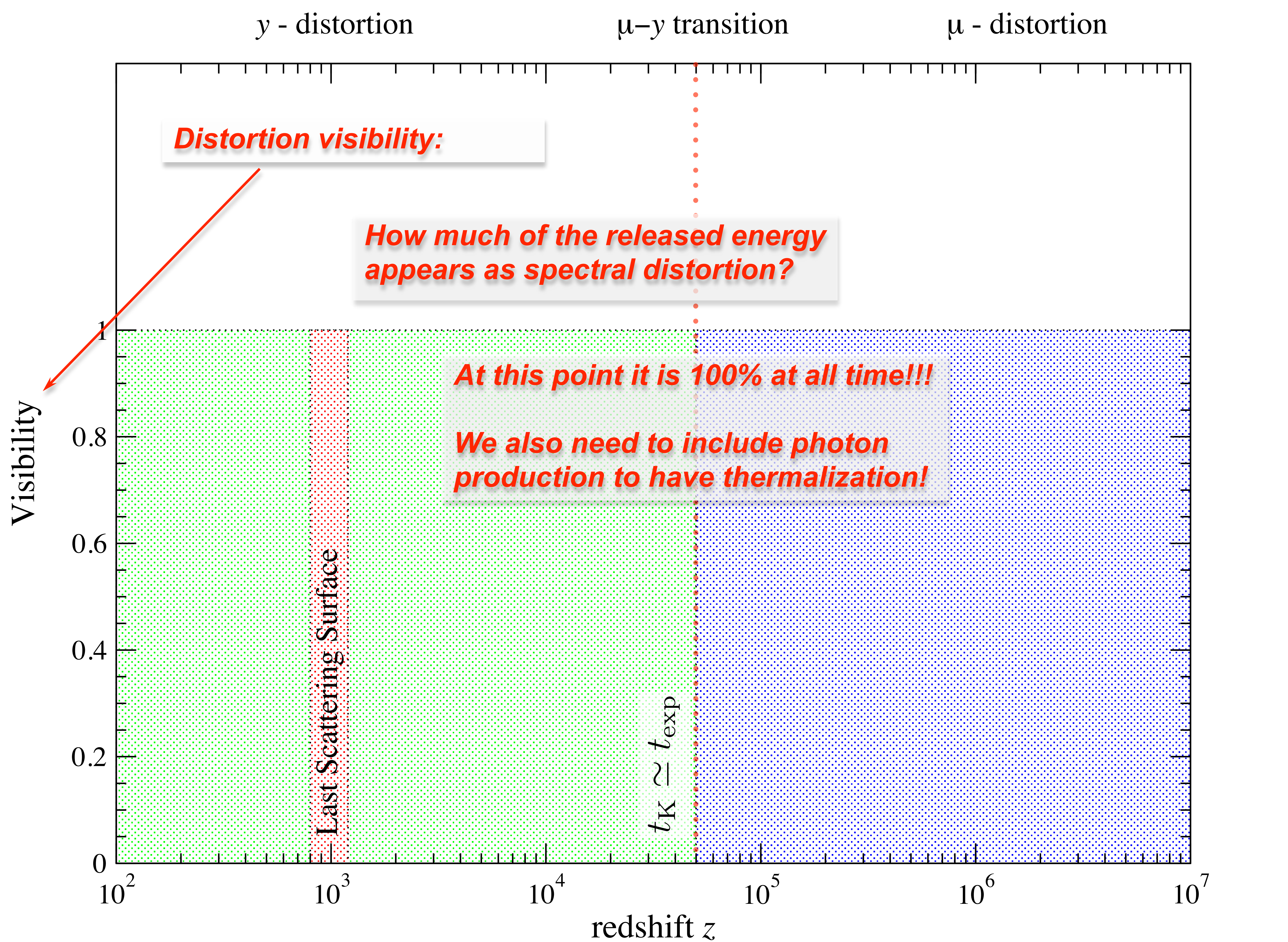

y - distortion
μ–y transition
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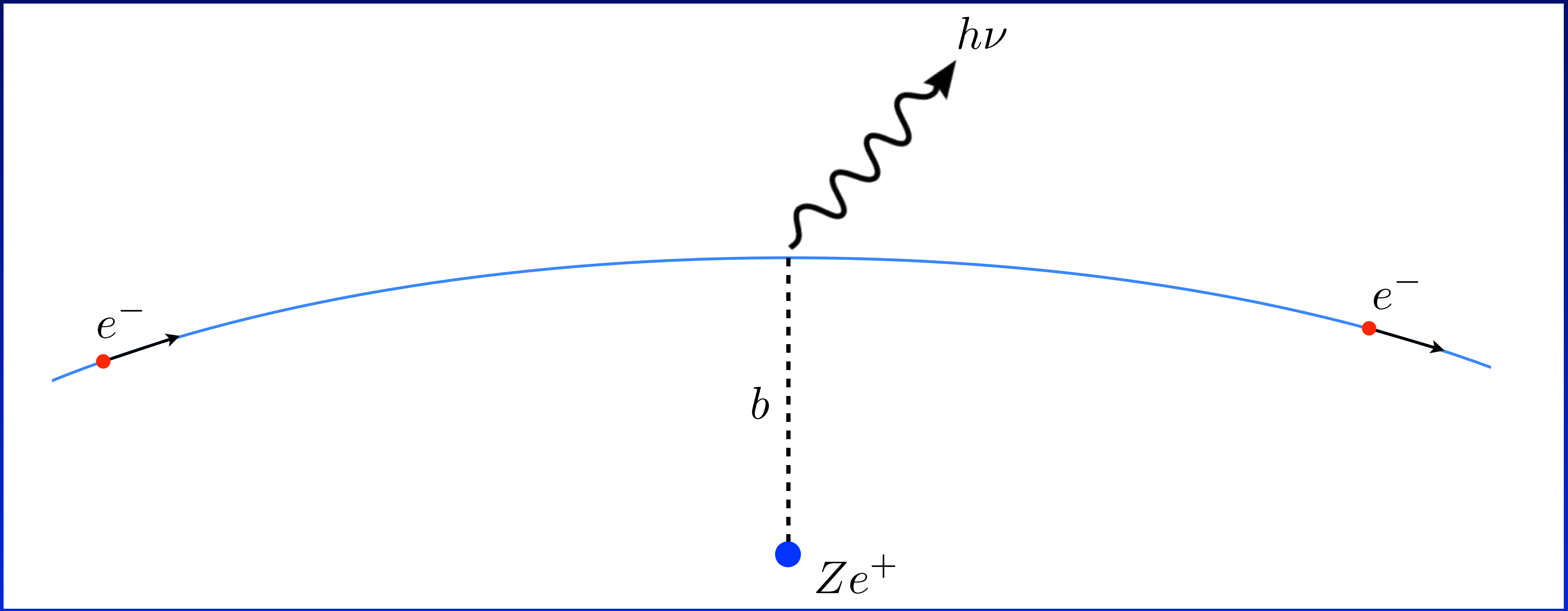
Distortion visibility:
How much of the released energy appears as spectral distortion?
At this point it is 100% at all time!!!
We also need to include photon production to have thermalization!
Visibility
0
0.2
0.4
0.6
0.8
1
Last Scattering Surface
$t_K \simeq t_{exp}$
10^2
10^3
10^4
10^5
10^6
10^7
redshift z

Photon production processes

Thermal Bremsstrahlung

- Reaction: $e + p \leftrightarrow e' + p + \gamma$



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- Reaction: $e + p \leftrightarrow e' + p + \gamma$

  → 1. order α correction to *Coulomb* scattering

  → production of low frequency photons

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- Reaction: $e + p \leftrightarrow e' + p + \gamma$

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  → production of low frequency photons

  → include dependence on composition (Gaunt-factors; ***simple fits***: **Itoh et al, 2000**)

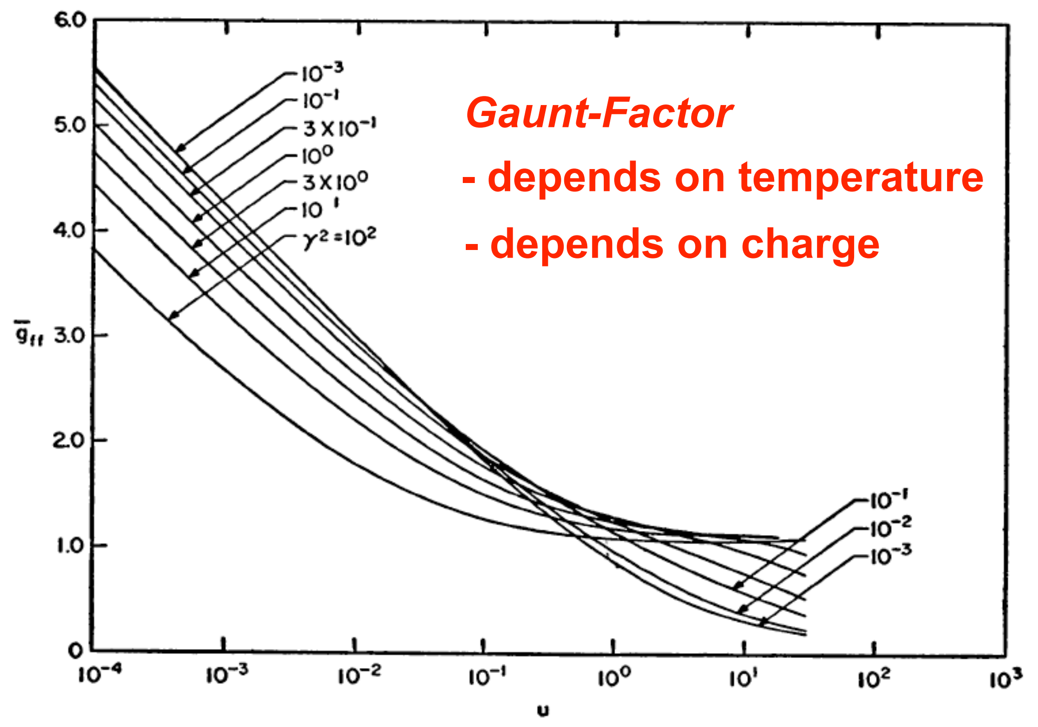

FIG. 5.—Temperature-averaged free-free Gaunt factor versus $u = h\nu/kT$ for various values of $\gamma^2 = Z^2Ry/kT$.

Karzas & Latter, 1961, ApJS, 6, 167

Thermal Bremsstrahlung

- Reaction: $e + p \leftrightarrow e' + p + \gamma$

→ 1. order α correction to *Coulomb* scattering

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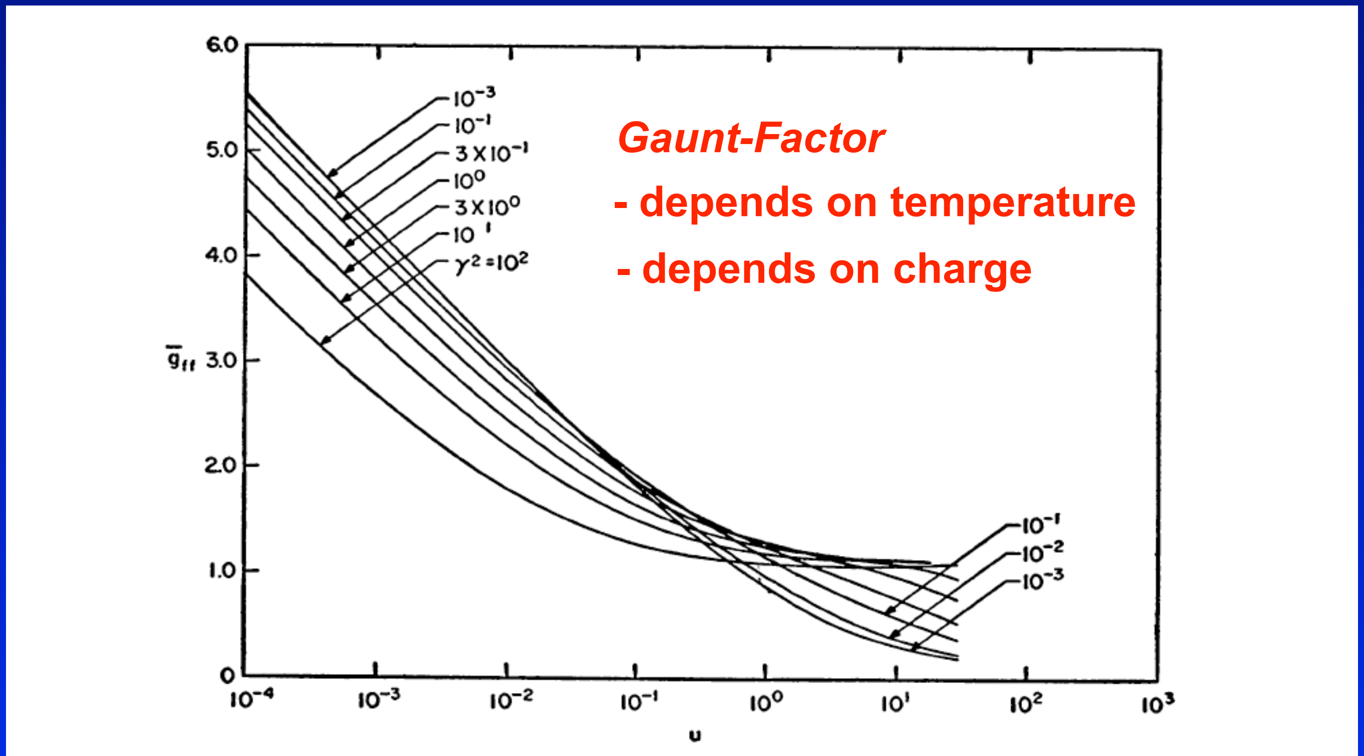

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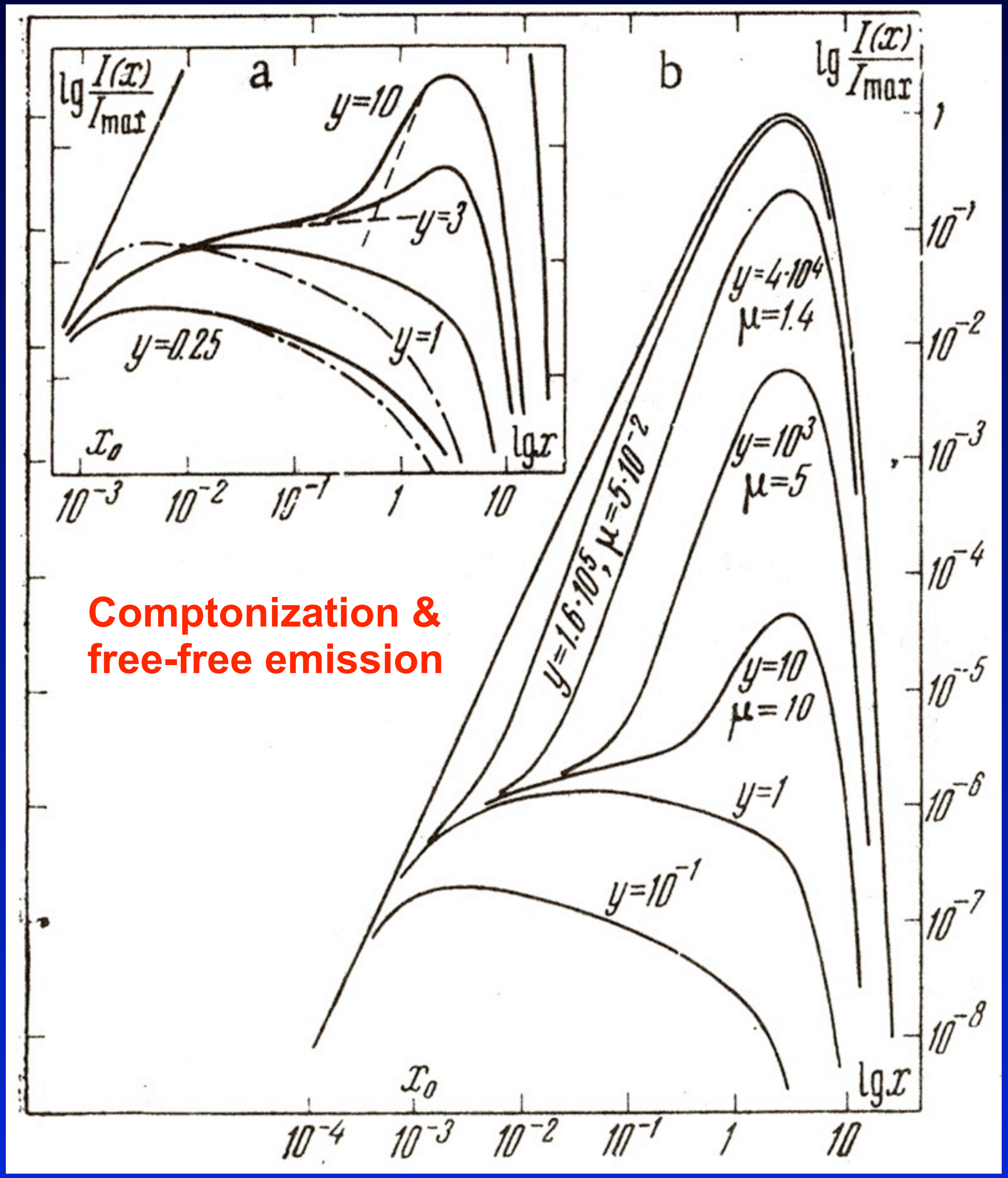

Illarionov & Sunyaev, 1975, Sov. Astr, 18, pp.413

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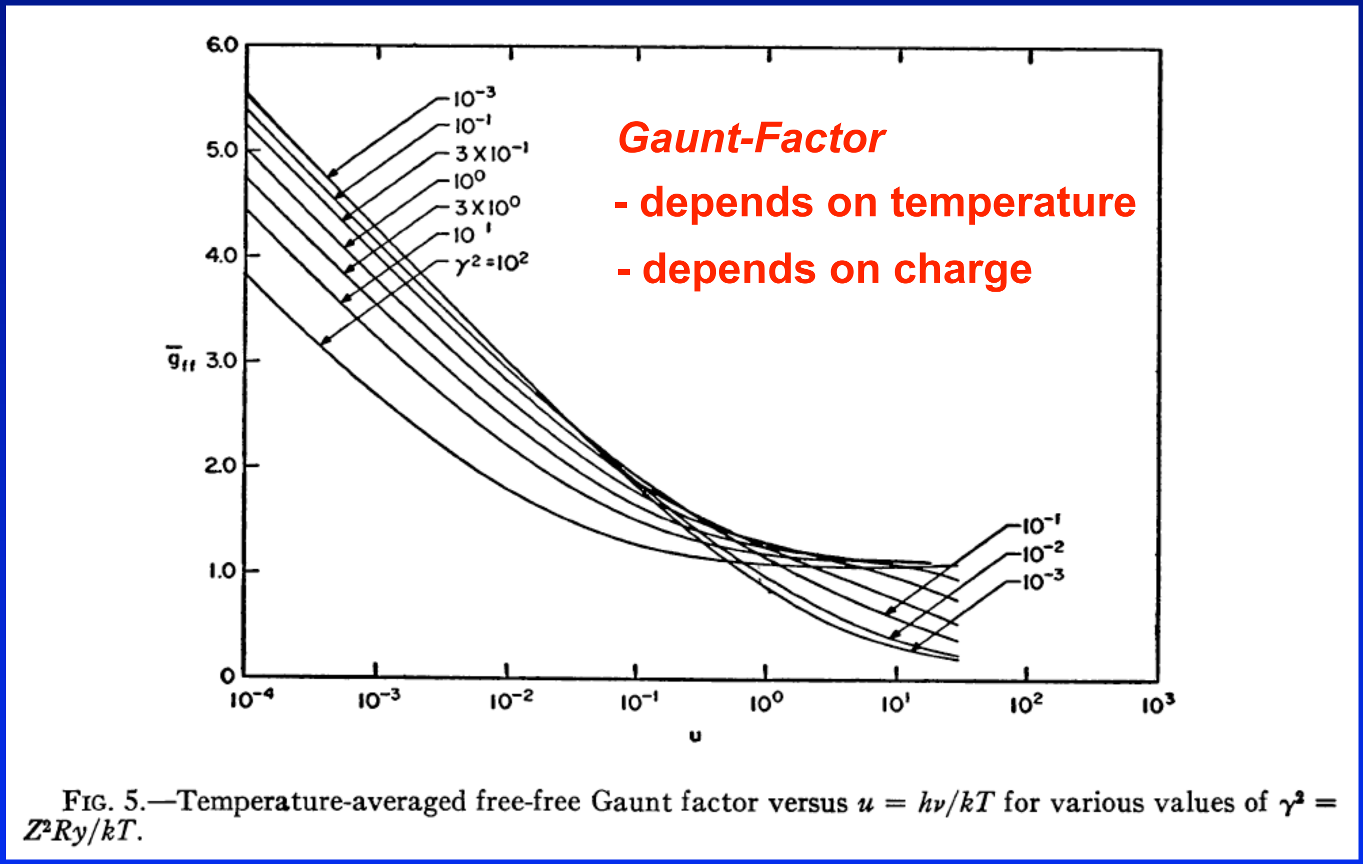

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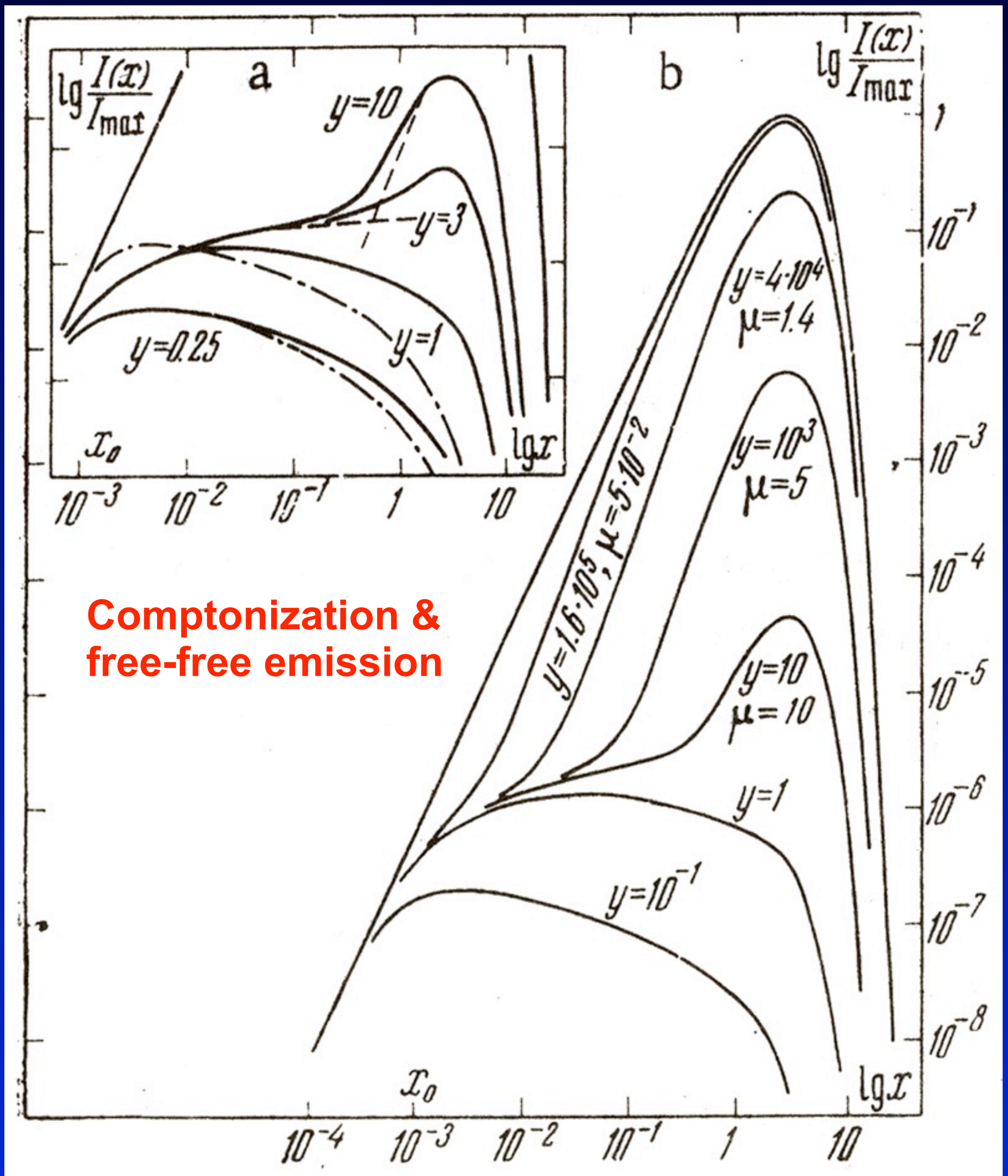

Illarionov & Sunyaev, 1975, Sov. Astr, 18, pp.413

With Bremsstrahlung alone thermalization inefficient already at $z \lesssim 10^7$!

Double Compton Emission

- Reaction: $e + \gamma \leftrightarrow e' + \gamma' + \gamma_2$

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$$K_{\rm DC} \propto N_{\rm e} N_\gamma \left(\frac{kT_\gamma}{m_{\rm e}c^2}\right)^{-1} \propto (1+z)^5 \iff K_{\rm BR} \propto N_{\rm e} N_{\rm b} \left(\frac{kT_{\rm e}}{m_{\rm e}c^2}\right)^{-7/2} \propto (1+z)^{5/2}$$

Double Compton Emissivity | **Bremsstrahlung Emissivity**

Double Compton Emission

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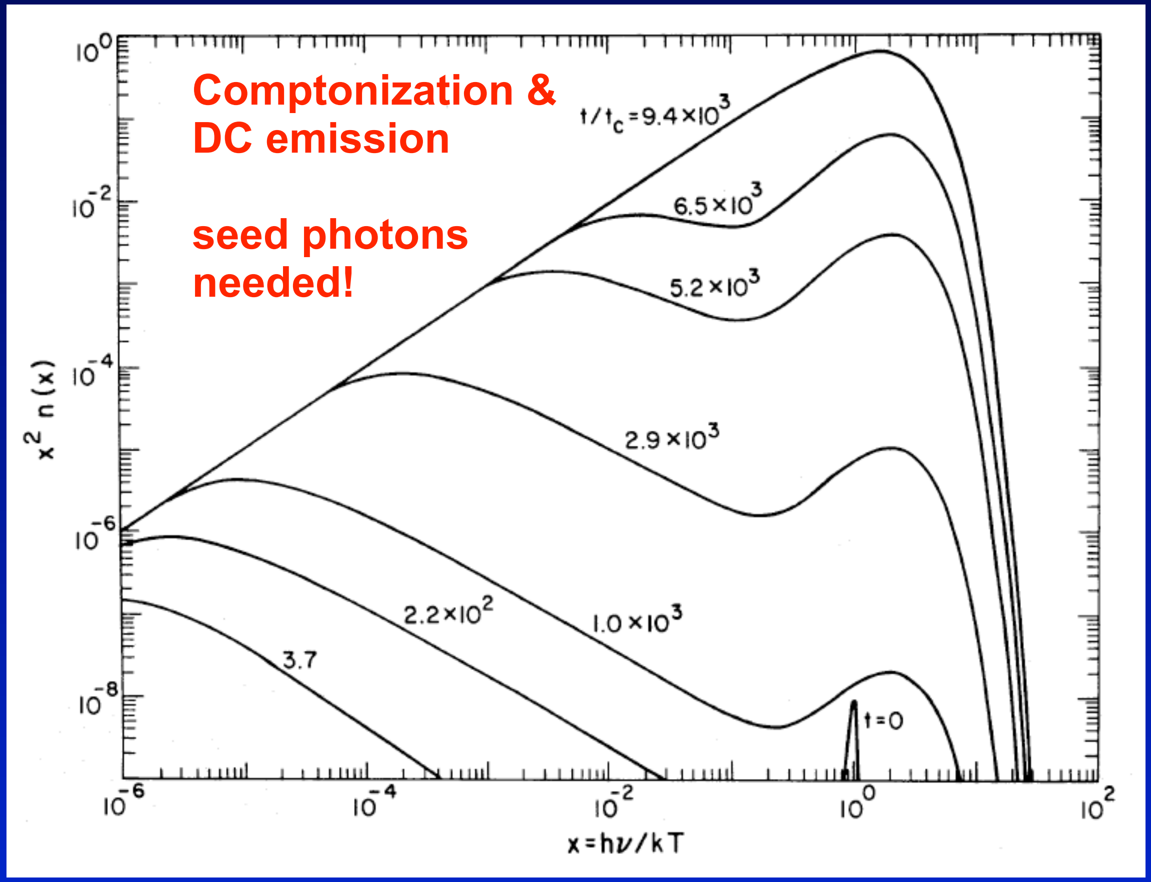

Lightman 1981

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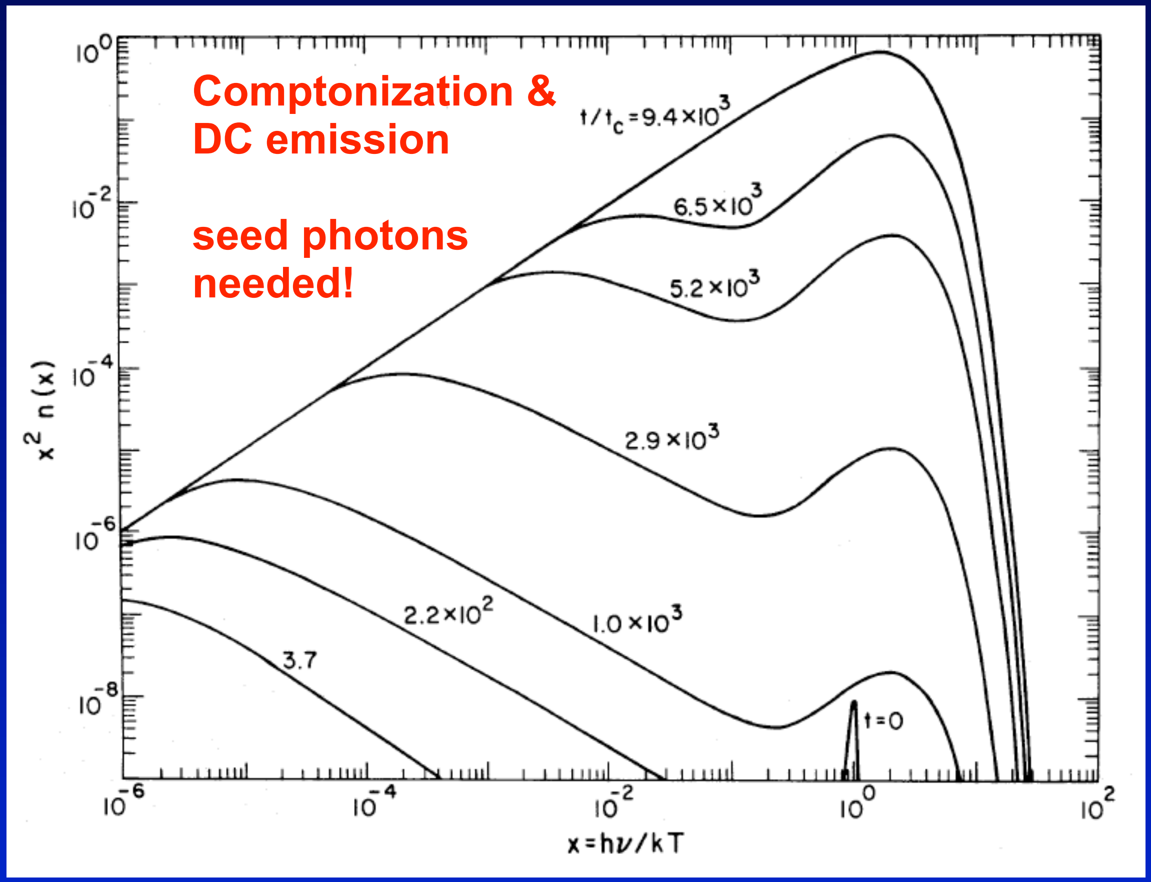
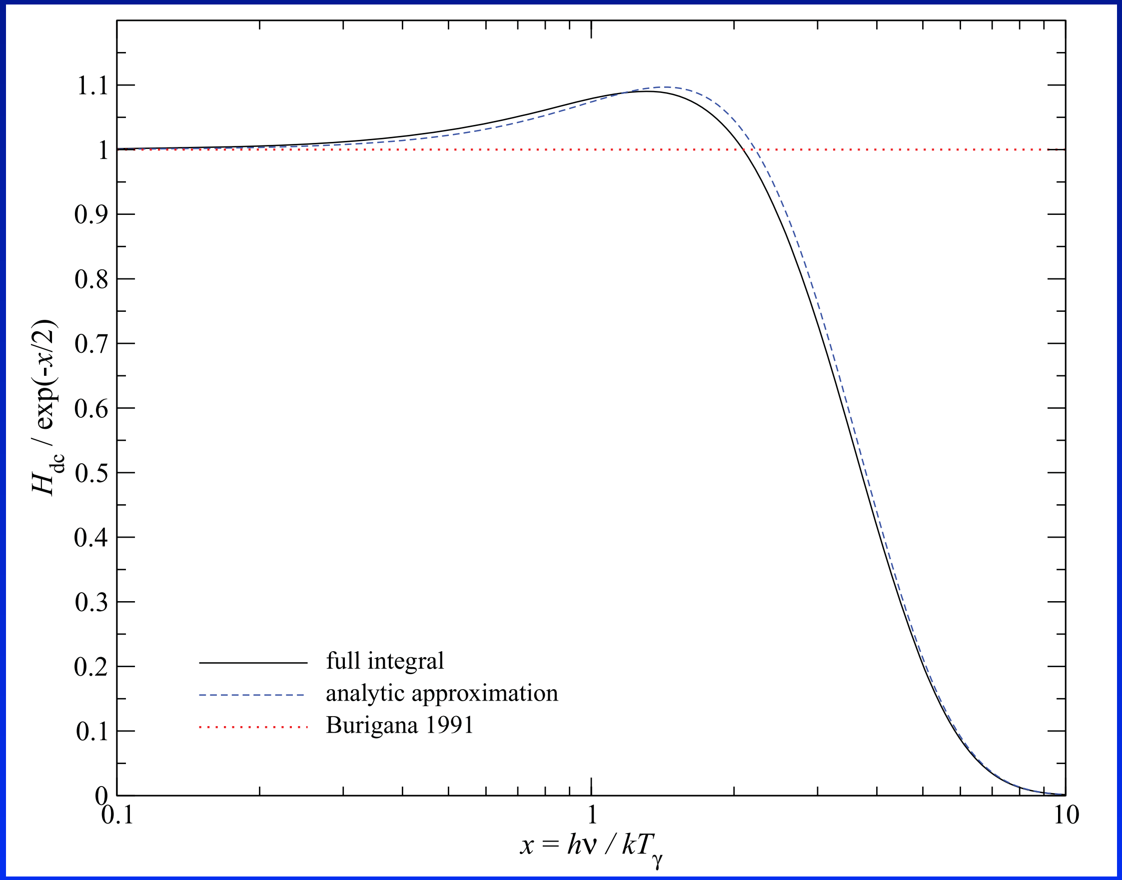


Lightman 1981



JC, 2005; JC, Sazonov & Sunyaev, 2007; JC & Sunyaev, 2012

→ was only included later **(Danese & De Zotti, 1982)**

→ DC Gaunt-factor and temperature corrections included by latest computations, but the effect is small

Example: *Energy release by decaying relict particle*

redshift

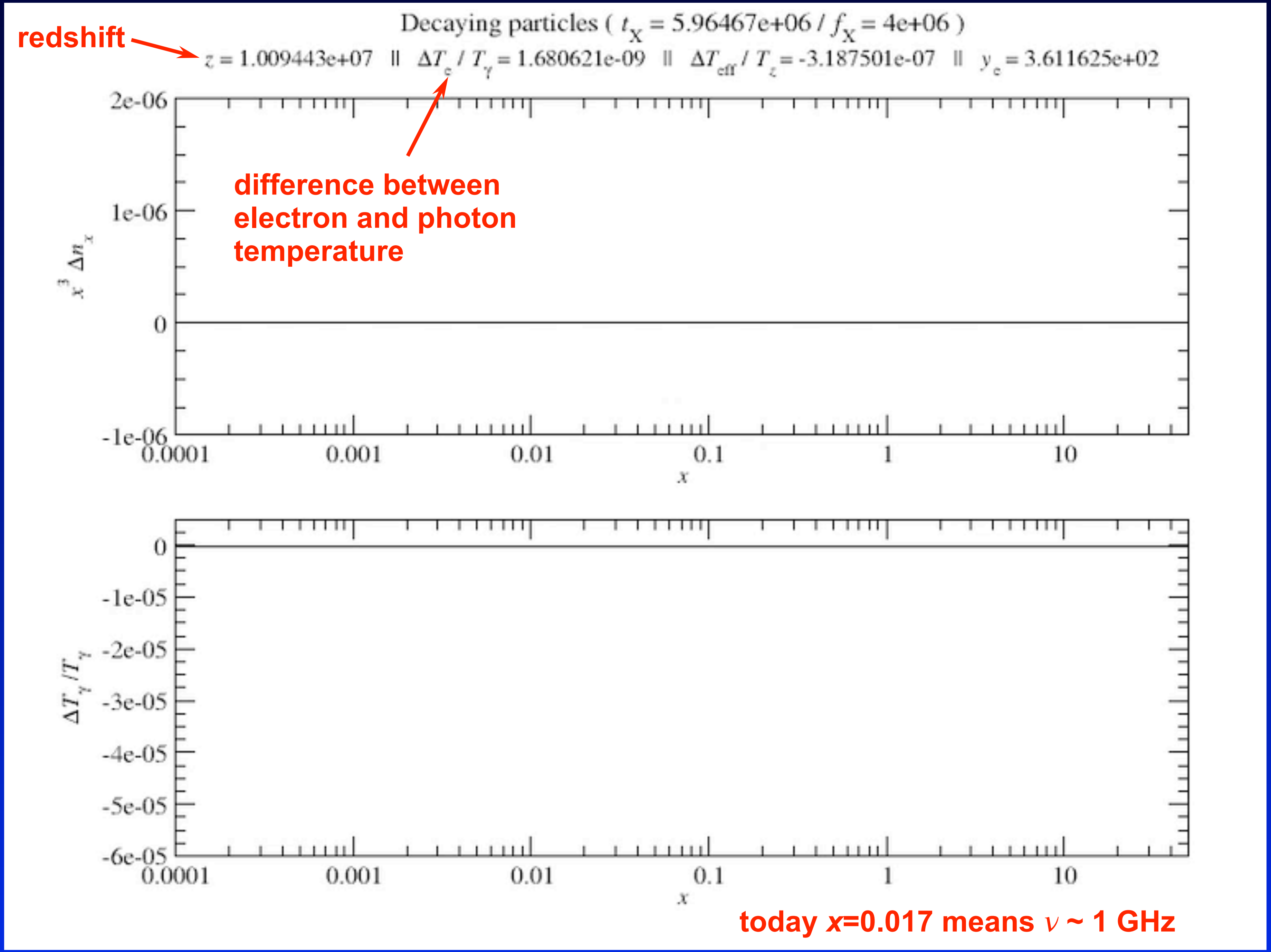
difference between electron and photon temperature

- initial condition: *full equilibrium*
- total energy release:
  $\Delta\rho/\rho \sim 1.3\text{x}10^{-6}$
- most of energy release around:
  $z_X \sim 2\text{x}10^6$
- positive μ-distortion
- high frequency distortion frozen around $z \simeq 5\text{x}10^5$
- late ($z<10^3$) free-free absorption at very low frequencies ($T_e<T_\gamma$)

today x=0.017 means ν ~ 1 GHz

Computation carried out with *CosmoTherm*
(JC & Sunyaev 2012)

Example: *Energy release by decaying relict particle*



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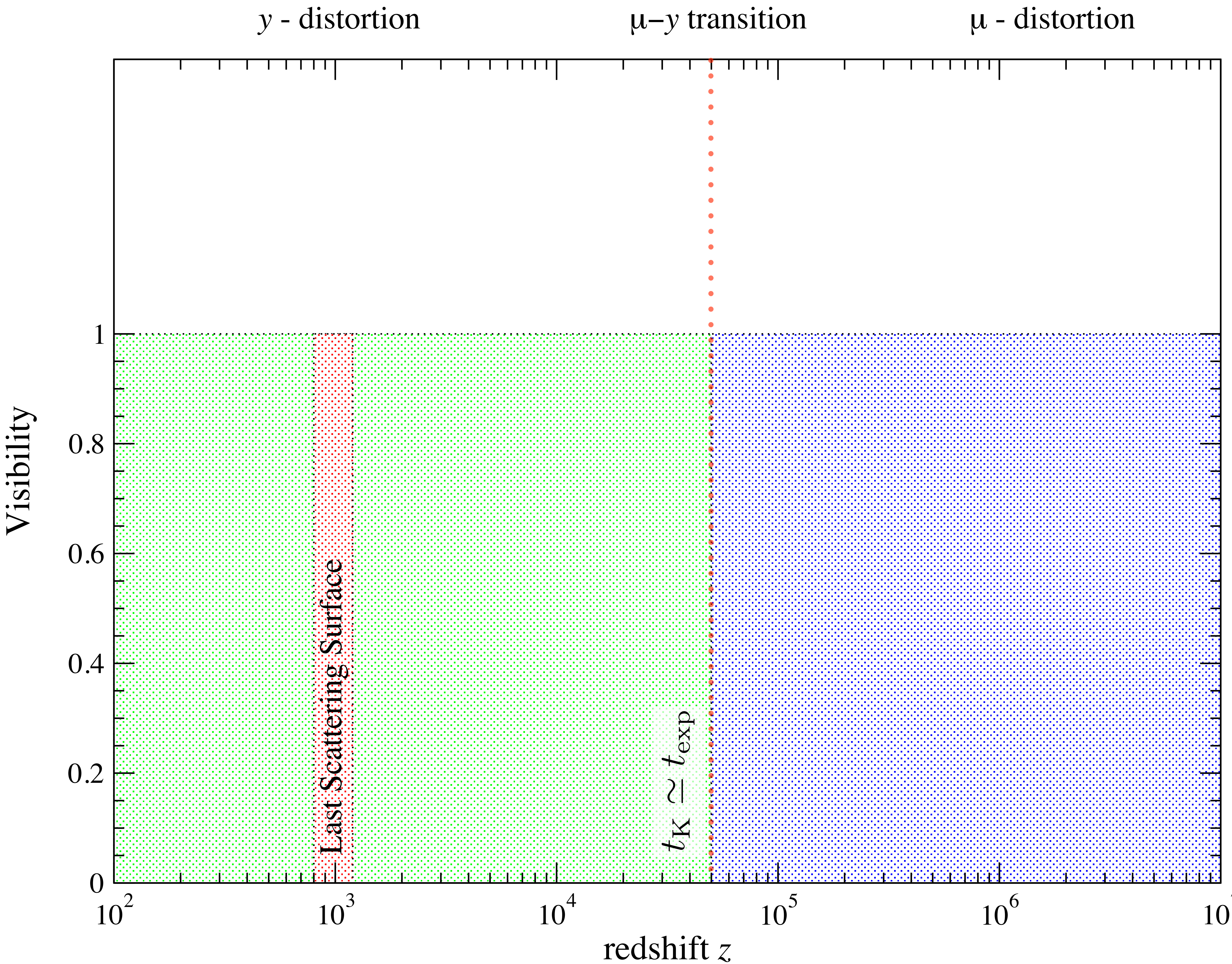
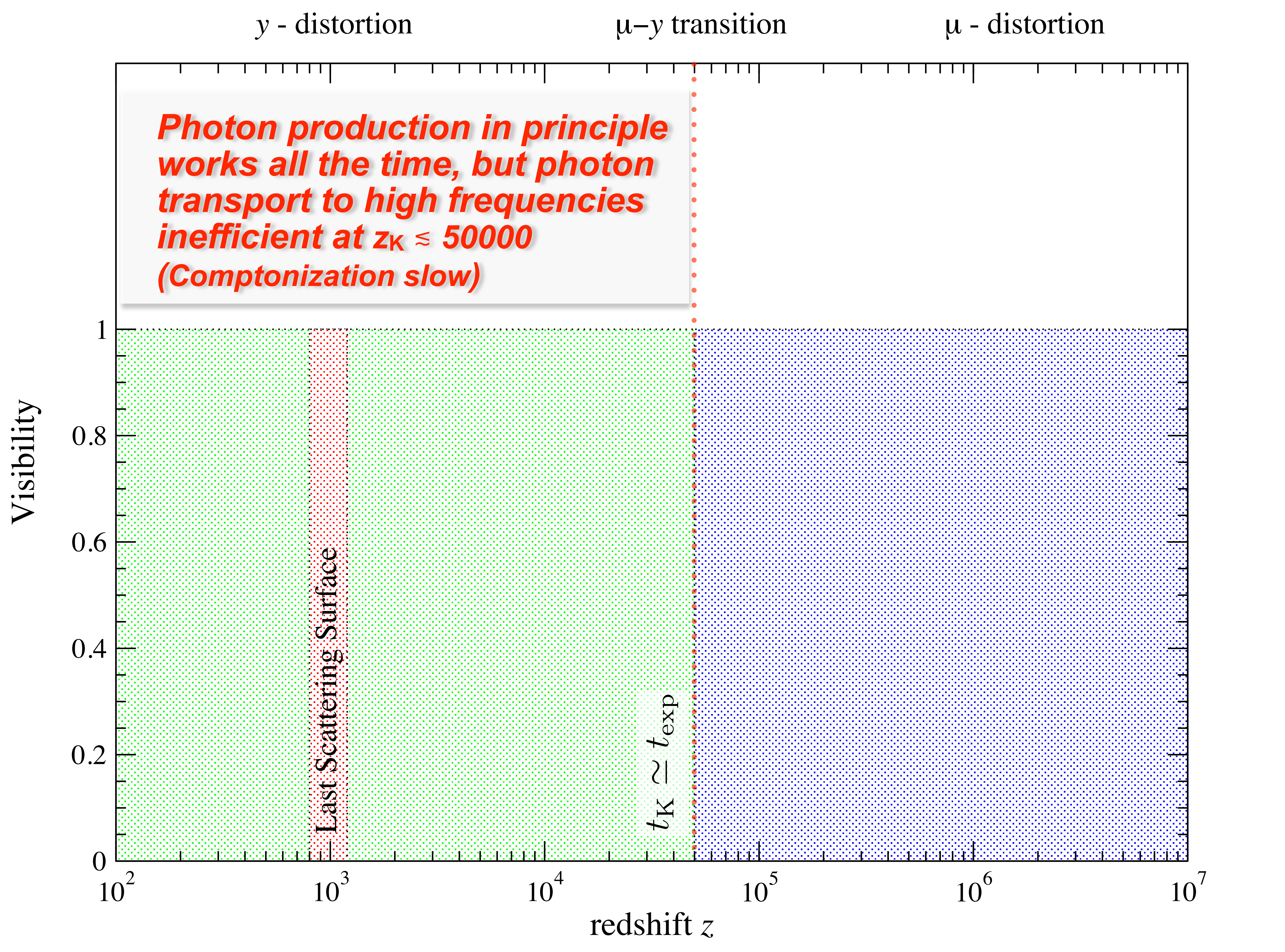
Let’s try to understand the evolution of distortions with photon production analytically!


