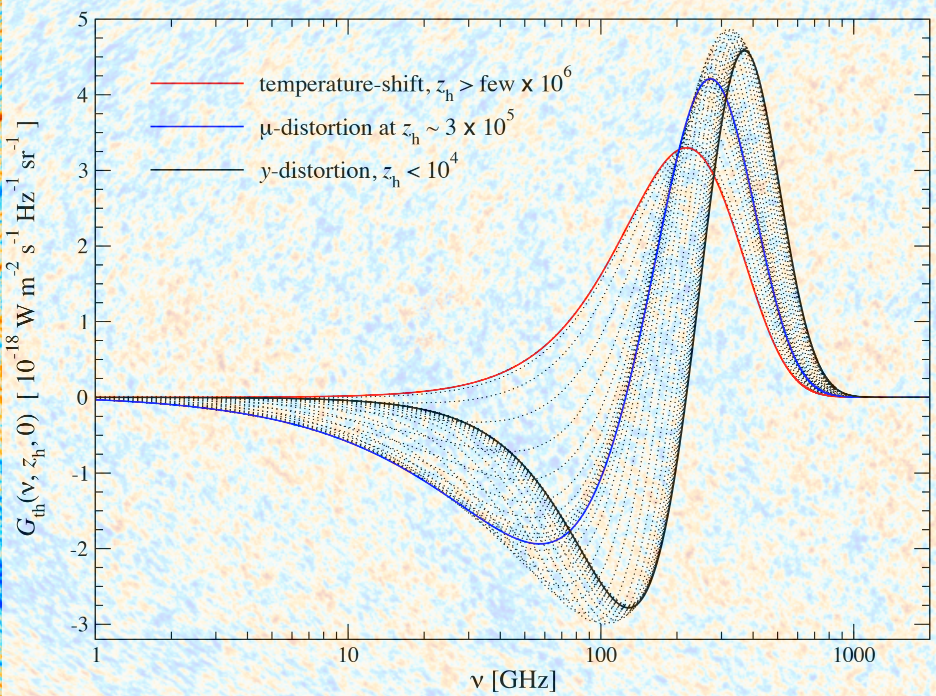
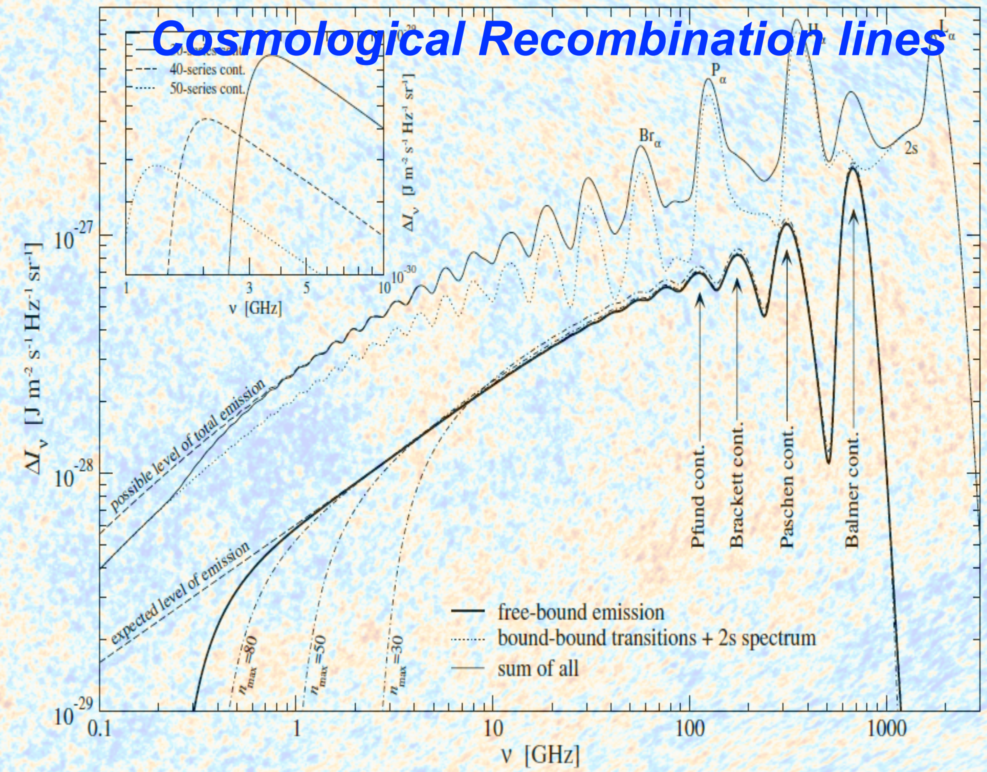
Spectral Distortions of the CMB

Primordial Distortions



Cosmological Recombination lines



JOHNS HOPKINS UNIVERSITY

Jens Chluba

46th GIF School: Cosmology after Planck

Paris, September 12th, 2014

CITA ICAT Canadian Institute for Theoretical Astrophysics L'institut canadien d'astrophysique theorique

Main Goals of this Lecture

- Convince you that future CMB distortions science will be *extremely* exciting!
- Explain how distortions evolve and thermalize
- Definition of different types of distortions
- Computations of spectral distortions *(you should be able to do this yourself afterwards!)*
- Provide an overview for different sources of primordial distortions
- Show you why the CMB spectrum provides a *complementary* probe of inflation and particle physics

References for the Theory of Spectral Distortions

- Original works
  - Zeldovich & Sunyaev, 1969, Ap&SS, 4, 301
  - Sunyaev & Zeldovich, 1970, Ap&SS, 7, 20
  - Illarionov & Sunyaev, 1975, Sov. Astr., 18, 413

Yakov Zeldovich

Rashid Sunyaev

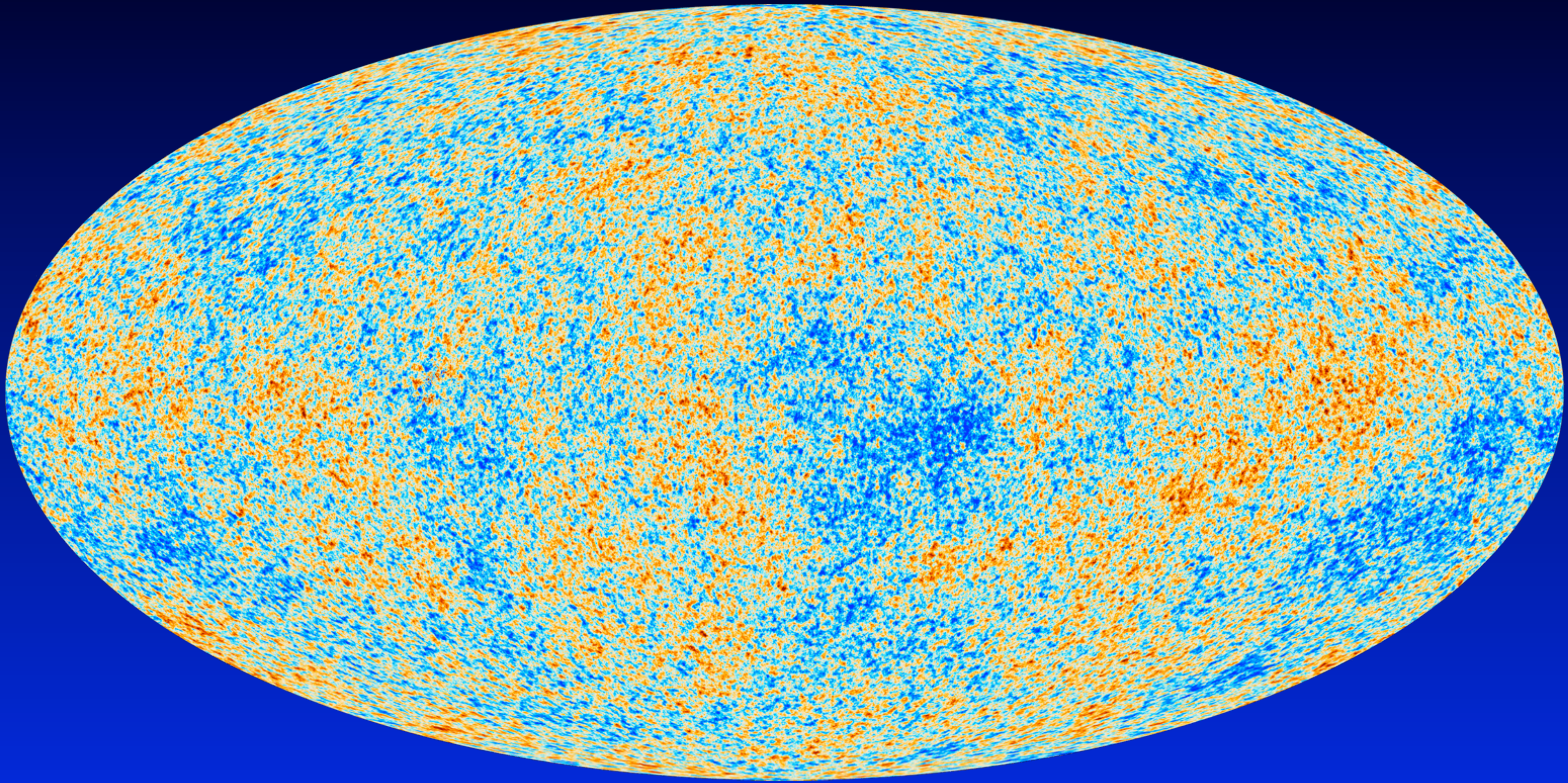
References for the Theory of Spectral Distortions

- Original works
  - Zeldovich & Sunyaev, 1969, Ap&SS, 4, 301
  - Sunyaev & Zeldovich, 1970, Ap&SS, 7, 20
  - Illarionov & Sunyaev, 1975, Sov. Astr., 18, 413
- Additional milestones
  - Danese & de Zotti, 1982, A&A, 107, 39
  - Burigana, Danese & de Zotti, 1991, ApJ, 379, 1
  - Hu & Silk, 1993, Phys. Rev. D, 48, 485
  - Hu, 1995, PhD thesis

References for the Theory of Spectral Distortions

- Original works
  - Zeldovich & Sunyaev, 1969, Ap&SS, 4, 301
  - Sunyaev & Zeldovich, 1970, Ap&SS, 7, 20
  - Illarionov & Sunyaev, 1975, Sov. Astr., 18, 413
- Additional milestones
  - Danese & de Zotti, 1982, A&A, 107, 39
  - Burigana, Danese & de Zotti, 1991, ApJ, 379, 1
  - Hu & Silk, 1993, Phys. Rev. D, 48, 485
  - Hu, 1995, PhD thesis
- More recent overviews
  - Sunyaev & JC, 2009, AN, 330, 657
  - JC & Sunyaev, 2012, MNRAS, 419, 1294
  - JC, MNRAS, 436, 2232 & ArXiv:1405.6938

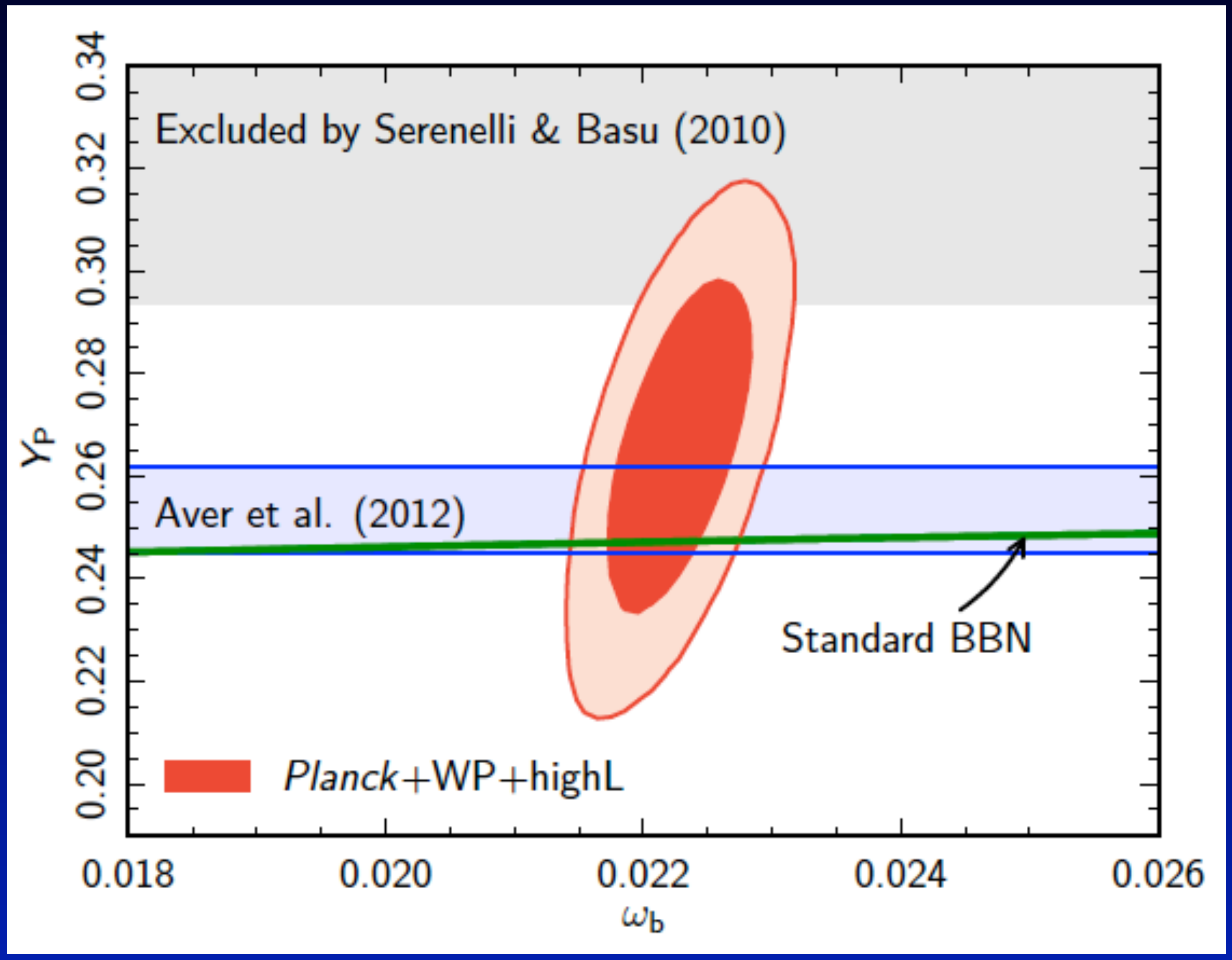
Cosmic Microwave Background Anisotropies

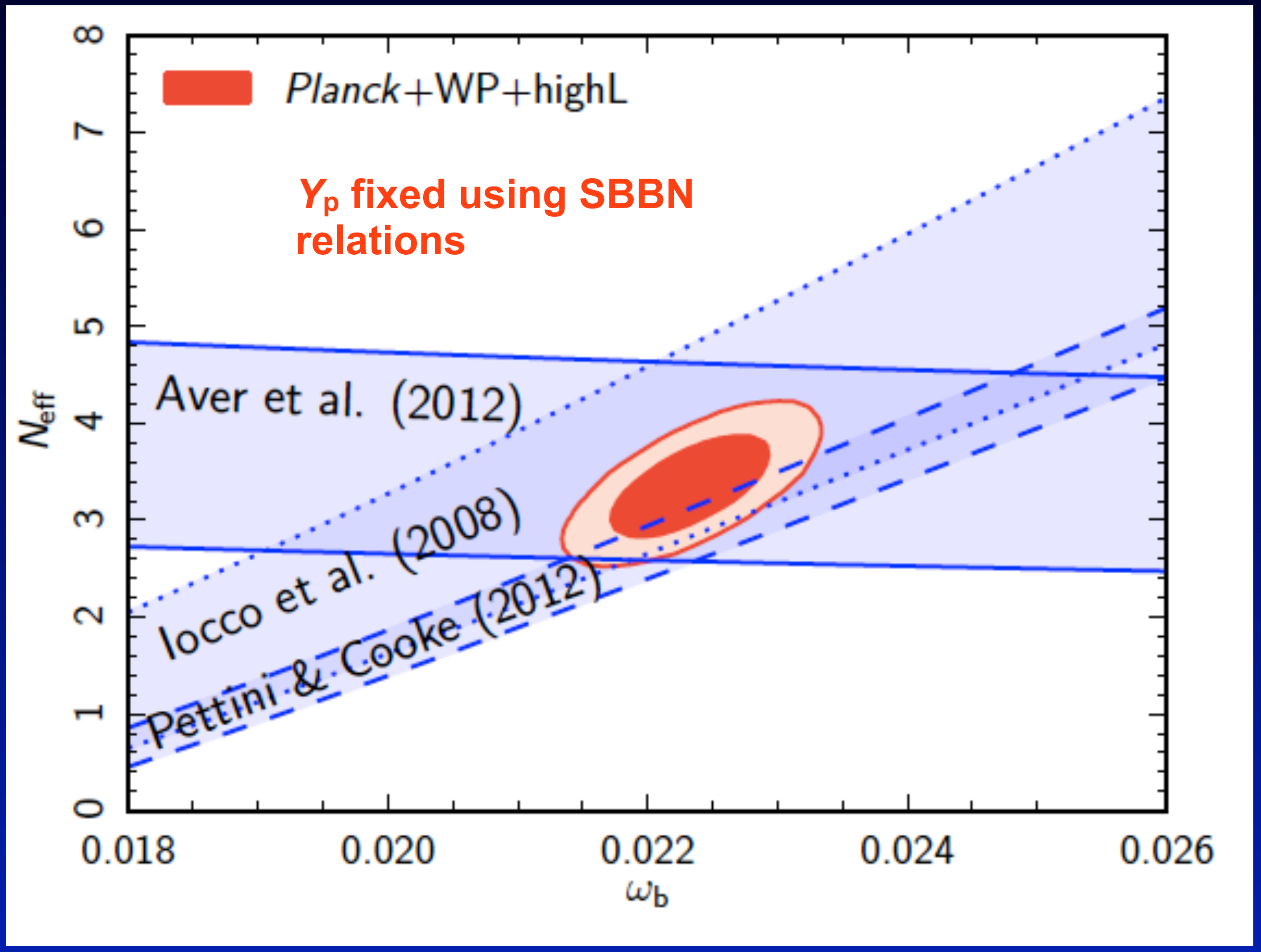


Planck all-sky temperature map

- CMB has a blackbody spectrum in every direction
- tiny variations of the CMB temperature $\Delta T/T \sim 10^{-5}$

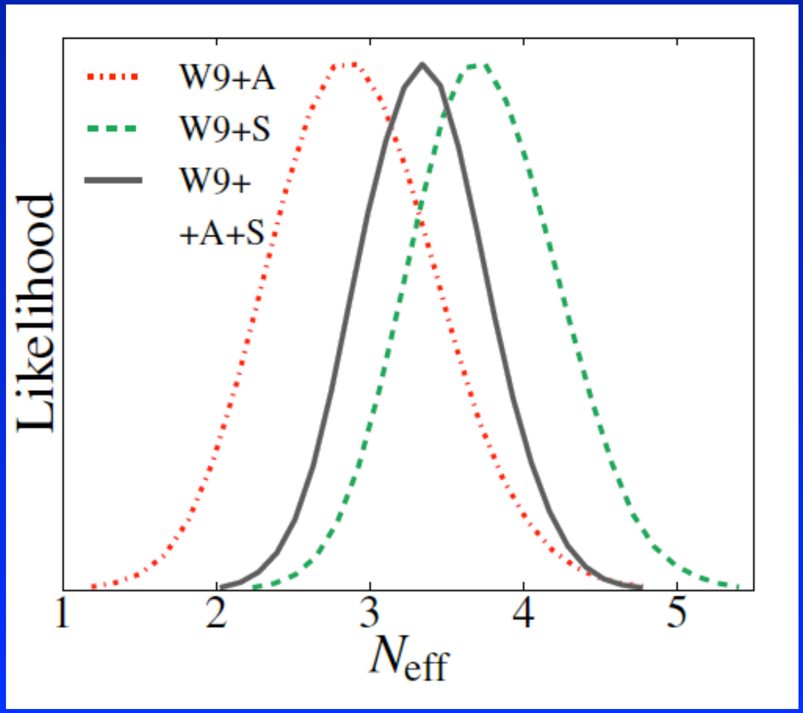
CMB constraints on N_{eff} and Y_p





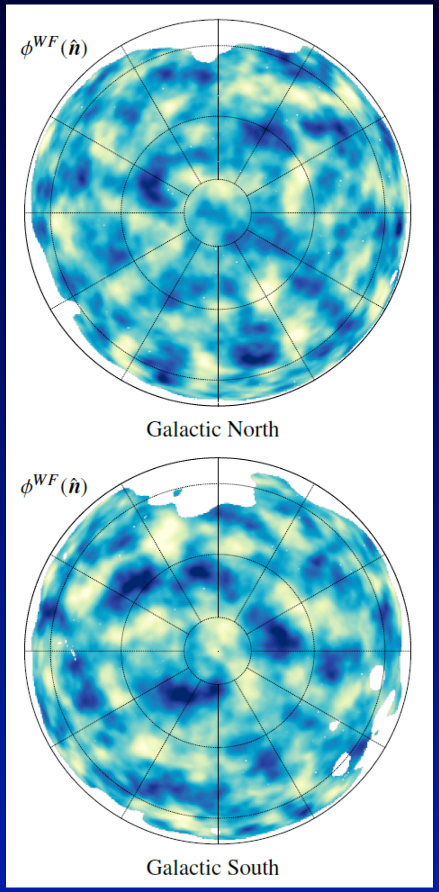
Planck Collaboration, 2013, paper XV

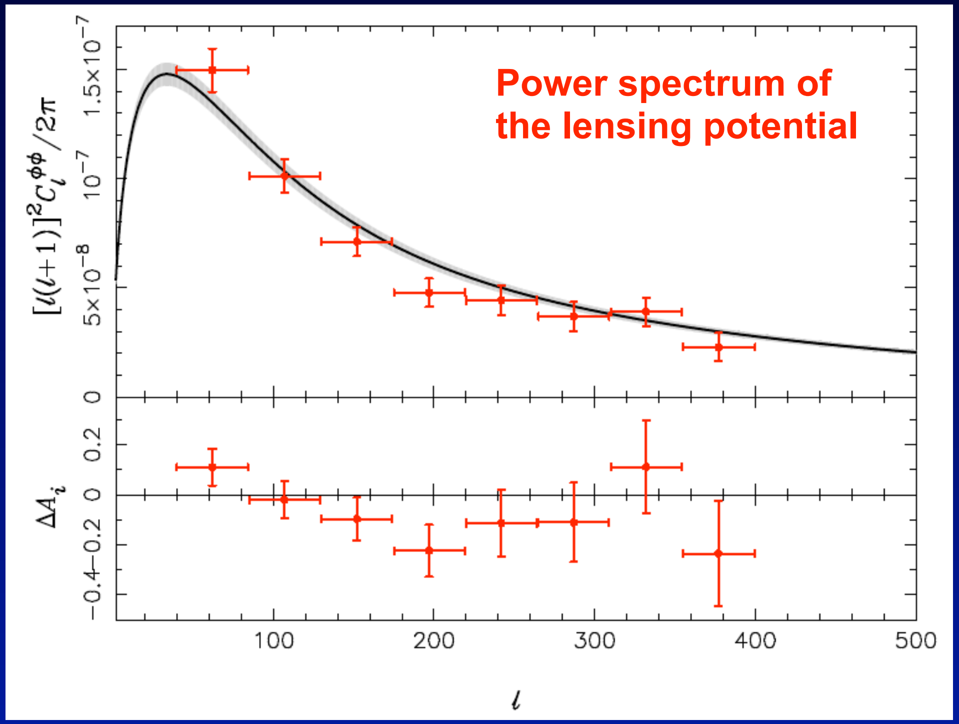
- *Helium determination from CMB consistent with SBNN prediction*
- *CMB constraint on N_{eff} competitive*
- *Partial degeneracy with Y_p and running*
- *Some tension between different data sets*



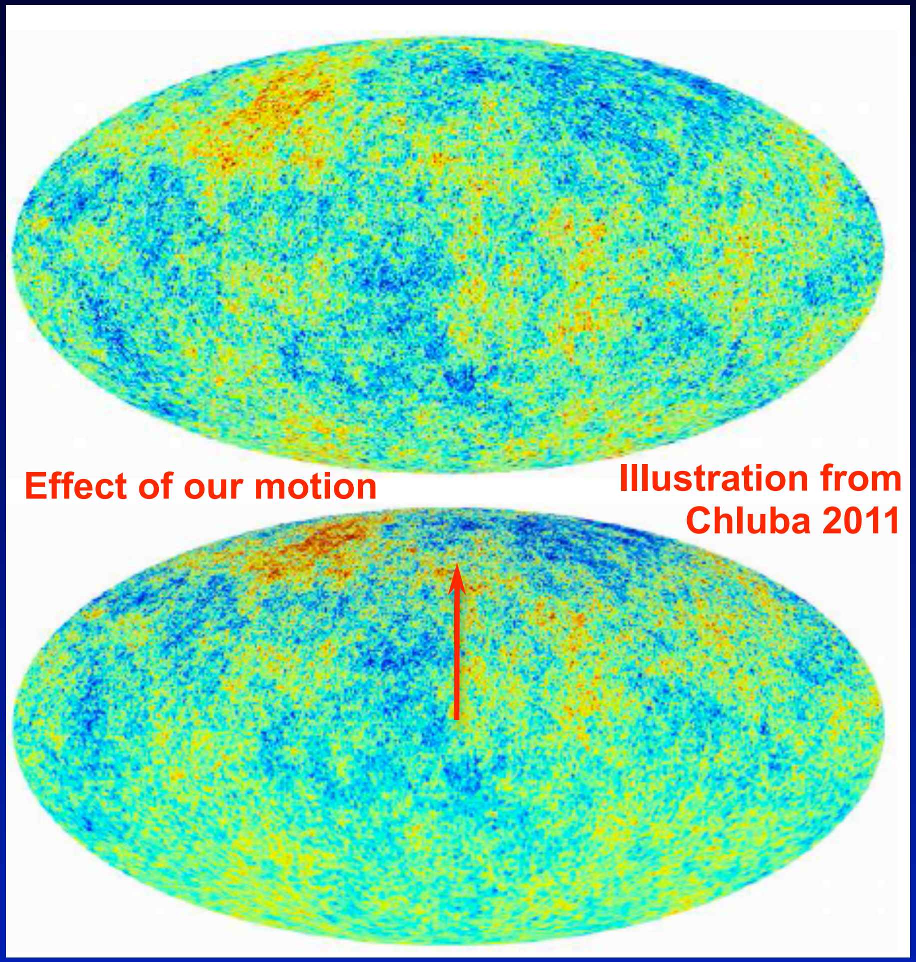
Calabrese et al. 2013

All kind of fun new science with the CMB!



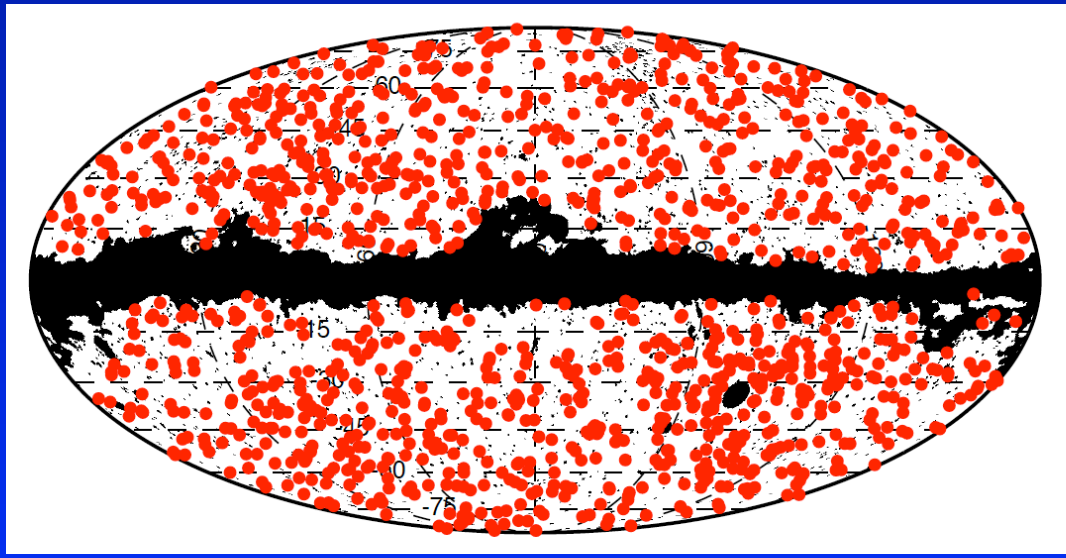


Planck Collaboration, 2013, paper XVII



Planck Collaboration, 2013, paper XXVII

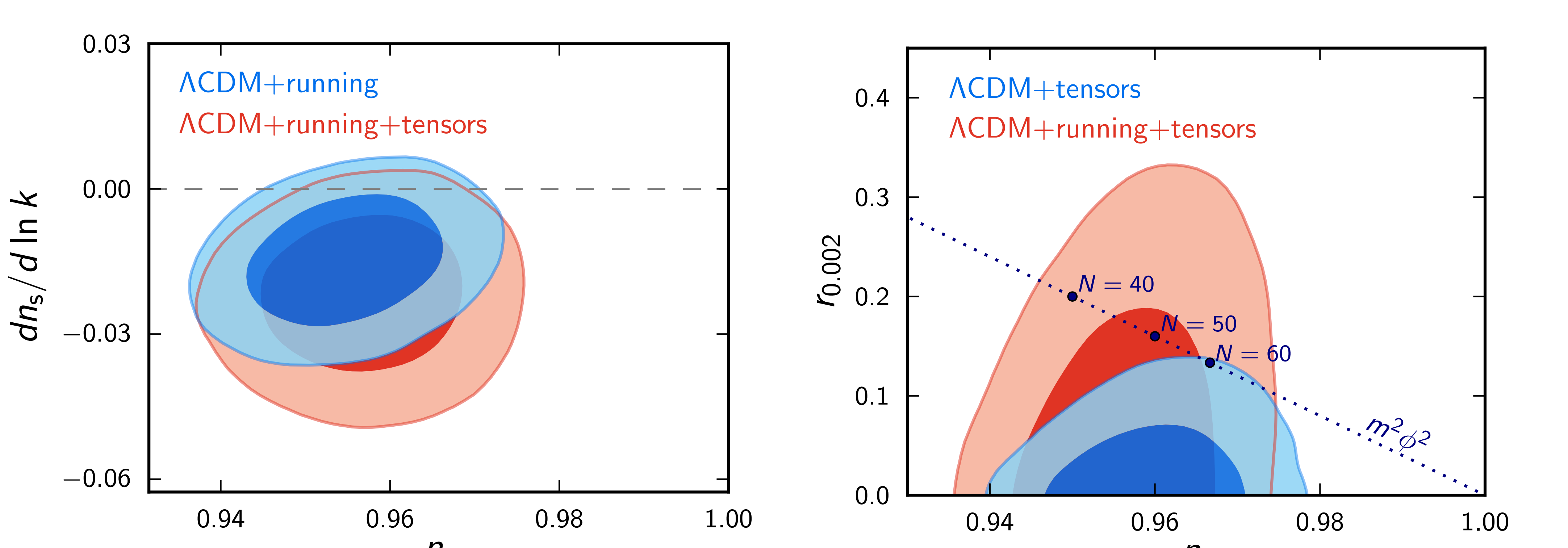
SZ clusters on the sky



Planck Collaboration, 2013, paper XXIV

- *Non-Gaussianity* (test of inflation models)
- *Topology*
- *CMB anomalies*
- *CIB* and *Galactic science*

CMB anisotropies as probe of Inflation

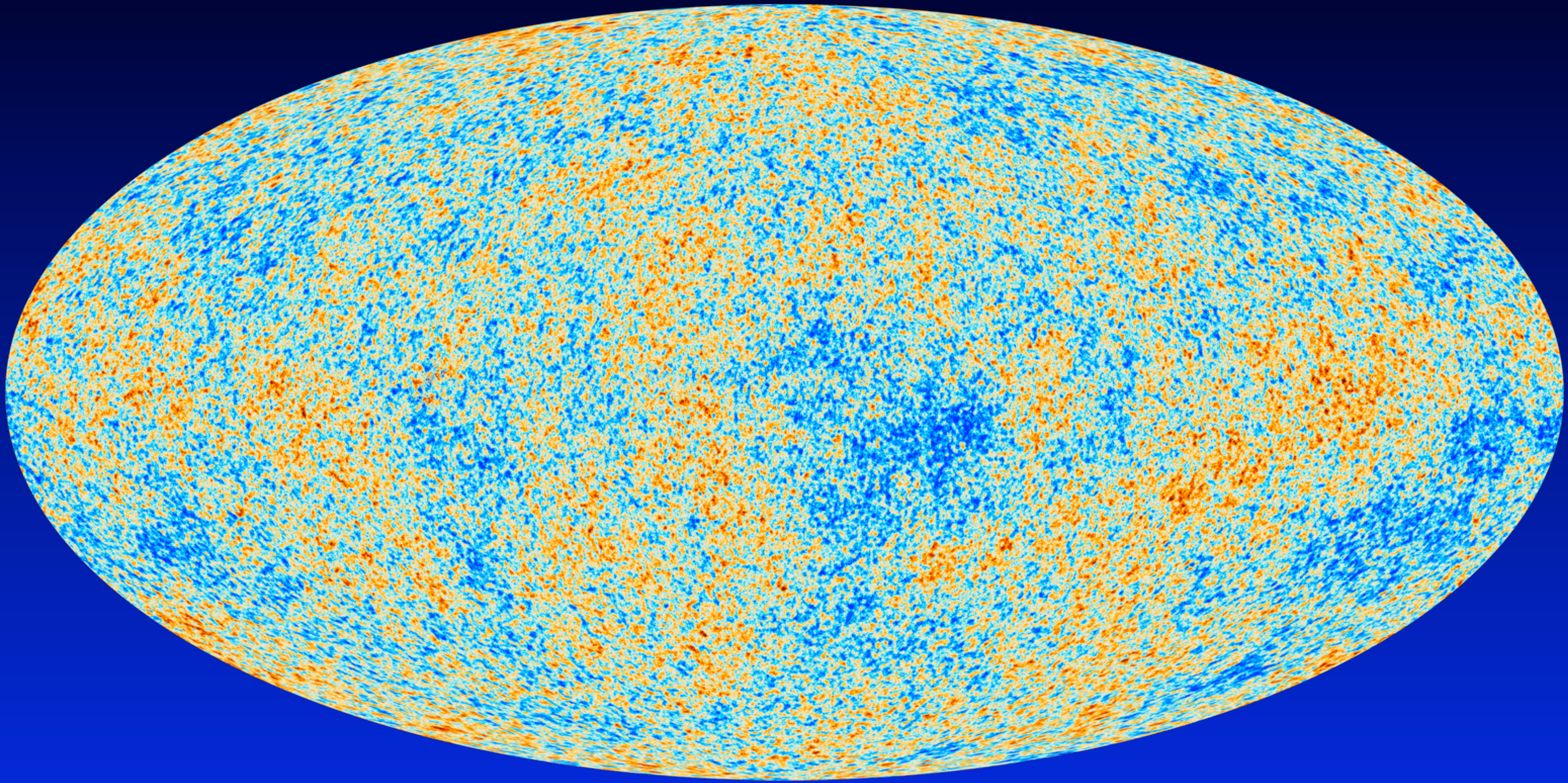


Planck Collaboration, 2013, paper XVI

- *Big goal/hope:* detection of B-polarization
- Plenty of progress over the next few years:

  ground/balloon: BICEP2, SPTpol, ACTpol, Spider, ...
  space: Planck, LiteBIRD, PIXIE, COrE+, ...?

