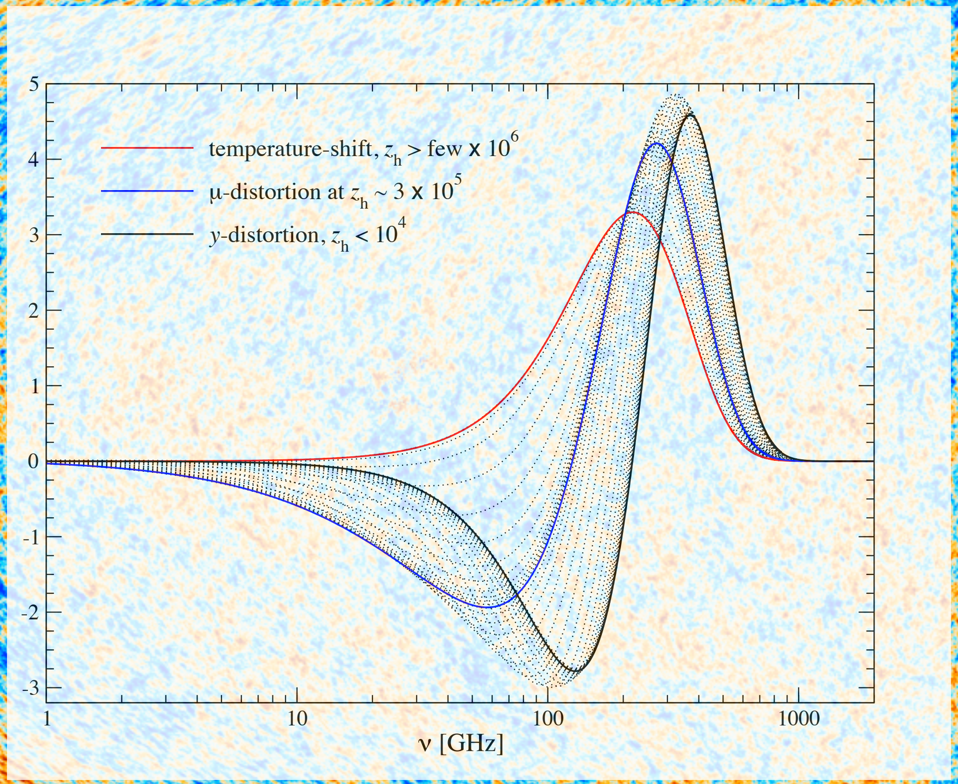
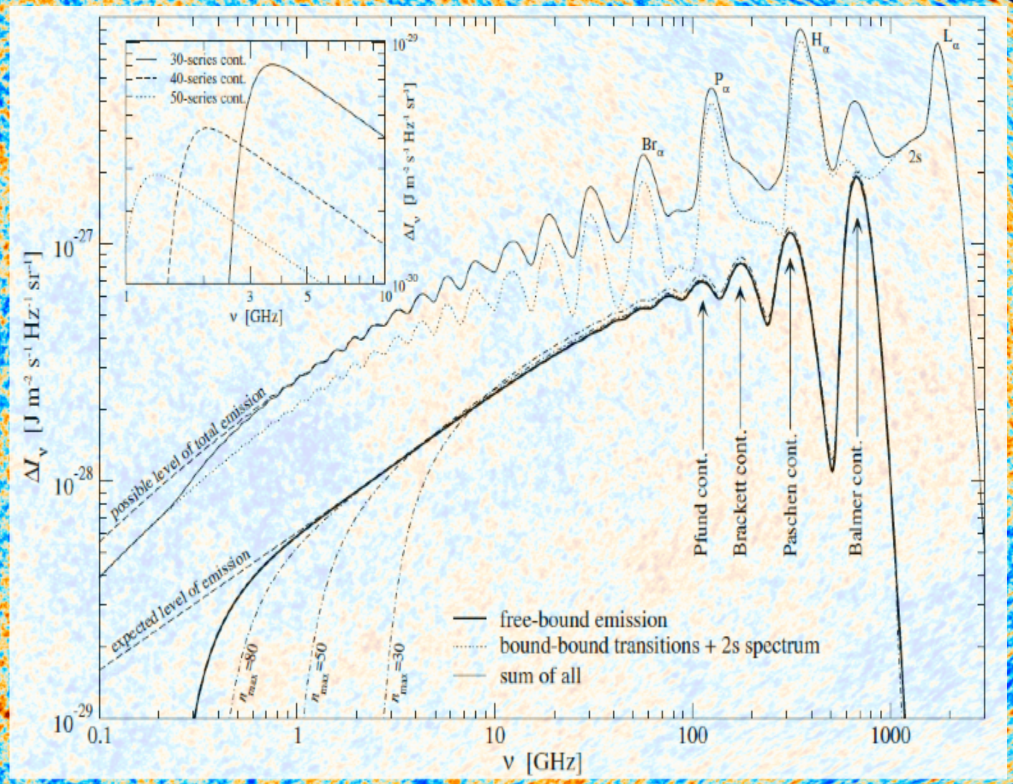
Cosmic Microwave Background and Spectral Distortions II:

Theory of CMB spectral distortions

Primordial Distortions



Cosmological Recombination lines



Jens Chluba

ICCUB School: “Hot Topics in Cosmology”

Barcelona, Spain, Oct. 23rd-26th, 2017

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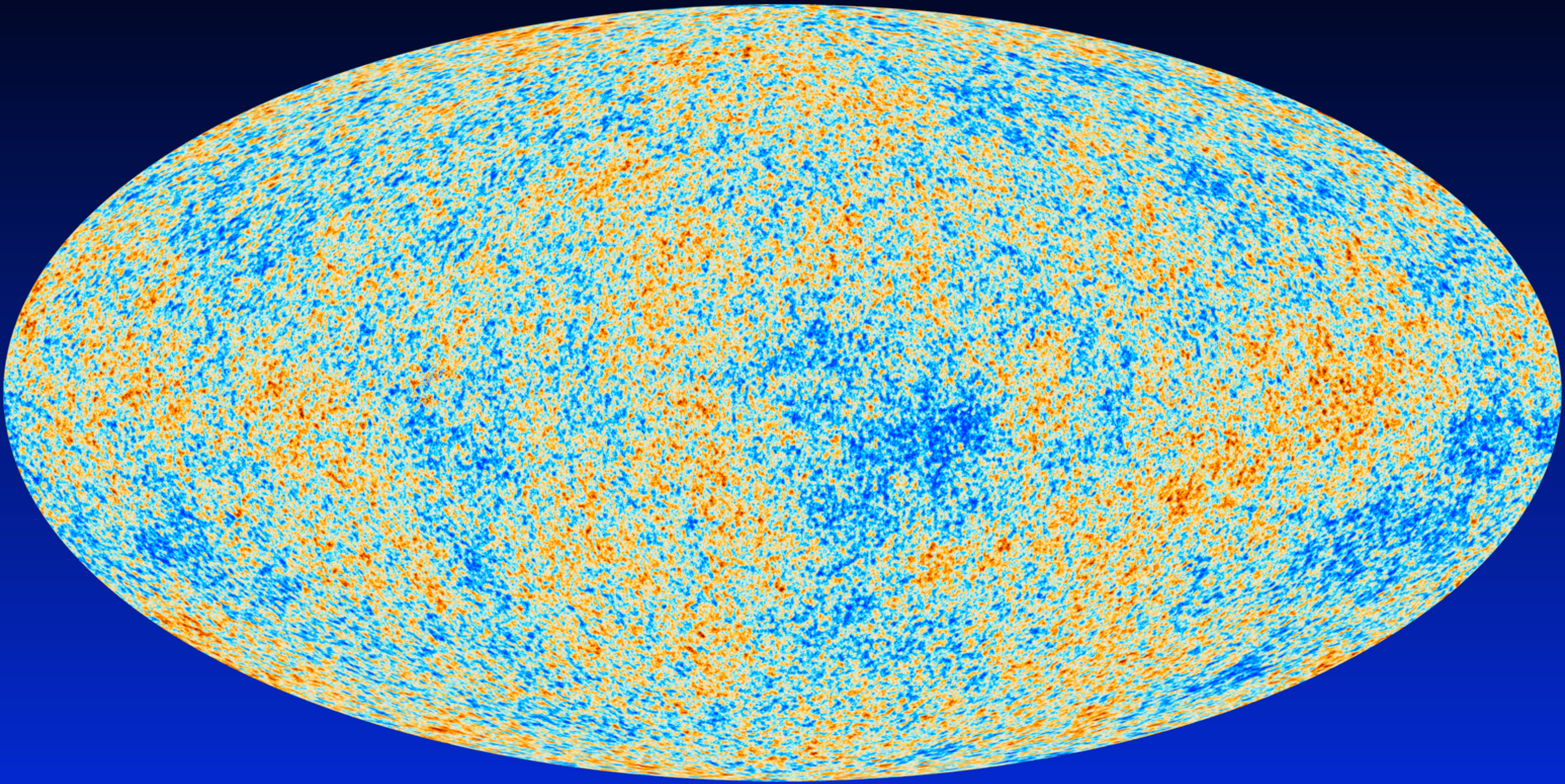
** CMB ≙ Cosmic Microwave Background*

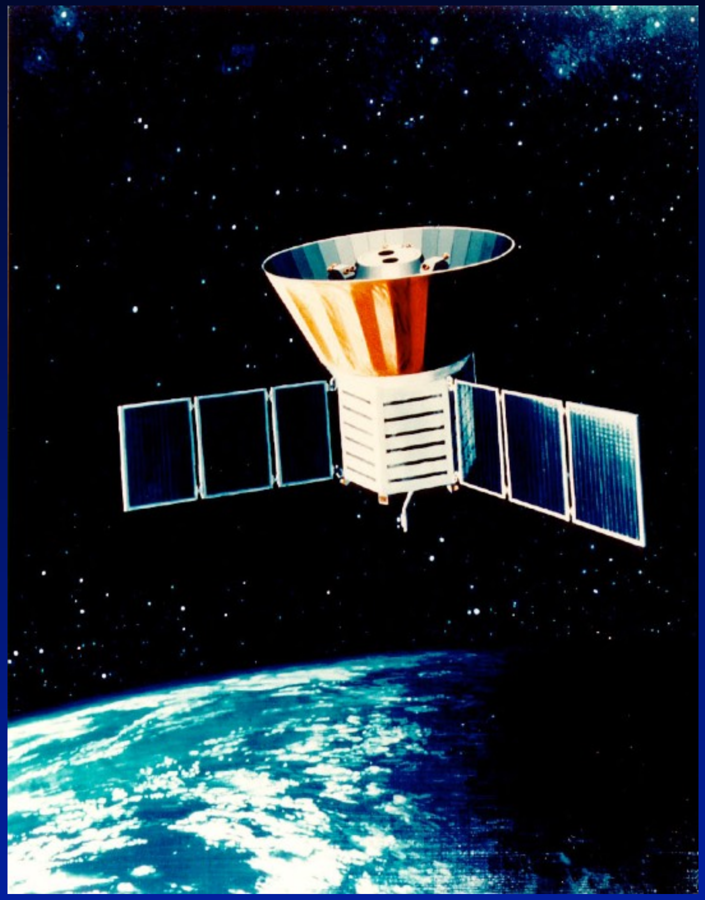
Cosmic Microwave Background Anisotropies



Planck all-sky temperature map

- CMB has a blackbody spectrum in every direction
- tiny variations of the CMB temperature $\Delta T/T \sim 10^{-5}$

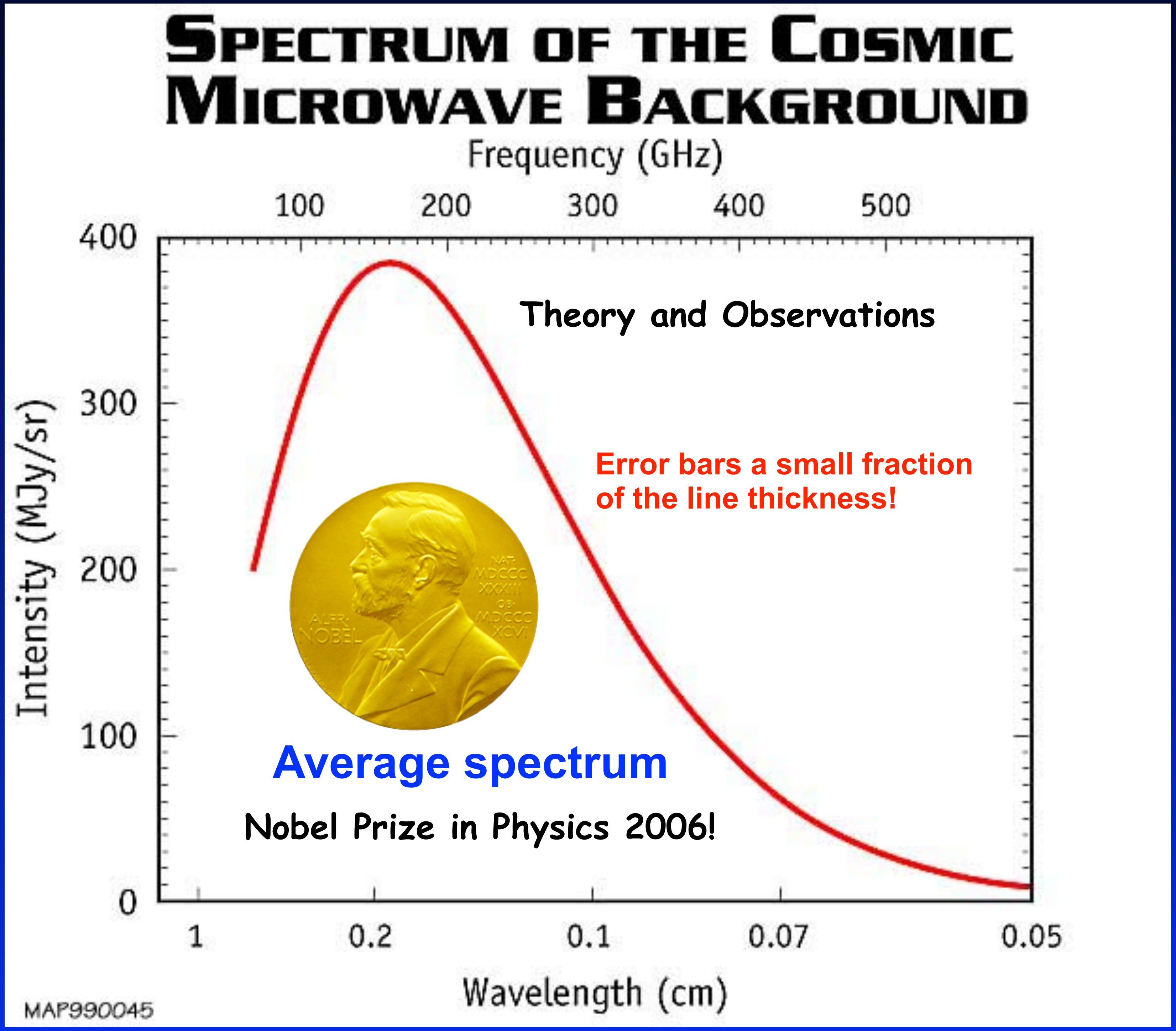
COBE / FIRAS (Far InfraRed Absolute Spectrophotometer)



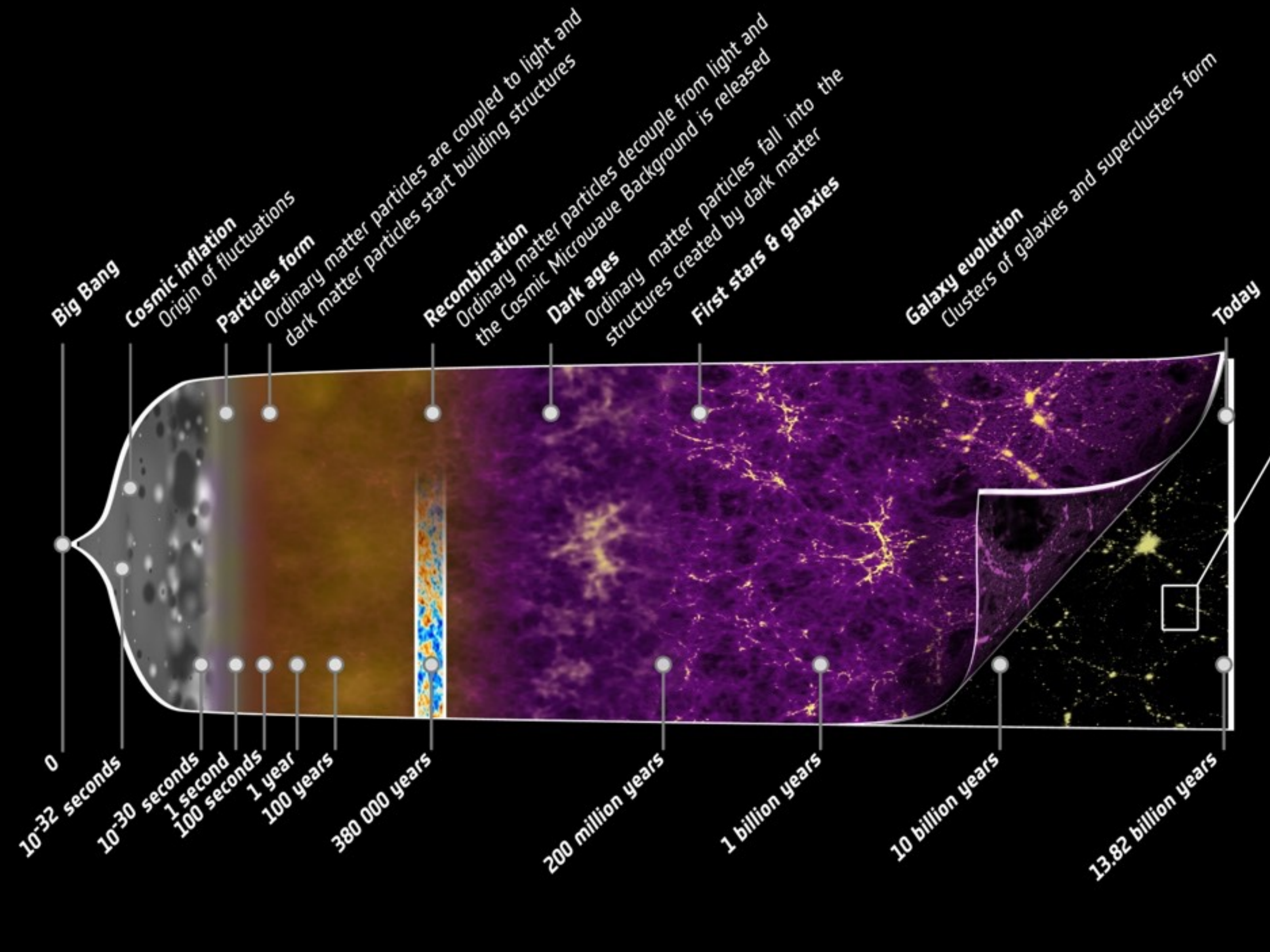
$$T_0 = 2.725 \pm 0.001\,\mathrm{K}$$

$$|\,y\,| \leq 1.5\times10^{-5}$$

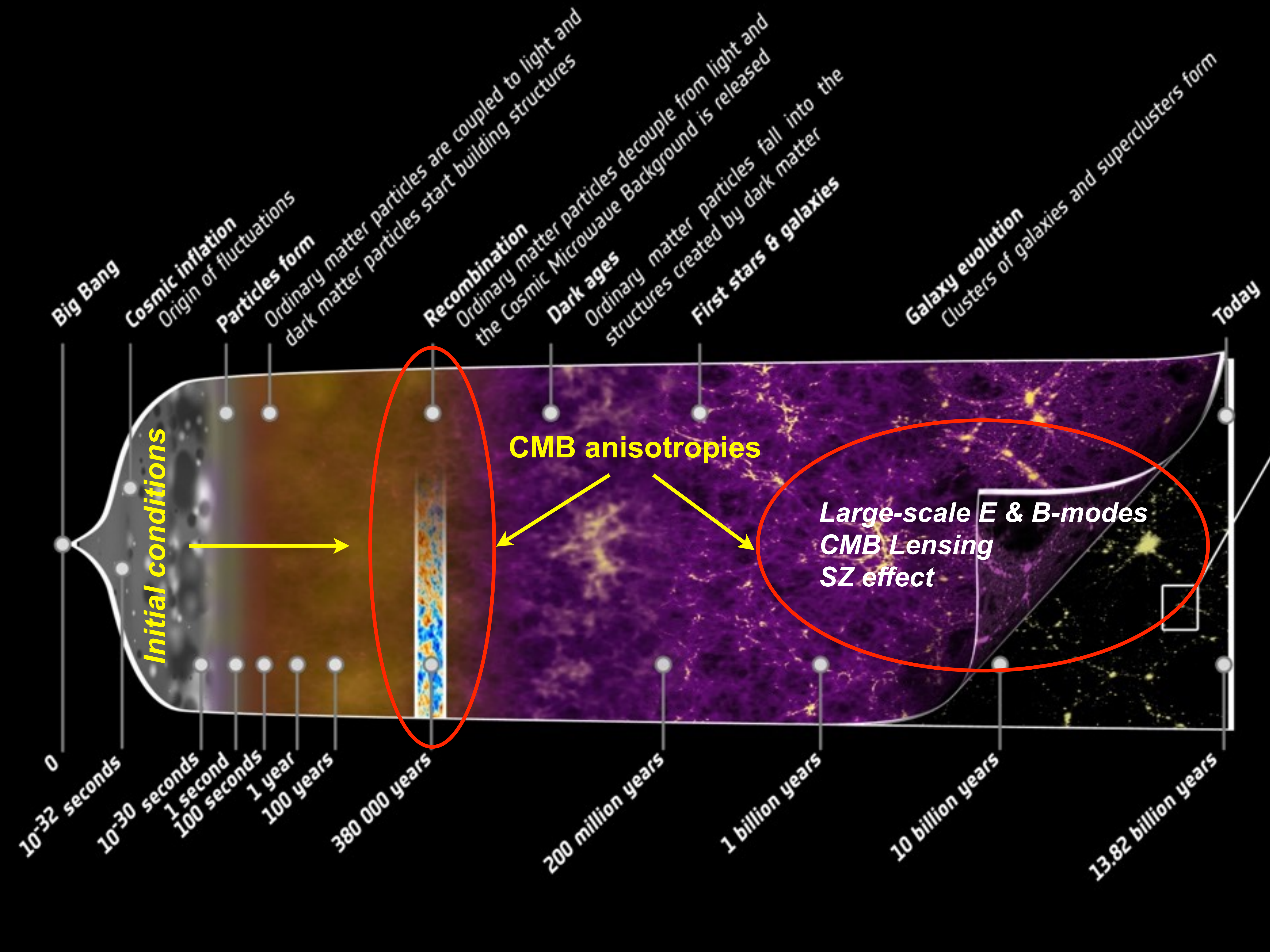
$$|\,\mu\,| \leq 9\times10^{-5}$$



Mather et al., 1994, ApJ, 420, 439
Fixsen et al., 1996, ApJ, 473, 576
Fixsen et al., 2003, ApJ, 594, 67



Big Bang
Cosmic inflation
Origin of fluctuations
Particles form
Ordinary matter particles are coupled to light and
dark matter particles start building structures
Recombination
Ordinary matter particles decouple from light and
the Cosmic Microwave Background is released
Dark ages
Ordinary matter particles fall into the
structures created by dark matter
First stars & galaxies
Galaxy evolution
Clusters of galaxies and superclusters form
Today
0
10-32 seconds
10-30 seconds
1 second
100 seconds
1 year
100 years
380 000 years
200 million years
1 billion years
10 billion years
13.82 billion years


Big Bang
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Galaxy evolution
Clusters of galaxies and superclusters form
Today
Initial conditions
CMB anisotropies
Large-scale E & B-modes
CMB Lensing
SZ effect
0
10-32 seconds
10-30 seconds
1 second
100 seconds
1 year
100 years
380 000 years
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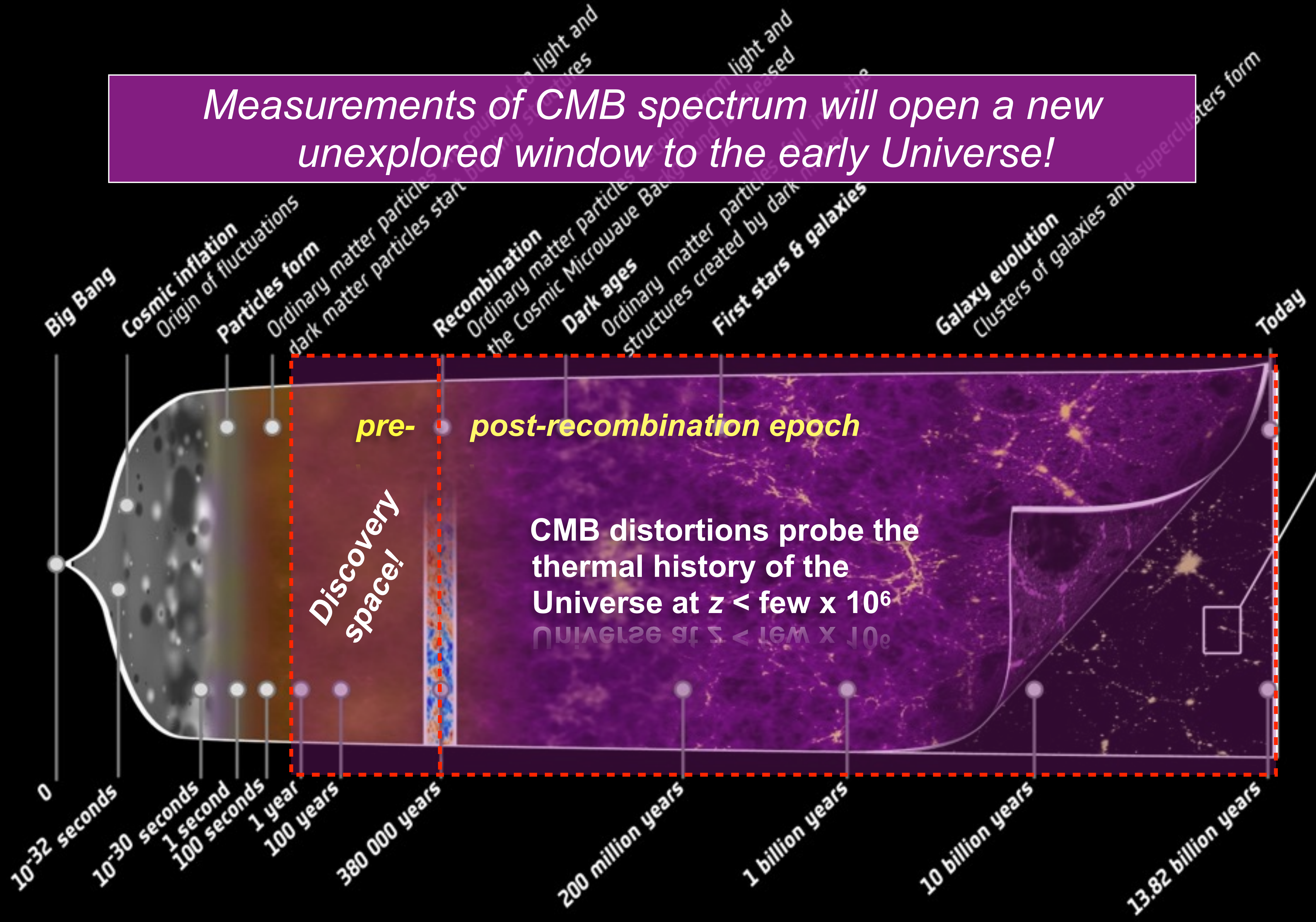
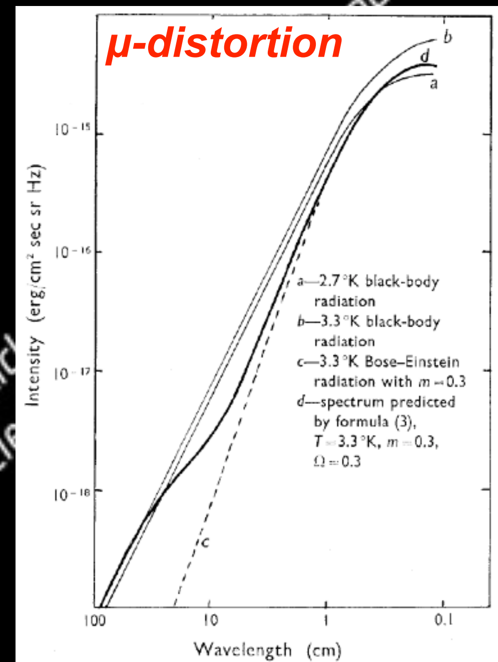
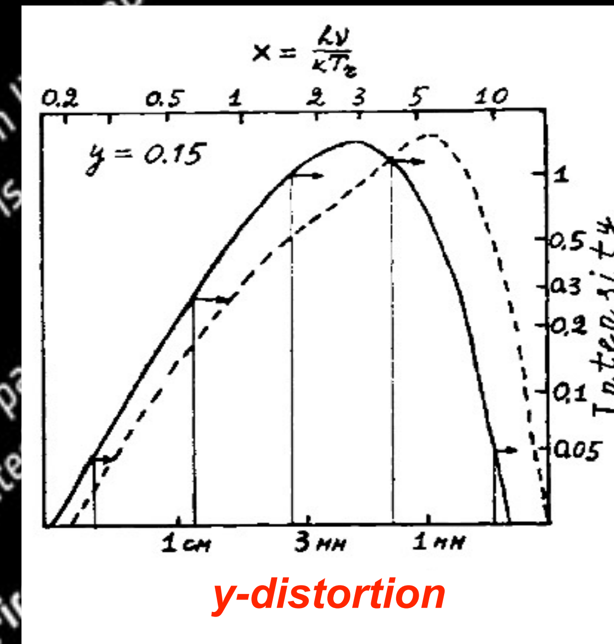
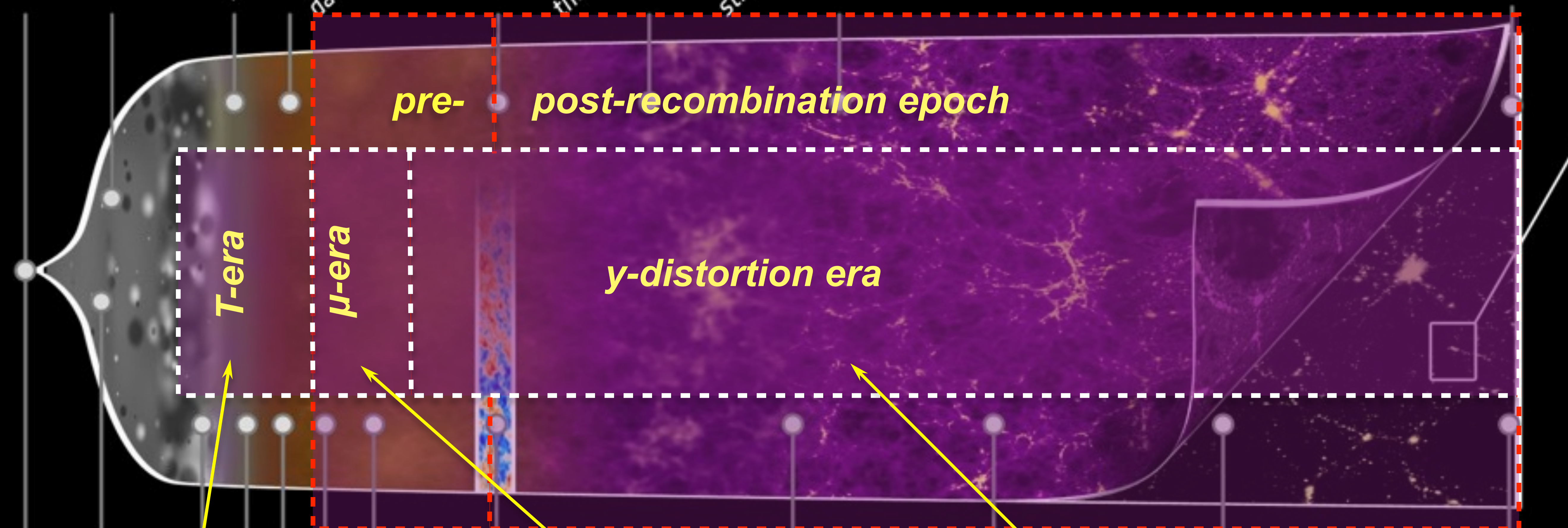
Measurements of CMB spectrum will open a new unexplored window to the early Universe!