y - distortion
μ−y transition
μ - distortion
Visibility
0
0.2
0.4
0.6
0.8
1
Last Scattering Surface
$t_K \simeq t_{exp}$
10^2
10^3
10^4
10^5
10^6
10^7
redshift z



y - distortion
μ–y transition
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Photon production in principle works all the time, but photon transport to high frequencies inefficient at $z_K \lesssim 50000$ (Comptonization slow)
Visibility
1
0.8
0.6
0.4
0.2
0
Last Scattering Surface
$t_K \simeq t_{exp}$
10^2
10^3
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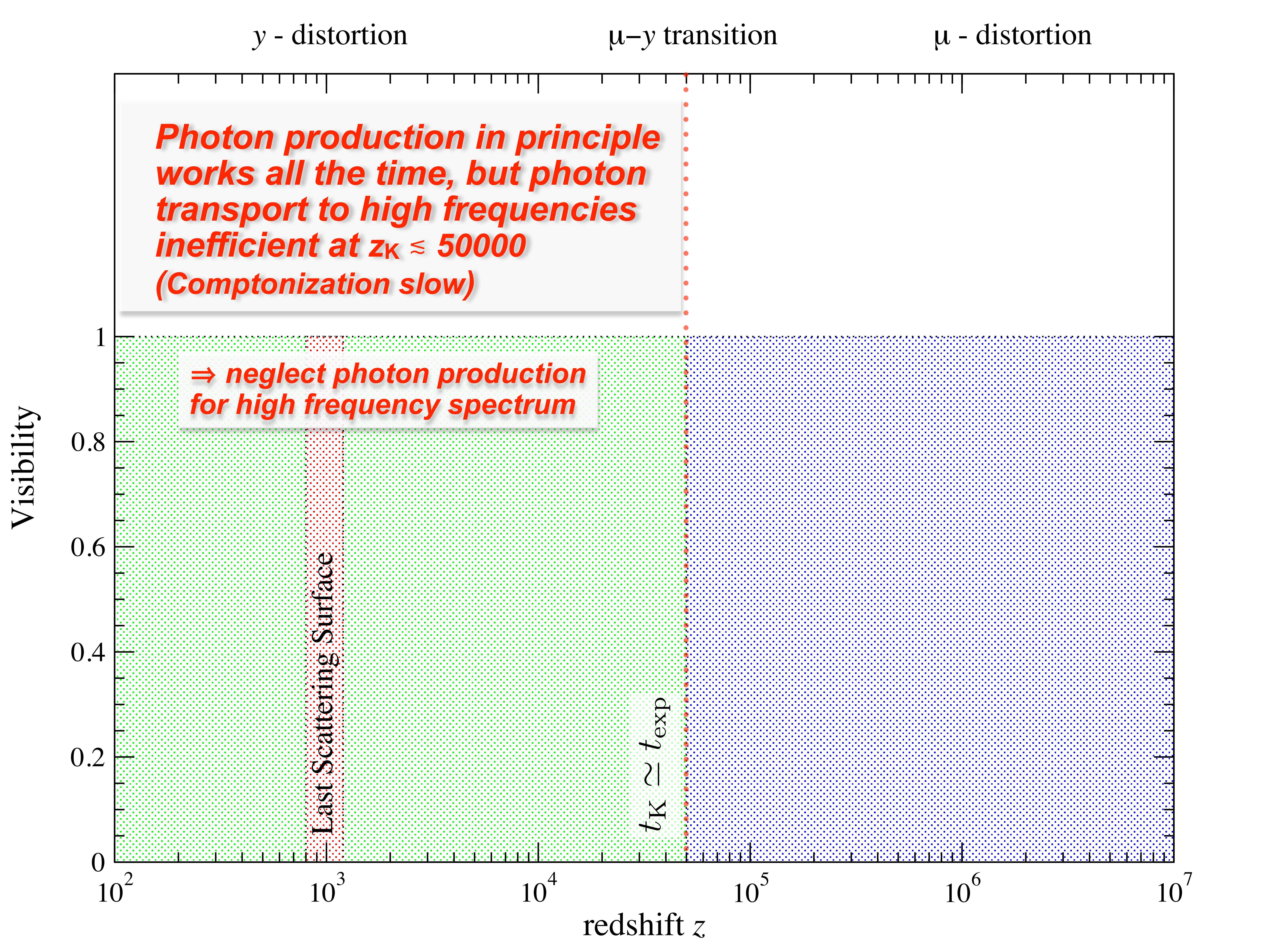
redshift z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Photon production in principle works all the time, but photon transport to high frequencies inefficient at $z_K \lesssim 50000$ (Comptonization slow)
⇒ neglect photon production for high frequency spectrum
Last Scattering Surface
$t_K \simeq t_{exp}$
Visibility
1
0.8
0.6
0.4
0.2
0
10^2
10^3
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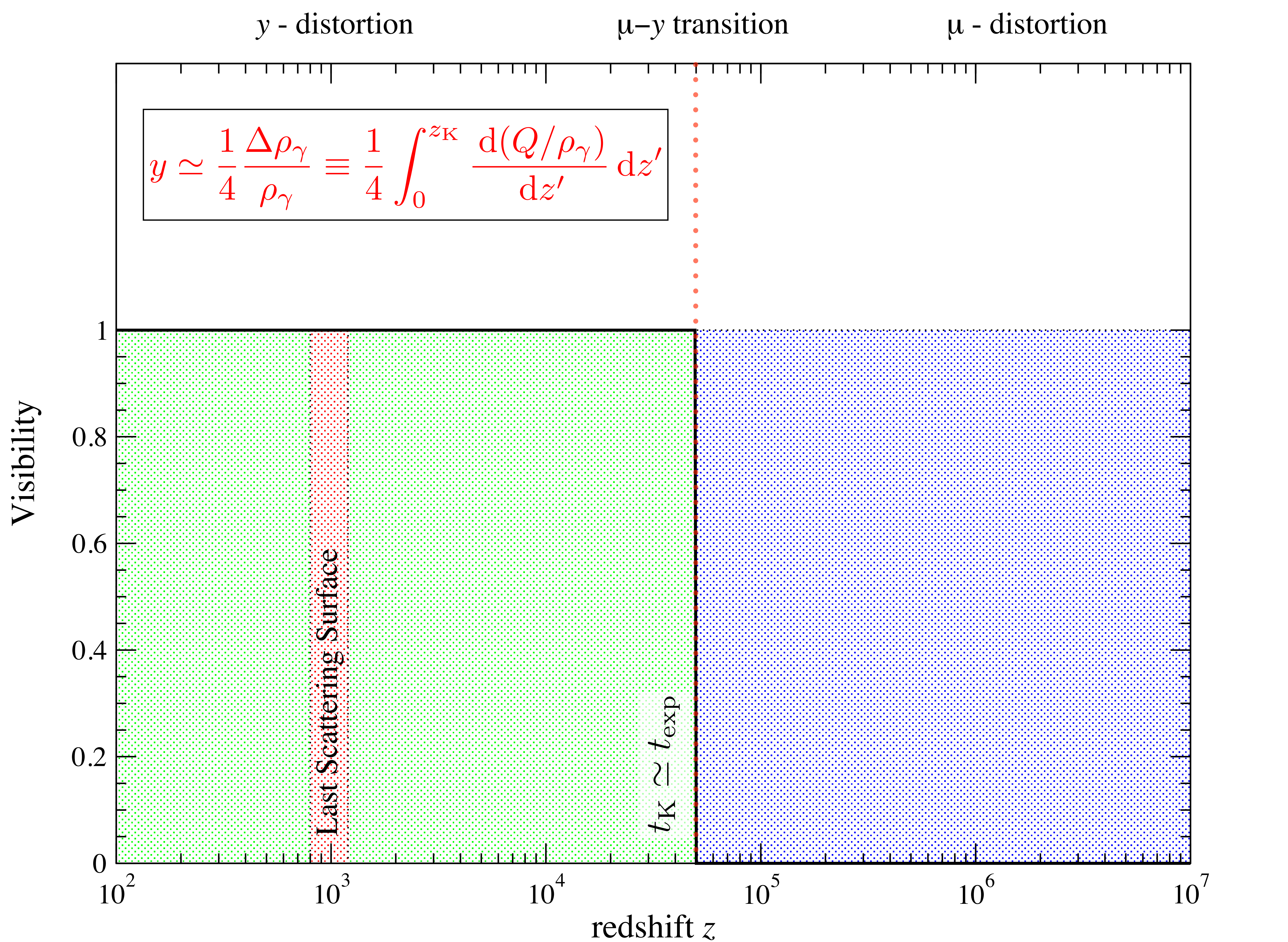
redshift z



y - distortion
μ–y transition
μ - distortion
$y \simeq \frac{1}{4}\frac{\Delta\rho_\gamma}{\rho_\gamma} \equiv \frac{1}{4}\int_0^{z_K} \frac{d(Q/\rho_\gamma)}{dz'}\,dz'$
Visibility
Last Scattering Surface
$t_K \simeq t_{exp}$
redshift z
0
0.2
0.4
0.6
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1
10^2
10^3
10^4
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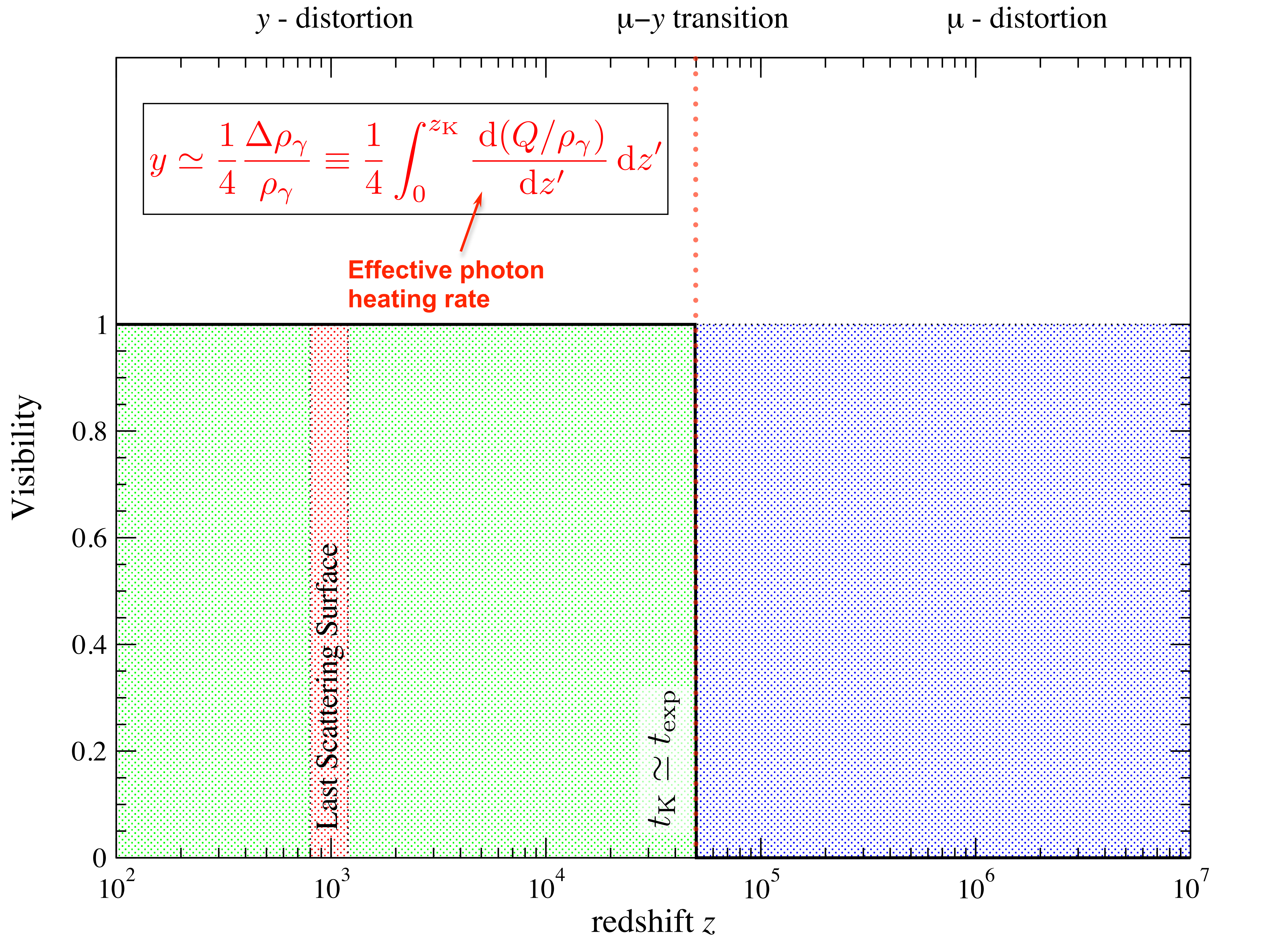
10^6
10^7



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Effective photon heating rate
Last Scattering Surface
$t_{\rm K} \simeq t_{\rm exp}$
Visibility
0
0.2
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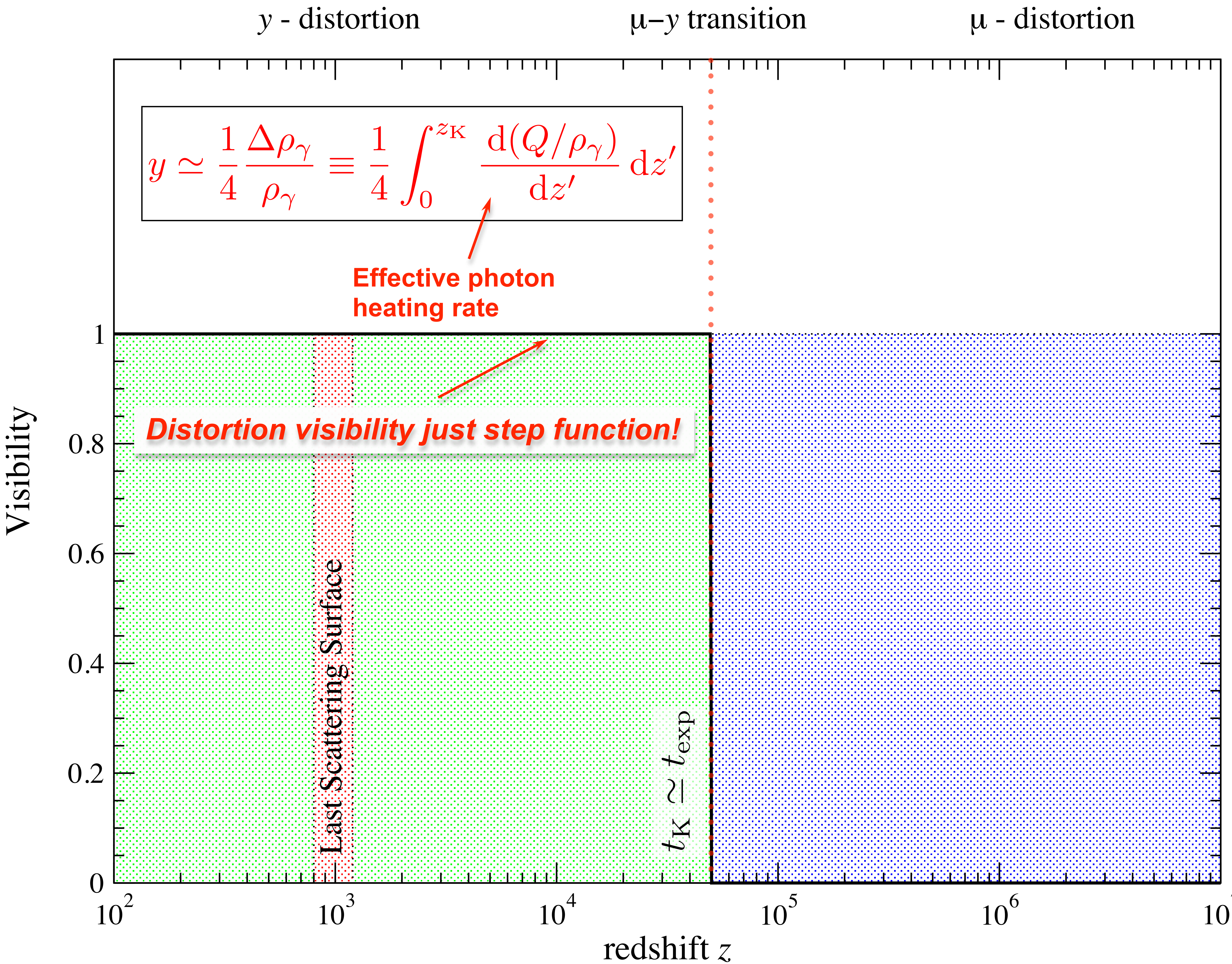
redshift z



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Effective photon heating rate
Distortion visibility just step function!
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Visibility
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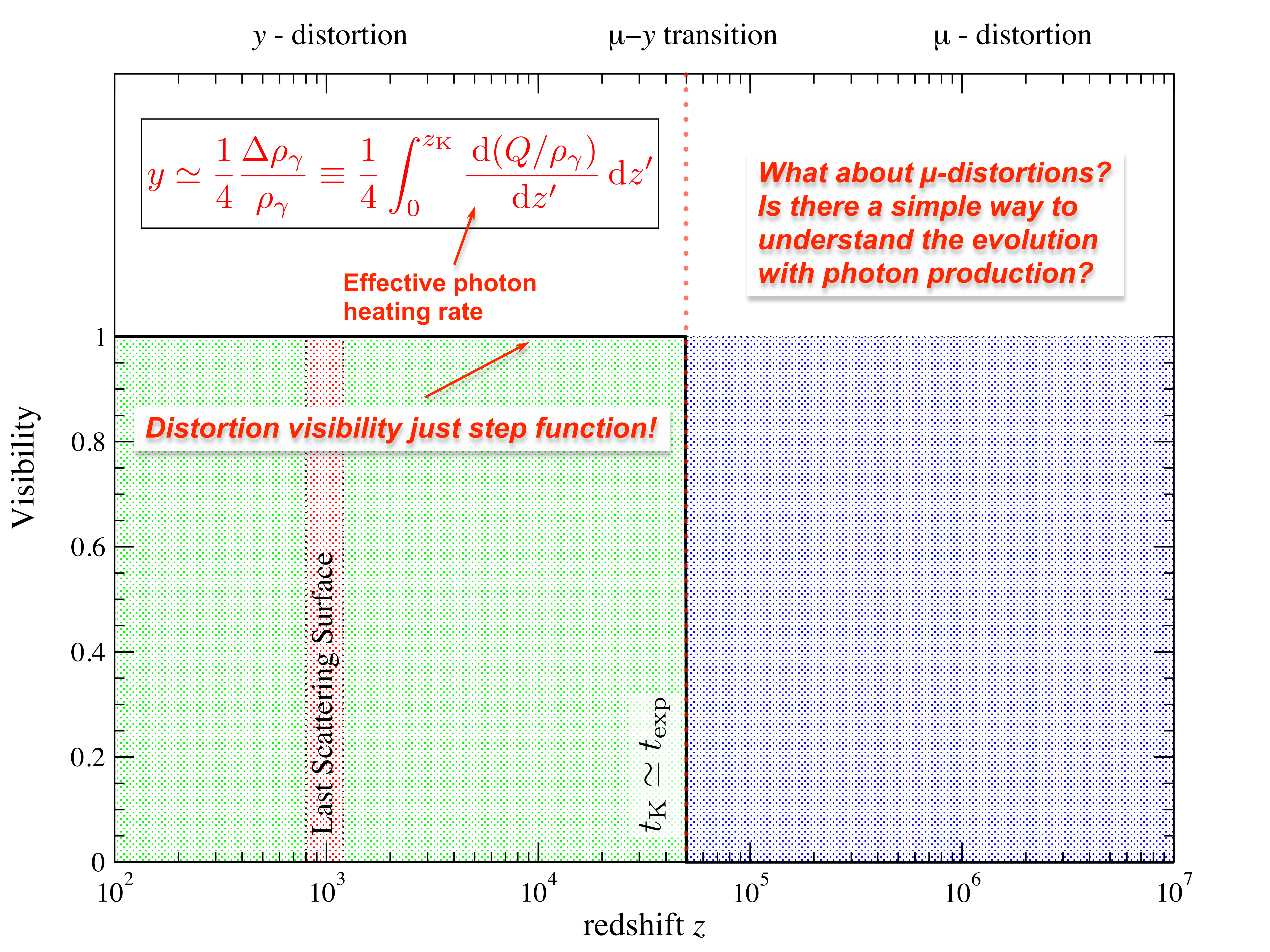

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Effective photon heating rate
What about μ-distortions? Is there a simple way to understand the evolution with photon production?
Distortion visibility just step function!
Last Scattering Surface
$t_\mathrm{K} \simeq t_\mathrm{exp}$
Visibility
0
0.2
0.4
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0.8
1
10^2
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redshift z

Analytic Approximation for μ-distortion

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- *Comptonization efficient!* $\Longrightarrow \quad \left.\frac{\mathrm{d}n}{\mathrm{d}\tau}\right|_{\mathrm{C}} + \left.\frac{\mathrm{d}n}{\mathrm{d}\tau}\right|_{\mathrm{em/abs}} \approx 0$

Analytic Approximation for μ-distortion

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- *low frequency limit & small distortion* $\Longrightarrow$ $$\mu(x, z) \approx \mu_0(z)\,\mathrm{e}^{-x_{\mathrm{c}}(z)/x}$$

(e.g., see Sunyaev & Zeldovich, 1970, ApSS, 7, 20; Hu 1995, PhD Thesis)

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chemical potential at high frequencies

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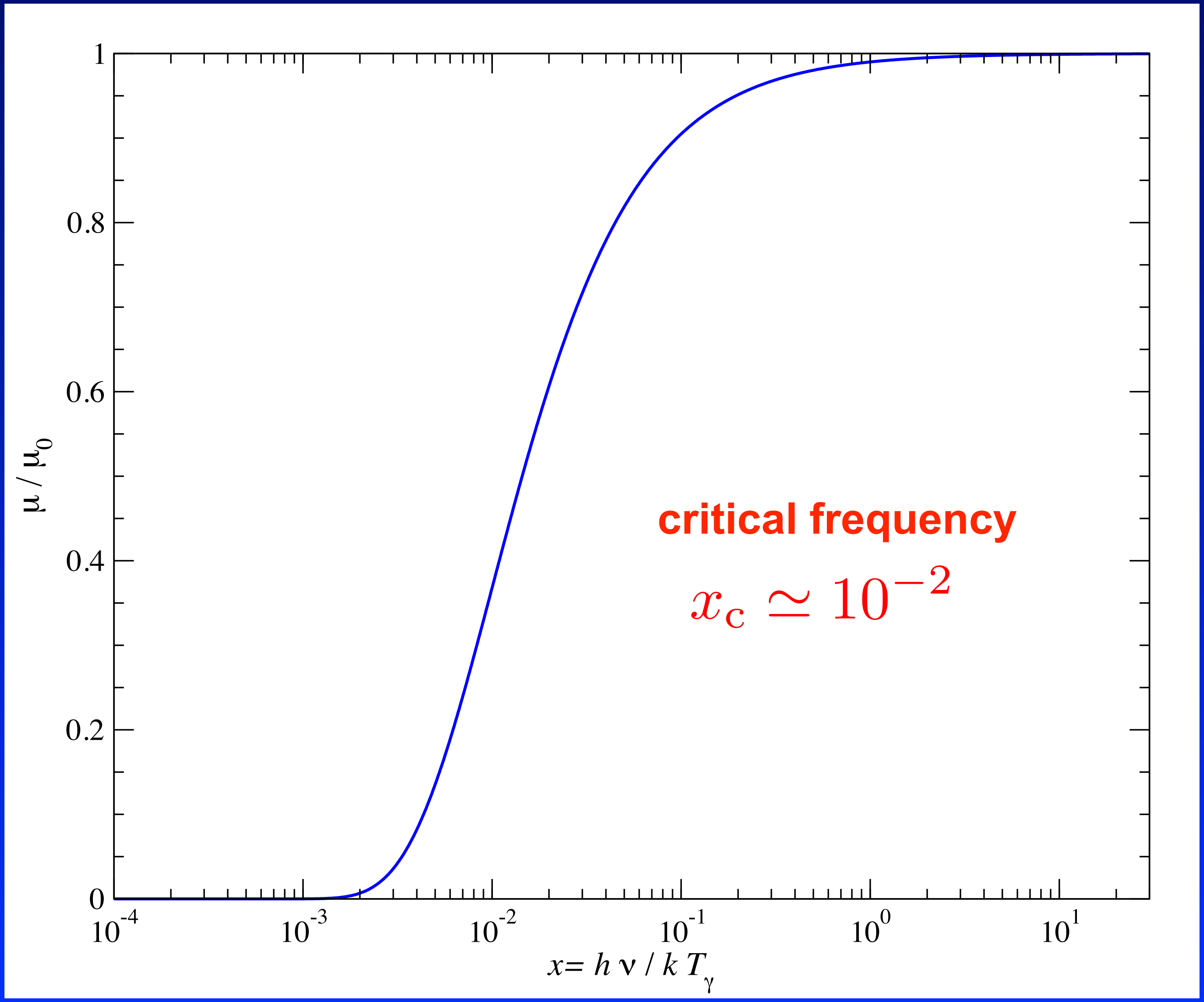

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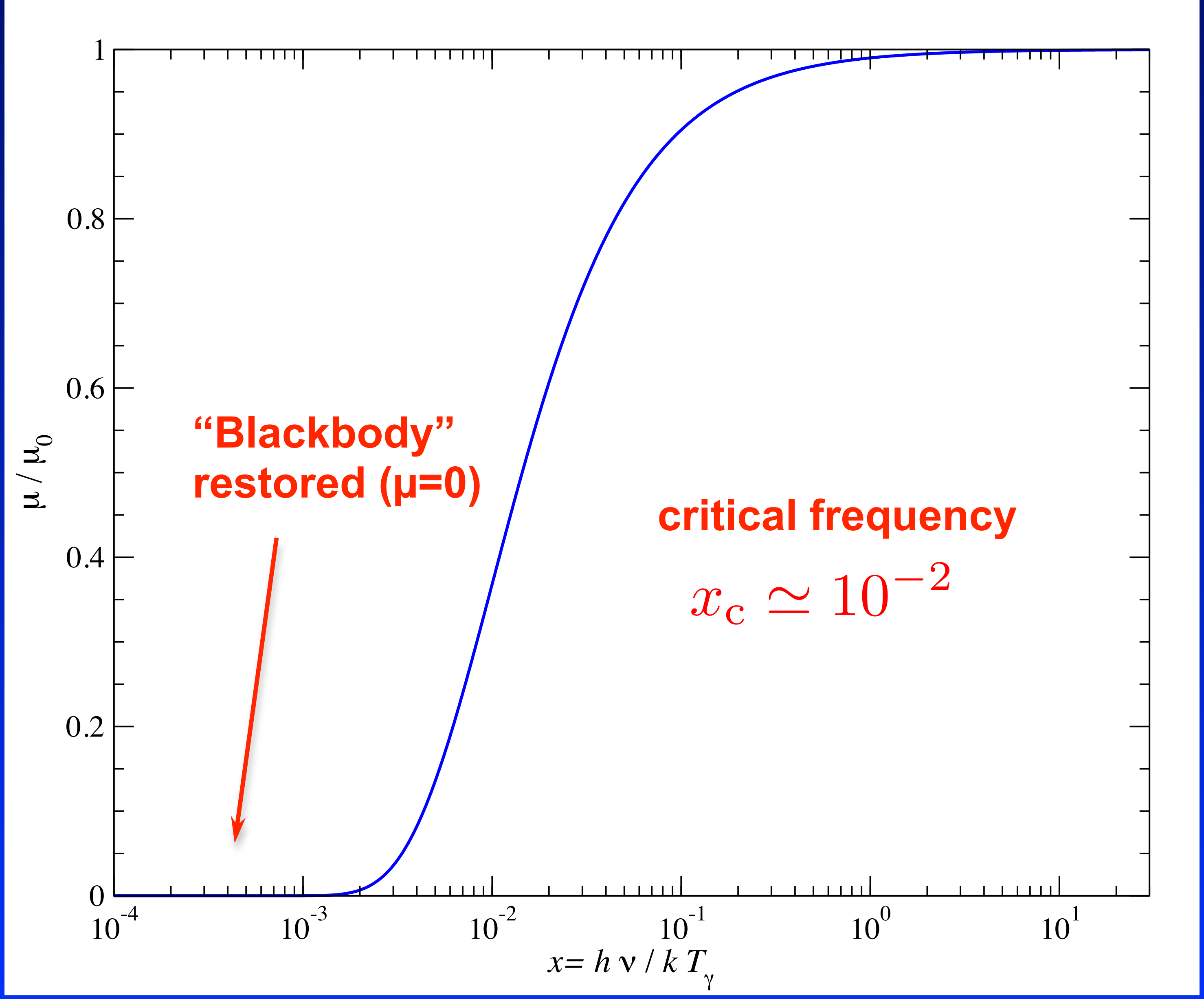

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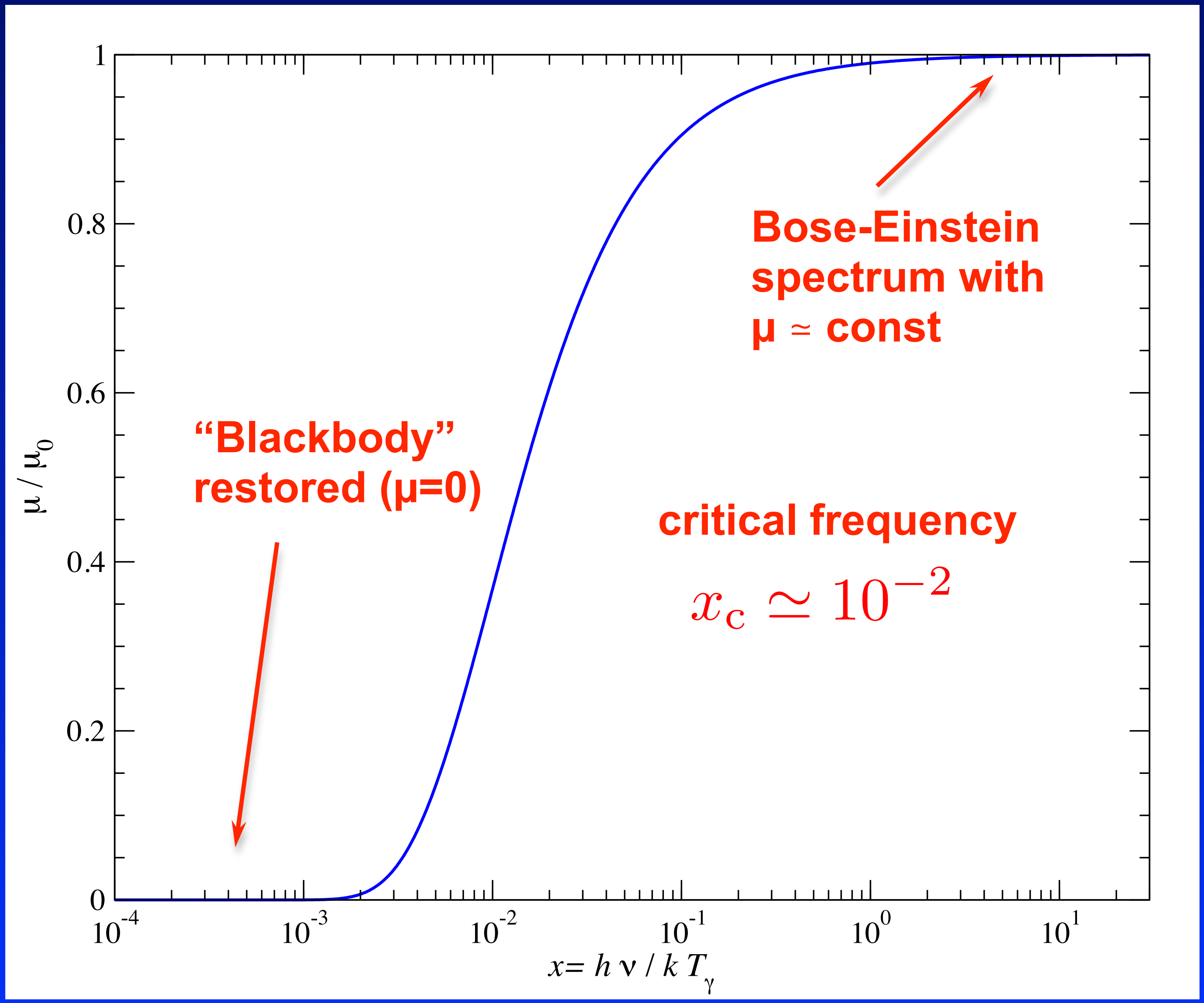

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(e.g., see Sunyaev & Zeldovich, 1970, ApSS, 7, 20; Hu 1995, PhD Thesis)

Last step: How does $\mu_0(z)$ depend on z?

Analytic Approximation for μ-distortion

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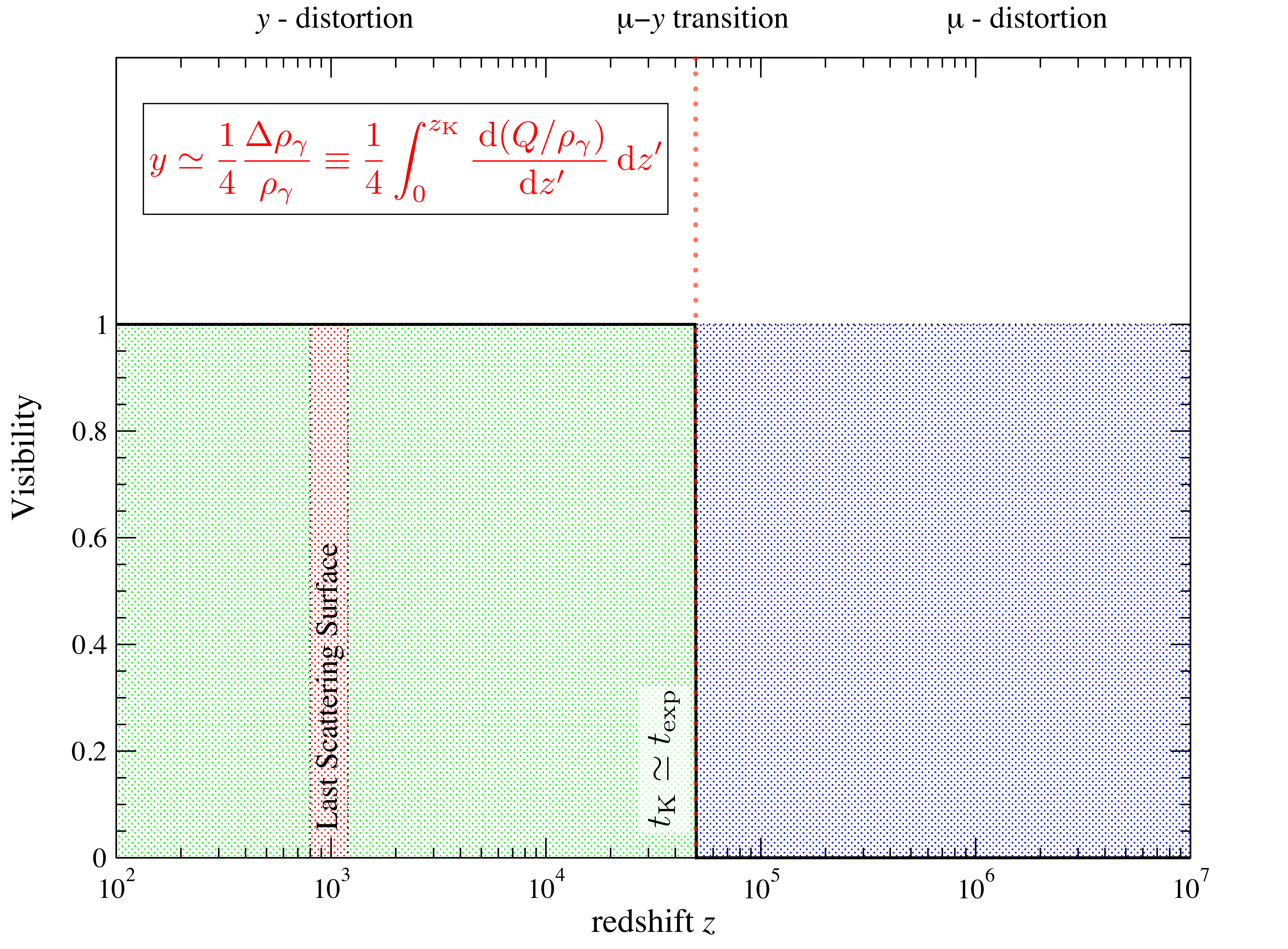
- *Transition between μ and y modeled as simple step function*



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Visibility
Last Scattering Surface
$t_K \simeq t_{exp}$
redshift z
0
0.2
0.4
0.6
0.8
1
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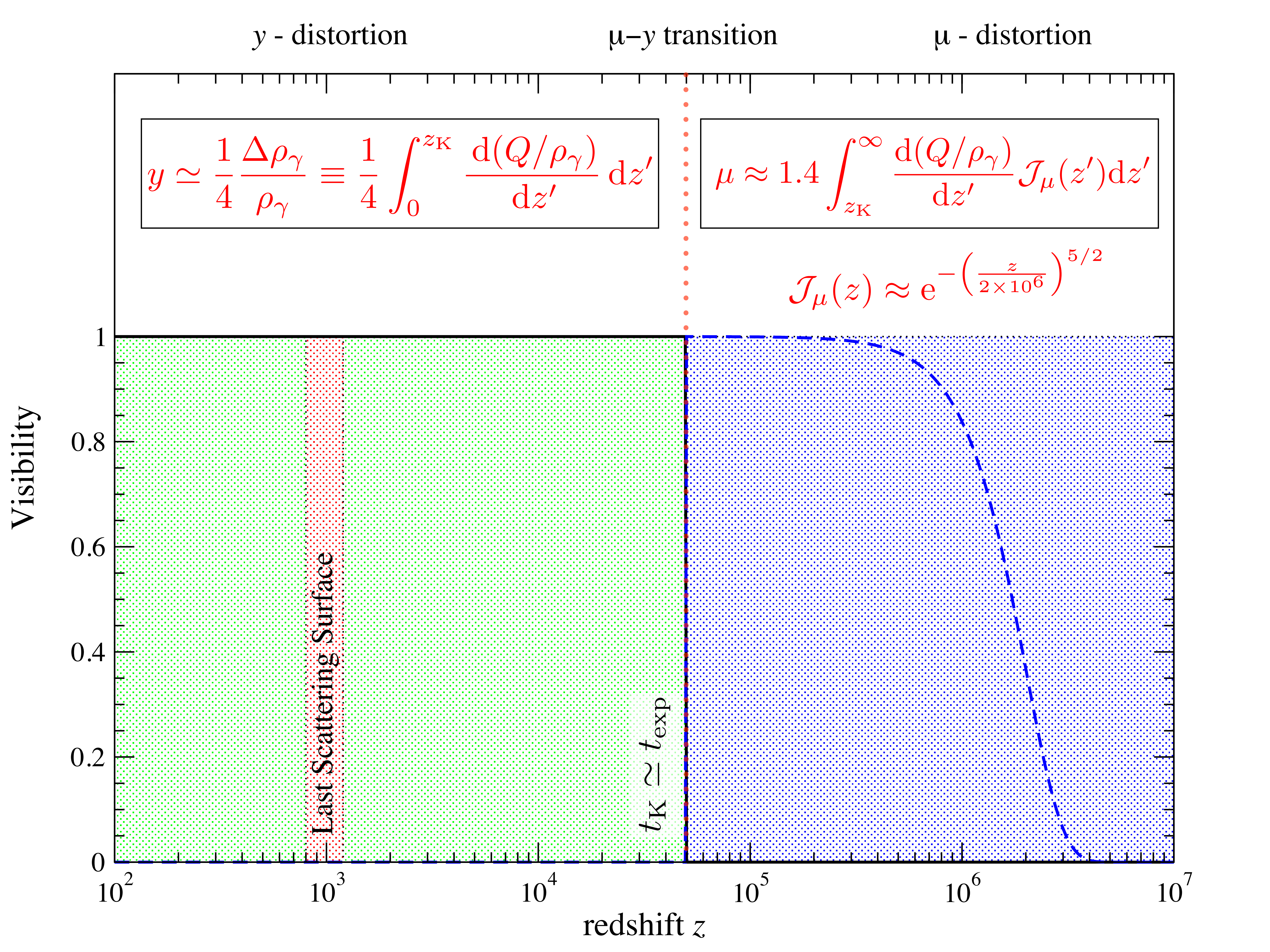
10^3
10^4
10^5
10^6
10^7



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$\mu \approx 1.4\int_{z_\mathrm{K}}^{\infty} \frac{\mathrm{d}(Q/\rho_\gamma)}{\mathrm{d}z'}\mathcal{J}_\mu(z')\mathrm{d}z'$
$\mathcal{J}_\mu(z) \approx \mathrm{e}^{-\left(\frac{z}{2\times10^6}\right)^{5/2}}$
Last Scattering Surface
$t_\mathrm{K} \simeq t_\mathrm{exp}$
Visibility
0
0.2
0.4
0.6
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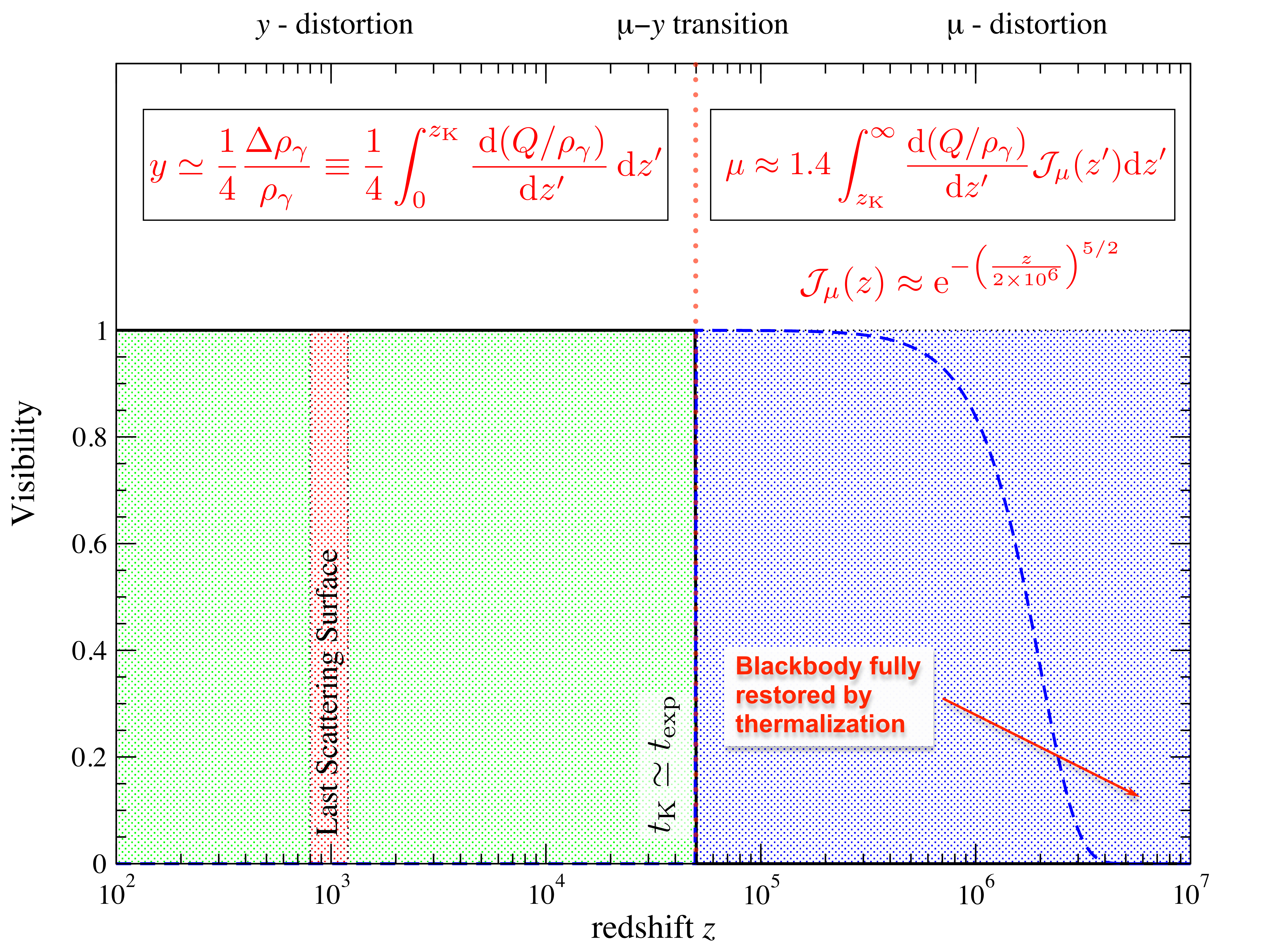
1
10^2
10^3
10^4
10^5
10^6
10^7
redshift z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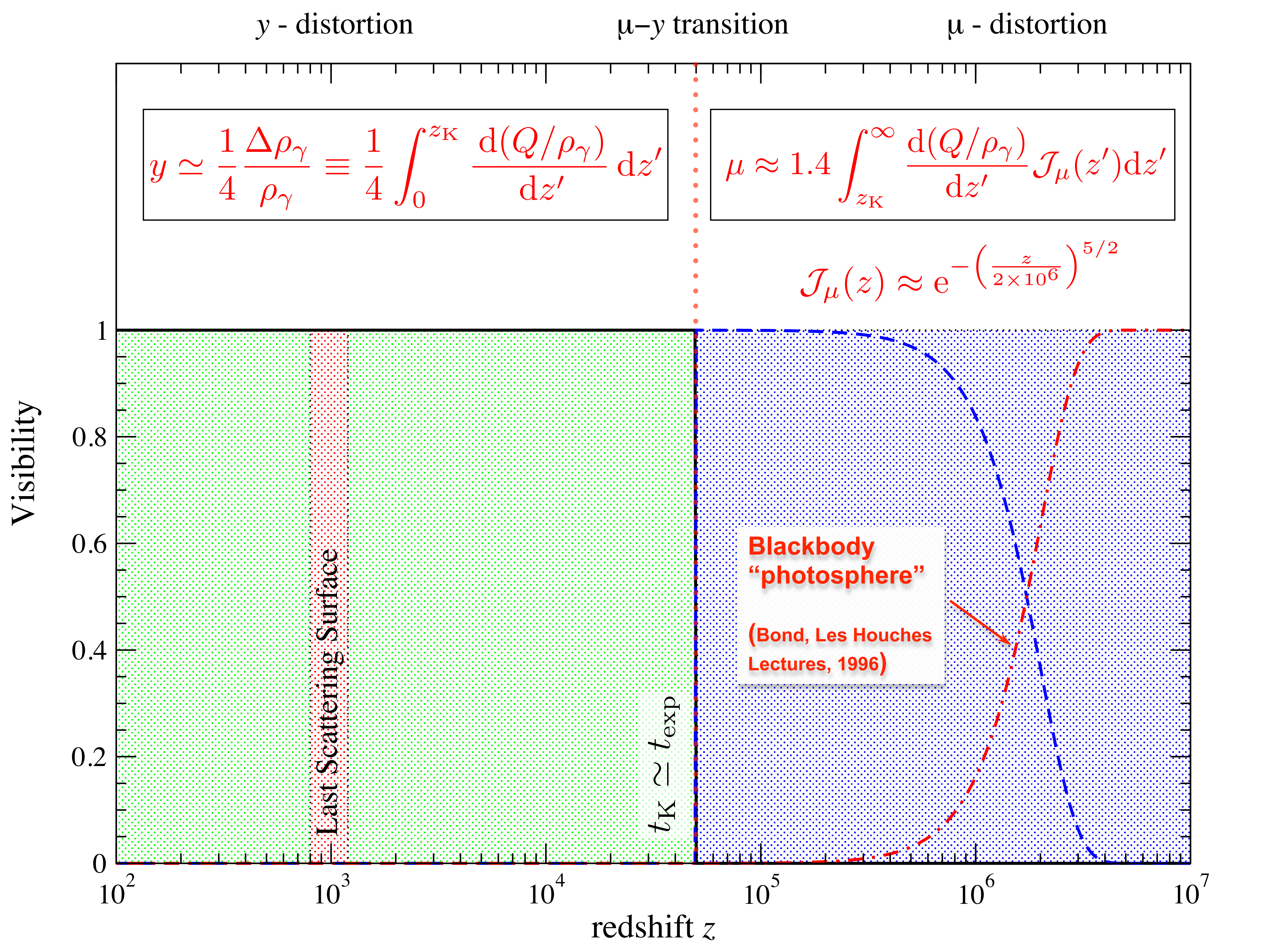
μ - distortion
$y \simeq \frac{1}{4}\frac{\Delta\rho_\gamma}{\rho_\gamma} \equiv \frac{1}{4}\int_0^{z_K} \frac{d(Q/\rho_\gamma)}{dz'}\,dz'$
$\mu \approx 1.4 \int_{z_K}^{\infty} \frac{d(Q/\rho_\gamma)}{dz'} \mathcal{J}_\mu(z')dz'$
$\mathcal{J}_\mu(z) \approx e^{-\left(\frac{z}{2\times10^6}\right)^{5/2}}$
Visibility
1
0.8
0.6
0.4
0.2
0
Last Scattering Surface
$t_K \simeq t_{exp}$
Blackbody fully
restored by
thermalization
10^2
10^3
10^4
10^5
10^6
10^7
redshift z



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μ–y transition
μ - distortion
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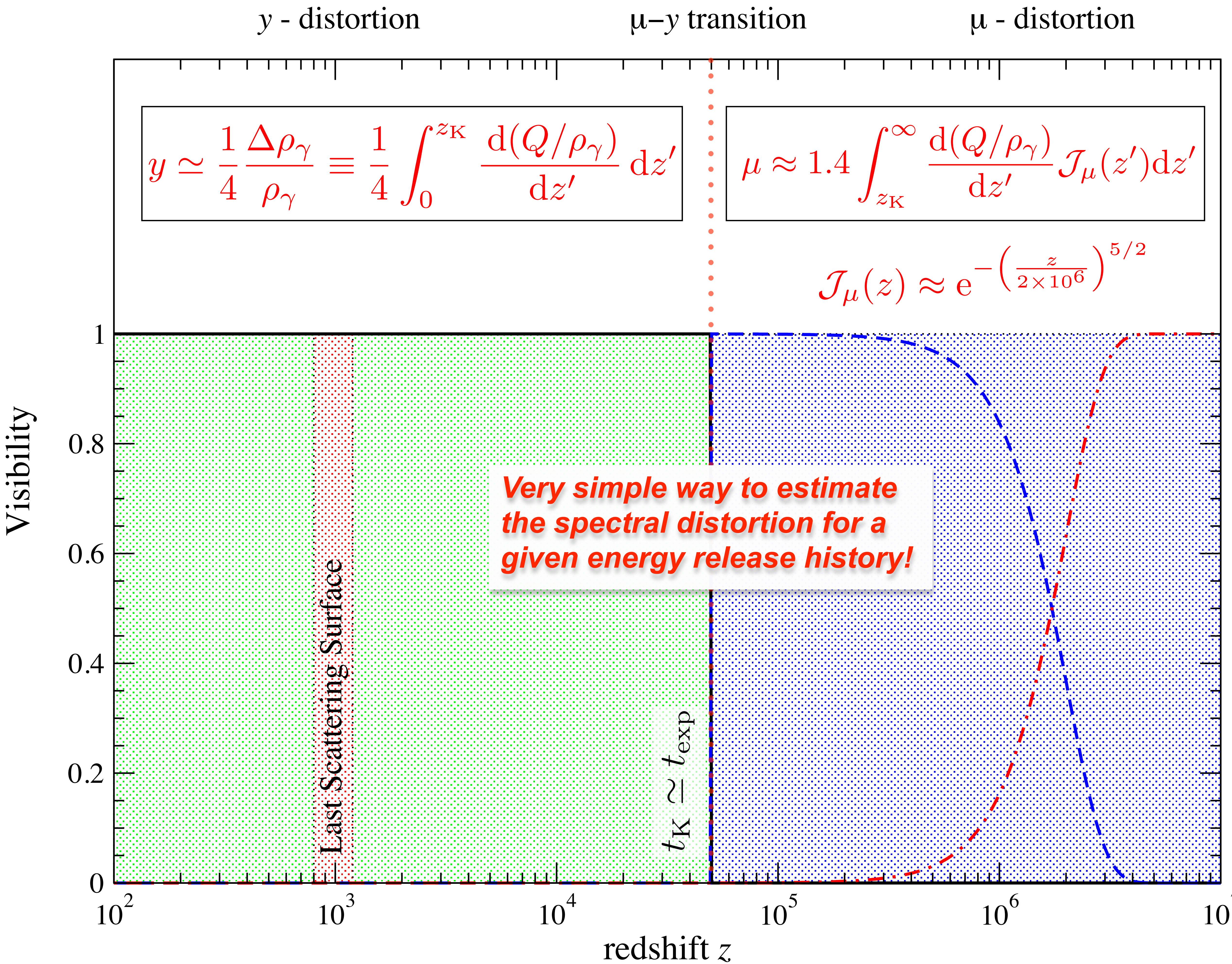
$\mathcal{J}_\mu(z) \approx e^{-\left(\frac{z}{2\times10^6}\right)^{5/2}}$
Visibility
redshift z
Last Scattering Surface
$t_K \simeq t_{exp}$
Blackbody "photosphere"
(Bond, Les Houches Lectures, 1996)



y - distortion
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$\mu \approx 1.4\int_{z_K}^{\infty} \frac{d(Q/\rho_\gamma)}{dz'}\mathcal{J}_\mu(z')dz'$
$\mathcal{J}_\mu(z) \approx e^{-\left(\frac{z}{2\times10^6}\right)^{5/2}}$
Very simple way to estimate the spectral distortion for a given energy release history!
Last Scattering Surface
$t_K \simeq t_{exp}$
Visibility
0
0.2
0.4
0.6
0.8
1
10^2
10^3
10^4
10^5
10^6
10^7
redshift z

Distortion visibility function

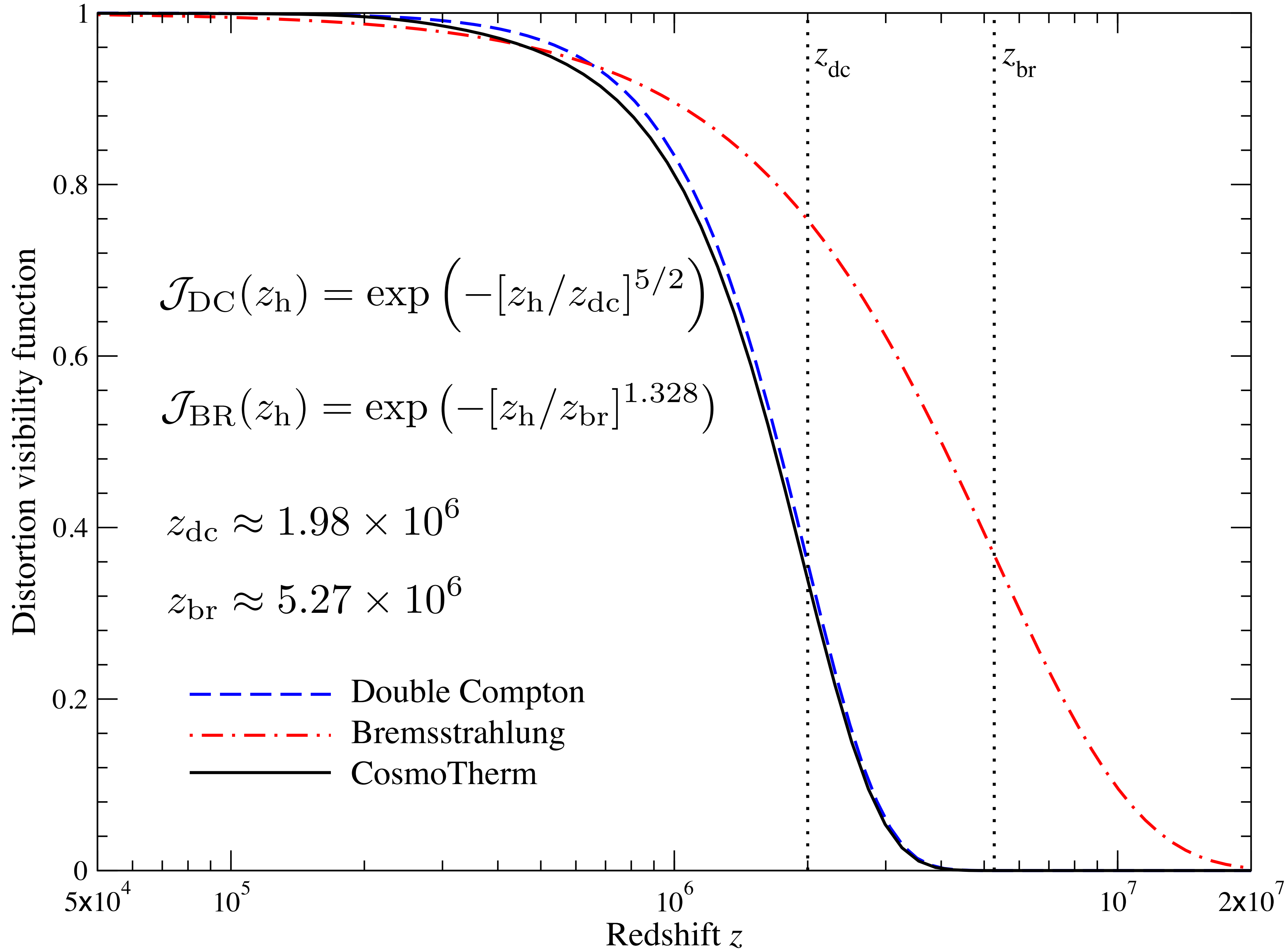


Figure 4.9: Distortion visibility function. We compare $\mathcal{J}_{\rm DC}(z_{\rm h})$, $\mathcal{J}_{\rm BR}(z_{\rm h})$ and the numerical result obtained with CosmoTherm. DC emission significantly change the thermalization efficiency. Deviations from the numerical result can be captured by adding several effects, as discussed in Sect. 4.6.