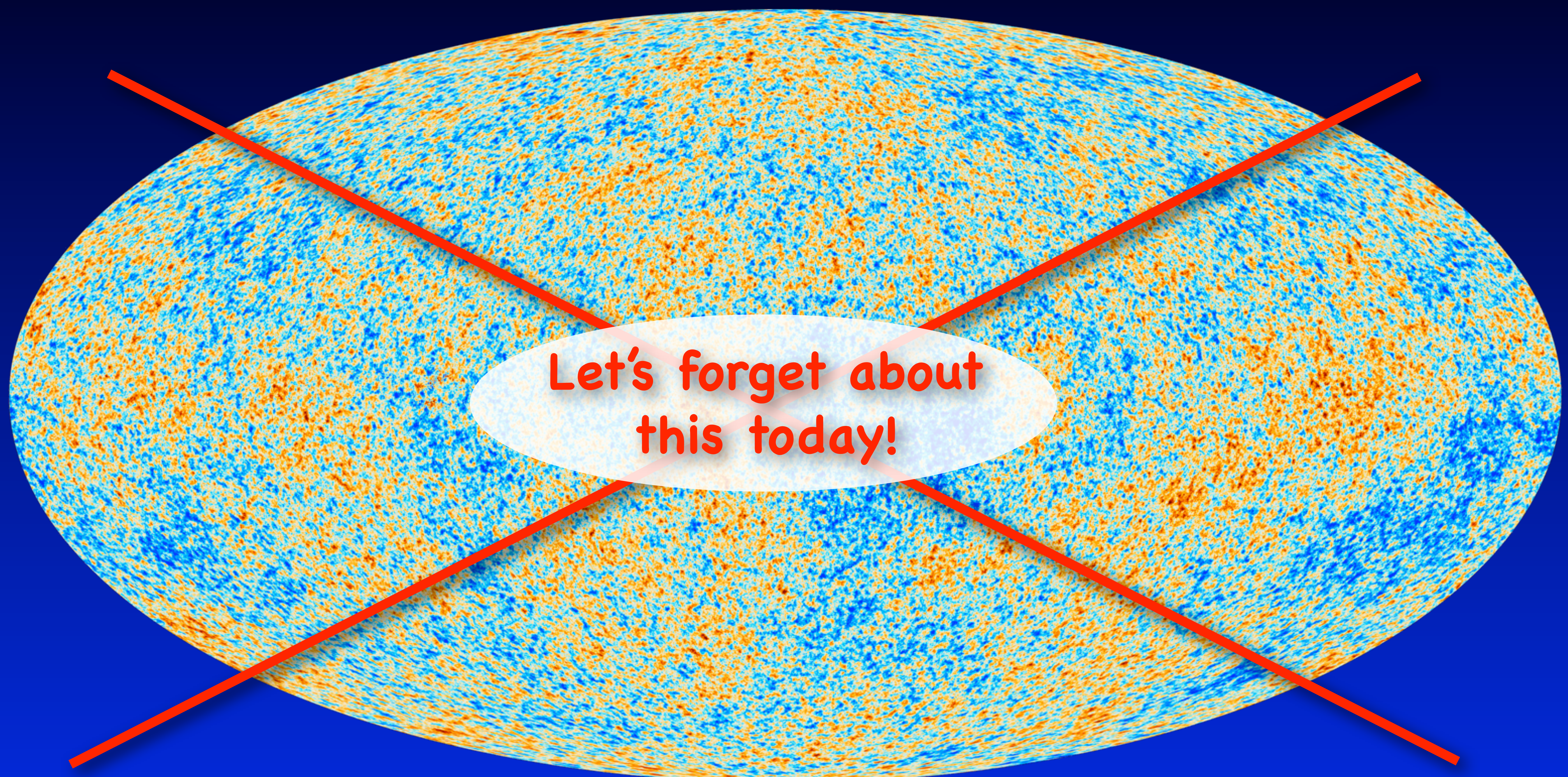
Cosmic Microwave Background Anisotropies



Planck all-sky temperature map

- CMB has a blackbody spectrum in every direction
- tiny variations of the CMB temperature $\Delta T/T \sim 10^{-5}$

Cosmic Microwave Background Anisotropies



Planck all-sky temperature map

- CMB has a blackbody spectrum in every direction
- tiny variations of the CMB temperature $\Delta T/T \sim 10^{-5}$

CMB provides another independent piece of information!

COBE/FIRAS

$$T_0 = (2.726 \pm 0.001)\,\mathrm{K}$$

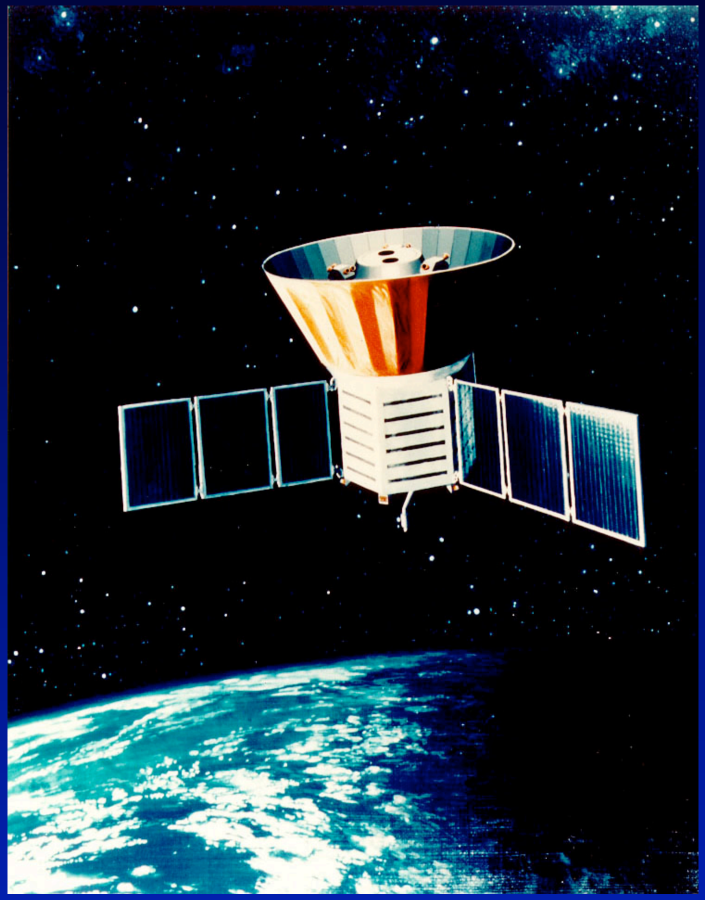
Absolute measurement required!

One has to go to space...

Mather et al., 1994, ApJ, 420, 439
Fixsen et al., 1996, ApJ, 473, 576
Fixsen, 2003, ApJ, 594, 67
Fixsen, 2009, ApJ, 707, 916

- CMB monopole is 10000 - 100000 times larger than the fluctuations

COBE / FIRAS (Far InfraRed Absolute Spectrophotometer)

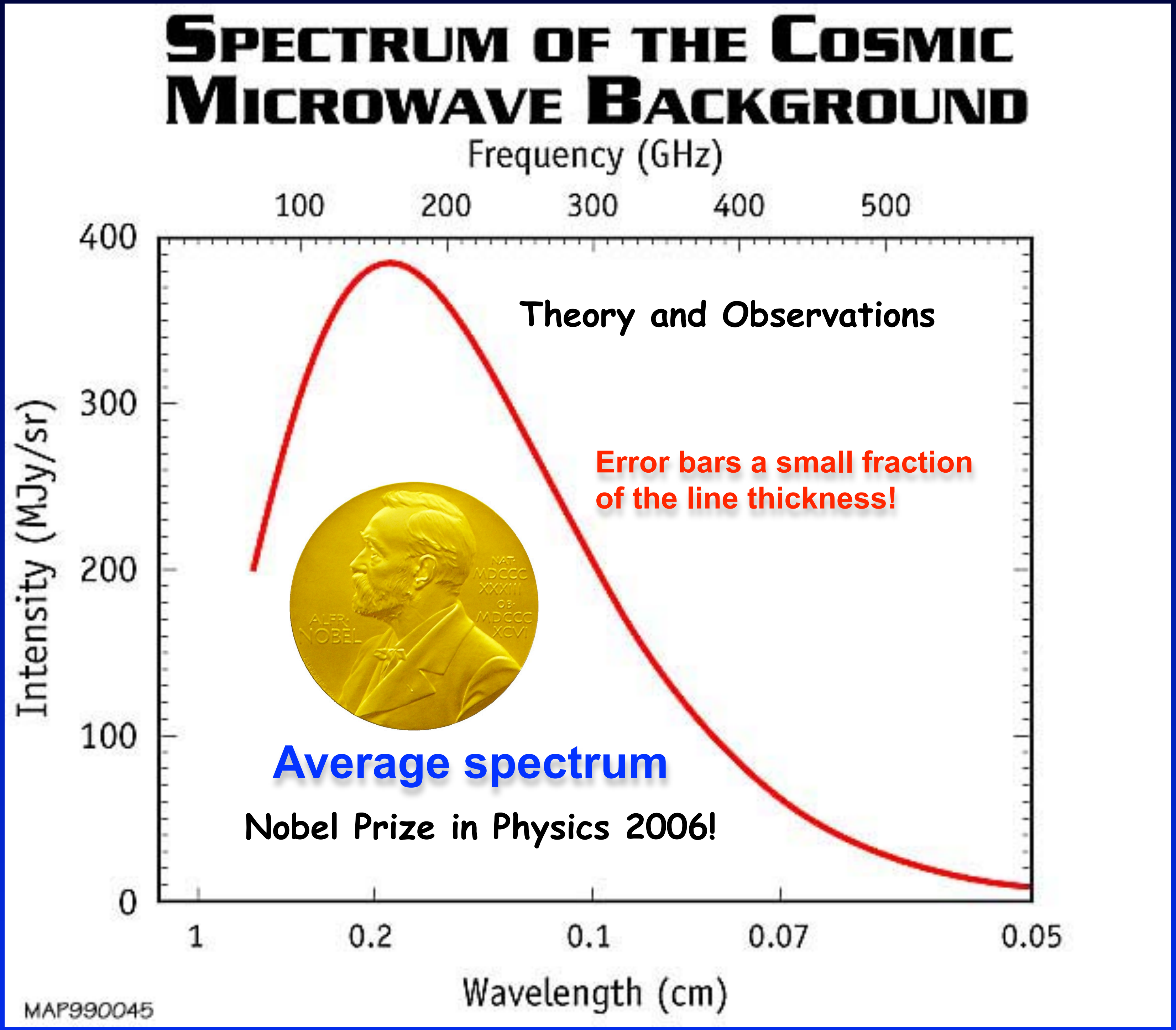


$$T_0 = 2.725 \pm 0.001\,\mathrm{K}$$

$$|\,y\,| \leq 1.5 \times 10^{-5}$$

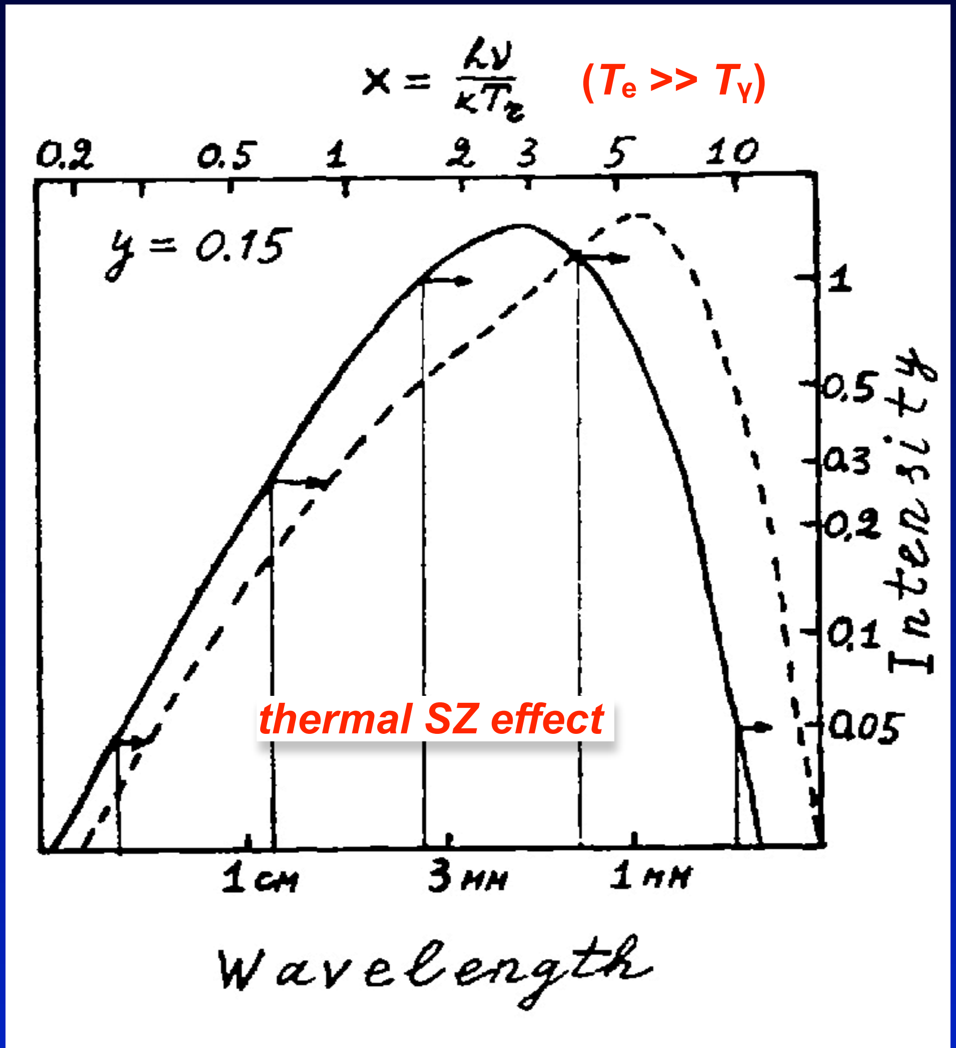
$$|\,\mu\,| \leq 9 \times 10^{-5}$$

Mather et al., 1994, ApJ, 420, 439
Fixsen et al., 1996, ApJ, 473, 576
Fixsen et al., 2003, ApJ, 594, 67



Standard types of primordial CMB distortions

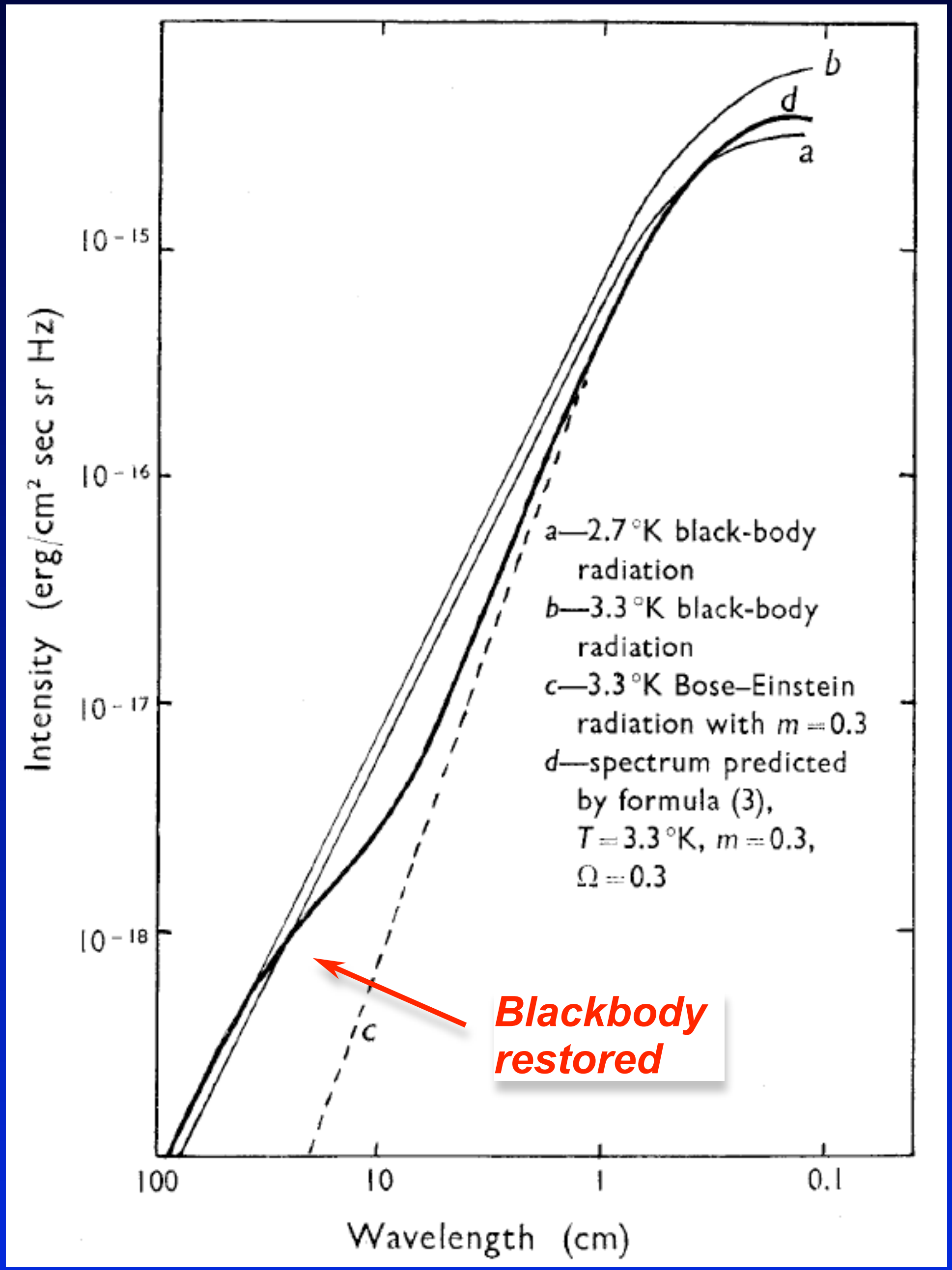
Compton y-distortion



Sunyaev & Zeldovich, 1980, ARAA, 18, 537

- also known from thSZ effect
- up-scattering of CMB photon
- important at late times (z<50000)
- scattering inefficient

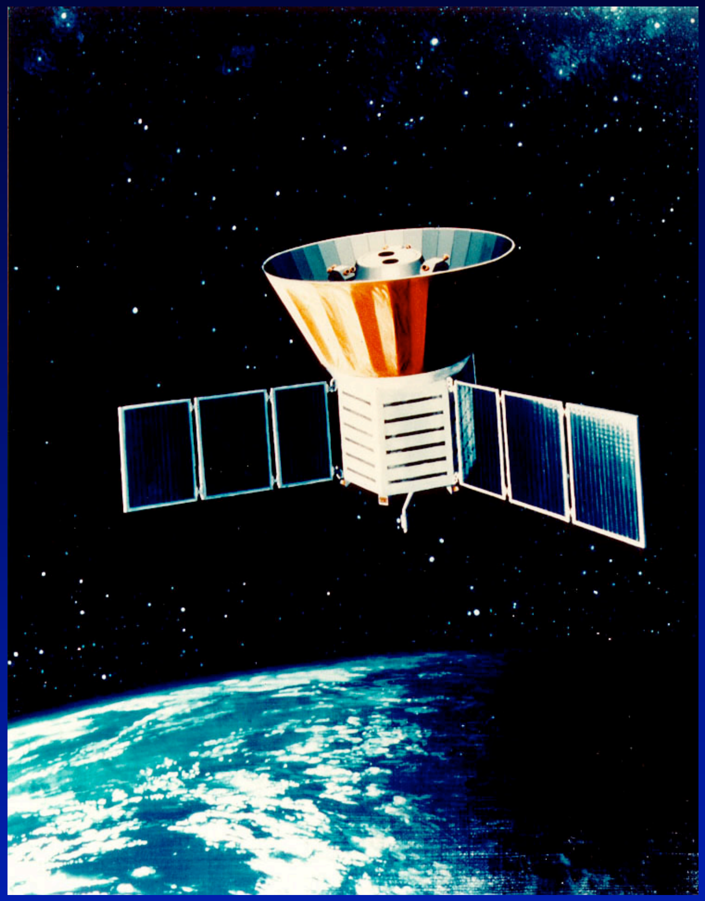
Chemical potential μ-distortion

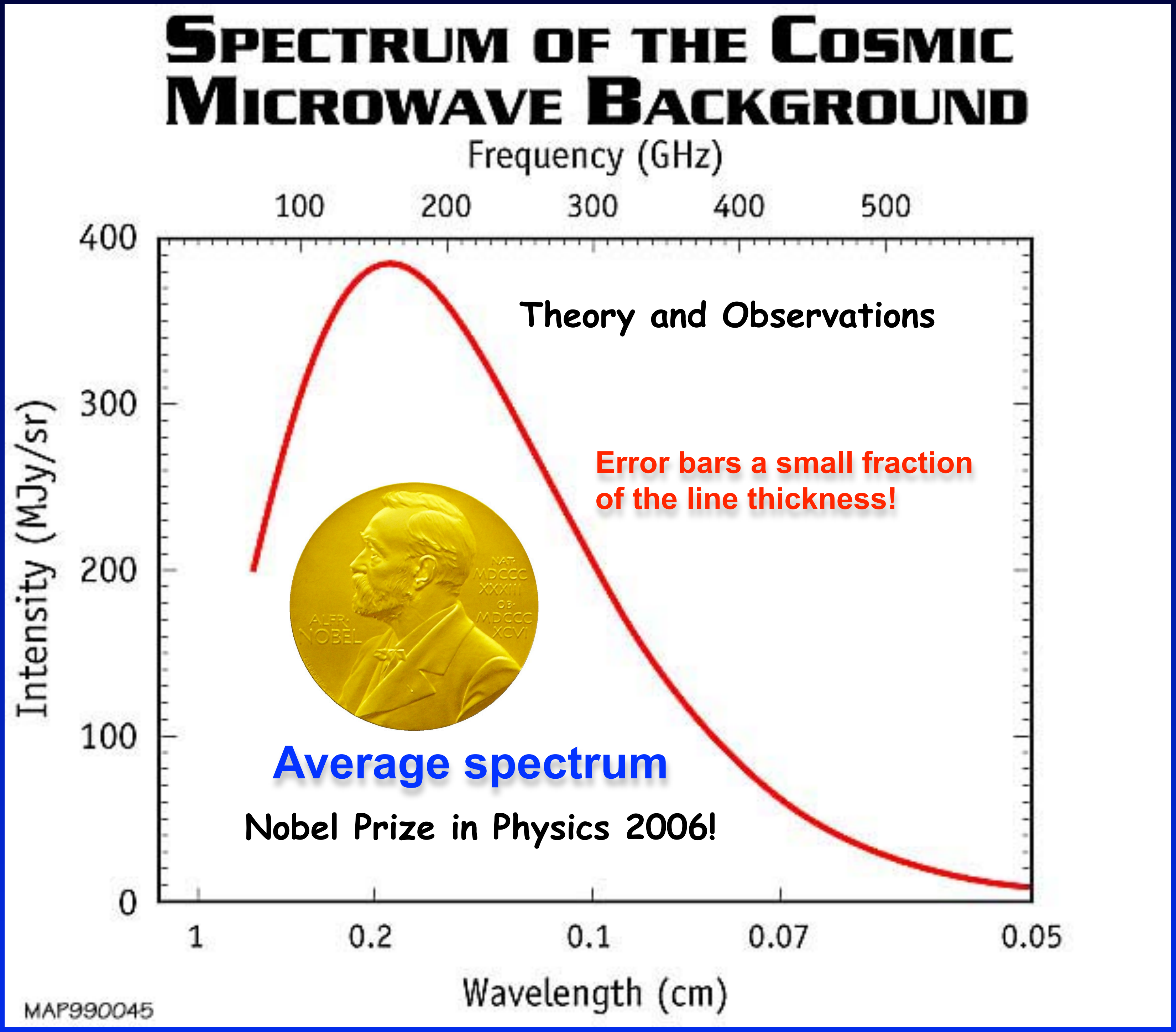


Sunyaev & Zeldovich, 1970, ApSS, 2, 66

- important at very times (z>50000)
- scattering very efficient

COBE / FIRAS (Far InfraRed Absolute Spectrophotometer)





$$T_0 = 2.725 \pm 0.001\,\mathrm{K}$$

$$|y| \le 1.5\times10^{-5}$$

$$|\mu| \le 9\times10^{-5}$$

Mather et al., 1994, ApJ, 420, 439
Fixsen et al., 1996, ApJ, 473, 576
Fixsen et al., 2003, ApJ, 594, 67

Only very small distortions of CMB spectrum are still allowed!

No primordial distortion found so far!? Why are we at all talking about this then?

Physical mechanisms that lead to spectral distortions

Standard sources of distortions

- *Cooling by adiabatically expanding ordinary matter*
  (JC, 2005; JC & Sunyaev 2011; Khatri, Sunyaev & JC, 2011)
- Heating by *decaying* or *annihilating* relic particles
  (Kawasaki et al., 1987; Hu & Silk, 1993; McDonald et al., 2001; JC, 2005; JC & Sunyaev, 2011; JC, 2013; JC & Jeong, 2013)
- *Evaporation of primordial black holes & superconducting strings*
  (Carr et al. 2010; Ostriker & Thompson, 1987; Tashiro et al. 2012; Pani & Loeb, 2013)
- *Dissipation of primordial acoustic modes & magnetic fields*
  (Sunyaev & Zeldovich, 1970; Daly 1991; Hu et al. 1994; JC & Sunyaev, 2011; JC et al. 2012 - Jedamzik et al. 2000; Kunze & Komatsu, 2013)
- *Cosmological recombination radiation*
  (Zeldovich et al., 1968; Peebles, 1968; Dubrovich, 1977; Rubino-Martin et al., 2006; JC & Sunyaev, 2006; Sunyaev & JC, 2009)

pre-recombination epoch

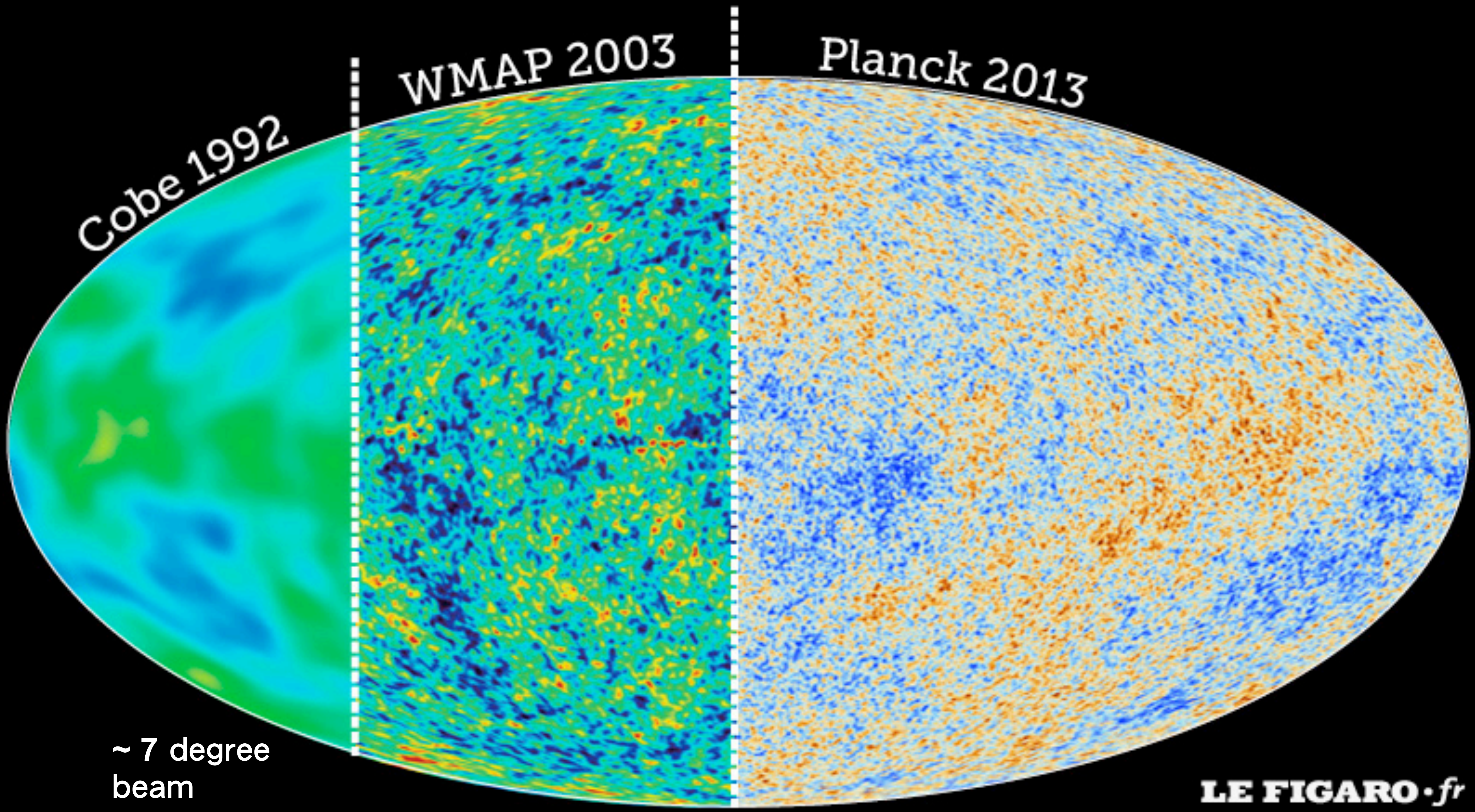
„high“ redshifts

„low“ redshifts

post-recombination

- Signatures due to first supernovae and their remnants
  (Oh, Cooray & Kamionkowski, 2003)
- Shock waves arising due to large-scale structure formation
  (Sunyaev & Zeldovich, 1972; Cen & Ostriker, 1999)
- SZ-effect from clusters; effects of reionization
  (Refregier et al., 2003; Zhang et al. 2004; Trac et al. 2008)
- more exotic processes
  (Lochan et al. 2012; Bull & Kamionkowski, 2013; Brax et al., 2013; Tashiro et al. 2013)

Dramatic improvements in angular resolution and sensitivity over the past decades!