$$\frac{\Delta T}{T} \simeq \frac{1}{4}\left.\frac{\Delta\rho_\gamma}{\rho_\gamma}\right|_T \qquad \mu \simeq 1.4\left.\frac{\Delta\rho_\gamma}{\rho_\gamma}\right|_\mu \qquad y \simeq \frac{1}{4}\left.\frac{\Delta\rho_\gamma}{\rho_\gamma}\right|_y$$

Part II: Theory of CMB spectral distortions

Some important conditions and assumptions

- Plasma fully ionized before recombination (z~1000)
  - → free electrons, protons and helium nuclei
  - → photon dominated (~2 Billion photons per baryon)
- Coulomb scattering $e + p \leftrightarrow e' + p$
  - → electrons in full thermal equilibrium with baryons
  - → electrons follow thermal Maxwell-Boltzmann distribution
  - → efficient down to very low redshifts (z ~ 10-100)
- Medium homogeneous and isotropic on large scales
  - → thermalization problem rather simple!
  - → in principle *allows very precise computations*
- Hubble expansion
  - → adiabatic cooling of photons [$T_\gamma \sim (1+z)$] and ordinary matter [$T_m \sim (1+z)^2$]
  - → redshifting of photons (no distortion…)

Photon Boltzmann Equation for Average Spectrum

$$\frac{\mathrm{d}n_\nu}{\mathrm{d}t} = \frac{\partial n_\nu}{\partial t} + \frac{\partial n_\nu}{\partial x_i} \cdot \frac{\mathrm{d}x_i}{\mathrm{d}t} + \frac{\partial n_\nu}{\partial p}\frac{\mathrm{d}p}{\mathrm{d}t} + \frac{\partial n_\nu}{\partial \hat{p}_i} \cdot \frac{\mathrm{d}\hat{p}_i}{\mathrm{d}t} = \mathcal{C}[n]$$

Photon occupation number

Liouville operator

Collision term

- Isotropy & Homogeneity: $\Longrightarrow$ $$\frac{\partial n_\nu}{\partial t} - H\nu\frac{\partial n_\nu}{\partial \nu} = \mathcal{C}[n]$$
  (⟺ *average* spectrum...)

redshifting term

- Collision term: $$\mathcal{C}[n] = \left.\frac{\mathrm{d}n_\nu}{\mathrm{d}t}\right|_{\mathrm{C}} + \left.\frac{\mathrm{d}n_\nu}{\mathrm{d}t}\right|_{\mathrm{BR}} + \left.\frac{\mathrm{d}n_\nu}{\mathrm{d}t}\right|_{\mathrm{DC}}$$

Compton Scattering

Bremsstrahlung

Double Compton

redistribution of photon over frequency

adjusting photon number

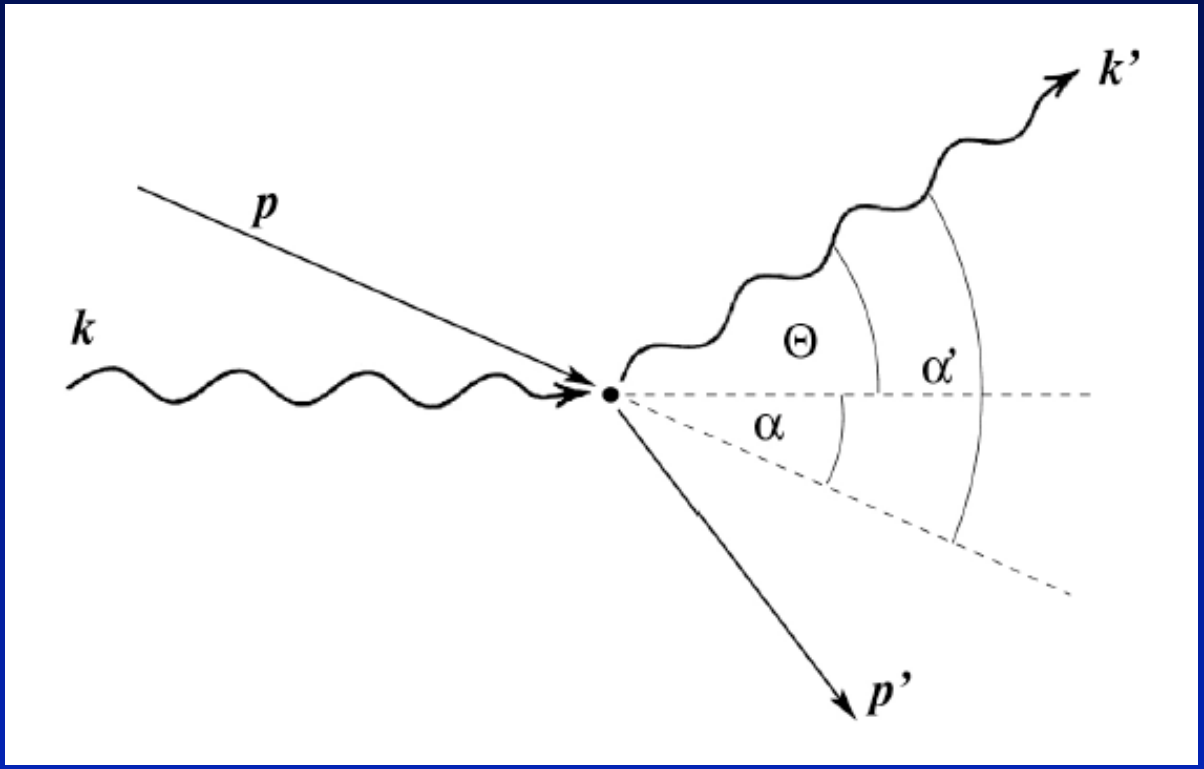
Photon Boltzmann Equation for Average Spectrum

$$\frac{\mathrm{d}n_\nu}{\mathrm{d}t} = \frac{\partial n_\nu}{\partial t} + \frac{\partial n_\nu}{\partial x_i} \cdot \frac{\mathrm{d}x_i}{\mathrm{d}t} + \frac{\partial n_\nu}{\partial p} \frac{\mathrm{d}p}{\mathrm{d}t} + \frac{\partial n_\nu}{\partial \hat{p}_i} \cdot \frac{\mathrm{d}\hat{p}_i}{\mathrm{d}t} = \mathcal{C}[n]$$

- Isotropy & Homogeneity: $\Longrightarrow \quad \frac{\partial n_\nu}{\partial t} - H\nu \frac{\partial n_\nu}{\partial \nu} = \mathcal{C}[n]$
- Collision term: $\mathcal{C}[n] = \left.\frac{\mathrm{d}n_\nu}{\mathrm{d}t}\right|_{\mathrm{C}} + \left.\frac{\mathrm{d}n_\nu}{\mathrm{d}t}\right|_{\mathrm{BR}} + \left.\frac{\mathrm{d}n_\nu}{\mathrm{d}t}\right|_{\mathrm{DC}}$
- Full equilibrium: $\mathcal{C}[n] \equiv 0 \;\Rightarrow$ blackbody spectrum conserved
- Energy release: $\mathcal{C}[n] \neq 0 \;\Rightarrow$ *thermalization process starts*

Redistribution of photons by Compton scattering

- Reaction: $\gamma + e \longleftrightarrow \gamma' + e'$



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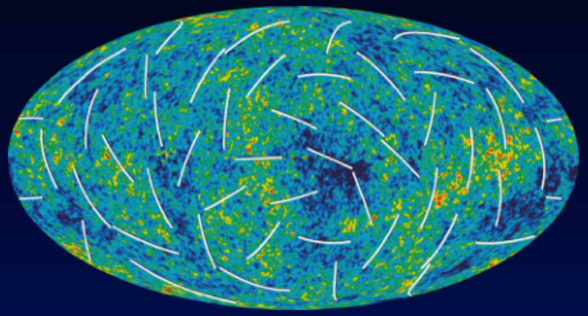
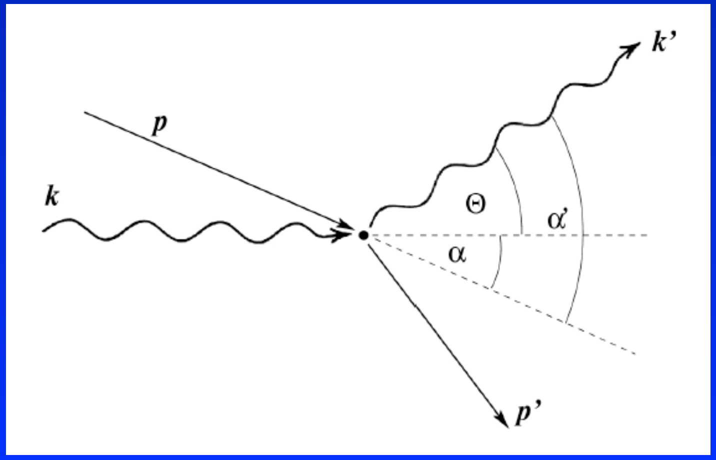


- Reaction: $\gamma + e \longleftrightarrow \gamma' + e'$

→ *no energy exchange ⇒ Thomson limit*

⇒ important for anisotropies

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \frac{3\sigma_\mathrm{T}}{16\pi}\left[1 + (\hat{\gamma}\cdot\hat{\gamma}')^2\right]$$



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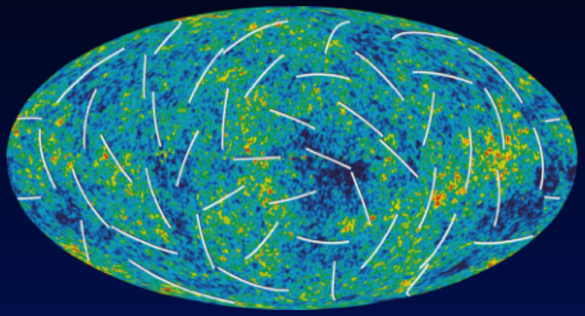

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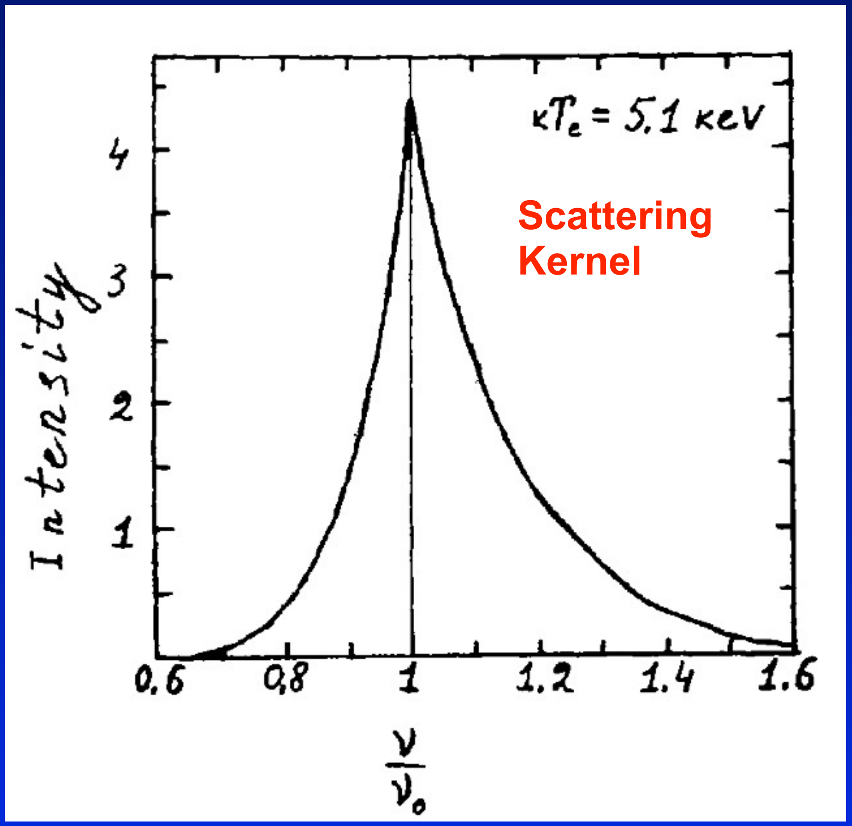
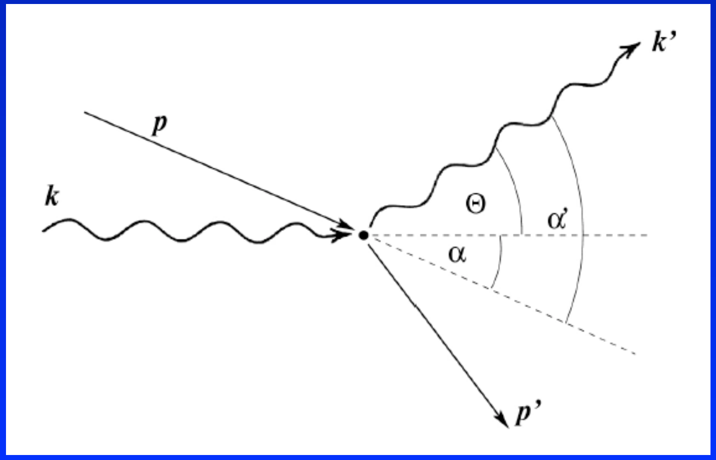
$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \frac{3\sigma_{\mathrm{T}}}{16\pi}\left[1 + (\hat{\gamma}\cdot\hat{\gamma}')^2\right]$$

→ *energy exchange included*

- up-scattering due to the ***Doppler*** effect for $h\nu < 4kT_{\mathrm{e}}$
- down-scattering because of ***recoil*** (and stimulated recoil) for $h\nu > 4kT_{\mathrm{e}}$
- ***Doppler*** broadening $\dfrac{\Delta\nu}{\nu} \simeq \sqrt{\dfrac{2kT_{\mathrm{e}}}{m_{\mathrm{e}}c^2}}$

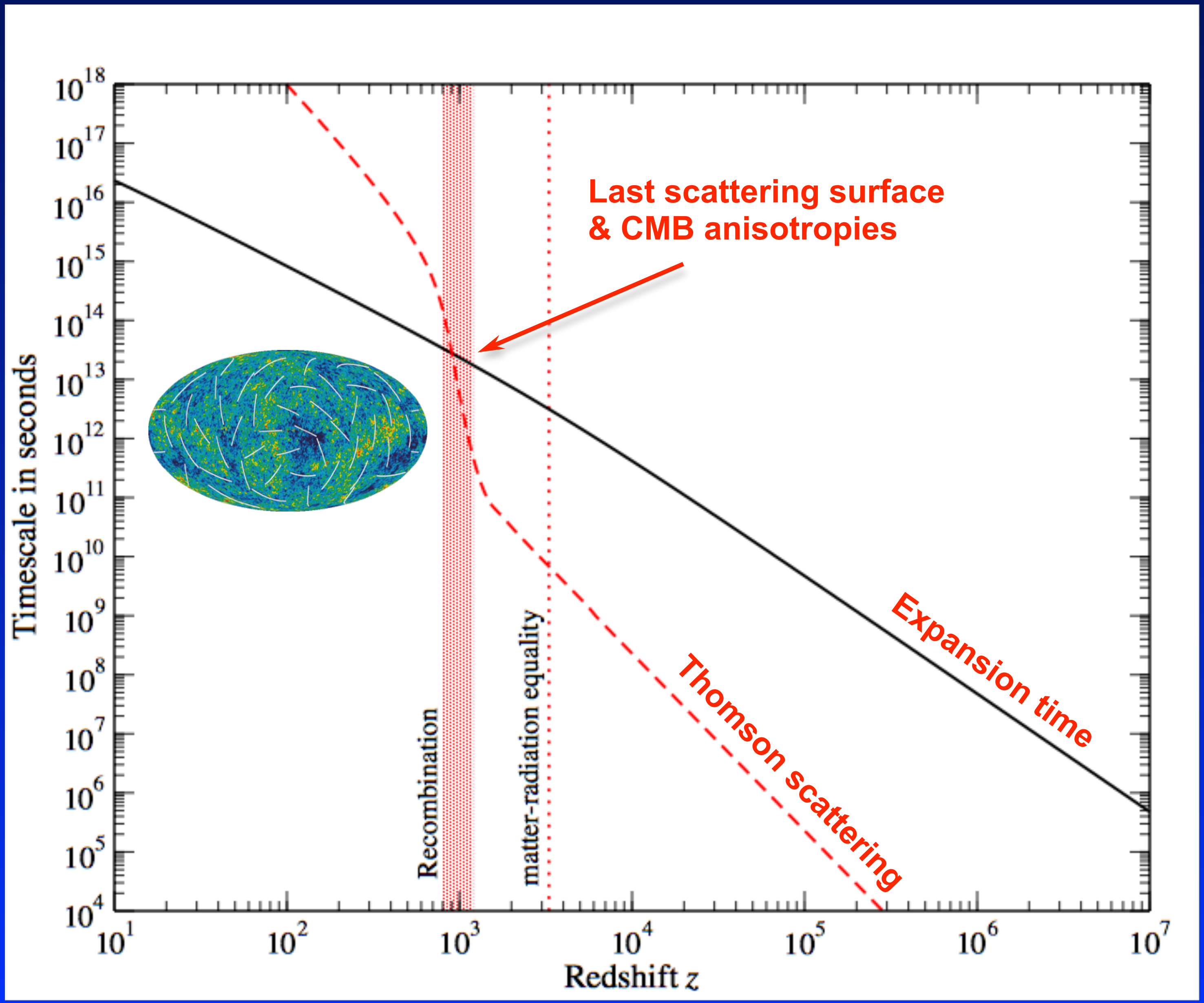


Sunyaev & Zeldovich, 1980, ARAA, 18, 537



Important Timescales for Compton Process

- *Thomson scattering* $t_{\rm C} = (\sigma_{\rm T} N_{\rm e} c)^{-1} \approx 2.3 \times 10^{20}\, \chi_{\rm e}^{-1} (1+z)^{-3}\,{\rm sec}$



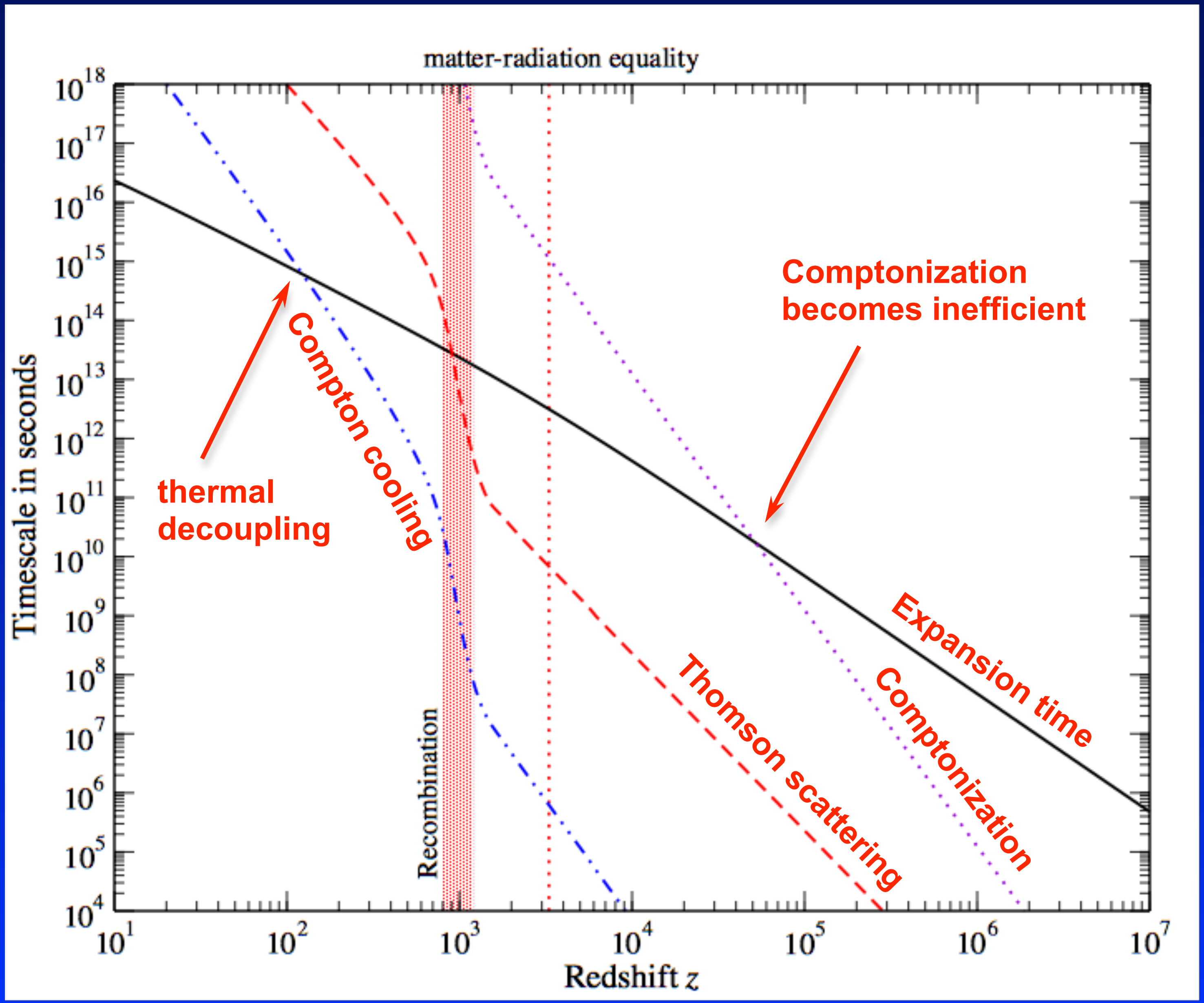
Radiation dominated

$$t_{\rm exp} = H^{-1} \simeq 4.8 \times 10^{19} (1+z)^{-2}\,{\rm sec}$$
$$\simeq 8.4 \times 10^{17} (1+z)^{-3/2}\,{\rm sec}$$

Matter dominated

Important Timescales for Compton Process

- *Thomson scattering* $t_\mathrm{C} = (\sigma_\mathrm{T} N_\mathrm{e} c)^{-1} \approx 2.3 \times 10^{20}\, \chi_\mathrm{e}^{-1} (1+z)^{-3}\,\mathrm{sec}$
- *Comptonization* $t_\mathrm{K} = \left(4 \frac{kT_\mathrm{e}}{m_\mathrm{e} c^2} \sigma_\mathrm{T} N_\mathrm{e} c\right)^{-1} \approx 1.2 \times 10^{29}\, \chi_\mathrm{e}^{-1} (T_\mathrm{e}/T_\gamma)^{-1} (1+z)^{-4}\,\mathrm{sec}$
- *Compton cooling* $t_\mathrm{cool} = \left(\frac{4\rho_\gamma}{m_\mathrm{e} c^2} \frac{\sigma_\mathrm{T} N_\mathrm{e} c}{(3/2)N}\right)^{-1} \approx 7.1 \times 10^{19}\, \chi_\mathrm{e}^{-1} (T_\mathrm{e}/T_\gamma)^{-1} (1+z)^{-4}\,\mathrm{sec}$



- *matter temperature starts deviating from Compton equilibrium temperature at z ≲ 100-200*
- *Comptonization becomes inefficient at $z_\mathrm{K} \simeq 50000$*

⇒ character of distortion changes at z_K ! $\mu \Leftrightarrow y$

Radiation dominated

$$t_\mathrm{exp} = H^{-1} \simeq 4.8 \times 10^{19} (1+z)^{-2}\,\mathrm{sec}$$
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Matter dominated