Distortion visibility function

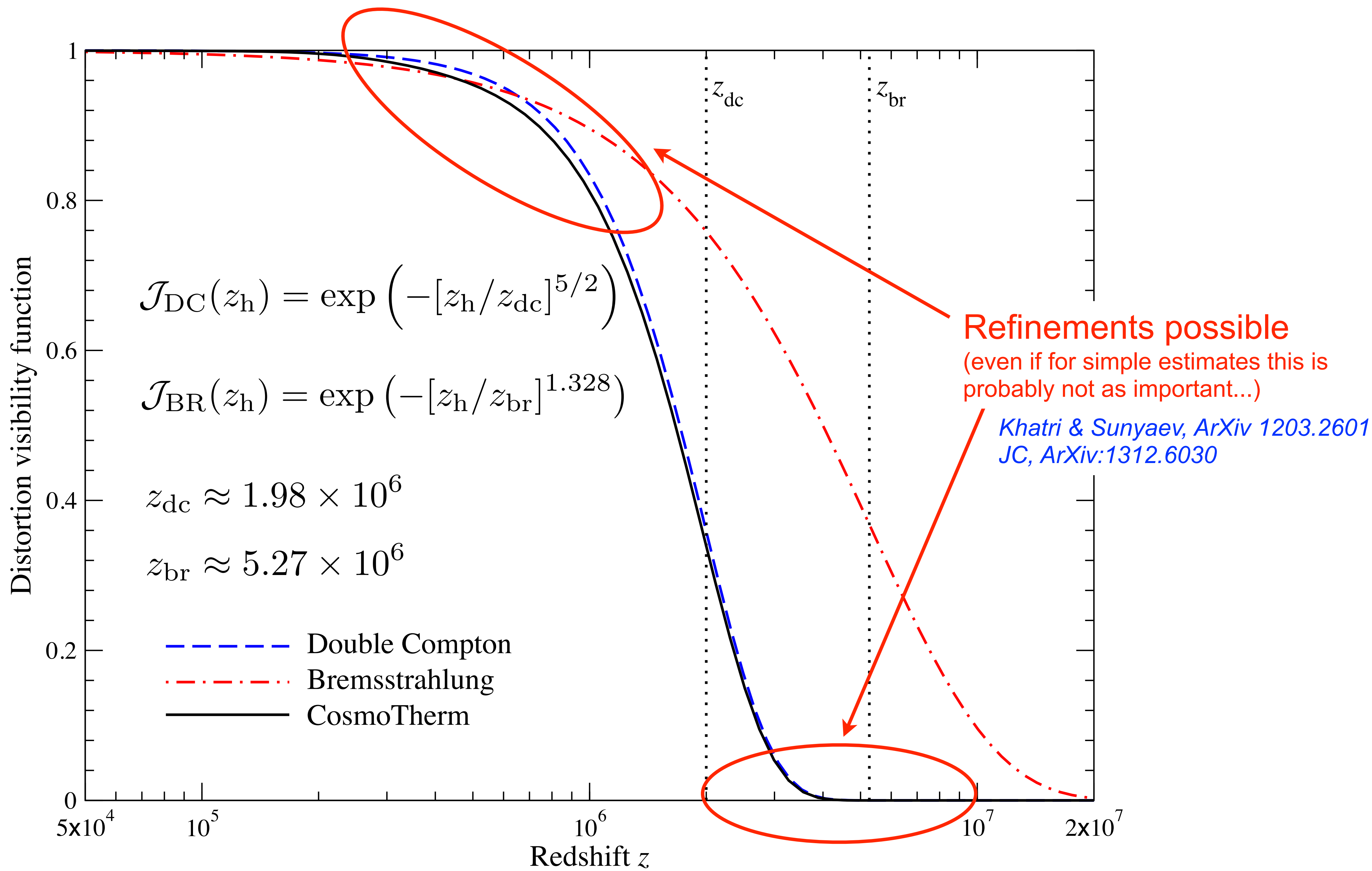
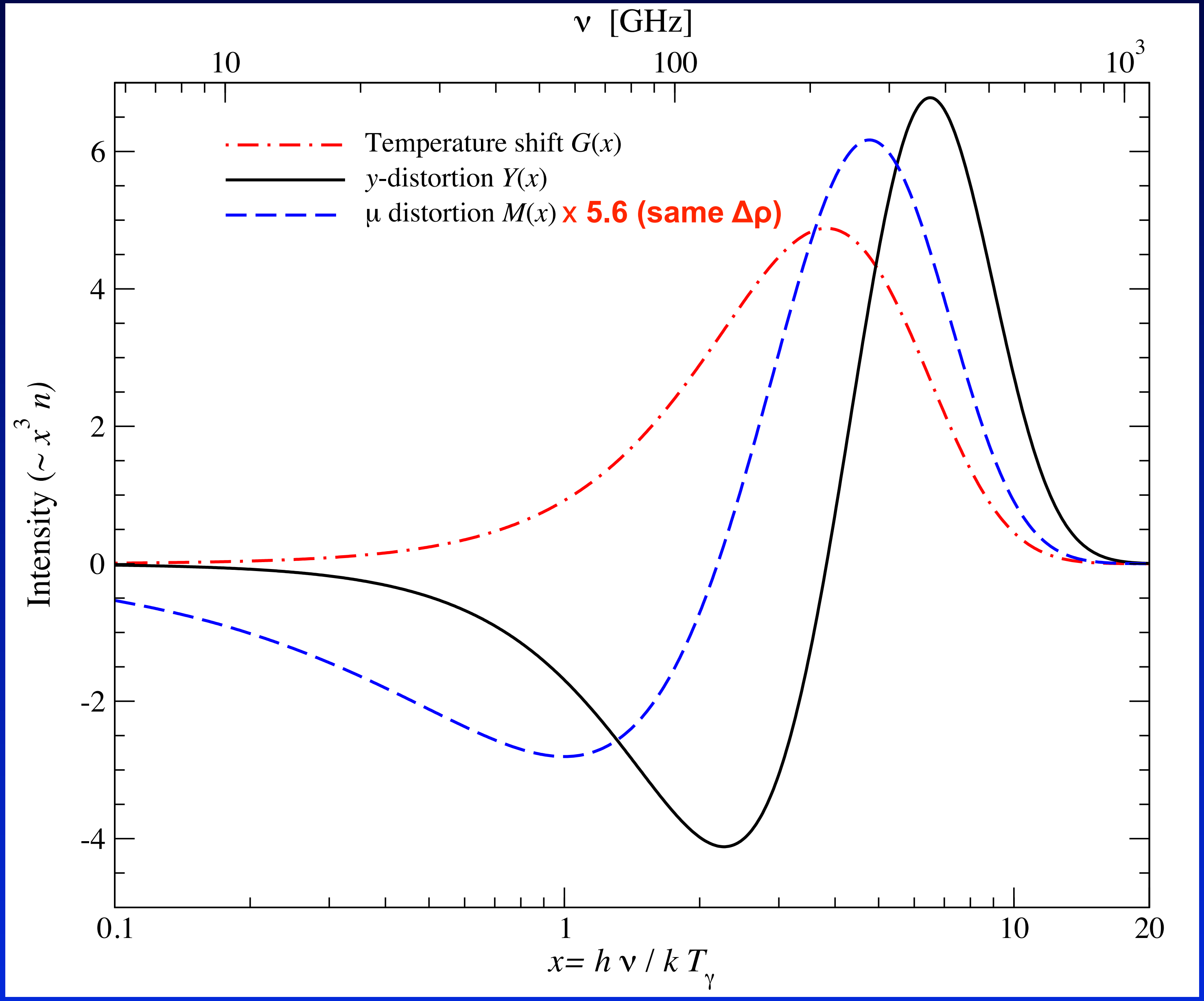


Figure 4.9: Distortion visibility function. We compare $\mathcal{J}_{\rm DC}(z_{\rm h})$, $\mathcal{J}_{\rm BR}(z_{\rm h})$ and the numerical result obtained with CosmoTherm. DC emission significantly change the thermalization efficiency. Deviations from the numerical result can be captured by adding several effects, as discussed in Sect. 4.6.

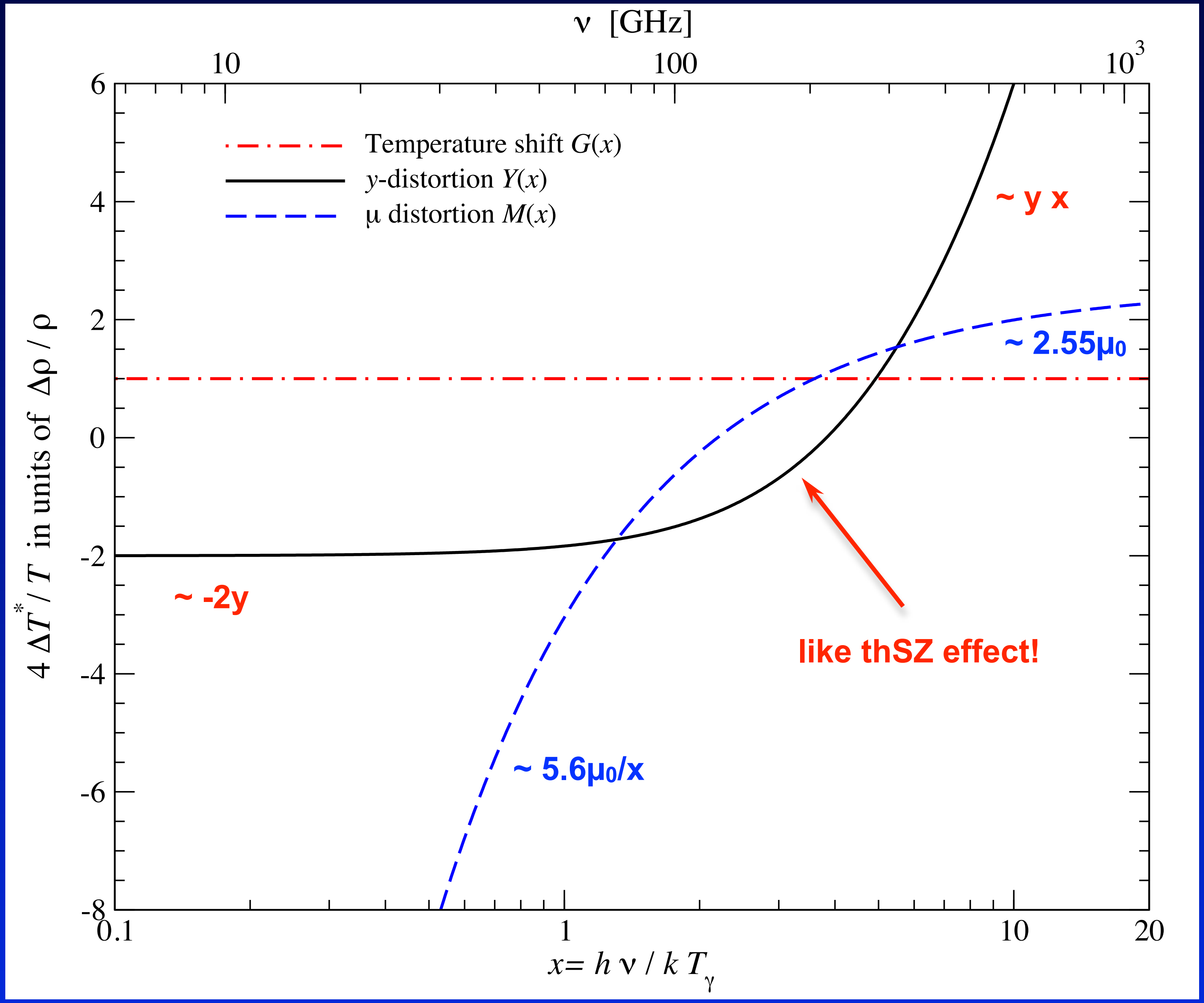
What about the μ-y transition regime?

Is the transition really as abrupt?

Temperature shift ↔ y-distortion ↔ μ-distortion



Temperature shift ↔ y-distortion ↔ μ-distortion



Same as before but as effective temperature

$$\frac{\Delta T^*}{T} \approx \frac{\Delta n(x)}{G(x)}$$

Transition from y-distortion → μ-distortion

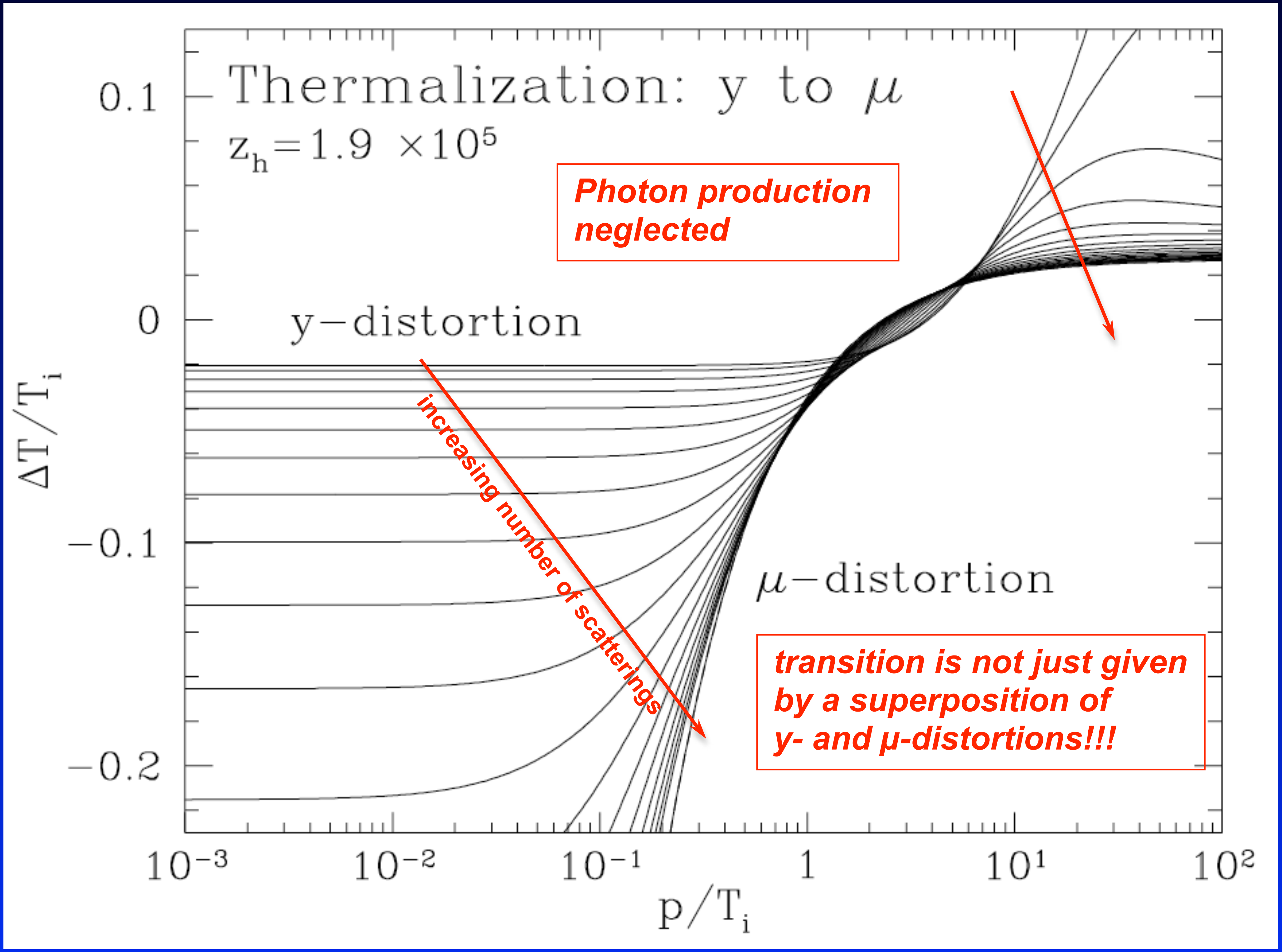
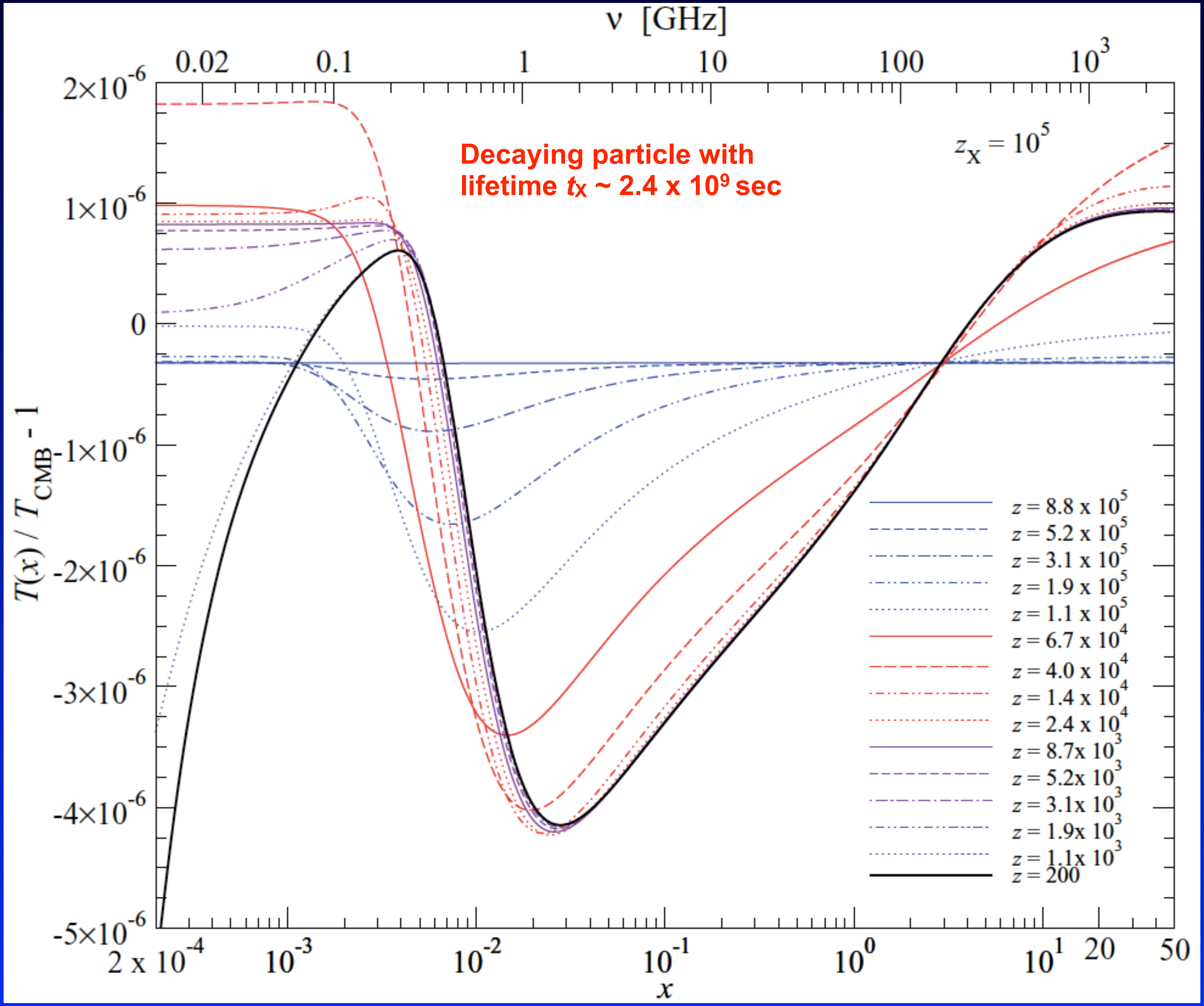


Figure from Wayne Hu's PhD thesis, 1995, but see also discussion in Burigana, 1991

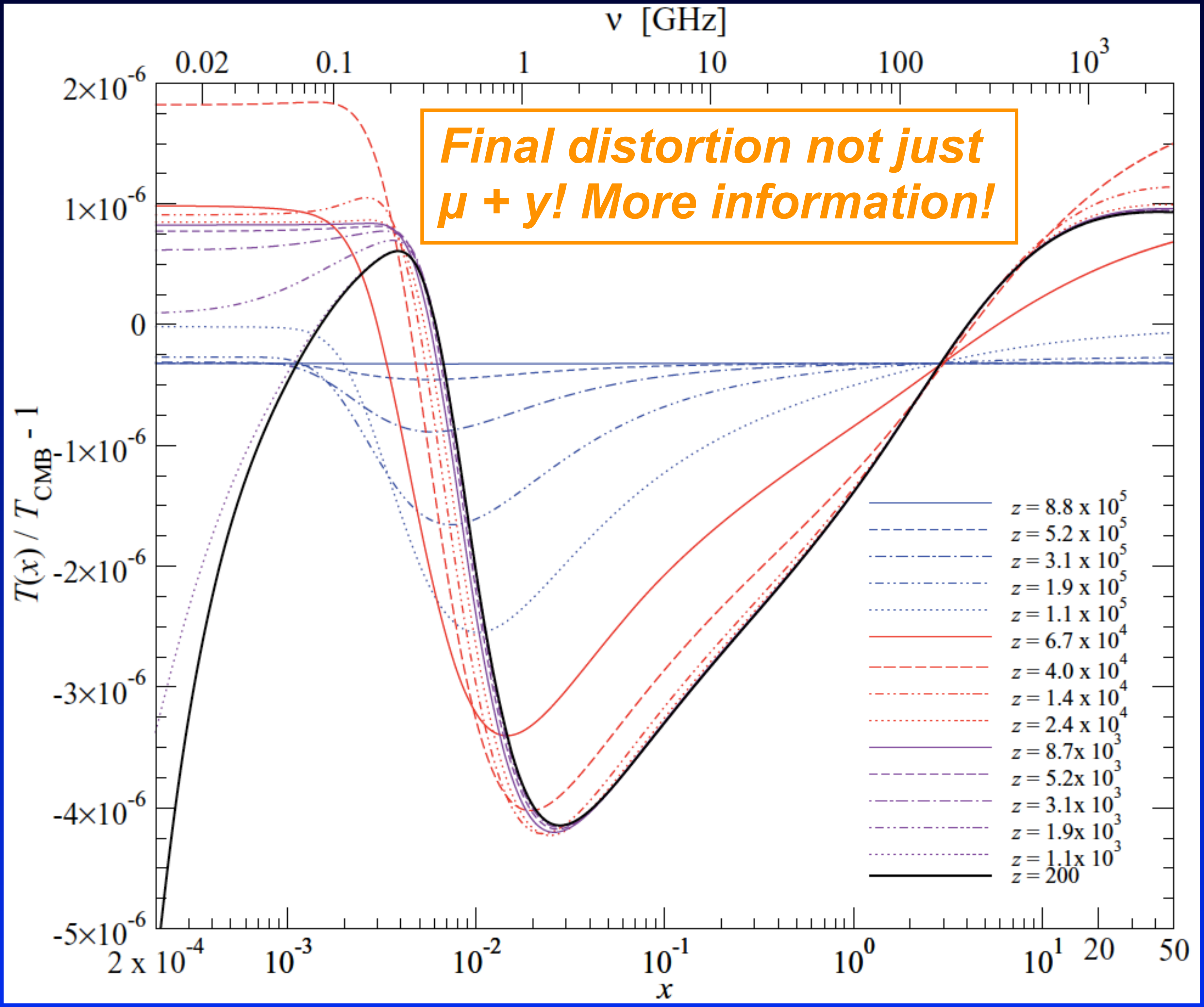
Distortion *not* just superposition of μ and y-distortion!



Computation carried out with *CosmoTherm*
(JC & Sunyaev 2011)

- First explicit calculation that showed that there is more!

Distortion *not* just superposition of μ and y-distortion!



Computation carried out with *CosmoTherm*
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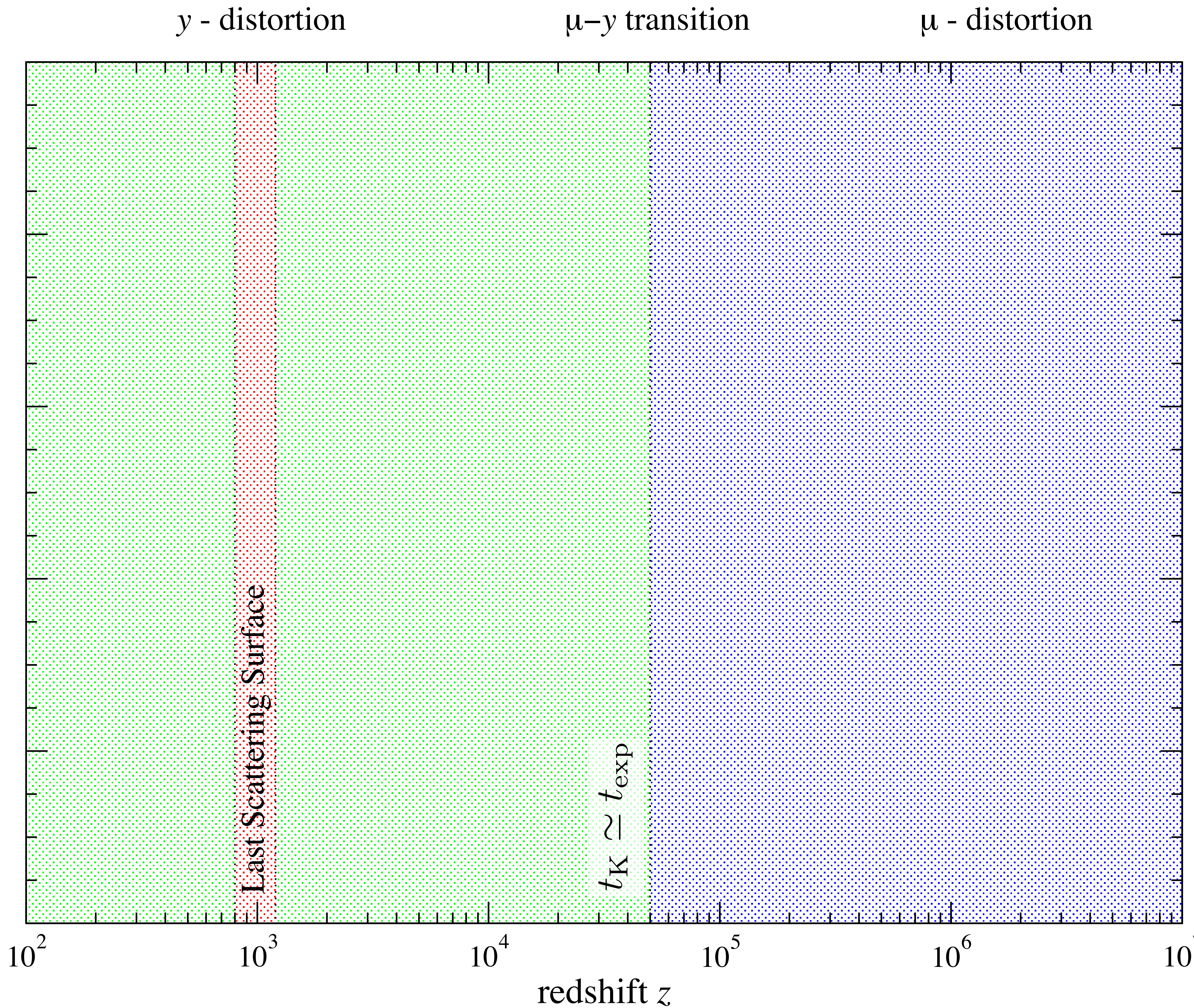
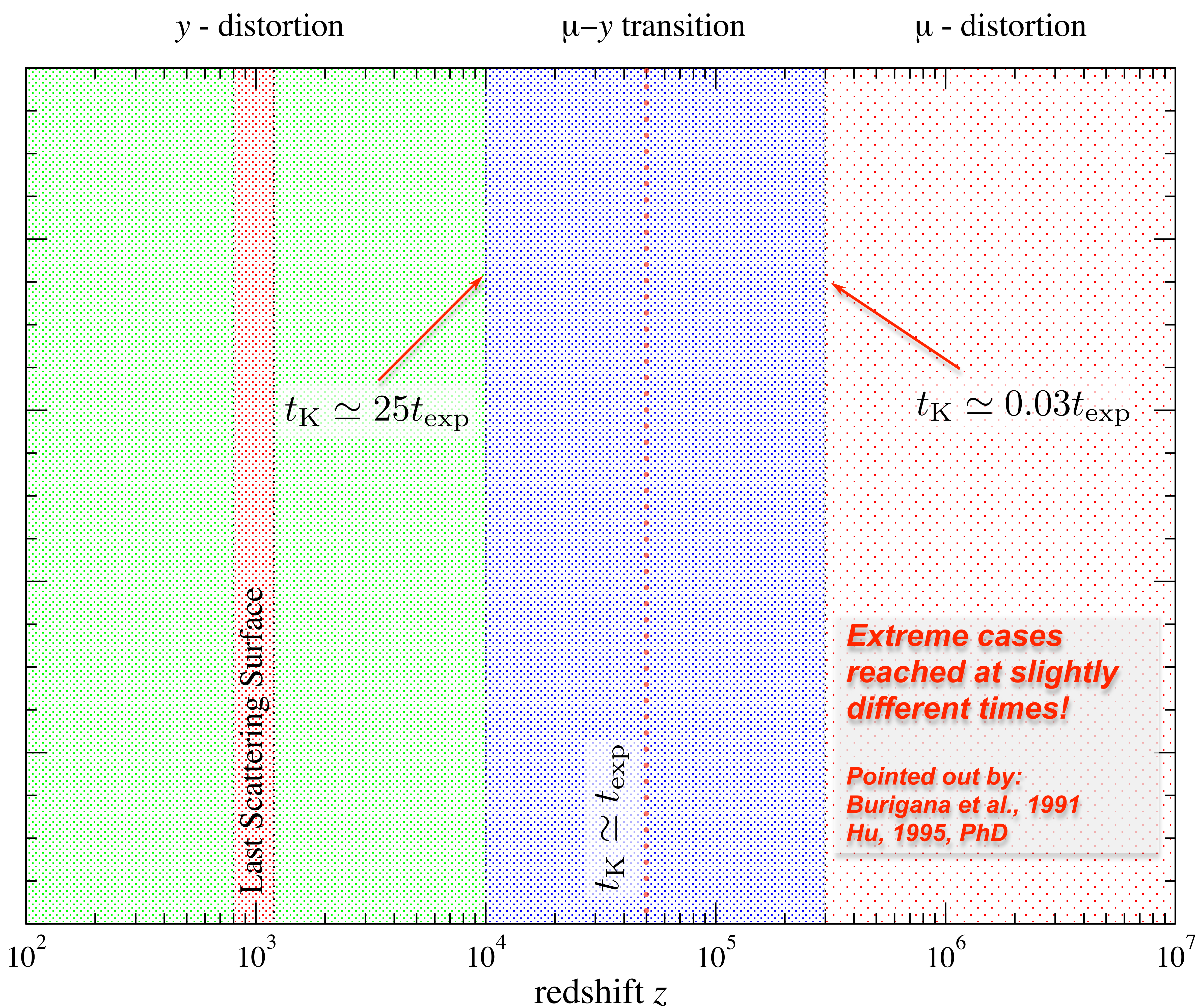
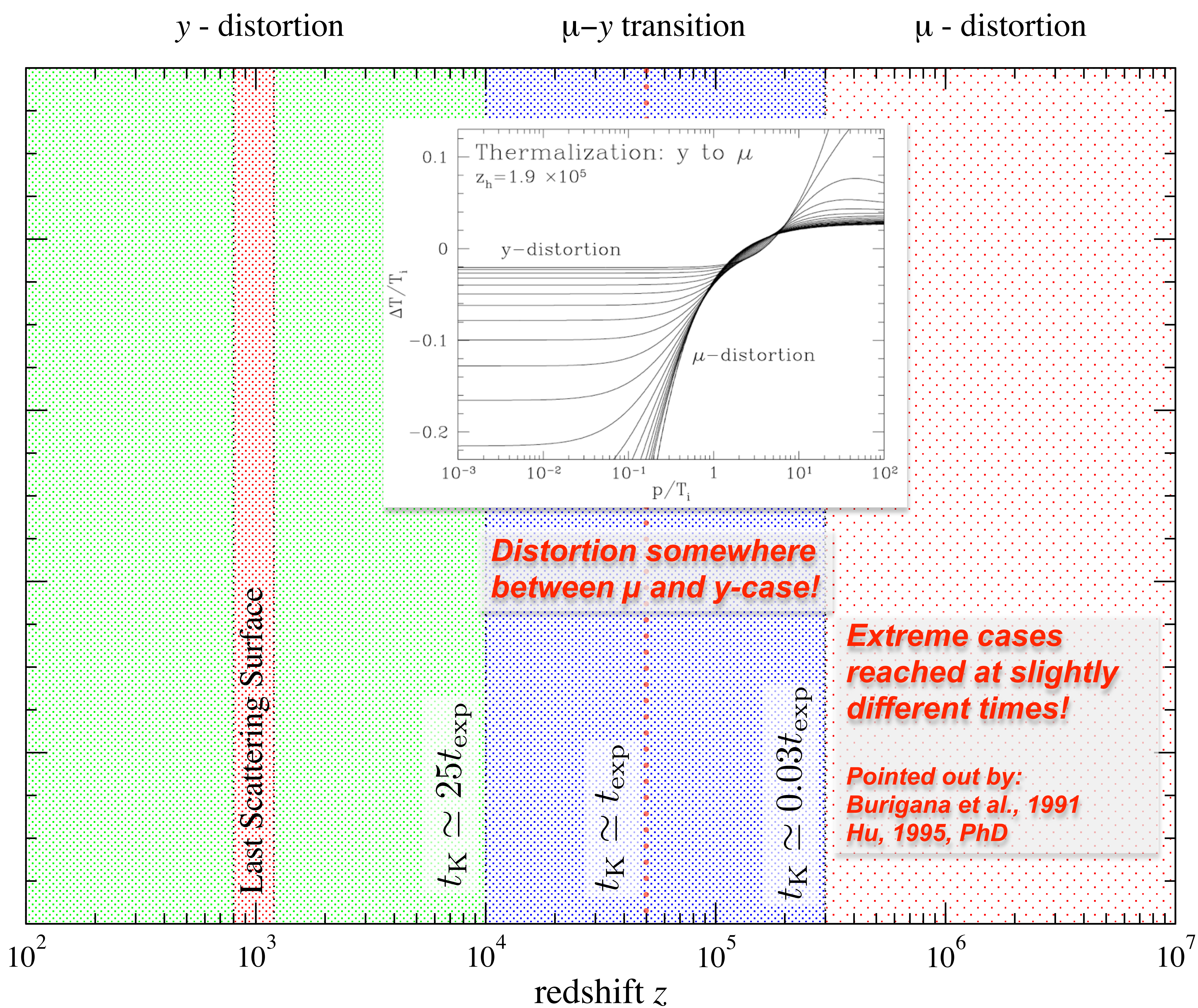
- First explicit calculation that showed that there is more!



y - distortion
μ−y transition
μ - distortion
Last Scattering Surface
$t_{\rm K} \simeq t_{\rm exp}$
10^2
10^3
10^4
10^5
10^6
10^7
redshift z



y - distortion
μ–y transition
μ - distortion
Last Scattering Surface
$t_K \simeq 25 t_{exp}$
$t_K \simeq t_{exp}$
$t_K \simeq 0.03 t_{exp}$
Extreme cases reached at slightly different times!
Pointed out by:
Burigana et al., 1991
Hu, 1995, PhD
10^2
10^3
10^4
10^5
10^6
10^7
redshift z



y - distortion
μ–y transition
μ - distortion
Thermalization: y to μ
$z_h = 1.9 \times 10^5$
y–distortion
μ–distortion
ΔT/T_i
p/T_i
Distortion somewhere between μ and y-case!
Extreme cases reached at slightly different times!
Pointed out by:
Burigana et al., 1991
Hu, 1995, PhD
Last Scattering Surface
$t_K \simeq 25 t_{exp}$
$t_K \simeq t_{exp}$
$t_K \simeq 0.03 t_{exp}$
redshift z

Quasi-Exact Treatment of the Thermalization Problem

- *For real forecasts of future prospects a precise & fast method for computing the spectral distortion is needed!*
- *Case-by-case computation of the distortion (e.g., with CosmoTherm, JC & Sunyaev, 2012, ArXiv:1109.6552) still rather time-consuming*
- *But: distortions are small ⇒ thermalization problem becomes linear!*
- *Simple solution: compute "response function" of the thermalization problem ⇒ Green's function approach (JC, 2013, ArXiv:1304.6120)*
- *Final distortion for fixed energy-release history given by*

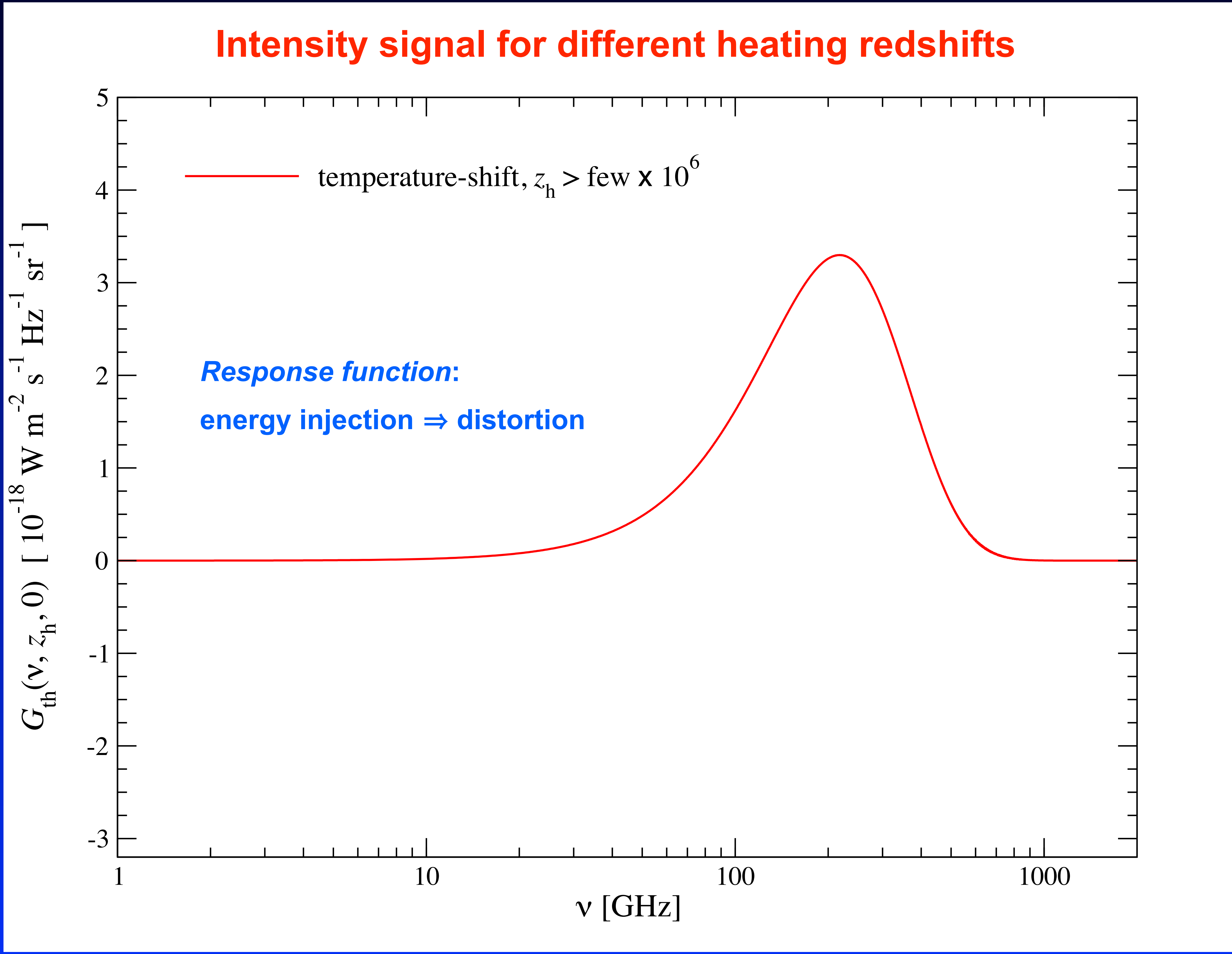
$$\Delta I_\nu \approx \int_0^\infty G_{\rm th}(\nu, z') \frac{\mathrm{d}(Q/\rho_\gamma)}{\mathrm{d}z'} \mathrm{d}z'$$

Thermalization Green's function

- *Fast and quasi-exact! No additional approximations!*

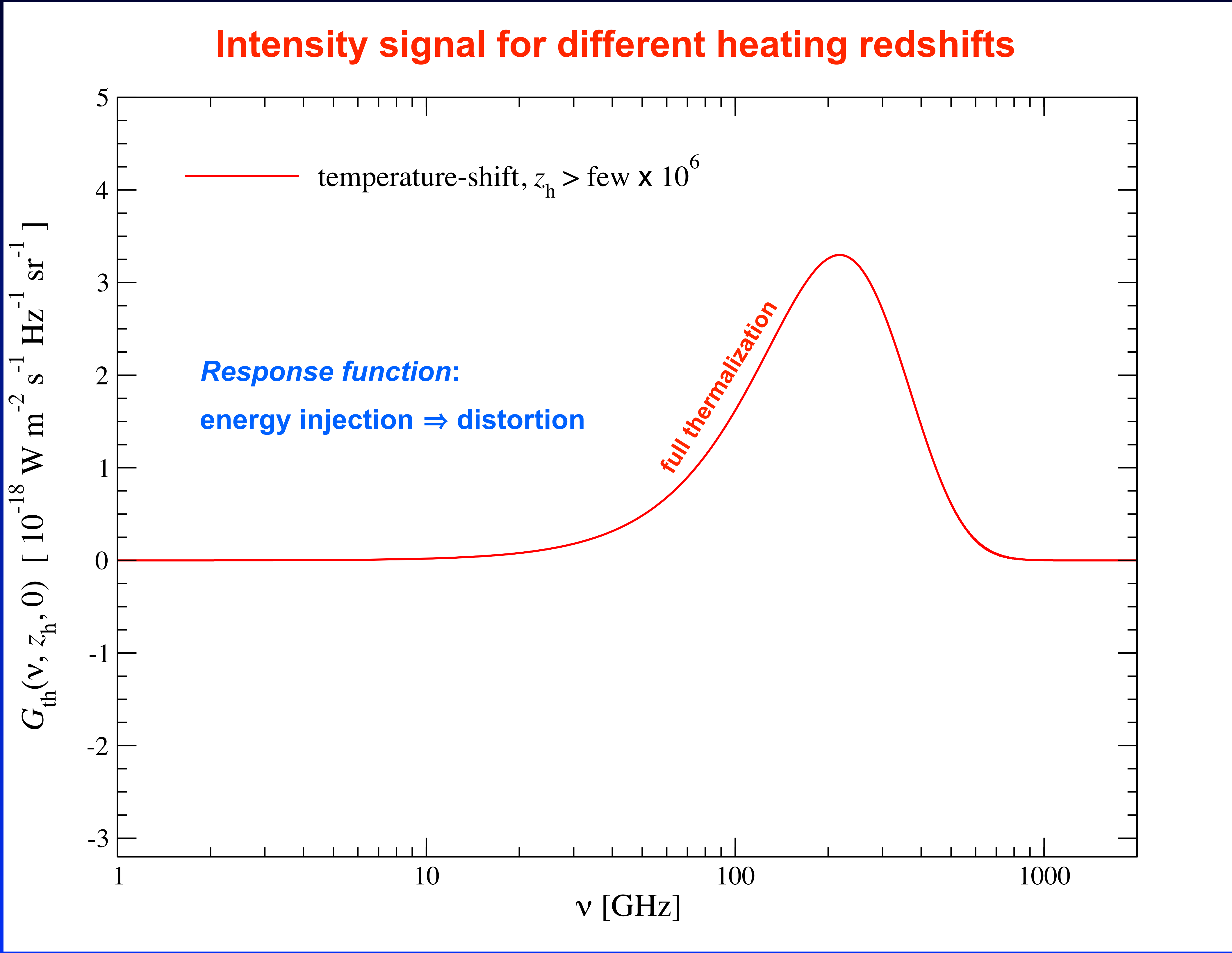
CosmoTherm available at: www.Chluba.de/CosmoTherm

What does the spectrum look like after energy injection?



JC & Sunyaev, 2012, ArXiv:1109.6552
JC, 2013, ArXiv:1304.6120

What does the spectrum look like after energy injection?



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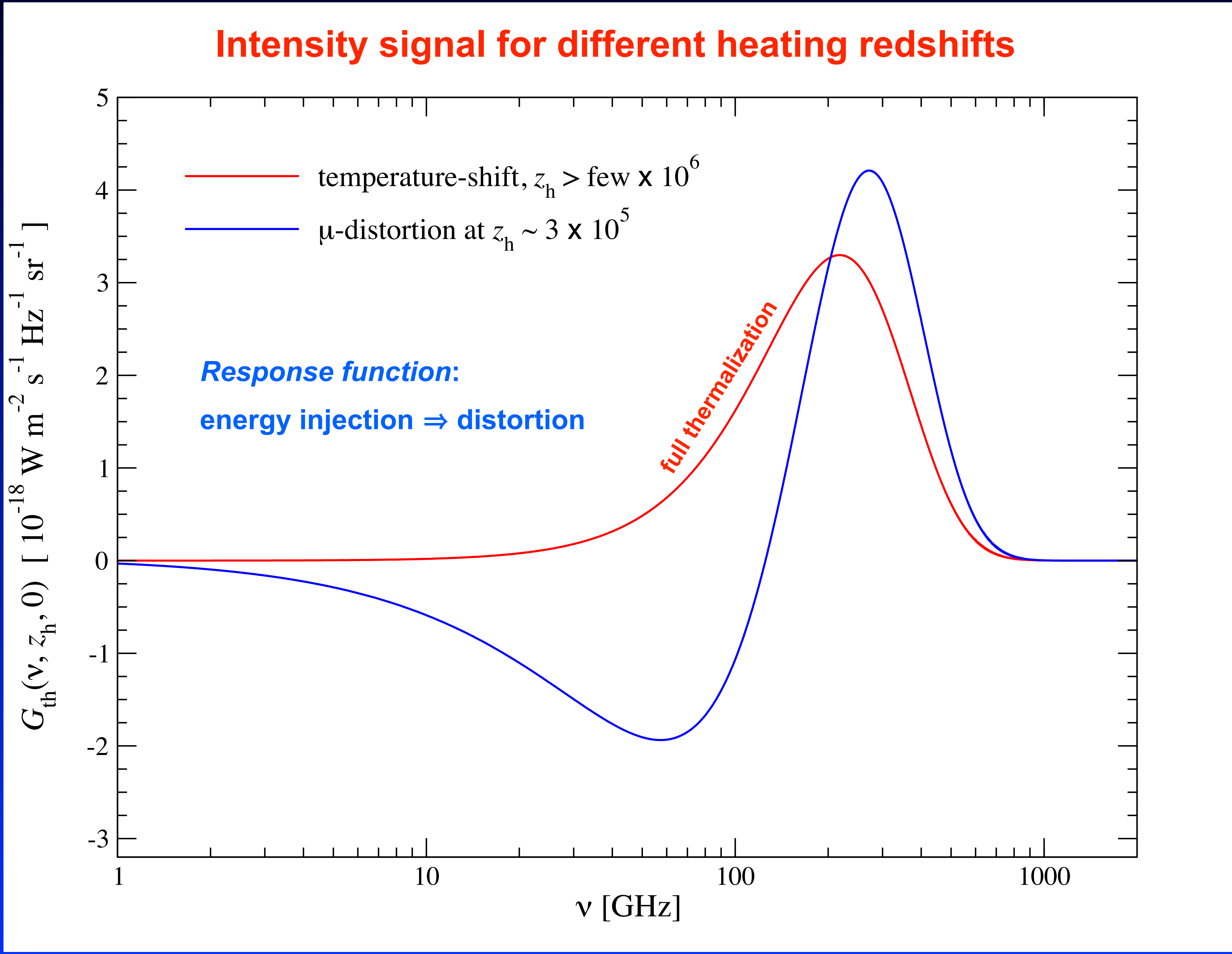

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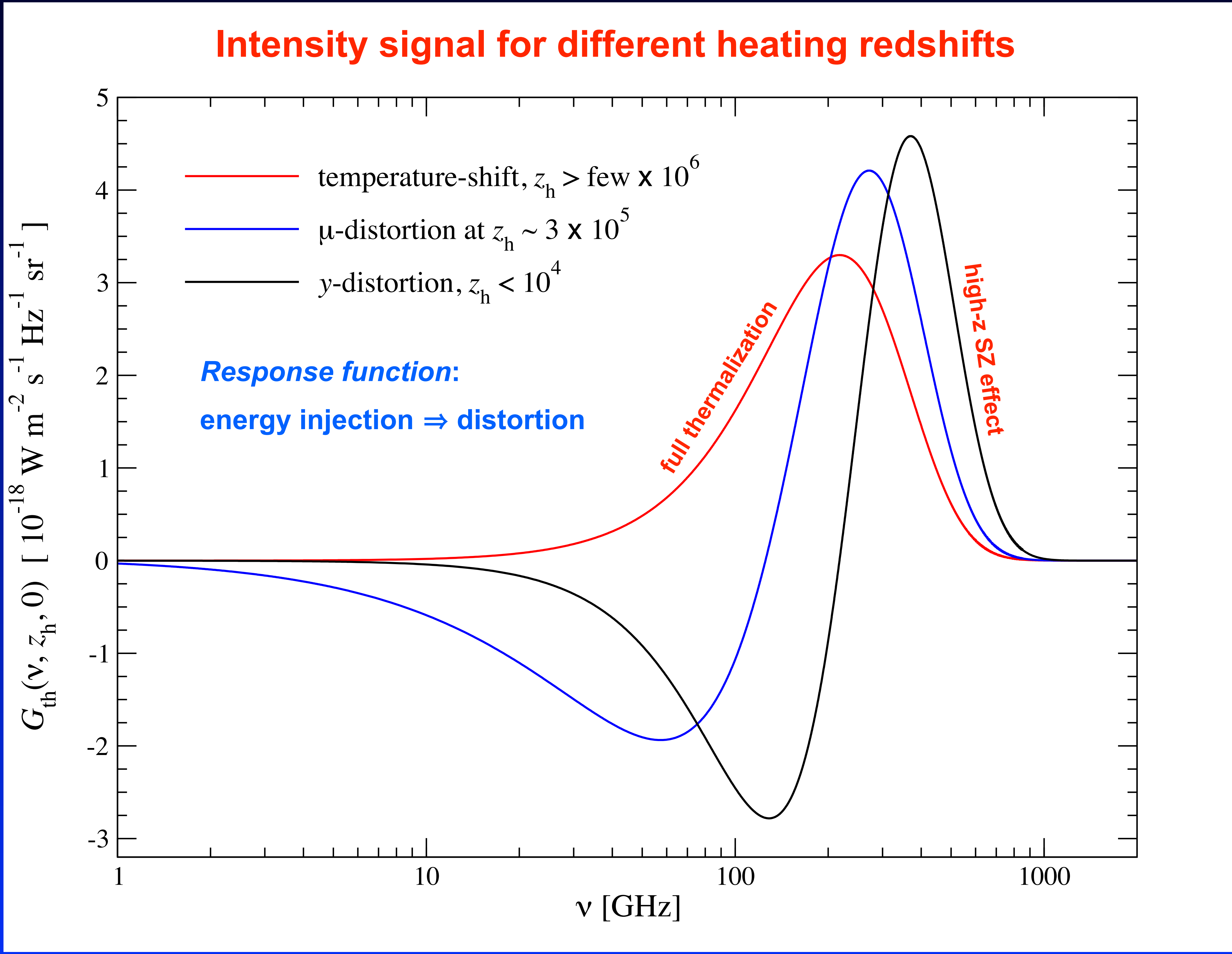

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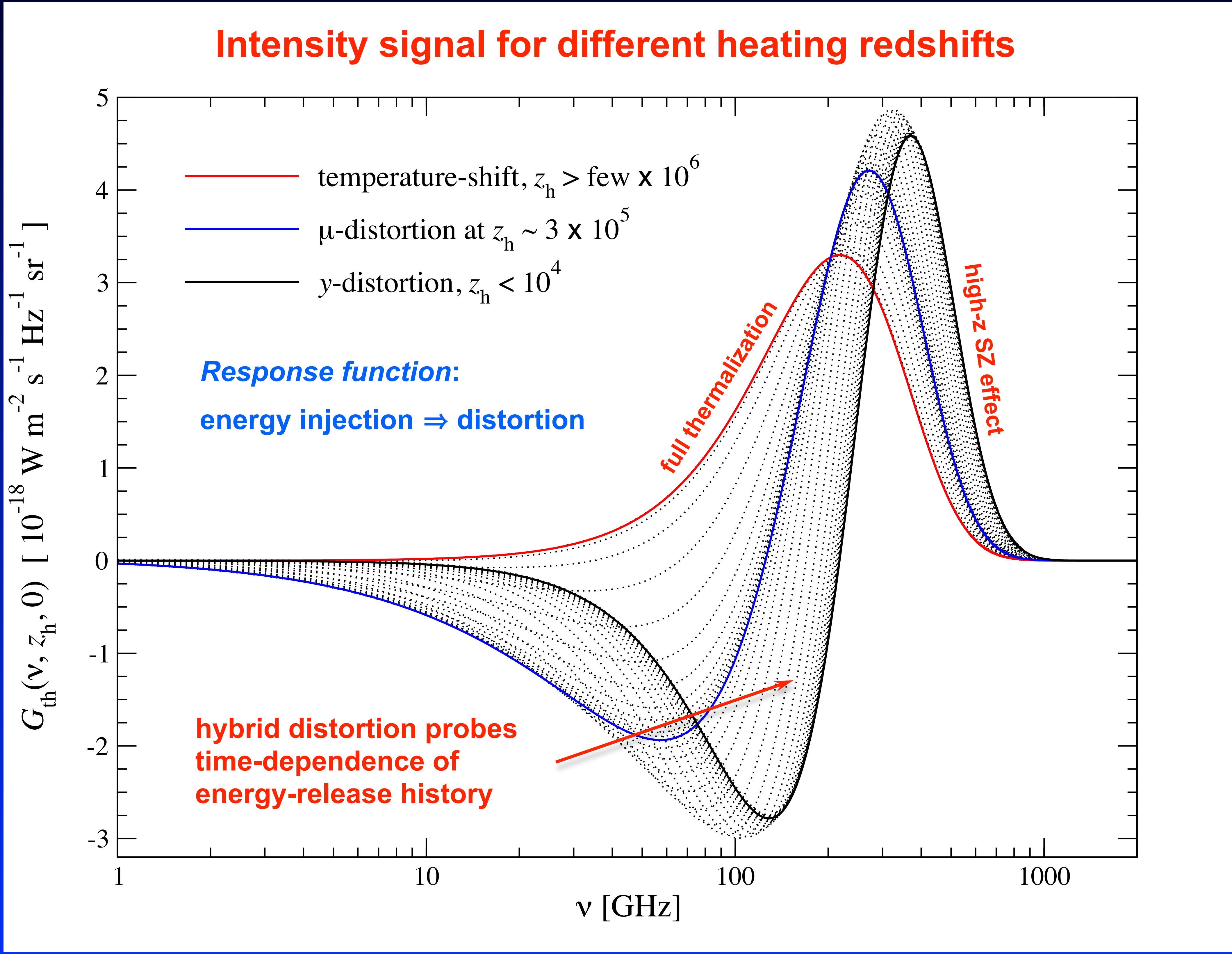