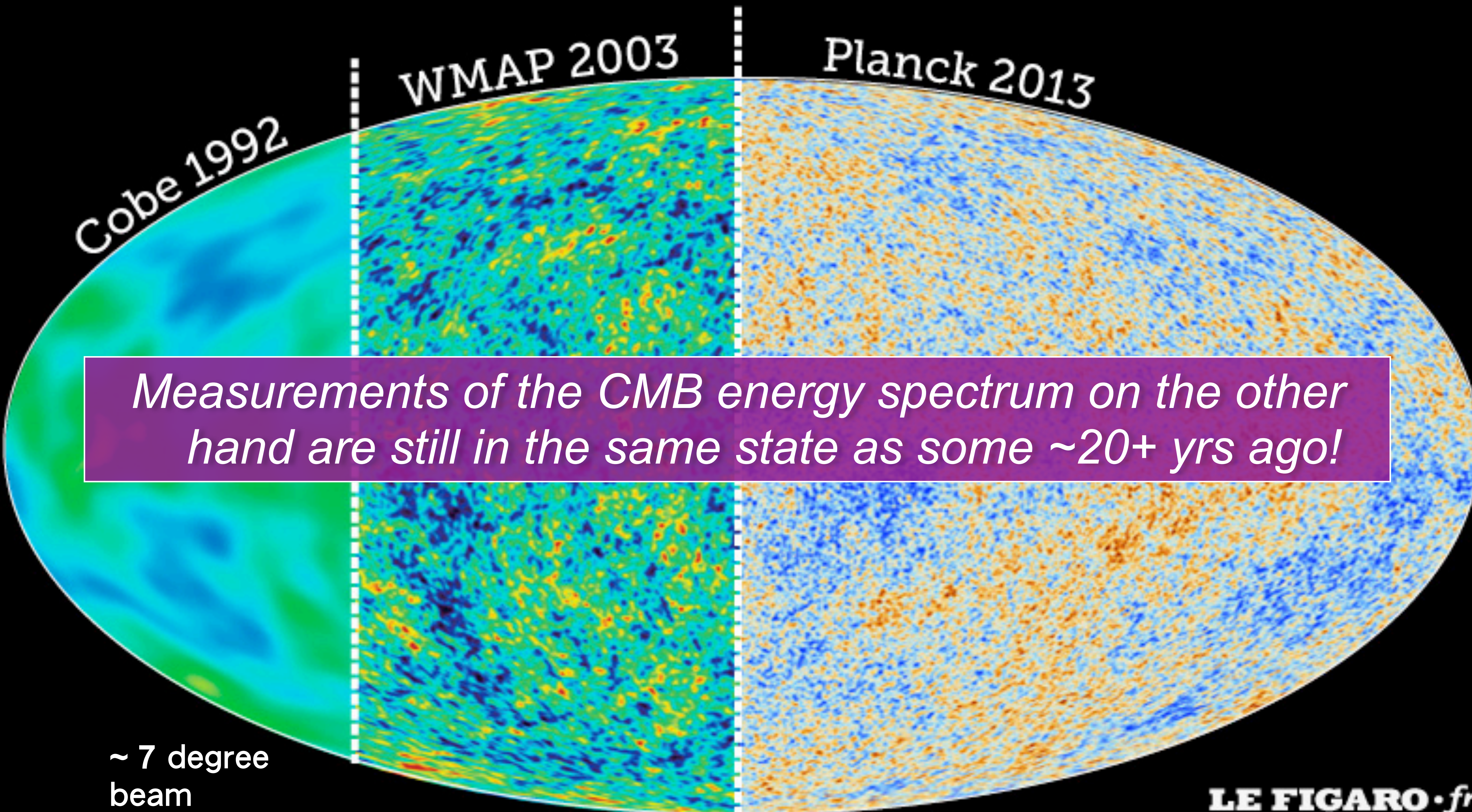


~ 7 degree beam

~ 0.3 degree beam

~ 0.08 degree beam

Dramatic improvements in angular resolution and sensitivity over the past decades!



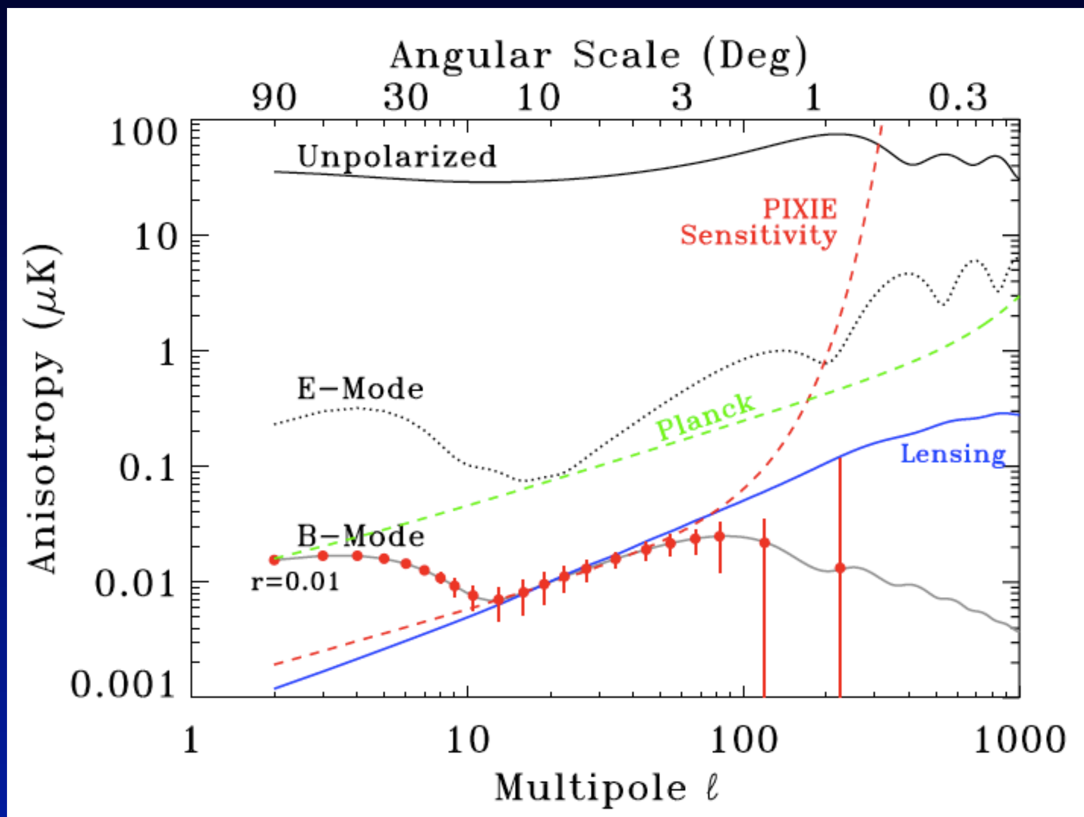
Measurements of the CMB energy spectrum on the other hand are still in the same state as some ~20+ yrs ago!

~ 7 degree beam

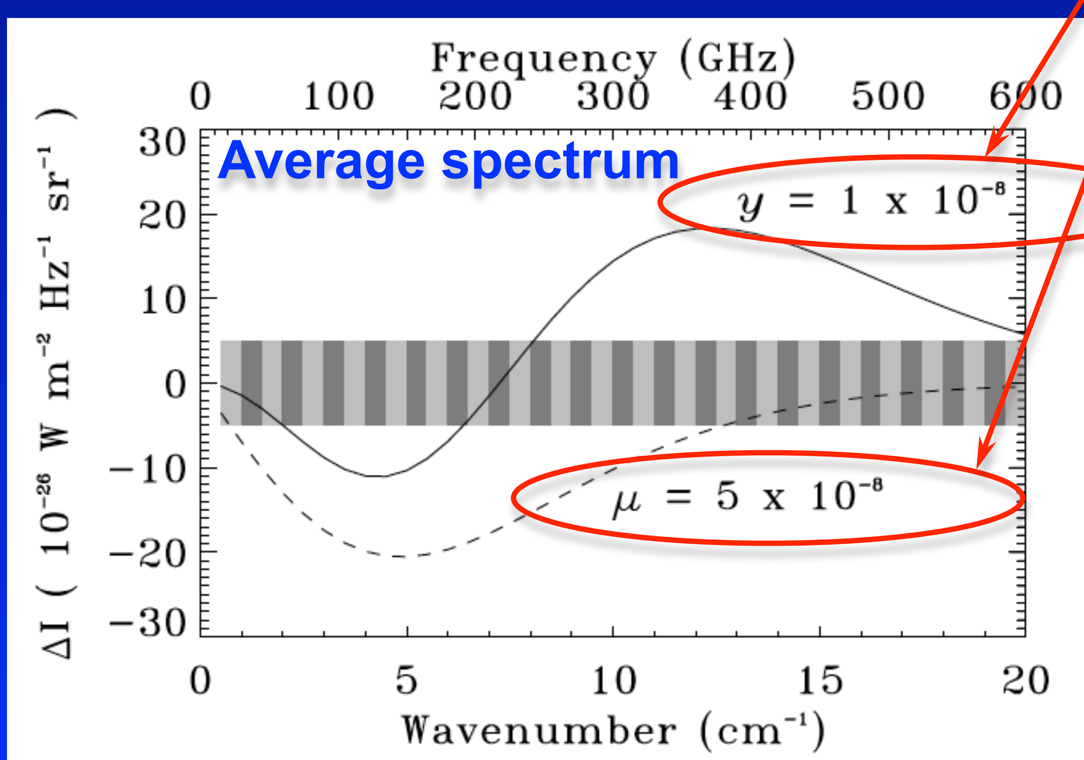
~ 0.3 degree beam

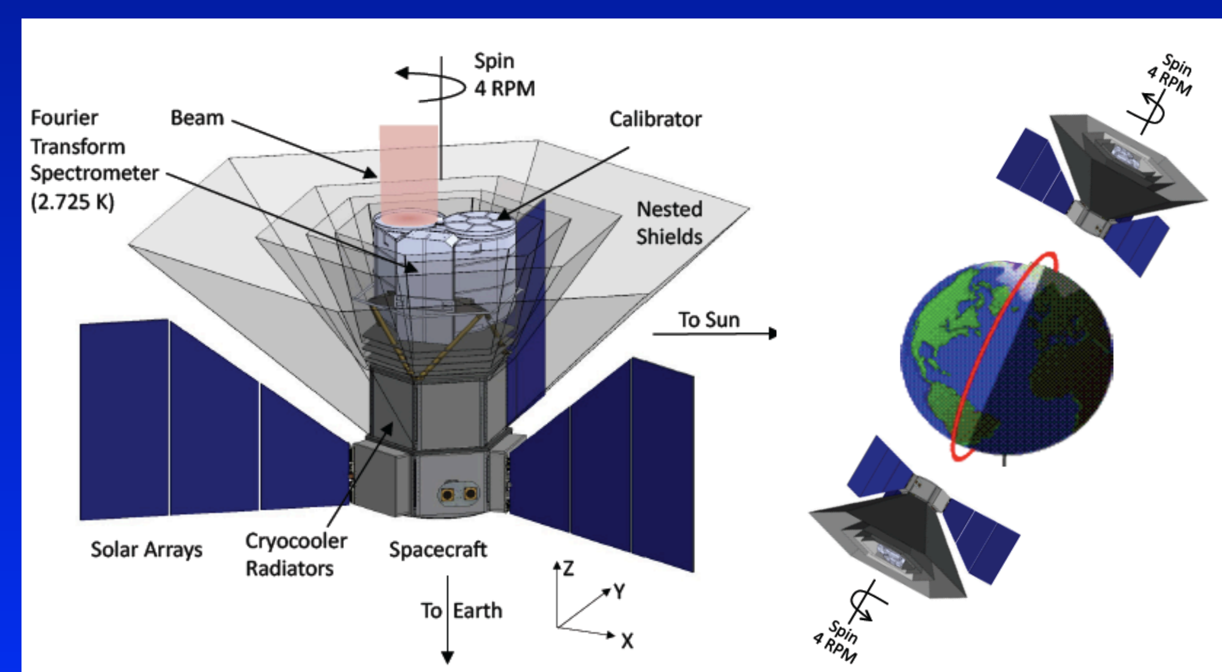
~ 0.08 degree beam

PIXIE: Primordial Inflation Explorer

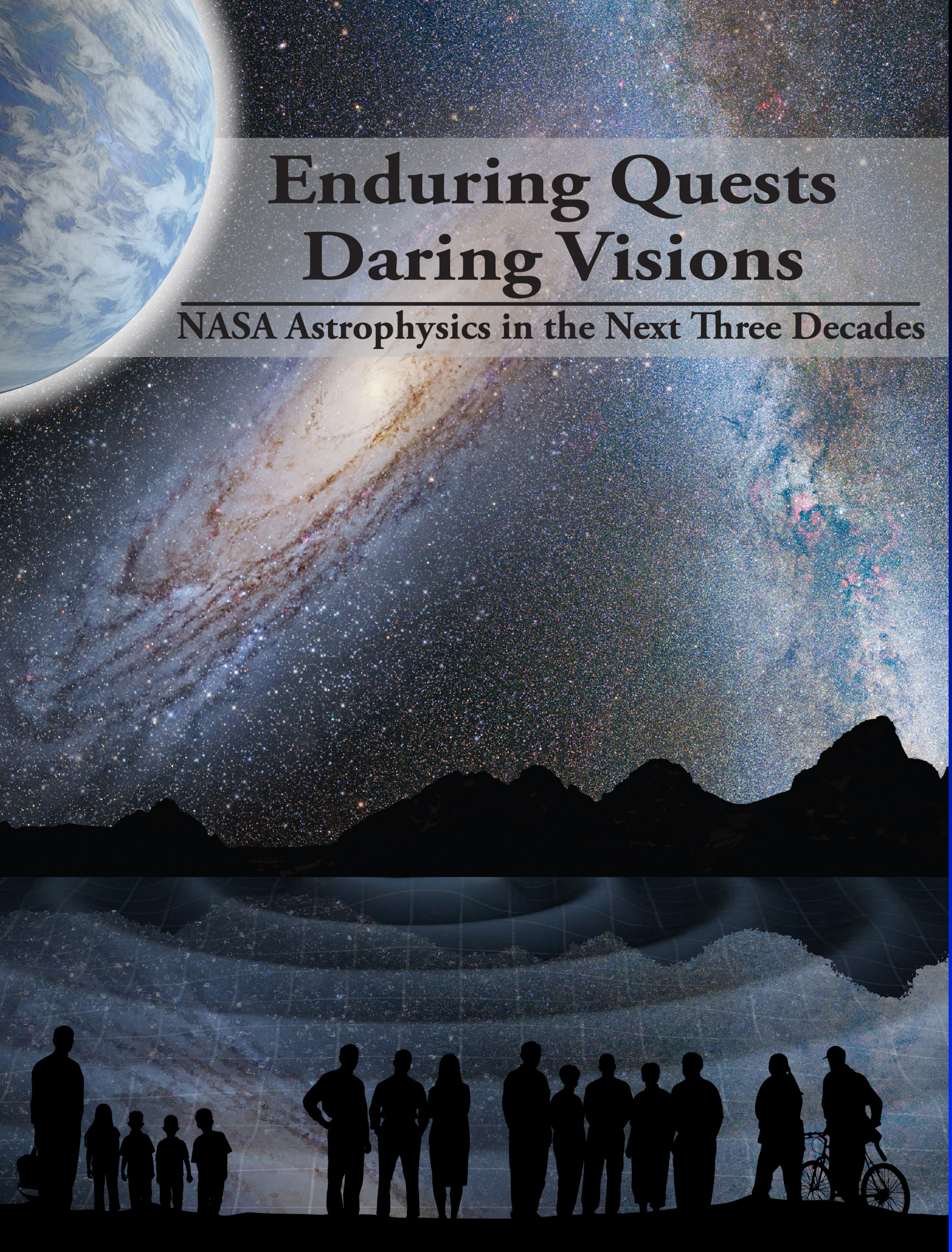


- 400 spectral channel in the frequency range 30 GHz and 6THz (Δv ~ 15GHz)
- about 1000 (!!!) times more sensitive than COBE/FIRAS
- B-mode polarization from inflation ($r \approx 10^{-3}$)
- improved limits on μ and y
- was proposed 2011 as NASA EX mission (i.e. cost ~ 200 M$)





Kogut et al, JCAP, 2011, arXiv:1105.2044



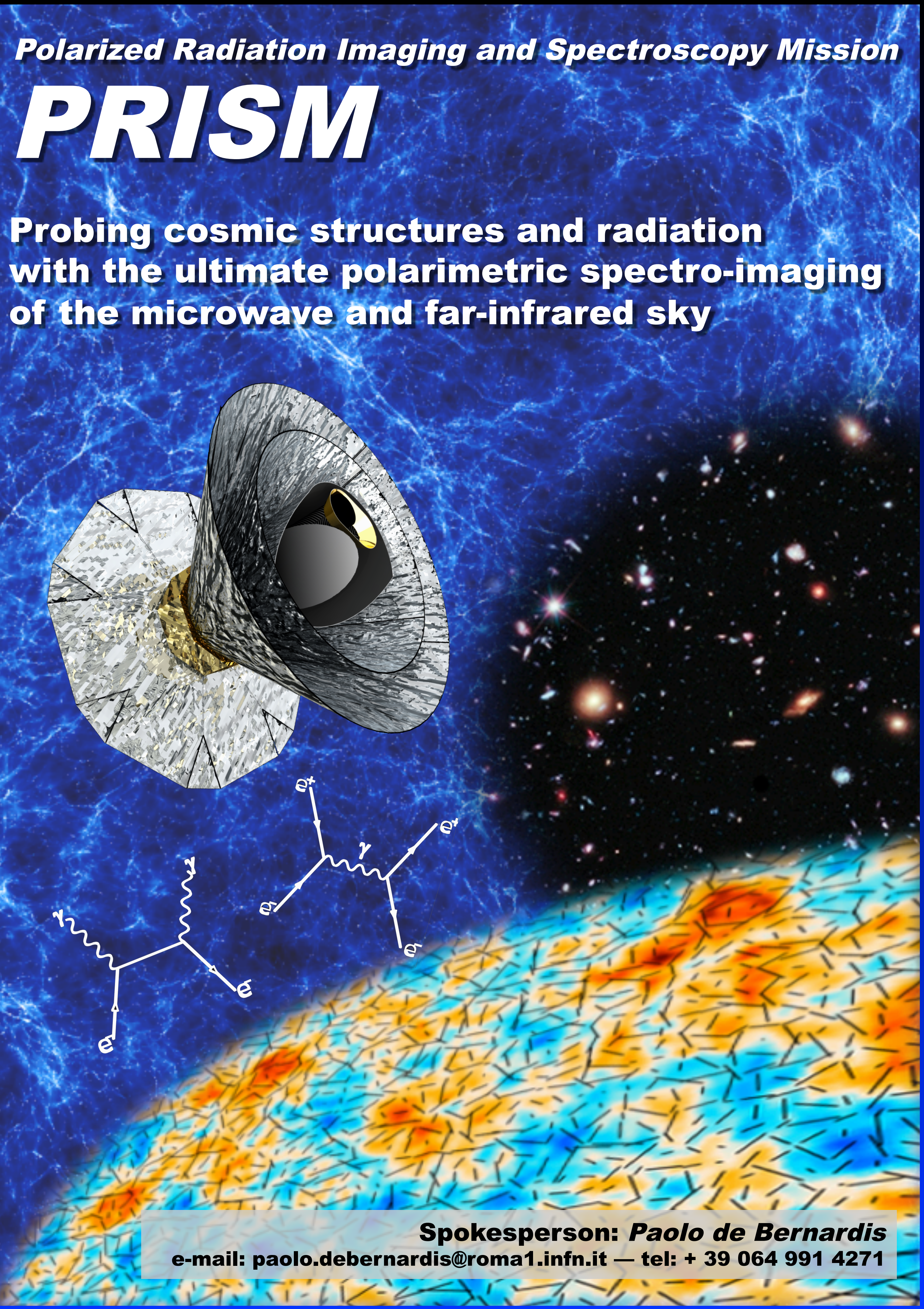
NASA 30-yr Roadmap Study

(published Dec 2013)

How does the Universe work?

"Measure the spectrum of the CMB with precision several orders of magnitude higher than COBE FIRAS, from a moderate-scale mission or an instrument on CMB Polarization Surveyor."

New call from NASA expected ~2-3 years from now



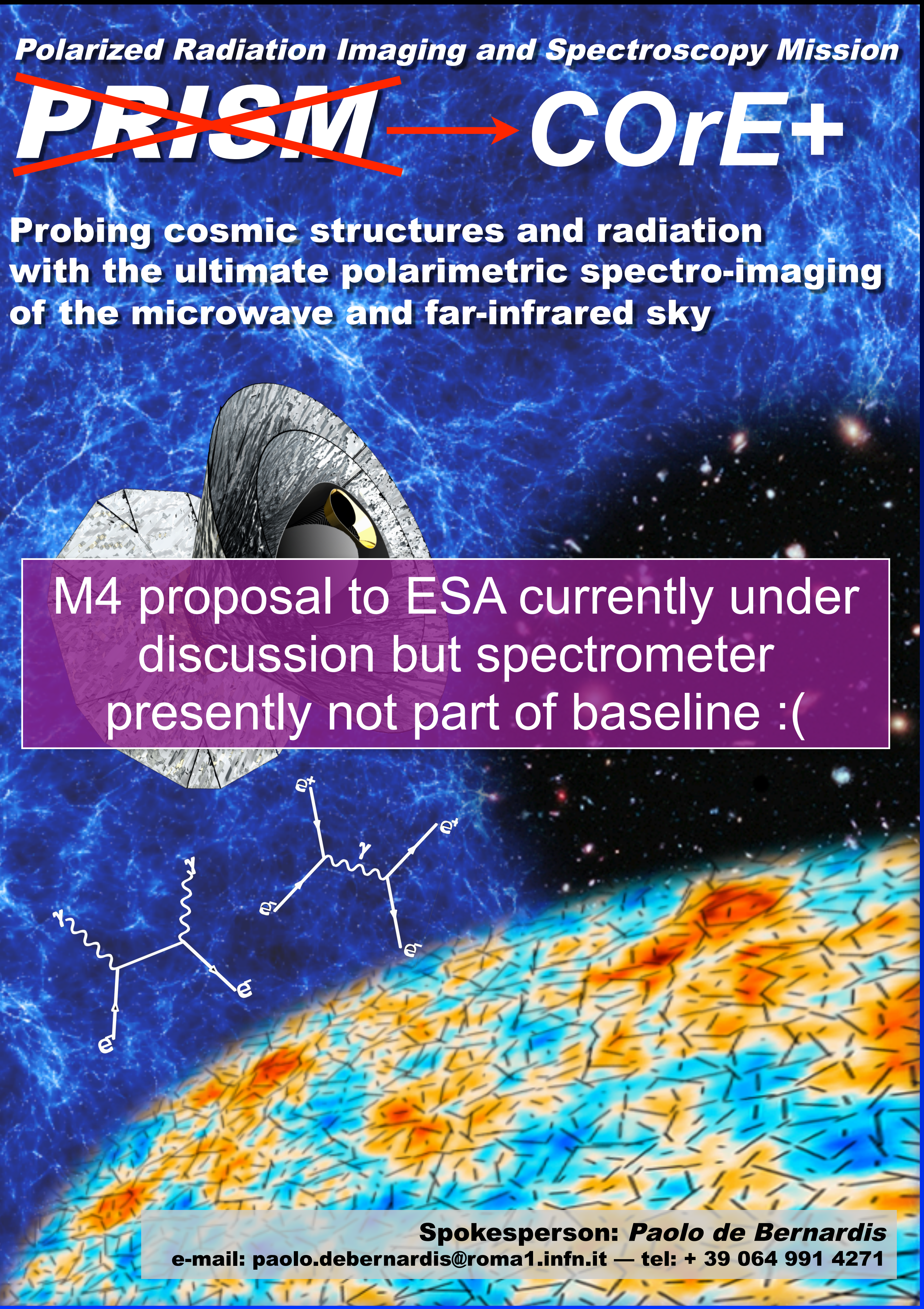
Instruments:

- L-class ESA mission
- White paper, May 24th, 2013
- Imager:
  - polarization sensitive
  - 3.5m telescope [arcmin resolution at highest frequencies]
  - 30GHz-6THz [30 broad ($\Delta\nu/\nu$~25%) and 300 narrow ($\Delta\nu/\nu$~2.5%) bands]
- Spectrometer:
  - FTS similar to PIXIE
  - 30GHz-6THz ($\Delta\nu$~15 & 0.5 GHz)

Some of the science goals:

- B-mode polarization from inflation ($r \approx 5\text{x}10^{-4}$)
- count all SZ clusters $>10^{14}$ M_{sun}
- CIB/large scale structure
- Galactic science
- *CMB spectral distortions*

More info at: http://www.prism-mission.org/



M4 proposal to ESA currently under discussion but spectrometer presently not part of baseline :(

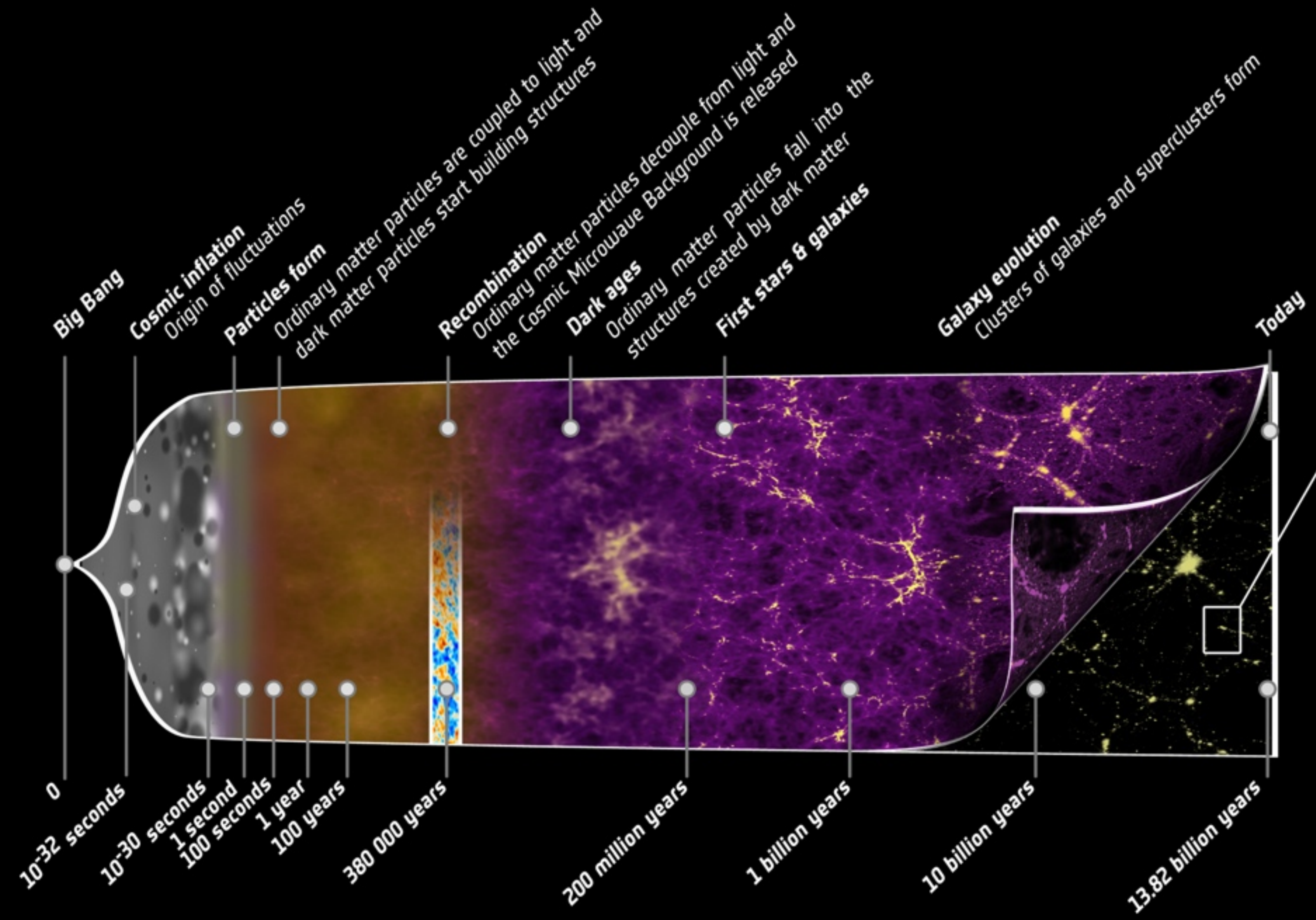
Instruments:

- L-class ESA mission
- White paper, May 24th, 2013
- Imager:
  - polarization sensitive
  - 3.5m telescope [arcmin resolution at highest frequencies]
  - 30GHz-6THz [30 broad ($\Delta\nu/\nu \sim 25\%$) and 300 narrow ($\Delta\nu/\nu \sim 2.5\%$) bands]
- ~~Spectrometer:~~
  - ~~FTS similar to PIXIE~~
  - ~~30GHz-6THz ($\Delta\nu \sim 15$ & 0.5 GHz)~~

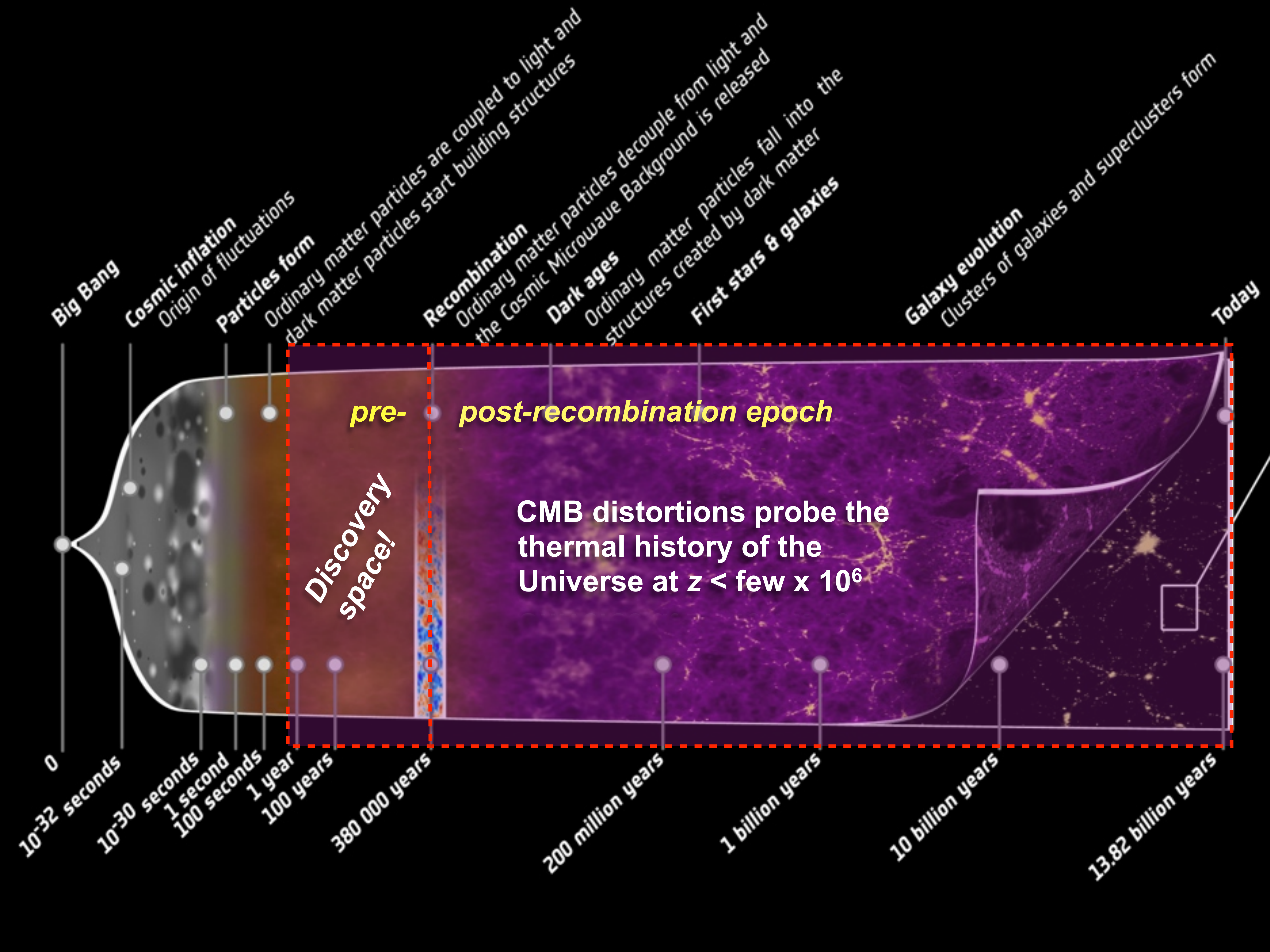
Some of the science goals:

- B-mode polarization from inflation ($r \approx 5\mathrm{x}10^{-4}$)
- count all SZ clusters $>10^{14}$ M_{sun}
- CIB/large scale structure
- Galactic science
- ~~*CMB spectral distortions*~~

More info at:
http://www.prism-mission.org/

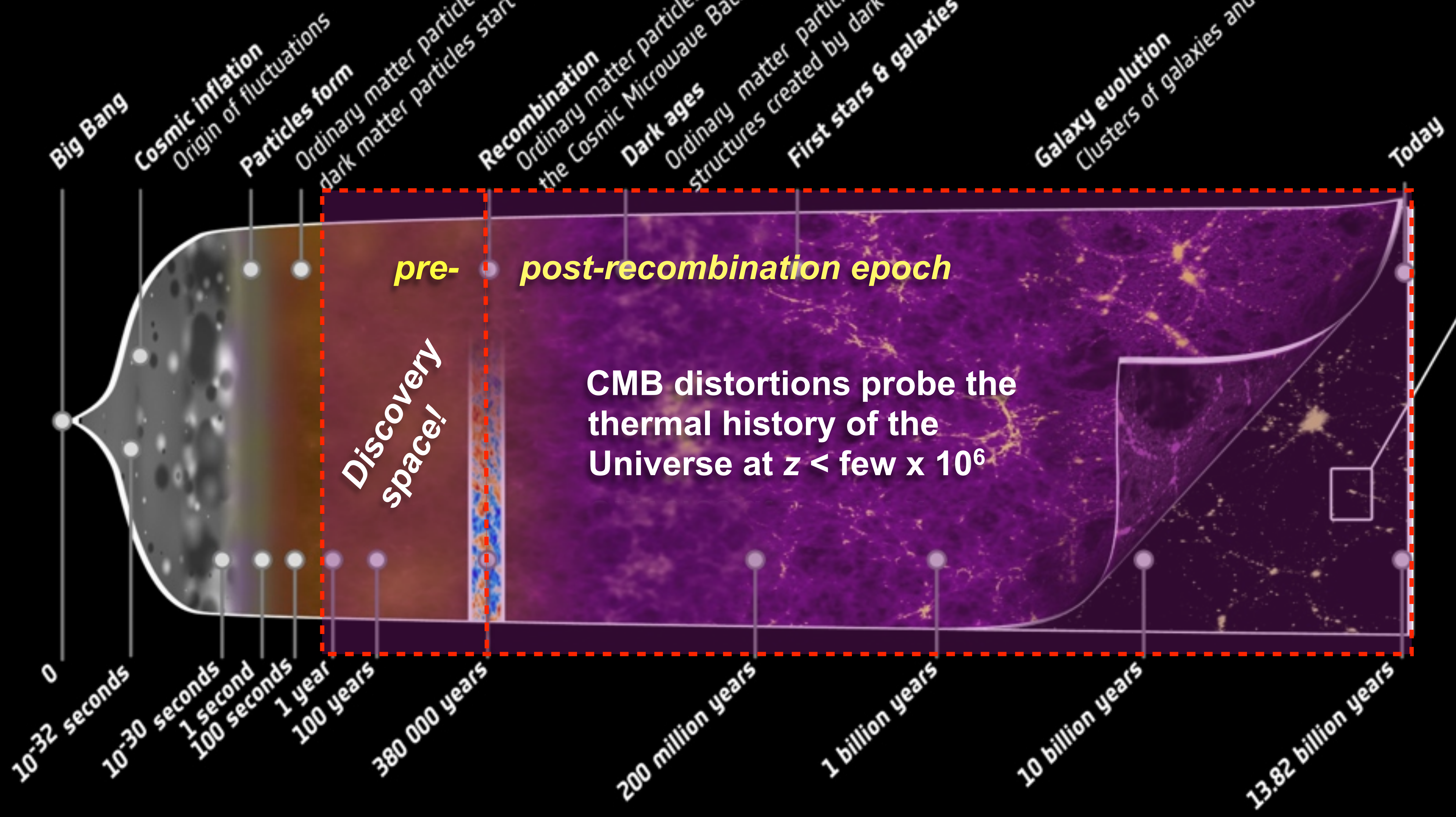


Big Bang
Cosmic inflation
Origin of fluctuations
Particles form
Ordinary matter particles are coupled to light and dark matter particles start building structures
Recombination
Ordinary matter particles decouple from light and the Cosmic Microwave Background is released
Dark ages
Ordinary matter particles fall into the structures created by dark matter
First stars & galaxies
Galaxy evolution
Clusters of galaxies and superclusters form
Today
0
10-32 seconds
10-30 seconds
1 second
100 seconds
1 year
100 years
380 000 years
200 million years
1 billion years
10 billion years
13.82 billion years



Big Bang
Cosmic inflation
Origin of fluctuations
Particles form
Ordinary matter particles are coupled to light and dark matter particles start building structures
Recombination
Ordinary matter particles decouple from light and the Cosmic Microwave Background is released
Dark ages
Ordinary matter particles fall into the structures created by dark matter
First stars & galaxies
Galaxy evolution
Clusters of galaxies and superclusters form
Today
pre-
post-recombination epoch
Discovery space!
CMB distortions probe the thermal history of the Universe at z < few x 10^6
0
10^-32 seconds
10^-30 seconds
1 second
100 seconds
1 year
100 years
380 000 years
200 million years
1 billion years
10 billion years
13.82 billion years

Measurements of CMB spectrum will open a new unexplored window to the early Universe!