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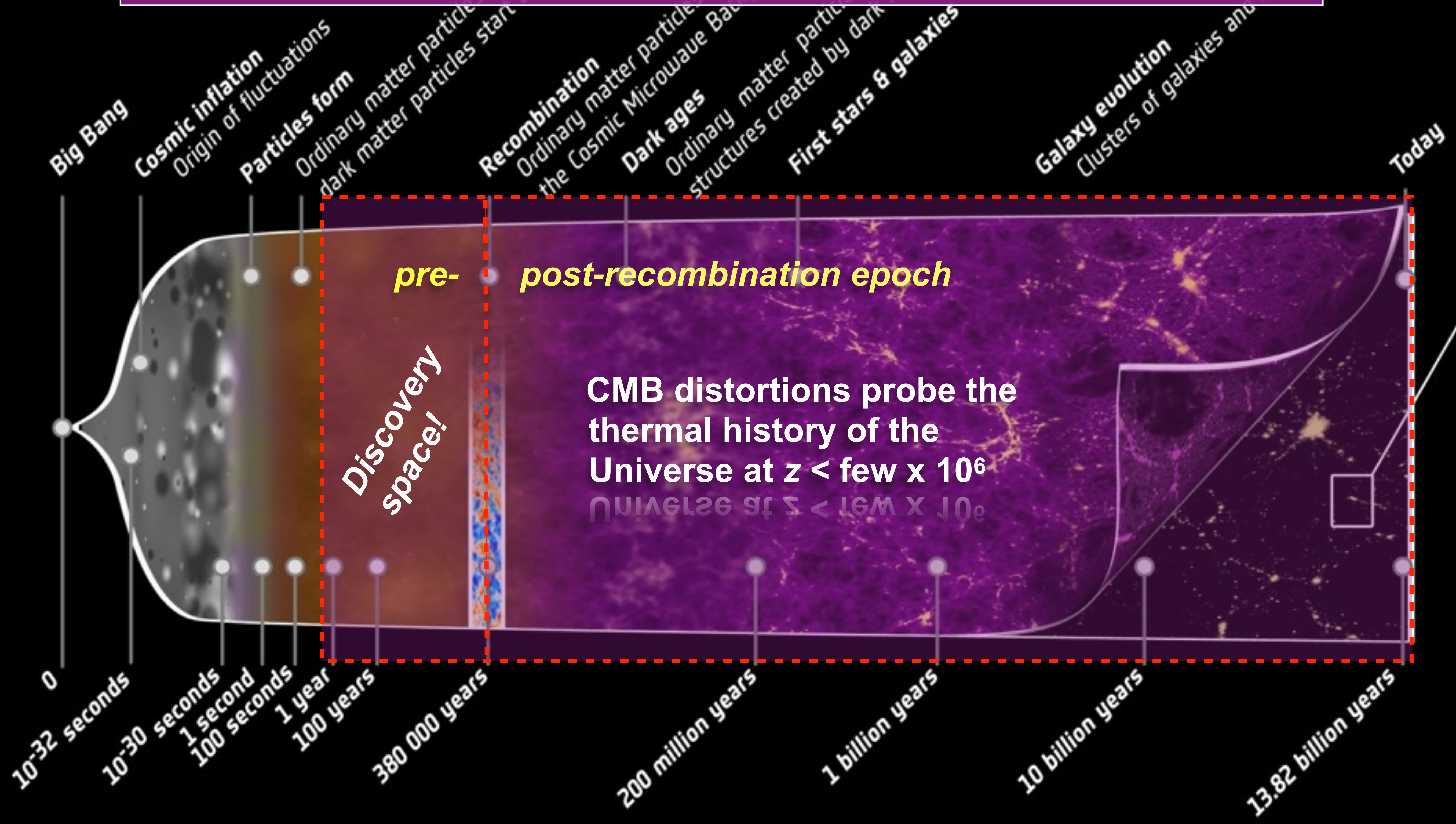

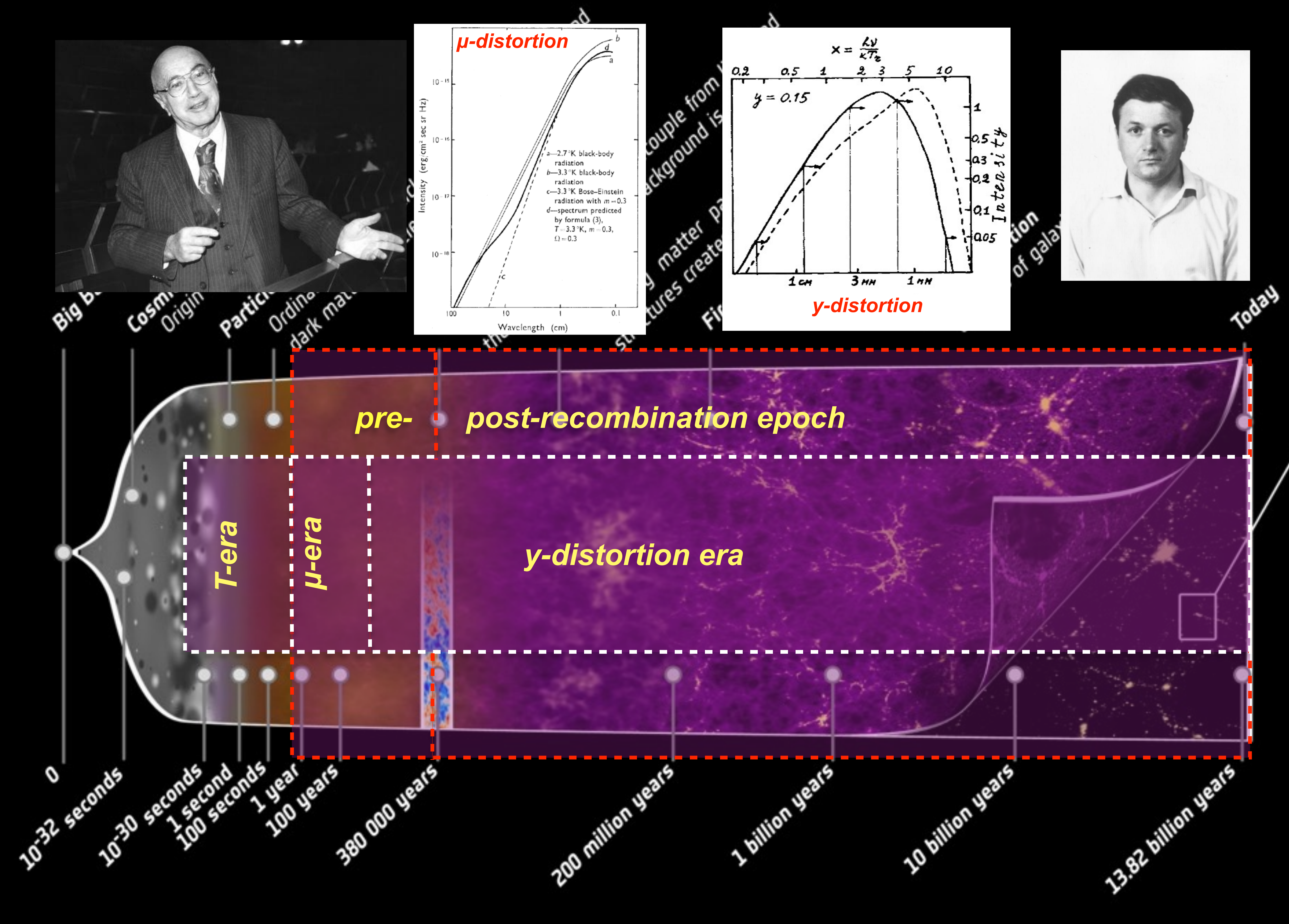
μ-distortion
Intensity (erg/cm² sec sr Hz)
Wavelength (cm)
a—2.7 °K black-body radiation
b—3.3 °K black-body radiation
c—3.3 °K Bose-Einstein radiation with m = 0.3
d—spectrum predicted by formula (3), T = 3.3 °K, m = 0.3, Ω = 0.3
y = 0.15
Intensity
1 cm
3 mm
1 mm
y-distortion
Today
pre-
post-recombination epoch
T-era
μ-era
y-distortion era
0
10-32 seconds
10-30 seconds
1 second
100 seconds
1 year
100 years
380 000 years
200 million years
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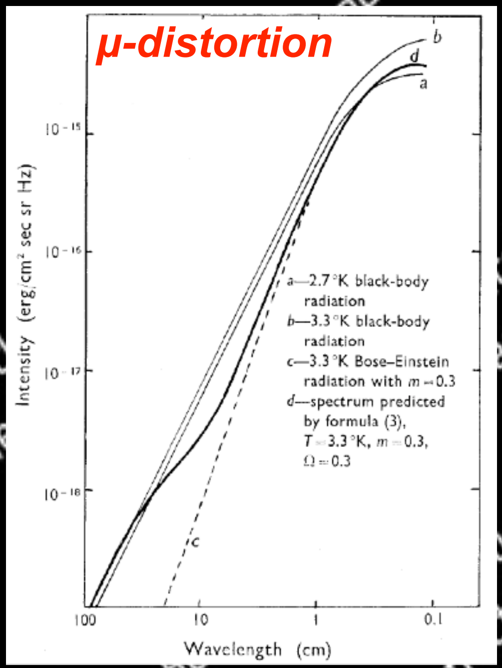
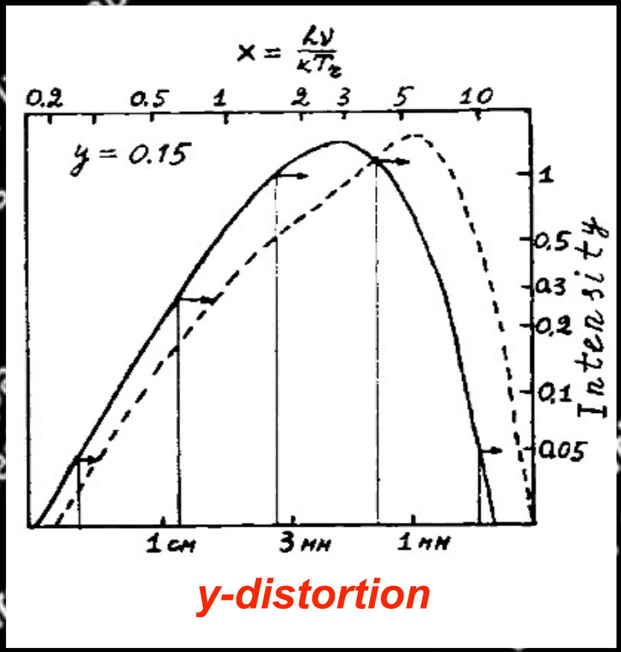
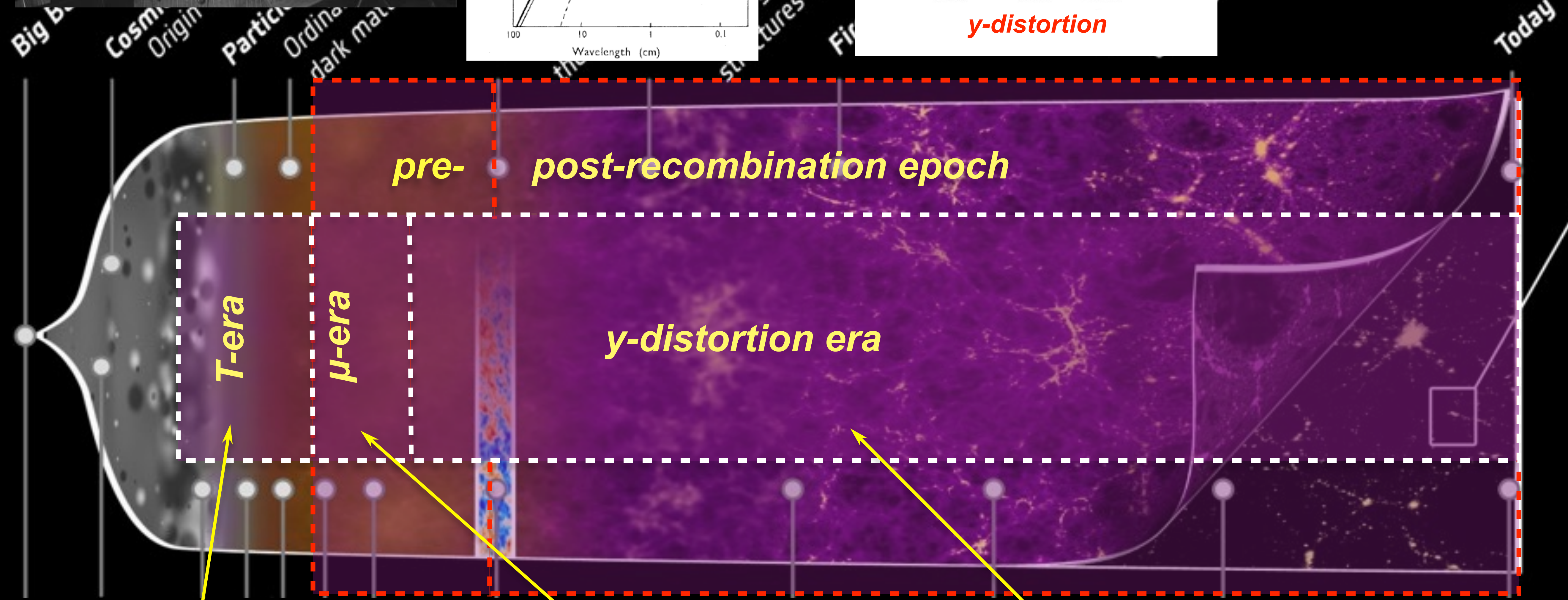






$$\frac{\Delta T}{T} \simeq \frac{1}{4} \left.\frac{\Delta\rho_\gamma}{\rho_\gamma}\right|_T \qquad \mu \simeq 1.4 \left.\frac{\Delta\rho_\gamma}{\rho_\gamma}\right|_\mu \qquad y \simeq \frac{1}{4} \left.\frac{\Delta\rho_\gamma}{\rho_\gamma}\right|_y$$

What are y- and μ-distortions?

Compton y-distortion / thermal SZ effect

- *Kompaneets equation:* $$\left.\frac{\mathrm{d}n}{\mathrm{d}\tau}\right|_{\rm C} \approx \frac{\theta_{\rm e}}{x^2}\frac{\partial}{\partial x} x^4 \left[\frac{\partial n}{\partial x} + \frac{T_\gamma}{T_{\rm e}} n(1+n)\right]$$

- *insert:* $n \approx n^{\rm bb} = 1/(\mathrm{e}^x - 1) \quad \Longrightarrow \quad \Delta n \approx y\,Y(x)$ *with* $y \ll 1$

$$y = \int \frac{k[T_{\rm e} - T_\gamma]}{m_{\rm e} c^2}\,\sigma_{\rm T} N_{\rm e} c\,\mathrm{d}t$$

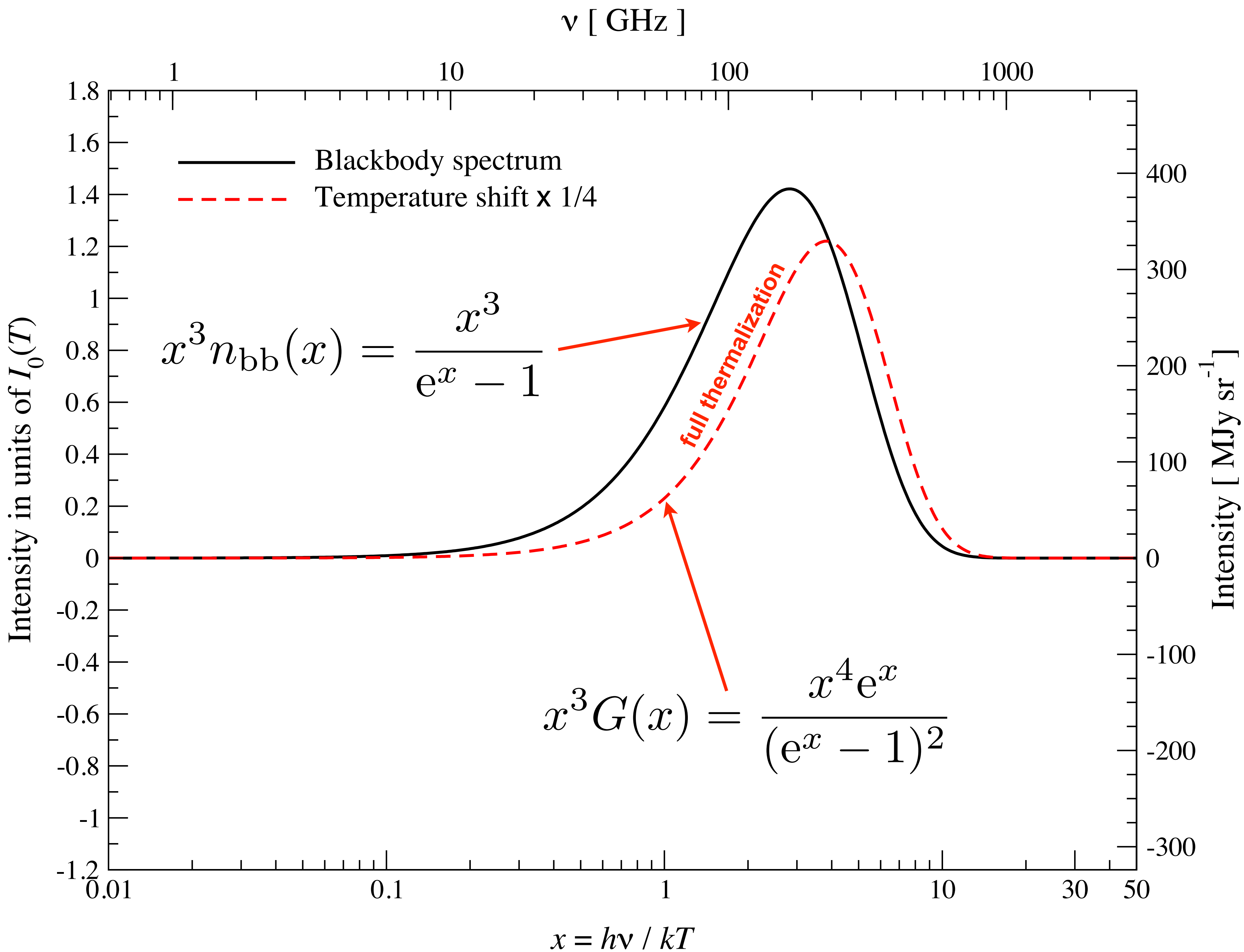
Compton y-parameter

$$\boxed{Y(x) = \frac{x\mathrm{e}^x}{(\mathrm{e}^x - 1)^2}\left[x\frac{\mathrm{e}^x + 1}{\mathrm{e}^x - 1} - 4\right]}$$

spectrum of y-distortion (↔ SZ effect)

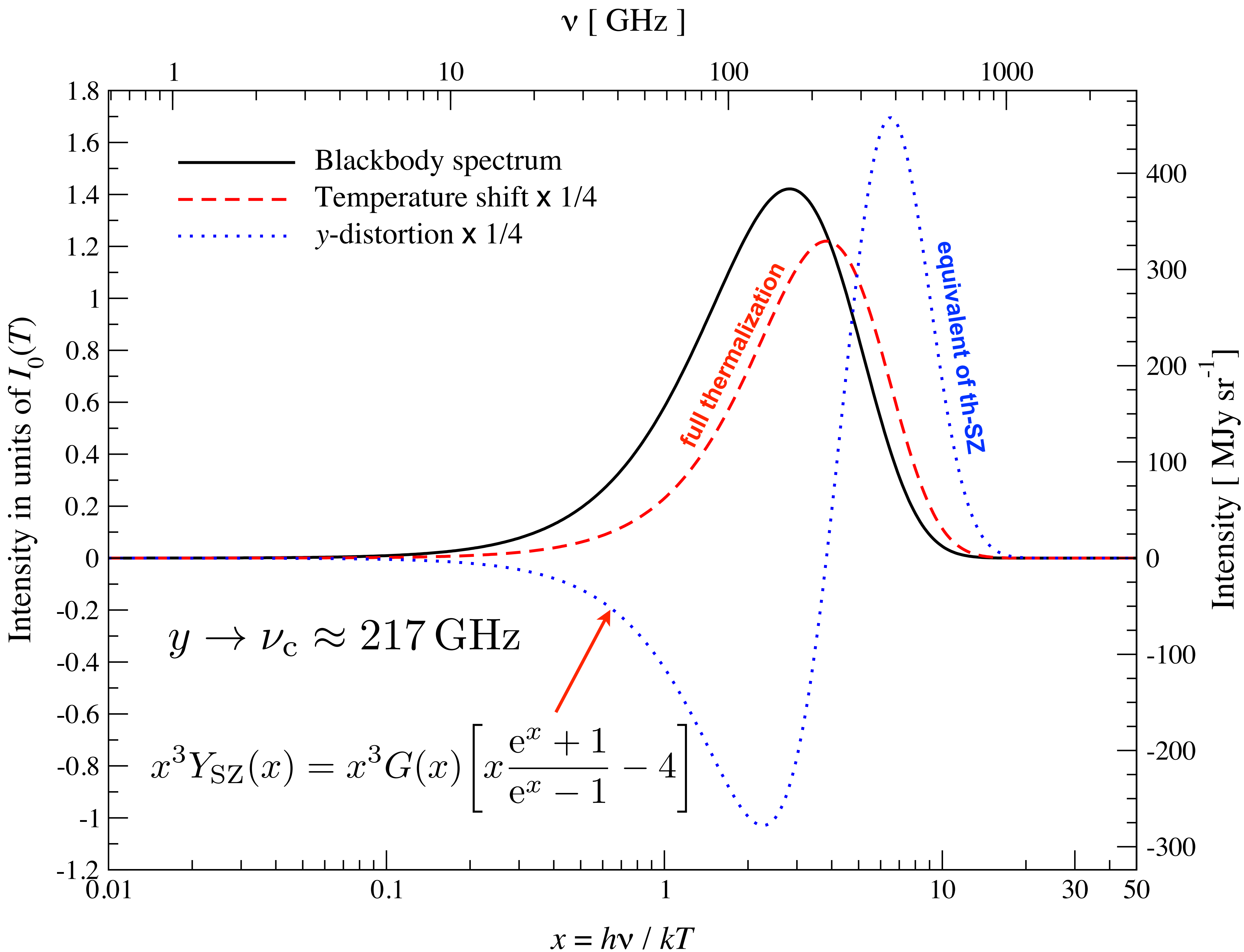
- *if* $T_{\rm e} = T_\gamma \quad \Longrightarrow \quad \left.\frac{\mathrm{d}n}{\mathrm{d}\tau}\right|_{\rm C} = 0$ *(kinetic equilibrium with electrons)*
- *if* $T_{\rm e} < T_\gamma \quad \Longrightarrow$ *down-scattering of photons / heating of electrons*
- *if* $T_{\rm e} > T_\gamma \quad \Longrightarrow$ *up-scattering of photons / cooling of electrons*
- *for* $T_{\rm e} \gg T_\gamma \quad \Longrightarrow$ *thermal Sunyaev-Zeldovich effect (up-scattering)*

Simplest spectral shapes



$$I_0 = (2h/c^2)(kT_0/h)^3 \approx 270\,\mathrm{MJy\,sr^{-1}}$$

Simplest spectral shapes



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Chemical Potential / μ-parameter

- *Limit of "many" scatterings* $\Longrightarrow$ $$\left.\frac{\mathrm{d}n}{\mathrm{d}\tau}\right|_{\mathrm{C}} \approx 0$$

"Kinetic equilibrium" to scattering

- *Kompaneets equation:* $\Longrightarrow$ $$\partial_x n \approx -\frac{T_\gamma}{T_{\mathrm{e}}} n(1+n)$$

$$\left.\frac{\mathrm{d}n}{\mathrm{d}\tau}\right|_{\mathrm{C}} \approx \frac{\theta_{\mathrm{e}}}{x^2}\frac{\partial}{\partial x} x^4 \left[\frac{\partial n}{\partial x} + \frac{T_\gamma}{T_{\mathrm{e}}} n(1+n)\right]$$

Chemical Potential / μ-parameter

- *Limit of "many" scatterings* $\Longrightarrow \quad \left.\frac{\mathrm{d}n}{\mathrm{d}\tau}\right|_{\mathrm{C}} \approx 0$ **"Kinetic equilibrium" to scattering**
- *Kompaneets equation:* $\Longrightarrow \quad \partial_x n \approx -\frac{T_\gamma}{T_\mathrm{e}} n(1+n)$
- *for* $T_\gamma = T_\mathrm{e} \quad \Longrightarrow \quad n = n^{\mathrm{bb}}(x) = 1/(\mathrm{e}^x - 1)$
- *any spectrum can be written as:* $n(x) = 1/(\mathrm{e}^{x+\mu(x)} - 1)$ **chemical potential parameter** (*"wrong" sign*)

$$\Longrightarrow \quad (1 + \partial_x \mu) = -\frac{T_\gamma}{T_\mathrm{e}} \quad \Longrightarrow \quad x + \mu = x\frac{T_\gamma}{T_\mathrm{e}} + \mu_0$$

constant

- *General equilibrium solution:* *Bose-Einstein spectrum with*

$T_\gamma = T_\mathrm{e} \equiv T_\mathrm{eq} \quad \text{and} \quad \mu_0 = \text{const}$ ($\equiv 0$ for blackbody)

Something is missing? How do you fix T_e and μ_0?

Final definition of μ-type distortion

- *initial condition:* $N_\gamma = N_\gamma^{\rm bb}(T_\gamma)$ and $\rho_\gamma = \rho_\gamma^{\rm bb}(T_\gamma)$
- *after energy release* ⇒

$$\approx 1.368$$

$$N_\gamma^{\rm bb}(T_\gamma) = N_\gamma^{\rm BE}(T_{\rm e}, \mu_0) \approx N_\gamma^{\rm bb}(T_\gamma)\left(1 + 3\frac{\Delta T}{T_\gamma} - \frac{\pi^2}{6\zeta(3)}\mu_0\right)$$

$$\approx 1.111$$

$$\rho_\gamma^{\rm bb}(T_\gamma) + \Delta\rho_\gamma = \rho_\gamma^{\rm BE}(T_{\rm e}, \mu_0) \approx \rho_\gamma^{\rm bb}(T_\gamma)\left(1 + 4\frac{\Delta T}{T_\gamma} - \frac{90\zeta(3)}{\pi^4}\mu_0\right)$$

- *Solution:* $\frac{\Delta T}{T_\gamma} \approx \frac{\pi^2}{18\zeta(3)}\mu_0 \approx 0.456\,\mu_0$ *and* $\boxed{\mu_0 \approx 1.401\,\frac{\Delta\rho_\gamma}{\rho_\gamma}}$

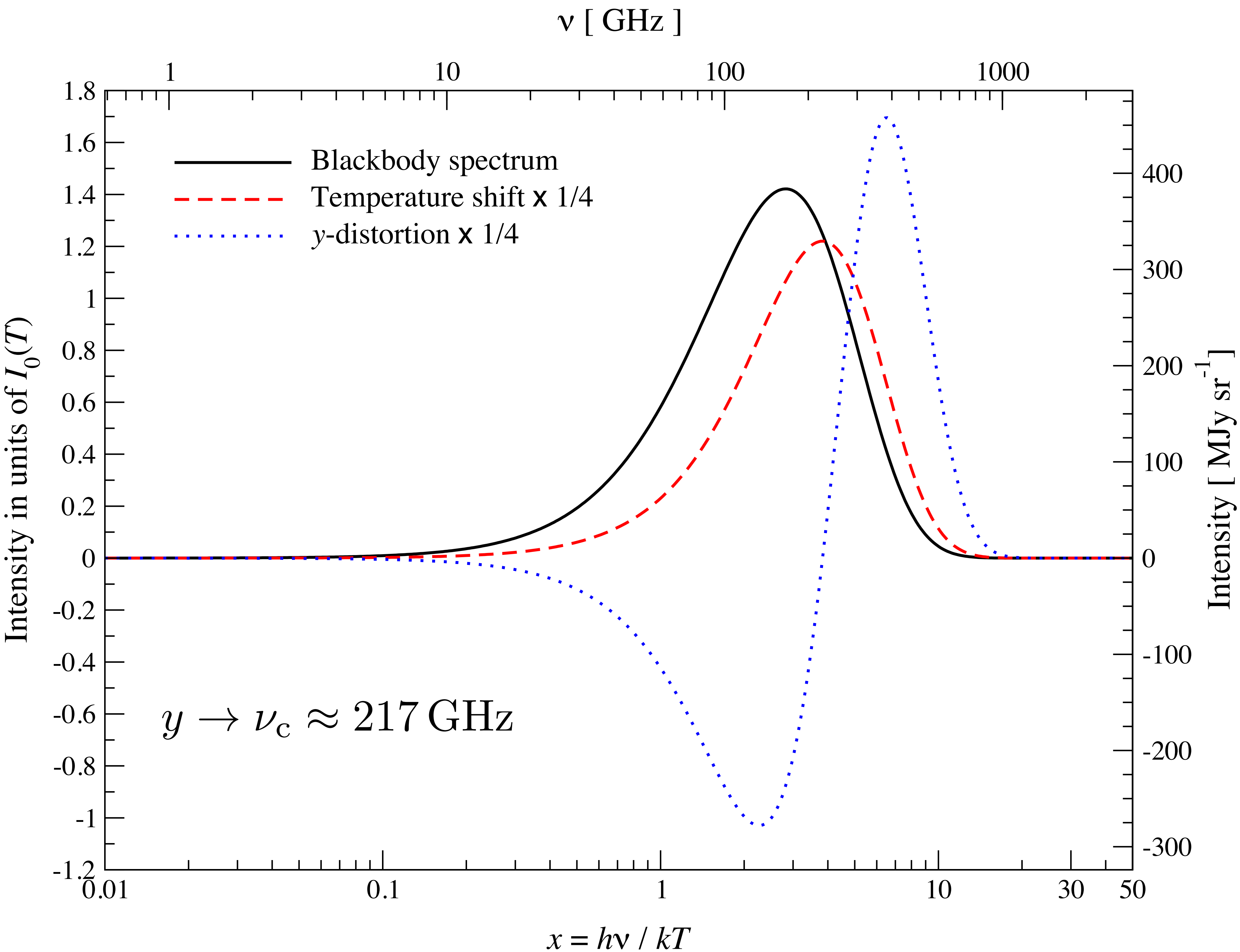
$$\Longrightarrow \boxed{M(x) = G(x)\left[\frac{\pi^2}{18\zeta(3)} - \frac{1}{x}\right]}$$