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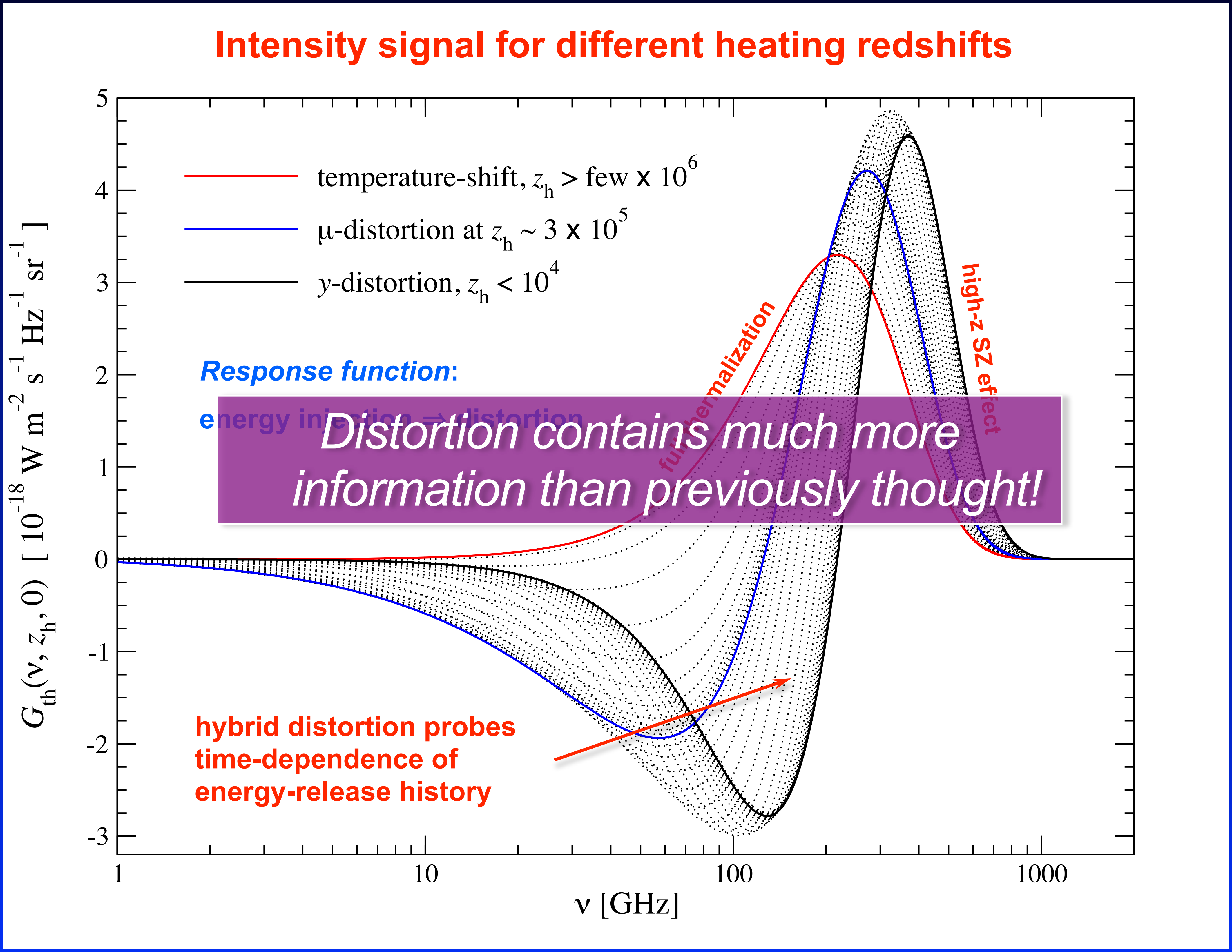
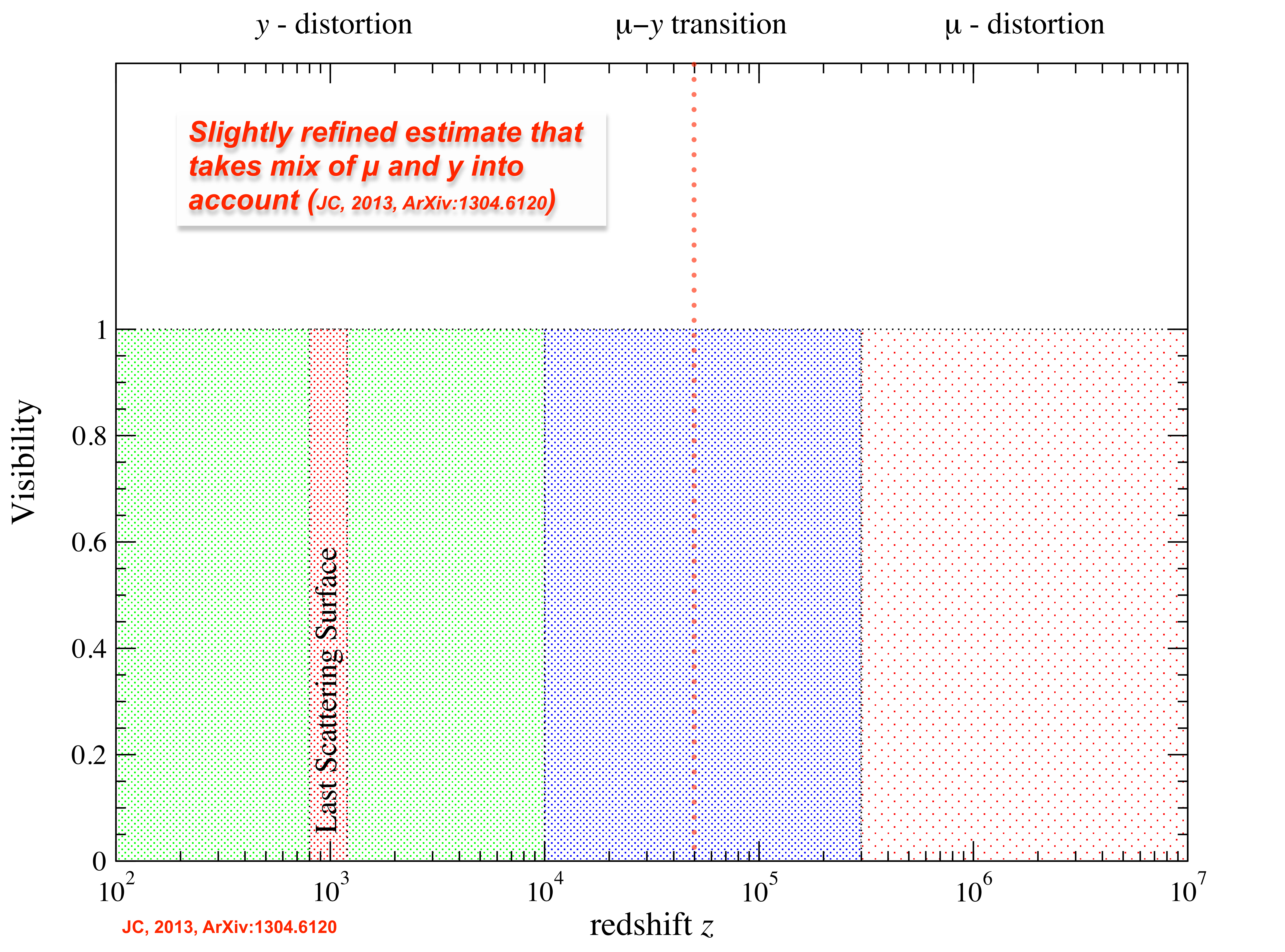
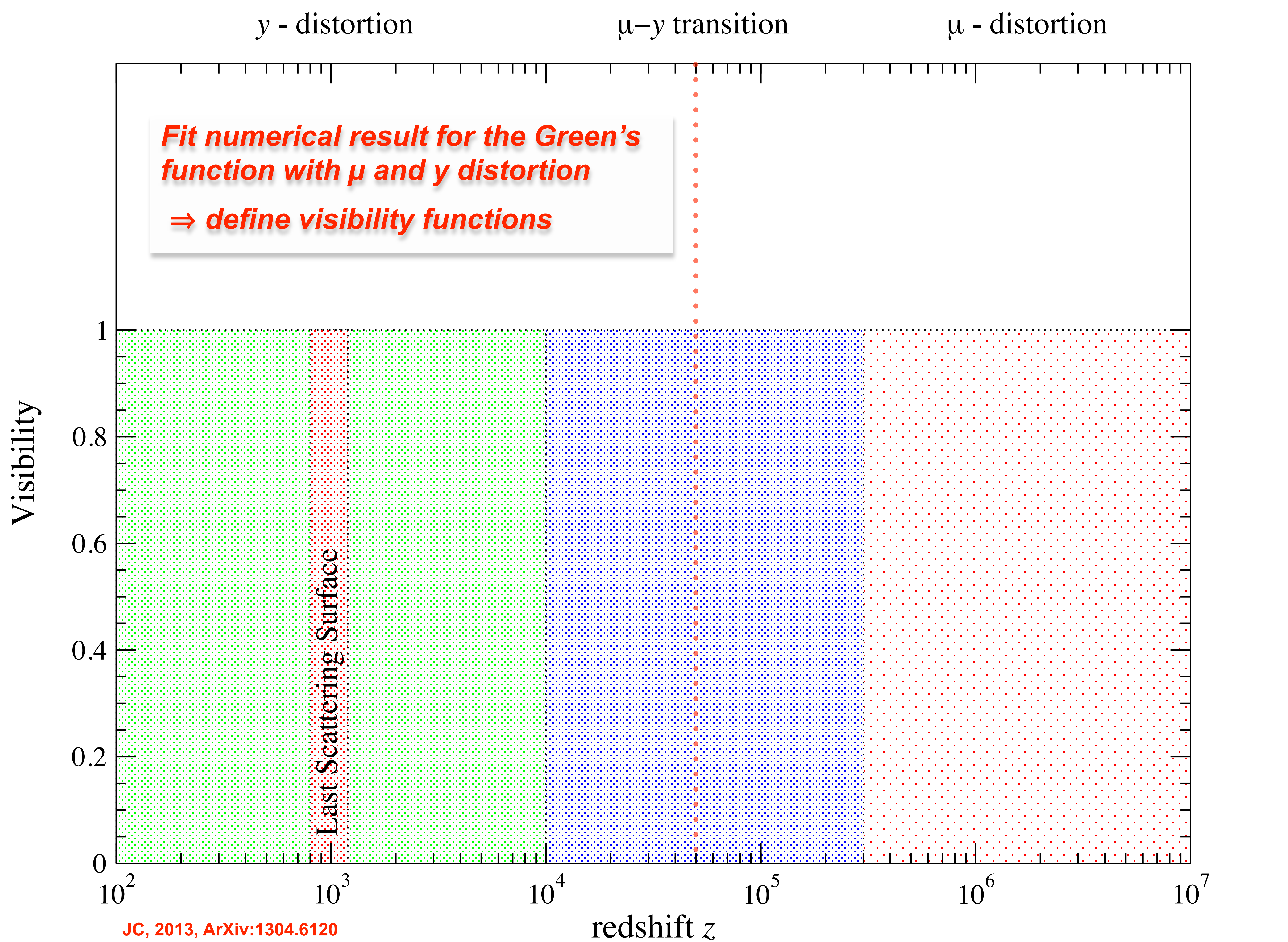


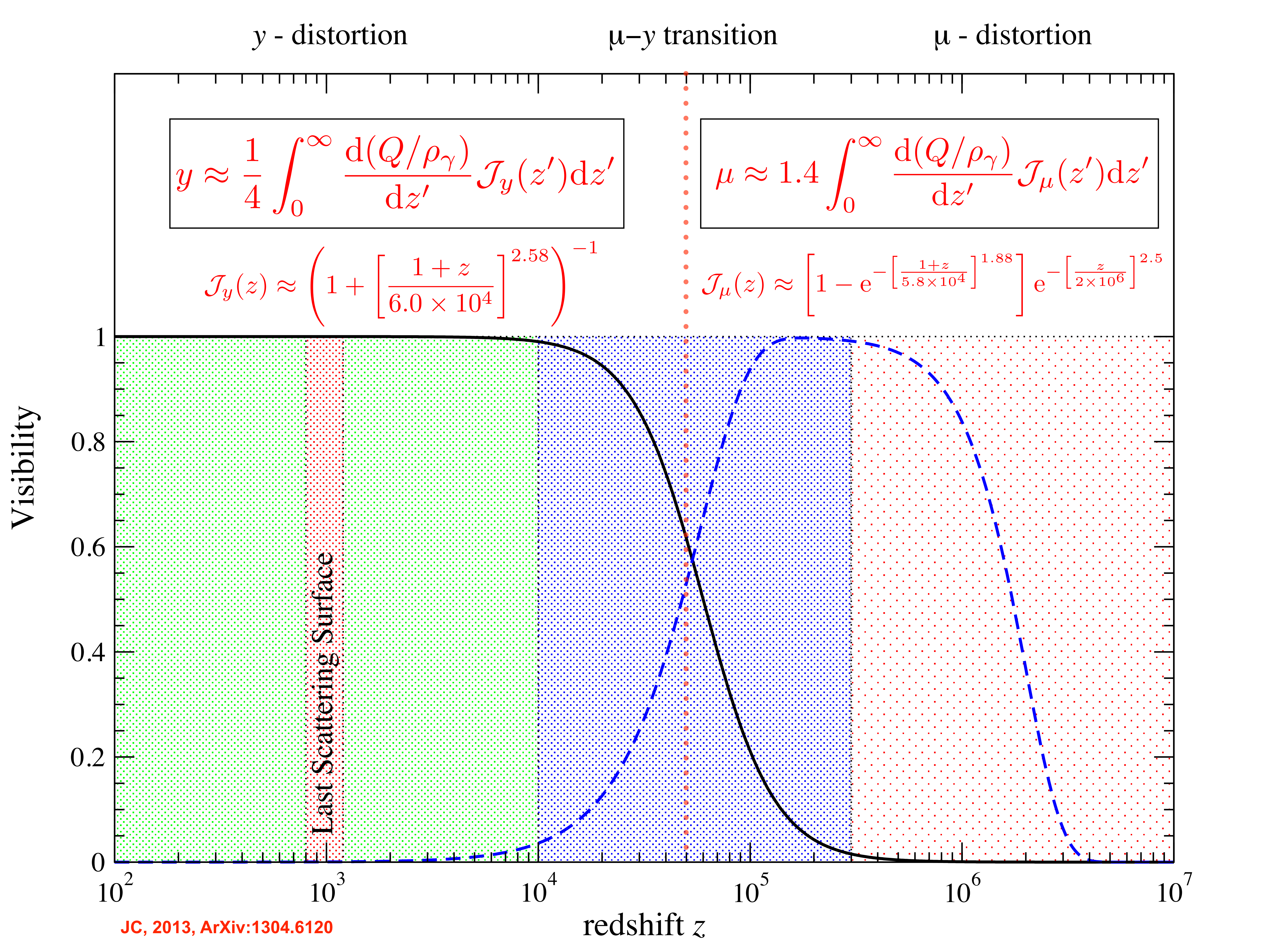
JC & Sunyaev, 2012, ArXiv:1109.6552
JC, 2013, ArXiv:1304.6120



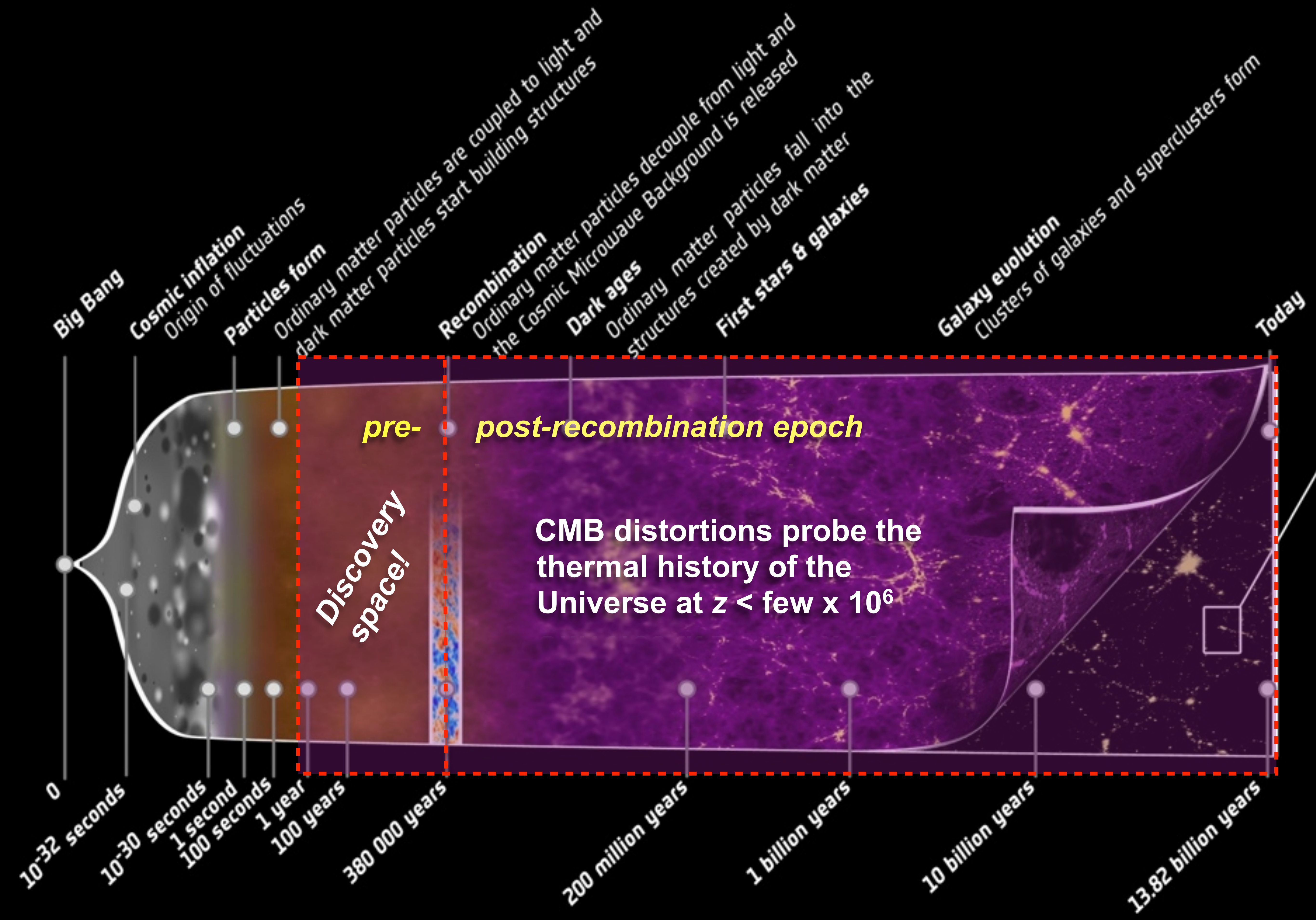
y - distortion
μ−y transition
μ - distortion
Slightly refined estimate that takes mix of μ and y into account (JC, 2013, ArXiv:1304.6120)
Last Scattering Surface
Visibility
0
0.2
0.4
0.6
0.8
1
10^2
10^3
10^4
10^5
10^6
10^7
redshift z
JC, 2013, ArXiv:1304.6120



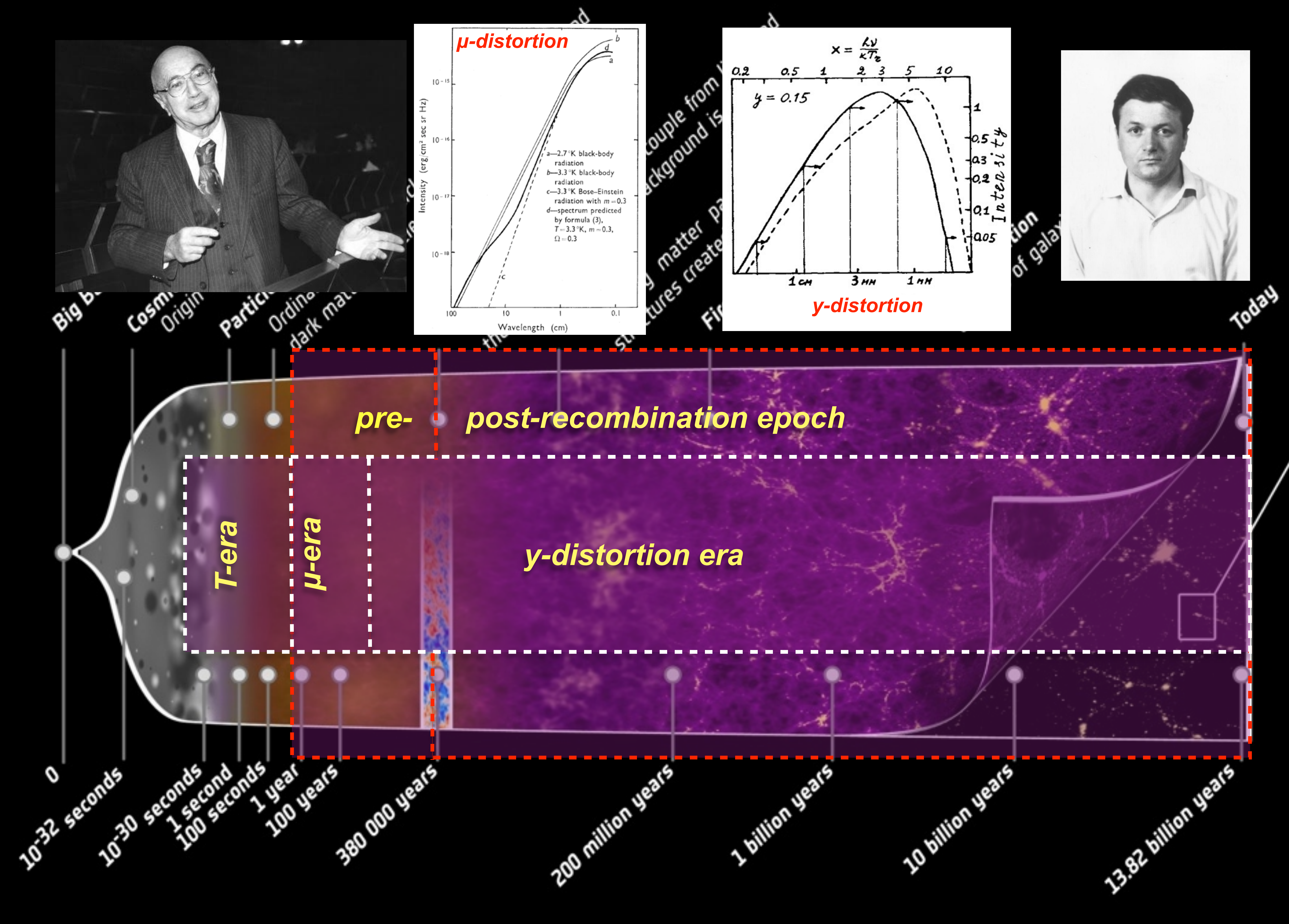
y - distortion
μ−y transition
μ - distortion
Fit numerical result for the Green's function with μ and y distortion
⇒ define visibility functions
Visibility
1
0.8
0.6
0.4
0.2
0
Last Scattering Surface
10^2
10^3
10^4
10^5
10^6
10^7
redshift z
JC, 2013, ArXiv:1304.6120



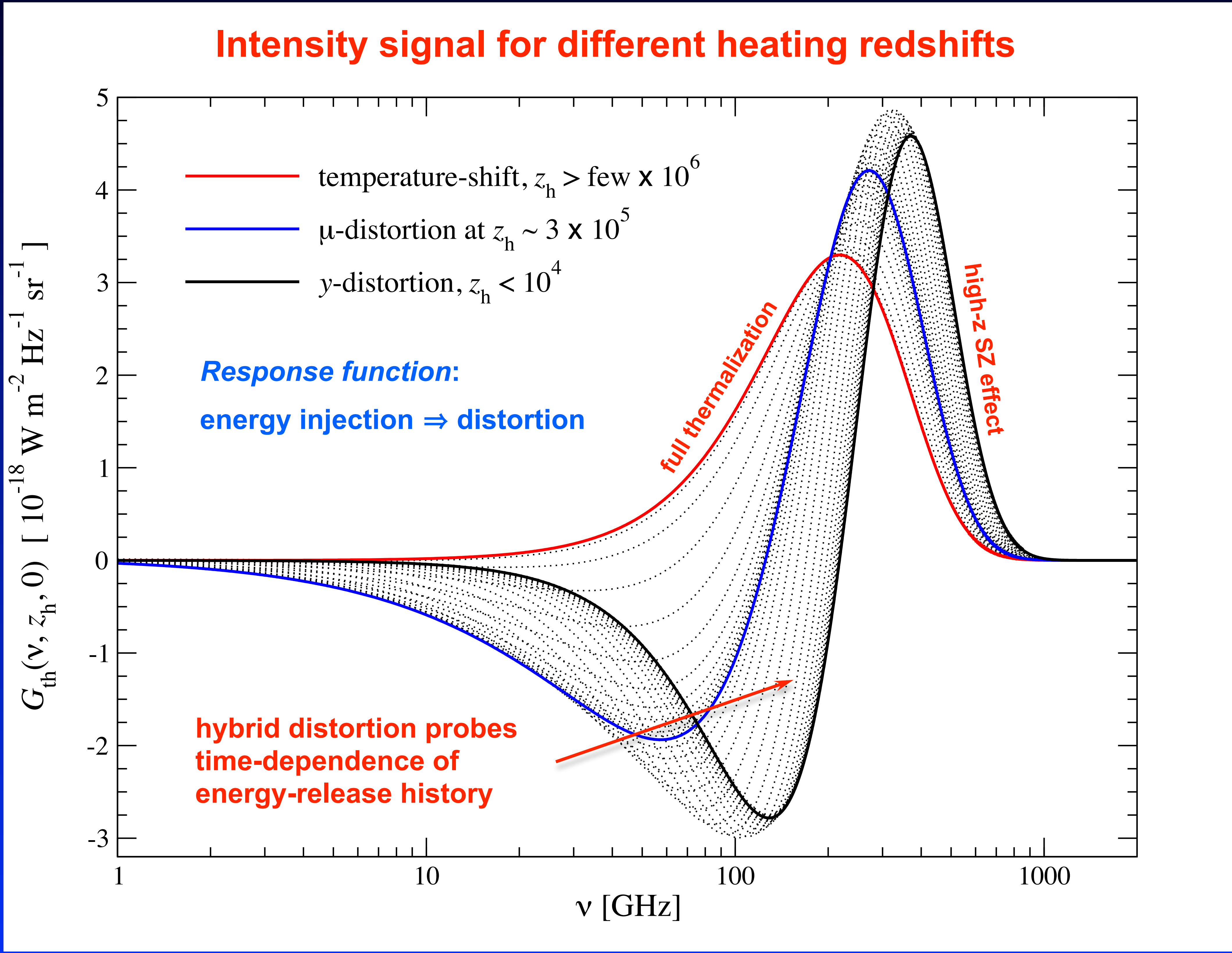
y - distortion
μ–y transition
μ - distortion
$y \approx \frac{1}{4}\int_0^\infty \frac{d(Q/\rho_\gamma)}{dz'}\mathcal{J}_y(z')dz'$
$\mu \approx 1.4\int_0^\infty \frac{d(Q/\rho_\gamma)}{dz'}\mathcal{J}_\mu(z')dz'$
$\mathcal{J}_y(z) \approx \left(1+\left[\frac{1+z}{6.0\times10^4}\right]^{2.58}\right)^{-1}$
$\mathcal{J}_\mu(z) \approx \left[1-e^{-\left[\frac{1+z}{5.8\times10^4}\right]^{1.88}}\right]e^{-\left[\frac{z}{2\times10^6}\right]^{2.5}}$
Visibility
Last Scattering Surface
redshift z
JC, 2013, ArXiv:1304.6120



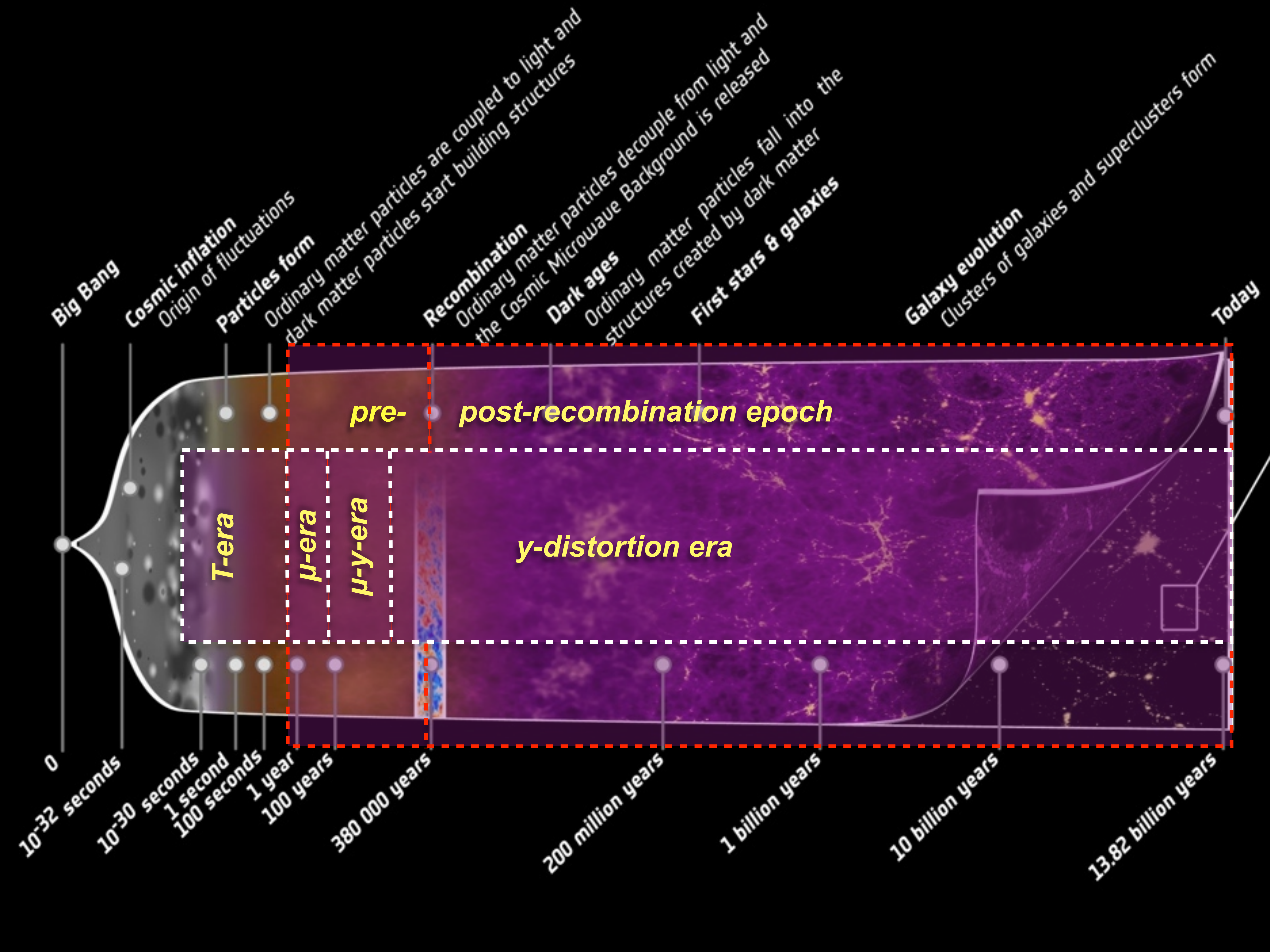
Big Bang
Cosmic inflation
Origin of fluctuations
Particles form
Ordinary matter particles are coupled to light and dark matter particles start building structures
Recombination
Ordinary matter particles decouple from light and the Cosmic Microwave Background is released
Dark ages
Ordinary matter particles fall into the structures created by dark matter
First stars & galaxies
Galaxy evolution
Clusters of galaxies and superclusters form
Today
pre-
post-recombination epoch
Discovery space!
CMB distortions probe the thermal history of the Universe at z < few x 10^6
0
10^-32 seconds
10^-30 seconds
1 second
100 seconds
1 year
100 years
380 000 years
200 million years
1 billion years
10 billion years
13.82 billion years


μ-distortion
Intensity (erg/cm² sec sr Hz)
Wavelength (cm)
a—2.7°K black-body radiation
b—3.3°K black-body radiation
c—3.3°K Bose–Einstein radiation with m=0.3
d—spectrum predicted by formula (3), T=3.3°K, m=0.3, Ω=0.3
x = hν/kT_e
y = 0.15
Intensity
1 cm
3 mm
1 mm
y-distortion
pre-
post-recombination epoch
T-era
μ-era
y-distortion era
Today
0
10-32 seconds
10-30 seconds
1 second
100 seconds
1 year
100 years
380 000 years
200 million years
1 billion years
10 billion years
13.82 billion years

What does the spectrum look like after energy injection?



JC & Sunyaev, 2012, ArXiv:1109.6552
JC, 2013, ArXiv:1304.6120



Big Bang
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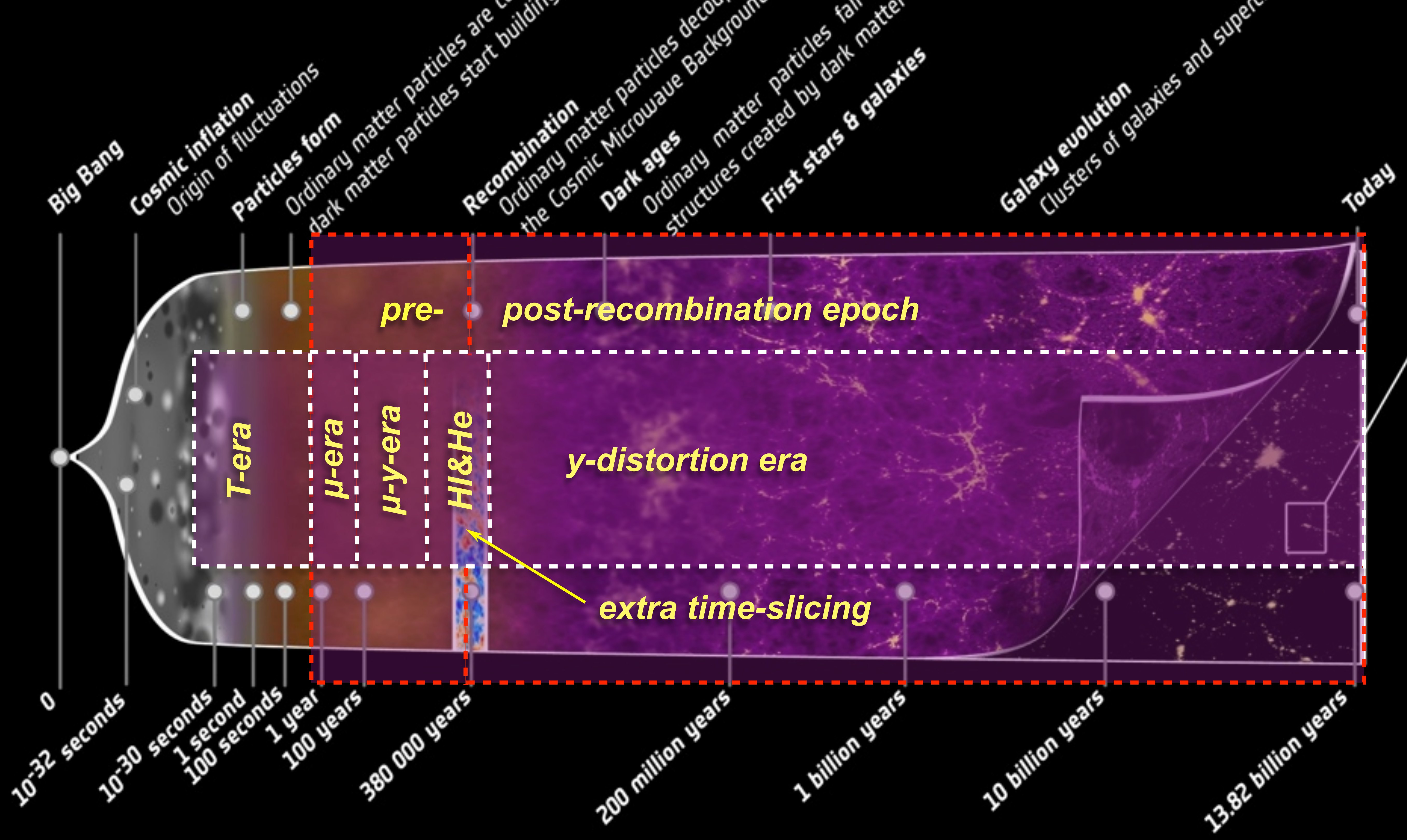
First stars & galaxies
Galaxy evolution
Clusters of galaxies and superclusters form
Today
pre-
post-recombination epoch
T-era
μ-era
μ-y-era
y-distortion era
0
10-32 seconds
10-30 seconds
1 second
100 seconds
1 year
100 years
380 000 years
200 million years
1 billion years
10 billion years
13.82 billion years

CMB spectrum adds another dimension to the problem!



Physical mechanisms that lead to spectral distortions

Standard sources of distortions

- *Cooling by adiabatically expanding ordinary matter*
  (JC, 2005; JC & Sunyaev 2011; Khatri, Sunyaev & JC, 2011)
- Heating by *decaying* or *annihilating* relic particles
  (Kawasaki et al., 1987; Hu & Silk, 1993; McDonald et al., 2001; JC, 2005; JC & Sunyaev, 2011; JC, 2013; JC & Jeong, 2013)
- *Evaporation of primordial black holes & superconducting strings*
  (Carr et al. 2010; Ostriker & Thompson, 1987; Tashiro et al. 2012; Pani & Loeb, 2013)
- *Dissipation of primordial acoustic modes & magnetic fields*
  (Sunyaev & Zeldovich, 1970; Daly 1991; Hu et al. 1994; JC & Sunyaev, 2011; JC et al. 2012 - Jedamzik et al. 2000; Kunze & Komatsu, 2013)
- *Cosmological recombination radiation*
  (Zeldovich et al., 1968; Peebles, 1968; Dubrovich, 1977; Rubino-Martin et al., 2006; JC & Sunyaev, 2006; Sunyaev & JC, 2009)

„high“ redshifts

pre-recombination epoch

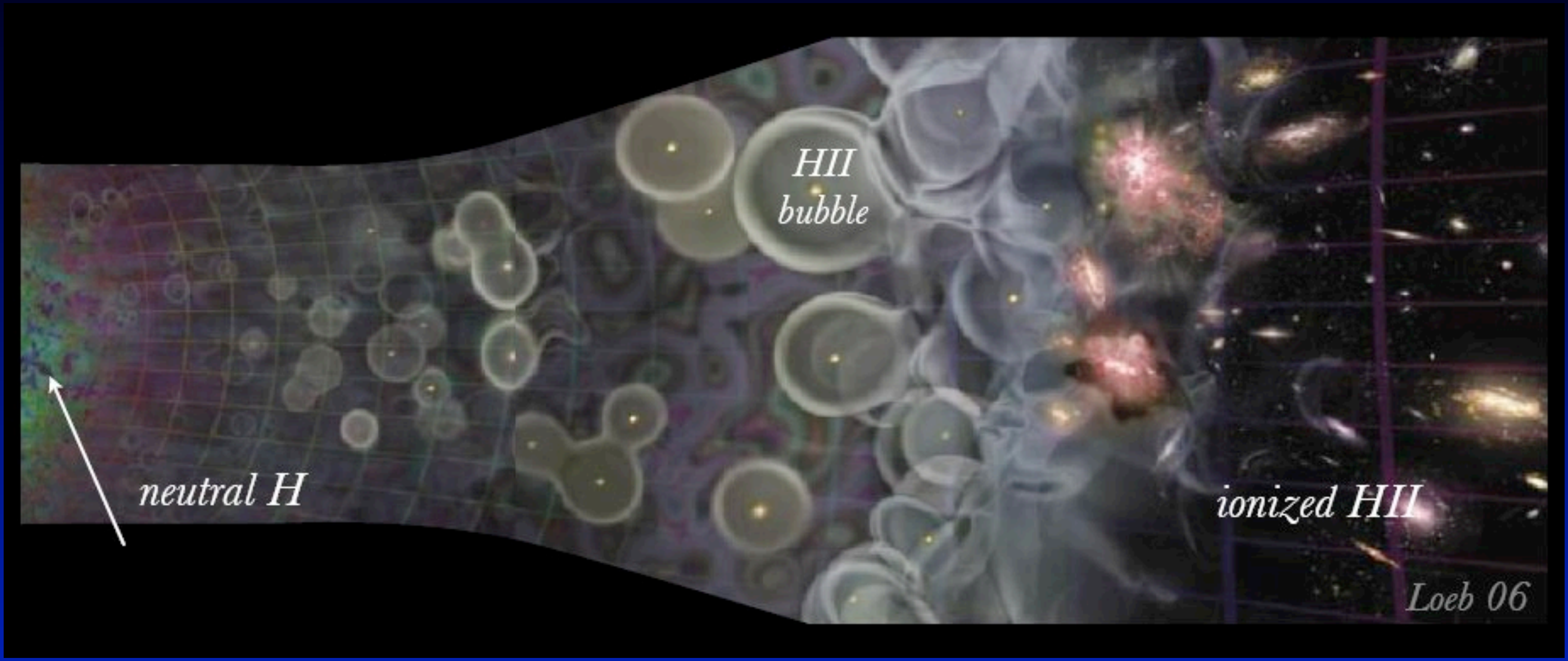
„low“ redshifts

post-recombination

- Signatures due to first supernovae and their remnants
  (Oh, Cooray & Kamionkowski, 2003)
- Shock waves arising due to large-scale structure formation
  (Sunyaev & Zeldovich, 1972; Cen & Ostriker, 1999)
- SZ-effect from clusters; effects of reionization
  (Refregier et al., 2003; Zhang et al. 2004; Trac et al. 2008)
- other exotic processes
  (Lochan et al. 2012; Bull & Kamionkowski, 2013; Brax et al., 2013; Tashiro et al. 2013)

Reionization and structure formation

Simple estimates for the distortion

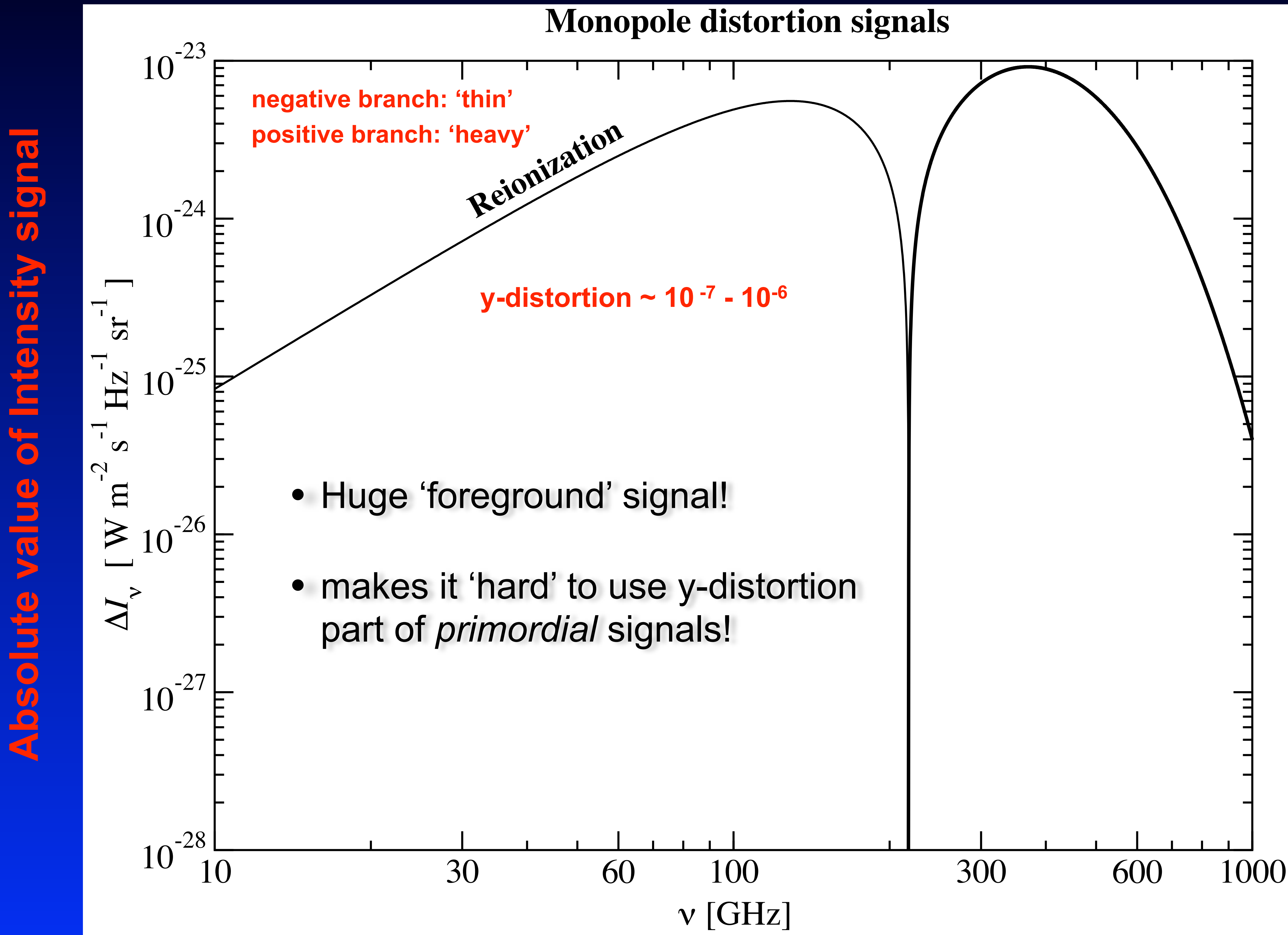


- Gas temperature T ≃ 10^4 K
- Thomson optical depth $\tau \simeq 0.1$

$$\Longrightarrow \quad y \simeq \frac{kT_{\rm e}}{m_{\rm e}c^2}\,\tau \approx 2\times10^{-7}$$

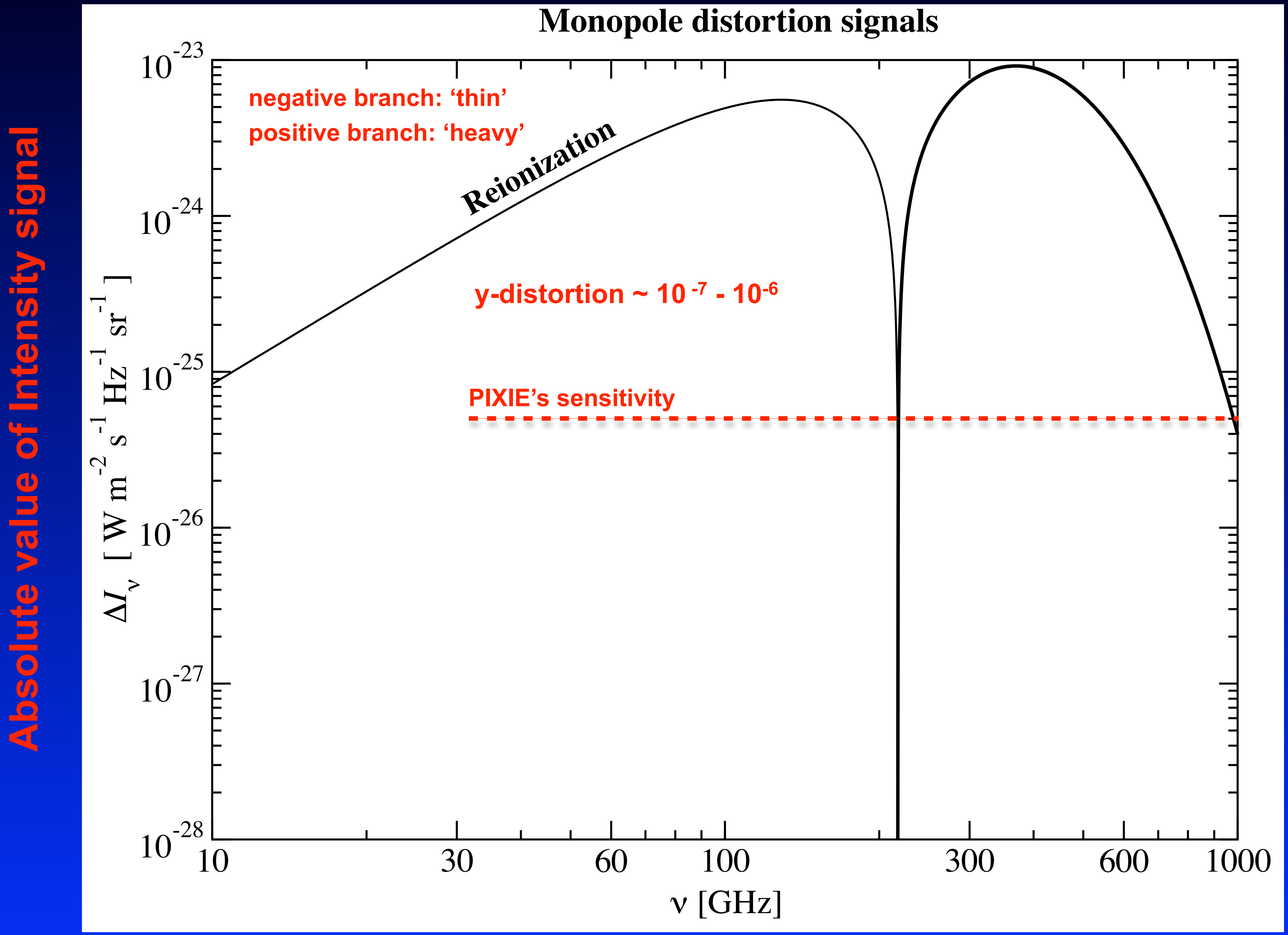
- second order Doppler effect $y \simeq$ few x 10^{-8}
- structure formation / SZ effect (e.g., Refregier et al., 2003) $y \simeq$ few x 10^{-7}-10^{-6}

Average CMB spectral distortions

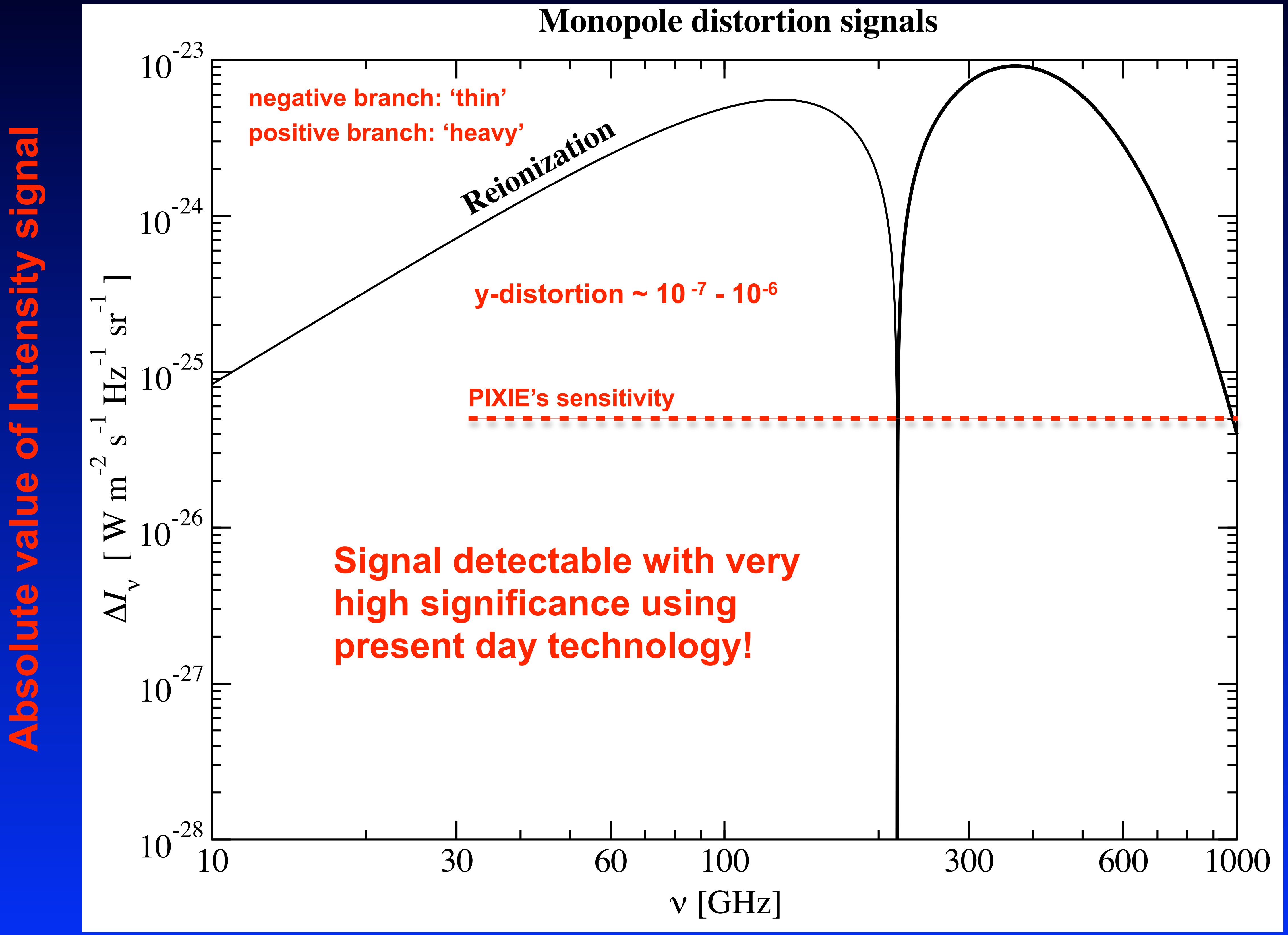


- Huge 'foreground' signal!
- makes it 'hard' to use y-distortion part of *primordial* signals!