Why should one expect some spectral distortion?

Why should one expect some spectral distortion?

Full thermodynamic equilibrium (certainly valid at very high redshift)

- CMB has a blackbody spectrum at every time (not affected by expansion)
- Photon number density and energy density determined by temperature T_γ

$T_\gamma \sim 2.726\,(1+z)$ K

$N_\gamma \sim 411$ cm^{-3} $(1+z)^3 \sim 2\times10^9\,N_b$ (*entropy density dominated by photons*)

$\rho_\gamma \sim 5.1\times10^{-7}\,m_e c^2$ cm^{-3} $(1+z)^4 \sim \rho_b$ x $(1+z)$ / 925 ~ 0.26 eV cm^{-3} $(1+z)^4$

Why should one expect some spectral distortion?

Full thermodynamic equilibrium (certainly valid at very high redshift)

- CMB has a blackbody spectrum at every time (not affected by expansion)
- Photon number density and energy density determined by temperature T_γ

$T_\gamma \sim 2.726\,(1+z)$ K

$N_\gamma \sim 411\ \text{cm}^{-3}\,(1+z)^3 \sim 2\times10^9\,N_\text{b}$ (*entropy density dominated by photons*)

$\rho_\gamma \sim 5.1\times10^{-7}\,m_e c^2\ \text{cm}^{-3}\,(1+z)^4 \sim \rho_\text{b} \times (1+z)\,/\,925 \sim 0.26\ \text{eV cm}^{-3}\,(1+z)^4$

Perturbing full equilibrium by

- Energy injection (interaction *matter* ←→ *photons*)
- Production of (energetic) photons and/or particles (i.e. change of entropy)

→ CMB spectrum deviates from a pure blackbody

→ thermalization process (partially) erases distortions

(Compton scattering, double Compton and Bremsstrahlung in the expanding Universe)

Why should one expect some spectral distortion?

Full thermodynamic equilibrium (certainly valid at very high redshift)

- CMB has a blackbody spectrum at every time (not affected by expansion)
- Photon number density and energy density determined by temperature T_γ

$T_\gamma \sim 2.726\,(1+z)$ K

$N_\gamma \sim 411$ cm^{-3} $(1+z)^3 \sim 2\times10^9\, N_b$ (*entropy density dominated by photons*)

$\rho_\gamma \sim 5.1\times10^{-7}\, m_e c^2$ cm^{-3} $(1+z)^4 \sim \rho_b$ x $(1+z)$ / 925 ~ 0.26 eV cm^{-3} $(1+z)^4$

Perturbing full equilibrium by

- Energy injection (interaction *matter* ←→ *photons*)
- Production of (energetic) photons and/or particles (i.e. change of entropy)

→ CMB spectrum deviates from a pure blackbody

→ thermalization process (partially) erases distortions

(Compton scattering, double Compton and Bremsstrahlung in the expanding Universe)

Measurements of CMB spectrum place very tight constraints on the thermal history of our Universe!

Spectrum of a Temperature Shift

- Which $\Delta n_\nu = c^2 \Delta N_\nu / \nu^2$ is needed to change one blackbody to another?

Spectrum of a Temperature Shift

- Which $\Delta n_\nu = c^2 \Delta N_\nu / \nu^2$ is needed to change one blackbody to another?

$$n_\nu^{\rm bb}(T') \approx n_\nu^{\rm bb}(T) + \frac{\partial n_\nu^{\rm bb}(T)}{\partial T}(T' - T) + \mathcal{O}[(T' - T)^2]$$

Spectrum of a Temperature Shift

- Which $\Delta n_\nu = c^2 \Delta N_\nu / \nu^2$ is needed to change one blackbody to another?

$$n_\nu^{\rm bb}(T') \approx n_\nu^{\rm bb}(T) + \frac{\partial n_\nu^{\rm bb}(T)}{\partial T}\,(T'-T) + \mathcal{O}[(T'-T)^2]$$

$$\Delta n_\nu = \frac{\partial n_\nu^{\rm bb}(T)}{\partial T}(T'-T) = \frac{x\,{\rm e}^x}{({\rm e}^x-1)^2}\,\frac{T'-T}{T} \equiv G(x)\frac{\Delta T}{T}$$

Defines spectrum of a temperature shift!

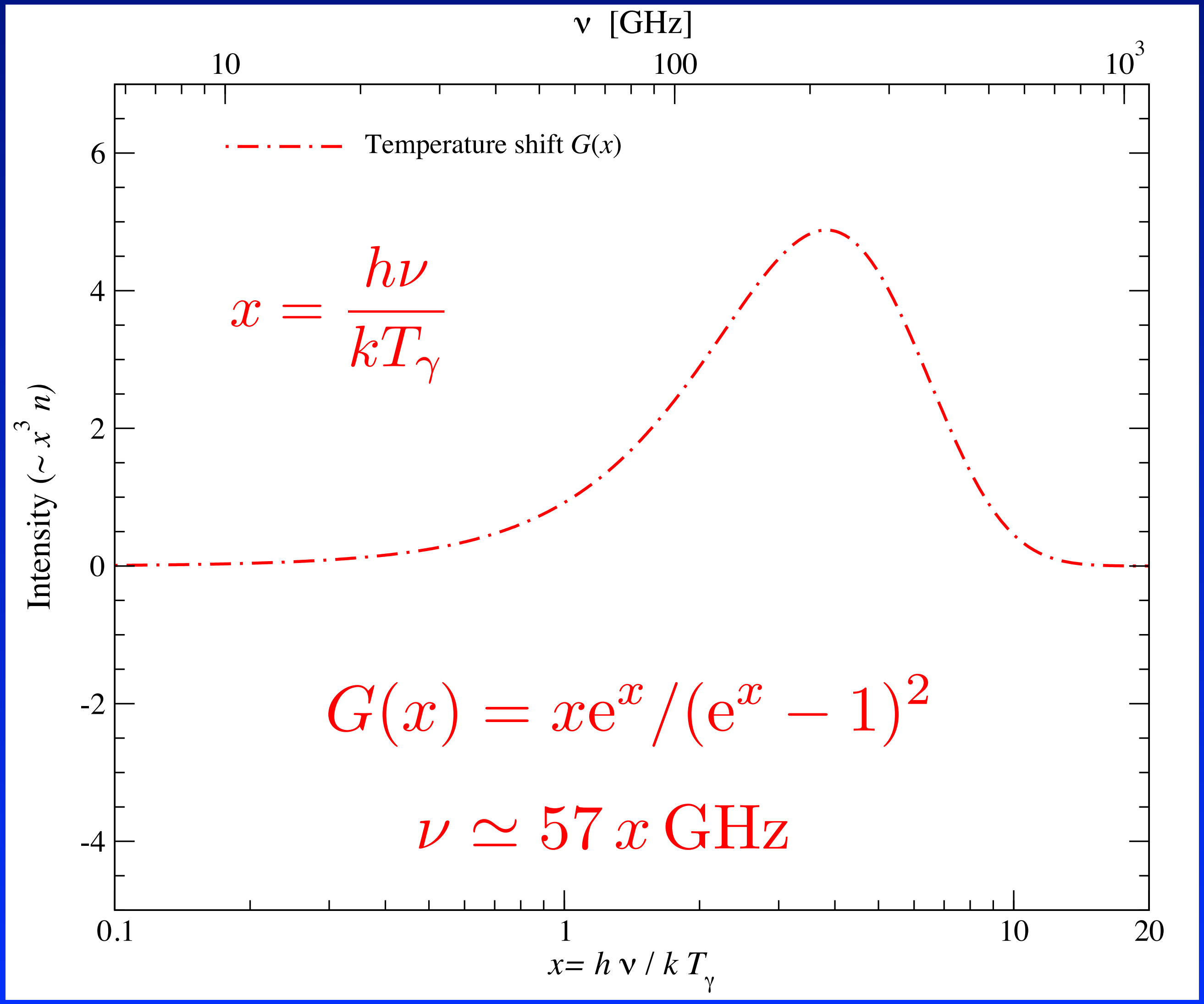
Spectrum of a Temperature Shift

- Which $\Delta n_\nu = c^2 \Delta N_\nu / \nu^2$ is needed to change one blackbody to another?

$$n_\nu^{\rm bb}(T') \approx n_\nu^{\rm bb}(T) + \frac{\partial n_\nu^{\rm bb}(T)}{\partial T}(T'-T) + \mathcal{O}[(T'-T)^2]$$

$$\Delta n_\nu = \frac{\partial n_\nu^{\rm bb}(T)}{\partial T}(T'-T) = \frac{x\,{\rm e}^x}{({\rm e}^x-1)^2}\frac{T'-T}{T} \equiv G(x)\frac{\Delta T}{T}$$

Defines spectrum of a temperature shift!



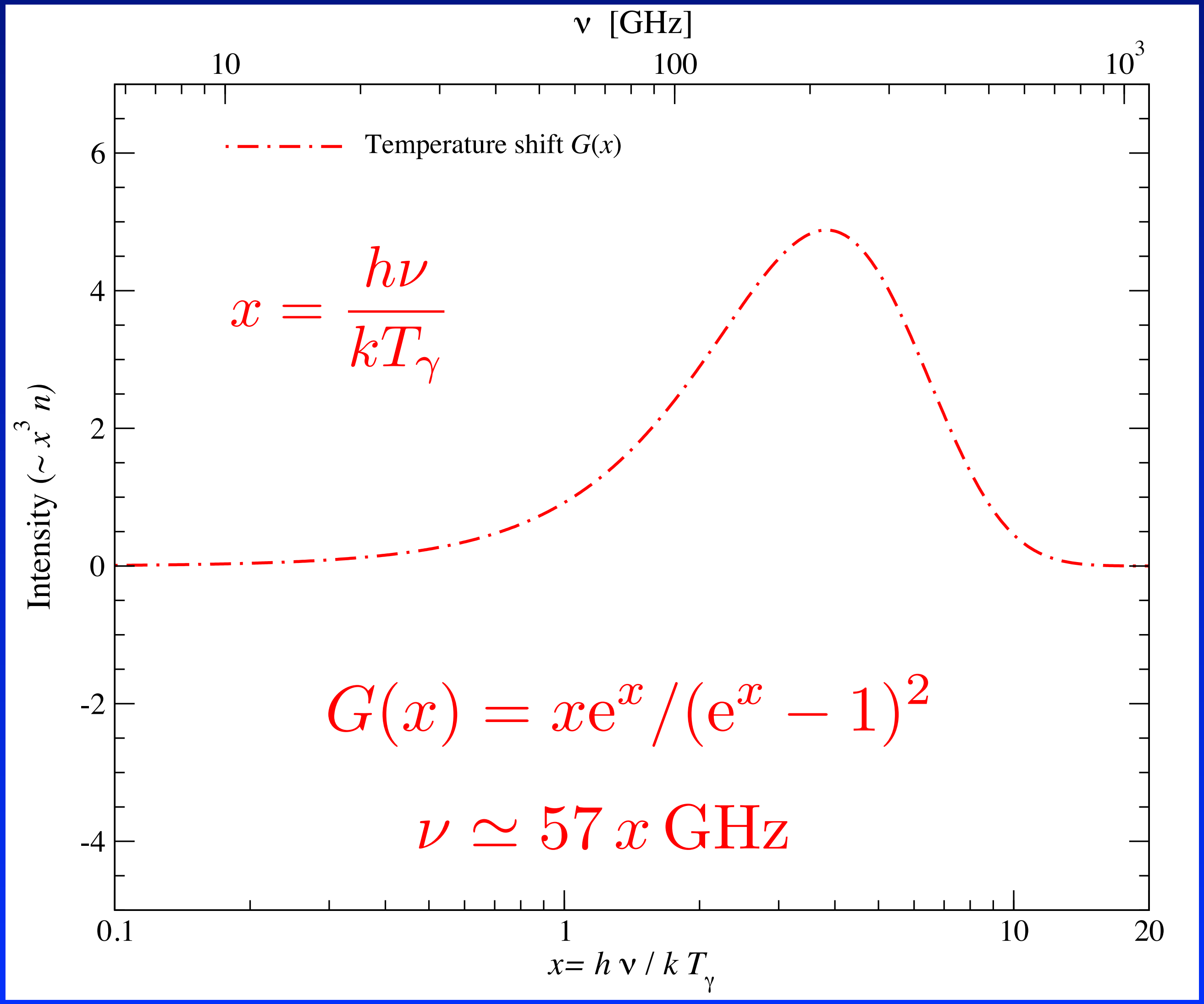
Spectrum of a Temperature Shift

- Which $\Delta n_\nu = c^2 \Delta N_\nu / \nu^2$ is needed to change one blackbody to another?

$$n_\nu^{\rm bb}(T') \approx n_\nu^{\rm bb}(T) + \frac{\partial n_\nu^{\rm bb}(T)}{\partial T}(T' - T) + \mathcal{O}[(T'-T)^2]$$

$$\Delta n_\nu = \frac{\partial n_\nu^{\rm bb}(T)}{\partial T}(T'-T) = \frac{x\,{\rm e}^x}{({\rm e}^x - 1)^2}\,\frac{T'-T}{T} \equiv G(x)\frac{\Delta T}{T}$$

Defines spectrum of a temperature shift!



- Adiabatic condition

$$\frac{\Delta\rho_\gamma}{\rho_\gamma^{\rm bb}} \approx \frac{4}{3}\,\frac{\Delta N_\gamma}{N_\gamma^{\rm bb}}$$

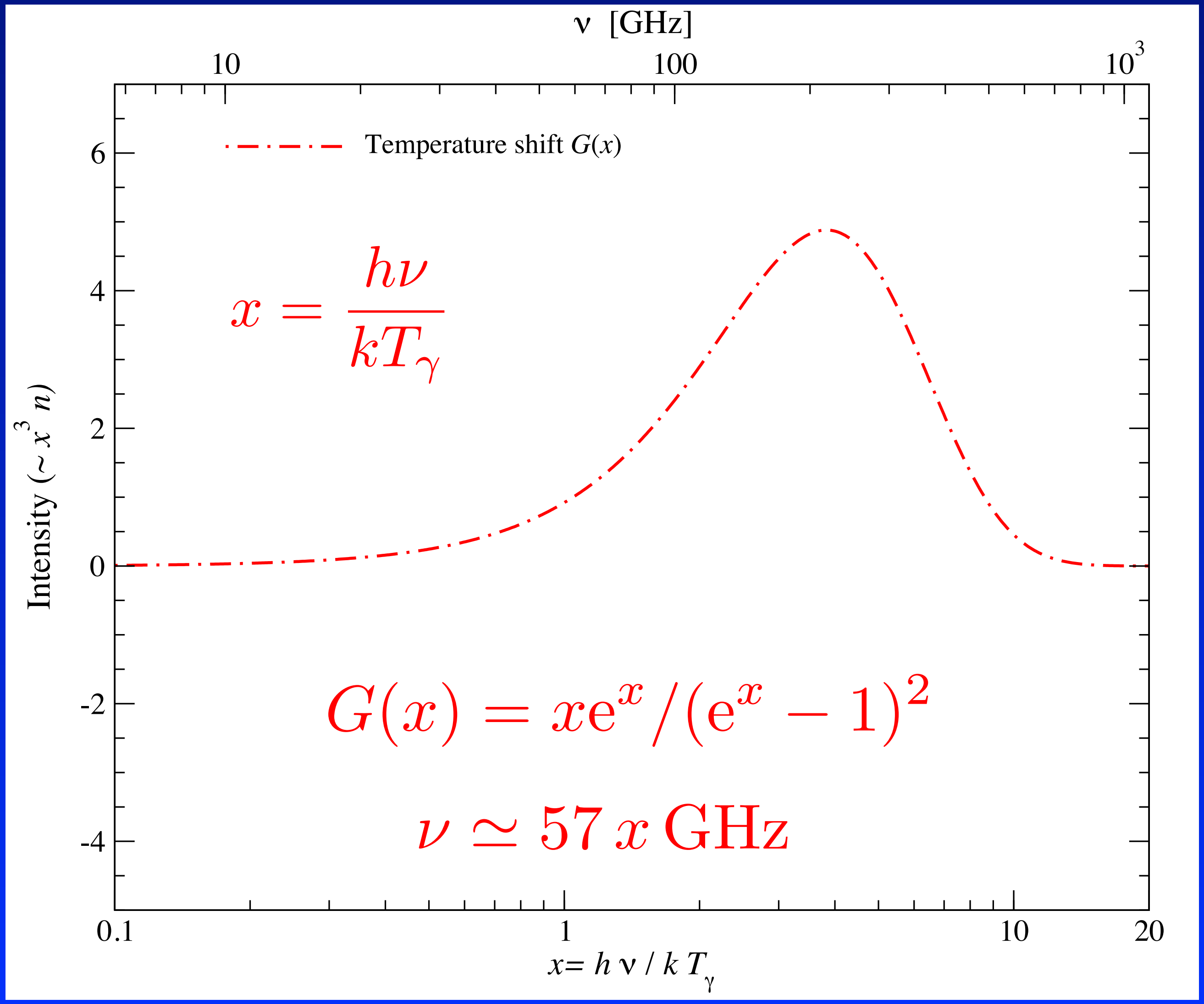
Spectrum of a Temperature Shift

- Which $\Delta n_\nu = c^2 \Delta N_\nu / \nu^2$ is needed to change one blackbody to another?

$$n_\nu^{\rm bb}(T') \approx n_\nu^{\rm bb}(T) + \frac{\partial n_\nu^{\rm bb}(T)}{\partial T}(T'-T) + \mathcal{O}[(T'-T)^2]$$

$$\Delta n_\nu = \frac{\partial n_\nu^{\rm bb}(T)}{\partial T}(T'-T) = \frac{x\,{\rm e}^x}{({\rm e}^x-1)^2}\,\frac{T'-T}{T} \equiv G(x)\frac{\Delta T}{T}$$

Defines spectrum of a temperature shift!



- Adiabatic condition

$$\frac{\Delta\rho_\gamma}{\rho_\gamma^{\rm bb}} \approx \frac{4}{3}\,\frac{\Delta N_\gamma}{N_\gamma^{\rm bb}}$$

- Efficient thermalization ↔ $G(x)$

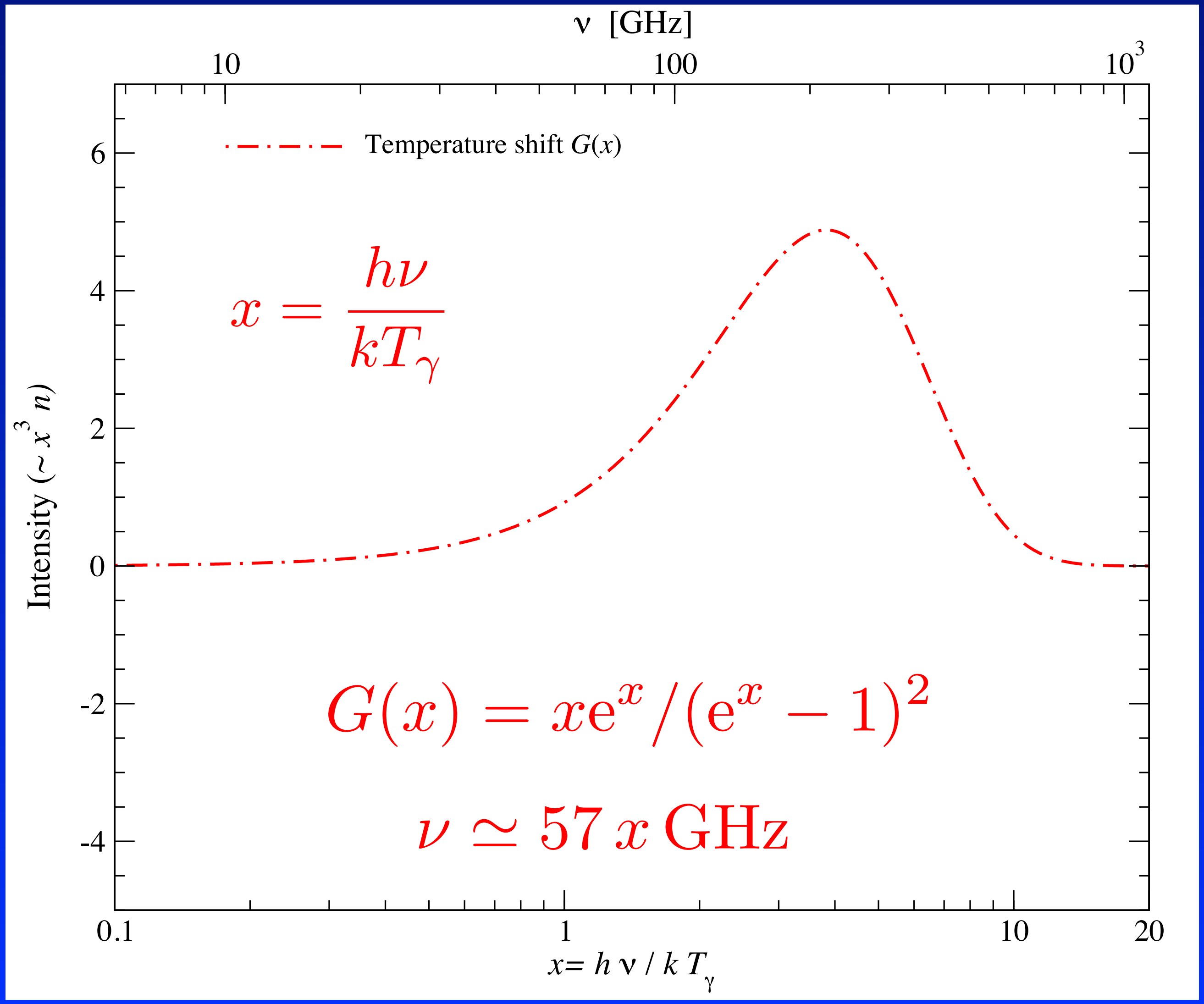
Spectrum of a Temperature Shift

- Which $\Delta n_\nu = c^2 \Delta N_\nu / \nu^2$ is needed to change one blackbody to another?

$$n_\nu^{\rm bb}(T') \approx n_\nu^{\rm bb}(T) + \frac{\partial n_\nu^{\rm bb}(T)}{\partial T}(T'-T) + \mathcal{O}[(T'-T)^2]$$

$$\Delta n_\nu = \frac{\partial n_\nu^{\rm bb}(T)}{\partial T}(T'-T) = \frac{x\,{\rm e}^x}{({\rm e}^x-1)^2}\,\frac{T'-T}{T} \equiv G(x)\frac{\Delta T}{T}$$

Defines spectrum of a temperature shift!



- Adiabatic condition

$$\frac{\Delta\rho_\gamma}{\rho_\gamma^{\rm bb}} \approx \frac{4}{3}\frac{\Delta N_\gamma}{N_\gamma^{\rm bb}}$$

- Efficient thermalization ↔ $G(x)$
- *But: Monopole temperature* does not really teach you anything

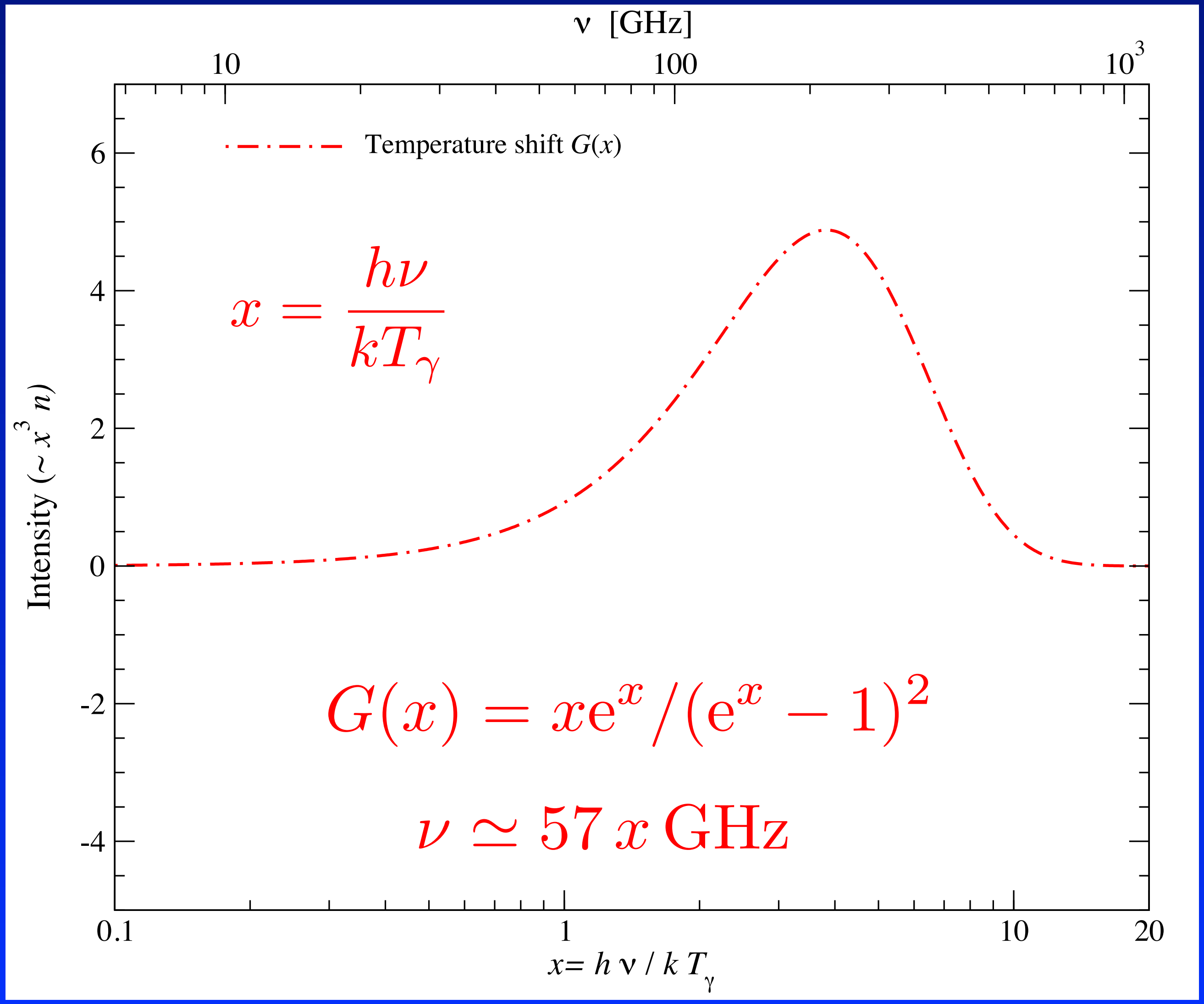
Spectrum of a Temperature Shift

- Which $\Delta n_\nu = c^2 \Delta N_\nu / \nu^2$ is needed to change one blackbody to another?

$$n_\nu^{\rm bb}(T') \approx n_\nu^{\rm bb}(T) + \frac{\partial n_\nu^{\rm bb}(T)}{\partial T}(T'-T) + \mathcal{O}[(T'-T)^2]$$

$$\Delta n_\nu = \frac{\partial n_\nu^{\rm bb}(T)}{\partial T}(T'-T) = \frac{x\,{\rm e}^x}{({\rm e}^x-1)^2}\,\frac{T'-T}{T} \equiv G(x)\frac{\Delta T}{T}$$

Defines spectrum of a temperature shift!



- Adiabatic condition

$$\frac{\Delta\rho_\gamma}{\rho_\gamma^{\rm bb}} \approx \frac{4}{3}\,\frac{\Delta N_\gamma}{N_\gamma^{\rm bb}}$$

- Efficient thermalization ↔ $G(x)$
- *But: Monopole temperature* does not really teach you anything
- (Only lowest order in $\Delta T/T$ → *superposition of blackbodies*)

How does the thermalization process work?