μ-distortion spectrum
(photon number conserved)

- $\mu_0 > 0 \Rightarrow$ *too few photons / too much energy*
- $\mu_0 < 0 \Rightarrow$ *too many photons / too little energy*

$$\frac{\Delta\rho_\gamma}{\rho_\gamma^{\rm bb}} \approx \frac{4}{3}\,\frac{\Delta N_\gamma}{N_\gamma^{\rm bb}}$$

Simplest spectral shapes



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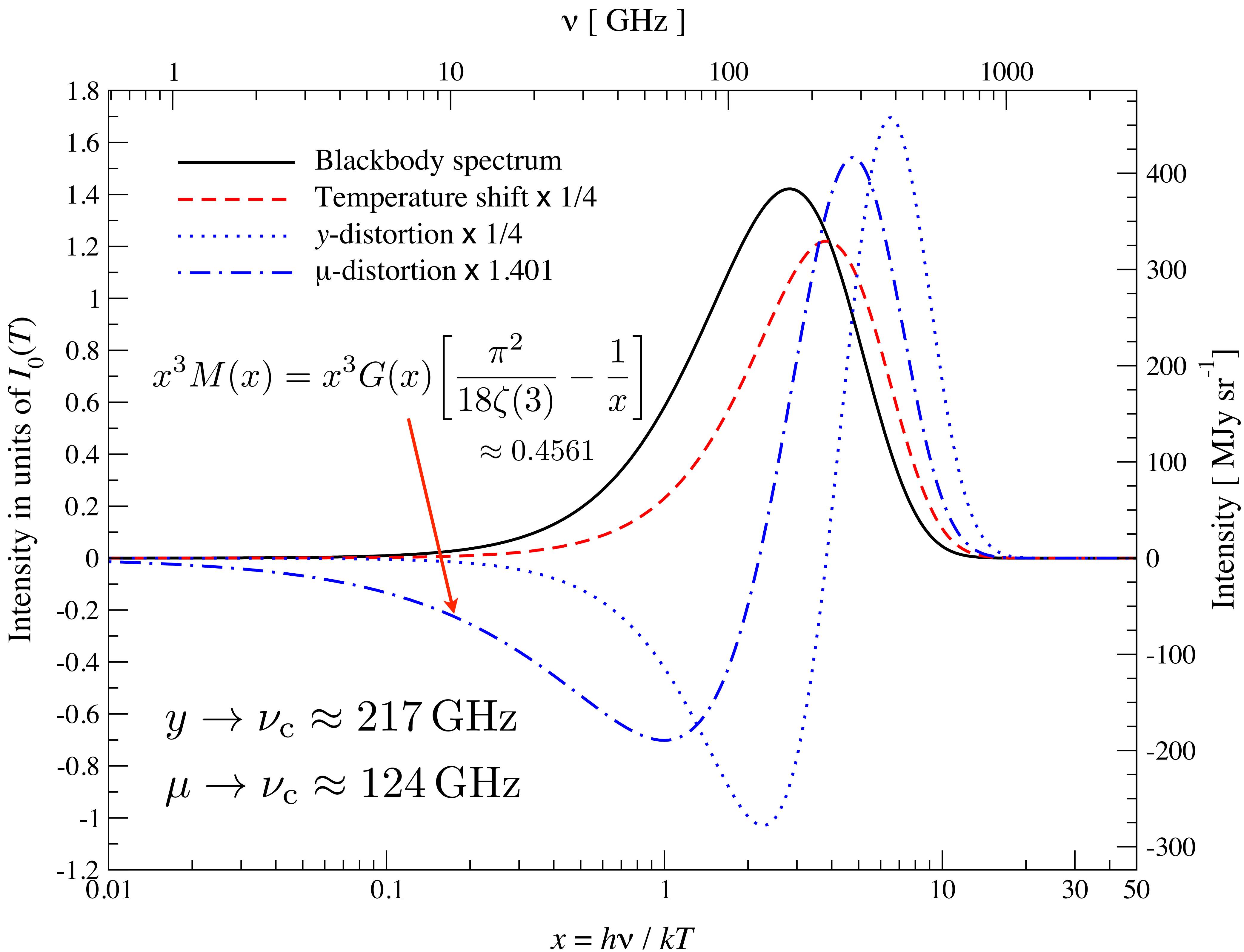
Simplest spectral shapes



$$I_0 = (2h/c^2)(kT_0/h)^3 \approx 270\,\mathrm{MJy\,sr^{-1}}$$

What about photon production processes?

Adjusting the photon number

- Bremsstrahlung $e + p \leftrightarrow e' + p + \gamma$
  - → 1. order α correction to *Coulomb* scattering
  - → production of low frequency photons
  - → important for the evolution of the distortion at low frequencies and late times (z< 2 x 10^5)

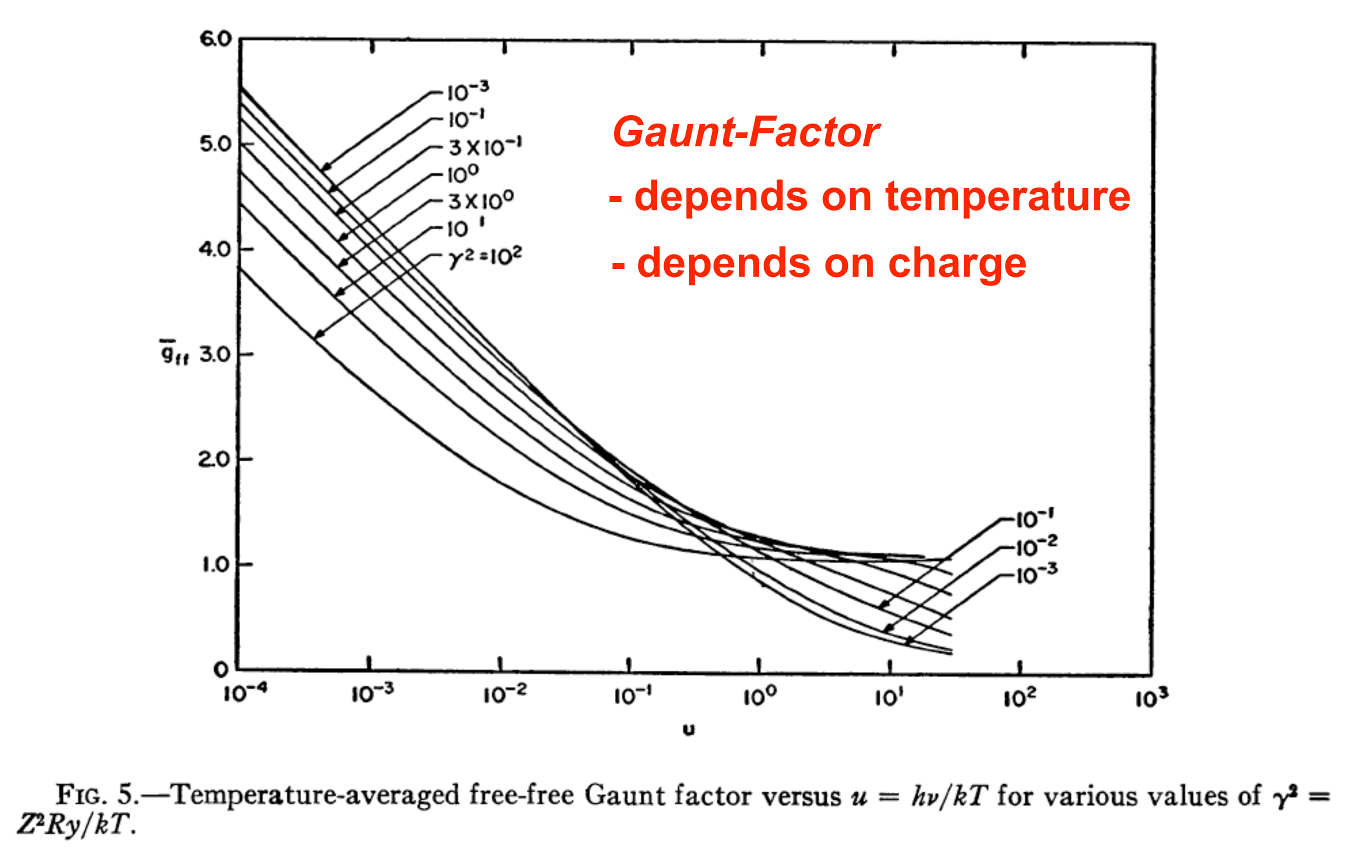
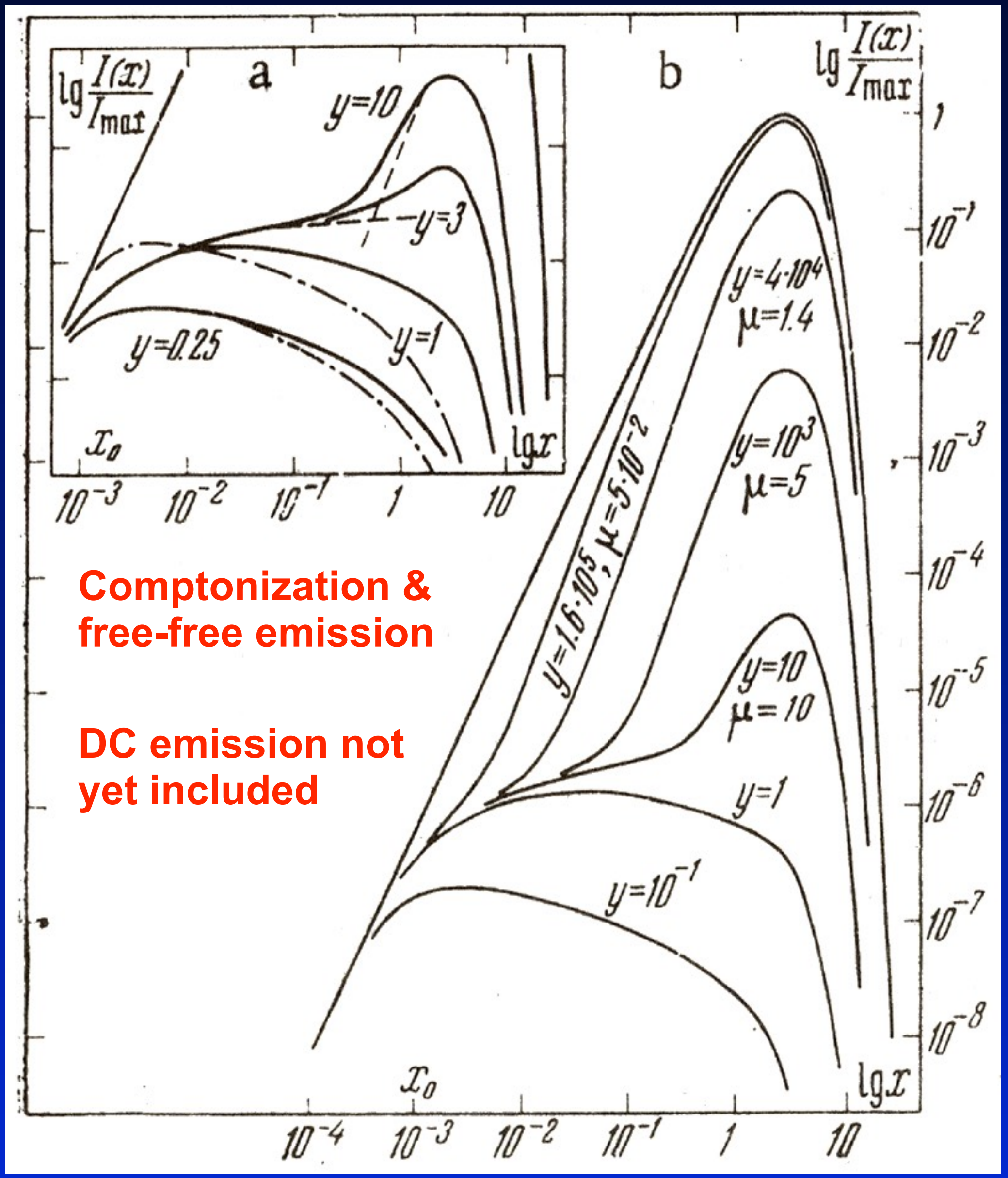


FIG. 5.—Temperature-averaged free-free Gaunt factor versus $u = h\nu/kT$ for various values of $\gamma^2 = Z^2Ry/kT$.

Karzas & Latter, 1961, ApJS, 6, 167



Illarionov & Sunyaev, 1975, Sov. Astr, 18, pp.413

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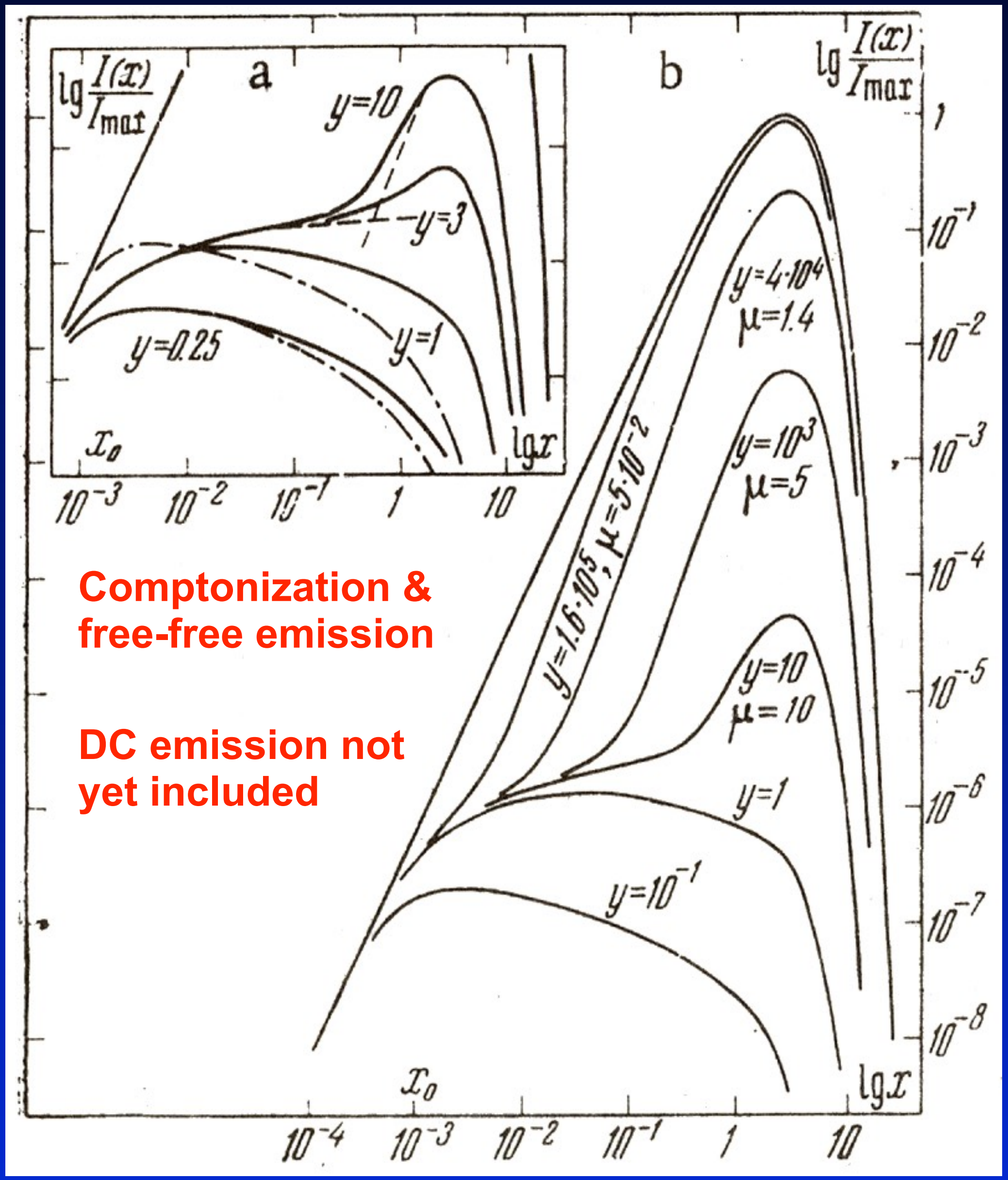


Illarionov & Sunyaev, 1975, Sov. Astr, 18, pp.413

- Double Compton scattering
  (Lightman 1981; Thorne, 1981)

$$e + \gamma \leftrightarrow e' + \gamma' + \gamma_2$$

  - → 1. order α correction to *Compton* scattering
  - → was only included later (Danese & De Zotti, 1982)
  - → production of low frequency photons
  - → very important at high redshifts (z > 2 x 10^5)

Final Set of evolution equations

Photon field

$$\frac{\partial f}{\partial \tau} \approx \frac{\theta_{\rm e}}{x^2}\frac{\partial}{\partial x}x^4\left[\frac{\partial}{\partial x}f + \frac{T_\gamma}{T_{\rm e}}\,f(1+f)\right] + \frac{K_{\rm BR}\,{\rm e}^{-x_{\rm e}}}{x_{\rm e}^3}[1 - f\,({\rm e}^{x_{\rm e}} - 1)] + \frac{K_{\rm DC}\,{\rm e}^{-2x}}{x^3}[1 - f\,({\rm e}^{x} - 1)] + S(\tau, x)$$

$$K_{\rm BR} = \frac{\alpha}{2\pi}\frac{\lambda_{\rm e}^3}{\sqrt{6\pi}\,\theta_{\rm e}^{7/2}}\sum_i Z_i^2 N_i\,\bar{g}_{\rm ff}(Z_i, T_{\rm e}, T_\gamma, x_{\rm e}), \qquad K_{\rm DC} = \frac{4\alpha}{3\pi}\,\theta_\gamma^2\,I_{\rm dc}\,g_{\rm dc}(T_{\rm e}, T_\gamma, x)$$

$$\bar{g}_{\rm ff}(x_{\rm e}) \approx \begin{cases} \frac{\sqrt{3}}{\pi}\ln\left(\frac{2.25}{x_{\rm e}}\right) & \text{for} \quad x_{\rm e} \le 0.37 \\ 1 & \text{otherwise} \end{cases}, \qquad g_{\rm dc} \approx \frac{1 + \frac{3}{2}x + \frac{29}{24}x^2 + \frac{11}{16}x^3 + \frac{5}{12}x^4}{1 + 19.739\theta_\gamma - 5.5797\theta_{\rm e}}.$$

$$I_{\rm dc} = \int x^4 f(1+f)\,{\rm d}x \approx 4\pi^4/15$$

Ordinary matter temperature

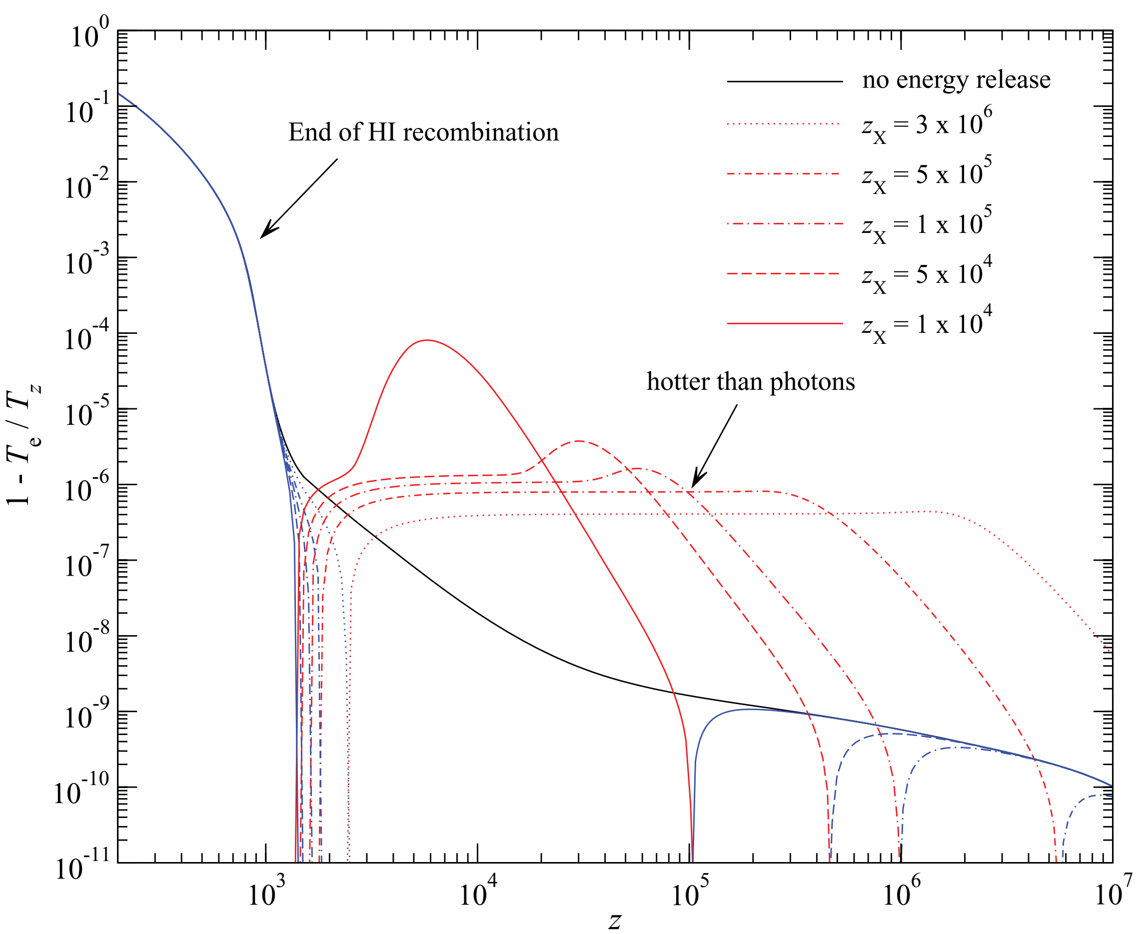
$$\frac{{\rm d}\rho_{\rm e}}{{\rm d}\tau} = \frac{{\rm d}(T_{\rm e}/T_\gamma)}{{\rm d}\tau} = \frac{t_{\rm T}\dot{Q}}{\alpha_{\rm h}\theta_\gamma} + \frac{4\tilde{\rho}_\gamma}{\alpha_{\rm h}}[\rho_{\rm e}^{\rm eq} - \rho_{\rm e}] - \frac{4\tilde{\rho}_\gamma}{\alpha_{\rm h}}\mathcal{H}_{\rm DC,BR}(\rho_{\rm e}) - H\,t_{\rm T}\,\rho_{\rm e}.$$

$$k\alpha_{\rm h} = \frac{3}{2}k[N_{\rm e} + N_{\rm H} + N_{\rm He}] = \frac{3}{2}kN_{\rm H}[1 + f_{\rm He} + X_{\rm e}] \qquad \rho_{\rm e}^{\rm eq} = T_{\rm e}^{\rm eq}/T_\gamma$$

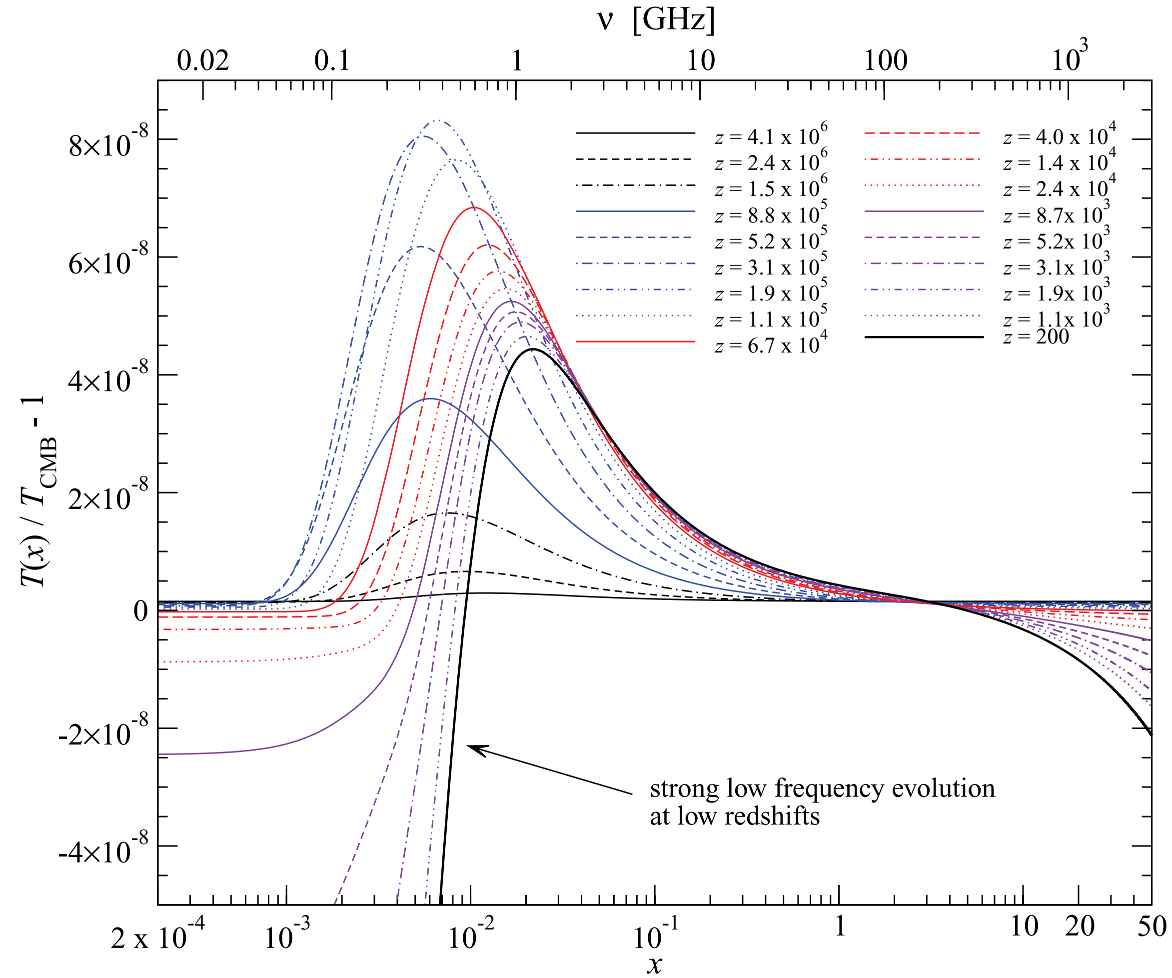
$$\tilde{\rho}_\gamma = \rho_\gamma/m_{\rm e}c^2 \qquad T_{\rm e}^{\rm eq} = T_\gamma\frac{\int x^4 f(1+f)\,{\rm d}x}{4\int x^3 f\,{\rm d}x} \equiv \frac{h}{k}\frac{\int \nu^4 f(1+f)\,{\rm d}\nu}{4\int \nu^3 f\,{\rm d}\nu}$$

CosmoTherm: *a new flexible thermalization code*

- Solve the thermalization problem for a *wide range* of energy release histories
- several scenarios already implemented *(decaying particles, damping of acoustic modes)*
- first *explicit* solution of time-dependent energy release scenarios
- open source code
- will be available at `www.Chluba.de/CosmoTherm/`
- Main reference: JC & Sunyaev, MNRAS, 2012 (arXiv:1109.6552)

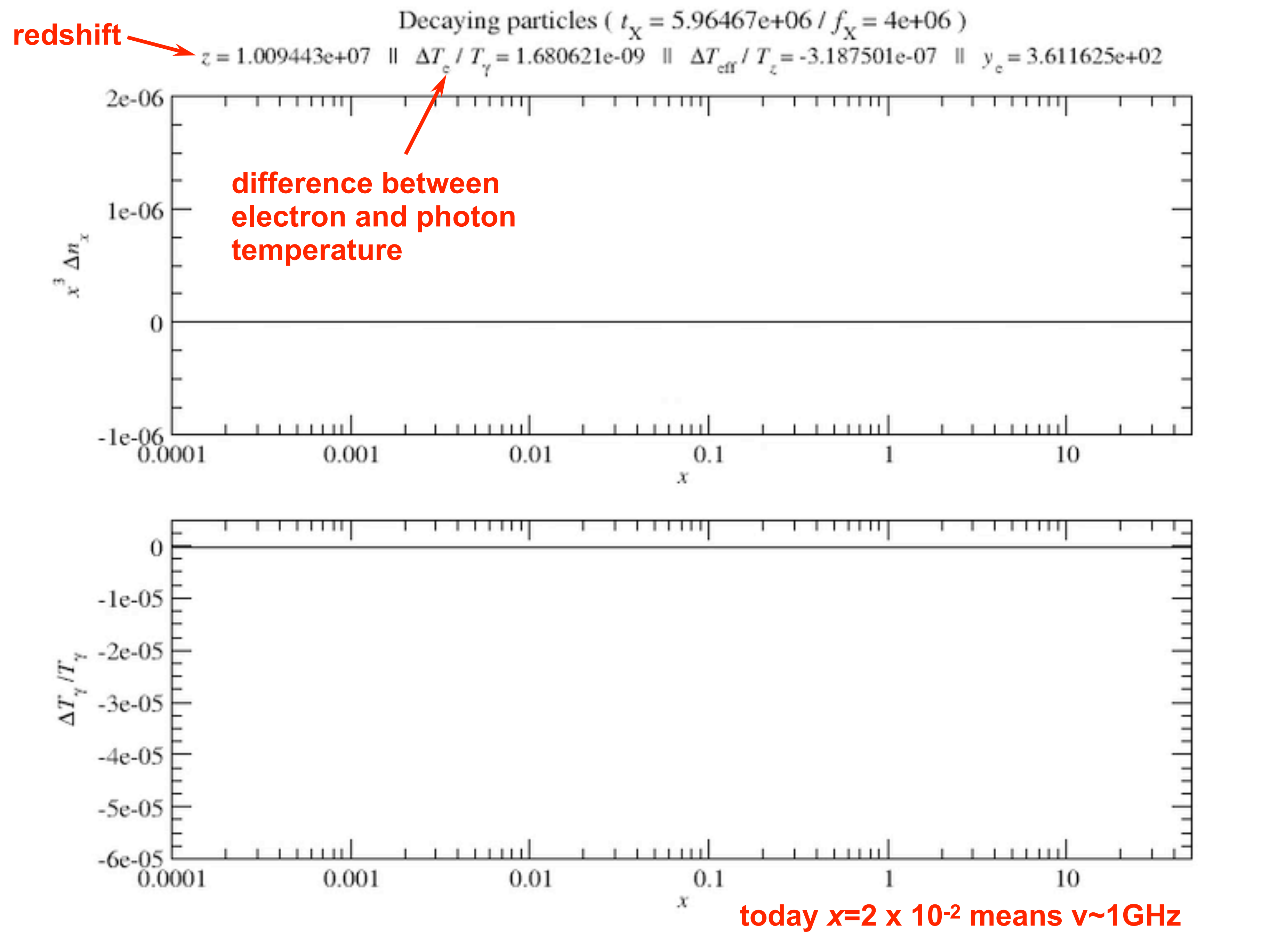


Electron temperature evolution



Evolution of distortion

Example: *Energy release by decaying relict particle*



- initial condition: *full equilibrium*
- total energy release: $\Delta\rho/\rho \sim 1.3 \times 10^{-6}$
- most of energy released around: $z_X \sim 2 \times 10^6$
- positive μ-distortion
- high frequency distortion frozen around $z \simeq 5 \times 10^5$
- late ($z < 10^3$) free-free absorption at very low frequencies ($T_e < T_\gamma$)

Computation carried out with *CosmoTherm*
(JC & Sunyaev 2012)

Is there a simple way to include the effect of photon production at low frequencies?