Average CMB spectral distortions



Average CMB spectral distortions



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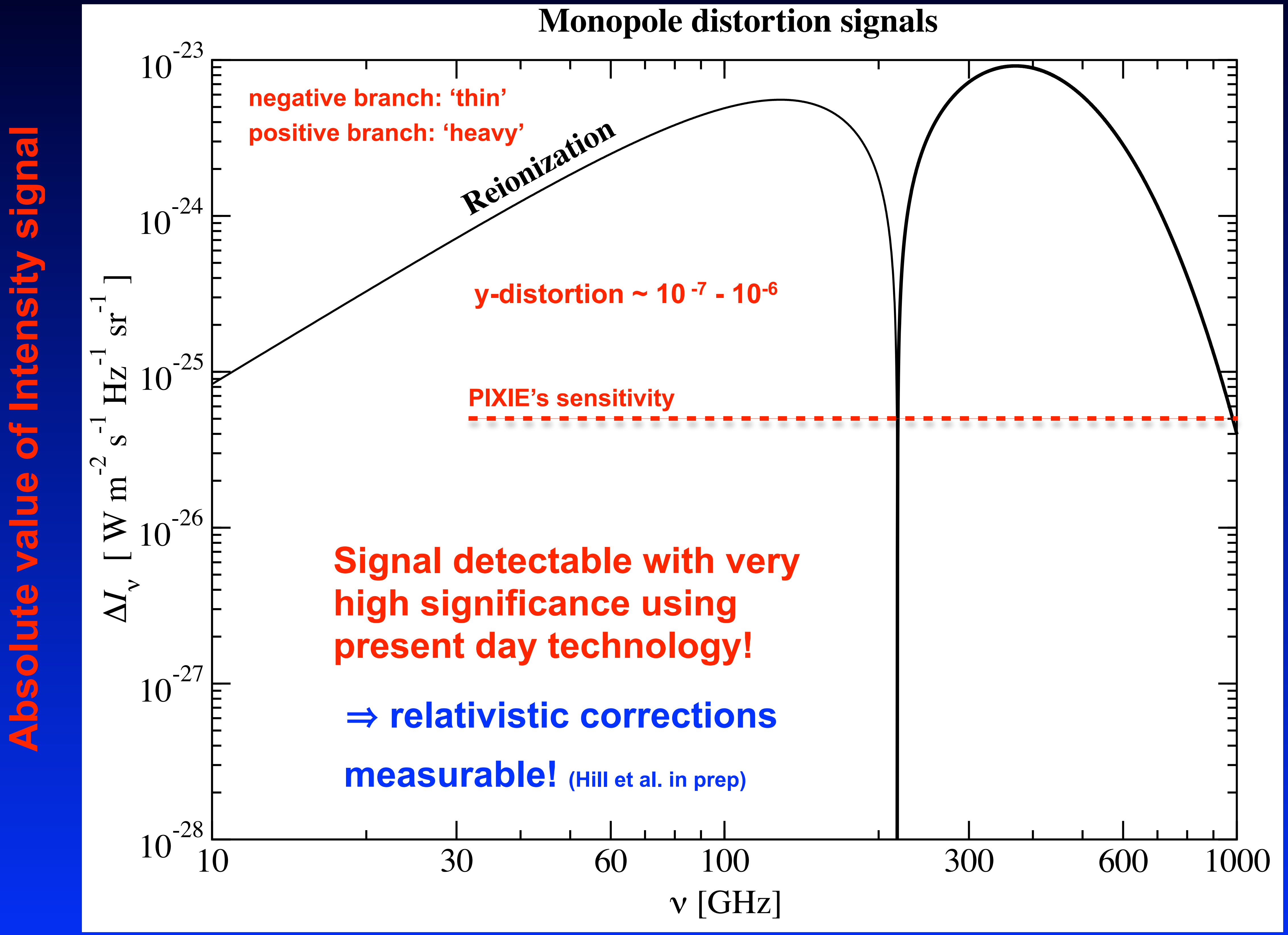
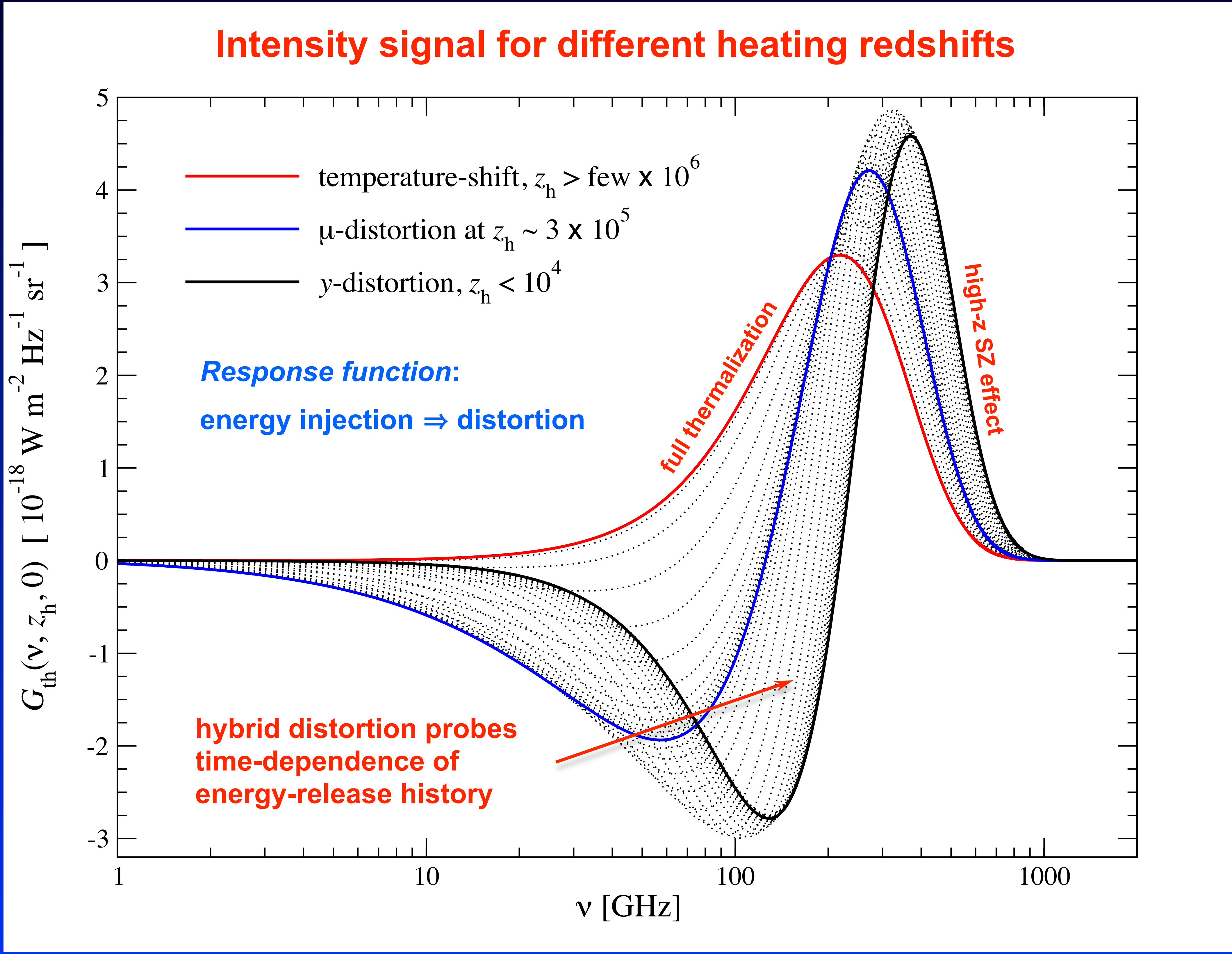
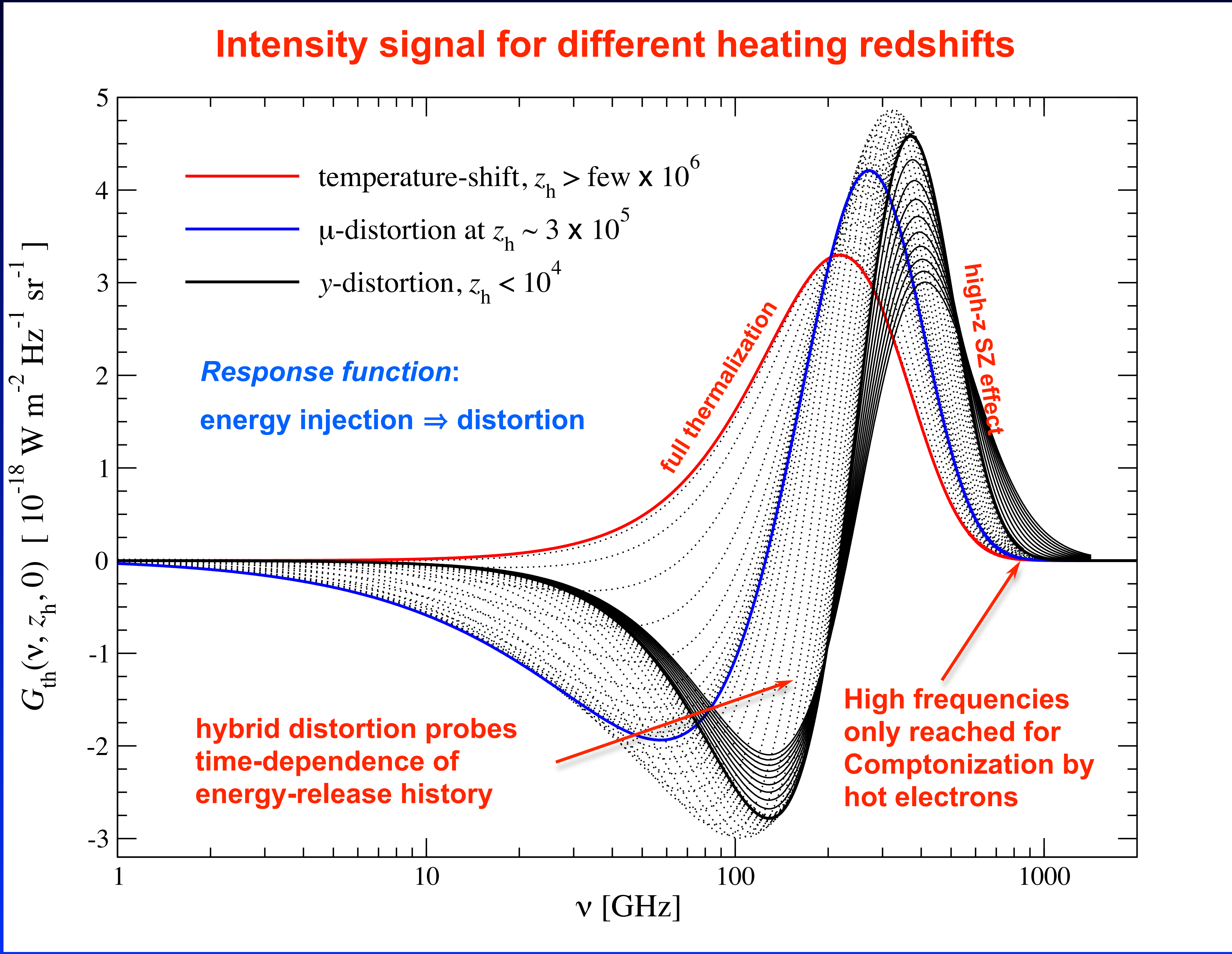


What does the spectrum look like after energy injection?



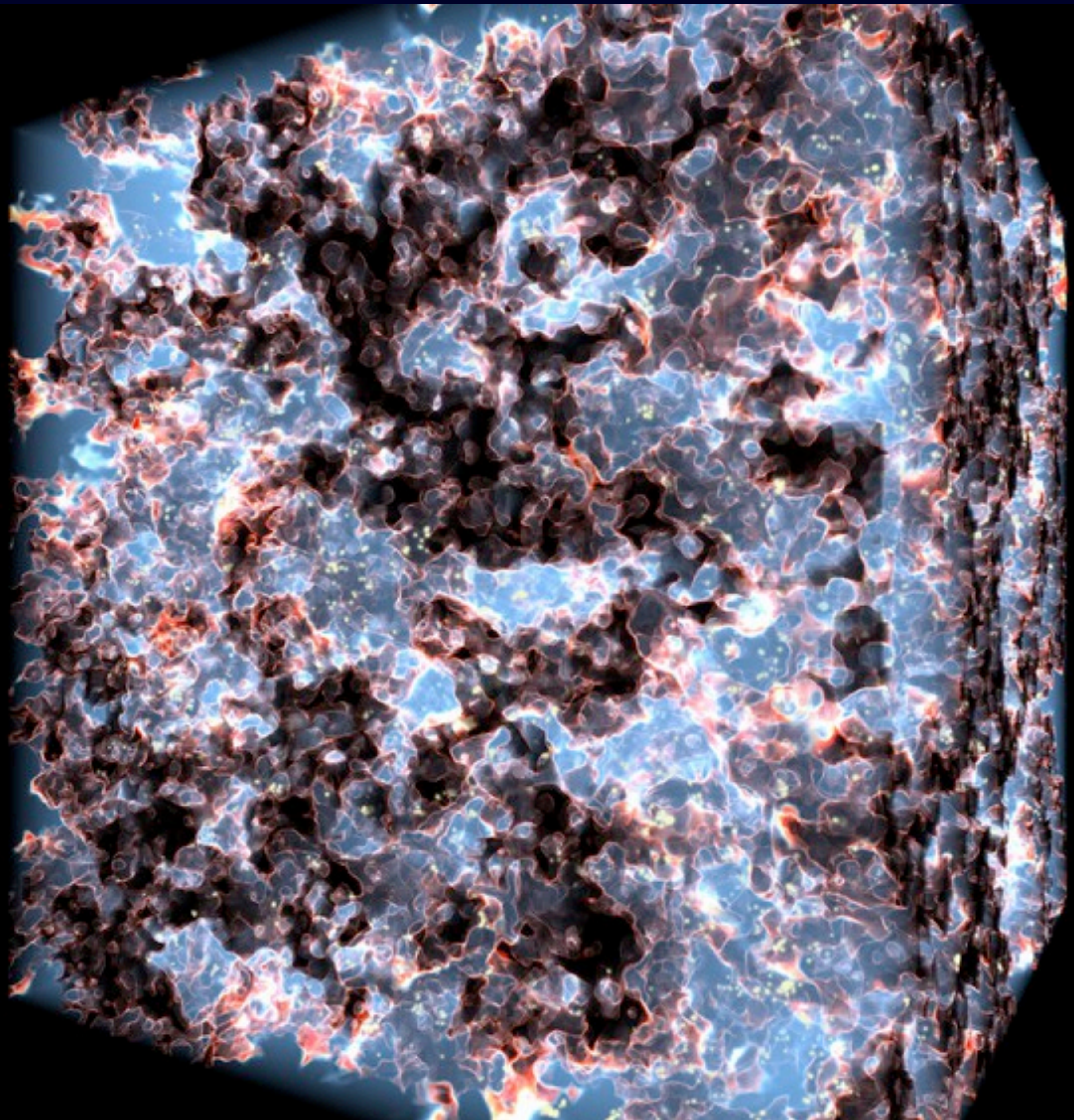
JC & Sunyaev, 2012, ArXiv:1109.6552
JC, 2013, ArXiv:1304.6120

What does the spectrum look like after energy injection?



JC & Sunyaev, 2012, ArXiv:1109.6552
JC, 2013, ArXiv:1304.6120

Fluctuations of the y-parameter at large scales



- spatial variations of the optical depth and temperature cause small-spatial variations of the y-parameter at different angular scales
- could tell us about the reionization sources and structure formation process
- additional independent piece of information!
- Cross-correlations with other signals

Example:
Simulation of reionization process (1Gpc/h) by *Alvarez* & *Abel*

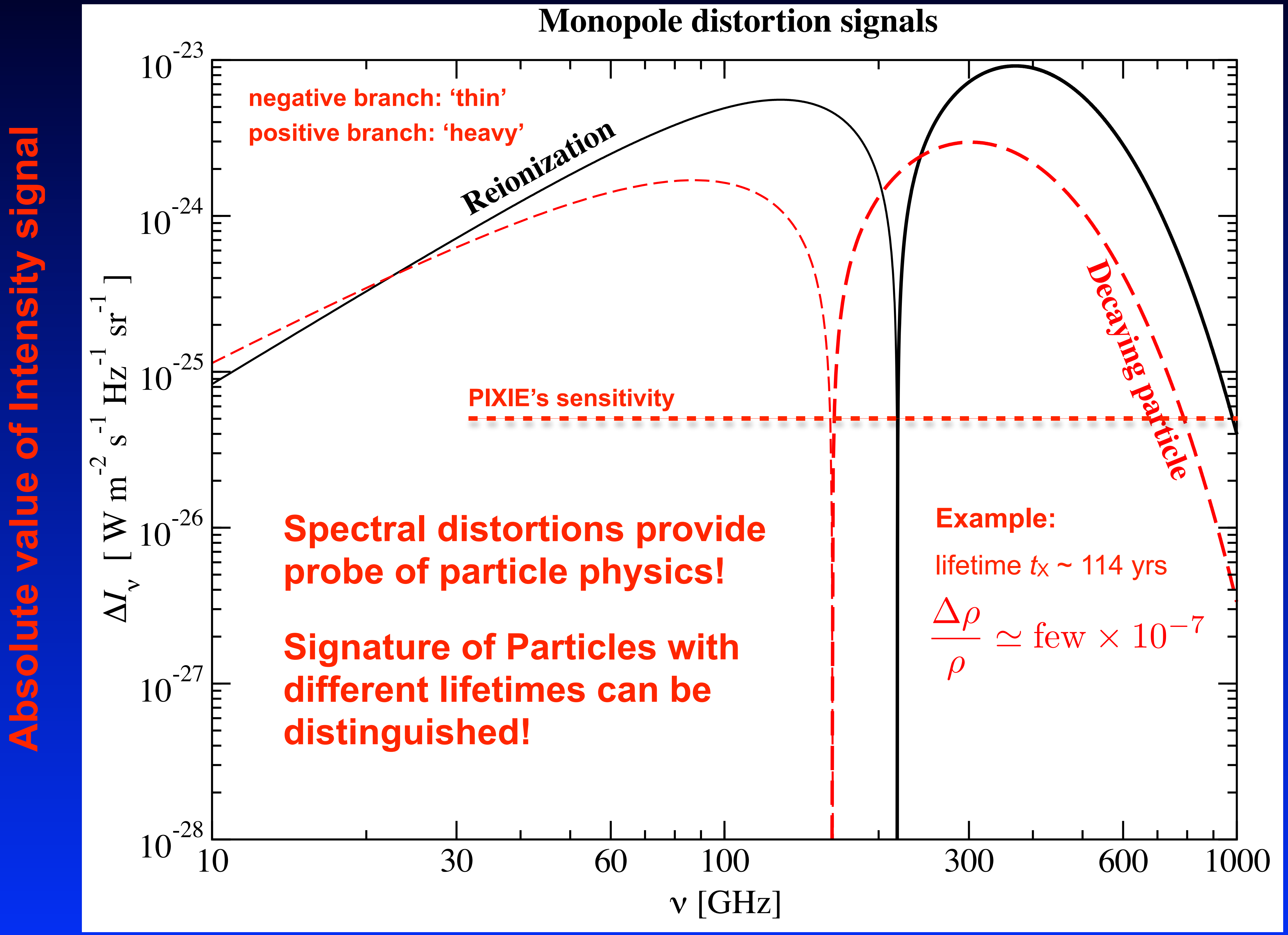
Decaying (dark matter) particles

Why is this interesting?

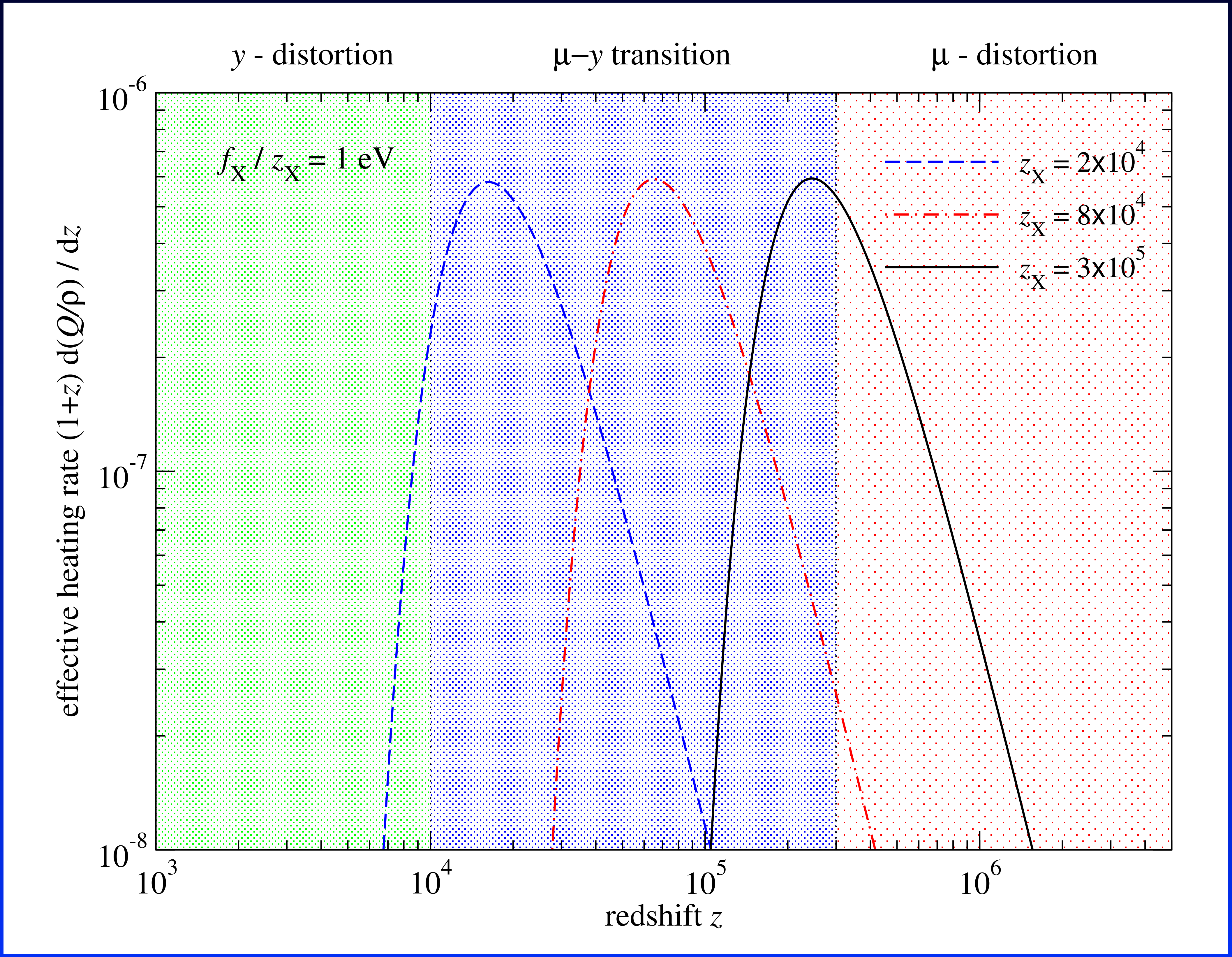
- A priori no specific particle in mind
- *But:* we do not know what *dark matter* is and where it really came from!
- Was *dark matter* thermally produced or possibly as a decay product of some heavy particle?
- is *dark matter* structureless or does it have internal (excited) states?
- sterile neutrinos? moduli? Or some other relic particle?
- From the theoretical point of view really no shortage of particles to play with...

CMB spectral distortions offer a new independent way to constrain these kind of models

Average CMB spectral distortions

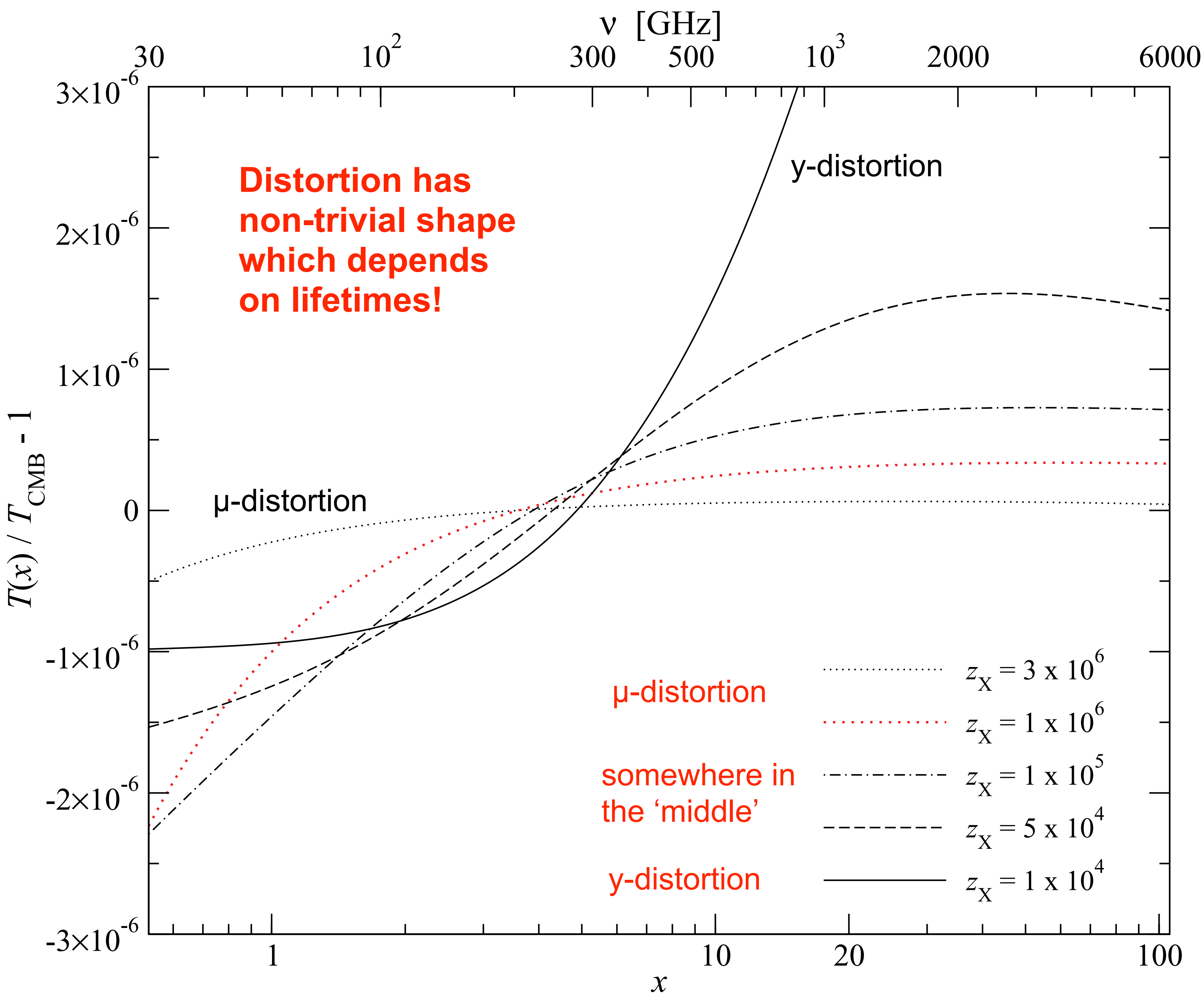


Decaying particle scenarios



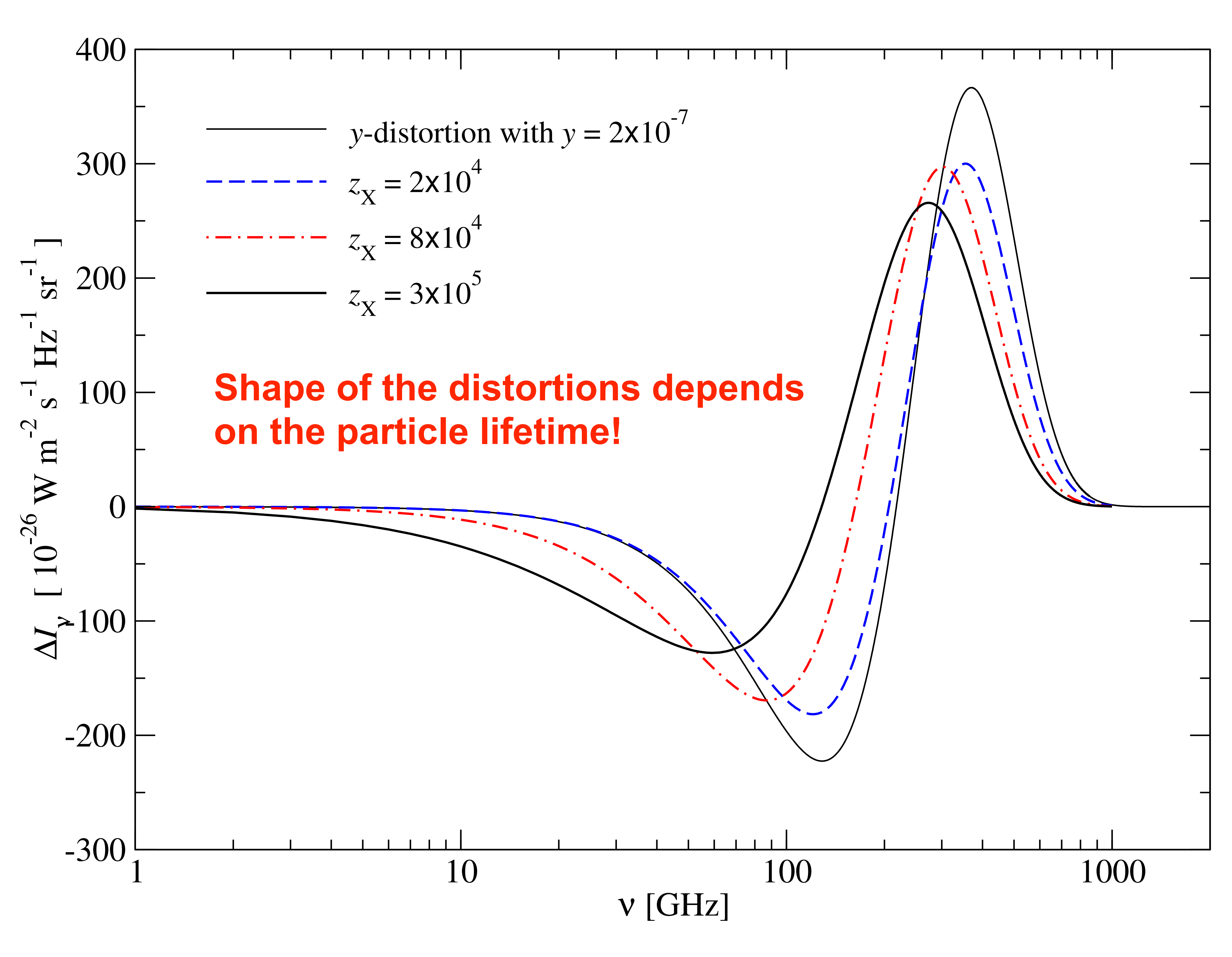
JC & Sunyaev, 2011, Arxiv:1109.6552
JC, 2013, Arxiv:1304.6120

High frequency distortion for decaying particle scenarios



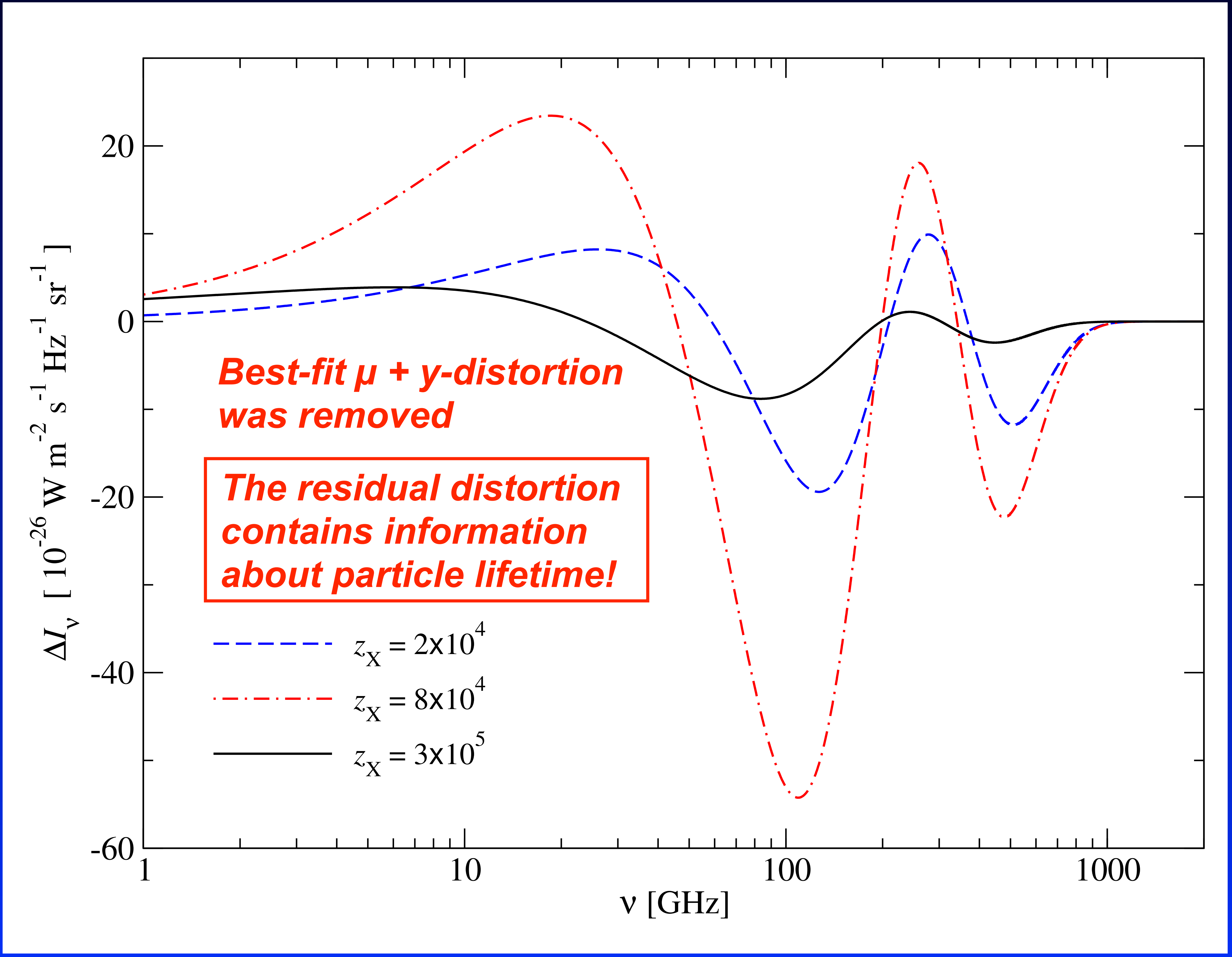
JC & Sunyaev, 2012

Decaying particle scenarios



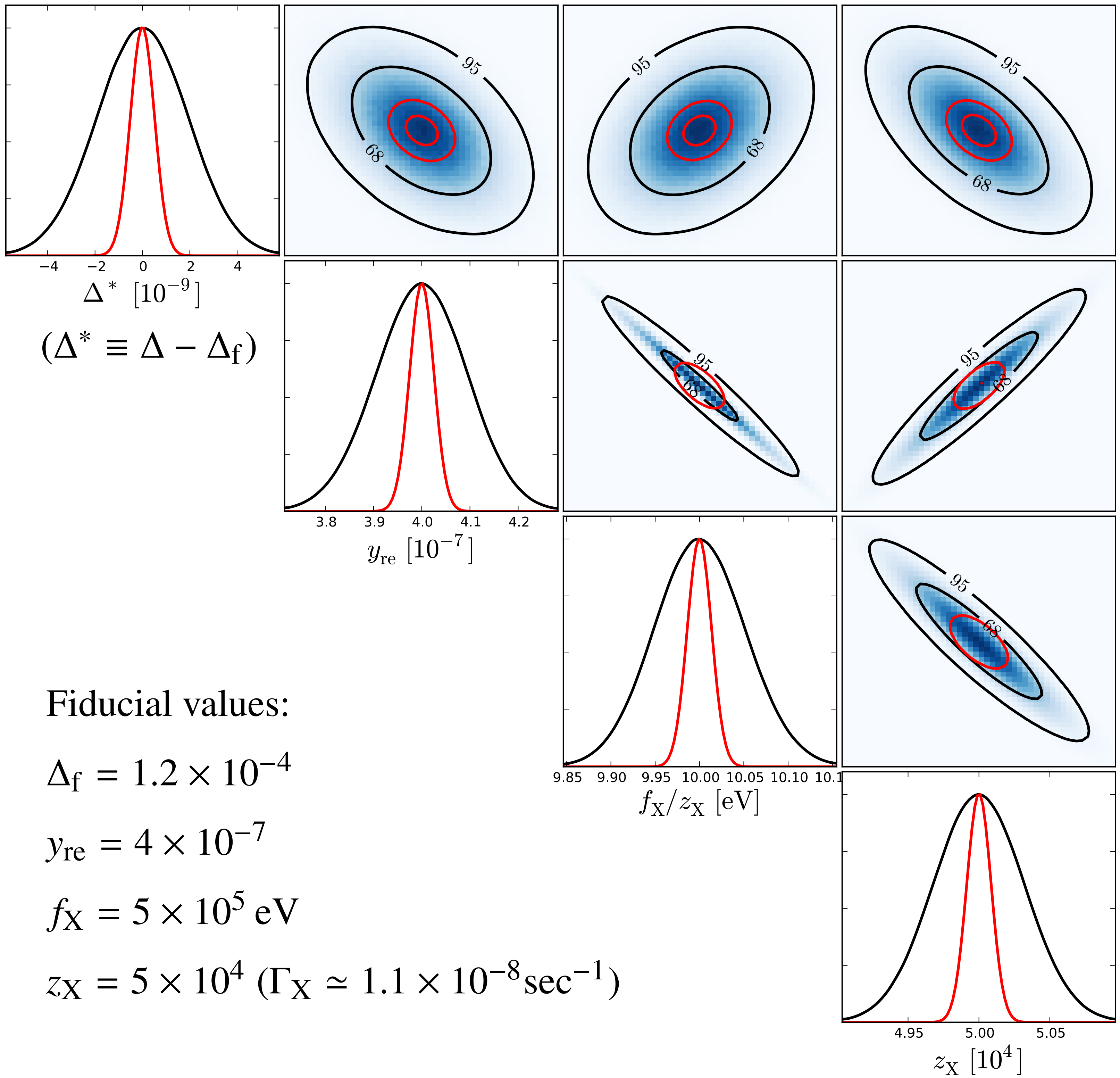
JC & Sunyaev, 2011, Arxiv:1109.6552
JC, 2013, Arxiv:1304.6120

Decaying particle scenarios (information in residual)



JC & Sunyaev, 2011, Arxiv:1109.6552
JC, 2013, Arxiv:1304.6120

Decaying particle scenarios



Fiducial values:

$\Delta_f = 1.2 \times 10^{-4}$

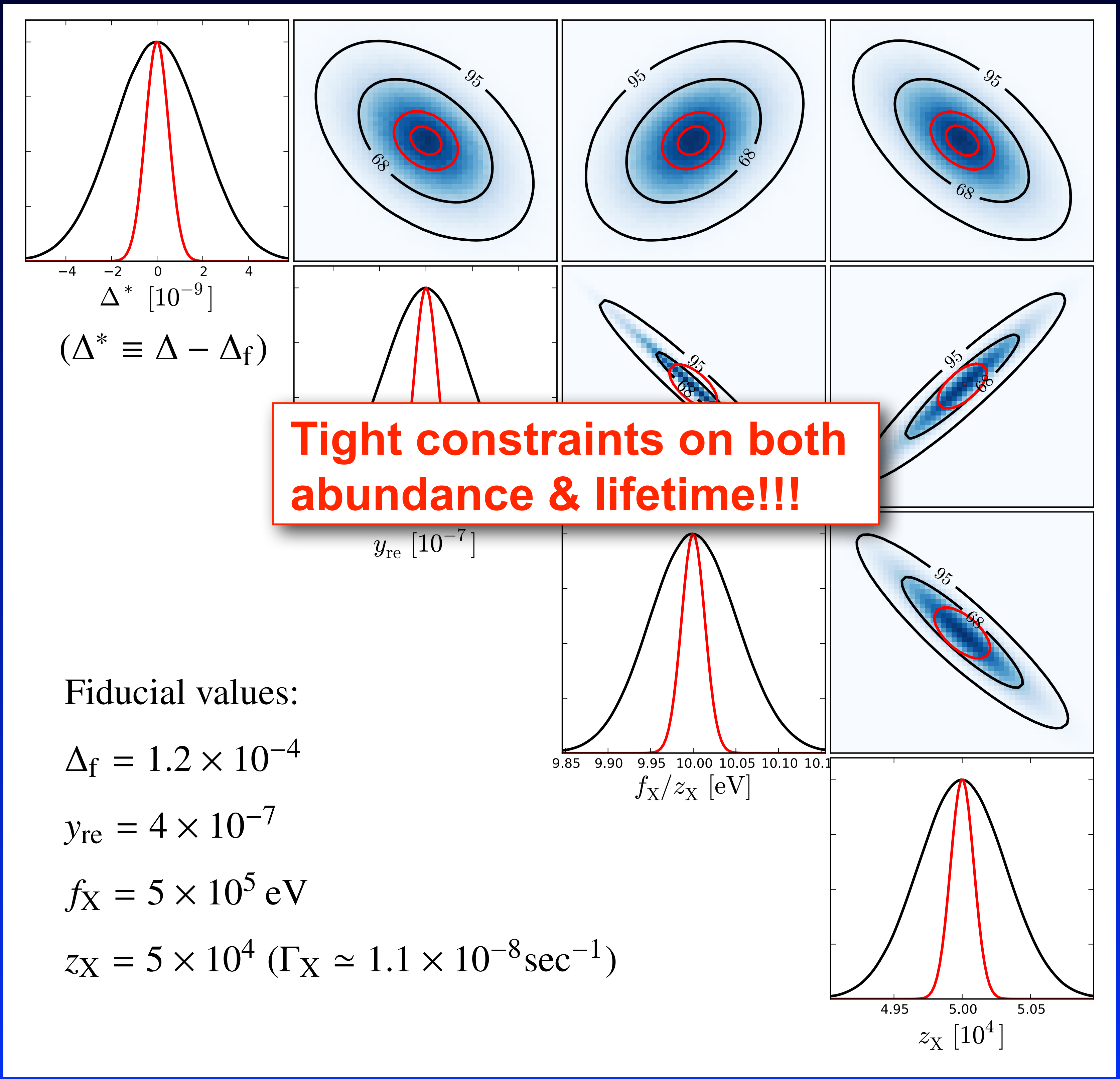
$y_{re} = 4 \times 10^{-7}$

$f_X = 5 \times 10^5$ eV

$z_X = 5 \times 10^4$ ($\Gamma_X \simeq 1.1 \times 10^{-8} \text{sec}^{-1}$)

JC, 2013, ArXiv:1304.6120

Decaying particle scenarios



Tight constraints on both abundance & lifetime!!!

Fiducial values:

$\Delta_f = 1.2 \times 10^{-4}$

$y_{re} = 4 \times 10^{-7}$

$f_X = 5 \times 10^5$ eV

$z_X = 5 \times 10^4 \; (\Gamma_X \simeq 1.1 \times 10^{-8} \mathrm{sec}^{-1})$

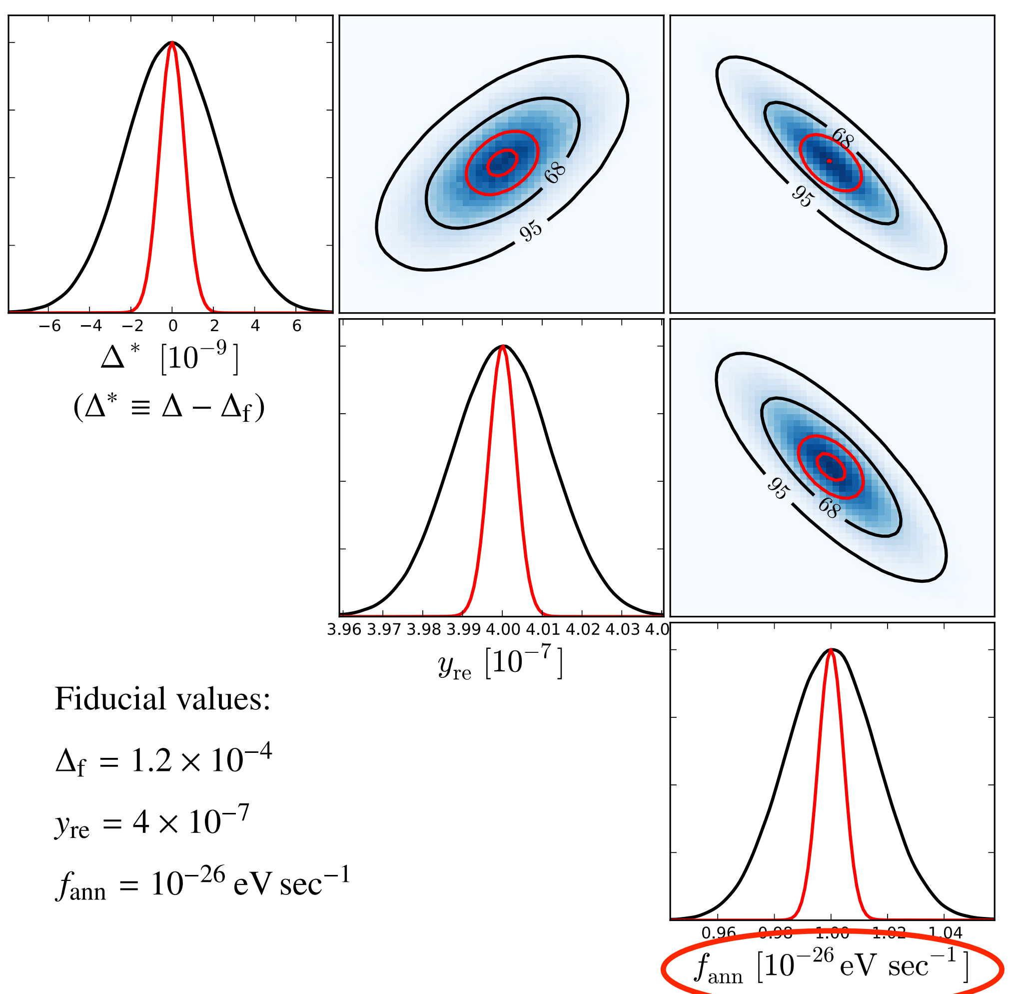
JC, 2013, ArXiv:1304.6120

Compressing the information of the distortion signal

Why model-independent approach to distortion signal

- Model-dependent analysis makes model-selection non-trivial
- Real information in the distortion signal limited by sensitivity and foregrounds
- *Principle Component Analysis* (PCA) can help optimizing this!
- useful for optimizing experimental designs (*frequencies; sensitivities,* ...)!

Annihilation scenario



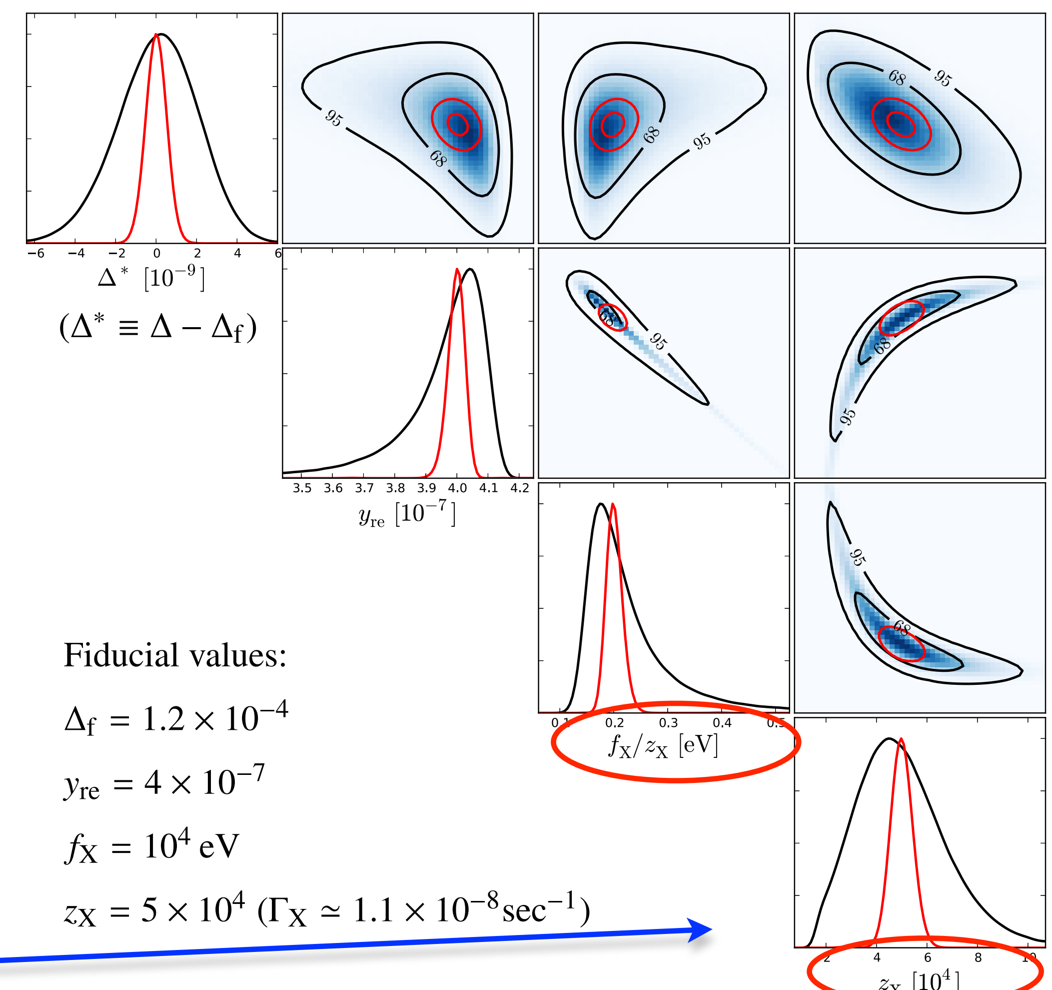
Fiducial values:

$\Delta_f = 1.2 \times 10^{-4}$

$y_{re} = 4 \times 10^{-7}$

$f_{ann} = 10^{-26}\,\mathrm{eV\,sec}^{-1}$

Decaying particle scenario



Fiducial values:

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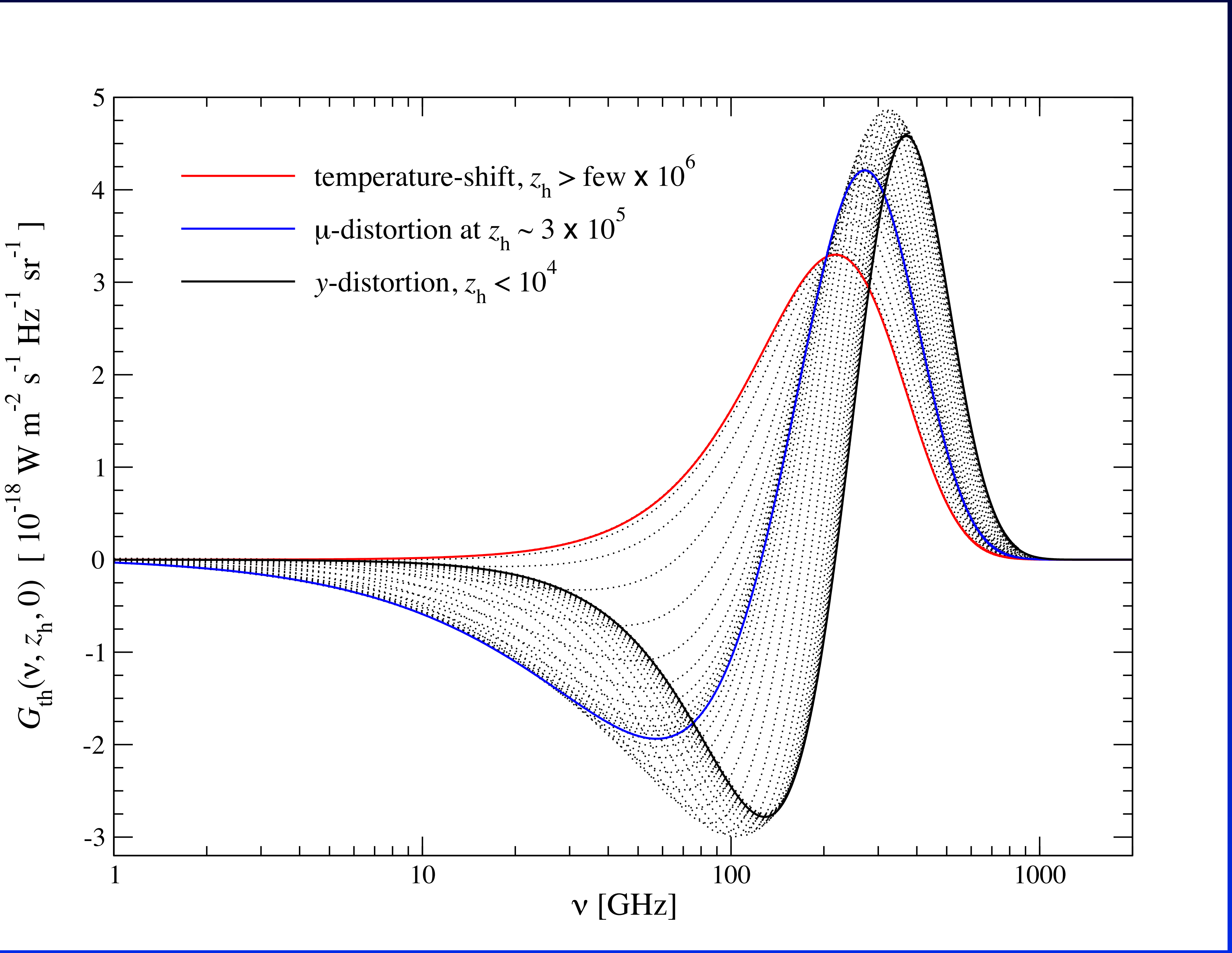
$f_X = 10^4$ eV

$z_X = 5 \times 10^4$ $(\Gamma_X \simeq 1.1 \times 10^{-8}\mathrm{sec}^{-1})$

How do we compare these?