Some important ingredients

- Plasma fully ionized before recombination (z~1000)
  - → free electrons, protons and helium nuclei
  - → photon dominated (~2 Billion photons per baryon)
- Coulomb scattering $e + p \leftrightarrow e' + p$
  - → electrons in full thermal equilibrium with baryons
  - → electrons follow thermal Maxwell-Boltzmann distribution
  - → efficient down to very low redshifts (z ~ 10-100)
- Medium homogeneous and isotropic on large scales
  - → thermalization problem rather simple!
  - → in principle *allows very precise computations*
- Hubble expansion
  - → adiabatic cooling of photons [$T_\gamma \sim (1+z)$] and ordinary matter [$T_m \sim (1+z)^2$]
  - → redshifting of photons

Photon Boltzmann Equation for Average Spectrum

$$\frac{\mathrm{d}n_\nu}{\mathrm{d}t} = \frac{\partial n_\nu}{\partial t} + \frac{\partial n_\nu}{\partial x_i} \cdot \frac{\partial x_i}{\partial t} + \frac{\partial n_\nu}{\partial p} \frac{\partial p}{\partial t} + \frac{\partial n_\nu}{\partial \hat{p}_i} \cdot \frac{\partial \hat{p}_i}{\partial t} = \mathcal{C}[n]$$

Photon occupation number

Photon Boltzmann Equation for Average Spectrum

$$\frac{\mathrm{d}n_\nu}{\mathrm{d}t} = \frac{\partial n_\nu}{\partial t} + \frac{\partial n_\nu}{\partial x_i} \cdot \frac{\partial x_i}{\partial t} + \frac{\partial n_\nu}{\partial p} \frac{\partial p}{\partial t} + \frac{\partial n_\nu}{\partial \hat{p}_i} \cdot \frac{\partial \hat{p}_i}{\partial t} = \mathcal{C}[n]$$

Photon occupation number

Liouville operator

Collision term

Photon Boltzmann Equation for Average Spectrum

$$\frac{\mathrm{d}n_\nu}{\mathrm{d}t} = \frac{\partial n_\nu}{\partial t} + \frac{\partial n_\nu}{\partial x_i} \cdot \frac{\partial x_i}{\partial t} + \frac{\partial n_\nu}{\partial p}\frac{\partial p}{\partial t} + \frac{\partial n_\nu}{\partial \hat{p}_i} \cdot \frac{\partial \hat{p}_i}{\partial t} = \mathcal{C}[n]$$

Photon occupation number

Liouville operator

Collision term

- Isotropy & Homogeneity: $\Longrightarrow \quad \frac{\partial n_\nu}{\partial t} - H\nu\frac{\partial n_\nu}{\partial \nu} = \mathcal{C}[n]$

redshifting term

Photon Boltzmann Equation for Average Spectrum

$$\frac{\mathrm{d}n_\nu}{\mathrm{d}t} = \frac{\partial n_\nu}{\partial t} + \frac{\partial n_\nu}{\partial x_i} \cdot \frac{\partial x_i}{\partial t} + \frac{\partial n_\nu}{\partial p}\frac{\partial p}{\partial t} + \frac{\partial n_\nu}{\partial \hat{p}_i} \cdot \frac{\partial \hat{p}_i}{\partial t} = \mathcal{C}[n]$$

Photon occupation number

Liouville operator

Collision term

- Isotropy & Homogeneity: $\implies \quad \frac{\partial n_\nu}{\partial t} - H\nu\frac{\partial n_\nu}{\partial \nu} = \mathcal{C}[n]$

redshifting term

- Collision term: $\mathcal{C}[n] = \left.\frac{\mathrm{d}n_\nu}{\mathrm{d}t}\right|_{\mathrm{C}} + \left.\frac{\mathrm{d}n_\nu}{\mathrm{d}t}\right|_{\mathrm{BR}} + \left.\frac{\mathrm{d}n_\nu}{\mathrm{d}t}\right|_{\mathrm{DC}}$

Photon Boltzmann Equation for Average Spectrum

$$\frac{\mathrm{d}n_\nu}{\mathrm{d}t} = \frac{\partial n_\nu}{\partial t} + \frac{\partial n_\nu}{\partial x_i}\cdot\frac{\partial x_i}{\partial t} + \frac{\partial n_\nu}{\partial p}\frac{\partial p}{\partial t} + \frac{\partial n_\nu}{\partial \hat{p}_i}\cdot\frac{\partial \hat{p}_i}{\partial t} = \mathcal{C}[n]$$

Photon occupation number — **Liouville operator** — **Collision term**

- Isotropy & Homogeneity: $\Longrightarrow \quad \frac{\partial n_\nu}{\partial t} - H\nu\frac{\partial n_\nu}{\partial \nu} = \mathcal{C}[n]$

redshifting term

- Collision term: $\mathcal{C}[n] = \left.\frac{\mathrm{d}n_\nu}{\mathrm{d}t}\right|_{\mathrm{C}} + \left.\frac{\mathrm{d}n_\nu}{\mathrm{d}t}\right|_{\mathrm{BR}} + \left.\frac{\mathrm{d}n_\nu}{\mathrm{d}t}\right|_{\mathrm{DC}}$

Compton Scattering — Bremsstrahlung — Double Compton

Photon Boltzmann Equation for Average Spectrum

$$\frac{\mathrm{d}n_\nu}{\mathrm{d}t} = \frac{\partial n_\nu}{\partial t} + \frac{\partial n_\nu}{\partial x_i} \cdot \frac{\partial x_i}{\partial t} + \frac{\partial n_\nu}{\partial p}\frac{\partial p}{\partial t} + \frac{\partial n_\nu}{\partial \hat{p}_i} \cdot \frac{\partial \hat{p}_i}{\partial t} = \mathcal{C}[n]$$

Photon occupation number

Liouville operator

Collision term

- Isotropy & Homogeneity: $\Longrightarrow \quad \frac{\partial n_\nu}{\partial t} - H\nu\frac{\partial n_\nu}{\partial \nu} = \mathcal{C}[n]$

redshifting term

- Collision term: $\mathcal{C}[n] = \left.\frac{\mathrm{d}n_\nu}{\mathrm{d}t}\right|_{\mathrm{C}} + \left.\frac{\mathrm{d}n_\nu}{\mathrm{d}t}\right|_{\mathrm{BR}} + \left.\frac{\mathrm{d}n_\nu}{\mathrm{d}t}\right|_{\mathrm{DC}}$

Compton Scattering — Bremsstrahlung — Double Compton

redistribution of photon over frequency

adjusting photon number

Photon Boltzmann Equation for Average Spectrum

$$\frac{\mathrm{d}n_\nu}{\mathrm{d}t} = \frac{\partial n_\nu}{\partial t} + \frac{\partial n_\nu}{\partial x_i} \cdot \frac{\partial x_i}{\partial t} + \frac{\partial n_\nu}{\partial p} \frac{\partial p}{\partial t} + \frac{\partial n_\nu}{\partial \hat{p}_i} \cdot \frac{\partial \hat{p}_i}{\partial t} = \mathcal{C}[n]$$

- Isotropy & Homogeneity: $\Longrightarrow \quad \frac{\partial n_\nu}{\partial t} - H\nu \frac{\partial n_\nu}{\partial \nu} = \mathcal{C}[n]$
- Collision term: $\mathcal{C}[n] = \left.\frac{\mathrm{d}n_\nu}{\mathrm{d}t}\right|_{\mathrm{C}} + \left.\frac{\mathrm{d}n_\nu}{\mathrm{d}t}\right|_{\mathrm{BR}} + \left.\frac{\mathrm{d}n_\nu}{\mathrm{d}t}\right|_{\mathrm{DC}}$
- Full equilibrium: $\mathcal{C}[n] \equiv 0 \;\Rightarrow$ blackbody spectrum conserved

Photon Boltzmann Equation for Average Spectrum

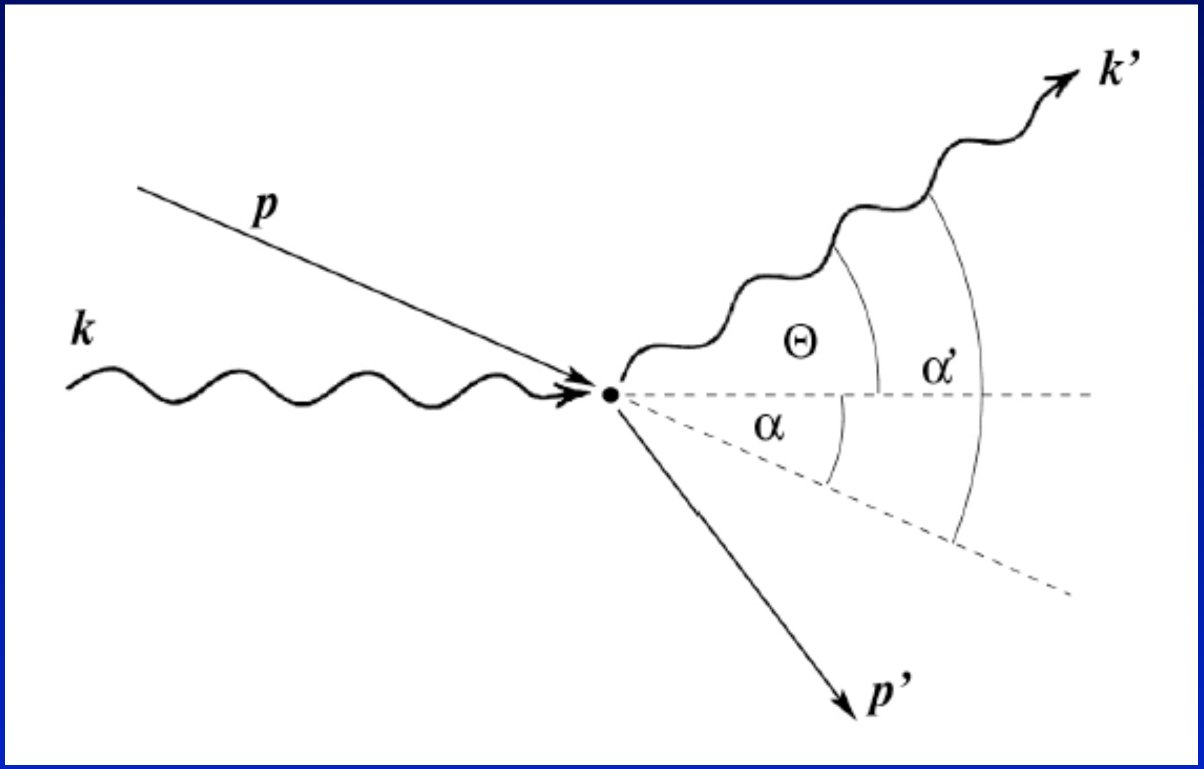
$$\frac{\mathrm{d}n_\nu}{\mathrm{d}t} = \frac{\partial n_\nu}{\partial t} + \frac{\partial n_\nu}{\partial x_i} \cdot \frac{\partial x_i}{\partial t} + \frac{\partial n_\nu}{\partial p} \frac{\partial p}{\partial t} + \frac{\partial n_\nu}{\partial \hat{p}_i} \cdot \frac{\partial \hat{p}_i}{\partial t} = \mathcal{C}[n]$$

- Isotropy & Homogeneity: $\Longrightarrow \quad \frac{\partial n_\nu}{\partial t} - H\nu \frac{\partial n_\nu}{\partial \nu} = \mathcal{C}[n]$
- Collision term: $\mathcal{C}[n] = \left.\frac{\mathrm{d}n_\nu}{\mathrm{d}t}\right|_{\mathrm{C}} + \left.\frac{\mathrm{d}n_\nu}{\mathrm{d}t}\right|_{\mathrm{BR}} + \left.\frac{\mathrm{d}n_\nu}{\mathrm{d}t}\right|_{\mathrm{DC}}$
- Full equilibrium: $\mathcal{C}[n] \equiv 0 \;\Rightarrow$ blackbody spectrum conserved
- Energy release: $\mathcal{C}[n] \neq 0 \;\Rightarrow$ *thermalization process starts*

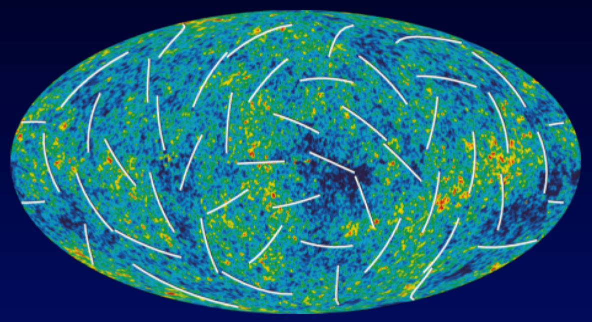
Compton scattering

Redistribution of photons by Compton scattering

- Reaction: $\gamma + e \longleftrightarrow \gamma' + e'$



Redistribution of photons by Compton scattering

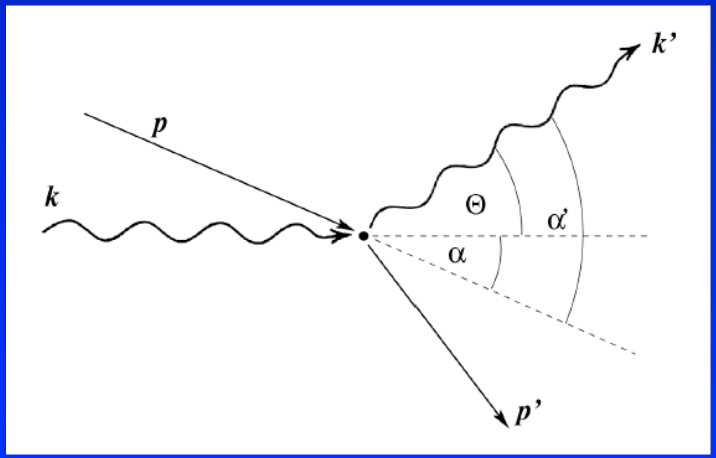


- Reaction: $\gamma + e \longleftrightarrow \gamma' + e'$

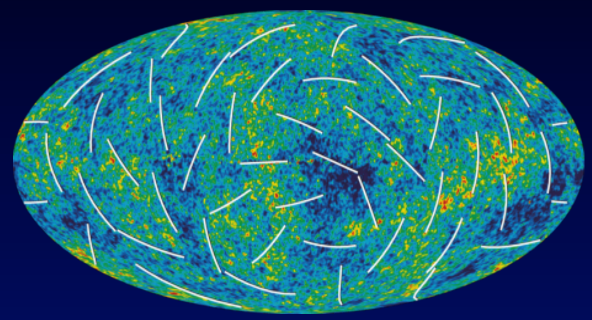
→ *no energy exchange* ⇒ *Thomson limit*

⇒ *important for anisotropies*

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \frac{3\sigma_{\mathrm{T}}}{16\pi}\left[1 + (\hat{\gamma}\cdot\hat{\gamma}')^2\right]$$



Redistribution of photons by Compton scattering



- Reaction: $\gamma + e \longleftrightarrow \gamma' + e'$

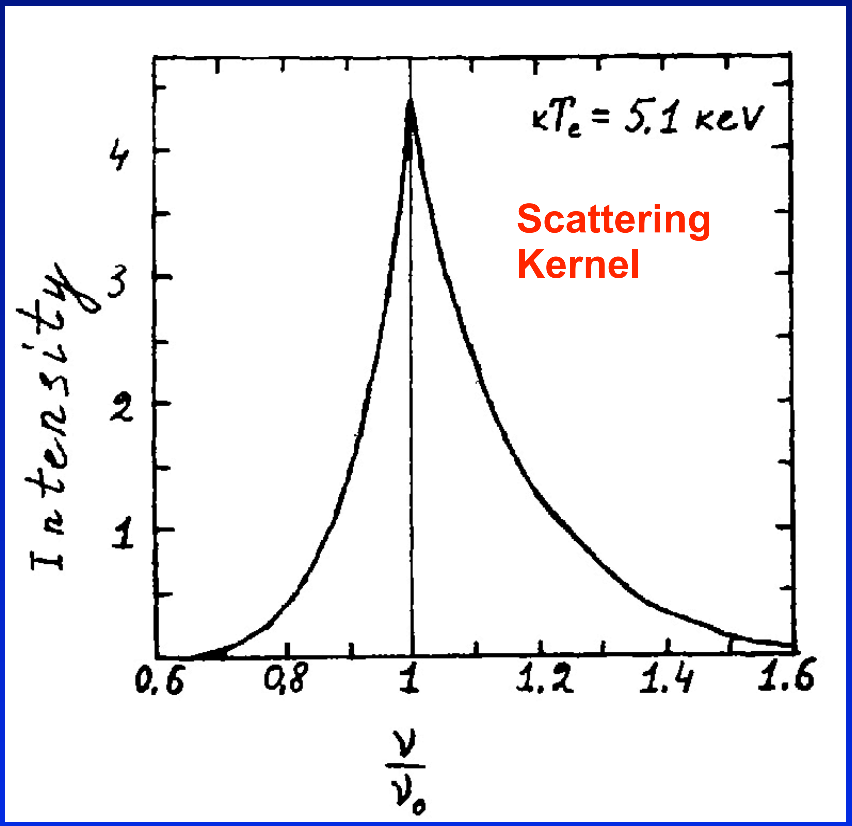
→ *no energy exchange* ⇒ *Thomson limit*

⇒ *important for anisotropies*

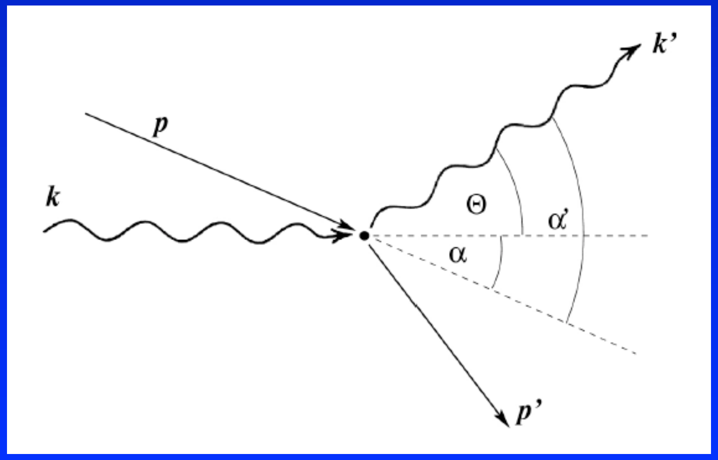
$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \frac{3\sigma_\mathrm{T}}{16\pi}\left[1 + (\hat{\gamma}\cdot\hat{\gamma}')^2\right]$$

→ *energy exchange included*

- up-scattering due to the ***Doppler*** effect for $h\nu < 4kT_\mathrm{e}$
- down-scattering because of ***recoil*** (and stimulated recoil) for $h\nu > 4kT_\mathrm{e}$
- ***Doppler*** broadening $\frac{\Delta\nu}{\nu} \simeq \sqrt{\frac{2kT_\mathrm{e}}{m_\mathrm{e}c^2}}$

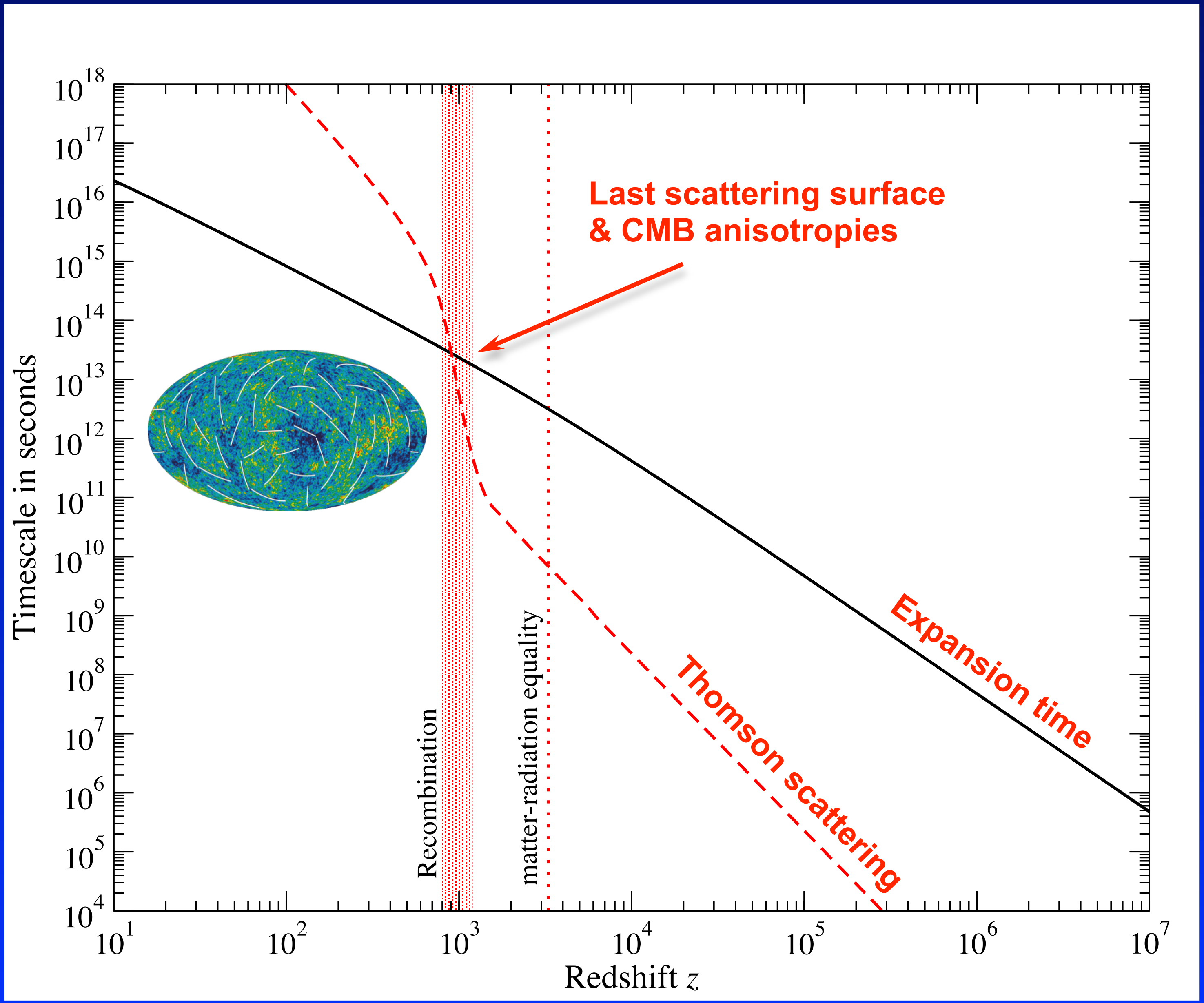


Sunyaev & Zeldovich, 1980, ARAA, 18, 537



Important Time-scales for Compton Process

- *Thomson scattering* $t_{\rm C} = (\sigma_{\rm T} N_{\rm e} c)^{-1} \approx 2.3 \times 10^{20} \, \chi_{\rm e}^{-1} (1+z)^{-3} \, {\rm sec}$



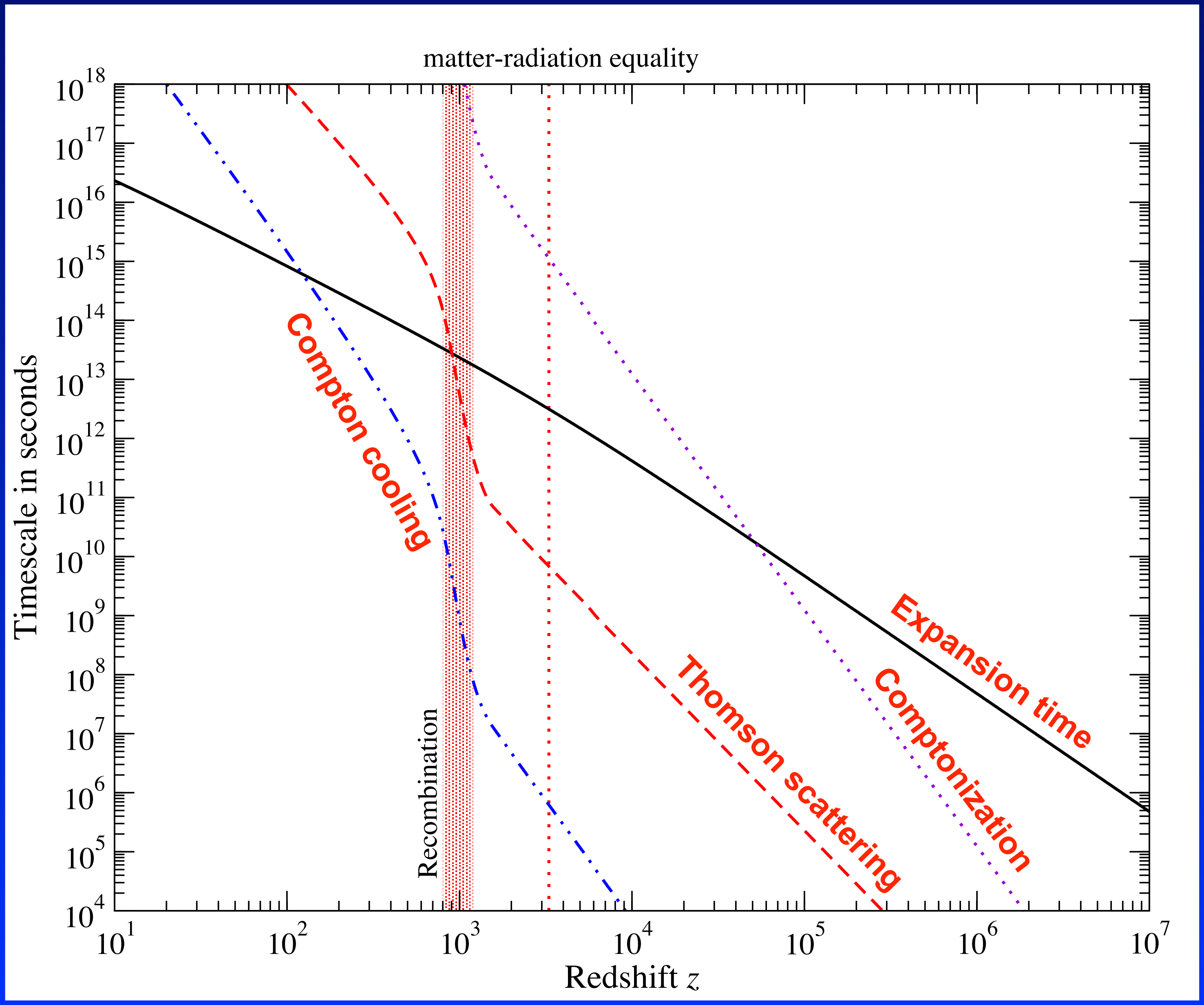
Radiation dominated

$$t_{\rm exp} = H^{-1} \simeq 4.8 \times 10^{19} (1+z)^{-2} \, {\rm sec}$$
$$\simeq 8.4 \times 10^{17} (1+z)^{-3/2} \, {\rm sec}$$

Matter dominated

Important Time-scales for Compton Process

- *Thomson scattering* $t_{\rm C} = (\sigma_{\rm T} N_{\rm e} c)^{-1} \approx 2.3 \times 10^{20}\, \chi_{\rm e}^{-1} (1+z)^{-3}\,{\rm sec}$
- *Comptonization* $t_{\rm K} = \left(4 \frac{kT_{\rm e}}{m_{\rm e}c^2} \sigma_{\rm T} N_{\rm e} c\right)^{-1} \approx 1.2 \times 10^{29}\, \chi_{\rm e}^{-1} (T_{\rm e}/T_\gamma)^{-1} (1+z)^{-4}\,{\rm sec}$
- *Compton cooling* $t_{\rm cool} = \left(\frac{4\rho_\gamma}{m_{\rm e}c^2} \frac{\sigma_{\rm T} N_{\rm e} c}{(3/2)N}\right)^{-1} \approx 7.1 \times 10^{19}\, \chi_{\rm e}^{-1} (T_{\rm e}/T_\gamma)^{-1} (1+z)^{-4}\,{\rm sec}$



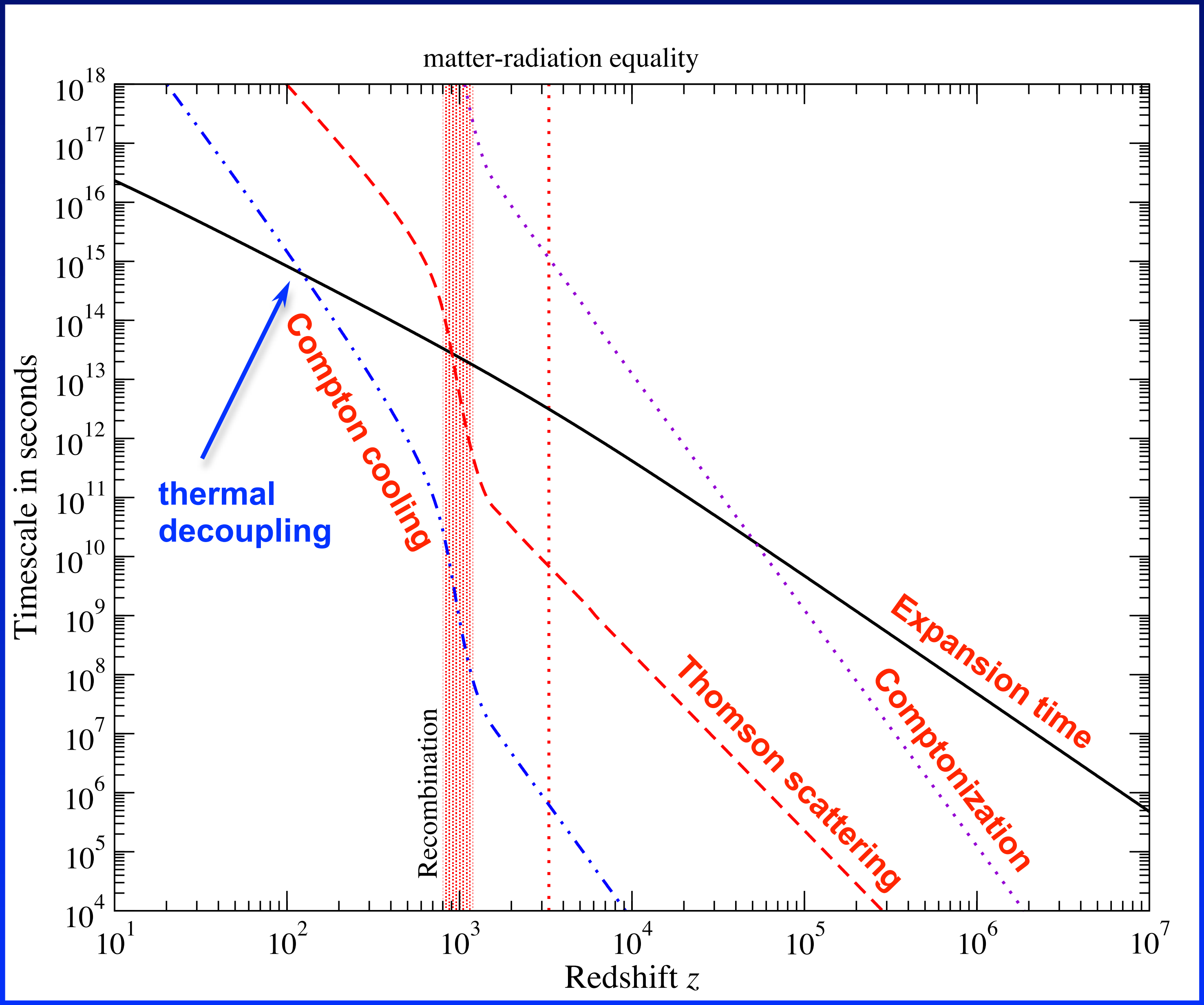
Radiation dominated

$$t_{\rm exp} = H^{-1} \simeq 4.8 \times 10^{19} (1+z)^{-2}\,{\rm sec}$$
$$\simeq 8.4 \times 10^{17} (1+z)^{-3/2}\,{\rm sec}$$

Matter dominated

Important Time-scales for Compton Process

- *Thomson scattering* $t_{\rm C} = (\sigma_{\rm T} N_{\rm e} c)^{-1} \approx 2.3 \times 10^{20}\, \chi_{\rm e}^{-1} (1+z)^{-3}\,{\rm sec}$
- *Comptonization* $t_{\rm K} = \left(4 \frac{kT_{\rm e}}{m_{\rm e}c^2} \sigma_{\rm T} N_{\rm e} c\right)^{-1} \approx 1.2 \times 10^{29}\, \chi_{\rm e}^{-1} (T_{\rm e}/T_\gamma)^{-1} (1+z)^{-4}\,{\rm sec}$
- *Compton cooling* $t_{\rm cool} = \left(\frac{4\rho_\gamma}{m_{\rm e}c^2} \frac{\sigma_{\rm T} N_{\rm e} c}{(3/2)N}\right)^{-1} \approx 7.1 \times 10^{19}\, \chi_{\rm e}^{-1} (T_{\rm e}/T_\gamma)^{-1} (1+z)^{-4}\,{\rm sec}$



- *matter temperature starts deviating from Compton equilibrium temperature at z ≲ 100-200*