Analytic Approximation for μ-distortion

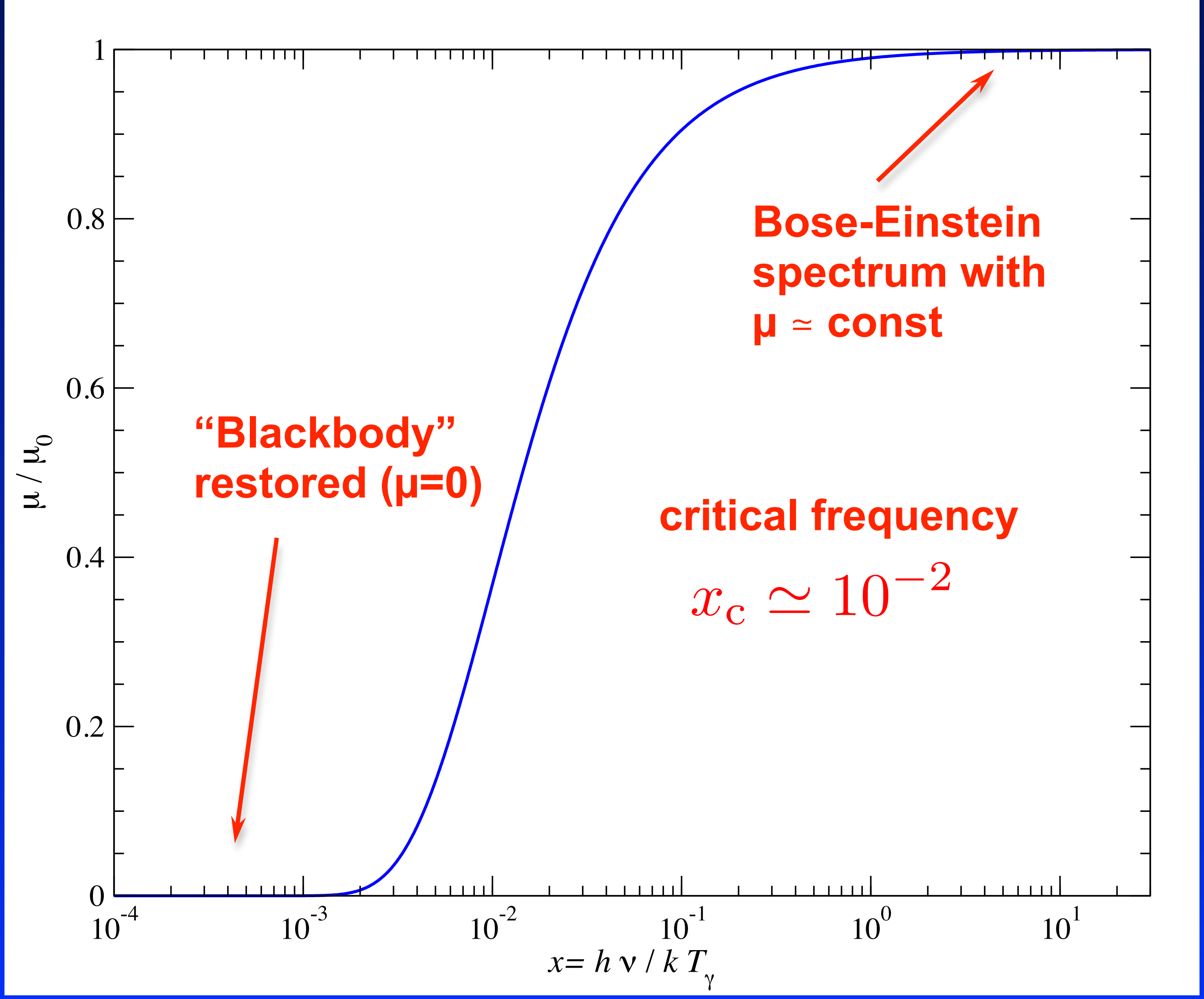
- *Comptonization efficient!* $\Longrightarrow$ $$\left.\frac{\mathrm{d}n}{\mathrm{d}\tau}\right|_{\mathrm{C}} + \left.\frac{\mathrm{d}n}{\mathrm{d}\tau}\right|_{\mathrm{em/abs}} \approx 0$$

- *low frequency limit & small distortion* $\Longrightarrow$ $$\mu(x,z) \approx \mu_0(z)\,\mathrm{e}^{-x_{\mathrm{c}}(z)/x}$$

critical frequency

chemical potential at high frequencies

(e.g., see Sunyaev & Zeldovich, 1970, ApSS, 7, 20; Hu 1995, PhD Thesis)



$$n = \frac{1}{\mathrm{e}^{x+\mu(x,z)} - 1}$$

Analytic Approximation for μ-distortion

- *Comptonization efficient!* $\Longrightarrow \quad \left.\frac{\mathrm{d}n}{\mathrm{d}\tau}\right|_{\mathrm{C}} + \left.\frac{\mathrm{d}n}{\mathrm{d}\tau}\right|_{\mathrm{em/abs}} \approx 0$

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critical frequency

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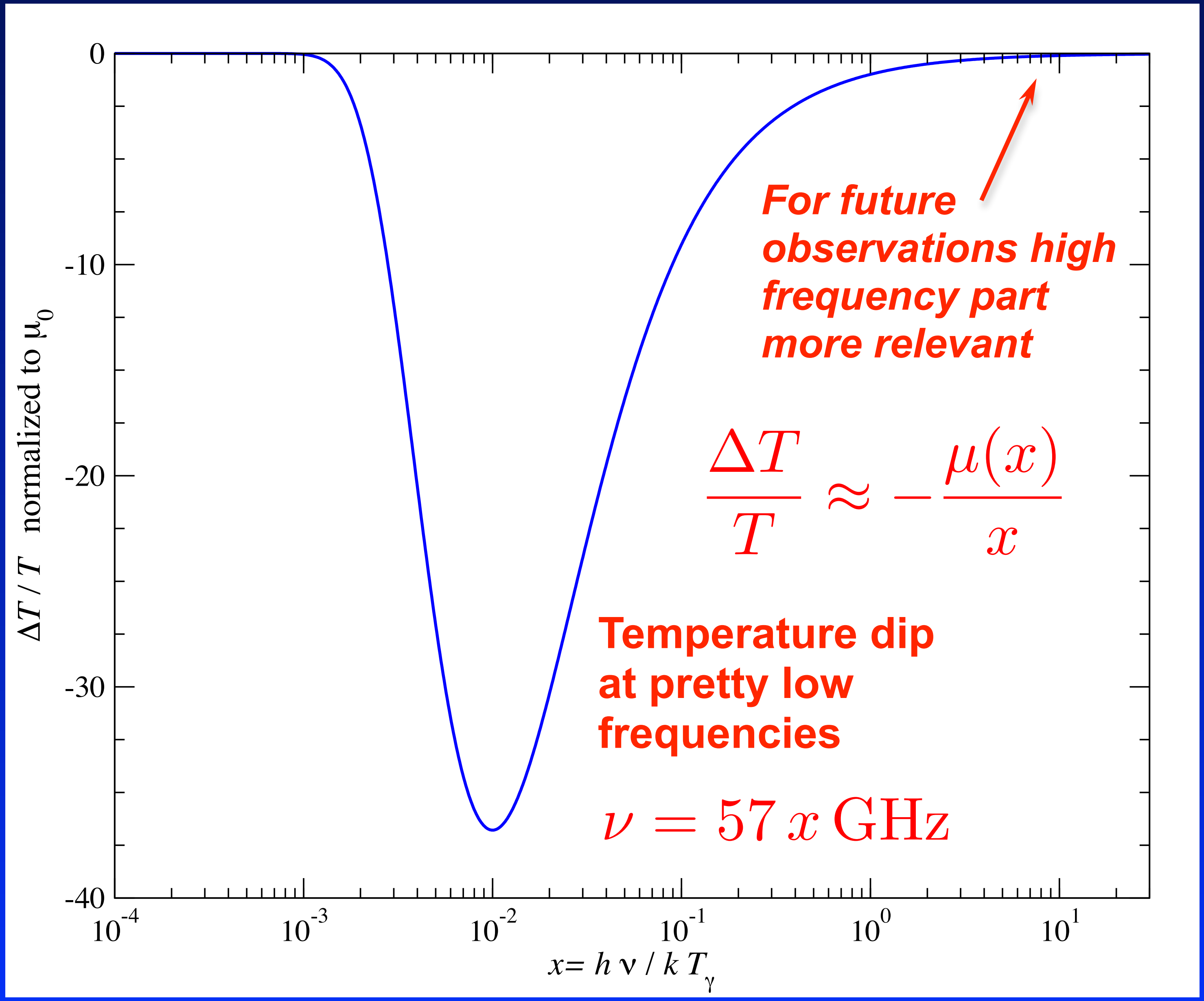

$$n = \frac{1}{\mathrm{e}^{x+\mu(x,z)} - 1}$$

Analytic Approximation for μ-distortion

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(e.g., see Sunyaev & Zeldovich, 1970, ApSS, 7, 20; Hu 1995, PhD Thesis)

Last step: How does $\mu_0(z)$ depend on z?

Analytic Approximation for μ-distortion

- *Comptonization efficient!* $\Longrightarrow$ $\left.\frac{\mathrm{d}n}{\mathrm{d}\tau}\right|_{\mathrm{C}} + \left.\frac{\mathrm{d}n}{\mathrm{d}\tau}\right|_{\mathrm{em/abs}} \approx 0$

- *low frequency limit & small distortion* $\Longrightarrow$ $\mu(x, z) \approx \mu_0(z)\,\mathrm{e}^{-x_{\mathrm{c}}(z)/x}$

(e.g., see Sunyaev & Zeldovich, 1970, ApSS, 7, 20; Hu 1995, PhD Thesis)

- *Use μ(x, z) to estimate the total photon production rate at low frequencies ⇒ determines at which rate μ_0 reduces*

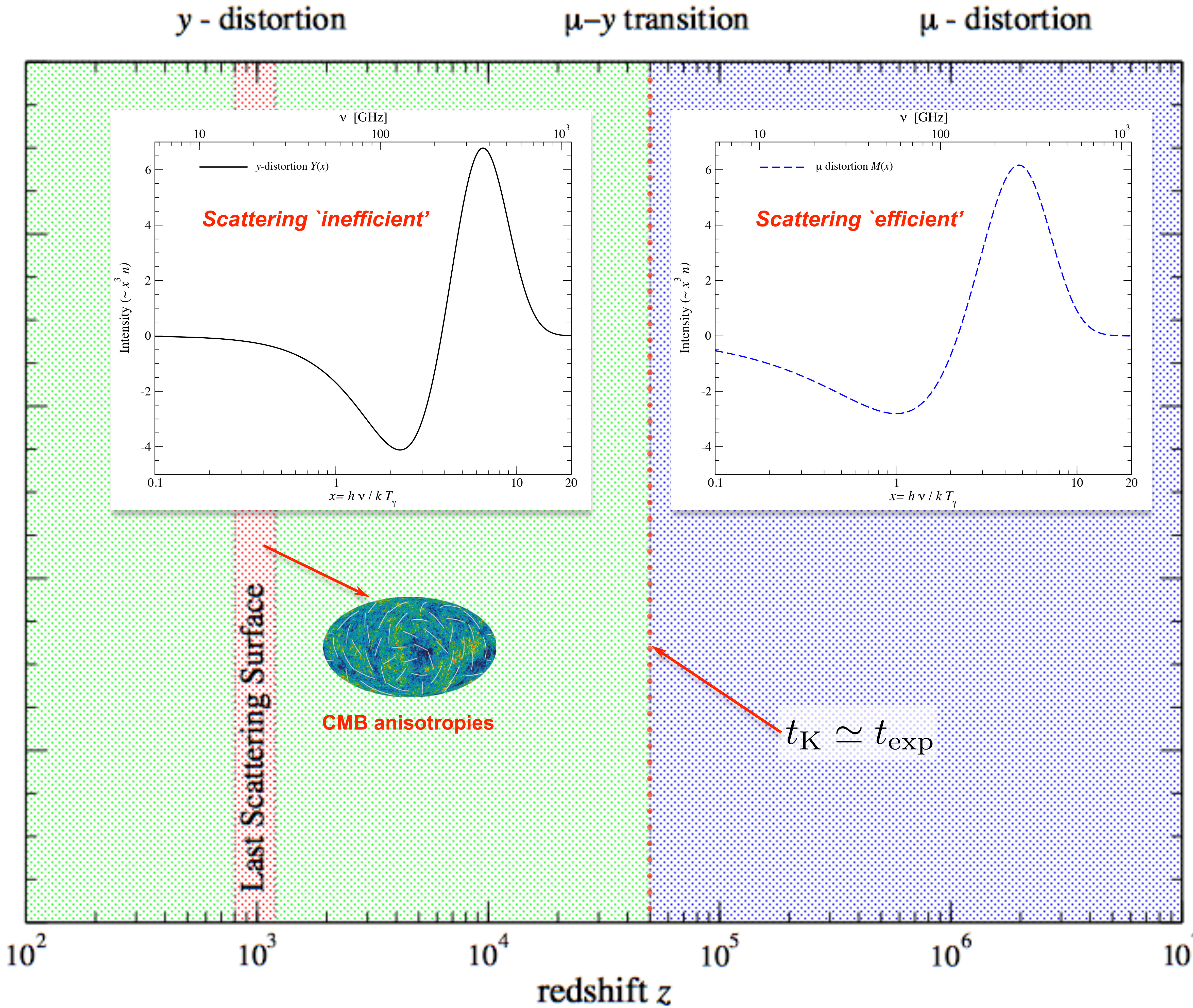
$$\mu_0 \approx 1.401\,\frac{\Delta\rho_\gamma}{\rho_\gamma} \quad \Longrightarrow \quad \mu_0 \approx 1.4 \int_{z_{\mathrm{K}}}^{\infty} \frac{\mathrm{d}(Q/\rho_\gamma)}{\mathrm{d}z'} \mathcal{J}_\mu(z')\mathrm{d}z'$$

Set by DC process

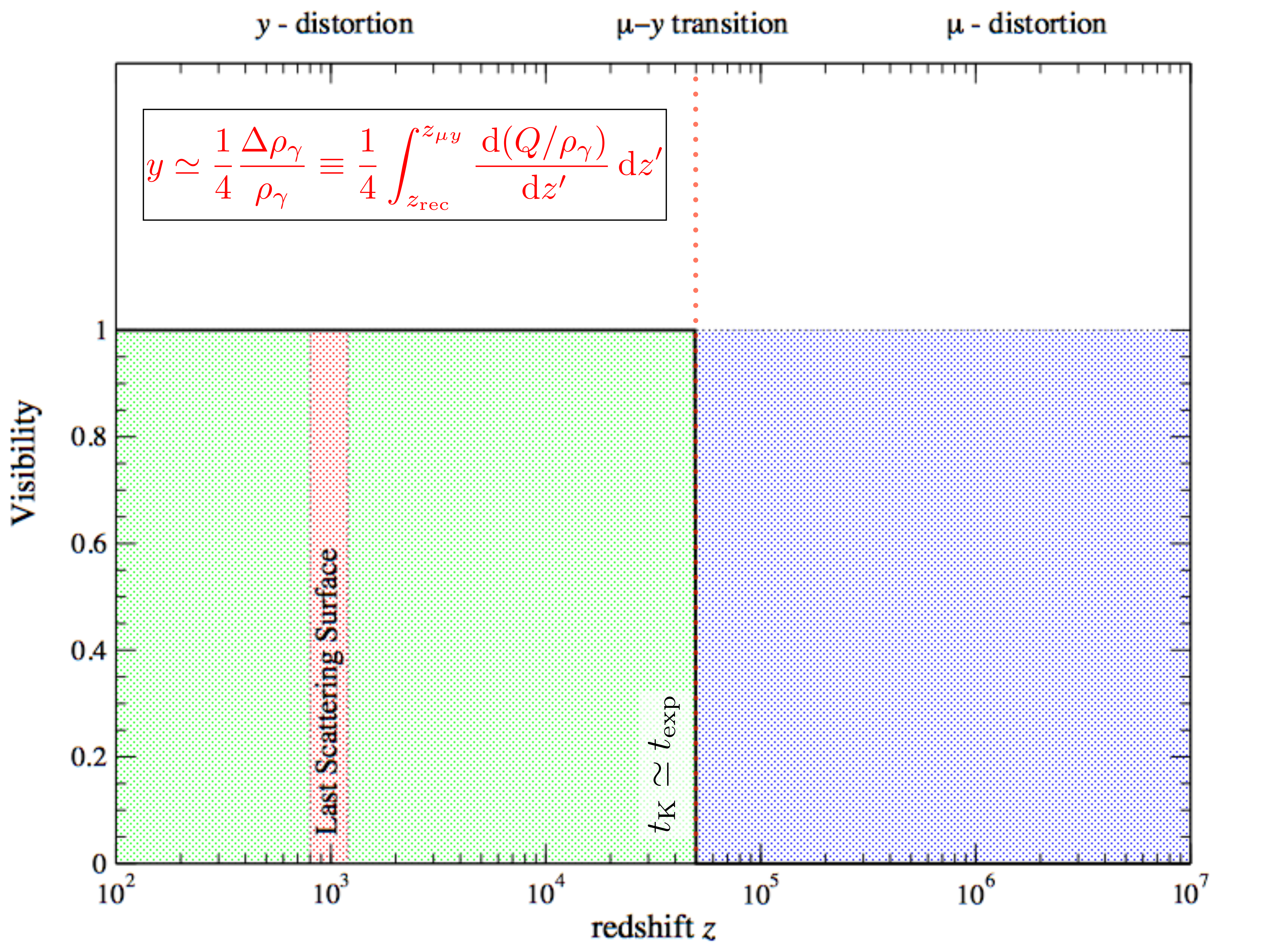
- *μ-distortion visibility function:* $\mathcal{J}_\mu(z) \approx \mathrm{e}^{-(z/z_\mu)^{5/2}}$ *with* $z_\mu \approx 2 \times 10^6$

- *Transition between μ and y modeled as simple step function*

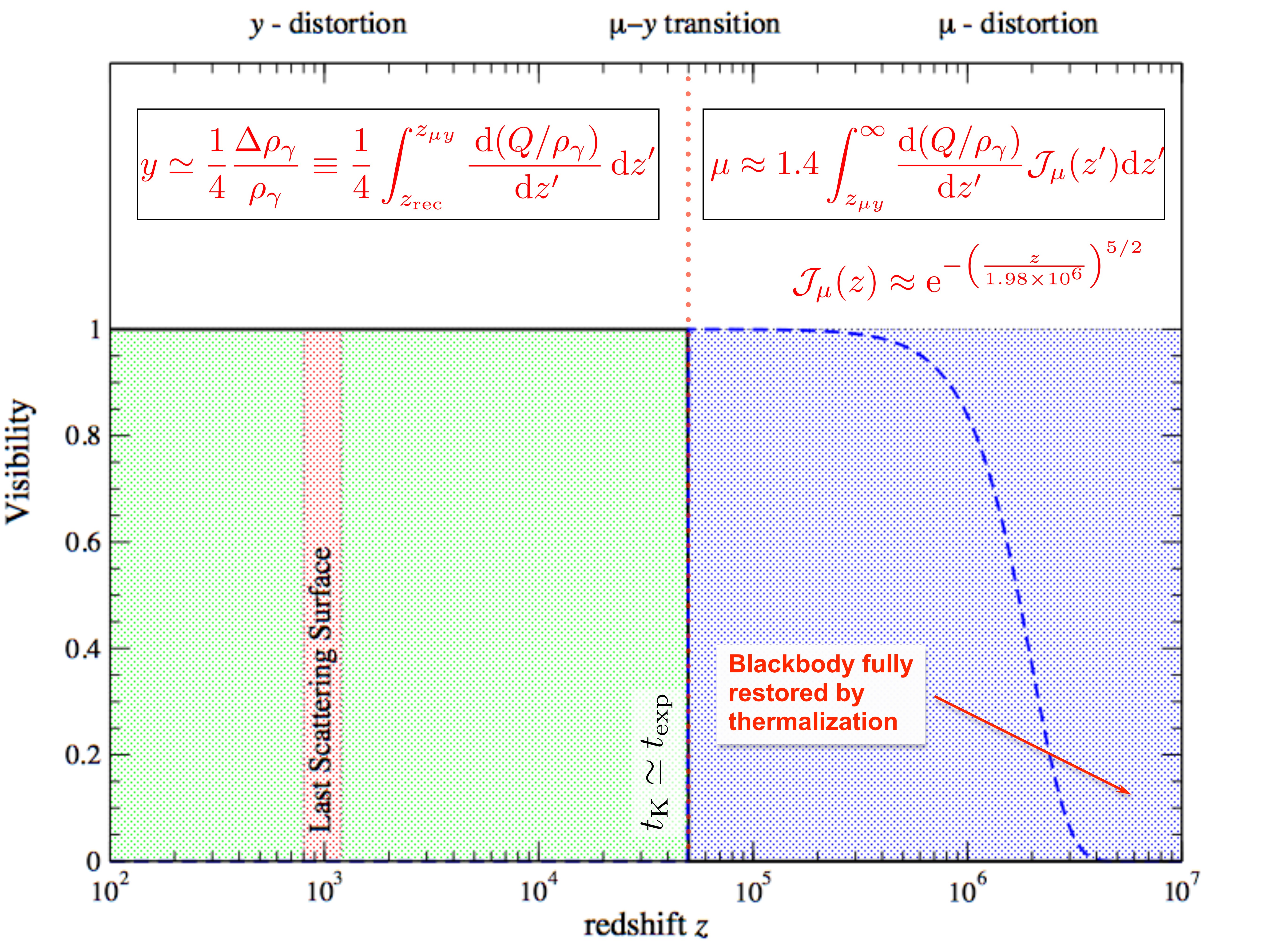
Classical approximations for μ and y



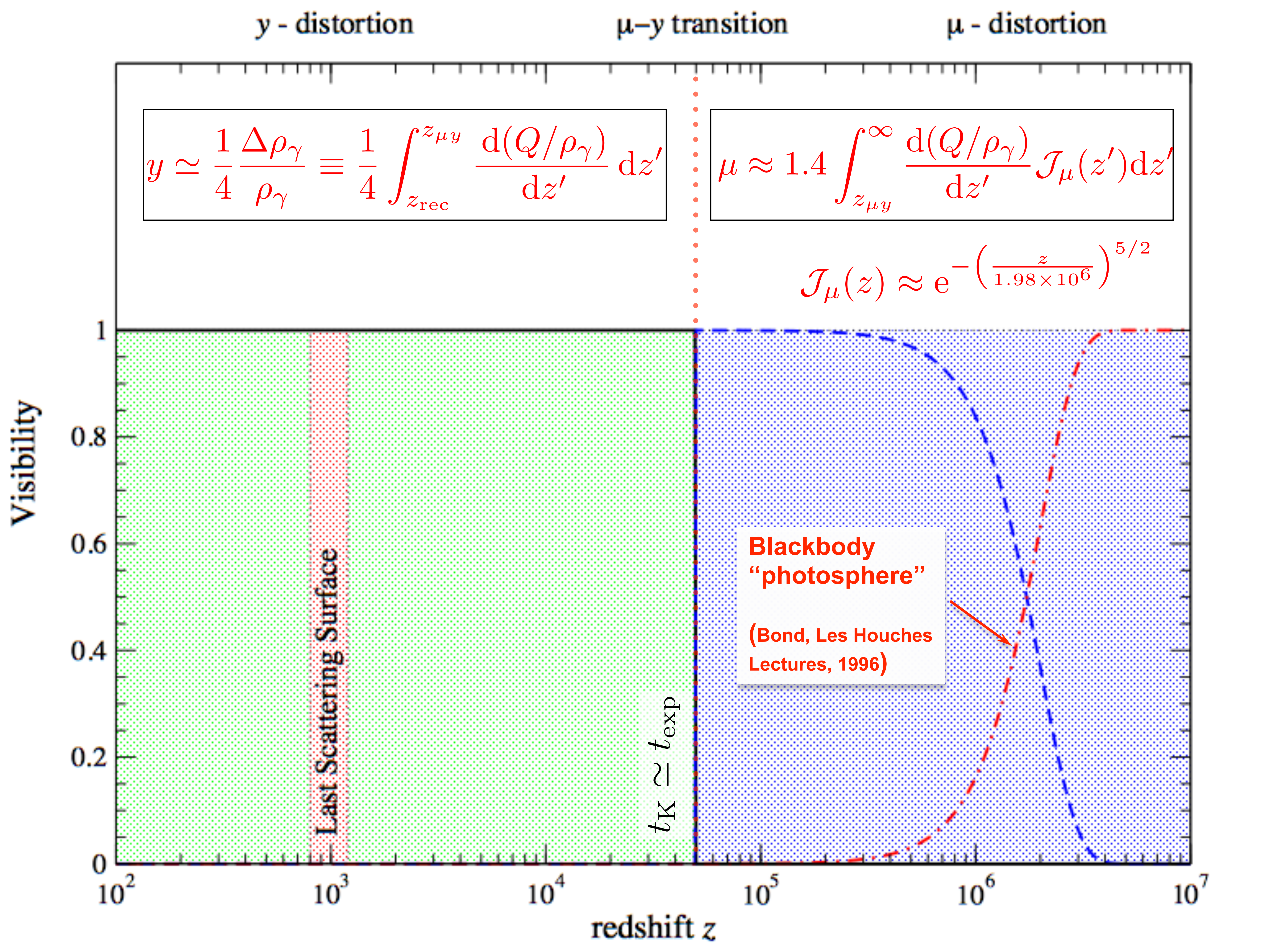
y - distortion
μ–y transition
μ - distortion
ν [GHz]
y-distortion Y(x)
Scattering `inefficient'
Intensity (~ x^3 n)
x= h ν / k T_γ
μ distortion M(x)
Scattering `efficient'
Last Scattering Surface
CMB anisotropies
$t_K \simeq t_{exp}$
redshift z