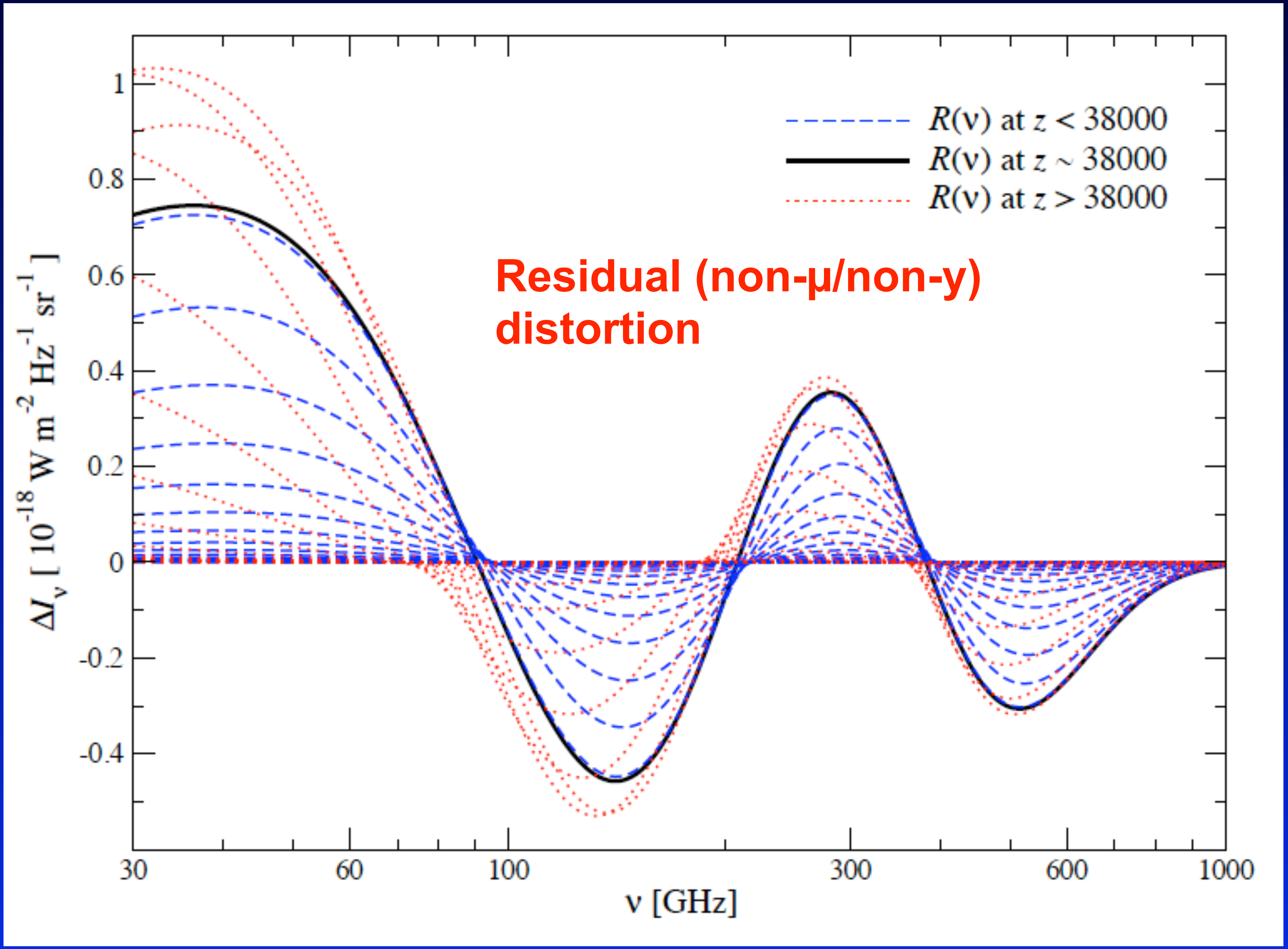
Using signal eigenmodes to compress the distortion data



- *Principle component decomposition* of the distortion signal
- compression of the useful information given instrumental settings

JC & Jeong, 2013

Using signal eigenmodes to compress the distortion data

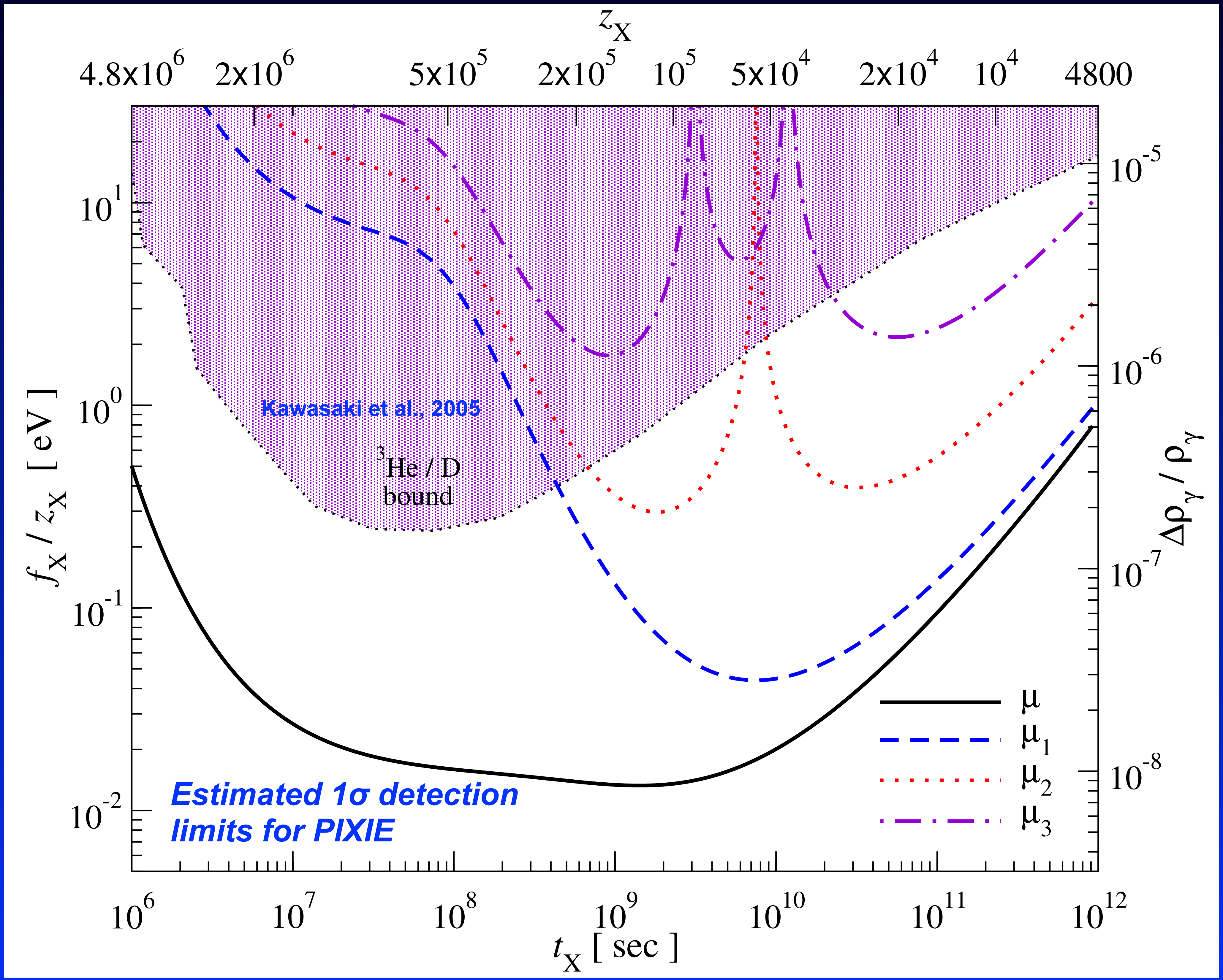


- *Principle component decomposition* of the distortion signal
- compression of the useful information given instrumental settings
- new set of observables

  $p=\{y, \mu, \mu_1, \mu_2, ...\}$
- model-comparison + forecasts of errors very simple!

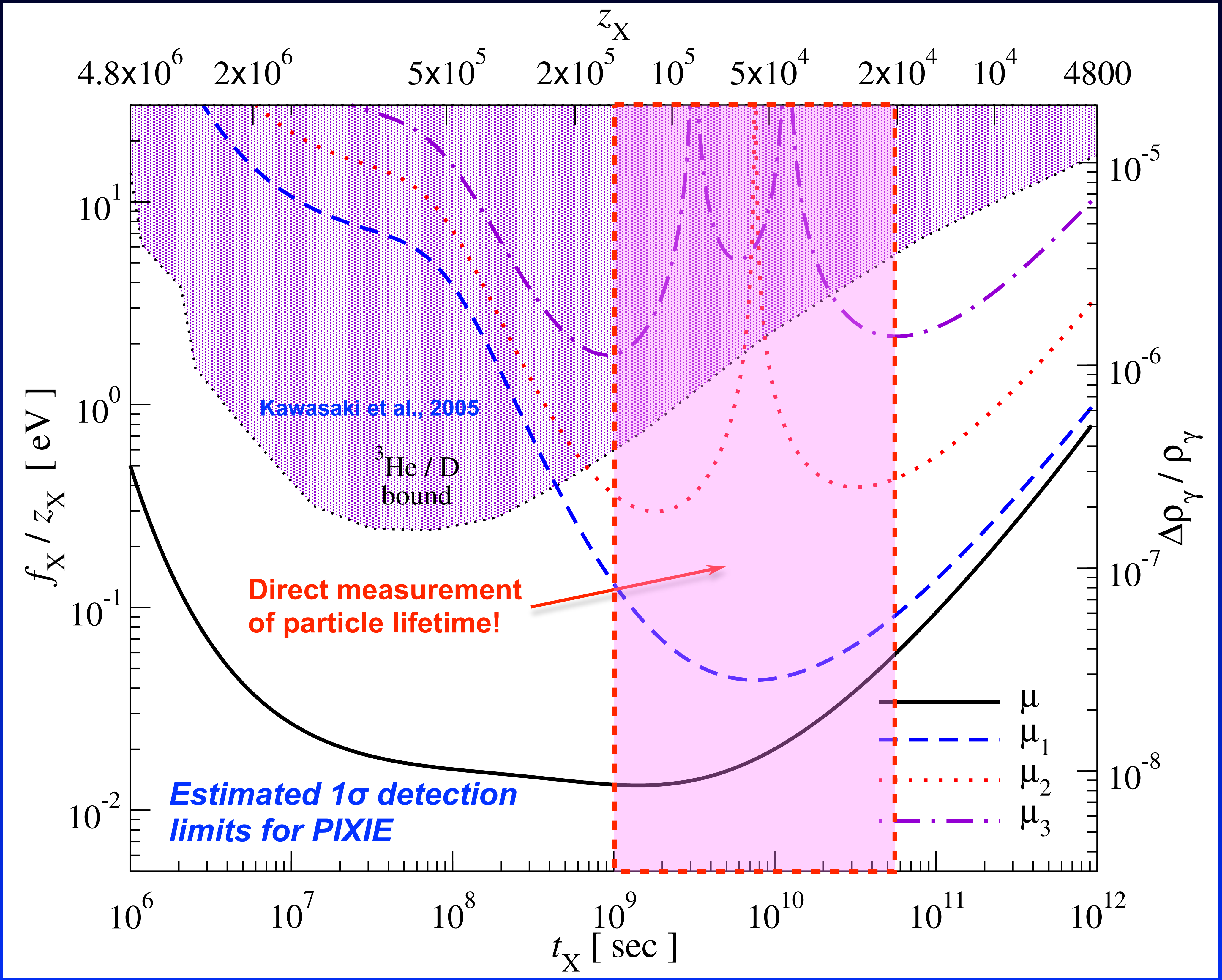
JC & Jeong, 2013

Distortions could shed light on decaying (DM) particles!



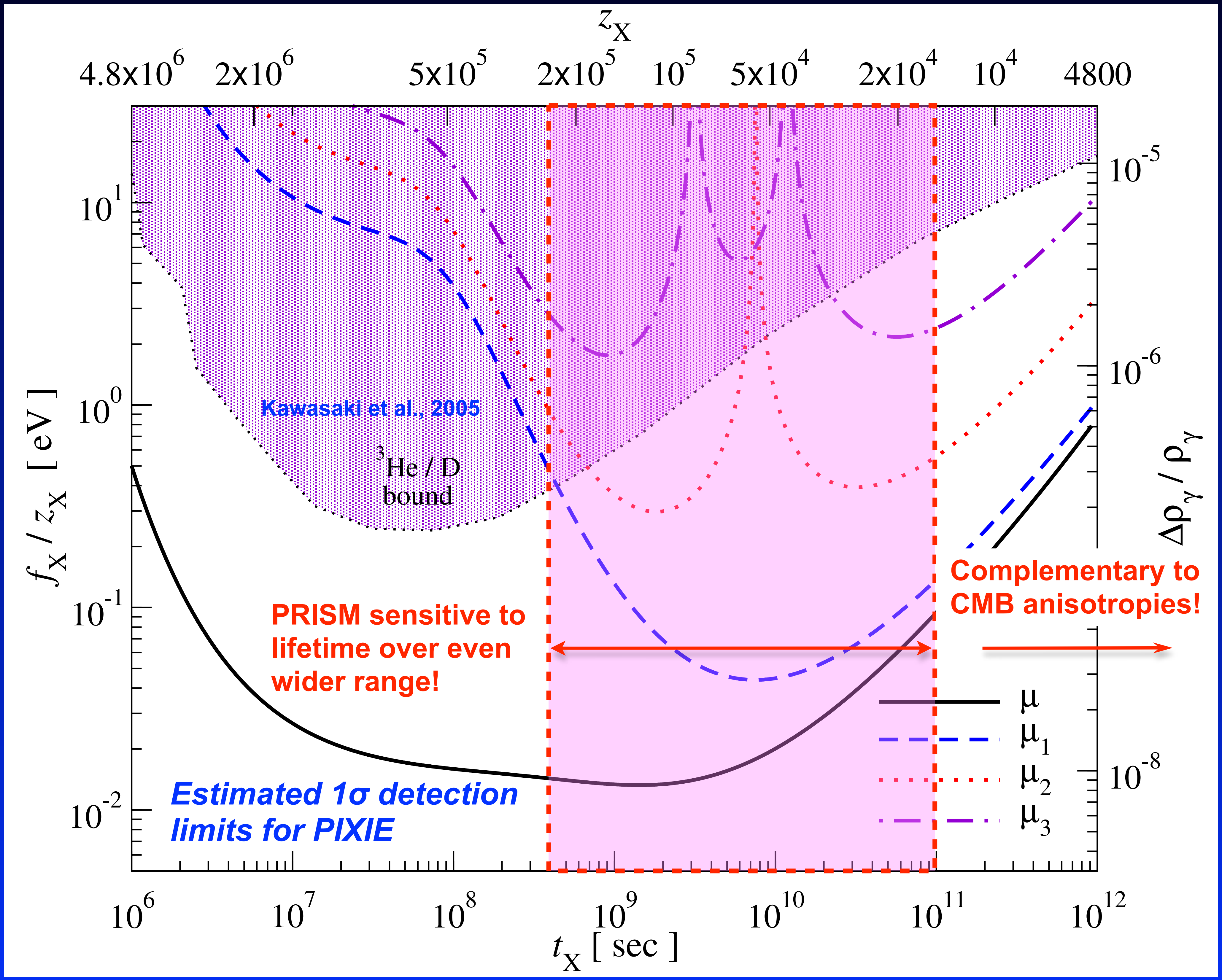
JC & Jeong, 2013

Distortions could shed light on decaying (DM) particles!



JC & Jeong, 2013

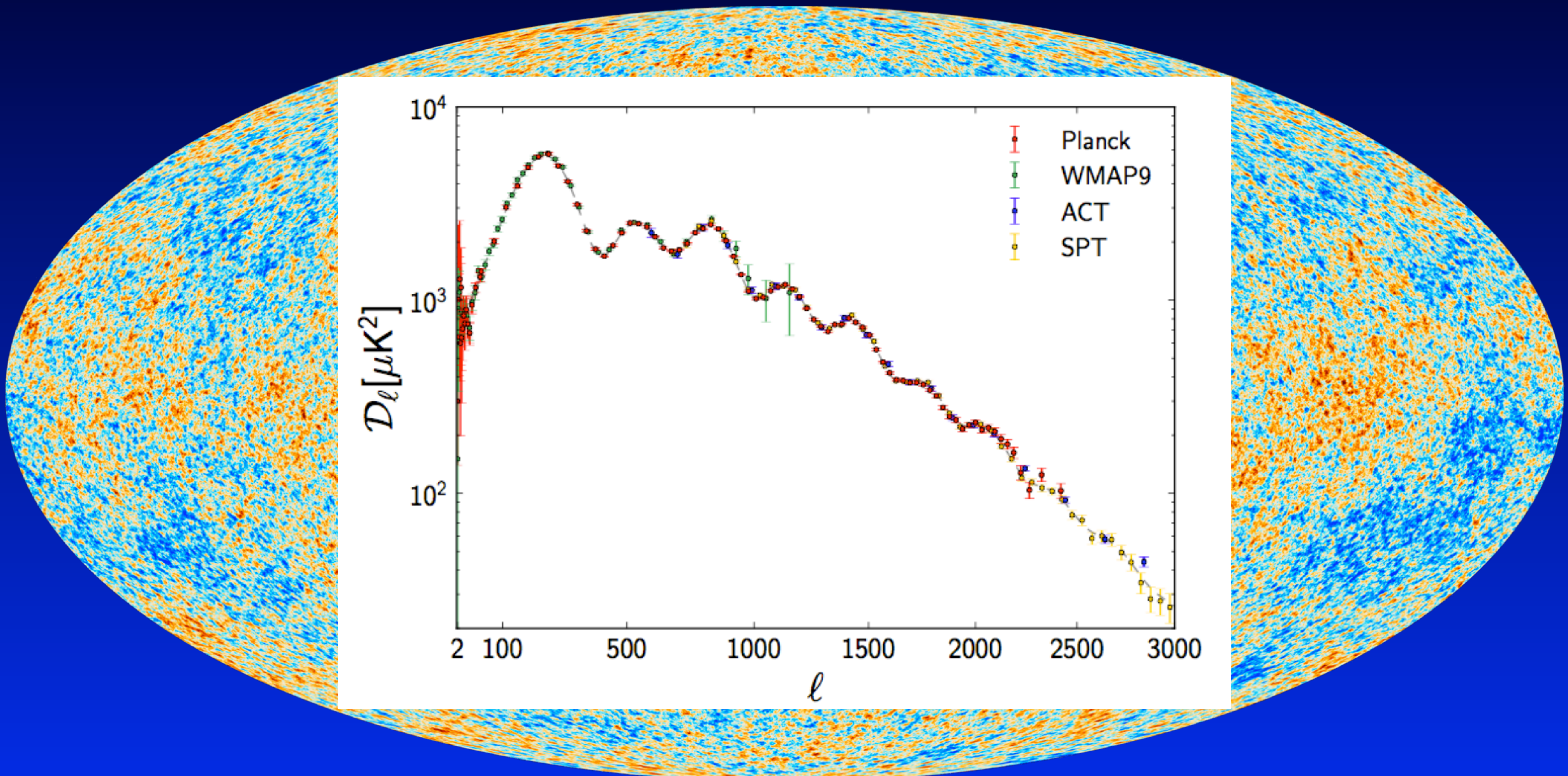
Distortions could shed light on decaying (DM) particles!



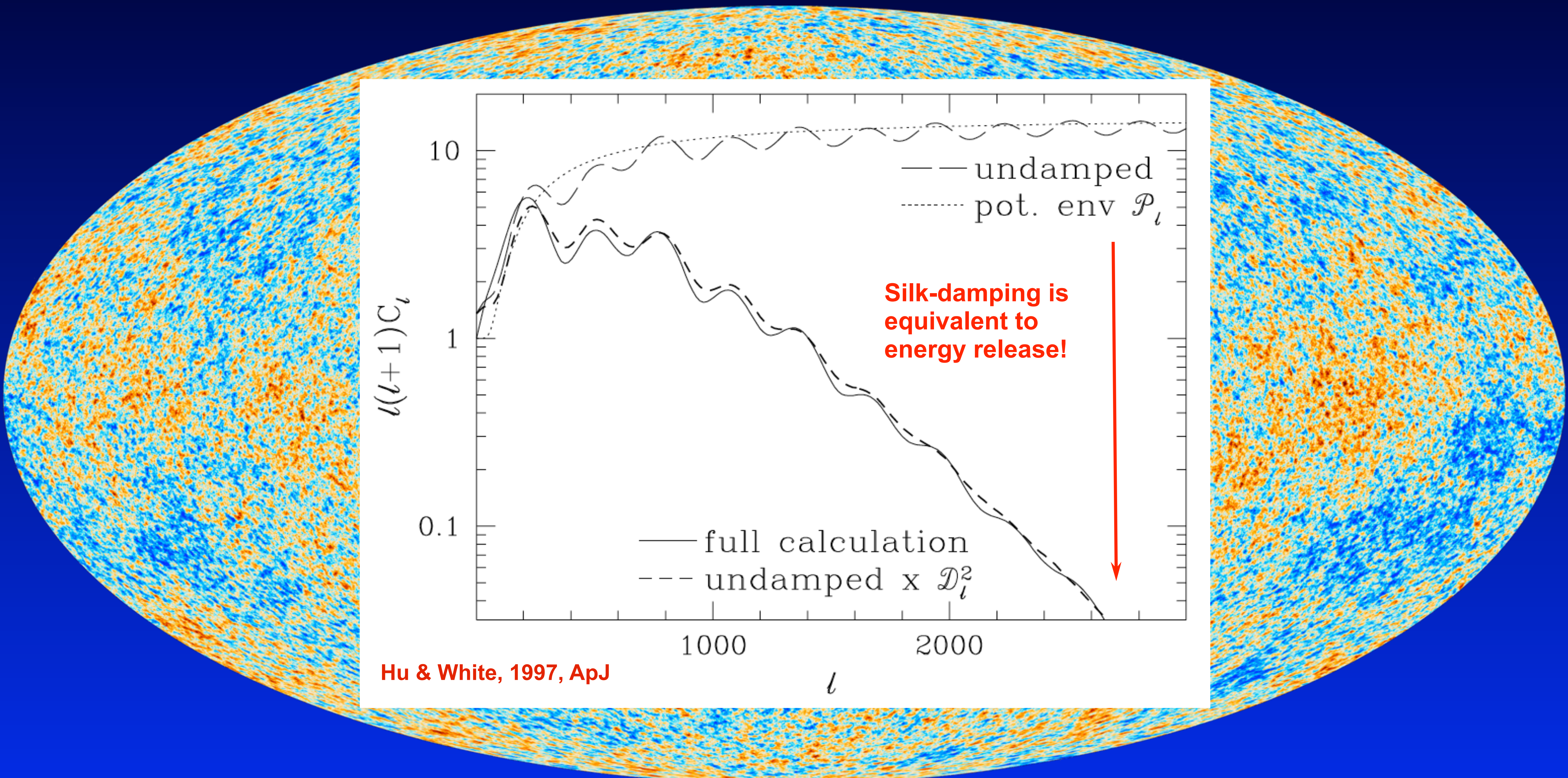
JC & Jeong, 2013

The dissipation of small-scale acoustic modes

Dissipation of small-scale acoustic modes



Dissipation of small-scale acoustic modes



Energy release caused by dissipation process

'Obvious' dependencies:

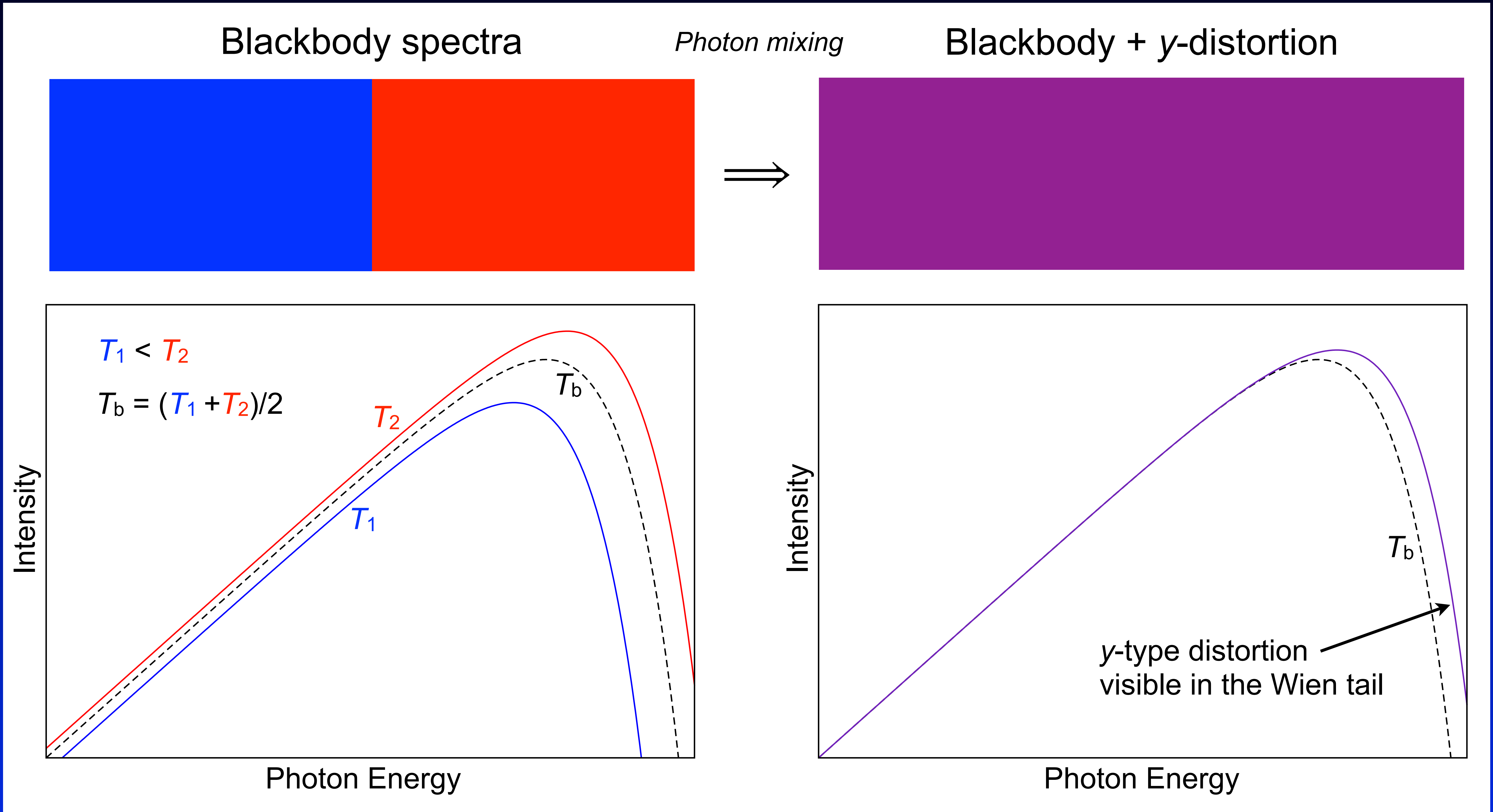
- *Amplitude* of the small-scale power spectrum
- *Shape* of the small-scale power spectrum
- *Dissipation scale* → $k_D \sim (H_0\, \Omega_{rel}^{1/2}\, N_{e,0})^{1/2}\, (1+z)^{3/2}$ at early times

not so 'obvious' dependencies:

- *primordial non-Gaussianity* in the ultra squeezed limit
  (Pajer & Zaldarriaga, 2012; Ganc & Komatsu, 2012)
- *Type* of the perturbations (adiabatic ↔ isocurvature)
  (Barrow & Coles, 1991; Hu et al., 1994; Dent et al, 2012, JC & Grin, 2012)
- *Neutrinos* (or any extra relativistic degree of freedom)

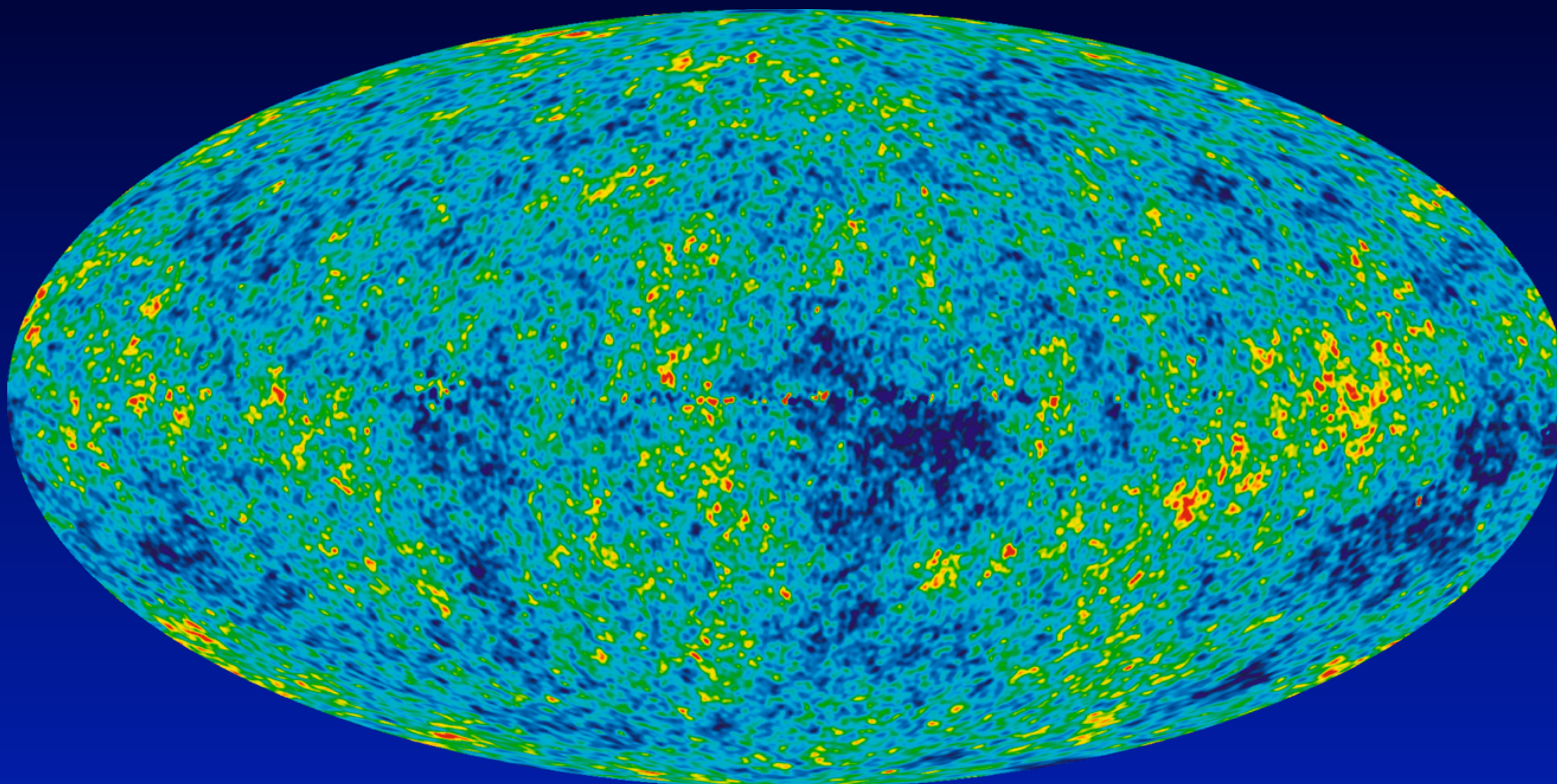
CMB Spectral distortions could add additional numbers beyond 'just' the tensor-to-scalar ratio from B-modes!

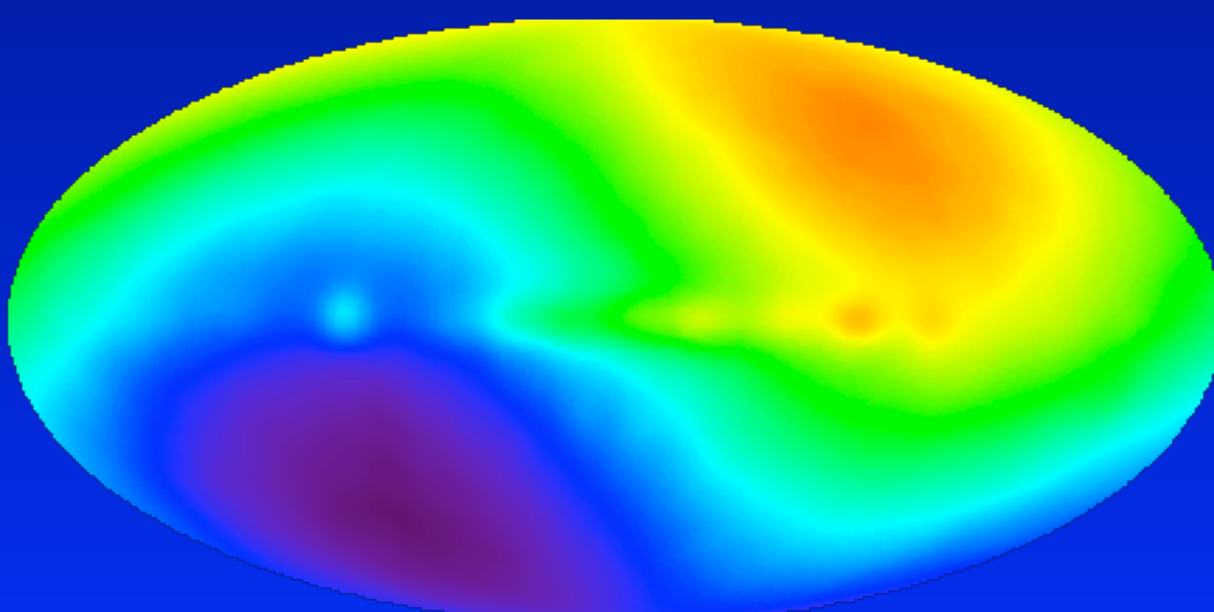
Distortion due to mixing of blackbodies



JC, Hamann & Patil, 2015

Distortion caused by superposition of blackbodies





- average spectrum

$$\Rightarrow \; y \simeq \frac{1}{2}\left\langle \left(\frac{\Delta T}{T}\right)^2 \right\rangle \approx 8\times 10^{-10}$$

$$\Delta T_{\rm sup} \simeq T\left\langle \left(\frac{\Delta T}{T}\right)^2 \right\rangle \approx 4.4{\rm nK}$$

- known with very high precision

- *CMB dipole* ($\beta_c \sim 1.23\text{x}10^{-3}$)

$$\Rightarrow \quad y \simeq \frac{\beta_{\rm c}^2}{6} \approx 2.6\times 10^{-7}$$

$$\Delta T_{\rm sup} \simeq T\,\frac{\beta_{\rm c}^2}{3} \approx 1.4\mu{\rm K}$$

- electrons are up-scattered
- can be taken out at the level of ~ 10^{-9}

JC & Sunyaev, 2004
JC, Khatri & Sunyaev, 2012

COBE/DMR: ΔT = 3.353 mK

Effective energy release caused by damping effect

- Effective heating rate from full 2x2 Boltzmann treatment **(JC, Khatri & Sunyaev, 2012)**

$$\frac{1}{a^4\rho_\gamma}\frac{\mathrm{d}a^4 Q_{\rm ac}}{\mathrm{d}t} = 4\sigma_{\rm T} N_{\rm e} c \left\langle \frac{(3\Theta_1-\beta)^2}{3} + \frac{9}{2}\Theta_2^2 - \frac{1}{2}\Theta_2(\Theta_0^{\rm P}+\Theta_2^{\rm P}) + \sum_{l\geq 3}(2l+1)\Theta_\ell^2 \right\rangle$$

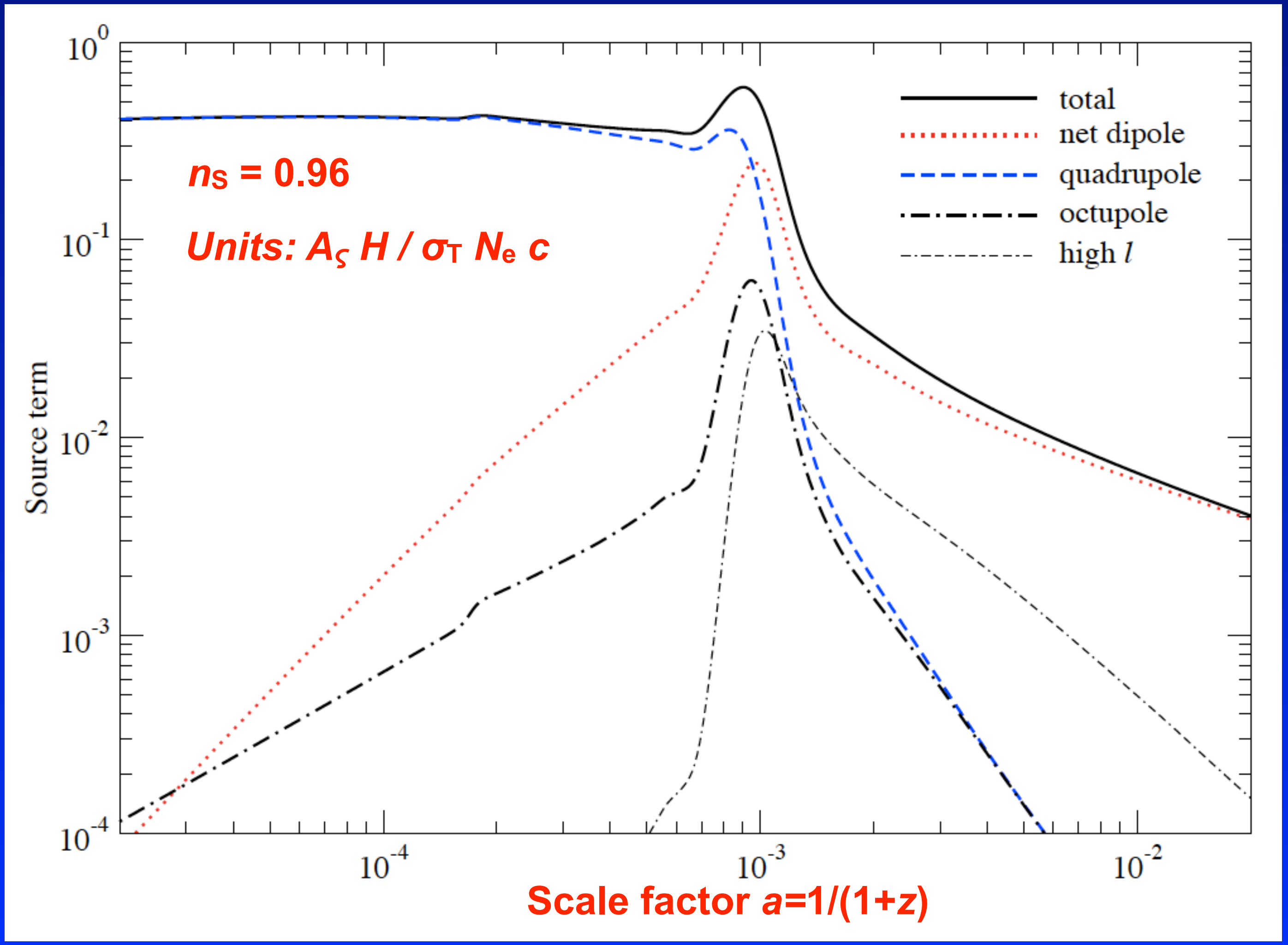
$$\Theta_\ell = \frac{1}{2}\int \Theta(\mu) P_\ell(\mu)\mathrm{d}\mu$$

gauge-independent dipole **effect of polarization** **higher multipoles**

$$\langle XY \rangle = \int \frac{k^2 \mathrm{d}k}{2\pi^2} P(k) X(k) Y(k)$$

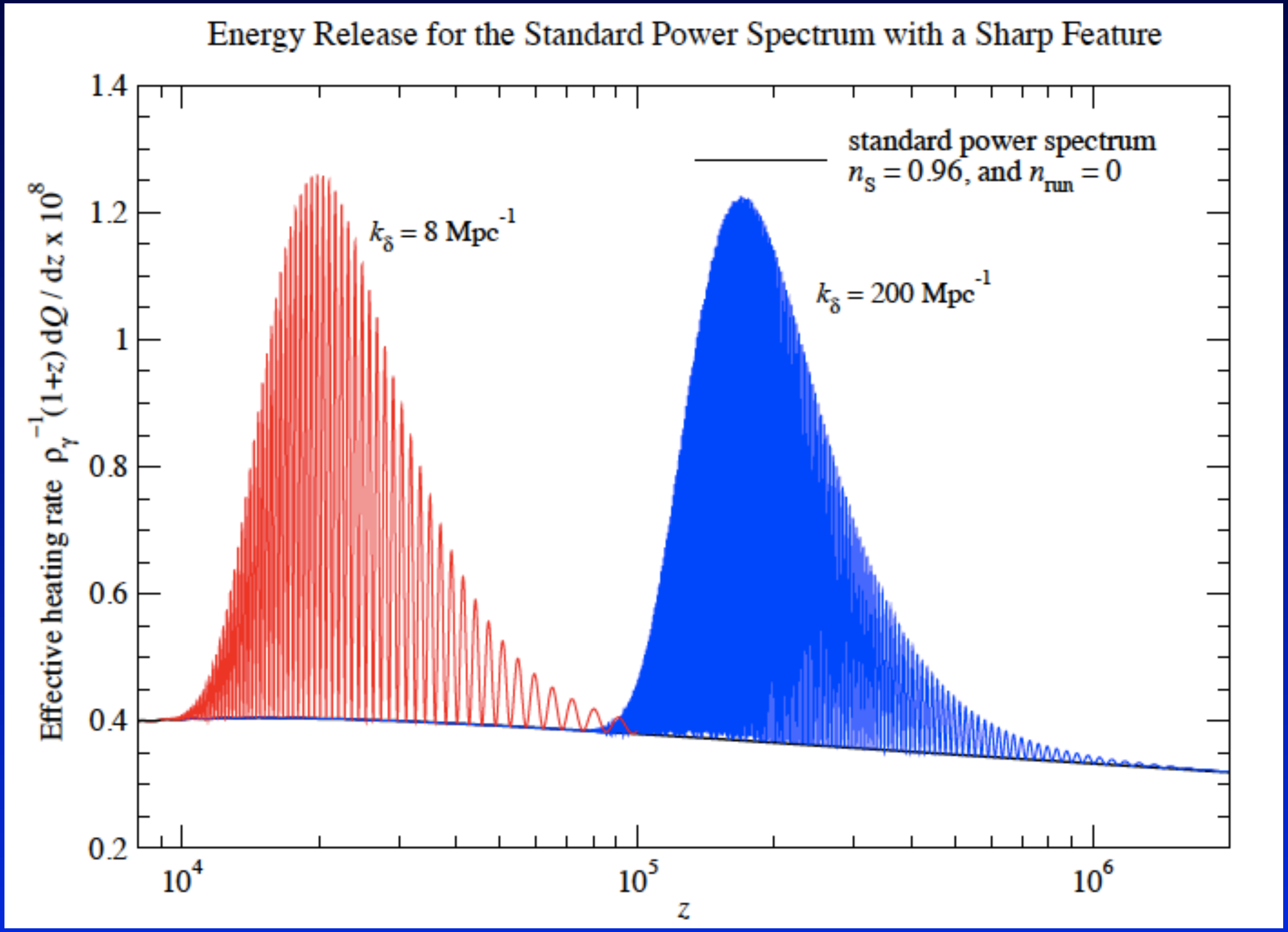
Primordial power spectrum

- *quadrupole* dominant at high *z*
- *net dipole* important only at low redshifts
- polarization ~5% effect
- contribution from higher multipoles rather small



JC, Khatri & Sunyaev, 2012

Which modes dissipate in the μ and y-eras?



JC, Erickcek & Ben-Dayan, 2012

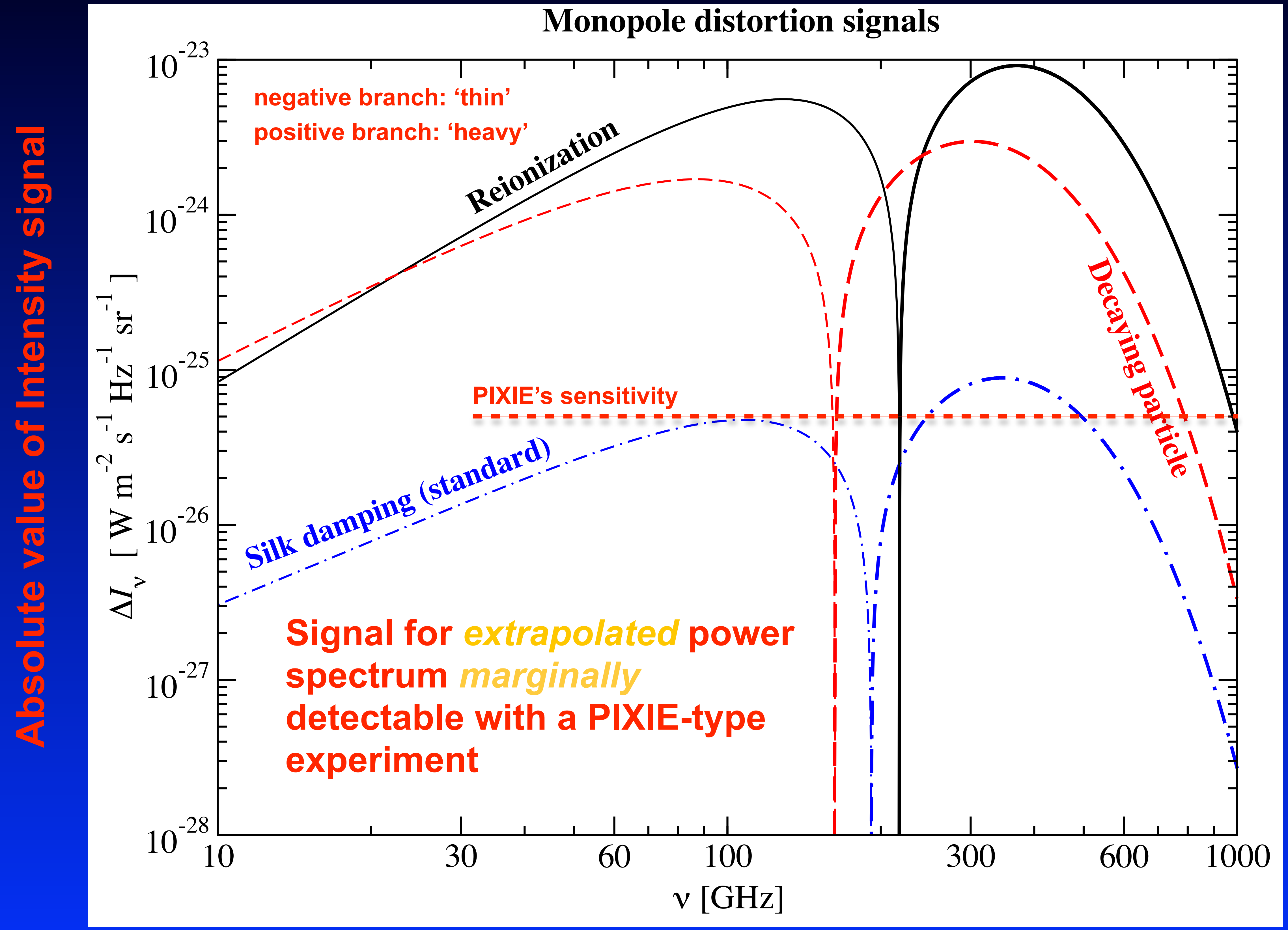
- Single mode with wavenumber k dissipates its energy at

  $z_d \sim 4.5 \times 10^5 (k\ \mathrm{Mpc}/10^3)^{2/3}$

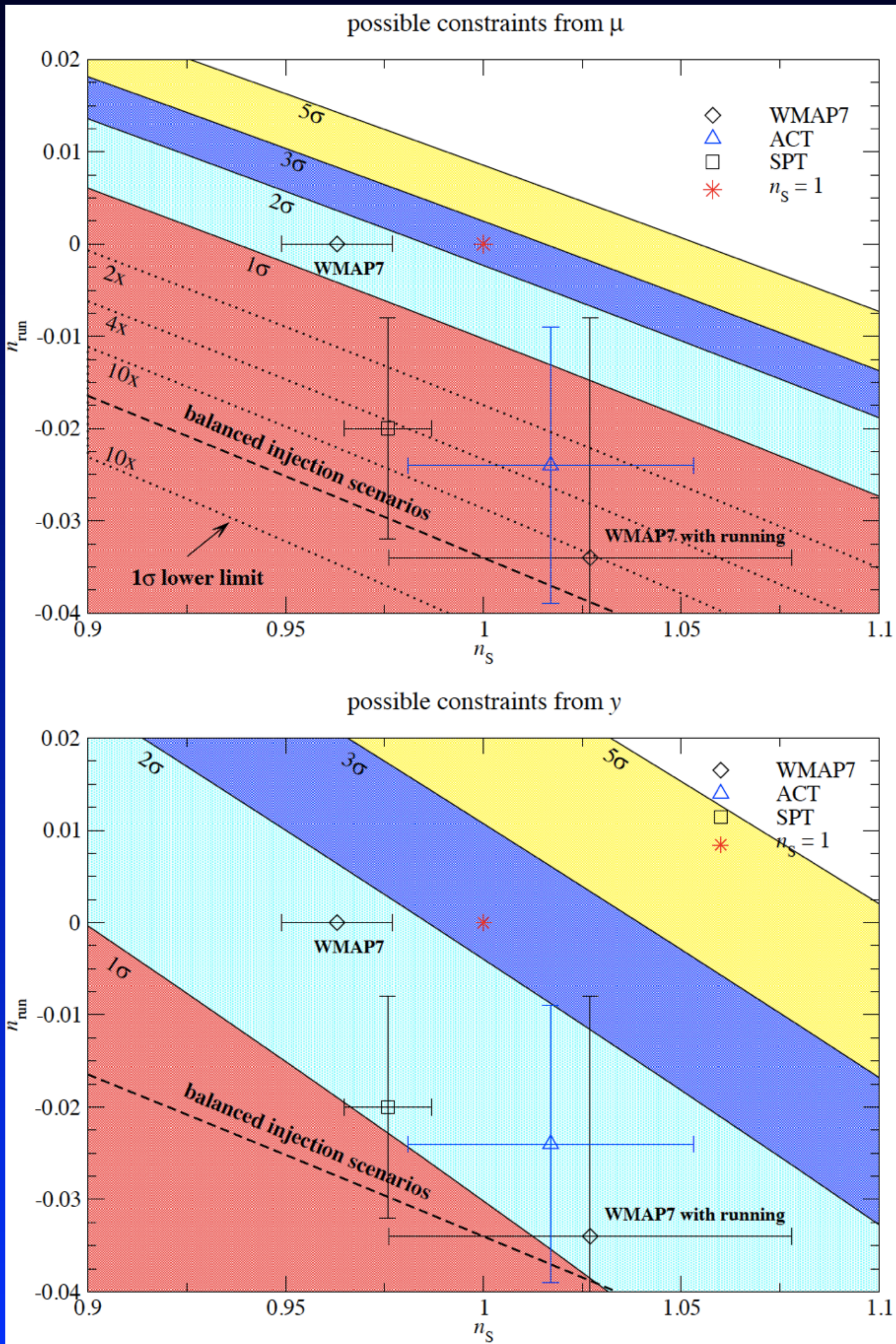
- Modes with wavenumber $50\ \mathrm{Mpc}^{-1} < k < 10^4\ \mathrm{Mpc}^{-1}$ dissipate their energy during the μ-era

- Modes with $k < 50\ \mathrm{Mpc}^{-1}$ cause y-distortion

Average CMB spectral distortions



Constraints on the standard primordial power spectrum



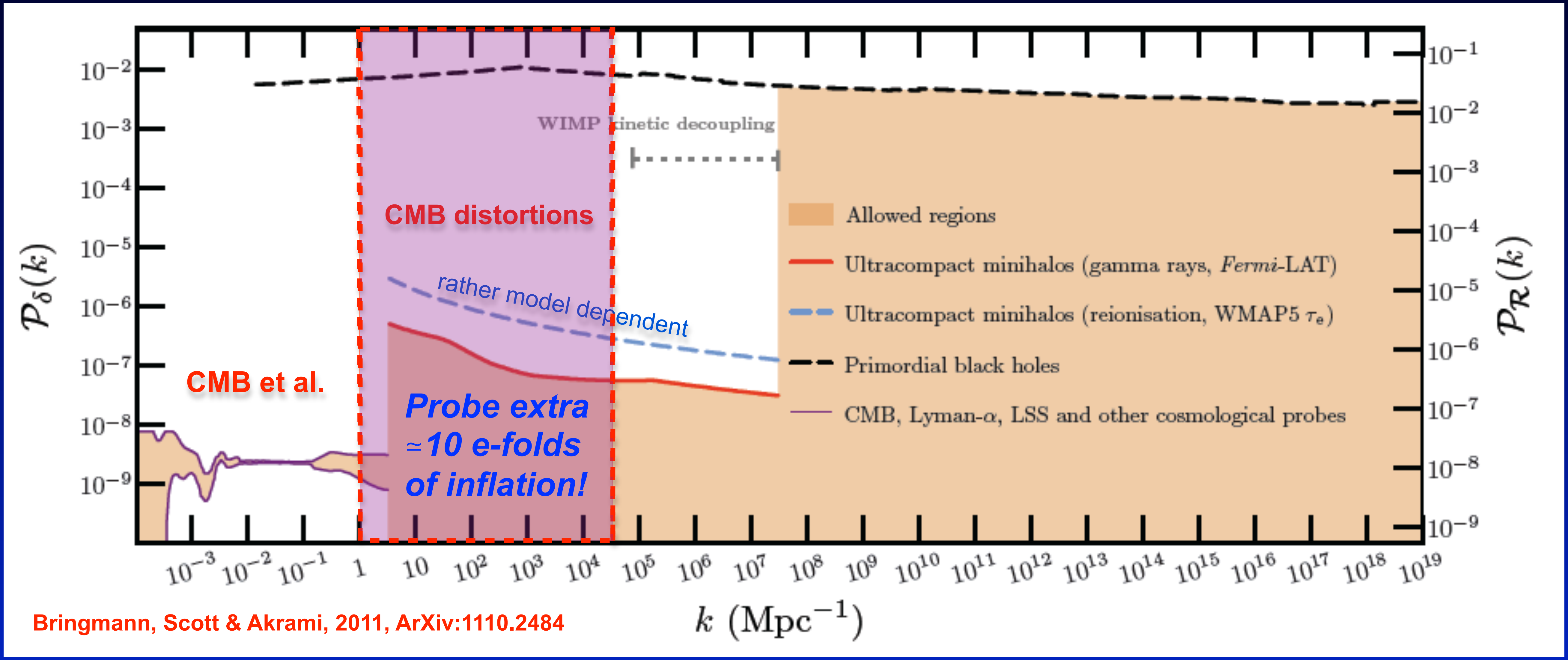
- For *any* given power spectrum very precise predictions are possible!
- The *physics* going into the computation are *well understood*
- For the standard power spectrum PIXIE might detect the *μ*-distortion caused by acoustic damping at ~ 1.5σ level
- PIXIE could *independently* rule out a scale-invariant power spectrum at ~ 2.5σ level
- *y*-distortion will be harder to measure, since many *other astrophysical processes* cause *y*-distortions at low redshift

$$P_\zeta(k) = 2\pi^2 A_\zeta k^{-3} (k/k_0)^{n_S - 1 + \frac{1}{2} n_{run} \ln(k/k_0)}$$

JC, Khatri & Sunyaev, 2012

But this is not all that one could look at !!!