Radiation dominated

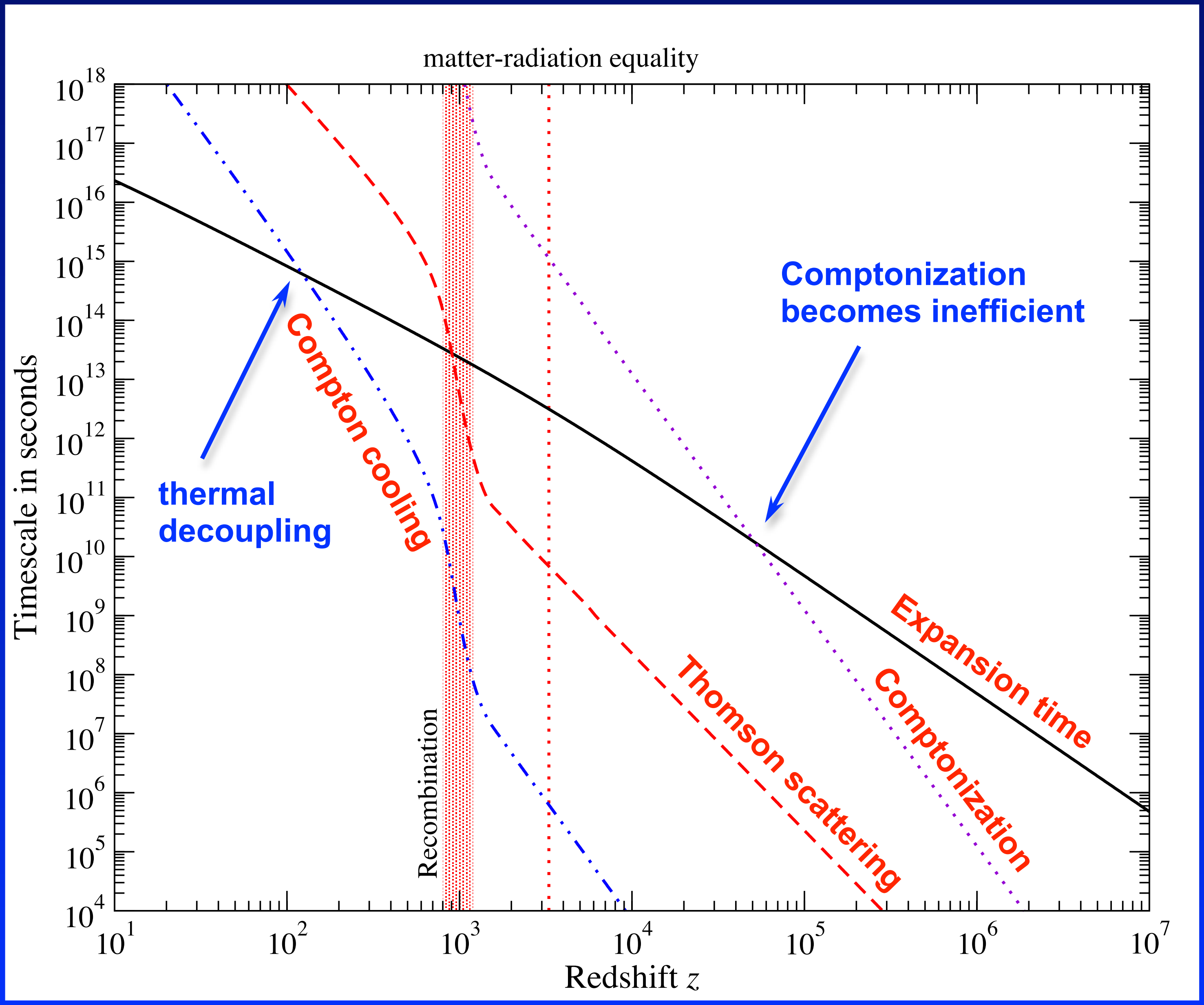
$$t_{\rm exp} = H^{-1} \simeq 4.8 \times 10^{19} (1+z)^{-2}\,{\rm sec}$$

$$\simeq 8.4 \times 10^{17} (1+z)^{-3/2}\,{\rm sec}$$

Matter dominated

Important Time-scales for Compton Process

- *Thomson scattering* $t_\mathrm{C} = (\sigma_\mathrm{T} N_\mathrm{e} c)^{-1} \approx 2.3 \times 10^{20}\, \chi_\mathrm{e}^{-1} (1+z)^{-3}\,\mathrm{sec}$
- *Comptonization* $t_\mathrm{K} = \left(4 \frac{kT_\mathrm{e}}{m_\mathrm{e} c^2} \sigma_\mathrm{T} N_\mathrm{e} c\right)^{-1} \approx 1.2 \times 10^{29}\, \chi_\mathrm{e}^{-1} (T_\mathrm{e}/T_\gamma)^{-1} (1+z)^{-4}\,\mathrm{sec}$
- *Compton cooling* $t_\mathrm{cool} = \left(\frac{4\rho_\gamma}{m_\mathrm{e} c^2} \frac{\sigma_\mathrm{T} N_\mathrm{e} c}{(3/2)N}\right)^{-1} \approx 7.1 \times 10^{19}\, \chi_\mathrm{e}^{-1} (T_\mathrm{e}/T_\gamma)^{-1} (1+z)^{-4}\,\mathrm{sec}$



- *matter temperature starts deviating from Compton equilibrium temperature at z ≲ 100-200*
- *Comptonization becomes inefficient at $z_\mathrm{K} \approx 50000$*

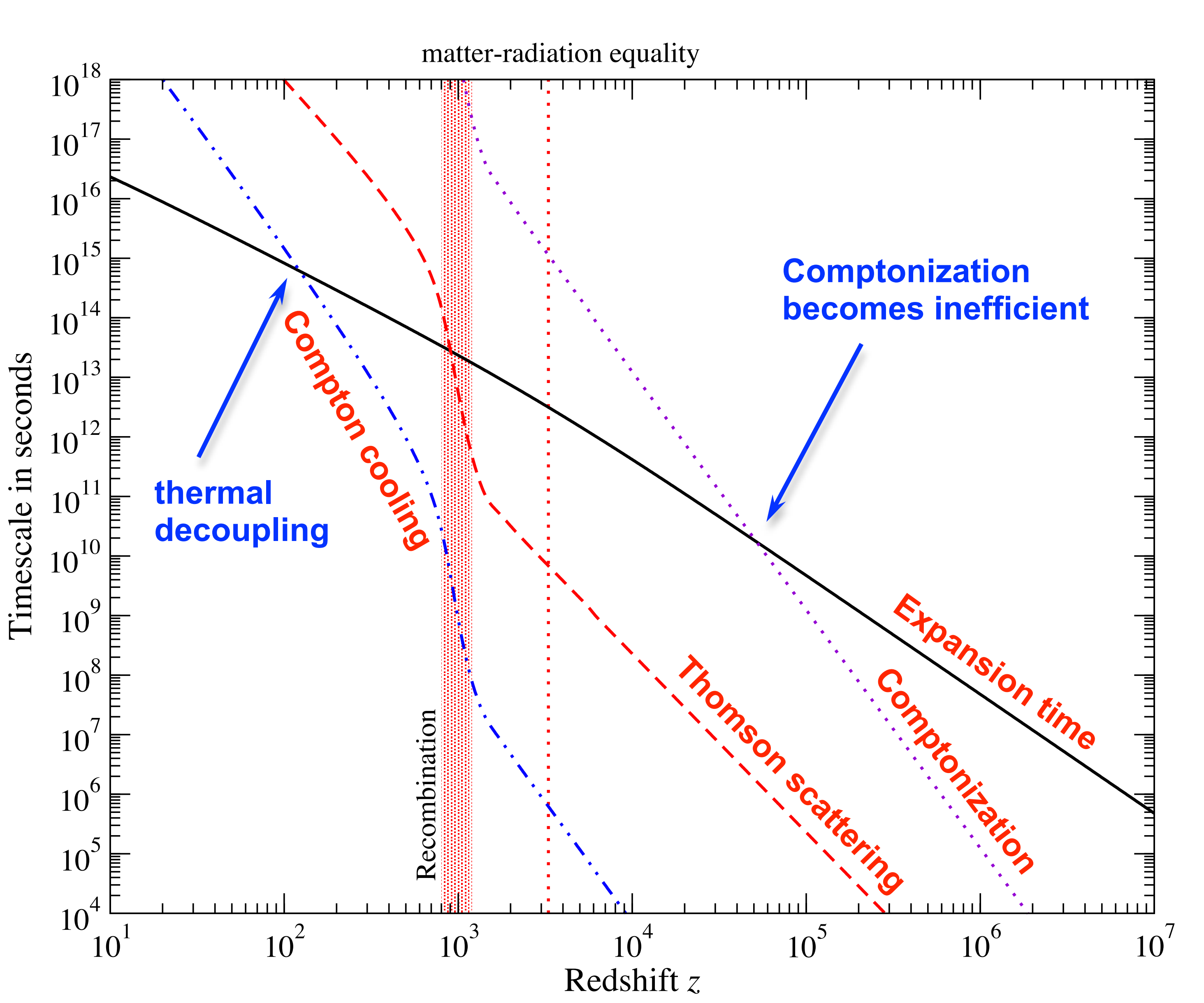
Radiation dominated

$$t_\mathrm{exp} = H^{-1} \simeq 4.8 \times 10^{19} (1+z)^{-2}\,\mathrm{sec}$$
$$\simeq 8.4 \times 10^{17} (1+z)^{-3/2}\,\mathrm{sec}$$

Matter dominated

Important Time-scales for Compton Process

- *Thomson scattering* $t_{\rm C} = (\sigma_{\rm T} N_{\rm e} c)^{-1} \approx 2.3 \times 10^{20}\, \chi_{\rm e}^{-1} (1+z)^{-3}\,\text{sec}$
- *Comptonization* $t_{\rm K} = \left(4 \frac{kT_{\rm e}}{m_{\rm e}c^2} \sigma_{\rm T} N_{\rm e} c\right)^{-1} \approx 1.2 \times 10^{29}\, \chi_{\rm e}^{-1} (T_{\rm e}/T_\gamma)^{-1} (1+z)^{-4}\,\text{sec}$
- *Compton cooling* $t_{\rm cool} = \left(\frac{4\rho_\gamma}{m_{\rm e}c^2} \frac{\sigma_{\rm T} N_{\rm e} c}{(3/2)N}\right)^{-1} \approx 7.1 \times 10^{19}\, \chi_{\rm e}^{-1} (T_{\rm e}/T_\gamma)^{-1} (1+z)^{-4}\,\text{sec}$



- *matter temperature starts deviating from Compton equilibrium temperature at z ≲ 100-200*
- *Comptonization becomes inefficient at $z_K \simeq 50000$*

⇒ character of distortion should change at z_K !

Radiation dominated

$$t_{\rm exp} = H^{-1} \simeq 4.8 \times 10^{19} (1+z)^{-2}\,\text{sec}$$
$$\simeq 8.4 \times 10^{17} (1+z)^{-3/2}\,\text{sec}$$

Matter dominated

What are y- and μ-distortions?

Compton y-distortion / thermal SZ effect

Compton y-distortion / thermal SZ effect

- *Kompaneets equation:*

$$\left.\frac{\mathrm{d}n}{\mathrm{d}\tau}\right|_{\mathrm{C}} \approx \frac{\theta_{\mathrm{e}}}{x^2}\frac{\partial}{\partial x}x^4\left[\frac{\partial n}{\partial x}+\frac{T_\gamma}{T_{\mathrm{e}}}\,n(1+n)\right]$$

Compton y-distortion / thermal SZ effect

- *Kompaneets equation:* $$\left.\frac{\mathrm{d}n}{\mathrm{d}\tau}\right|_{\mathrm{C}} \approx \frac{\theta_{\mathrm{e}}}{x^2}\frac{\partial}{\partial x}x^4\left[\frac{\partial n}{\partial x}+\frac{T_\gamma}{T_{\mathrm{e}}}\,n(1+n)\right]$$
- *insert:* $n \approx n^{\mathrm{bb}} = 1/(\mathrm{e}^x - 1)$

Compton y-distortion / thermal SZ effect

- *Kompaneets equation:* $$\left.\frac{\mathrm{d}n}{\mathrm{d}\tau}\right|_{\mathrm{C}} \approx \frac{\theta_{\mathrm{e}}}{x^2}\frac{\partial}{\partial x}x^4\left[\frac{\partial n}{\partial x} + \frac{T_\gamma}{T_{\mathrm{e}}}n(1+n)\right]$$
- *insert:* $n \approx n^{\mathrm{bb}} = 1/(\mathrm{e}^x - 1) \quad \Longrightarrow \quad \Delta n \approx y\,Y(x)$ *with* $y \ll 1$

Compton y-distortion / thermal SZ effect

- *Kompaneets equation:* $$\left.\frac{\mathrm{d}n}{\mathrm{d}\tau}\right|_{\mathrm{C}} \approx \frac{\theta_{\mathrm{e}}}{x^2}\frac{\partial}{\partial x}x^4\left[\frac{\partial n}{\partial x}+\frac{T_\gamma}{T_{\mathrm{e}}}n(1+n)\right]$$
- *insert:* $n \approx n^{\mathrm{bb}} = 1/(\mathrm{e}^x - 1) \quad \Longrightarrow \quad \Delta n \approx y\,Y(x)$ *with* $y \ll 1$

$$y = \int \frac{k[T_{\mathrm{e}} - T_\gamma]}{m_{\mathrm{e}}c^2}\,\sigma_{\mathrm{T}} N_{\mathrm{e}} c\,\mathrm{d}t$$

Compton y-parameter

Compton y-distortion / thermal SZ effect

- *Kompaneets equation:* $$\left.\frac{\mathrm{d}n}{\mathrm{d}\tau}\right|_{\mathrm{C}} \approx \frac{\theta_{\mathrm{e}}}{x^2}\frac{\partial}{\partial x}x^4\left[\frac{\partial n}{\partial x}+\frac{T_\gamma}{T_{\mathrm{e}}}n(1+n)\right]$$

- *insert:* $n \approx n^{\mathrm{bb}} = 1/(\mathrm{e}^x-1) \quad \Longrightarrow \quad \Delta n \approx y\,Y(x)$ *with* $y \ll 1$

$$y = \int \frac{k[T_{\mathrm{e}}-T_\gamma]}{m_{\mathrm{e}}c^2}\sigma_{\mathrm{T}}N_{\mathrm{e}}c\,\mathrm{d}t \qquad\qquad Y(x) = \frac{x\mathrm{e}^x}{(\mathrm{e}^x-1)^2}\left[x\frac{\mathrm{e}^x+1}{\mathrm{e}^x-1}-4\right]$$

Compton y-parameter — *spectrum of y-distortion*

Compton y-distortion / thermal SZ effect

- *Kompaneets equation:* $$\left.\frac{\mathrm{d}n}{\mathrm{d}\tau}\right|_{\mathrm{C}} \approx \frac{\theta_{\mathrm{e}}}{x^2}\frac{\partial}{\partial x}x^4\left[\frac{\partial n}{\partial x} + \frac{T_\gamma}{T_{\mathrm{e}}}n(1+n)\right]$$
- *insert:* $n \approx n^{\mathrm{bb}} = 1/(\mathrm{e}^x - 1) \quad \Longrightarrow \quad \Delta n \approx y\,Y(x)$ *with* $y \ll 1$

$$y = \int \frac{k[T_{\mathrm{e}} - T_\gamma]}{m_{\mathrm{e}}c^2}\sigma_{\mathrm{T}}N_{\mathrm{e}}c\,\mathrm{d}t \qquad Y(x) = \frac{x\mathrm{e}^x}{(\mathrm{e}^x-1)^2}\left[x\frac{\mathrm{e}^x+1}{\mathrm{e}^x-1} - 4\right]$$

Compton y-parameter *spectrum of y-distortion*

- *if* $T_{\mathrm{e}} = T_\gamma \quad \Longrightarrow \quad \left.\frac{\mathrm{d}n}{\mathrm{d}\tau}\right|_{\mathrm{C}} = 0$ *(thermal equilibrium with electrons)*

Compton y-distortion / thermal SZ effect

- *Kompaneets equation:* $$\left.\frac{\mathrm{d}n}{\mathrm{d}\tau}\right|_{\rm C} \approx \frac{\theta_{\rm e}}{x^2}\frac{\partial}{\partial x}x^4\left[\frac{\partial n}{\partial x}+\frac{T_\gamma}{T_{\rm e}}n(1+n)\right]$$
- *insert:* $n \approx n^{\rm bb} = 1/(\mathrm{e}^x - 1) \quad \Longrightarrow \quad \Delta n \approx y\,Y(x)$ *with* $y \ll 1$

$$y = \int \frac{k[T_{\rm e} - T_\gamma]}{m_{\rm e}c^2}\,\sigma_{\rm T} N_{\rm e} c\,\mathrm{d}t \qquad Y(x) = \frac{x\mathrm{e}^x}{(\mathrm{e}^x-1)^2}\left[x\frac{\mathrm{e}^x+1}{\mathrm{e}^x-1} - 4\right]$$

Compton y-parameter — *spectrum of y-distortion*

- *if* $T_{\rm e} = T_\gamma \quad \Longrightarrow \quad \left.\frac{\mathrm{d}n}{\mathrm{d}\tau}\right|_{\rm C} = 0$ *(thermal equilibrium with electrons)*
- *if* $T_{\rm e} < T_\gamma \quad \Longrightarrow$ *down-scattering of photons / heating of electrons*

Compton y-distortion / thermal SZ effect

- *Kompaneets equation:* $$\left.\frac{\mathrm{d}n}{\mathrm{d}\tau}\right|_{\mathrm{C}} \approx \frac{\theta_{\mathrm{e}}}{x^2}\frac{\partial}{\partial x}x^4\left[\frac{\partial n}{\partial x} + \frac{T_\gamma}{T_{\mathrm{e}}}n(1+n)\right]$$
- *insert:* $n \approx n^{\mathrm{bb}} = 1/(\mathrm{e}^x - 1) \quad \Longrightarrow \quad \Delta n \approx y\,Y(x)$ *with* $y \ll 1$

$$y = \int \frac{k[T_{\mathrm{e}} - T_\gamma]}{m_{\mathrm{e}}c^2}\sigma_{\mathrm{T}} N_{\mathrm{e}} c\,\mathrm{d}t \qquad\qquad Y(x) = \frac{x\mathrm{e}^x}{(\mathrm{e}^x-1)^2}\left[x\frac{\mathrm{e}^x+1}{\mathrm{e}^x-1} - 4\right]$$

Compton y-parameter — *spectrum of y-distortion*

- *if* $T_{\mathrm{e}} = T_\gamma \quad \Longrightarrow \quad \left.\frac{\mathrm{d}n}{\mathrm{d}\tau}\right|_{\mathrm{C}} = 0$ *(thermal equilibrium with electrons)*
- *if* $T_{\mathrm{e}} < T_\gamma \quad \Longrightarrow$ *down-scattering of photons / heating of electrons*
- *if* $T_{\mathrm{e}} > T_\gamma \quad \Longrightarrow$ *up-scattering of photons / cooling of electrons*

Compton y-distortion / thermal SZ effect

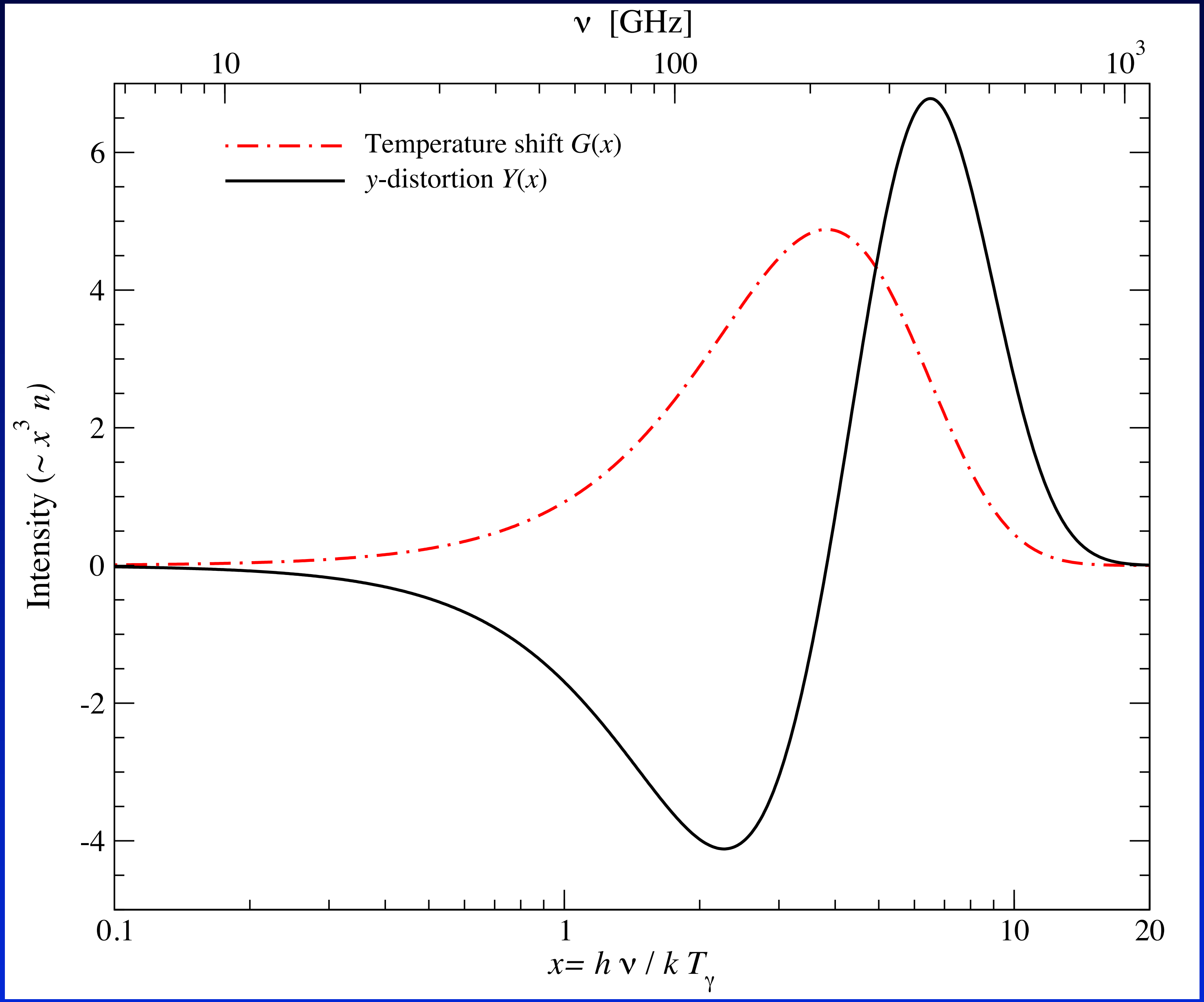
- *Kompaneets equation:* $$\left.\frac{\mathrm{d}n}{\mathrm{d}\tau}\right|_{\mathrm{C}} \approx \frac{\theta_{\mathrm{e}}}{x^2}\frac{\partial}{\partial x}x^4\left[\frac{\partial n}{\partial x} + \frac{T_\gamma}{T_{\mathrm{e}}}n(1+n)\right]$$
- *insert:* $n \approx n^{\mathrm{bb}} = 1/(\mathrm{e}^x - 1) \quad \Longrightarrow \quad \Delta n \approx y\,Y(x)$ *with* $y \ll 1$

$$y = \int \frac{k[T_{\mathrm{e}} - T_\gamma]}{m_{\mathrm{e}}c^2}\,\sigma_{\mathrm{T}}N_{\mathrm{e}}c\,\mathrm{d}t \qquad Y(x) = \frac{x\mathrm{e}^x}{(\mathrm{e}^x-1)^2}\left[x\frac{\mathrm{e}^x+1}{\mathrm{e}^x-1} - 4\right]$$

Compton y-parameter — *spectrum of y-distortion*

- *if* $T_{\mathrm{e}} = T_\gamma \quad \Longrightarrow \quad \left.\frac{\mathrm{d}n}{\mathrm{d}\tau}\right|_{\mathrm{C}} = 0$ *(thermal equilibrium with electrons)*
- *if* $T_{\mathrm{e}} < T_\gamma \quad \Longrightarrow$ *down-scattering of photons / heating of electrons*
- *if* $T_{\mathrm{e}} > T_\gamma \quad \Longrightarrow$ *up-scattering of photons / cooling of electrons*
- *for* $T_{\mathrm{e}} \gg T_\gamma \quad \Longrightarrow$ *thermal Sunyaev-Zeldovich effect (up-scattering)*

Temperature shift ↔ y-distortion



$$Y(x) = \frac{x e^x}{(e^x - 1)^2}\left[x\frac{e^x + 1}{e^x - 1} - 4\right]$$

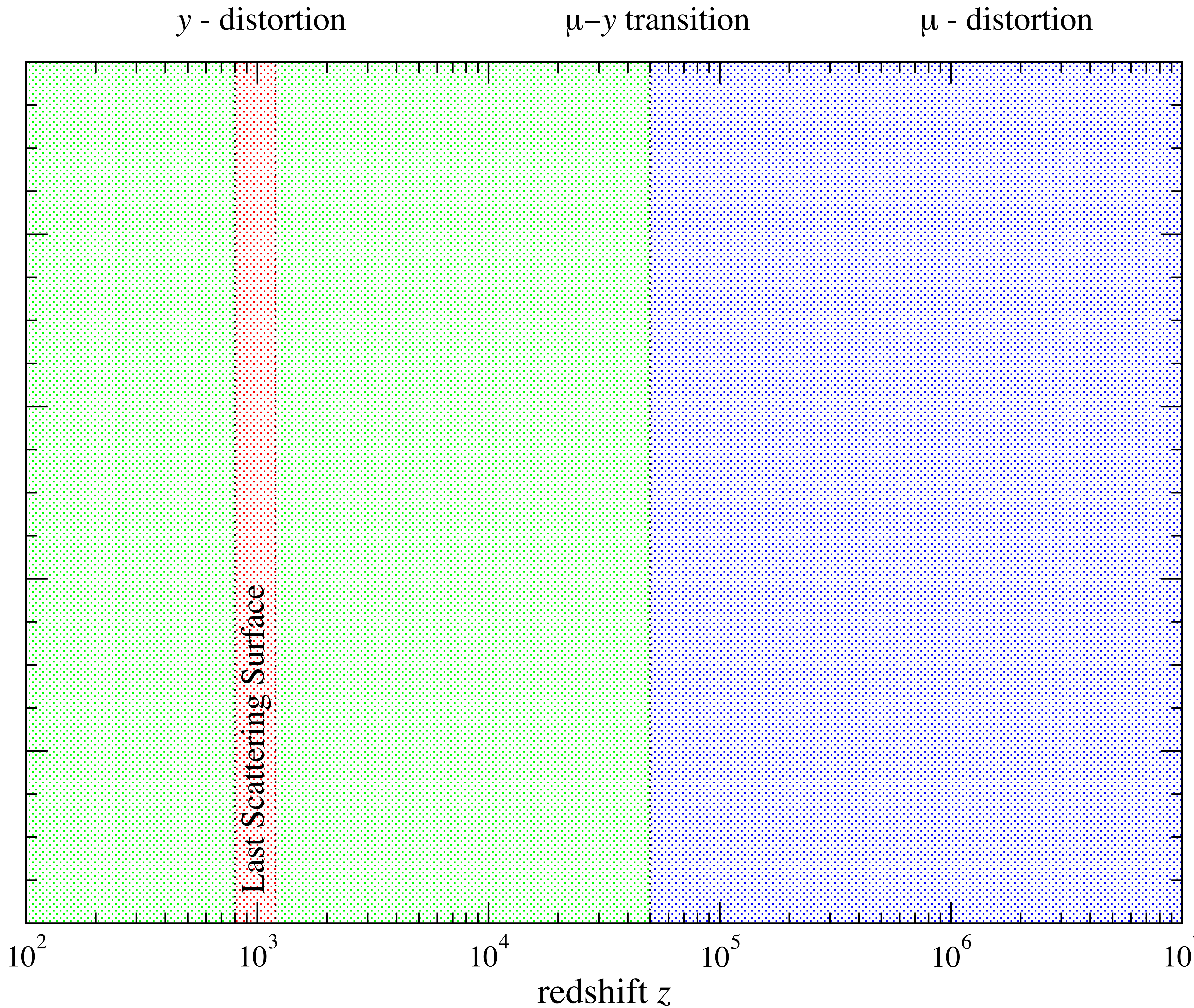
- thermal SZ spectrum (non-relativistic)
- *important for* $y \ll 1$
- null at ν ~ 217 GHz (x~3.83)
- photon number conserved

$$\int x^2 Y(x) \mathrm{d}x = 0$$

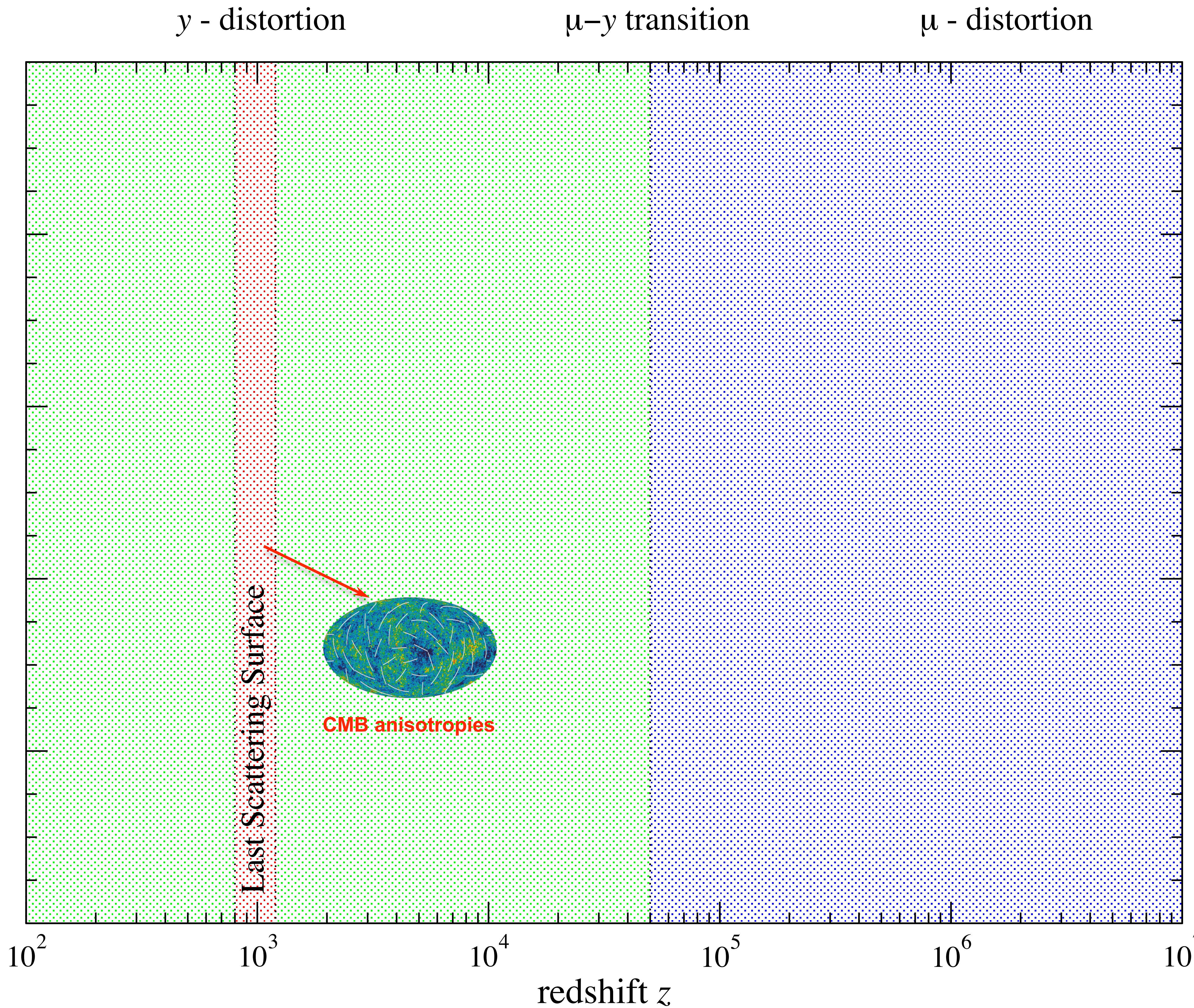
- energy exchange

$$\int x^3 Y(x) \mathrm{d}x = \frac{4\pi^4}{15} \leftrightarrow 4\rho_\gamma$$

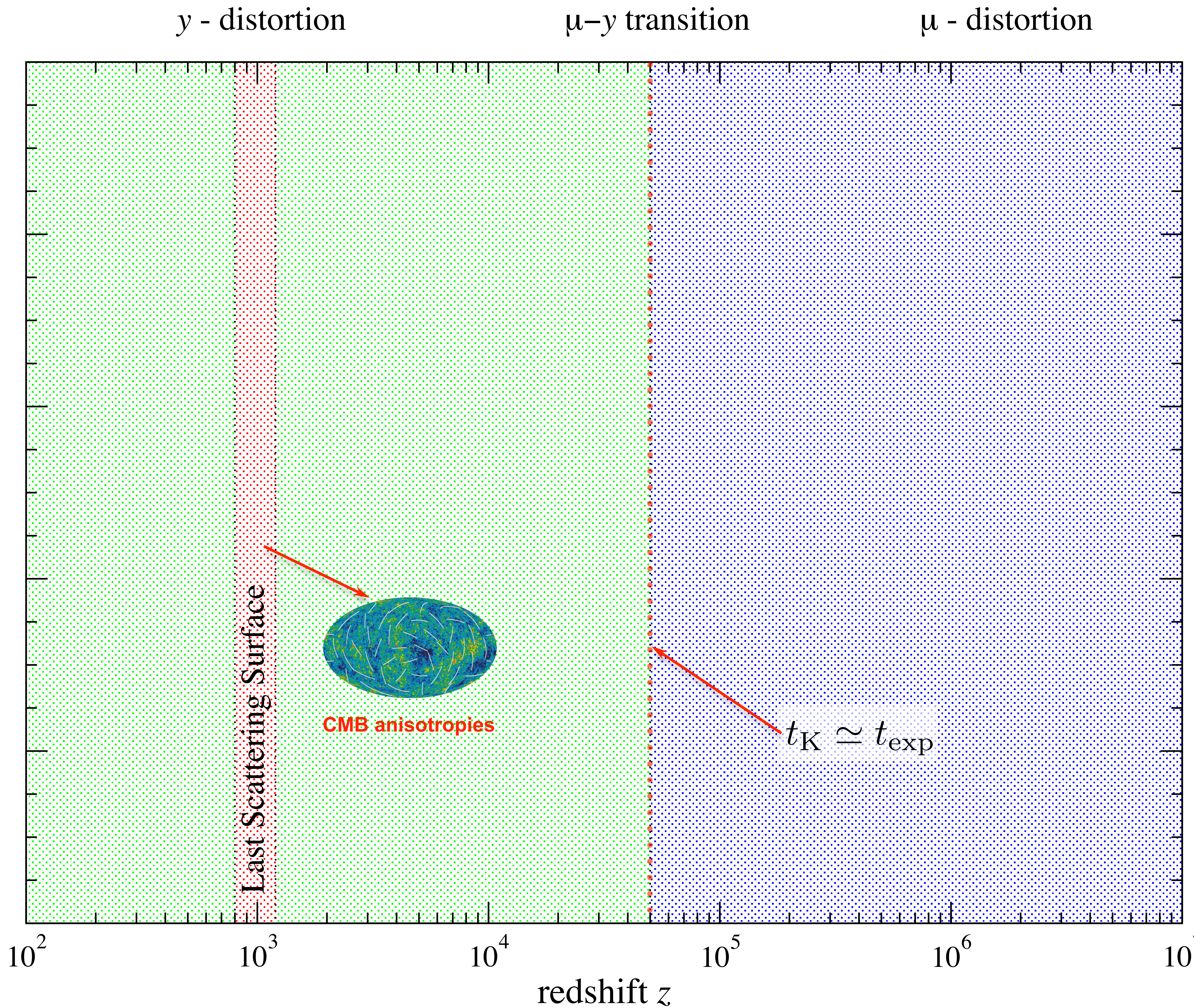
- *direction of energy flow* depends on difference between T_e and T_γ