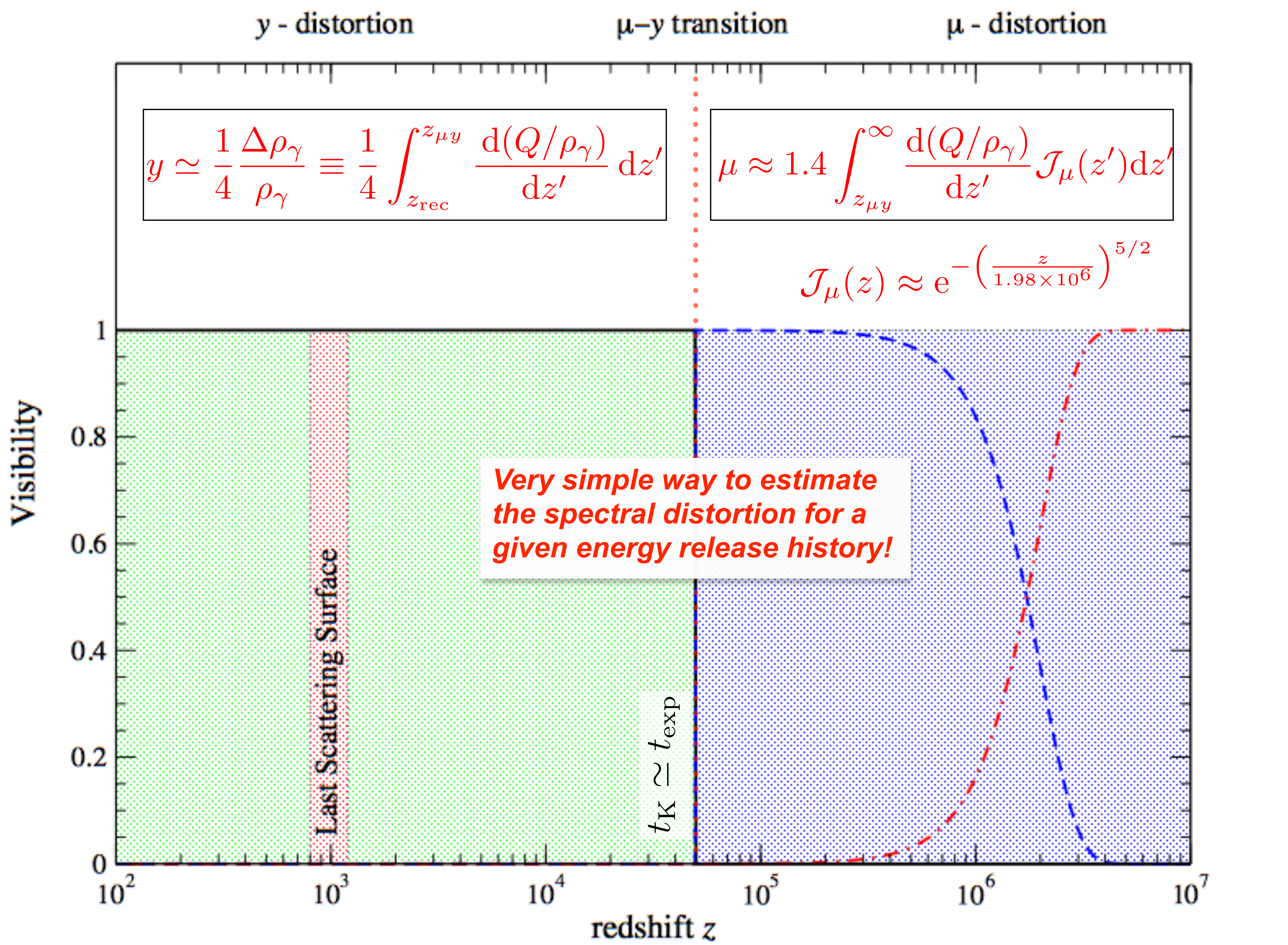
y - distortion
μ–y transition
μ - distortion
$y \simeq \frac{1}{4}\frac{\Delta\rho_\gamma}{\rho_\gamma} \equiv \frac{1}{4}\int_{z_{\rm rec}}^{z_{\mu y}} \frac{\mathrm{d}(Q/\rho_\gamma)}{\mathrm{d}z'}\,\mathrm{d}z'$
Visibility
0
0.2
0.4
0.6
0.8
1
Last Scattering Surface
$t_{\rm K} \simeq t_{\rm exp}$
10^2
10^3
10^4
10^5
10^6
10^7
redshift z



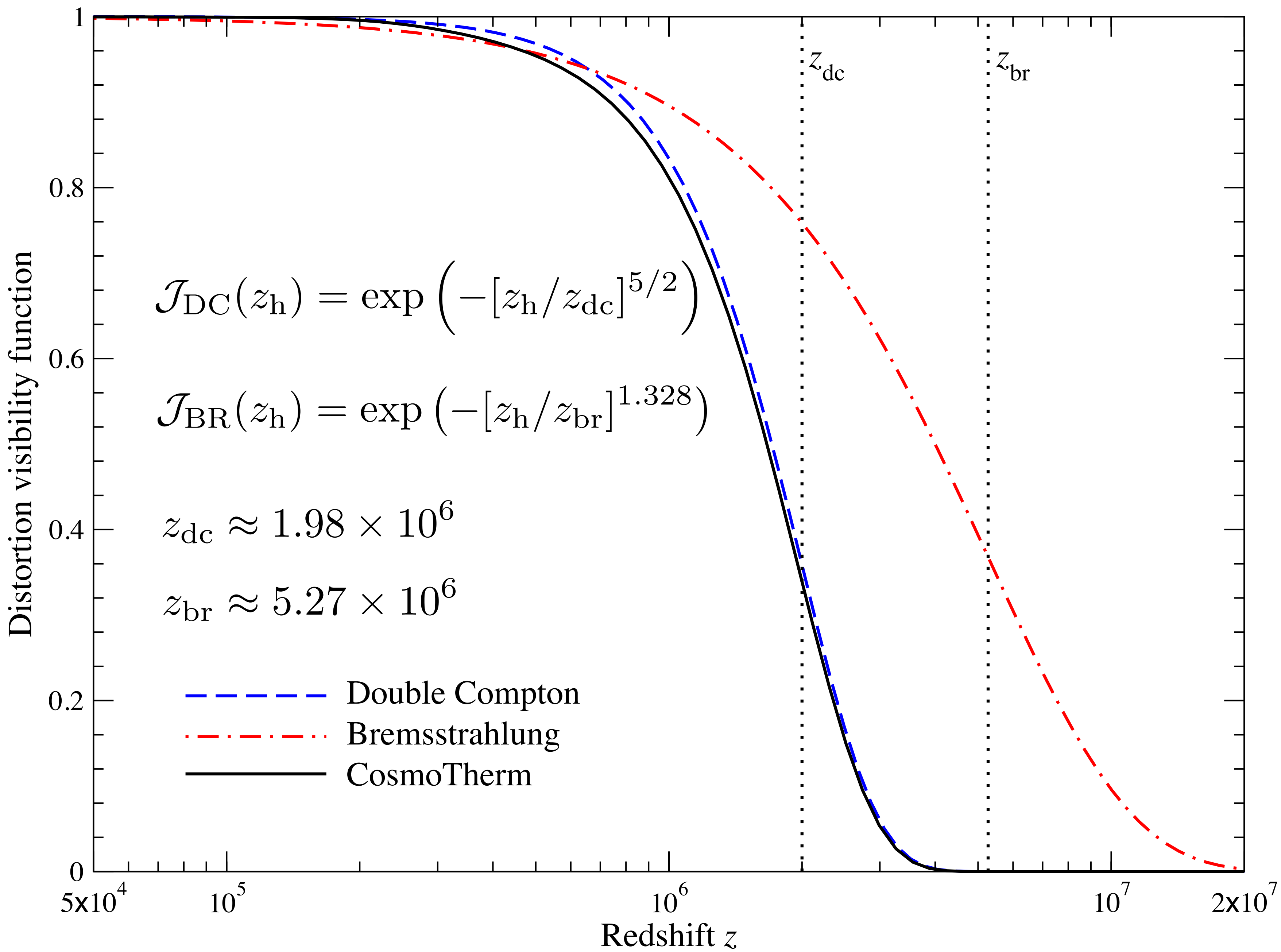
y - distortion
μ–y transition
μ - distortion
$y \simeq \frac{1}{4}\frac{\Delta\rho_\gamma}{\rho_\gamma} \equiv \frac{1}{4}\int_{z_{\rm rec}}^{z_{\mu y}} \frac{{\rm d}(Q/\rho_\gamma)}{{\rm d}z'}\,{\rm d}z'$
$\mu \approx 1.4 \int_{z_{\mu y}}^{\infty} \frac{{\rm d}(Q/\rho_\gamma)}{{\rm d}z'} \mathcal{J}_\mu(z'){\rm d}z'$
$\mathcal{J}_\mu(z) \approx {\rm e}^{-\left(\frac{z}{1.98\times10^6}\right)^{5/2}}$
Visibility
1
0.8
0.6
0.4
0.2
0
Last Scattering Surface
$t_{\rm K} \simeq t_{\rm exp}$
Blackbody fully restored by thermalization
10^2
10^3
10^4
10^5
10^6
10^7
redshift z



y - distortion
μ–y transition
μ - distortion
$y \simeq \frac{1}{4}\frac{\Delta\rho_\gamma}{\rho_\gamma} \equiv \frac{1}{4}\int_{z_{\rm rec}}^{z_{\mu y}} \frac{d(Q/\rho_\gamma)}{dz'}\,dz'$
$\mu \approx 1.4 \int_{z_{\mu y}}^{\infty} \frac{d(Q/\rho_\gamma)}{dz'} \mathcal{J}_\mu(z')dz'$
$\mathcal{J}_\mu(z) \approx e^{-\left(\frac{z}{1.98\times10^6}\right)^{5/2}}$
Last Scattering Surface
$t_K \simeq t_{\rm exp}$
Blackbody "photosphere"
(Bond, Les Houches Lectures, 1996)
Visibility
redshift z

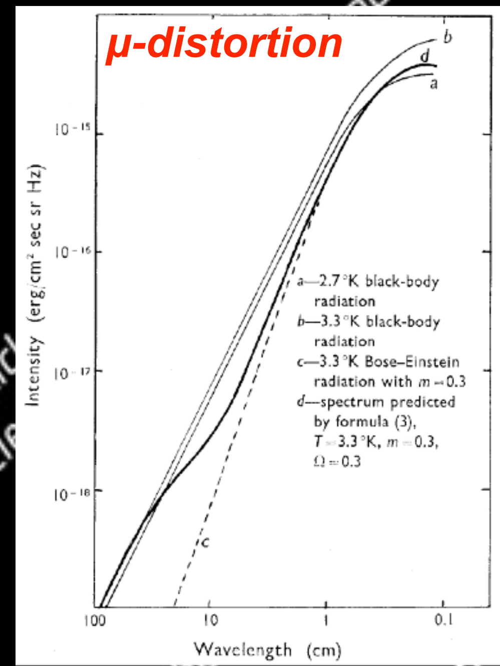

y - distortion
μ–y transition
μ - distortion
$y \simeq \frac{1}{4}\frac{\Delta\rho_\gamma}{\rho_\gamma} \equiv \frac{1}{4}\int_{z_{\rm rec}}^{z_{\mu y}} \frac{\mathrm{d}(Q/\rho_\gamma)}{\mathrm{d}z'}\,\mathrm{d}z'$
$\mu \approx 1.4\int_{z_{\mu y}}^{\infty} \frac{\mathrm{d}(Q/\rho_\gamma)}{\mathrm{d}z'}\mathcal{J}_\mu(z')\mathrm{d}z'$
$\mathcal{J}_\mu(z) \approx \mathrm{e}^{-\left(\frac{z}{1.98\times10^6}\right)^{5/2}}$
Very simple way to estimate
the spectral distortion for a
given energy release history!
Last Scattering Surface
$t_{\rm K} \simeq t_{\rm exp}$
Visibility
redshift z
1
0.8
0.6
0.4
0.2
0
10^2
10^3
10^4
10^5
10^6
10^7

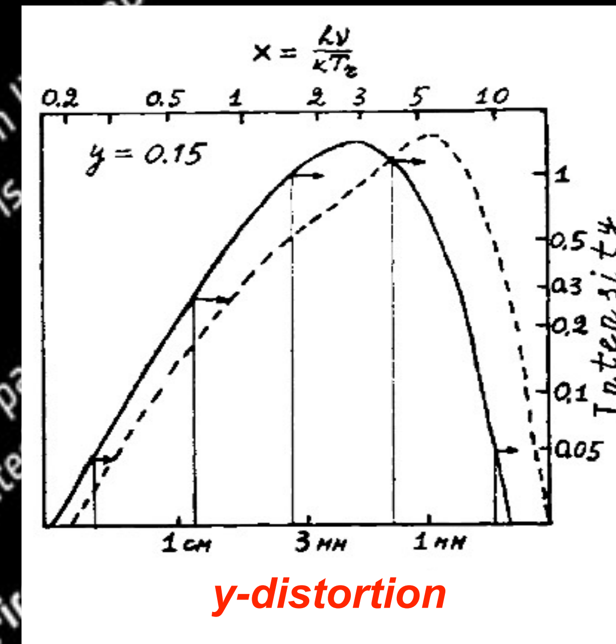
Distortion visibility for BR and DC

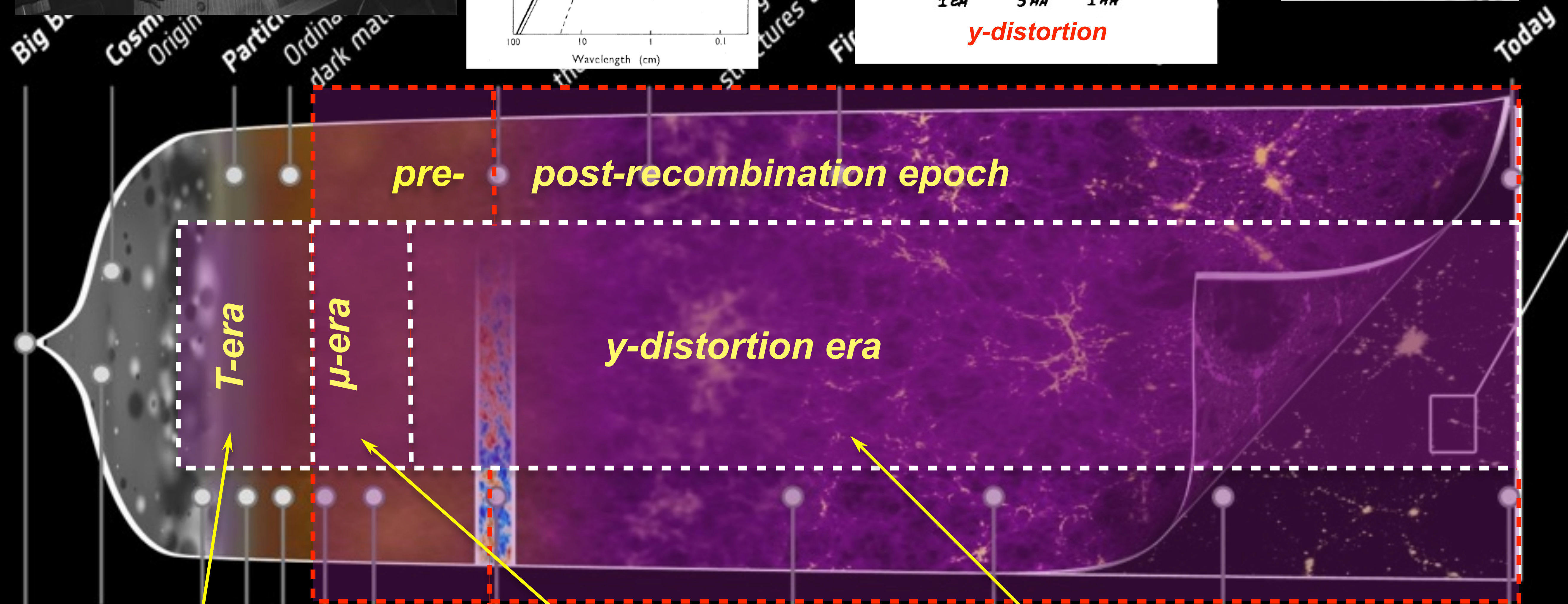


- Original estimates only included the effect of BR
- Double Compton emission was first included by Danese & de Zotti, 1982
- DC changes the distortion visibility quite strongly

Double Compton emission is really crucial !!!







$$\frac{\Delta T}{T} \simeq \frac{1}{4}\left.\frac{\Delta\rho_\gamma}{\rho_\gamma}\right|_T \qquad \mu \simeq 1.4\left.\frac{\Delta\rho_\gamma}{\rho_\gamma}\right|_\mu \qquad y \simeq \frac{1}{4}\left.\frac{\Delta\rho_\gamma}{\rho_\gamma}\right|_y$$

What about the μ-y transition regime?

Is the transition really as abrupt?

Quasi-Exact Treatment of the Thermalization Problem

- *For real forecasts of future prospects a precise & fast method for computing the spectral distortion is needed!*
- *Case-by-case computation of the distortion (e.g., with CosmoTherm, JC & Sunyaev, 2012, ArXiv:1109.6552) still rather time-consuming*
- *But: distortions are small ⇒ thermalization problem becomes linear!*
- *Simple solution: compute "response function" of the thermalization problem ⇒ Green's function approach (JC, 2013, ArXiv:1304.6120)*
- *Final distortion for fixed energy-release history given by*

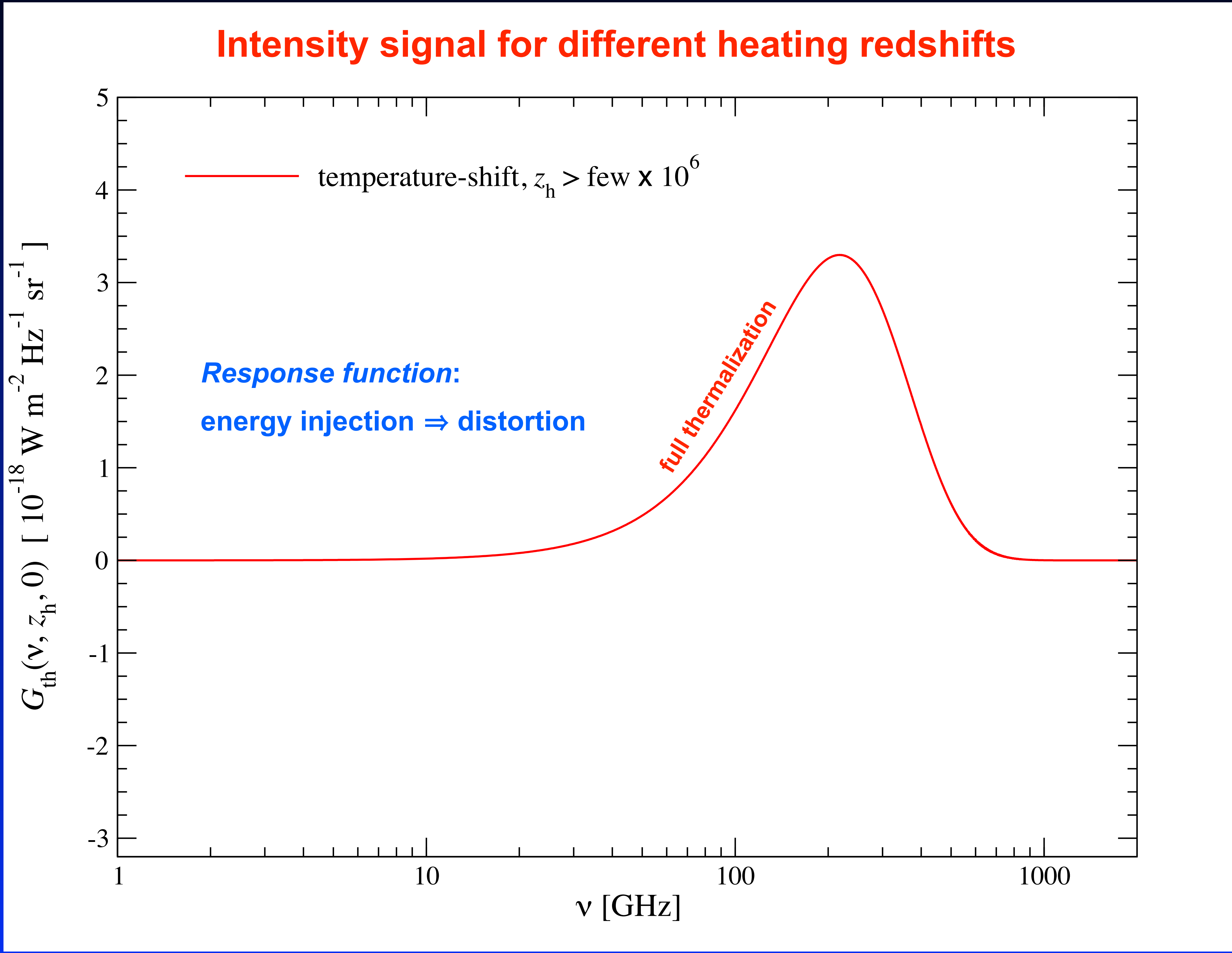
$$\Delta I_\nu \approx \int_0^\infty G_{\rm th}(\nu, z') \frac{\mathrm{d}(Q/\rho_\gamma)}{\mathrm{d}z'} \mathrm{d}z'$$

Thermalization Green's function

- *Fast and quasi-exact! No additional approximations!*

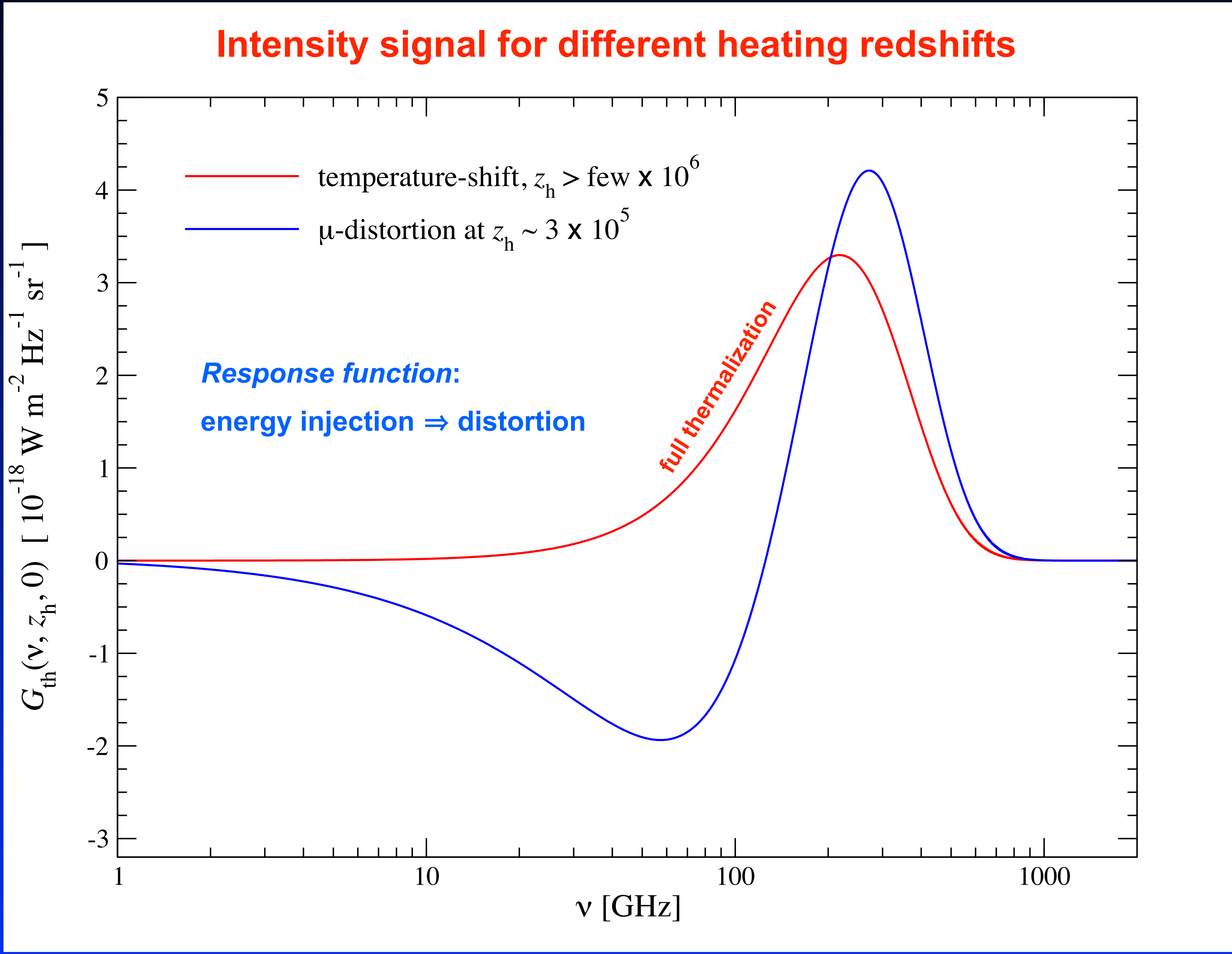
CosmoTherm available at: www.Chluba.de/CosmoTherm

What does the spectrum look like after energy injection?



JC & Sunyaev, 2012, ArXiv:1109.6552
JC, 2013, ArXiv:1304.6120

What does the spectrum look like after energy injection?