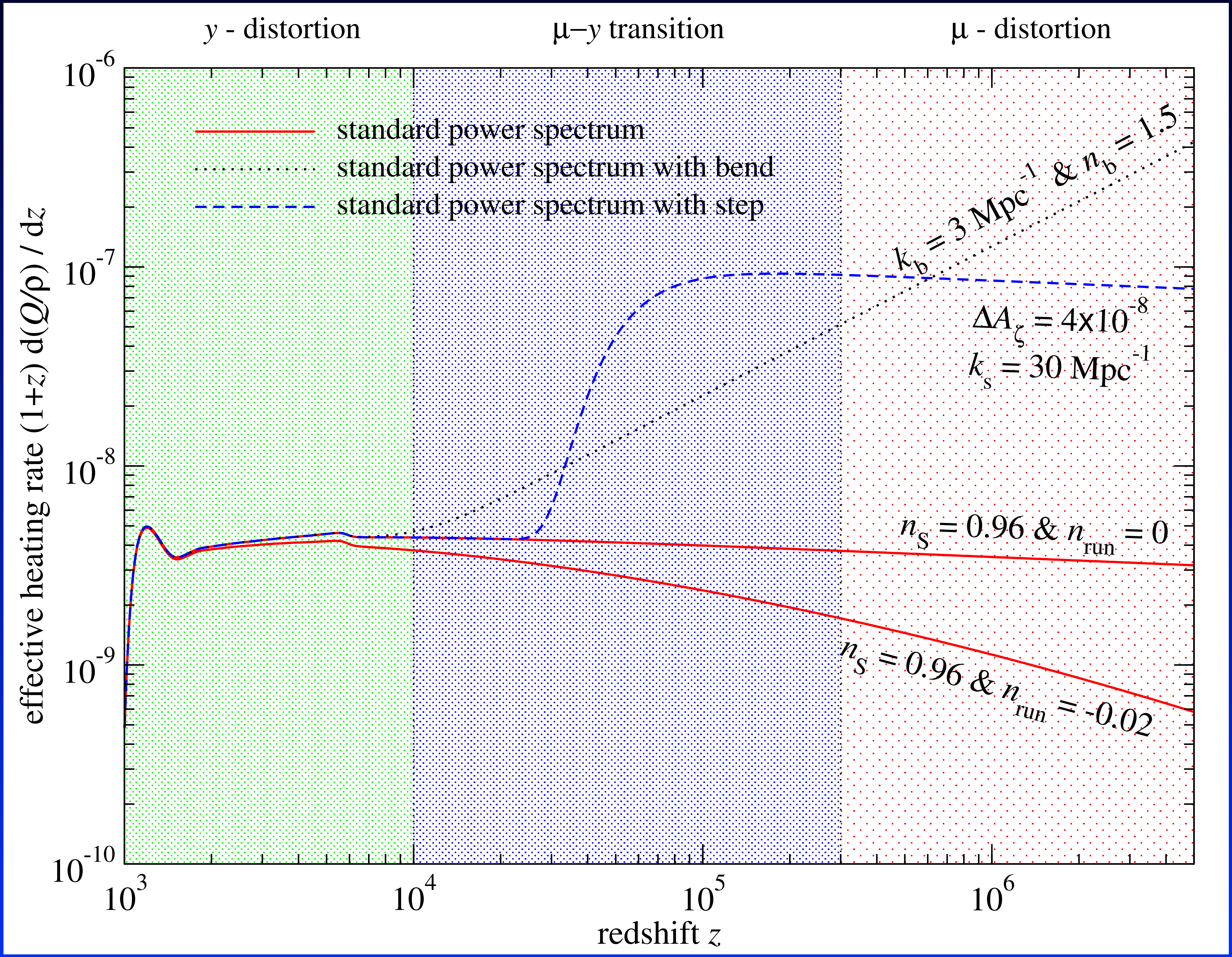
Distortions provide additional power spectrum constraints!



- Amplitude of power spectrum rather uncertain at $k > 3$ Mpc^{-1}
- improved limits at smaller scales can *rule out* many *inflationary models*
- CMB spectral distortions would *extend* our *lever arm* to $k \sim 10^4$ Mpc^{-1}
- very *complementary* piece of information about early-universe physics

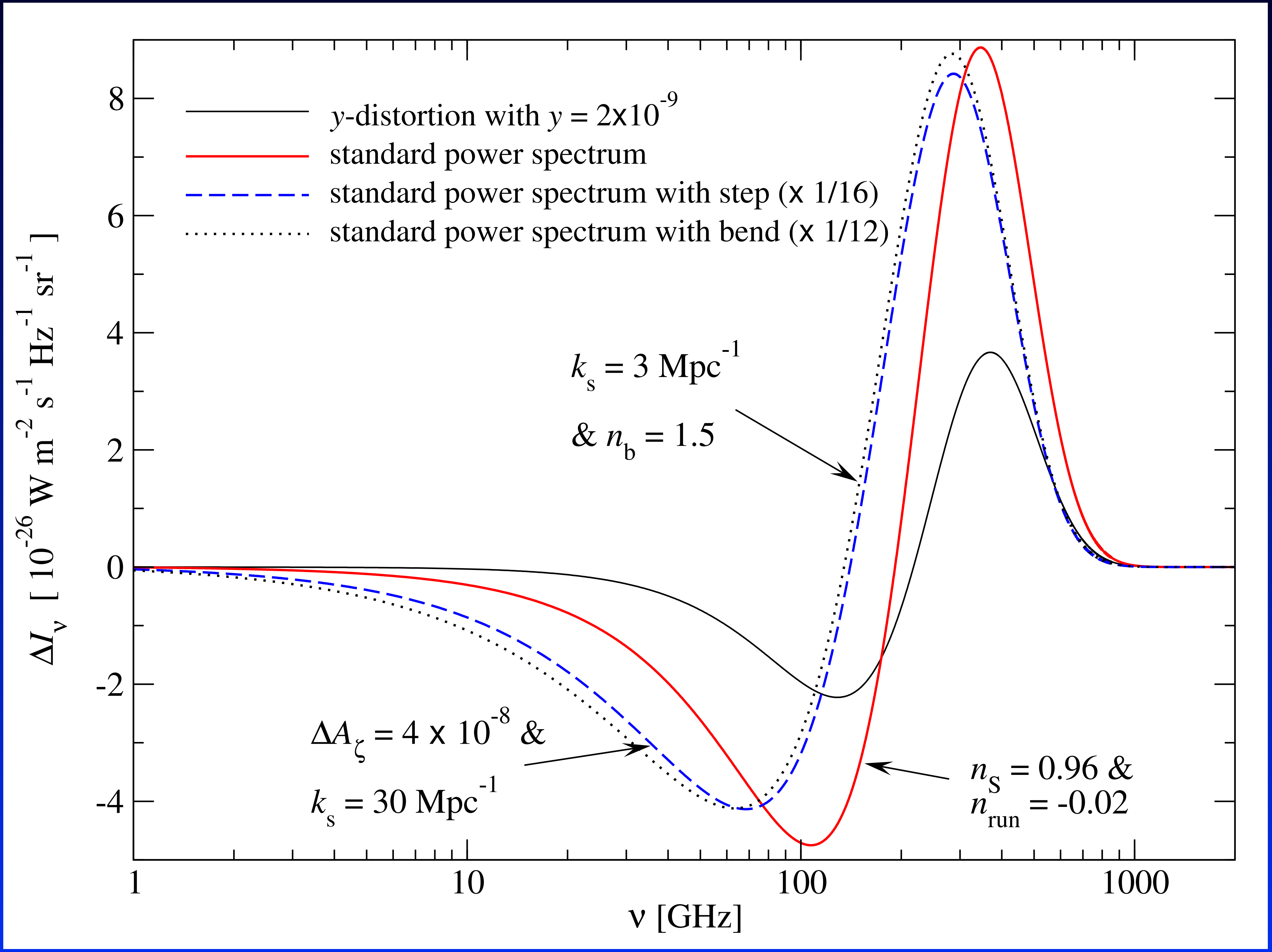
e.g., JC, Khatri & Sunyaev, 2012; JC, Erickcek & Ben-Dayan, 2012; JC & Jeong, 2013

Probing the small-scale power spectrum



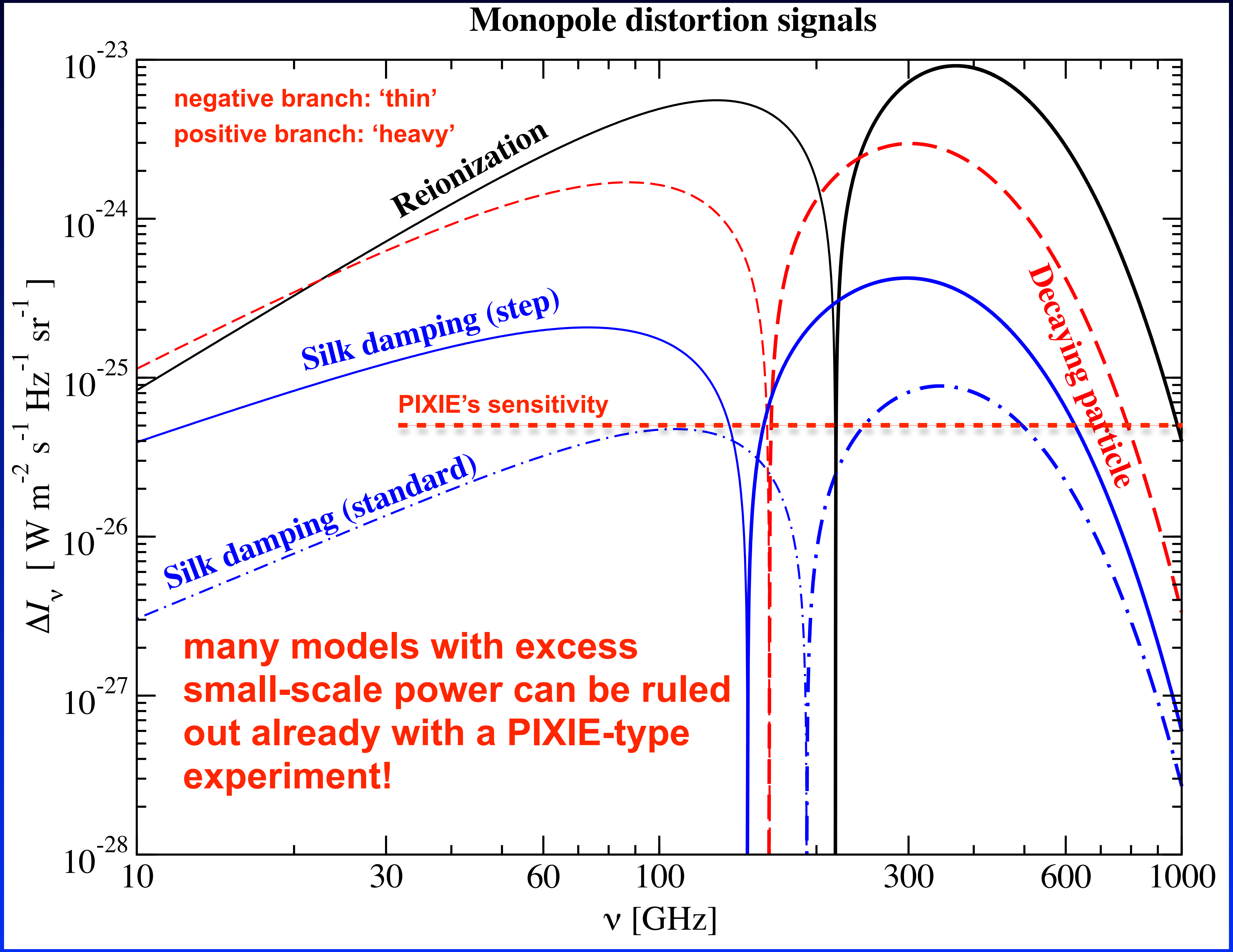
JC, 2013, Arxiv:1304.6120

Probing the small-scale power spectrum



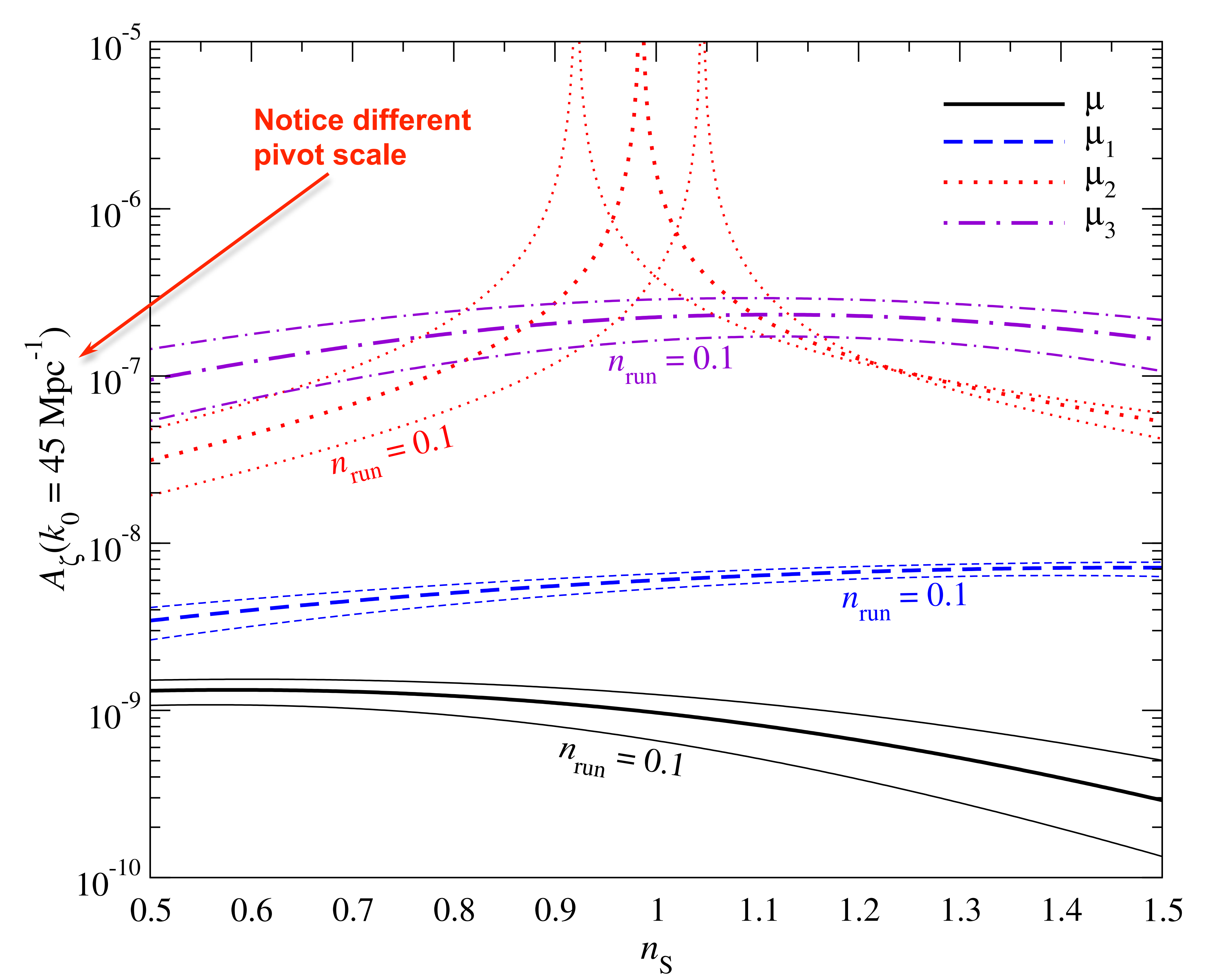
JC, 2013, Arxiv:1304.6120

Average CMB spectral distortions



Absolute value of Intensity signal

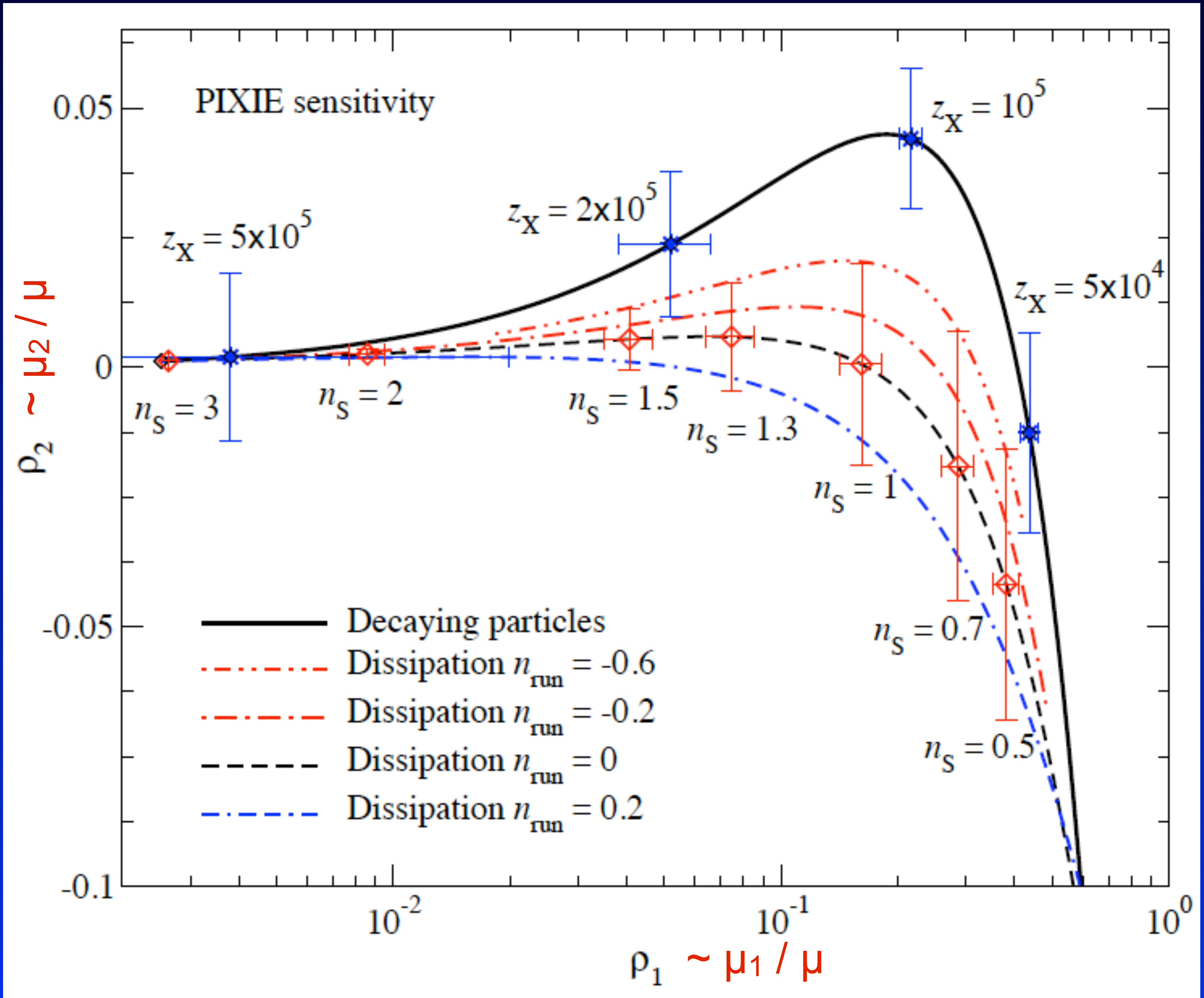
Dissipation scenario: 1σ-detection limits for PIXIE



JC & Jeong, 2013

$$P_\zeta(k) = 2\pi^2 A_\zeta k^{-3}(k/k_0)^{n_S-1+\frac{1}{2}n_{\mathrm{run}}\ln(k/k_0)}$$

Distinguishing dissipation and decaying particle scenarios

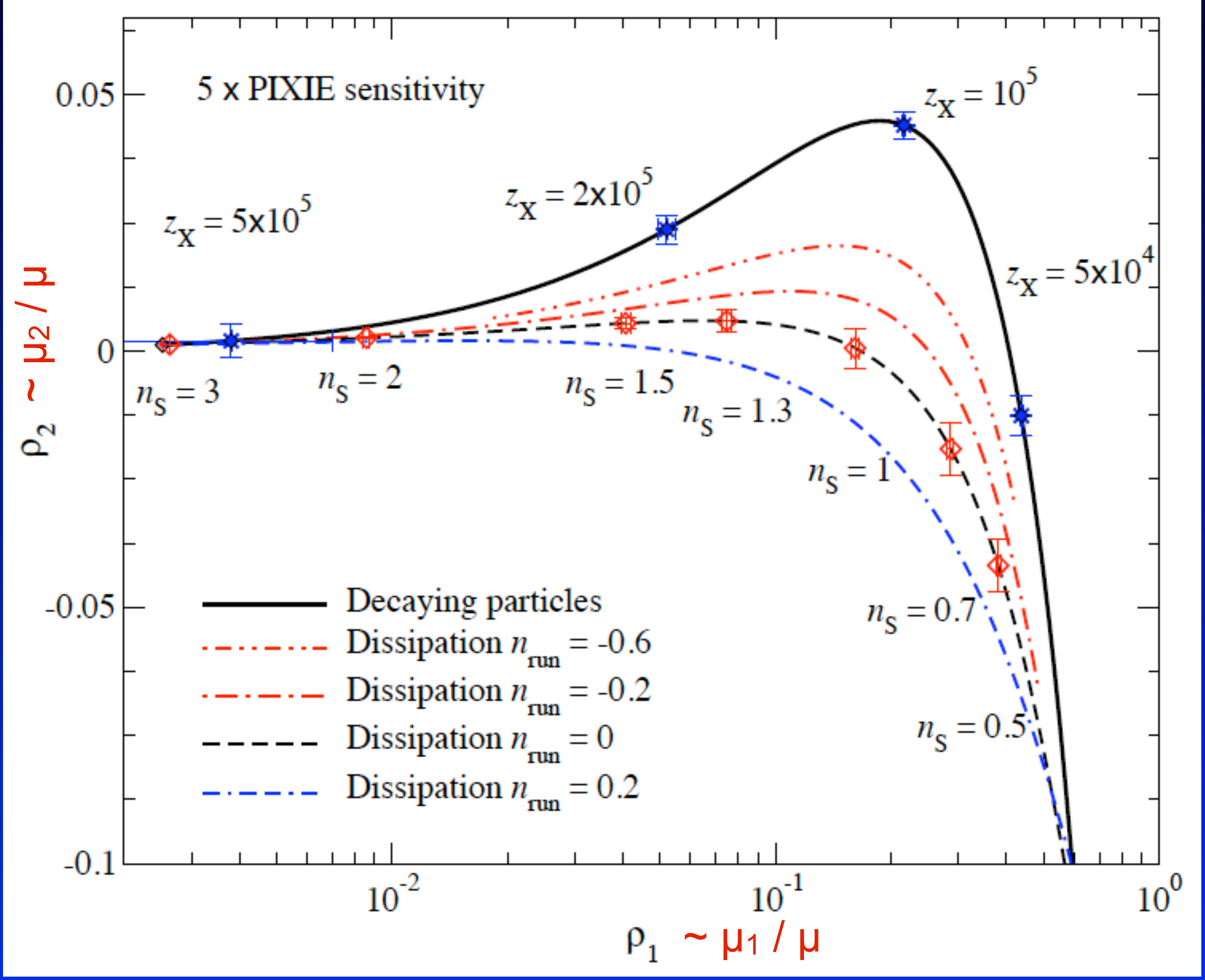


- measurement of μ, μ_1 & μ_2
- trajectories of decaying particle and dissipation scenarios differ!
- scenarios can in principle be distinguished

$$A_\zeta = 5 \times 10^{-8}$$

JC & Jeong, 2013

Distinguishing dissipation and decaying particle scenarios

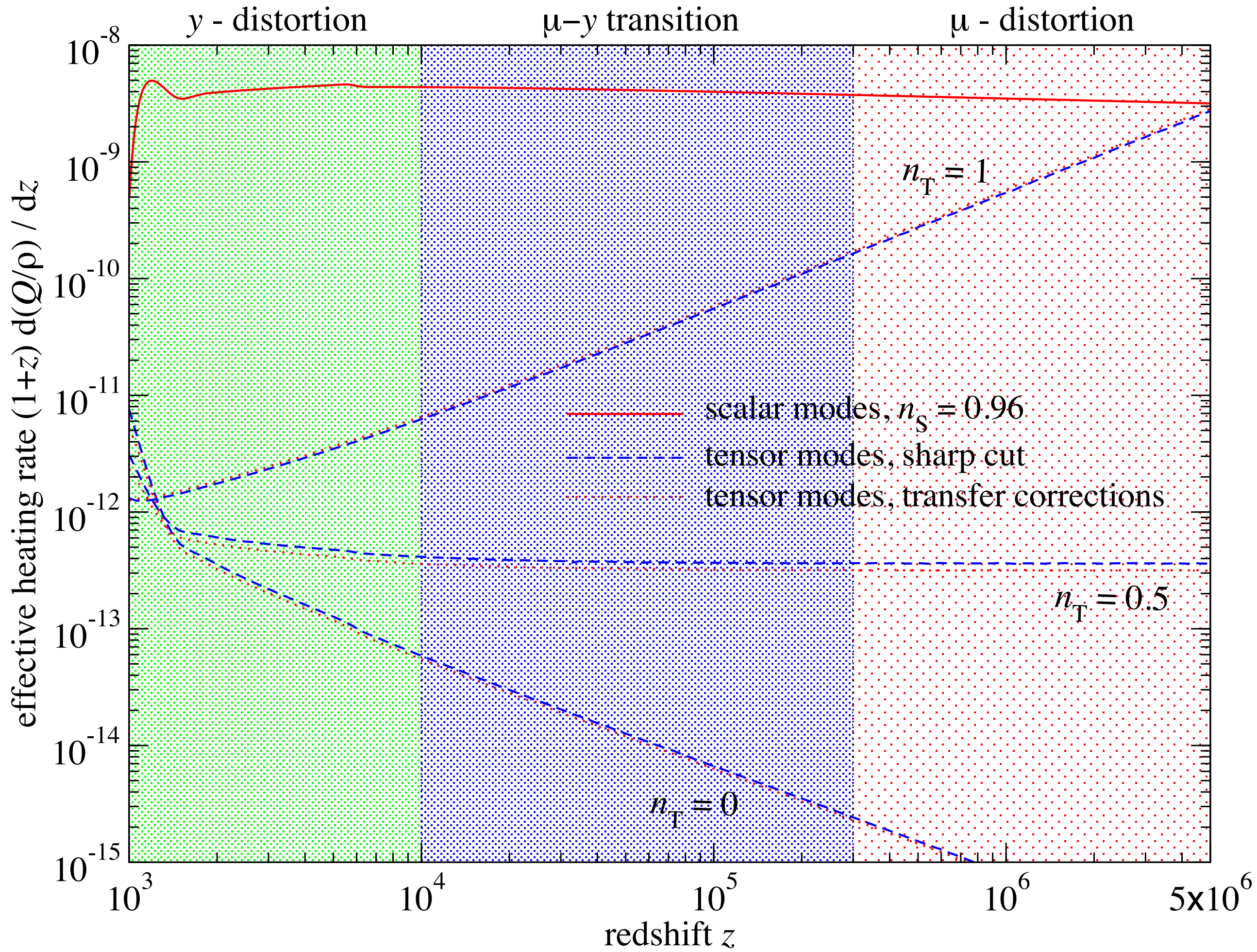


- measurement of μ, μ_1 & μ_2
- trajectories of decaying particle and dissipation scenarios differ!
- scenarios can in principle be distinguished

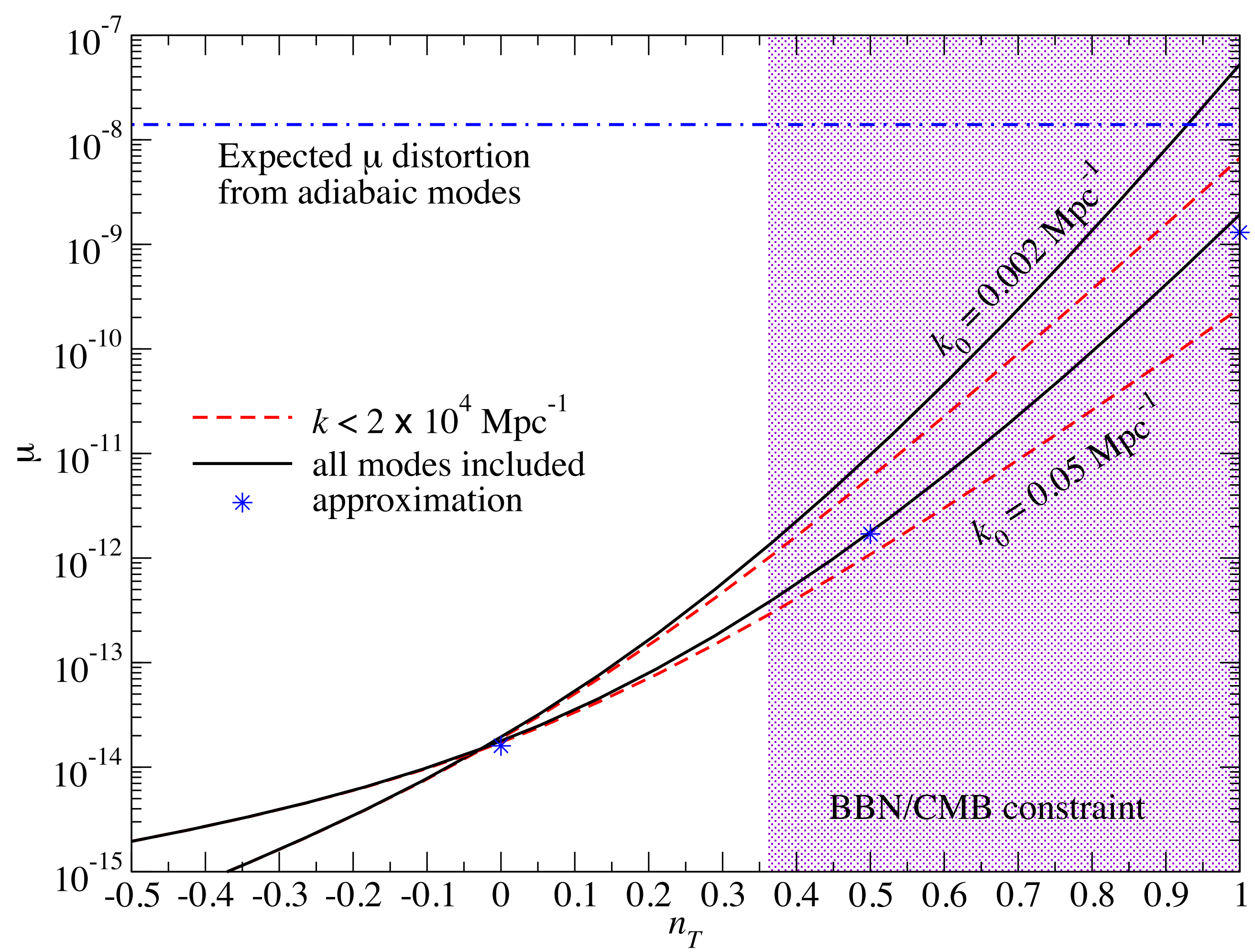
$$A_\zeta = 5 \times 10^{-8}$$

JC & Jeong, 2013

Dissipation of tensor perturbations



- distortion signal very small compared to adiabatic modes
- no severe contamination in simplest cases
- models with 'large' distortion already constrained by BBN/CMB



- heating rate can be computed similar to adiabatic modes
- heating rate much smaller than for scalar perturbations
- roughly constant per dlnz for n_T~0.5

JC et al., 2014, ArXiv:1407.3653

The cosmological recombination radiation

Simple estimates for hydrogen recombination

Hydrogen recombination:

- per recombined hydrogen atom an energy of ~ 13.6 eV in form of photons is released
- at z ~ 1100 → $\Delta\varepsilon/\varepsilon \sim 13.6\ \mathrm{eV}\ N_b / (N_\gamma\, 2.7 k T_r) \sim 10^{-9} - 10^{-8}$

→ recombination occurs at redshifts $z < 10^4$

→ At that time the *thermalization* process doesn't work anymore!

→ There should be some *small* spectral distortion due to additional Ly-α and 2s-1s photons!
(Zeldovich, Kurt & Sunyaev, 1968, ZhETF, 55, 278; Peebles, 1968, ApJ, 153, 1)

→ In 1975 ***Viktor Dubrovich*** emphasized the possibility to observe the recombinational lines from n > 3 and $\Delta n \ll n$!

First recombination computations completed in 1968!

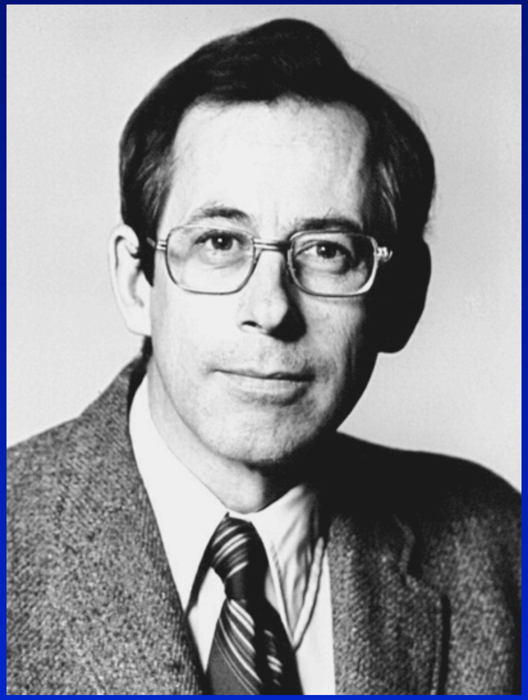
Yakov Zeldovich

Vladimir Kurt
(UV astronomer)

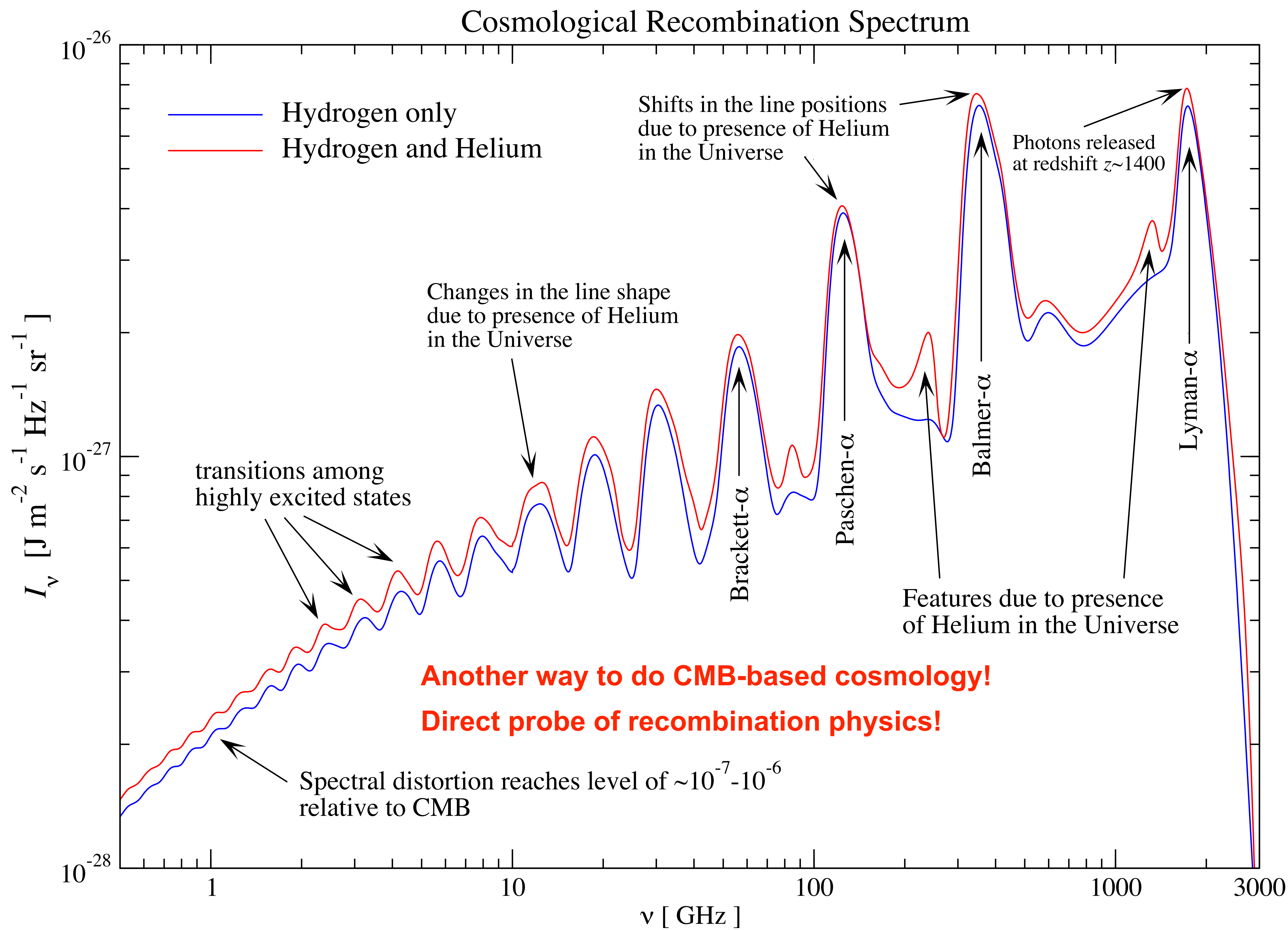
Moscow

Rashid Sunyaev

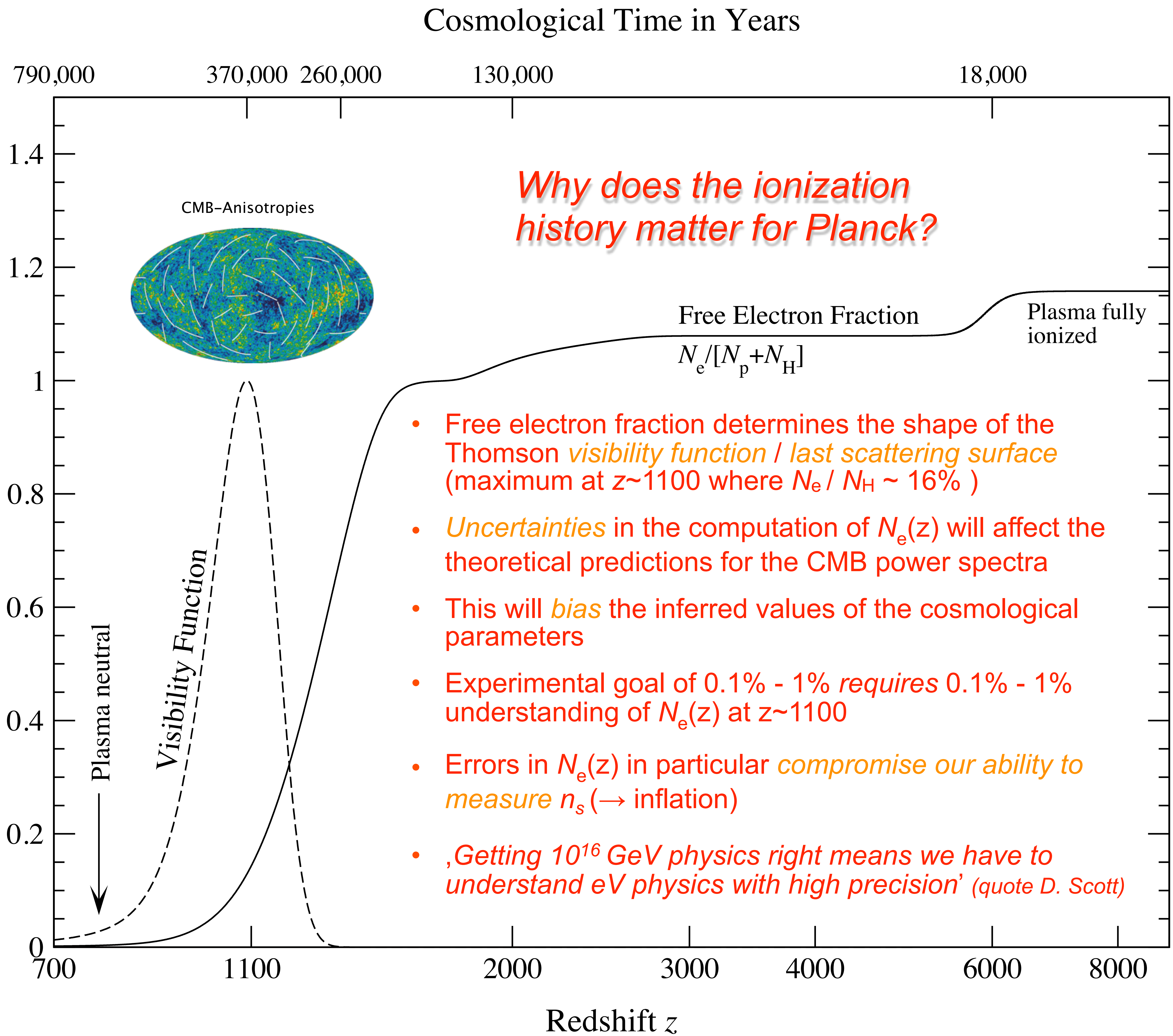
Princeton



Jim Peebles

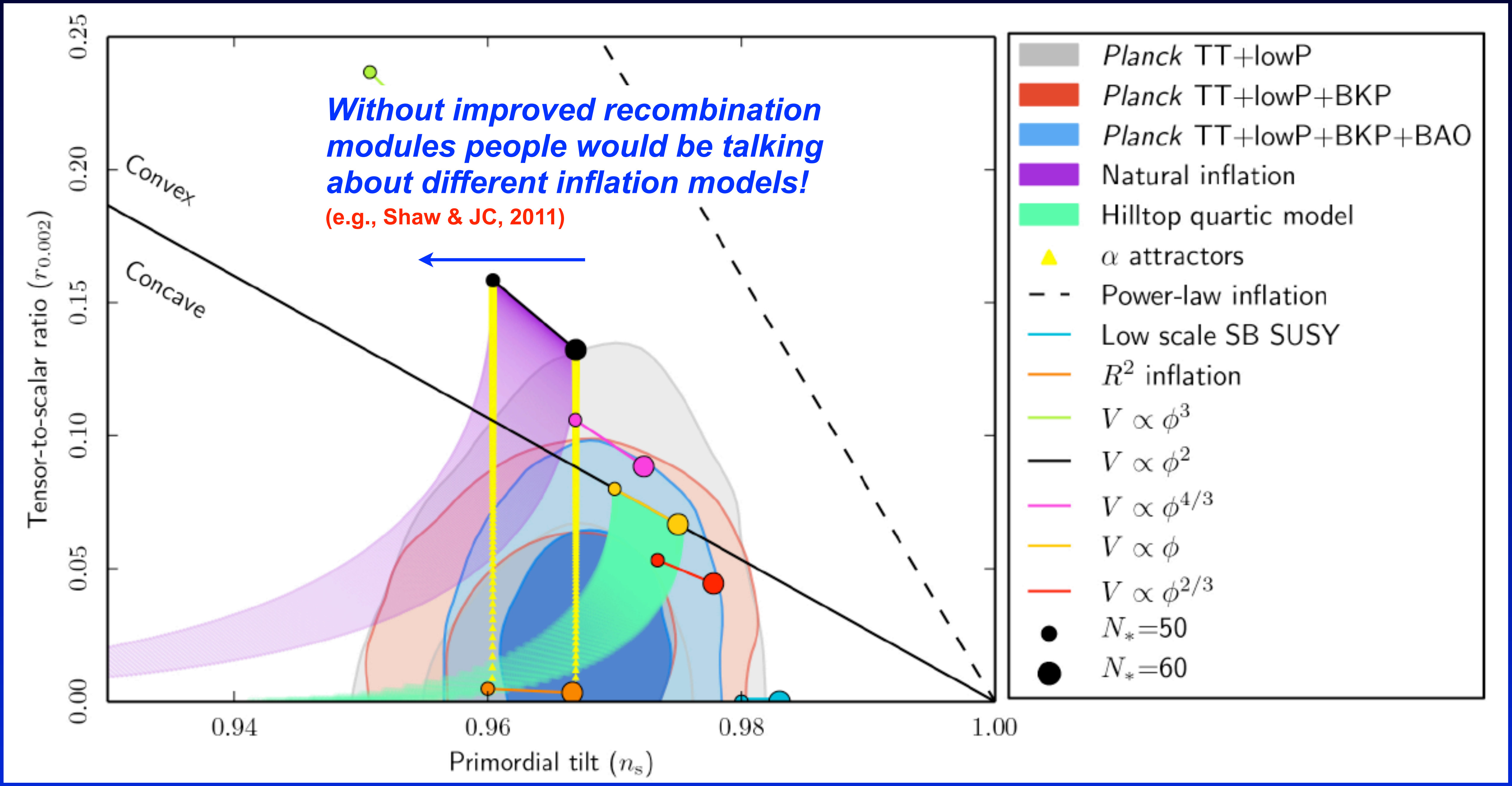


Rubino-Martin et al. 2006, 2008; Sunyaev & JC, 2009



Cosmological Time in Years
790,000
370,000
260,000
130,000
18,000
Why does the ionization history matter for Planck?
CMB-Anisotropies
Free Electron Fraction
$N_e/[N_p+N_H]$
Plasma fully ionized
Plasma neutral
Visibility Function
1.4
1.2
1
0.8
0.6
0.4
0.2
0
700
1100
2000
3000
4000
6000
8000
Redshift z
• Free electron fraction determines the shape of the Thomson *visibility function / last scattering surface* (maximum at z~1100 where N_e / N_H ~ 16%)
• *Uncertainties* in the computation of N_e(z) will affect the theoretical predictions for the CMB power spectra
• This will *bias* the inferred values of the cosmological parameters
• Experimental goal of 0.1% - 1% *requires* 0.1% - 1% understanding of N_e(z) at z~1100
• Errors in N_e(z) in particular *compromise our ability to measure* n_s (→ inflation)
• *‚Getting 10^{16} GeV physics right means we have to understand eV physics with high precision*' *(quote D. Scott)*