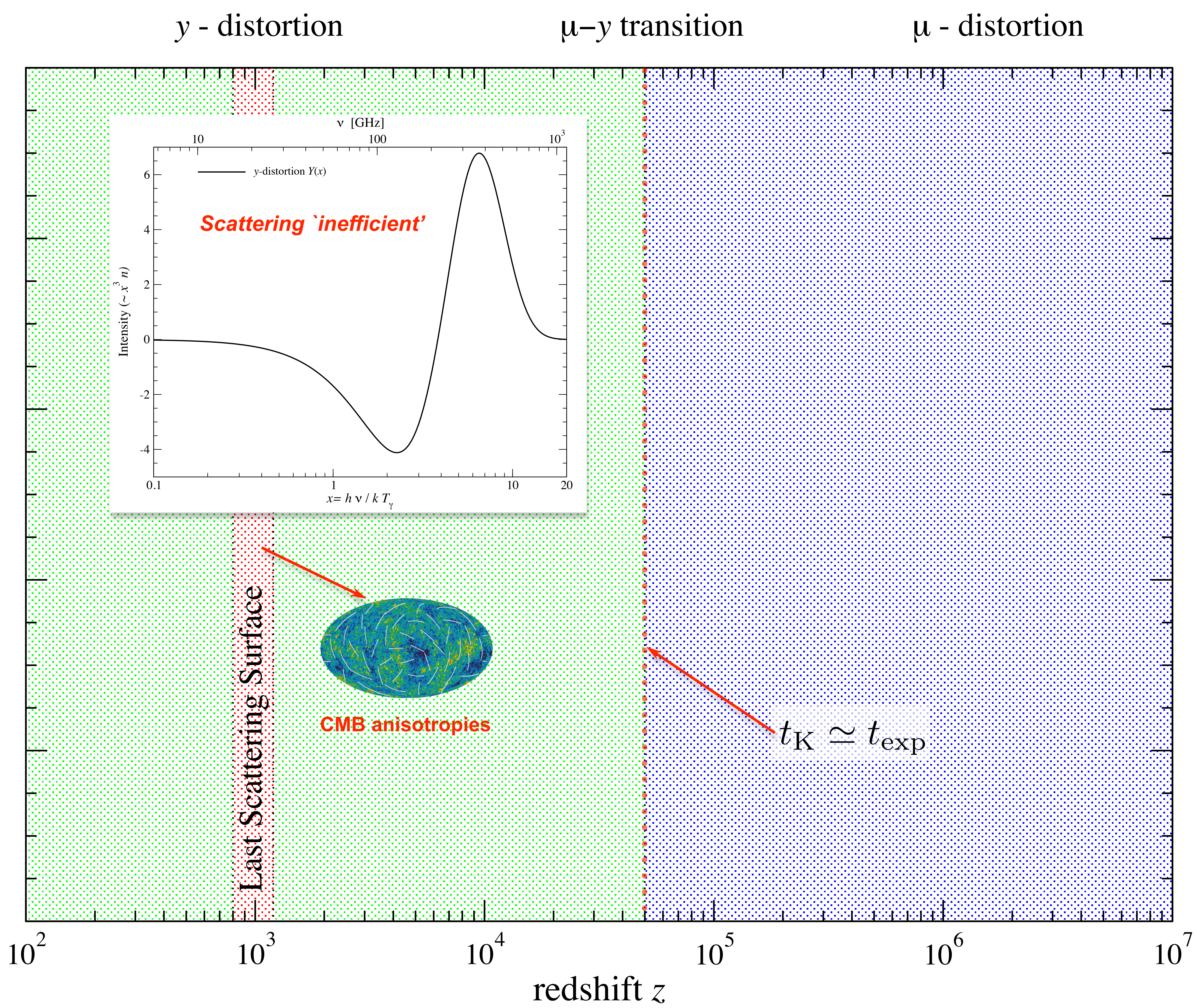
y - distortion
μ−y transition
μ - distortion
Last Scattering Surface
10^2
10^3
10^4
10^5
10^6
10^7
redshift z



y - distortion
μ−y transition
μ - distortion
Last Scattering Surface
CMB anisotropies
10^2
10^3
10^4
10^5
10^6
10^7
redshift z



y - distortion
μ−y transition
μ - distortion
Last Scattering Surface
CMB anisotropies
$t_{\rm K} \simeq t_{\rm exp}$
10^2
10^3
10^4
10^5
10^6
10^7
redshift z



y - distortion
μ–y transition
μ - distortion
ν [GHz]
10
100
10^3
y-distortion Y(x)
Scattering `inefficient'
Intensity (~ x^3 n)
6
4
2
0
-2
-4
0.1
1
10
20
x= h ν / k T_γ
Last Scattering Surface
CMB anisotropies
$t_{\rm K} \simeq t_{\rm exp}$
10^2
10^3
10^4
10^5
10^6
10^7
redshift z

Chemical Potential / μ-parameter

Chemical Potential / μ-parameter

- *Limit of "many" scatterings*

Chemical Potential / μ-parameter

- *Limit of "many" scatterings* $\Longrightarrow \quad \left.\frac{\mathrm{d}n}{\mathrm{d}\tau}\right|_{\mathrm{C}} \approx 0$

"Kinetic equilibrium" to scattering

Chemical Potential / μ-parameter

- *Limit of "many" scatterings* $\Longrightarrow$ $$\left.\frac{\mathrm{d}n}{\mathrm{d}\tau}\right|_{\mathrm{C}} \approx 0$$ **"Kinetic equilibrium" to scattering**

- *Kompaneets equation:* $\Longrightarrow$ $$\partial_x n \approx -\frac{T_\gamma}{T_{\mathrm{e}}} n(1+n)$$

$$\left.\frac{\mathrm{d}n}{\mathrm{d}\tau}\right|_{\mathrm{C}} \approx \frac{\theta_{\mathrm{e}}}{x^2}\frac{\partial}{\partial x} x^4 \left[\frac{\partial n}{\partial x} + \frac{T_\gamma}{T_{\mathrm{e}}} n(1+n)\right]$$

Chemical Potential / μ-parameter

- *Limit of "many" scatterings* $\Longrightarrow$ $\left.\frac{\mathrm{d}n}{\mathrm{d}\tau}\right|_{\mathrm{C}} \approx 0$ **"Kinetic equilibrium" to scattering**
- *Kompaneets equation:* $\Longrightarrow$ $\partial_x n \approx -\frac{T_\gamma}{T_{\mathrm{e}}} n(1+n)$
- *for* $T_\gamma = T_{\mathrm{e}}$ $\Longrightarrow$ $n = n^{\mathrm{bb}}(x) = 1/(\mathrm{e}^x - 1)$

Chemical Potential / μ-parameter

- *Limit of "many" scatterings* $\Longrightarrow$ $\left.\frac{\mathrm{d}n}{\mathrm{d}\tau}\right|_{\mathrm{C}} \approx 0$ **"Kinetic equilibrium" to scattering**
- *Kompaneets equation:* $\Longrightarrow$ $\partial_x n \approx -\frac{T_\gamma}{T_\mathrm{e}} n(1+n)$
- *for* $T_\gamma = T_\mathrm{e}$ $\Longrightarrow$ $n = n^{\mathrm{bb}}(x) = 1/(\mathrm{e}^x - 1)$
- *any spectrum can be written as:* $n(x) = 1/(\mathrm{e}^{x+\mu(x)} - 1)$

Chemical Potential / μ-parameter

- *Limit of "many" scatterings* $\Longrightarrow$ $\left.\frac{\mathrm{d}n}{\mathrm{d}\tau}\right|_{\mathrm{C}} \approx 0$ **"Kinetic equilibrium" to scattering**
- *Kompaneets equation:* $\Longrightarrow$ $\partial_x n \approx -\frac{T_\gamma}{T_\mathrm{e}} n(1+n)$
- *for* $T_\gamma = T_\mathrm{e}$ $\Longrightarrow$ $n = n^{\mathrm{bb}}(x) = 1/(\mathrm{e}^x - 1)$
- *any spectrum can be written as:* $n(x) = 1/(\mathrm{e}^{x+\mu(x)} - 1)$

chemical potential parameter (*"wrong" sign*)

Chemical Potential / μ-parameter

- *Limit of "many" scatterings* $\Longrightarrow$ $\left.\frac{\mathrm{d}n}{\mathrm{d}\tau}\right|_{\mathrm{C}} \approx 0$ **"Kinetic equilibrium" to scattering**
- *Kompaneets equation:* $\Longrightarrow$ $\partial_x n \approx -\frac{T_\gamma}{T_\mathrm{e}} n(1+n)$
- *for* $T_\gamma = T_\mathrm{e}$ $\Longrightarrow$ $n = n^{\mathrm{bb}}(x) = 1/(\mathrm{e}^x - 1)$
- *any spectrum can be written as:* $n(x) = 1/(\mathrm{e}^{x+\mu(x)} - 1)$ ← **chemical potential parameter** (*"wrong" sign*)

$$\Longrightarrow \quad (1 + \partial_x \mu) = -\frac{T_\gamma}{T_\mathrm{e}} \quad \Longrightarrow \quad x + \mu = x\frac{T_\gamma}{T_\mathrm{e}} + \mu_0$$

← **constant** (μ_0)

Chemical Potential / μ-parameter

- *Limit of "many" scatterings* $\Longrightarrow$ $\left.\frac{\mathrm{d}n}{\mathrm{d}\tau}\right|_{\mathrm{C}} \approx 0$ **"Kinetic equilibrium" to scattering**
- *Kompaneets equation:* $\Longrightarrow$ $\partial_x n \approx -\frac{T_\gamma}{T_\mathrm{e}} n(1+n)$
- *for* $T_\gamma = T_\mathrm{e}$ $\Longrightarrow$ $n = n^{\mathrm{bb}}(x) = 1/(\mathrm{e}^x - 1)$
- *any spectrum can be written as:* $n(x) = 1/(\mathrm{e}^{x+\mu(x)} - 1)$

chemical potential parameter (*"wrong" sign*)

$$\Longrightarrow \quad (1 + \partial_x \mu) = -\frac{T_\gamma}{T_\mathrm{e}} \quad \Longrightarrow \quad x + \mu = x\frac{T_\gamma}{T_\mathrm{e}} + \mu_0$$

constant

- *General equilibrium solution:* *Bose-Einstein spectrum with*

$T_\gamma = T_\mathrm{e} \equiv T_\mathrm{eq}$ and $\mu_0 = \mathrm{const}$ ($\equiv 0$ for blackbody)

Chemical Potential / μ-parameter

- *Limit of "many" scatterings* $\Longrightarrow \quad \left.\frac{\mathrm{d}n}{\mathrm{d}\tau}\right|_{\mathrm{C}} \approx 0$ **"Kinetic equilibrium" to scattering**
- *Kompaneets equation:* $\Longrightarrow \quad \partial_x n \approx -\frac{T_\gamma}{T_\mathrm{e}} n(1+n)$
- *for* $T_\gamma = T_\mathrm{e} \quad \Longrightarrow \quad n = n^{\mathrm{bb}}(x) = 1/(\mathrm{e}^x - 1)$
- *any spectrum can be written as:* $n(x) = 1/(\mathrm{e}^{x+\mu(x)} - 1)$ — **chemical potential parameter** (*"wrong" sign*)

$$\Longrightarrow \quad (1+\partial_x\mu) = -\frac{T_\gamma}{T_\mathrm{e}} \quad \Longrightarrow \quad x + \mu = x\frac{T_\gamma}{T_\mathrm{e}} + \mu_0$$

constant (μ_0)

- *General equilibrium solution:* *Bose-Einstein spectrum with*

$$T_\gamma = T_\mathrm{e} \equiv T_\mathrm{eq} \quad \text{and} \quad \mu_0 = \text{const} \ (\equiv 0 \text{ for blackbody})$$

Something is missing? How do you fix T_e and μ_0?

Final definition of μ-type distortion

- *initial condition:* $N_\gamma = N_\gamma^{\rm bb}(T_\gamma)$ and $\rho_\gamma = \rho_\gamma^{\rm bb}(T_\gamma)$

Final definition of μ-type distortion

- *initial condition:* $N_\gamma = N_\gamma^{\rm bb}(T_\gamma)$ and $\rho_\gamma = \rho_\gamma^{\rm bb}(T_\gamma)$

- *after energy release* ⇒

$$\approx 1.368$$

$$N_\gamma^{\rm bb}(T_\gamma) = N_\gamma^{\rm BE}(T_{\rm e}, \mu_0) \approx N_\gamma^{\rm bb}(T_\gamma)\left(1 + 3\frac{\Delta T}{T_\gamma} - \frac{\pi^2}{6\zeta(3)}\mu_0\right)$$

Final definition of μ-type distortion

- *initial condition:* $N_\gamma = N_\gamma^{\rm bb}(T_\gamma)$ and $\rho_\gamma = \rho_\gamma^{\rm bb}(T_\gamma)$

- *after energy release* $\Rightarrow$

$$\approx 1.368$$

$$N_\gamma^{\rm bb}(T_\gamma) = N_\gamma^{\rm BE}(T_{\rm e}, \mu_0) \approx N_\gamma^{\rm bb}(T_\gamma)\left(1 + 3\frac{\Delta T}{T_\gamma} - \frac{\pi^2}{6\zeta(3)}\mu_0\right)$$

$$\approx 1.111$$

$$\rho_\gamma^{\rm bb}(T_\gamma) + \Delta\rho_\gamma = \rho_\gamma^{\rm BE}(T_{\rm e}, \mu_0) \approx \rho_\gamma^{\rm bb}(T_\gamma)\left(1 + 4\frac{\Delta T}{T_\gamma} - \frac{90\zeta(3)}{\pi^4}\mu_0\right)$$

Final definition of μ-type distortion

- *initial condition:* $N_\gamma = N_\gamma^{\rm bb}(T_\gamma)$ and $\rho_\gamma = \rho_\gamma^{\rm bb}(T_\gamma)$
- *after energy release* $\Rightarrow$

$$\approx 1.368$$

$$N_\gamma^{\rm bb}(T_\gamma) = N_\gamma^{\rm BE}(T_{\rm e}, \mu_0) \approx N_\gamma^{\rm bb}(T_\gamma)\left(1 + 3\frac{\Delta T}{T_\gamma} - \frac{\pi^2}{6\zeta(3)}\mu_0\right)$$

$$\approx 1.111$$

$$\rho_\gamma^{\rm bb}(T_\gamma) + \Delta\rho_\gamma = \rho_\gamma^{\rm BE}(T_{\rm e}, \mu_0) \approx \rho_\gamma^{\rm bb}(T_\gamma)\left(1 + 4\frac{\Delta T}{T_\gamma} - \frac{90\zeta(3)}{\pi^4}\mu_0\right)$$

- *Solution:* $\frac{\Delta T}{T_\gamma} \approx \frac{\pi^2}{18\zeta(3)}\mu_0 \approx 0.456\,\mu_0$ *and* $\mu_0 \approx 1.401\,\frac{\Delta\rho_\gamma}{\rho_\gamma}$

Final definition of μ-type distortion

- *initial condition:* $N_\gamma = N_\gamma^{\rm bb}(T_\gamma)$ and $\rho_\gamma = \rho_\gamma^{\rm bb}(T_\gamma)$

- *after energy release* ⇒

≈ 1.368

$$N_\gamma^{\rm bb}(T_\gamma) = N_\gamma^{\rm BE}(T_{\rm e}, \mu_0) \approx N_\gamma^{\rm bb}(T_\gamma)\left(1 + 3\frac{\Delta T}{T_\gamma} - \frac{\pi^2}{6\zeta(3)}\mu_0\right)$$

≈ 1.111

$$\rho_\gamma^{\rm bb}(T_\gamma) + \Delta\rho_\gamma = \rho_\gamma^{\rm BE}(T_{\rm e}, \mu_0) \approx \rho_\gamma^{\rm bb}(T_\gamma)\left(1 + 4\frac{\Delta T}{T_\gamma} - \frac{90\zeta(3)}{\pi^4}\mu_0\right)$$

- *Solution:* $\frac{\Delta T}{T_\gamma} \approx \frac{\pi^2}{18\zeta(3)}\mu_0 \approx 0.456\,\mu_0$ *and* $\mu_0 \approx 1.401\,\frac{\Delta\rho_\gamma}{\rho_\gamma}$

$$\Longrightarrow \quad M(x) = G(x)\left[\frac{\pi^2}{18\zeta(3)} - \frac{1}{x}\right]$$

μ-distortion spectrum
(photon number conserved)

Final definition of μ-type distortion

- *initial condition:* $N_\gamma = N_\gamma^{\rm bb}(T_\gamma)$ and $\rho_\gamma = \rho_\gamma^{\rm bb}(T_\gamma)$
- *after energy release* ⇒

$$\approx 1.368$$

$$N_\gamma^{\rm bb}(T_\gamma) = N_\gamma^{\rm BE}(T_{\rm e}, \mu_0) \approx N_\gamma^{\rm bb}(T_\gamma)\left(1 + 3\frac{\Delta T}{T_\gamma} - \frac{\pi^2}{6\zeta(3)}\mu_0\right)$$

$$\approx 1.111$$

$$\rho_\gamma^{\rm bb}(T_\gamma) + \Delta\rho_\gamma = \rho_\gamma^{\rm BE}(T_{\rm e}, \mu_0) \approx \rho_\gamma^{\rm bb}(T_\gamma)\left(1 + 4\frac{\Delta T}{T_\gamma} - \frac{90\zeta(3)}{\pi^4}\mu_0\right)$$

- *Solution:* $\dfrac{\Delta T}{T_\gamma} \approx \dfrac{\pi^2}{18\zeta(3)}\mu_0 \approx 0.456\,\mu_0$ *and* $\mu_0 \approx 1.401\,\dfrac{\Delta\rho_\gamma}{\rho_\gamma}$

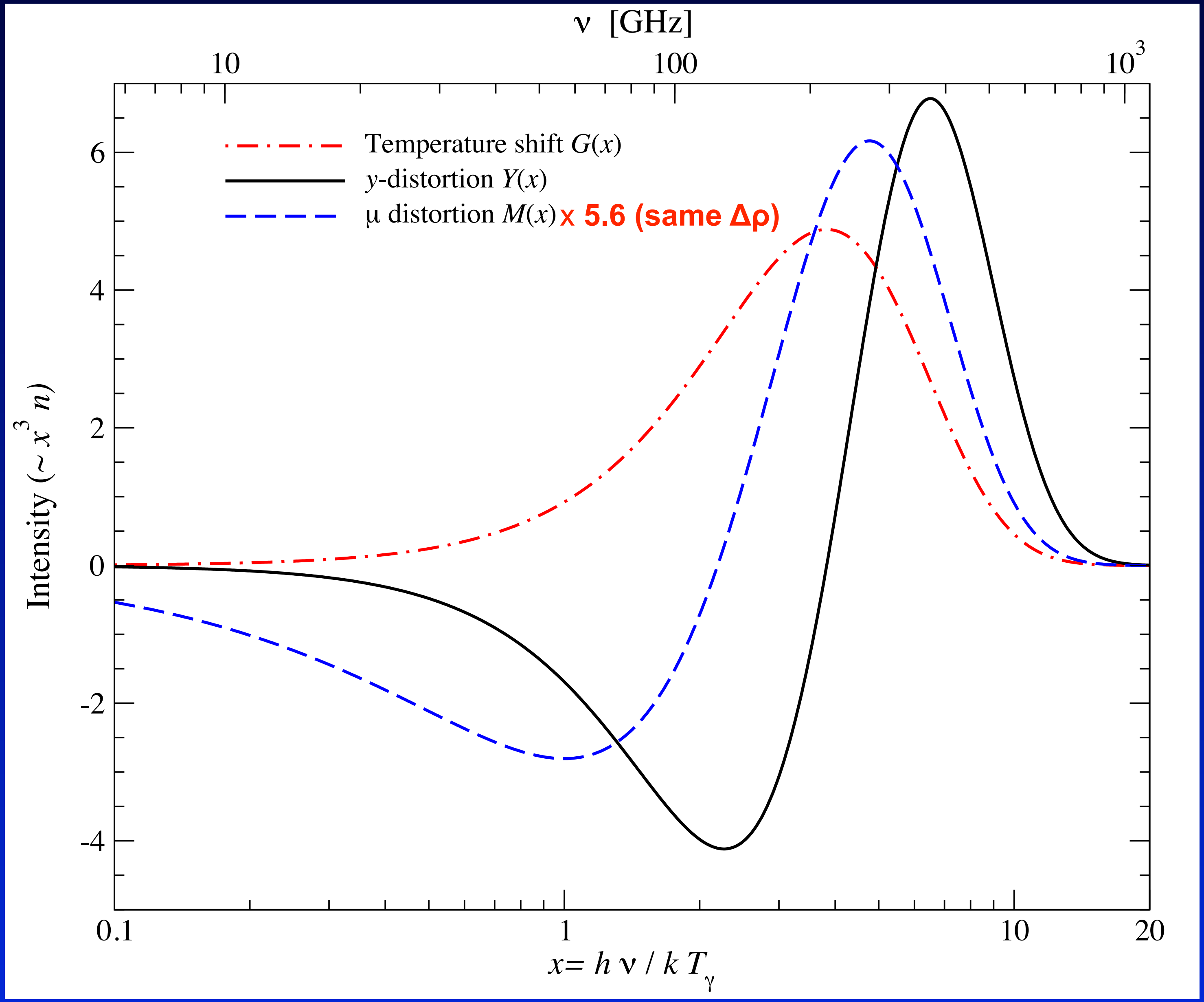
$$\Longrightarrow \quad M(x) = G(x)\left[\frac{\pi^2}{18\zeta(3)} - \frac{1}{x}\right]$$

μ-distortion spectrum
(photon number conserved)

- $\mu_0 > 0$ ⇒ *too few photons / too much energy*
- $\mu_0 < 0$ ⇒ *too many photons / too little energy*

$$\frac{\Delta\rho_\gamma}{\rho_\gamma^{\rm bb}} \approx \frac{4}{3}\frac{\Delta N_\gamma}{N_\gamma^{\rm bb}}$$

Temperature shift ↔ y-distortion ↔ μ-distortion



$$M(x) = G(x)\left[\frac{\pi^2}{18\zeta(3)} - \frac{1}{x}\right]$$

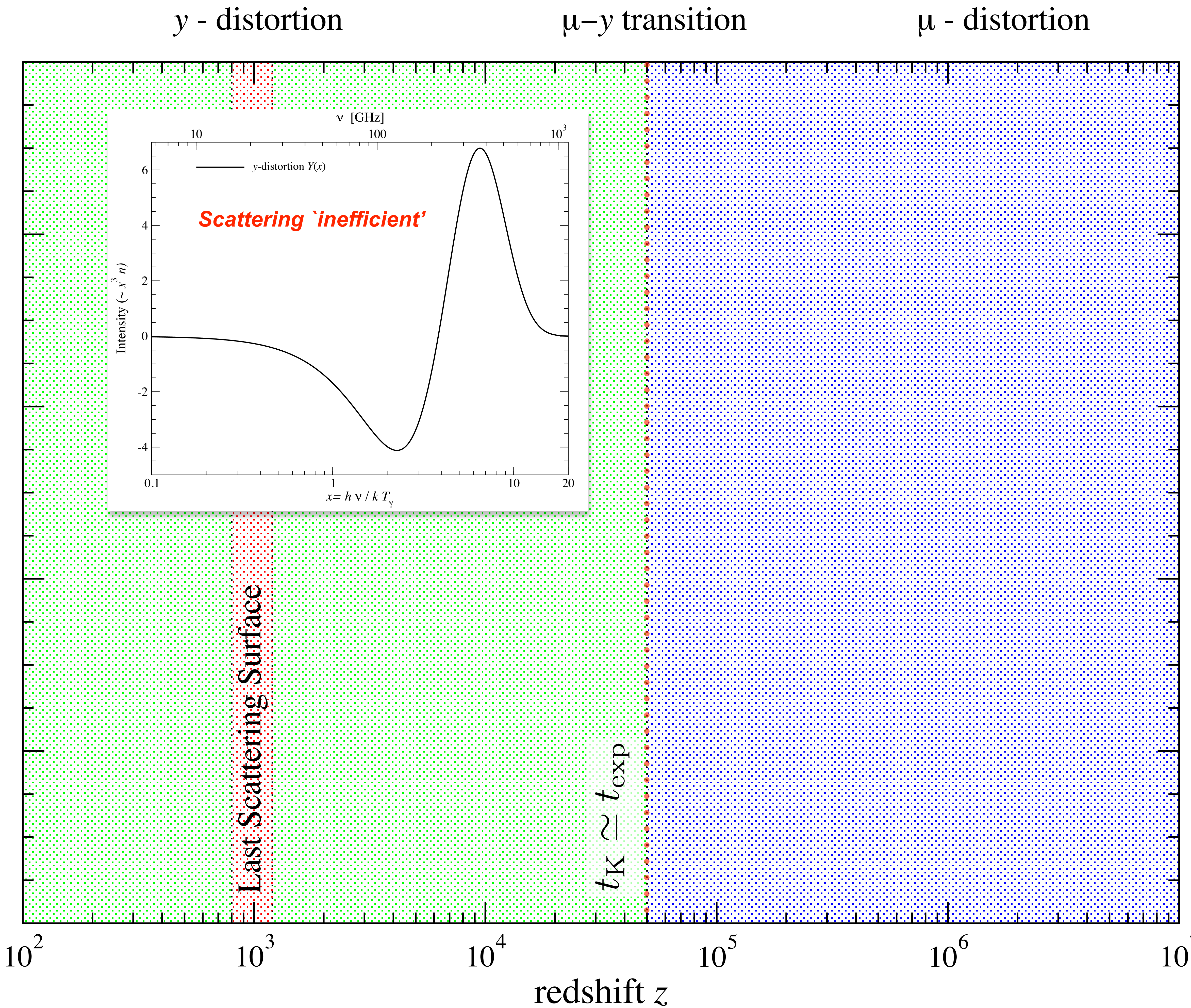
- *important for in the limit of many scatterings*
- null at ν ~ 125 GHz (x~2.19)
- photon number conserved

$$\int x^2 M(x)\mathrm{d}x = 0$$

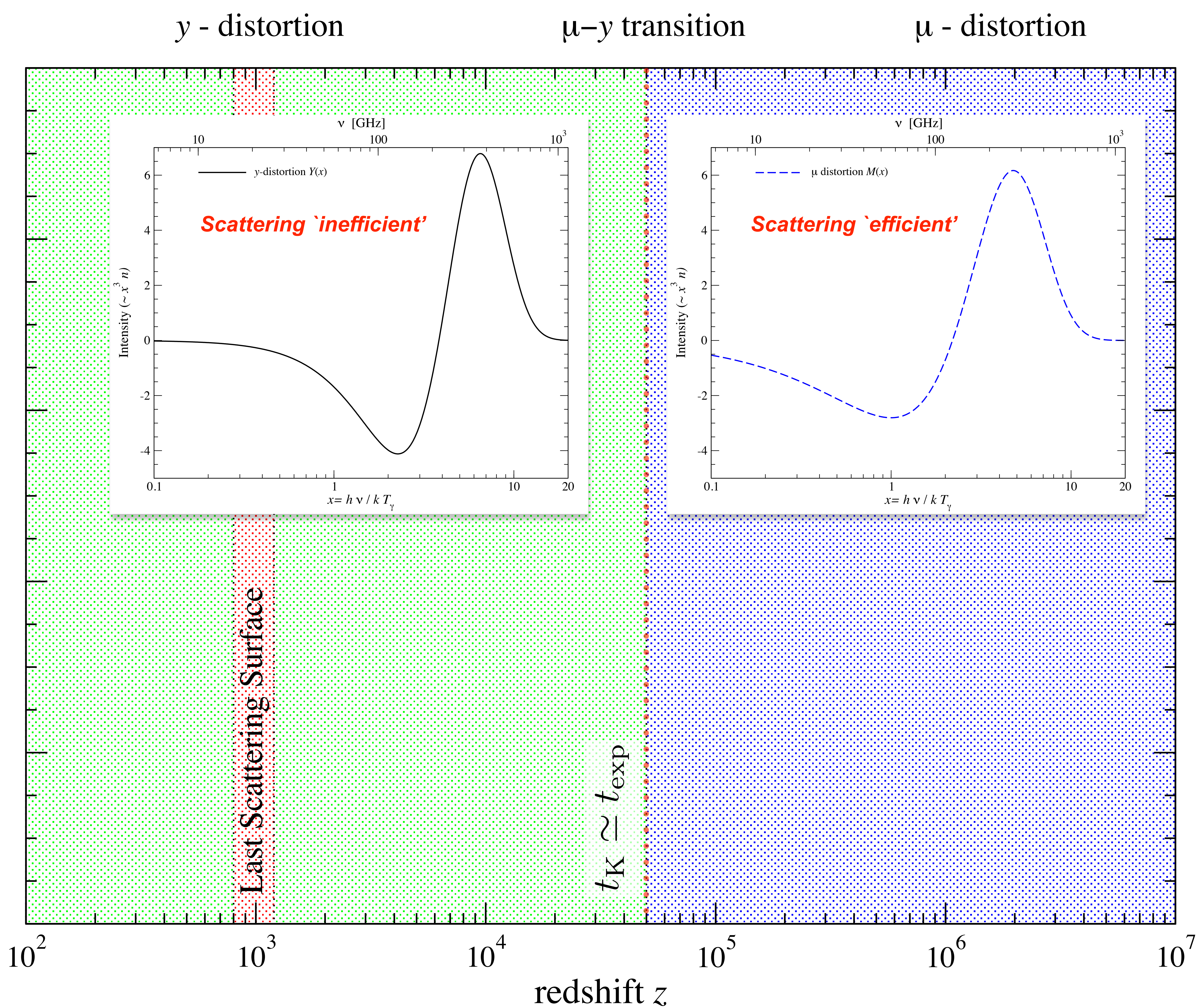
- energy exchange

$$\int x^3 M(x)\mathrm{d}x \approx \frac{\pi^4/15}{1.401} \leftrightarrow \frac{\rho_\gamma}{1.401}$$