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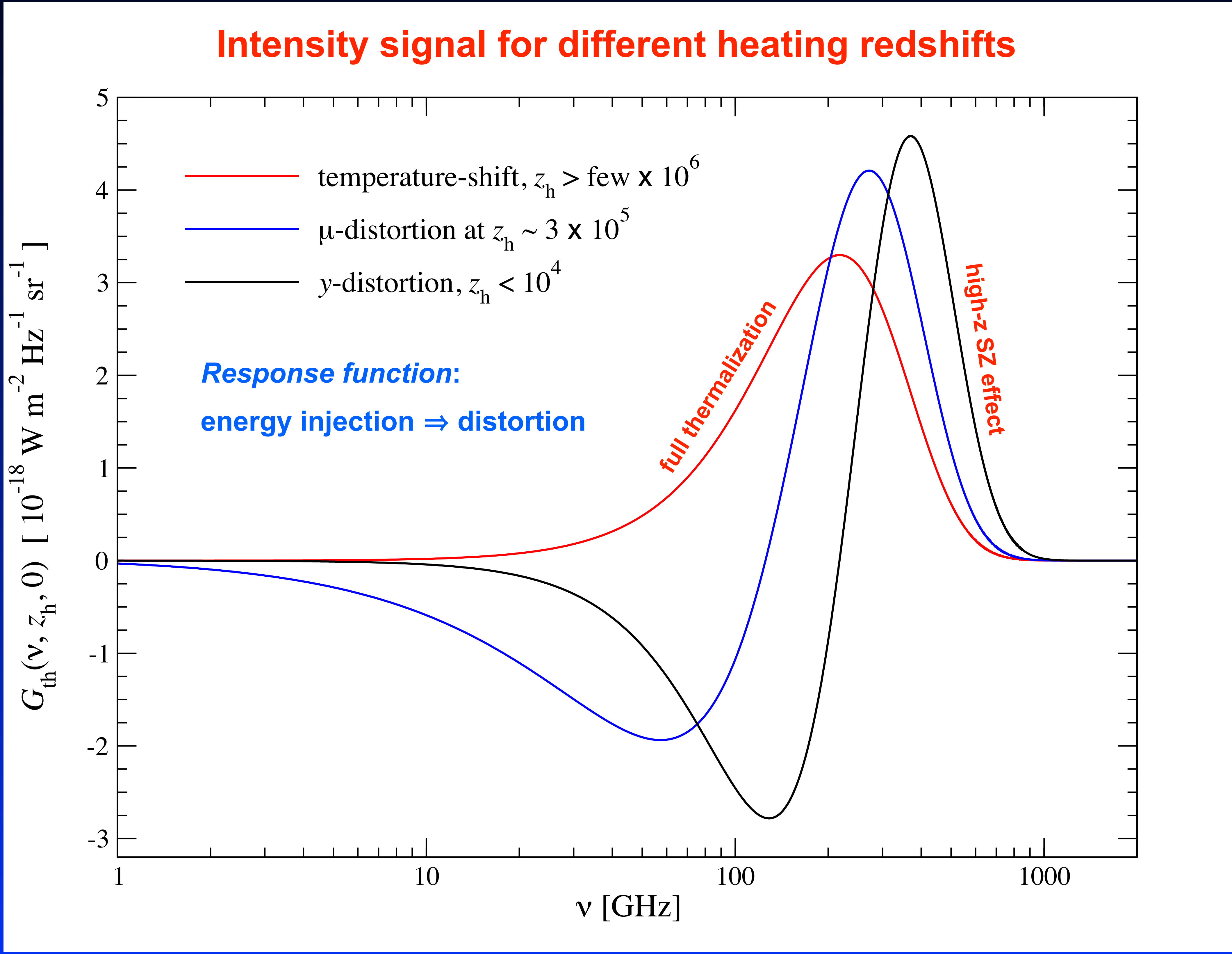

JC & Sunyaev, 2012, ArXiv:1109.6552
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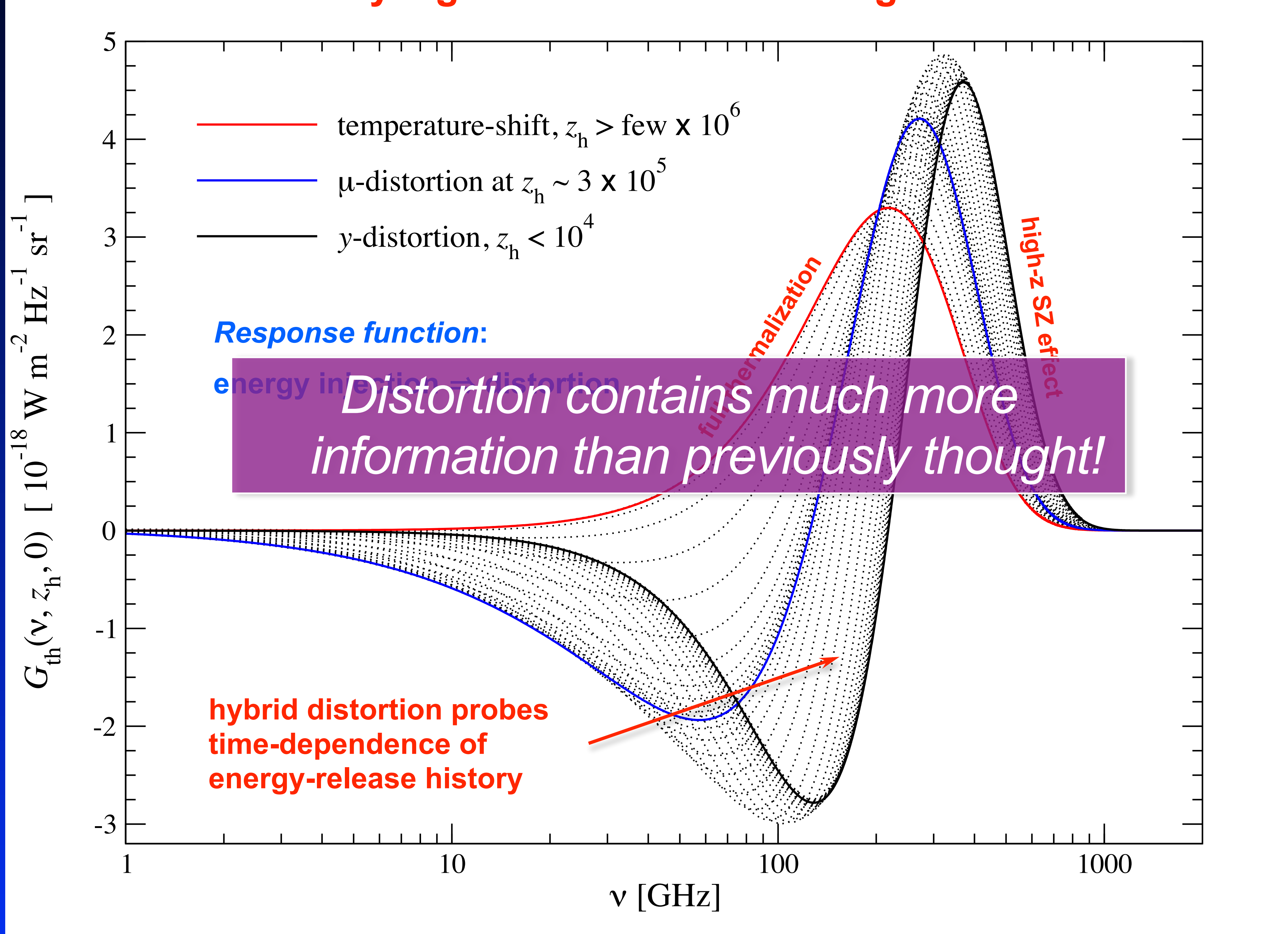
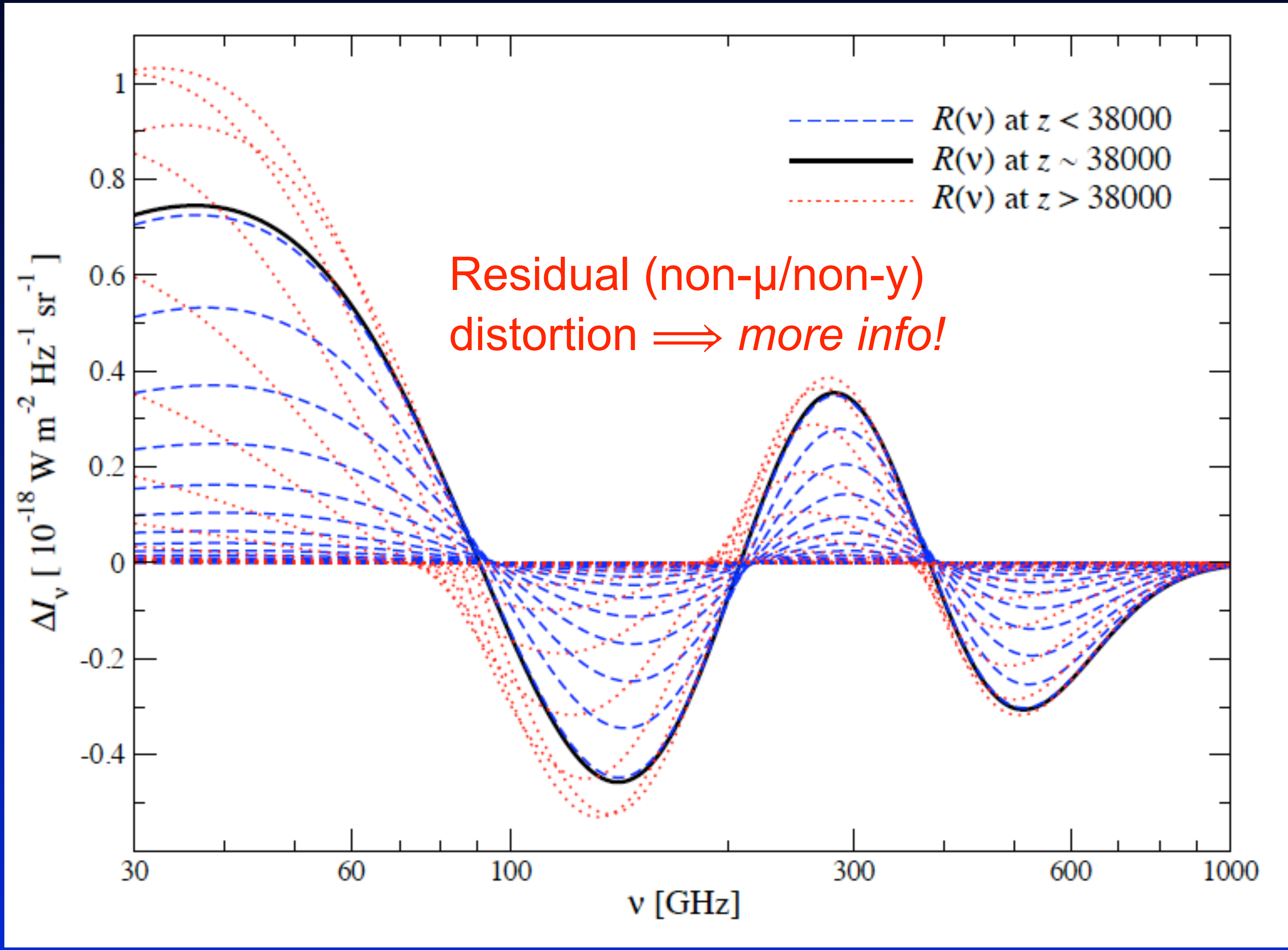
What does the spectrum look like after energy injection?



Distortion contains much more information than previously thought!

JC & Sunyaev, 2012, ArXiv:1109.6552
JC, 2013, ArXiv:1304.6120

Explicitly taking out the superposition of *T*, μ & y distortion



- *Allows us to distinguish different energy release scenarios!*

JC & Sunyaev, 2012, ArXiv:1109.6552
JC, 2013, ArXiv:1304.6120; JC, 2013, ArXiv:1304.6121; JC & Jeong, 2013

Transition from y-distortion → μ-distortion

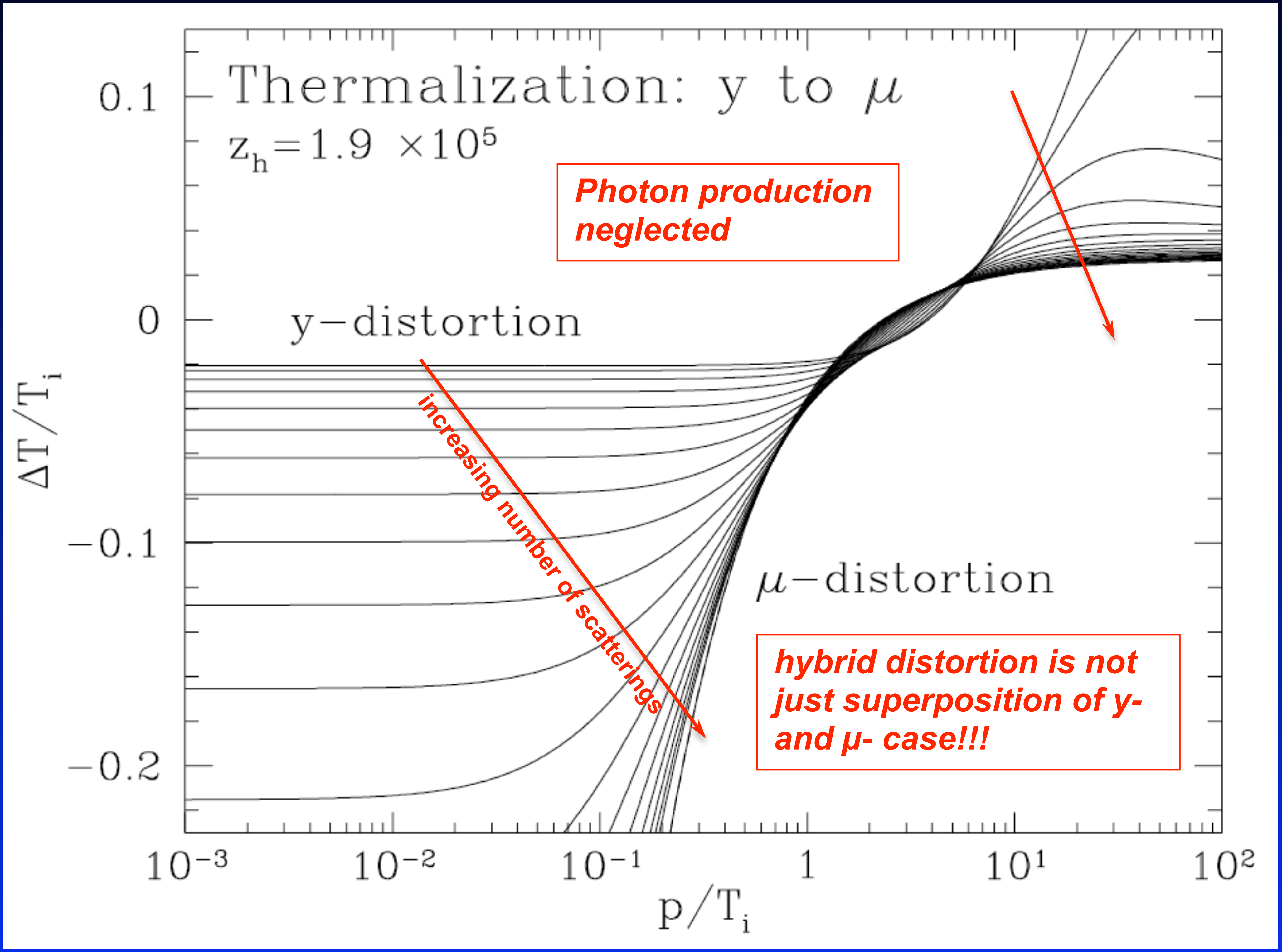
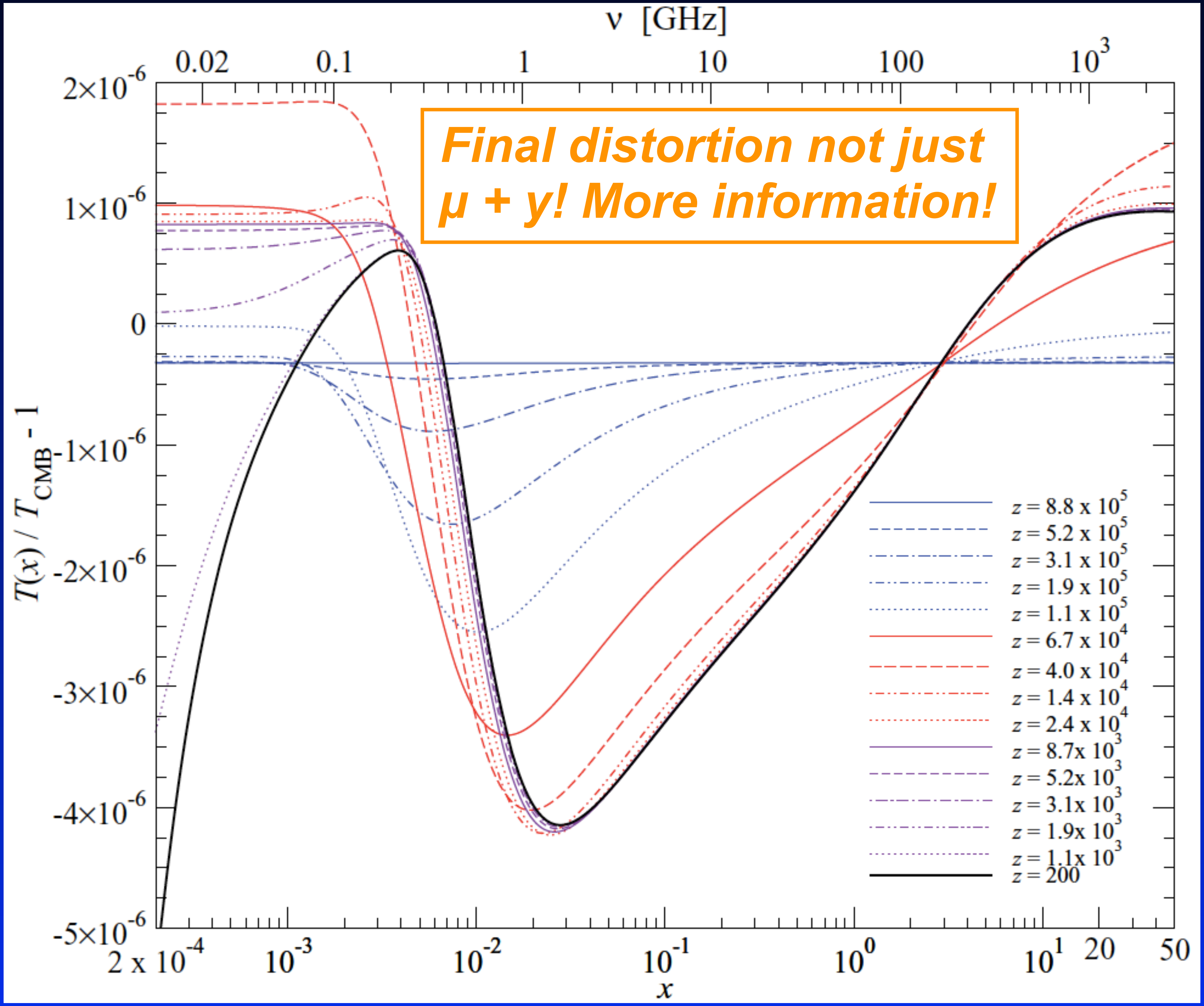


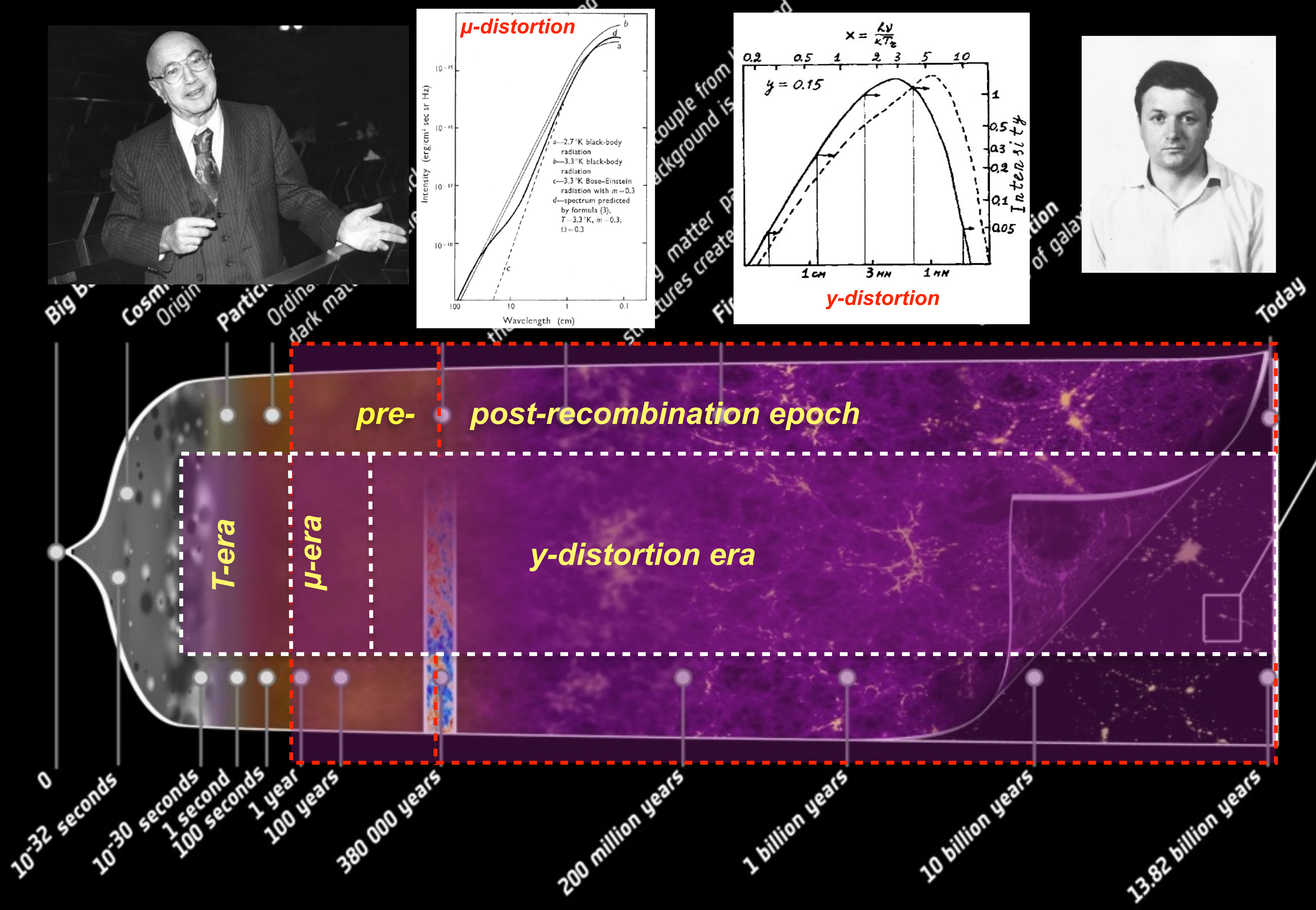
Figure from Wayne Hu's PhD thesis, 1995, but see also discussion in Burigana, 1991

Distortion *not* just superposition of μ and y-distortion!

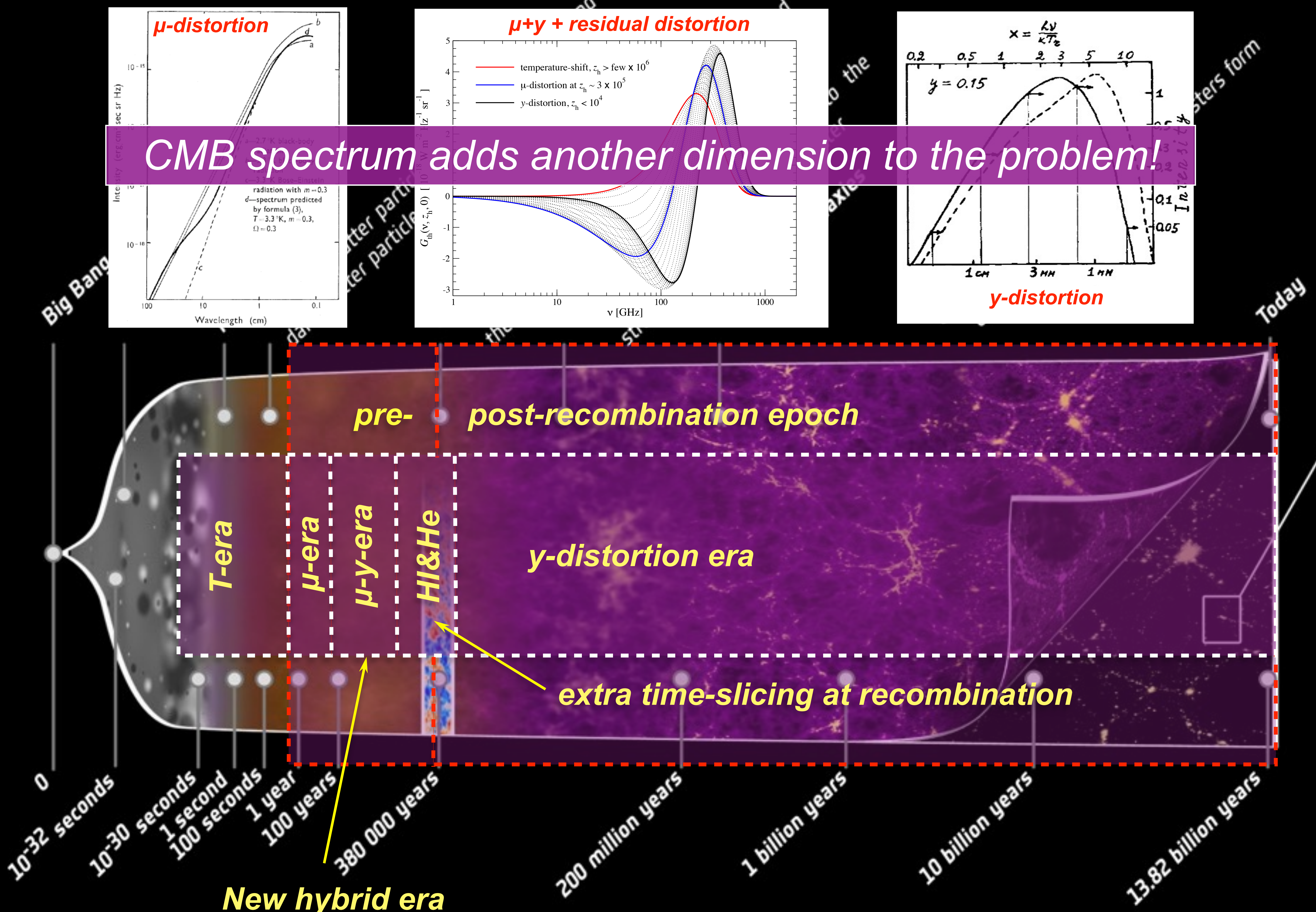


Computation carried out with *CosmoTherm* (JC & Sunyaev 2011)

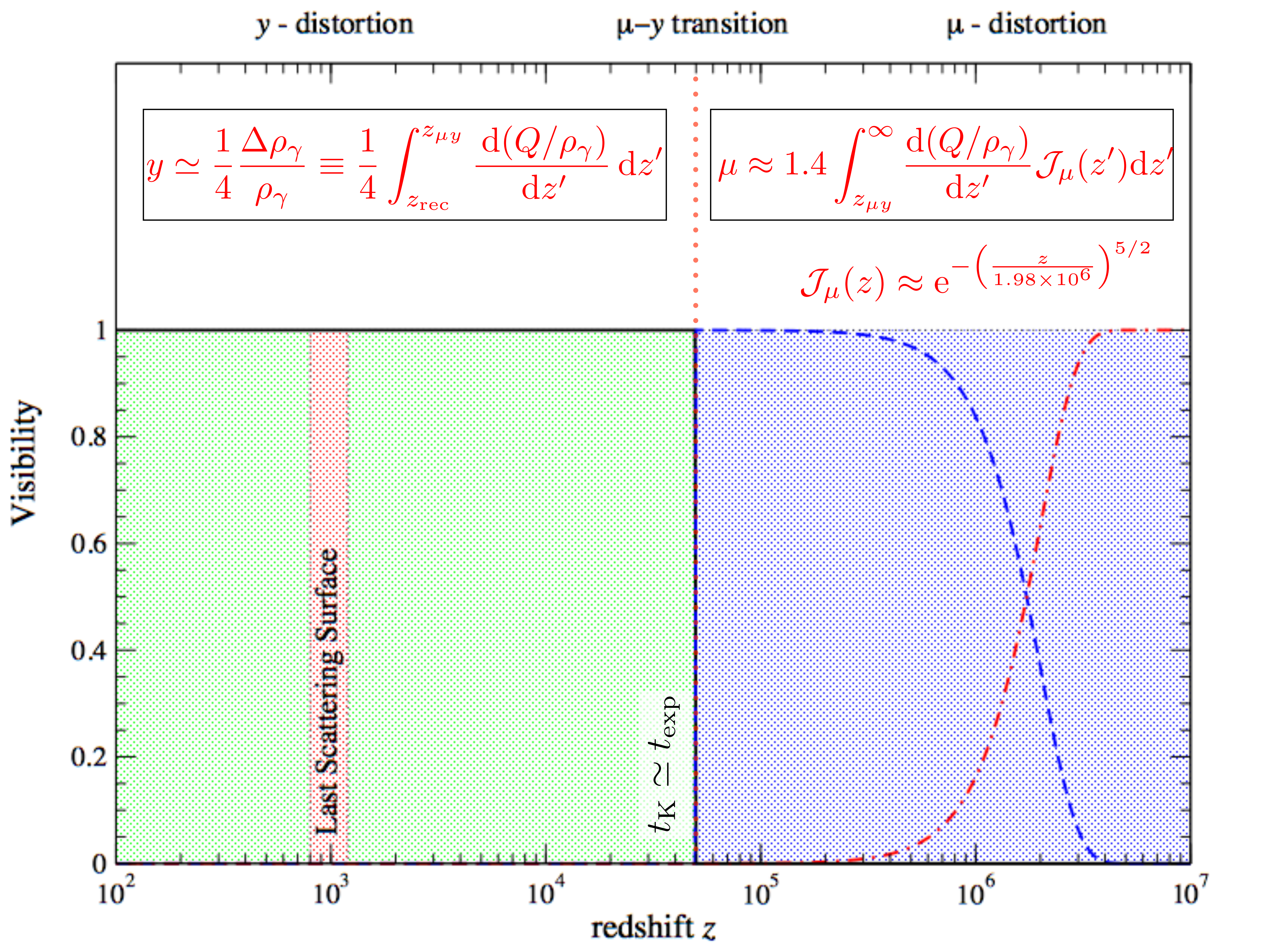
First explicit calculation that showed that there is more!