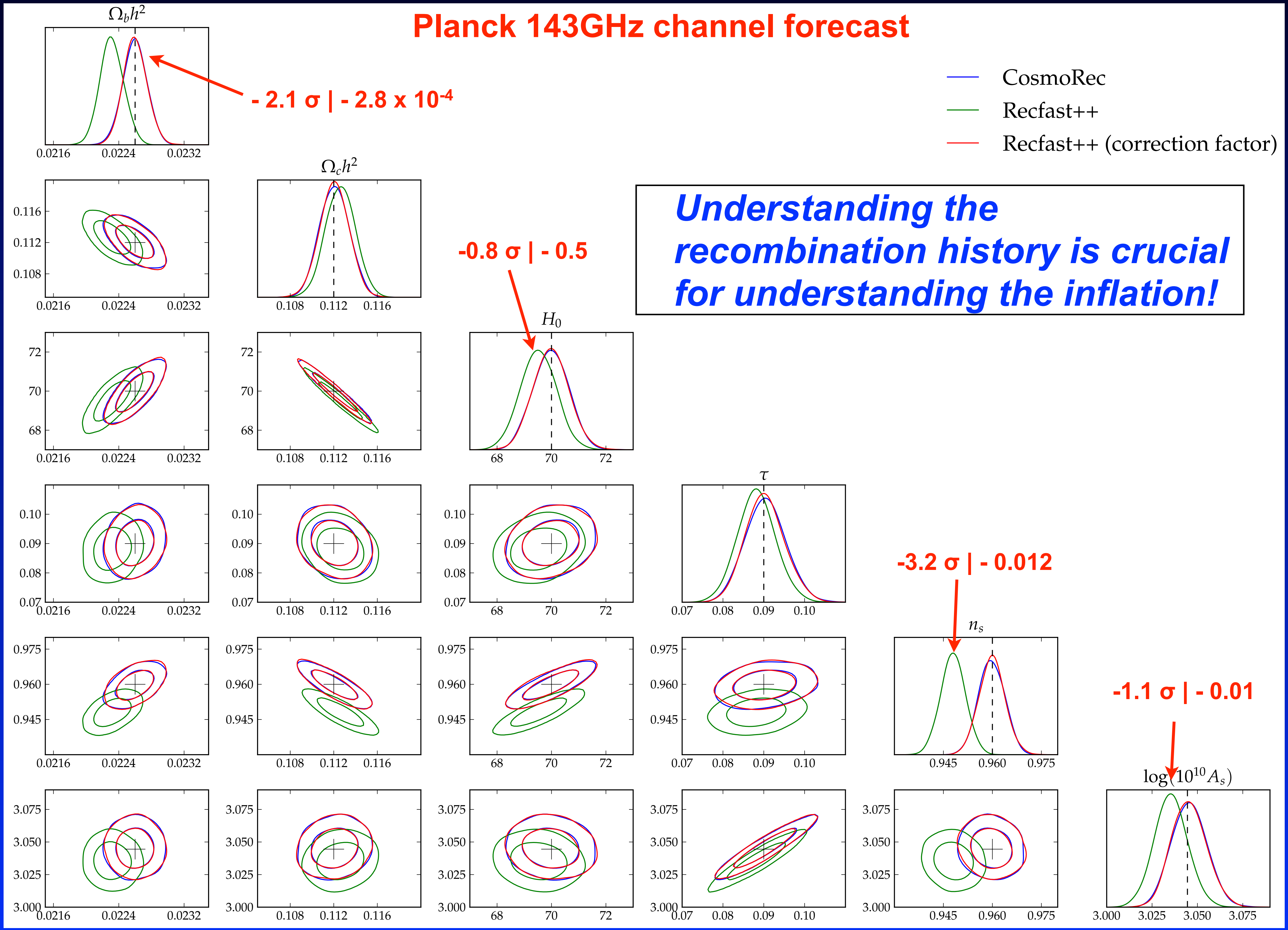
Importance of recombination for inflation constraints



Planck Collaboration, 2015, paper XX

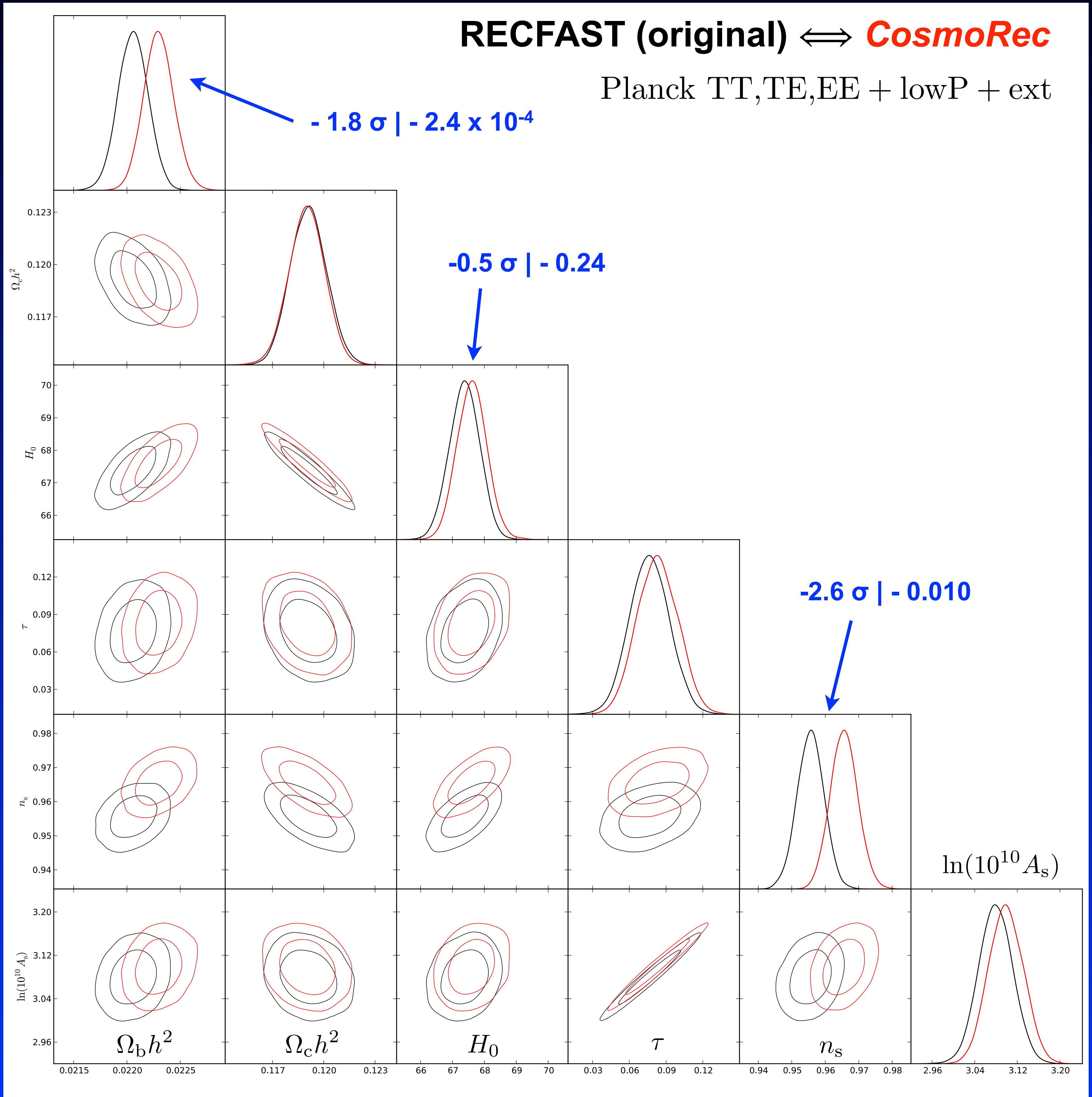
- Analysis uses refined recombination model (CosmoRec/HyRec)

Importance of recombination



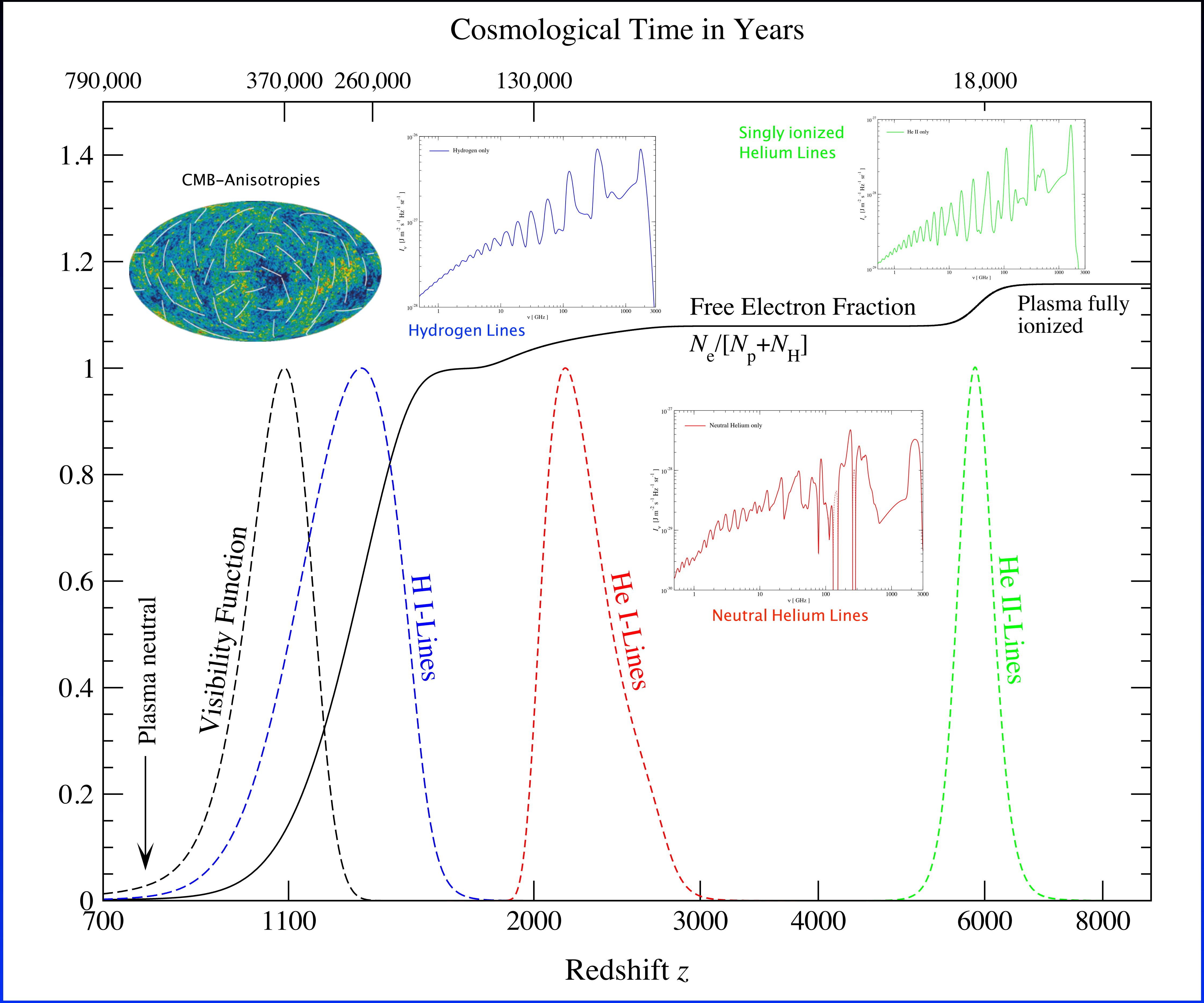
Shaw & JC, 2011, and references therein

Biases as they *would* have been for *Planck*



- Biases a little less significant with real *Planck* data
- absolute biases very similar
- In particular n_s would be biased significantly

Planck Collaboration, XIII 2015



Cosmological Time in Years
790,000
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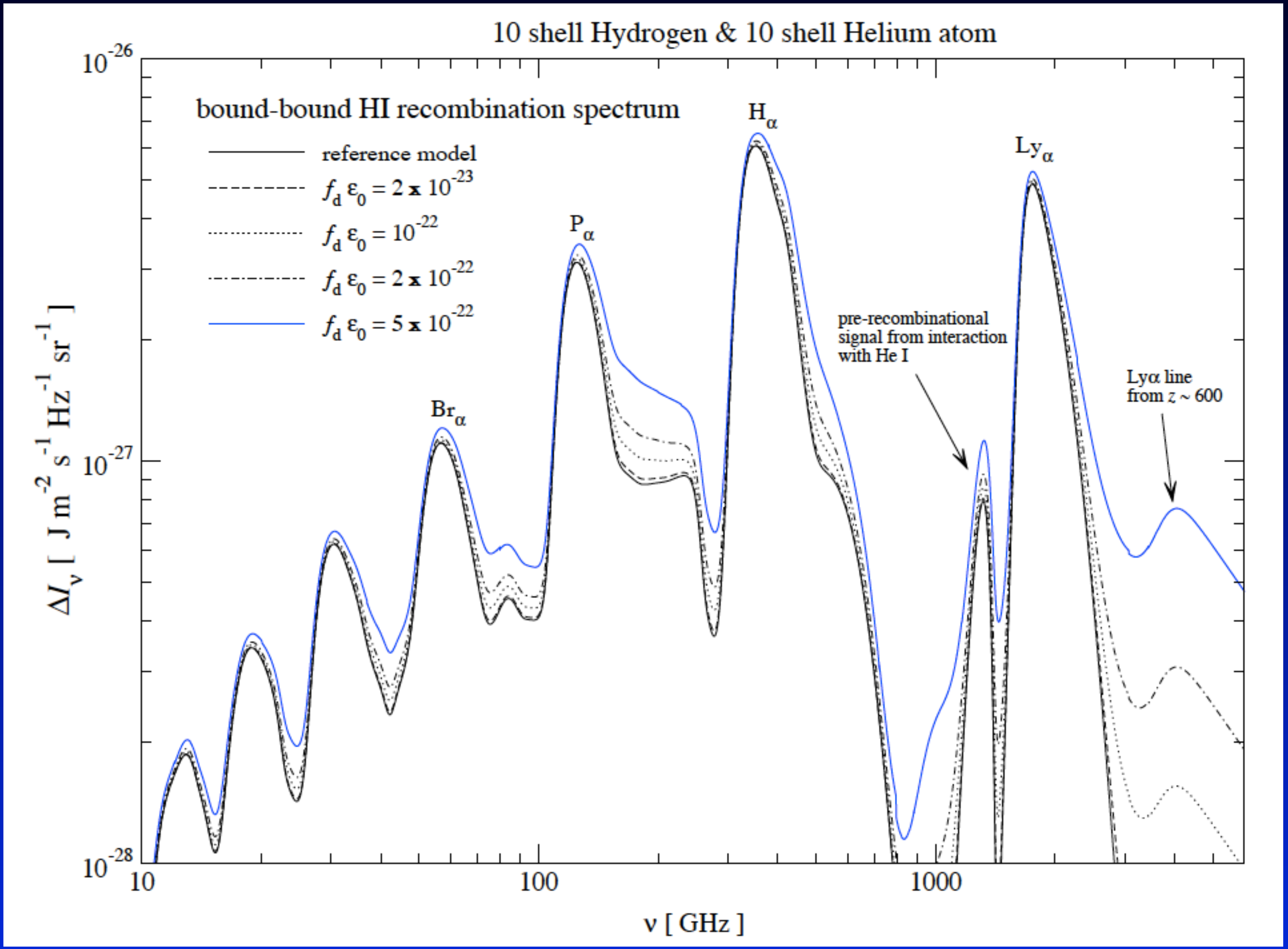
1.4
1.2
1
0.8
0.6
0.4
0.2
0
700
1100
2000
3000
4000
6000
8000
Redshift z
CMB-Anisotropies
Hydrogen only
Hydrogen Lines
Singly ionized Helium Lines
He II only
Free Electron Fraction
$N_e/[N_p+N_H]$
Plasma fully ionized
Neutral Helium only
Neutral Helium Lines
Plasma neutral
Visibility Function
H I-Lines
He I-Lines
He II-Lines

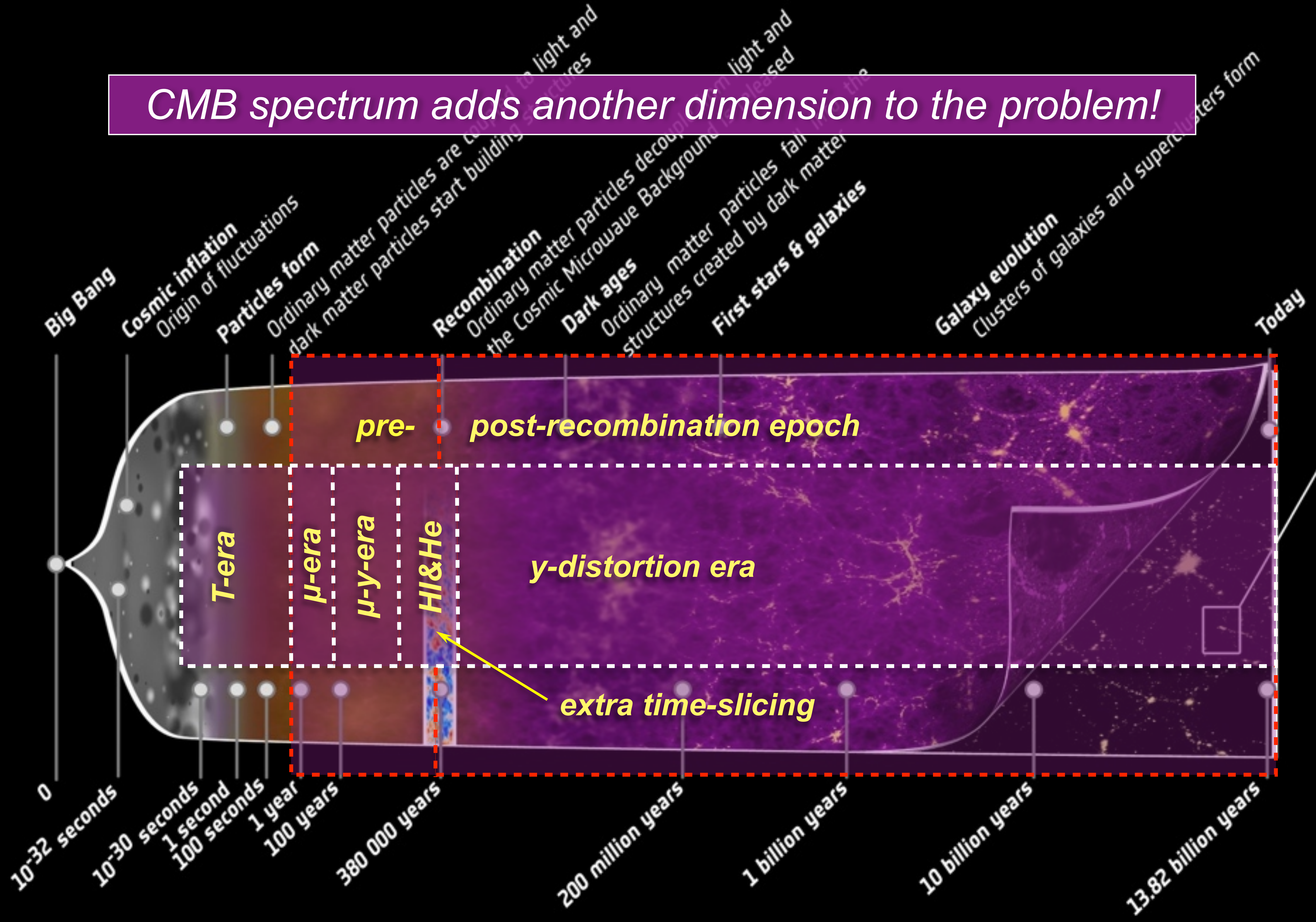
Dark matter annihilations / decays



JC, 2009, arXiv:0910.3663

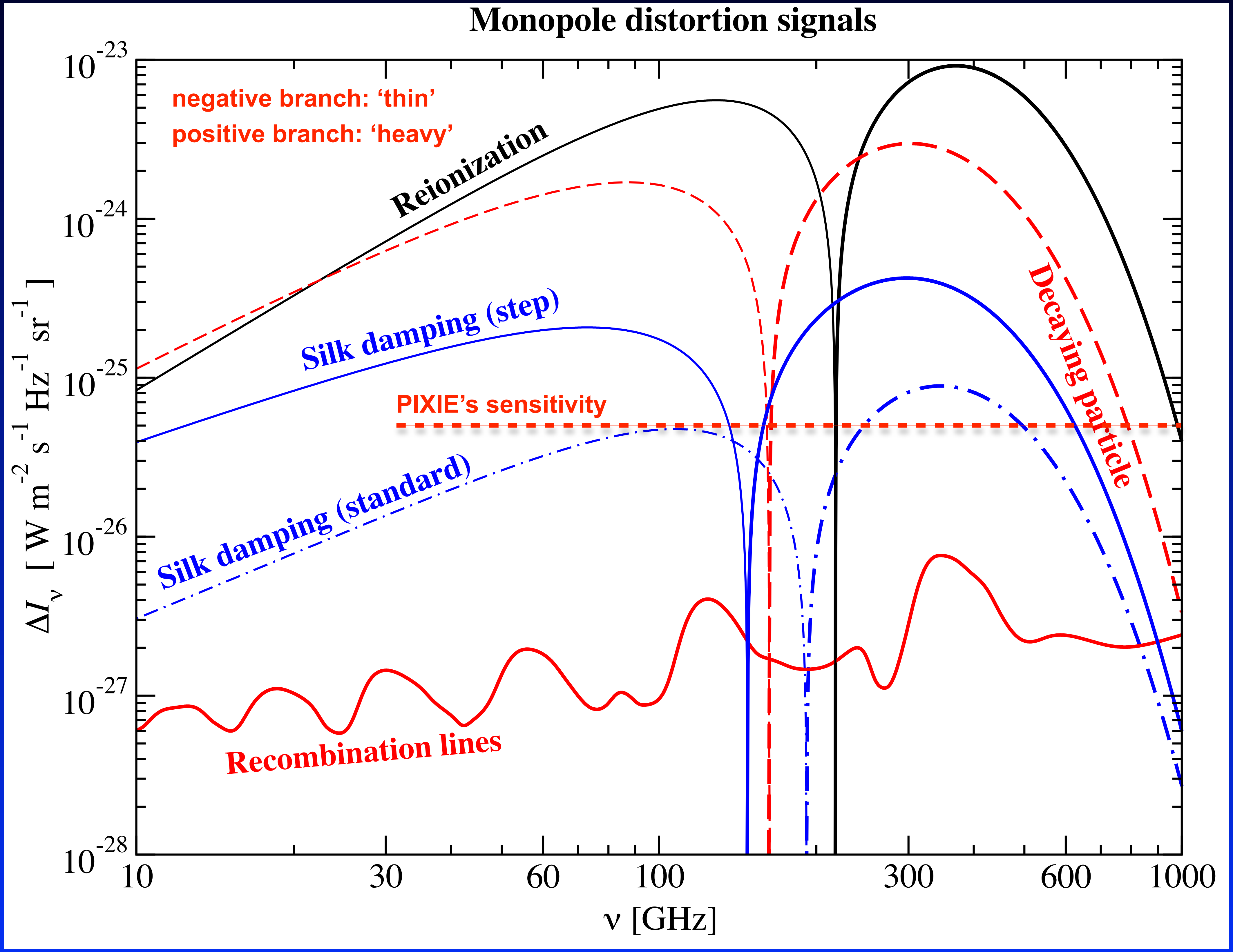
- Additional photons at all frequencies
- Broadening of spectral features
- Shifts in the positions

CMB spectrum adds another dimension to the problem!



Average CMB spectral distortions

Absolute value of Intensity signal



Average CMB spectral distortions

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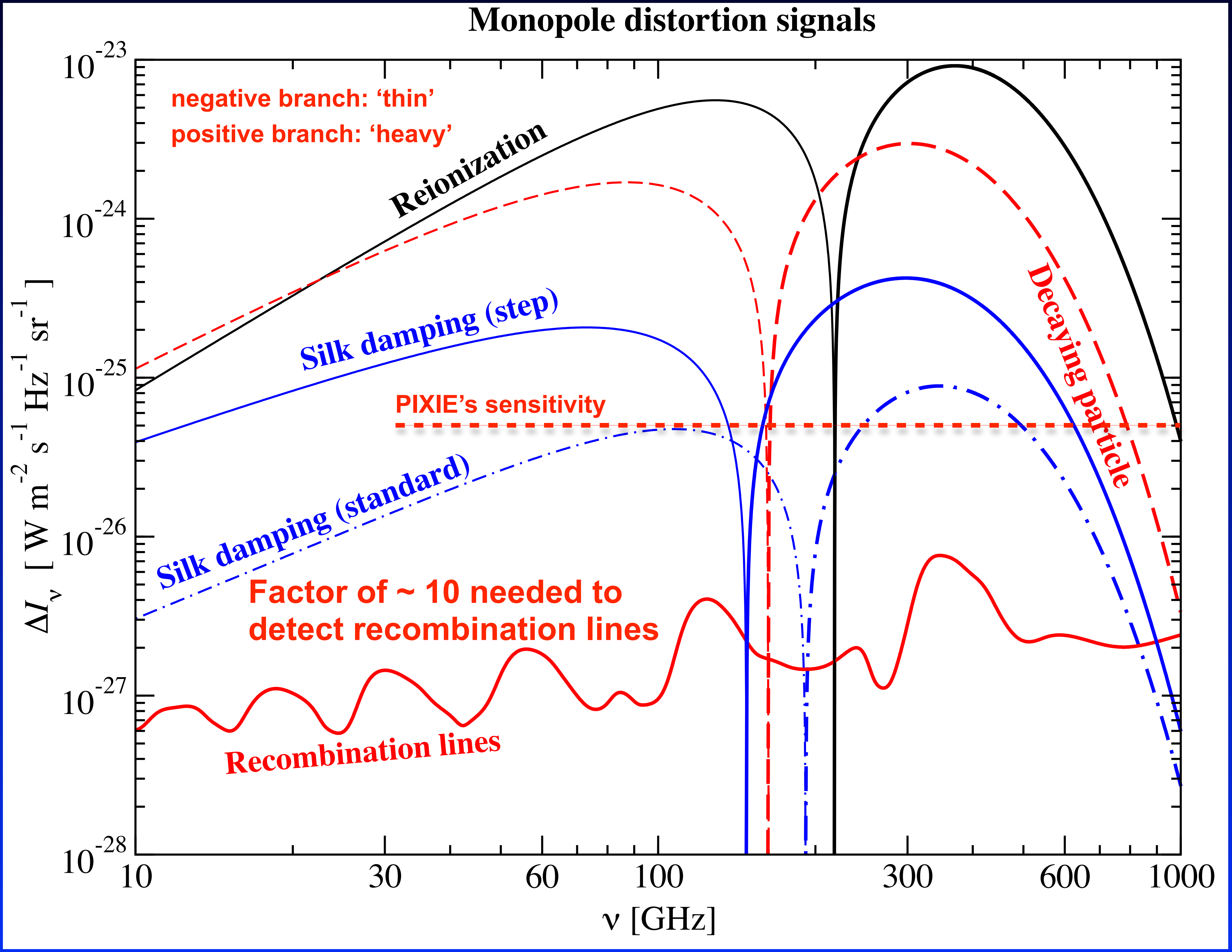
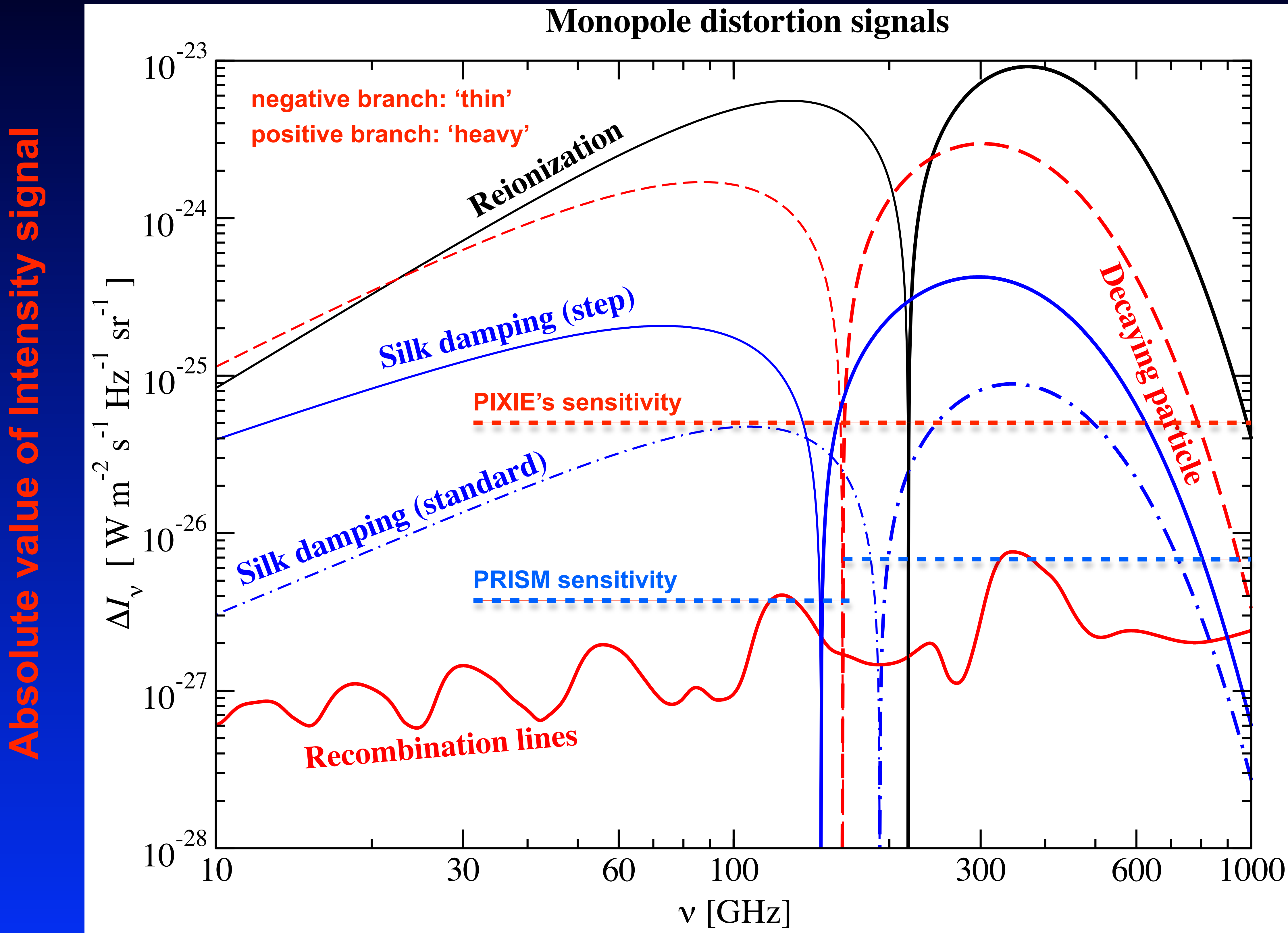


Average CMB spectral distortions



What would we actually learn by doing such hard job?

Cosmological Recombination Spectrum opens a way to measure:

→ **the specific *entropy* of our universe (related to $\Omega_b h^2$)**

→ **the CMB *monopole* temperature T_0**

→ ***the pre-stellar abundance of helium Y_p***

→ ***If recombination occurs as we think it does, then the lines can be predicted with very high accuracy!***

→ ***In principle allows us to directly check our understanding of the standard recombination physics***

If something unexpected or non-standard happened:

→ ***non-standard thermal histories should leave some measurable traces***

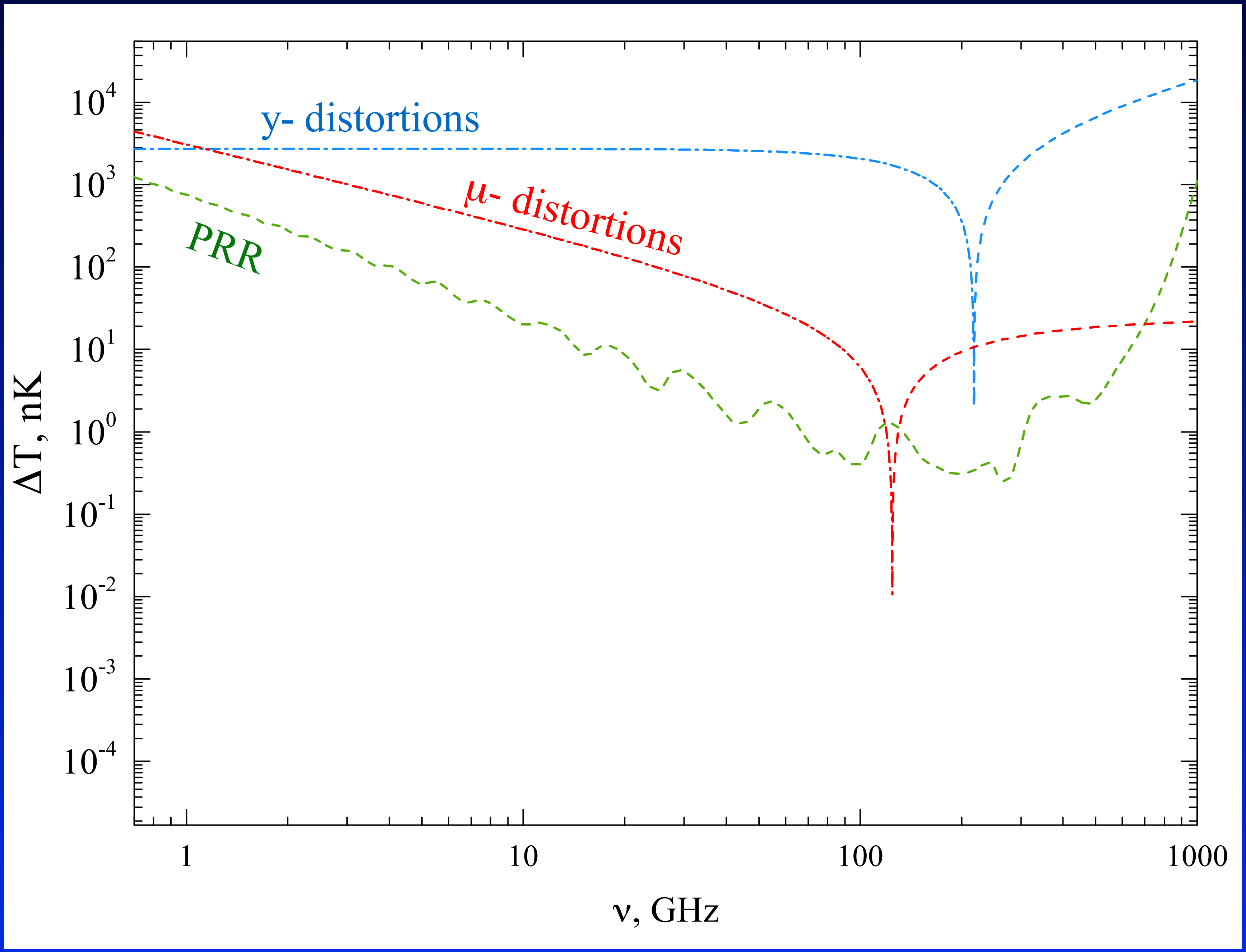
→ ***direct way to measure/reconstruct the recombination history!***

→ **possibility to distinguish *pre-* and *post-recombination y*-type distortions**

→ ***sensitive to energy release during recombination***

→ **variation of fundamental constants**

Spectral distortions of the CMB dipole



- motion with respect to CMB blackbody monopole

⇒ *CMB temperature dipole*

- including primordial distortions of the CMB

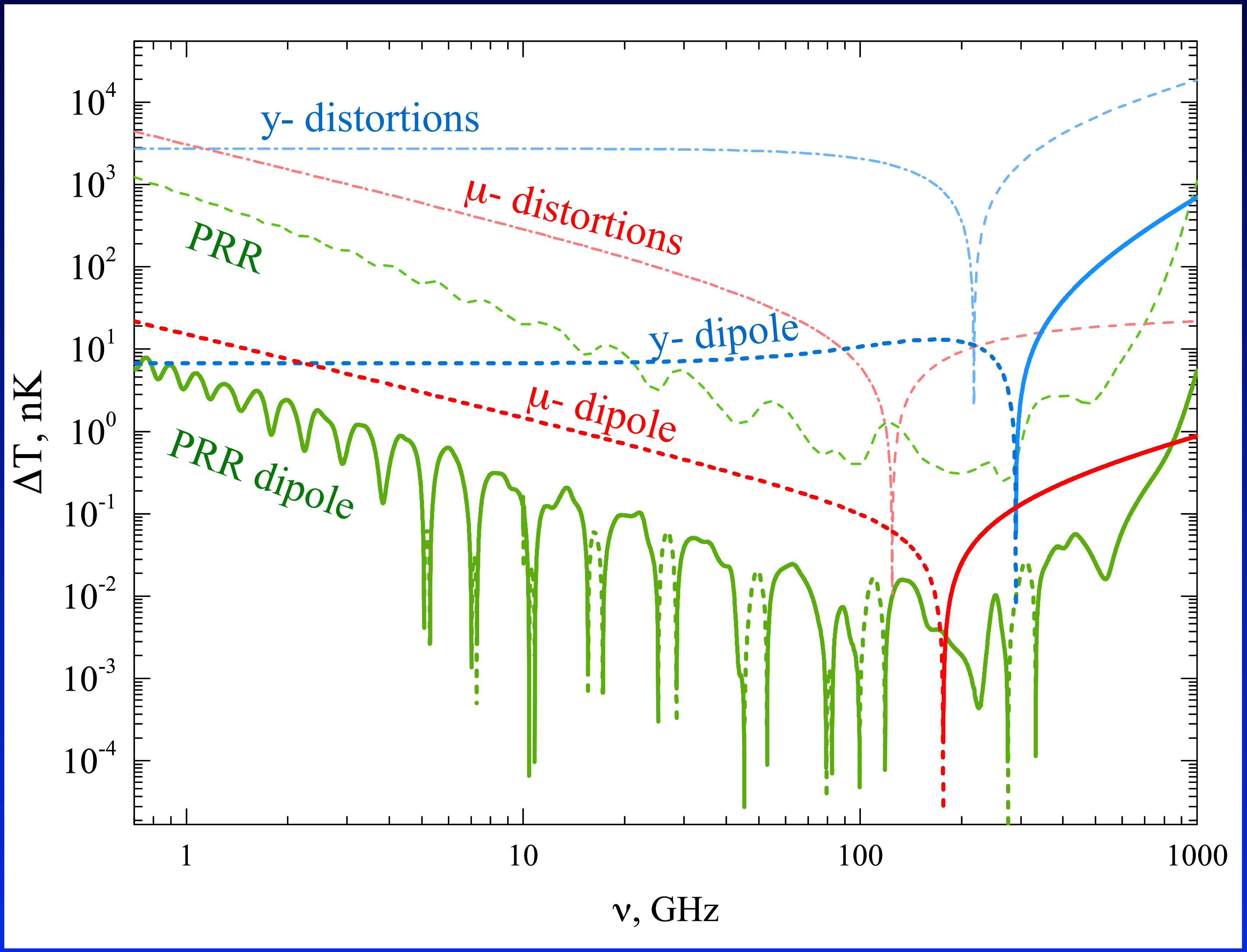
⇒ *CMB dipole is distorted*

$$\eta_{\rm d}(\nu, \mathbf{n}) \approx -\nu \partial_\nu \eta_{\rm m}(\nu)\, \beta \cos \Theta$$

- spectrum of the dipole is sensitive to the *derivative* of the monopole spectrum
- anisotropy does not need *absolute* calibration but just *inter-channel* calibration
- *but* signal is ~1000 times smaller...
- foregrounds will also leak into the dipole in this way

Balashev, Kholupenko, JC, Ivanchik & Varshalovich, ArXiv:1505.06028

Spectral distortions of the CMB dipole



- motion with respect to CMB blackbody monopole

⇒ *CMB temperature dipole*

- including primordial distortions of the CMB

⇒ *CMB dipole is distorted*

$$\eta_{\rm d}(\nu, \mathbf{n}) \approx -\nu \partial_\nu \eta_{\rm m}(\nu)\, \beta \cos\Theta$$

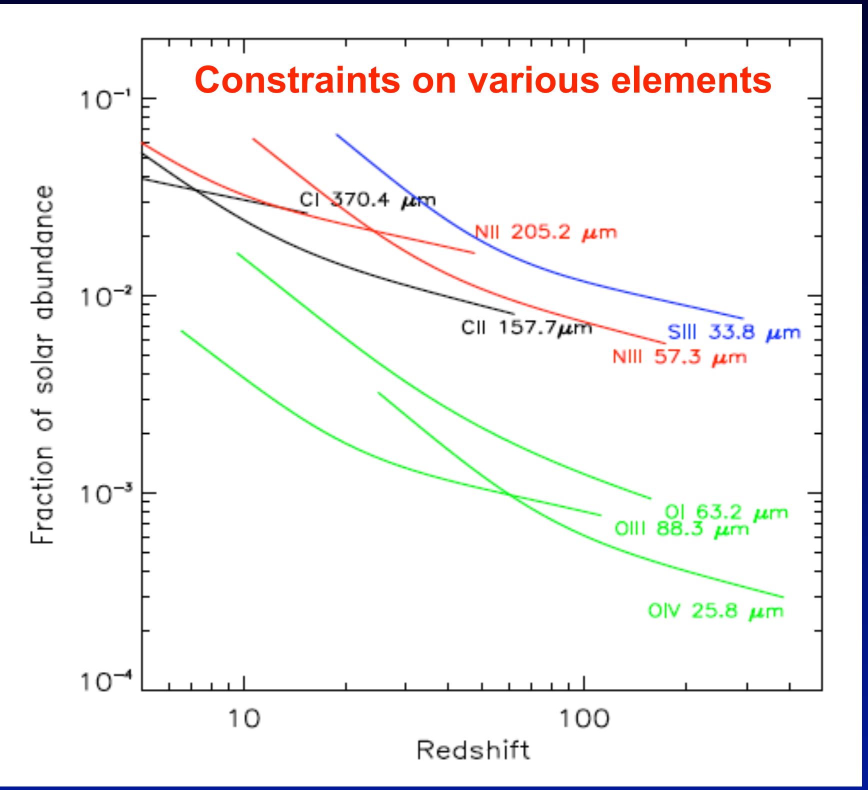
- spectrum of the dipole is sensitive to the *derivative* of the monopole spectrum
- anisotropy does not need *absolute* calibration but just *inter-channel* calibration
- *but* signal is ~1000 times smaller...
- foregrounds will also leak into the dipole in this way

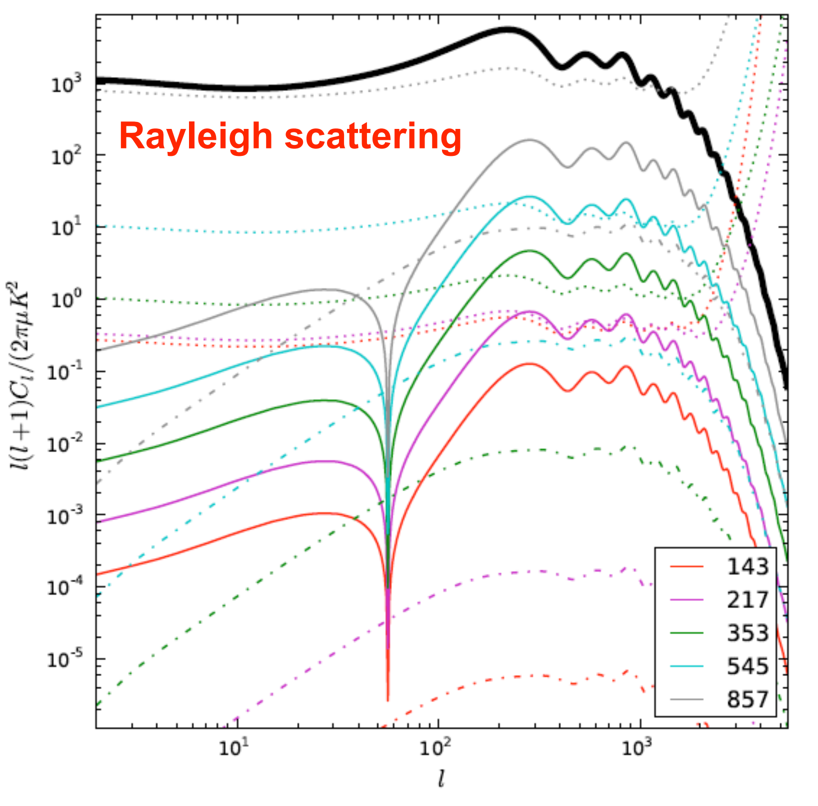
Balashev, Kholupenko, JC, Ivanchik & Varshalovich, ArXiv:1505.06028

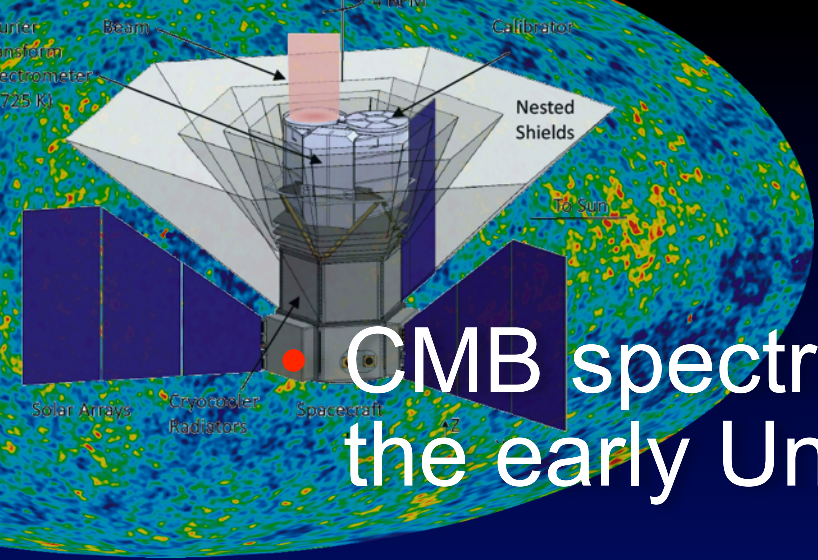
Other extremely interesting new signals

- Scattering signals from the dark ages
  (e.g., Basu et al., 2004; Hernandez-Monteagudo et al., 2007; Schleicher et al., 2009)
  - constrain abundances of chemical elements at high redshift
  - learn about star formation history
- Rayleigh / HI scattering signals
  (e.g., Yu et al., 2001; Rubino-Martin et al., 2005; Lewis 2013)
  - provides way to constrain recombination history
  - important when asking questions about $N_{\rm eff}$ and $Y_{\rm p}$
- Free-free signals from reionization
  (e.g., Burigana et al. 1995; Trombetti & Burigana, 2013)
  - constrains reionization history
  - depends on clumpiness of the medium

All these effects give spectral-spatial signals, and an absolute spectrometer will help with channel cross calibration!



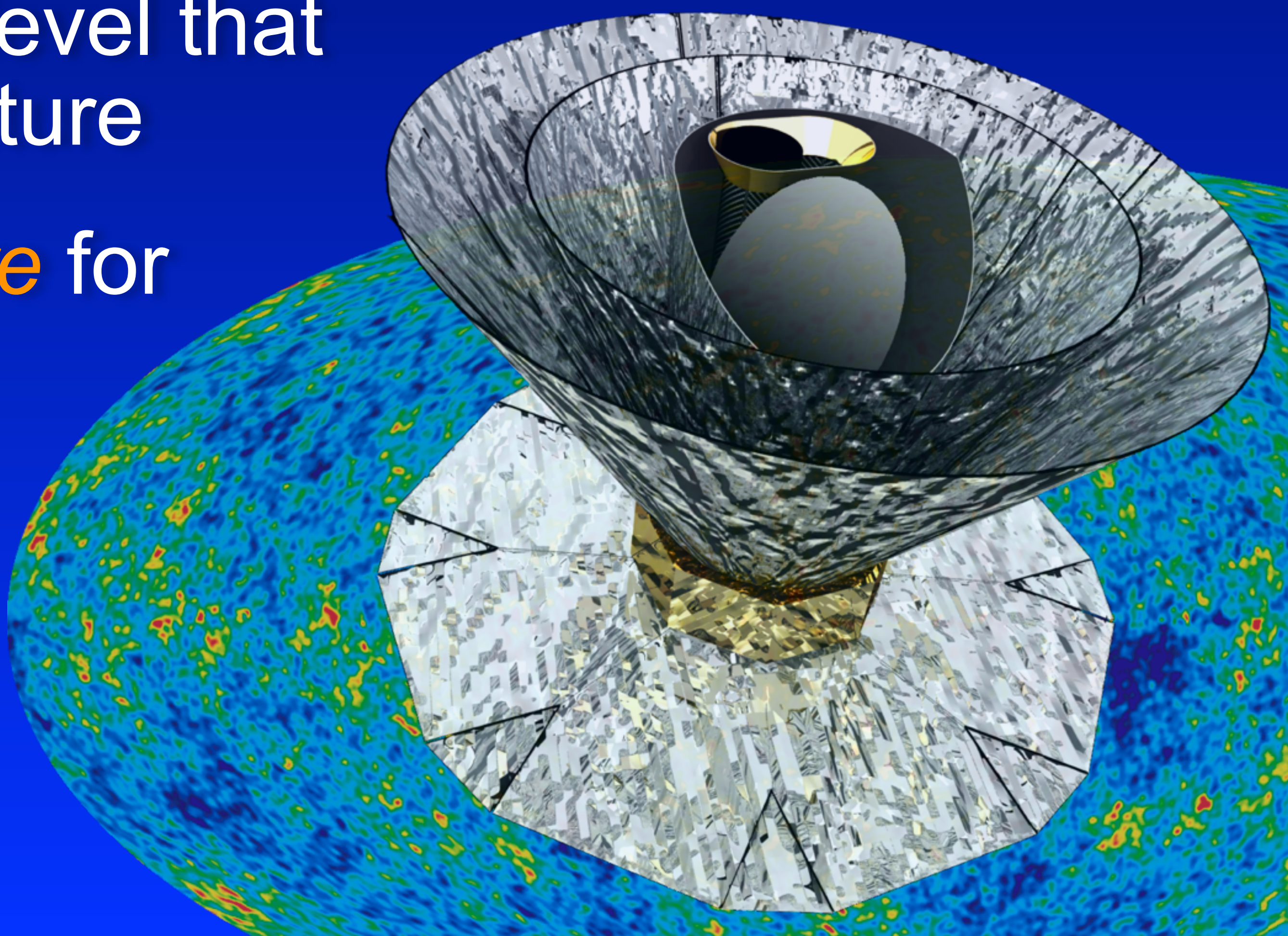


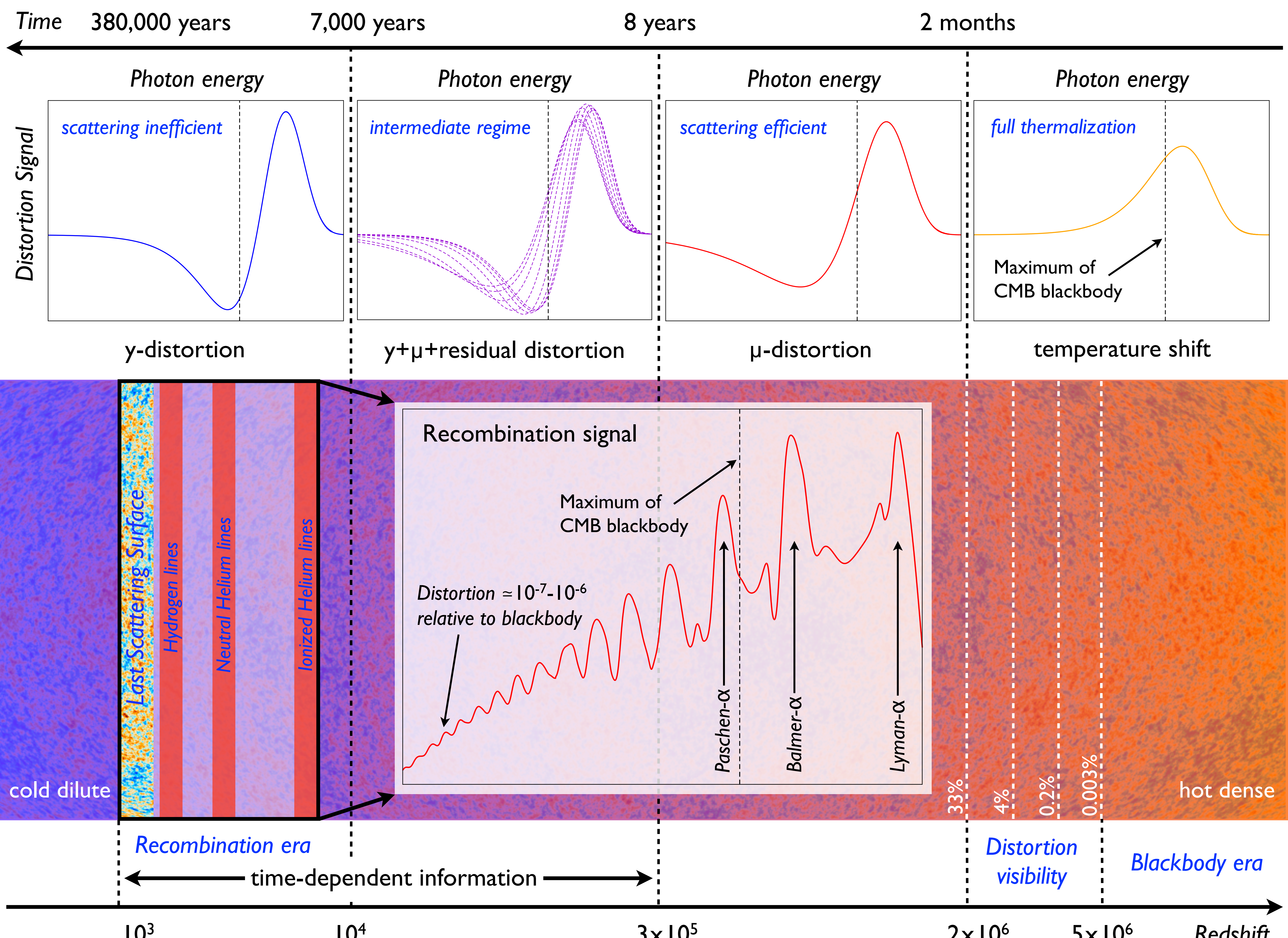


Conclusions

- CMB spectral distortions *will* open a *new window* to the early Universe
- new probe of the *inflation epoch* and *particle physics*
- *complementary* and *independent* source of information *not* just confirmation
- in *standard cosmology* several processes lead to *early energy release* at a level that will be detectable in the future
- extremely interesting *future* for CMB-based science!

We should make use of all this information!





Silk & Chluba, Science, 2014