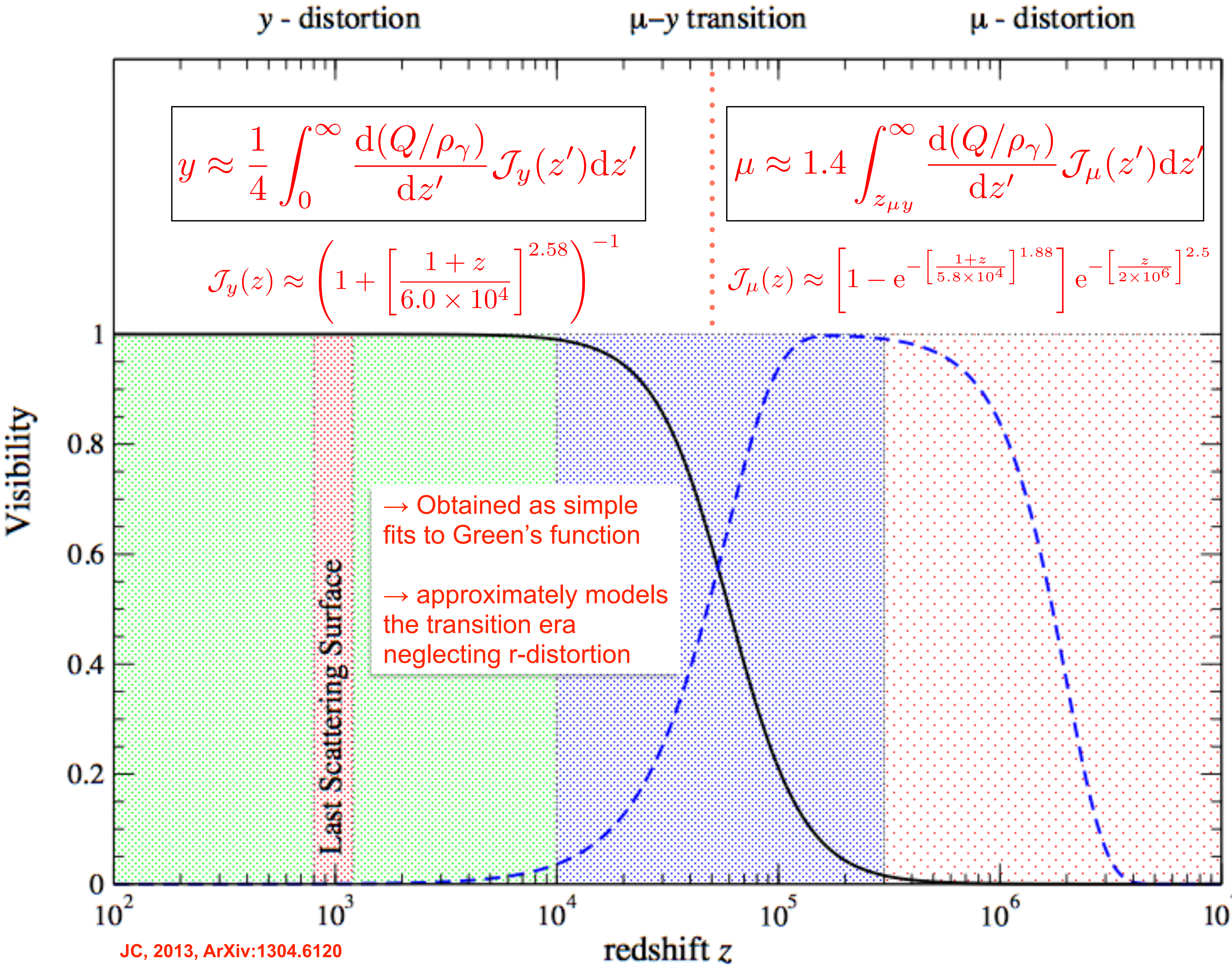


μ-distortion
Intensity (erg/cm² sec sr Hz)
Wavelength (cm)
a—2.7 °K black-body radiation
b—3.3 °K black-body radiation
c—3.3 °K Bose-Einstein radiation with m = 0.3
d—spectrum predicted by formula (3), T = 3.3 °K, m = 0.3, Ω = 0.3
$x = \frac{h\nu}{kT_e}$
y = 0.15
Intensity
1 см
3 мм
1 мм
y-distortion
Today
pre-
post-recombination epoch
T-era
μ-era
y-distortion era
0
10^{-32} seconds
10^{-30} seconds
1 second
100 seconds
1 year
100 years
380 000 years
200 million years
1 billion years
10 billion years
13.82 billion years

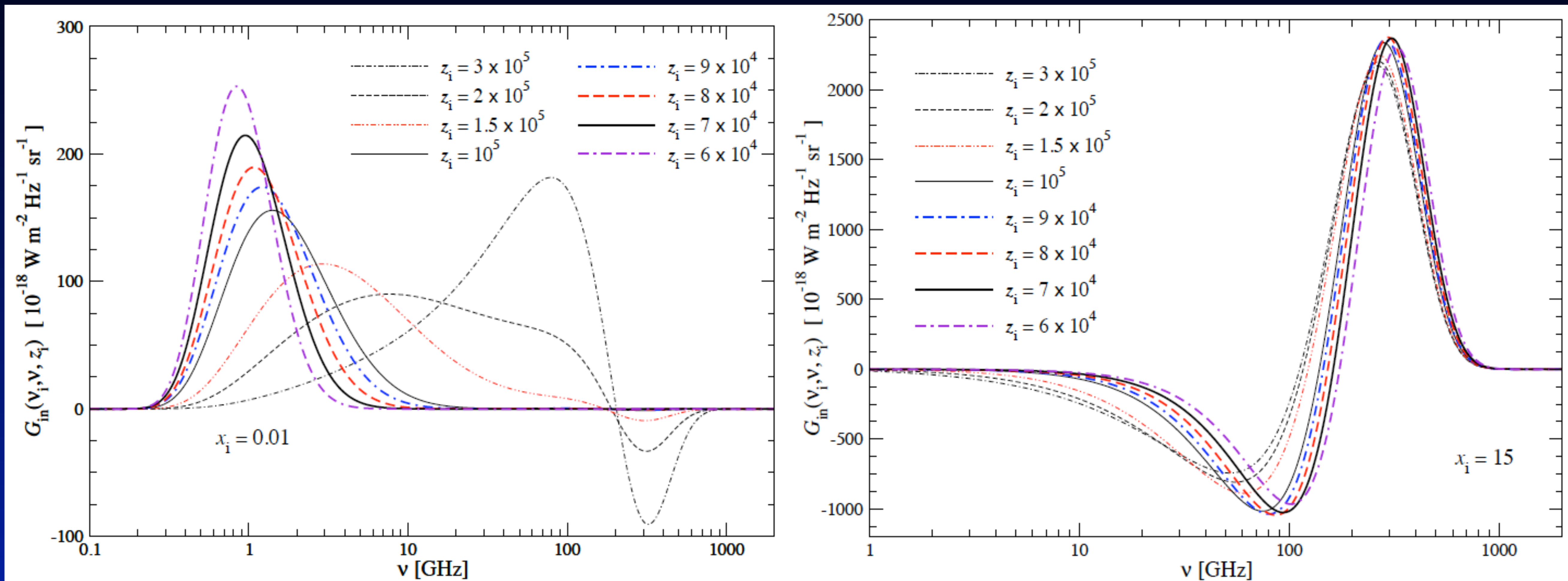


CMB spectrum adds another dimension to the problem!
μ-distortion
μ+y + residual distortion
y-distortion
temperature-shift, z_h > few x 10^6
μ-distortion at $z_h \sim 3 \times 10^5$
y-distortion, $z_h < 10^4$
ν [GHz]
Wavelength (cm)
y = 0.15
Big Bang
Today
pre-
post-recombination epoch
T-era
μ-era
μ-y-era
HI&He
y-distortion era
extra time-slicing at recombination
New hybrid era
0
10^{-32} seconds
10^{-30} seconds
1 second
100 seconds
1 year
100 years
380 000 years
200 million years
1 billion years
10 billion years
13.82 billion years


y - distortion
μ–y transition
μ - distortion
$y \simeq \frac{1}{4}\frac{\Delta\rho_\gamma}{\rho_\gamma} \equiv \frac{1}{4}\int_{z_{\rm rec}}^{z_{\mu y}} \frac{{\rm d}(Q/\rho_\gamma)}{{\rm d}z'}\,{\rm d}z'$
$\mu \approx 1.4\int_{z_{\mu y}}^{\infty} \frac{{\rm d}(Q/\rho_\gamma)}{{\rm d}z'}\mathcal{J}_\mu(z'){\rm d}z'$
$\mathcal{J}_\mu(z) \approx {\rm e}^{-\left(\frac{z}{1.98\times 10^6}\right)^{5/2}}$
Visibility
Last Scattering Surface
$t_{\rm K} \simeq t_{\rm exp}$
redshift z
0
0.2
0.4
0.6
0.8
1
10^2
10^3
10^4
10^5
10^6
10^7


y - distortion
μ–y transition
μ - distortion
$y \approx \frac{1}{4}\int_0^\infty \frac{\mathrm{d}(Q/\rho_\gamma)}{\mathrm{d}z'}\mathcal{J}_y(z')\mathrm{d}z'$
$\mathcal{J}_y(z) \approx \left(1+\left[\frac{1+z}{6.0\times10^4}\right]^{2.58}\right)^{-1}$
$\mu \approx 1.4\int_{z_{\mu y}}^\infty \frac{\mathrm{d}(Q/\rho_\gamma)}{\mathrm{d}z'}\mathcal{J}_\mu(z')\mathrm{d}z'$
$\mathcal{J}_\mu(z) \approx \left[1-\mathrm{e}^{-\left[\frac{1+z}{5.8\times10^4}\right]^{1.88}}\right]\mathrm{e}^{-\left[\frac{z}{2\times10^6}\right]^{2.5}}$
→ Obtained as simple fits to Green's function
→ approximately models the transition era neglecting r-distortion
Last Scattering Surface
Visibility
1
0.8
0.6
0.4
0.2
0
10^2
10^3
10^4
10^5
10^6
10^7
redshift z
JC, 2013, ArXiv:1304.6120

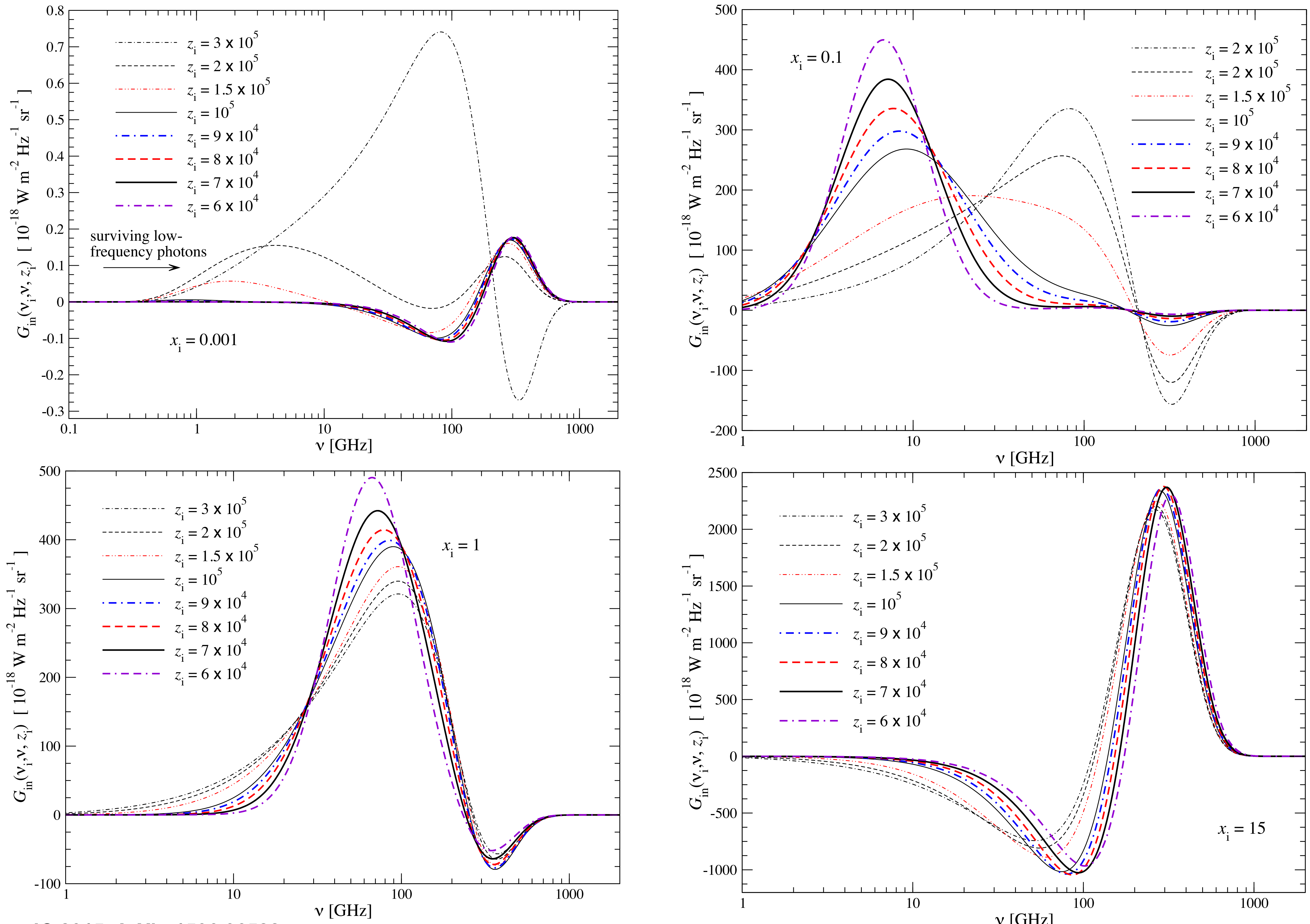
Green's function for photon injection



- Photon injection Green's function gives even richer phenomenology of distortion signals
- Depends on the details of the photon production process for redshifts z < few x 10^5
- difference between high and low frequency photon injection

JC 2015, ArXiv:1506.06582

Photon injection at later times



JC 2015, ArXiv:1506.06582

Physical mechanisms that lead to spectral distortions

Standard sources of distortions

- *Cooling by adiabatically expanding ordinary matter*
  (JC, 2005; JC & Sunyaev 2011; Khatri, Sunyaev & JC, 2011)
- Heating by *decaying* or *annihilating* relic particles
  (Kawasaki et al., 1987; Hu & Silk, 1993; McDonald et al., 2001; JC, 2005; JC & Sunyaev, 2011; JC, 2013; JC & Jeong, 2013)
- *Evaporation of primordial black holes & superconducting strings*
  (Carr et al. 2010; Ostriker & Thompson, 1987; Tashiro et al. 2012; Pani & Loeb, 2013)
- *Dissipation of primordial acoustic modes & magnetic fields*
  (Sunyaev & Zeldovich, 1970; Daly 1991; Hu et al. 1994; JC & Sunyaev, 2011; JC et al. 2012 - Jedamzik et al. 2000; Kunze & Komatsu, 2013)
- *Cosmological recombination radiation*
  (Zeldovich et al., 1968; Peebles, 1968; Dubrovich, 1977; Rubino-Martin et al., 2006; JC & Sunyaev, 2006; Sunyaev & JC, 2009)

pre-recombination epoch

„high“ redshifts

„low“ redshifts

- Signatures due to first supernovae and their remnants
  (Oh, Cooray & Kamionkowski, 2003)
- Shock waves arising due to large-scale structure formation
  (Sunyaev & Zeldovich, 1972; Cen & Ostriker, 1999)
- SZ-effect from clusters; effects of reionization
  (Refregier et al., 2003; Zhang et al. 2004; Trac et al. 2008)
- other exotic processes
  (Lochan et al. 2012; Bull & Kamionkowski, 2013; Brax et al., 2013; Tashiro et al. 2013)

post-recombination

Physical mechanisms that lead to spectral distortions

Photon injection

- *Cooling by adiabatically expanding ordinary matter*
  (JC, 2005; JC & Sunyaev 2011; Khatri, Sunyaev & JC, 2011)
- Heating by *decaying* or *annihilating* relic particles
  (Kawasaki et al., 1987; Hu & Silk, 1993; McDonald et al., 2001; JC, 2005; JC & Sunyaev, 2011; JC, 2013; JC & Jeong, 2013)
- *Evaporation of primordial black holes & superconducting strings*
  (Carr et al. 2010; Ostriker & Thompson, 1987; Tashiro et al. 2012; Pani & Loeb, 2013)
- *Dissipation of primordial acoustic modes & magnetic fields*
  (Sunyaev & Zeldovich, 1970; Daly 1991; Hu et al. 1994; JC & Sunyaev, 2011; JC et al. 2012 - Jedamzik et al. 2000; Kunze & Komatsu, 2013)
- *Cosmological recombination radiation*
  (Zeldovich et al., 1968; Peebles, 1968; Dubrovich, 1977; Rubino-Martin et al., 2006; JC & Sunyaev, 2006; Sunyaev & JC, 2009)

pre-recombination epoch

„high“ redshifts

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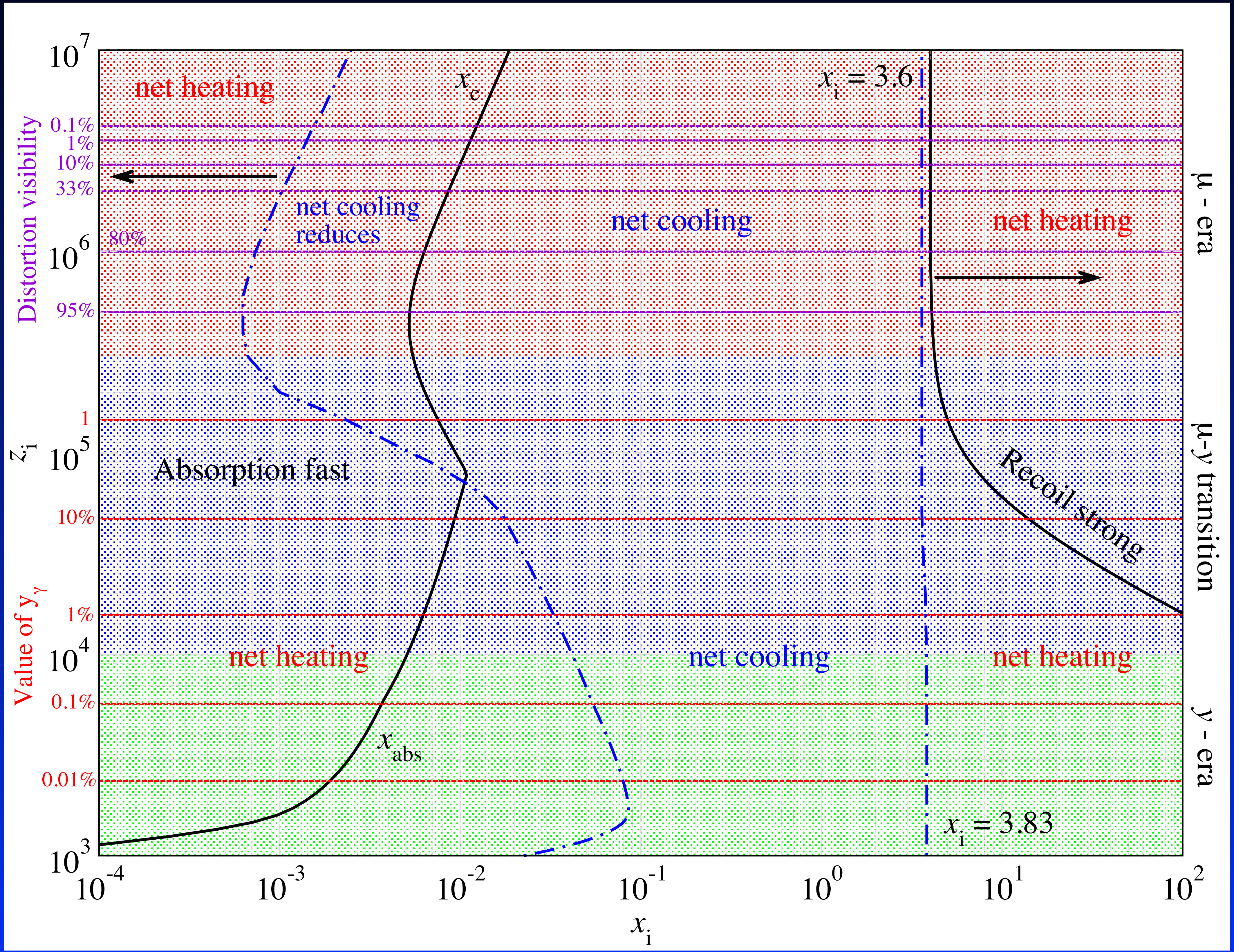
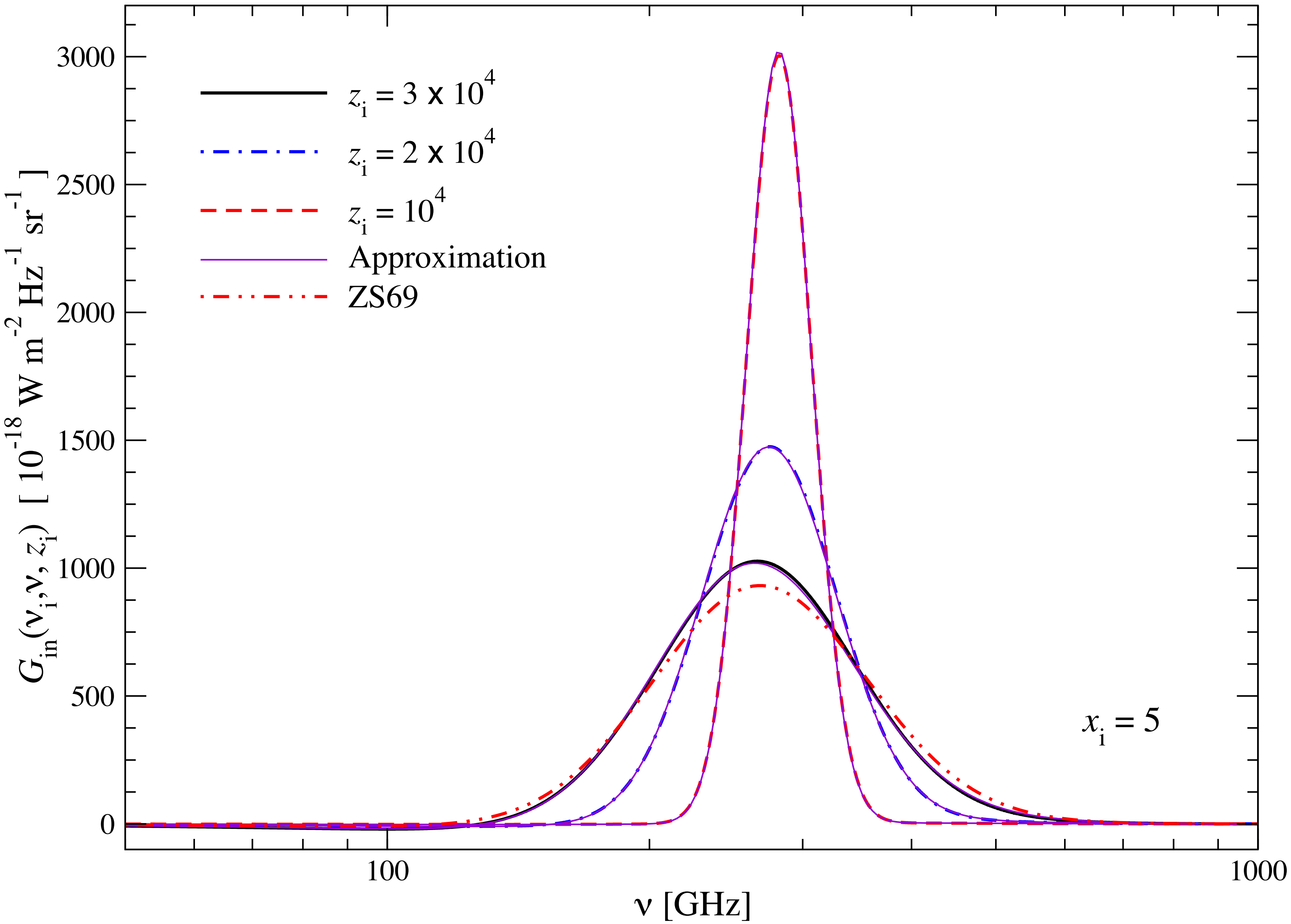
post-recombination

Different regimes for photon injection



JC 2015, ArXiv:1506.06582

Regime with very few scatterings



Classical solution by Zeldovich & Sunyaev, 1969

$$\Delta n(x, y) = \frac{A}{\sqrt{4\pi y}} \frac{\mathrm{e}^{-[\ln(x/x_{\rm i})-3y]^2/4y}}{x^3}$$

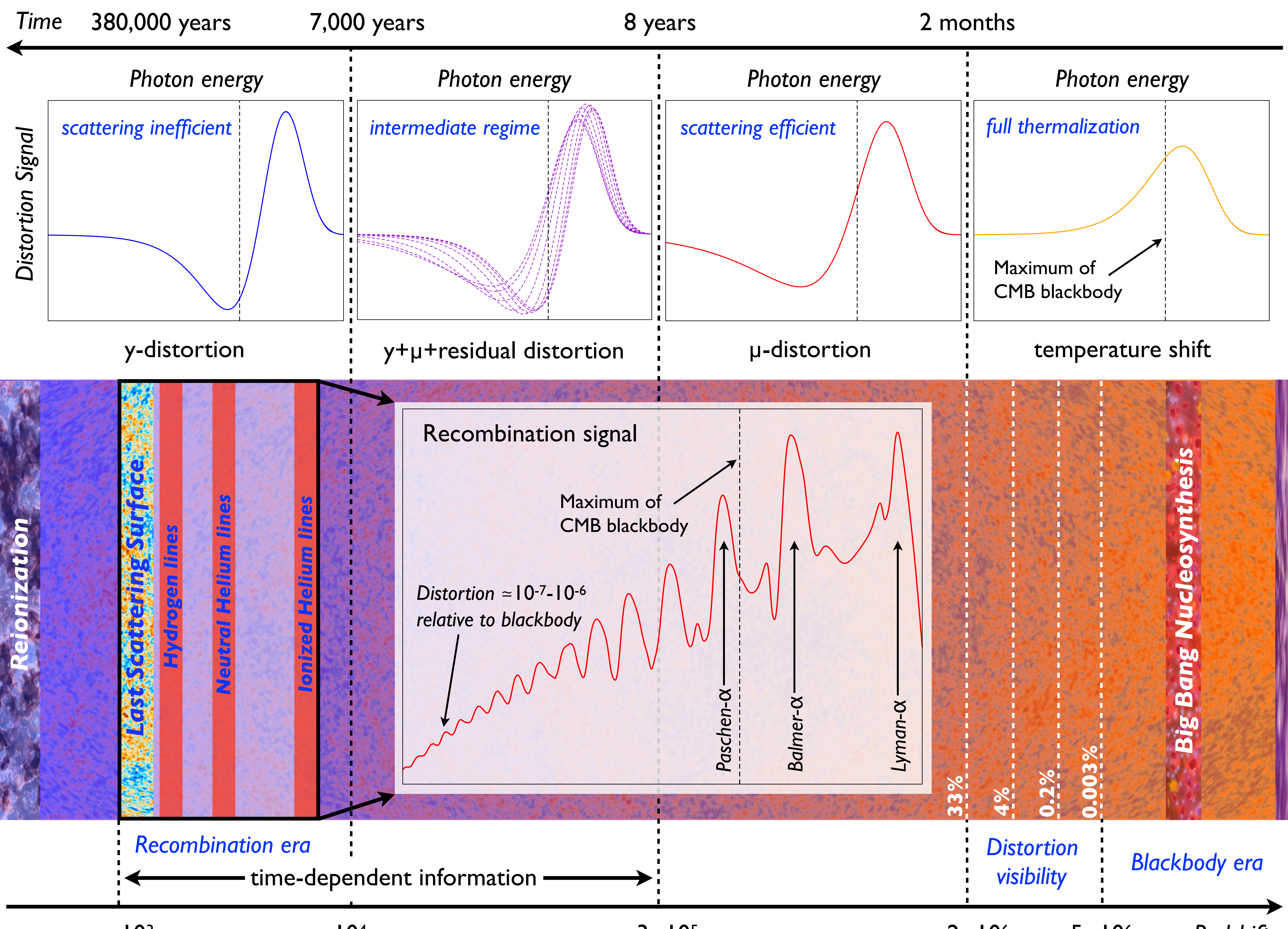
Improved solution capturing recoil and stimulated scattering

$$\Delta n^*(x, y) = \frac{A}{\sqrt{4\pi y\,\beta(x_{\rm i}, y)}} \frac{\mathrm{e}^{-\left[\ln(x/x_{\rm i})-\alpha(x_{\rm i},y)\,y+\ln(1+x_{\rm i}y)\right]^2/4y\,\beta(x_{\rm i},y)}}{x^3}$$

$$\alpha = [3 - 2f(x_{\rm i})]/\sqrt{1 + x_{\rm i}y} \qquad f(x_{\rm i}) = \mathrm{e}^{-x_{\rm i}}(1 + x_{\rm i}^2/2)$$

$$\beta = 1/[1 + x_{\rm i}y(1 - f(x_{\rm i}))]$$

JC 2015, ArXiv:1506.06582



Silk & Chluba, Science, 2014

Physical mechanisms that lead to spectral distortions

Standard sources of distortions

- *Cooling by adiabatically expanding ordinary matter*
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To be continued…