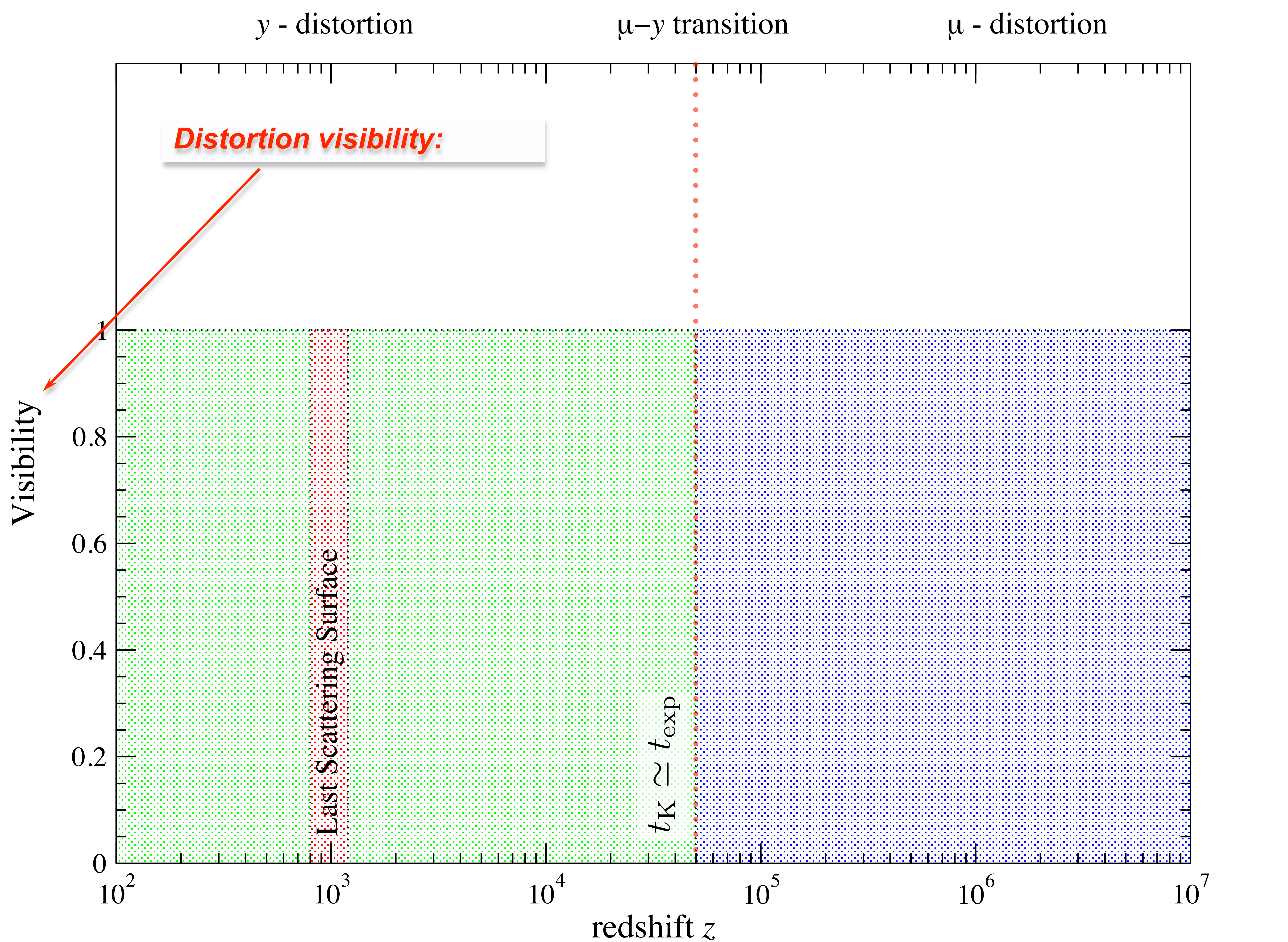
- *can only be created at very early times*



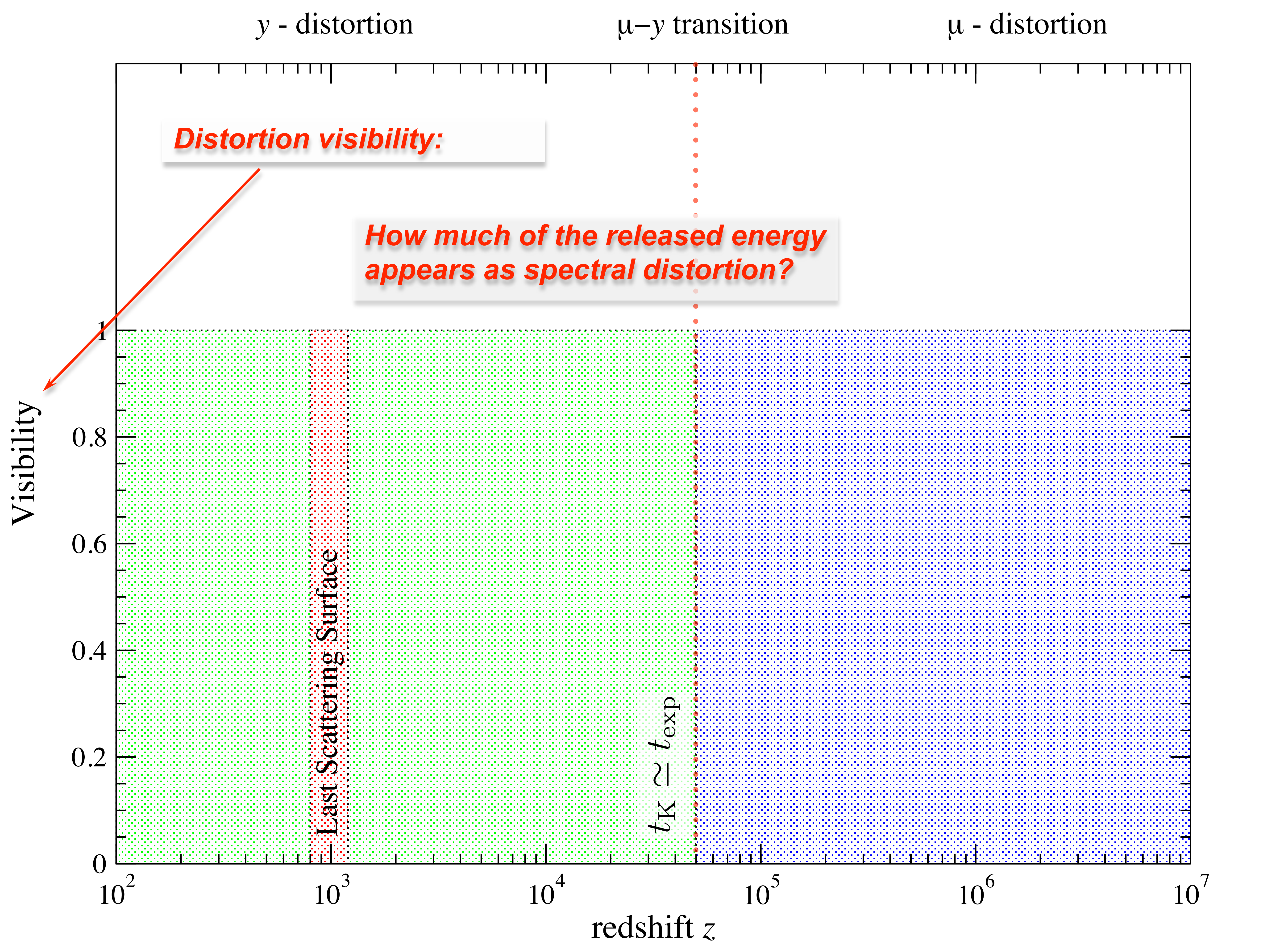
y - distortion
μ−y transition
μ - distortion
ν [GHz]
10
100
10^3
y-distortion Y(x)
Scattering `inefficient'
Intensity (~ x^3 n)
6
4
2
0
-2
-4
0.1
1
10
20
$x = h\nu / k T_\gamma$
Last Scattering Surface
$t_K \simeq t_{exp}$
10^2
10^3
10^4
10^5
10^6
10^7
redshift z



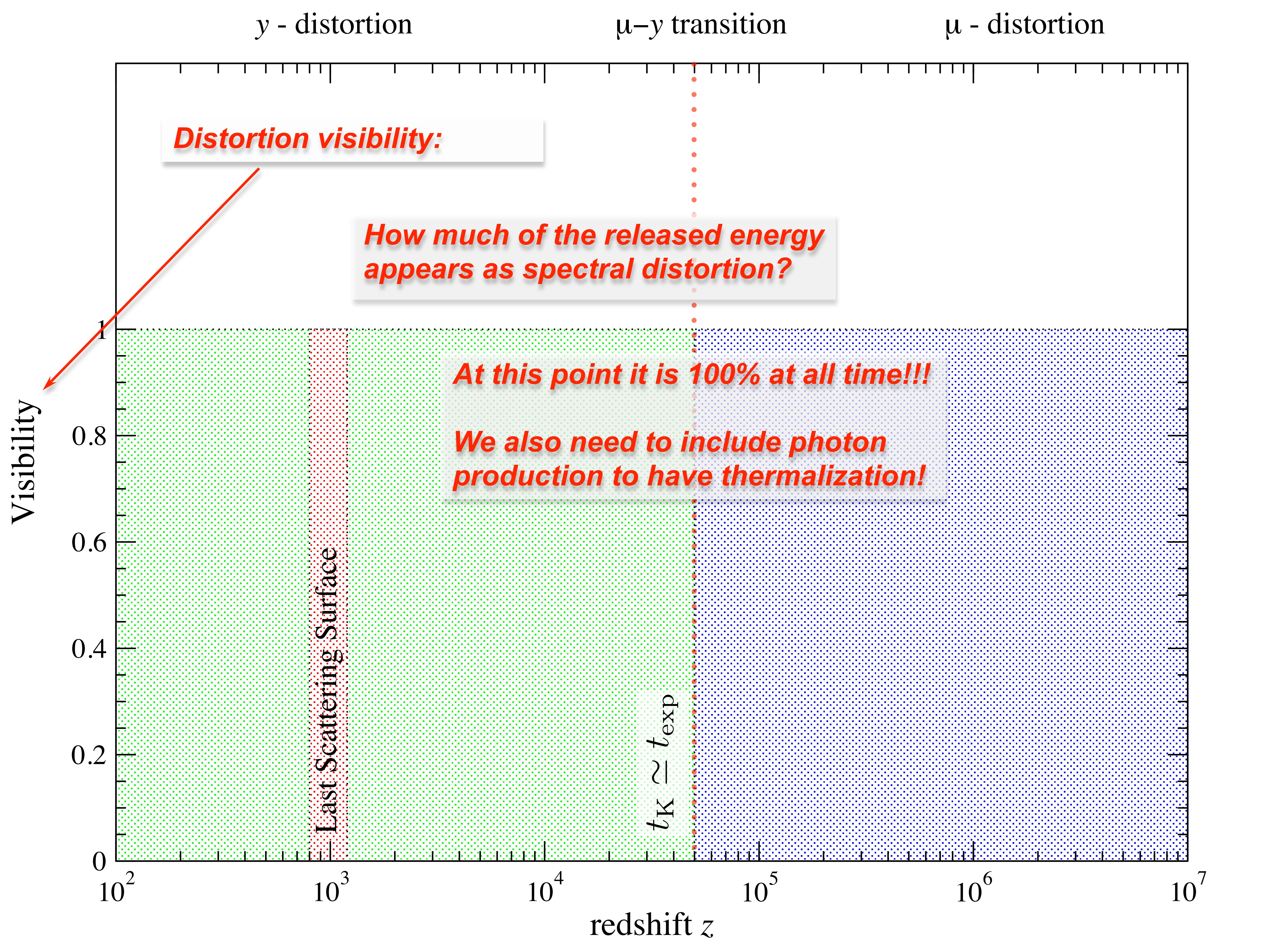
y - distortion
μ−y transition
μ - distortion
ν [GHz]
10
100
10^3
y-distortion Y(x)
Scattering `inefficient'
Intensity (~ x^3 n)
x= h ν / k T_γ
μ distortion M(x)
Scattering `efficient'
Last Scattering Surface
t_K ≃ t_exp
10^2
10^3
10^4
10^5
10^6
10^7
redshift z



y - distortion
μ–y transition
μ - distortion
Distortion visibility:
Visibility
0
0.2
0.4
0.6
0.8
1
Last Scattering Surface
$t_K \simeq t_{exp}$
10^2
10^3
10^4
10^5
10^6
10^7
redshift z



y - distortion
μ−y transition
μ - distortion
Distortion visibility:
How much of the released energy appears as spectral distortion?
Visibility
Last Scattering Surface
$t_K \simeq t_{exp}$
redshift z
0
0.2
0.4
0.6
0.8
1
10^2
10^3
10^4
10^5
10^6
10^7



y - distortion
μ–y transition
μ - distortion
Distortion visibility:
How much of the released energy appears as spectral distortion?
At this point it is 100% at all time!!!
We also need to include photon production to have thermalization!
Visibility
Last Scattering Surface
$t_K \simeq t_{exp}$
redshift z
0
0.2
0.4
0.6
0.8
1
10^2
10^3
10^4
10^5
10^6
10^7

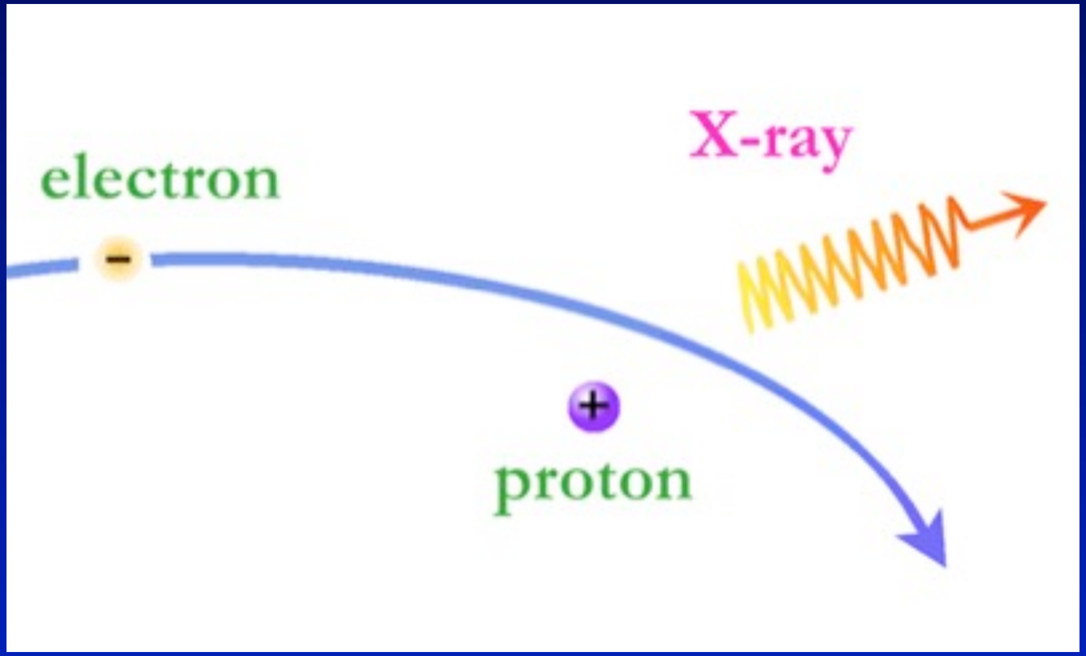
Photon production processes

Thermal Bremsstrahlung

- Reaction: $e + p \leftrightarrow e' + p + \gamma$

Thermal Bremsstrahlung

- Reaction: $e + p \leftrightarrow e' + p + \gamma$



Thermal Bremsstrahlung

- Reaction: $e + p \leftrightarrow e' + p + \gamma$

  → 1. order α correction to *Coulomb* scattering

  → production of low frequency photons

Thermal Bremsstrahlung

- Reaction: $e + p \leftrightarrow e' + p + \gamma$

→ 1. order α correction to *Coulomb* scattering

→ production of low frequency photons

→ include dependence on composition
(Gaunt-factors; ***simple fits***: **Itoh et al, 2000**)

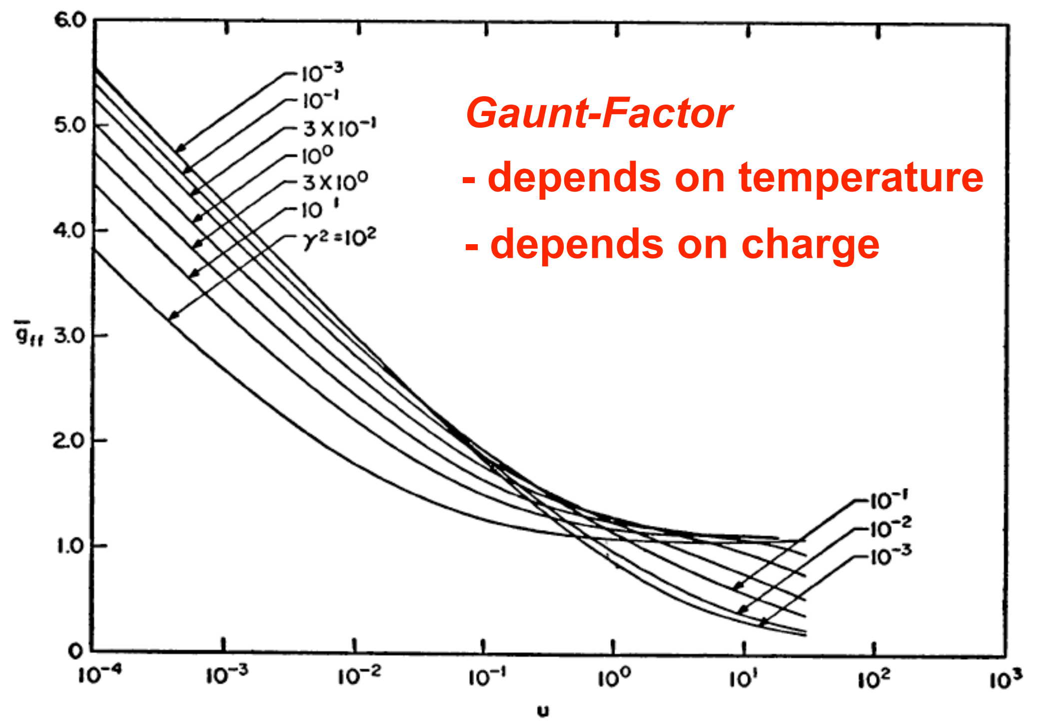


FIG. 5.—Temperature-averaged free-free Gaunt factor versus $u = h\nu/kT$ for various values of $\gamma^2 = Z^2 Ry/kT$.

Karzas & Latter, 1961, ApJS, 6, 167

Thermal Bremsstrahlung

- Reaction: $e + p \leftrightarrow e' + p + \gamma$
  - → 1. order α correction to *Coulomb* scattering
  - → production of low frequency photons
  - → include dependence on composition (Gaunt-factors; ***simple fits*: Itoh et al, 2000**)

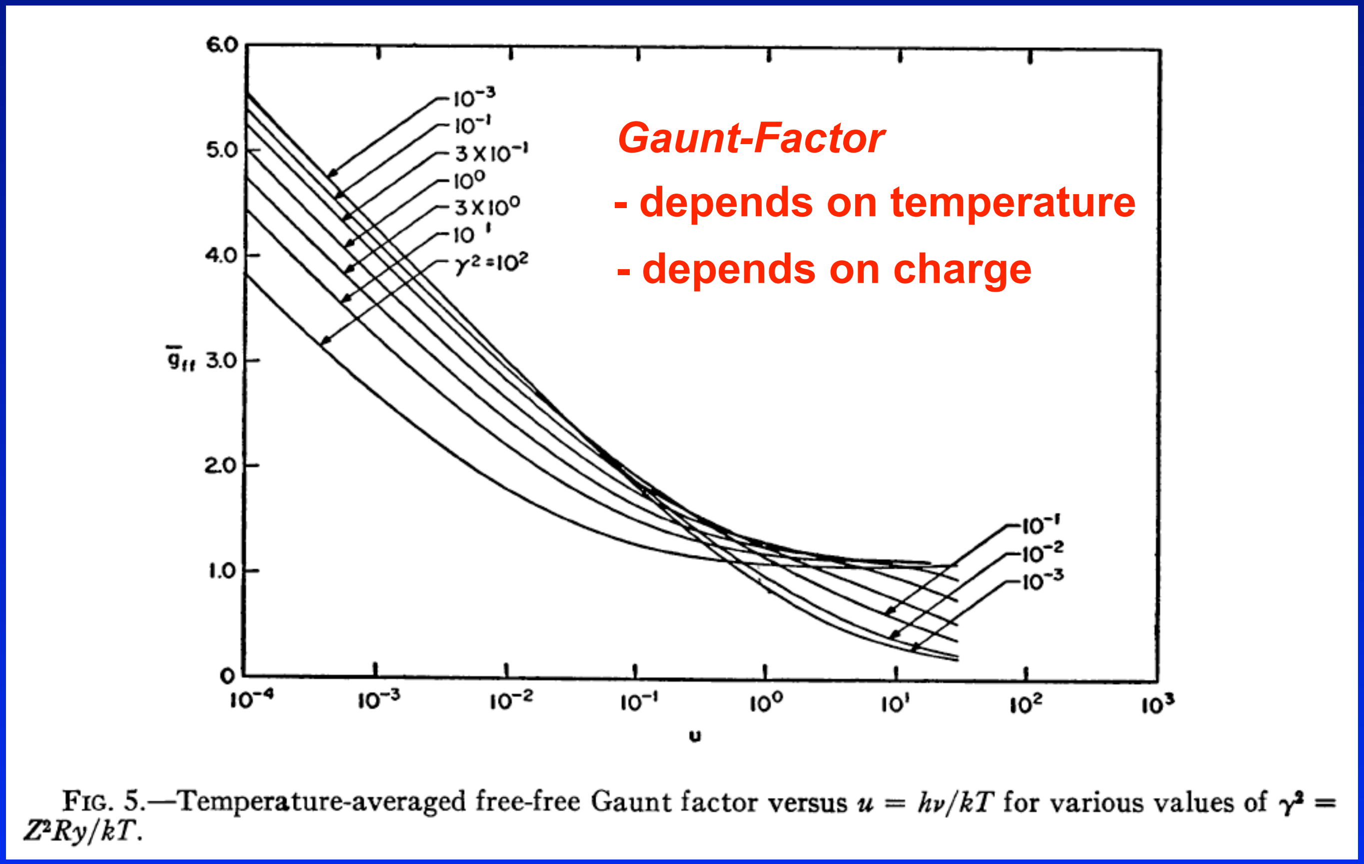
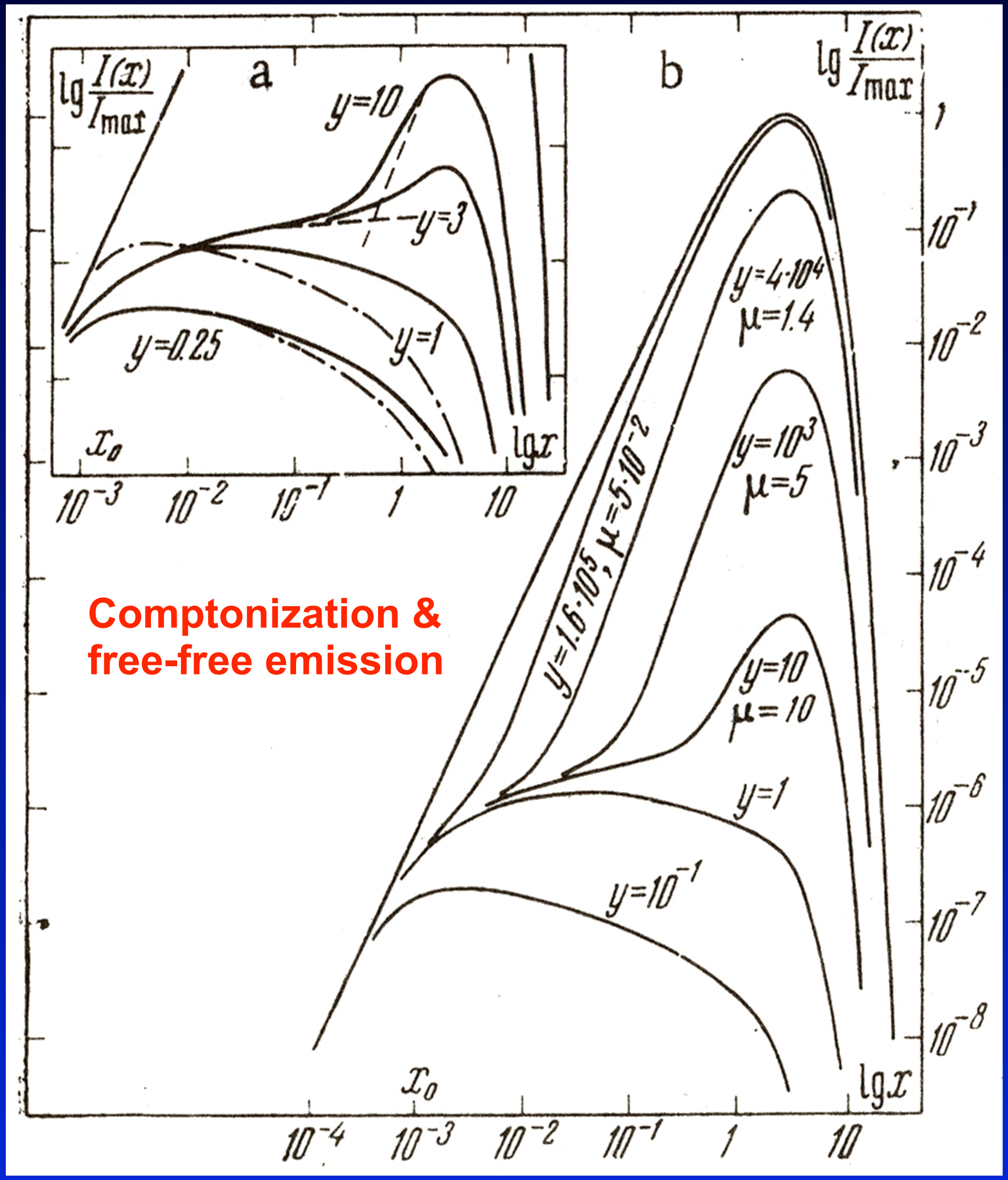


FIG. 5.—Temperature-averaged free-free Gaunt factor versus $u = h\nu/kT$ for various values of $\gamma^2 = Z^2Ry/kT$.

Karzas & Latter, 1961, ApJS, 6, 167



Illarionov & Sunyaev, 1975, Sov. Astr, 18, pp.413

Thermal Bremsstrahlung

- Reaction: $e + p \leftrightarrow e' + p + \gamma$
  - → 1. order α correction to *Coulomb* scattering
  - → production of low frequency photons
  - → include dependence on composition (Gaunt-factors; ***simple fits***: **Itoh et al, 2000**)

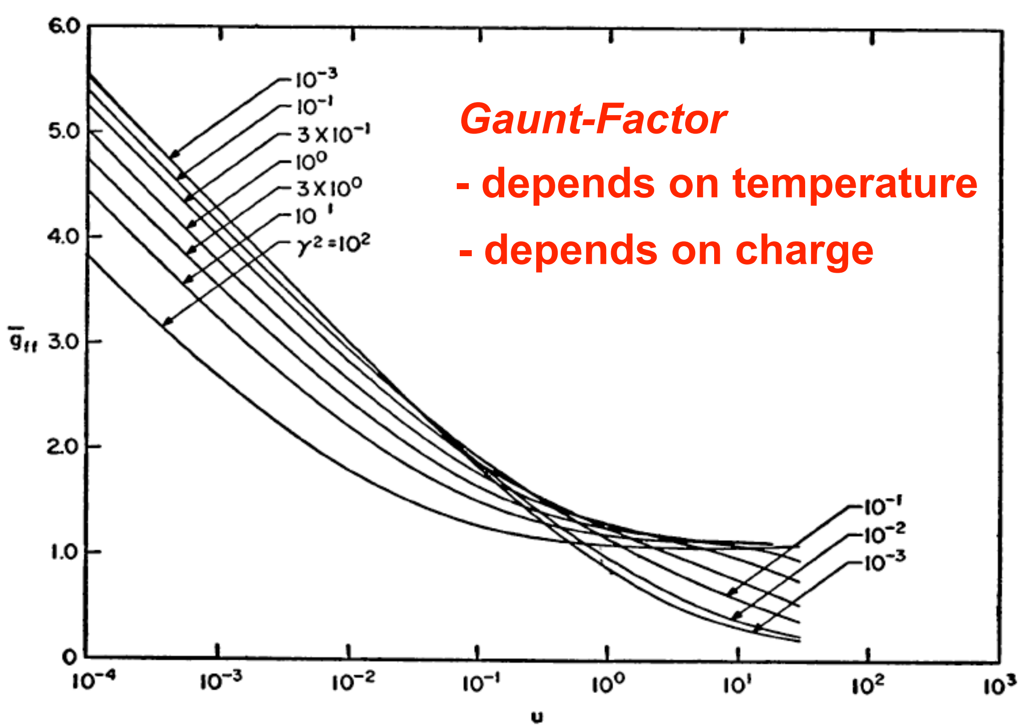
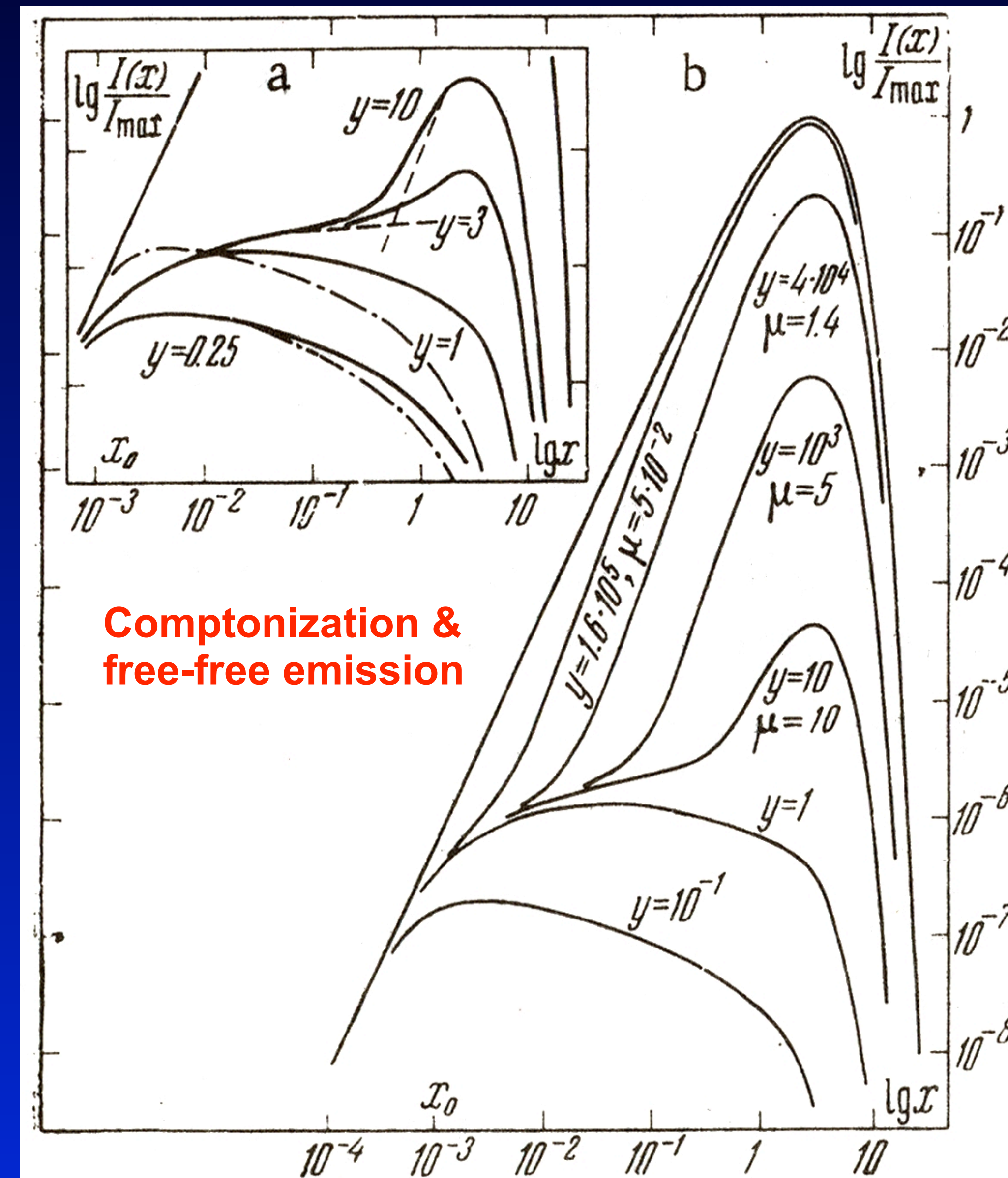


FIG. 5.—Temperature-averaged free-free Gaunt factor versus $u = h\nu/kT$ for various values of $\gamma^2 = Z^2Ry/kT$.

Karzas & Latter, 1961, ApJS, 6, 167



Illarionov & Sunyaev, 1975, Sov. Astr, 18, pp.413

Thermalization inefficient already at z ≲ 10^7 with Bremsstrahlung alone!

Double Compton Emission

- Reaction: $e + \gamma \leftrightarrow e' + \gamma' + \gamma_2$

Double Compton Emission

- Reaction: $e + \gamma \leftrightarrow e' + \gamma' + \gamma_2$
  - → 1. order α correction to *Compton* scattering

Double Compton Emission

- Reaction: $e + \gamma \leftrightarrow e' + \gamma' + \gamma_2$
  - → 1. order α correction to *Compton* scattering
  - → production of low frequency photons

Double Compton Emission

- Reaction: $e+\gamma \leftrightarrow e'+\gamma'+\gamma_2$
  - → 1. order α correction to *Compton* scattering
  - → production of low frequency photons

Important source of photons for photon-dominated plasmas, like our Universe at early times!

Double Compton Emission

- Reaction: $e + \gamma \leftrightarrow e' + \gamma' + \gamma_2$
  - → 1. order α correction to *Compton* scattering
  - → production of low frequency photons

Important source of photons for photon-dominated plasmas, like our Universe at early times!

$$K_{\rm DC} \propto N_{\rm e} N_\gamma \left(\frac{kT_\gamma}{m_{\rm e}c^2}\right)^{-1} \propto (1+z)^5 \iff K_{\rm BR} \propto N_{\rm e} N_{\rm b} \left(\frac{kT_{\rm e}}{m_{\rm e}c^2}\right)^{-7/2} \propto (1+z)^{5/2}$$

Double Compton Emission

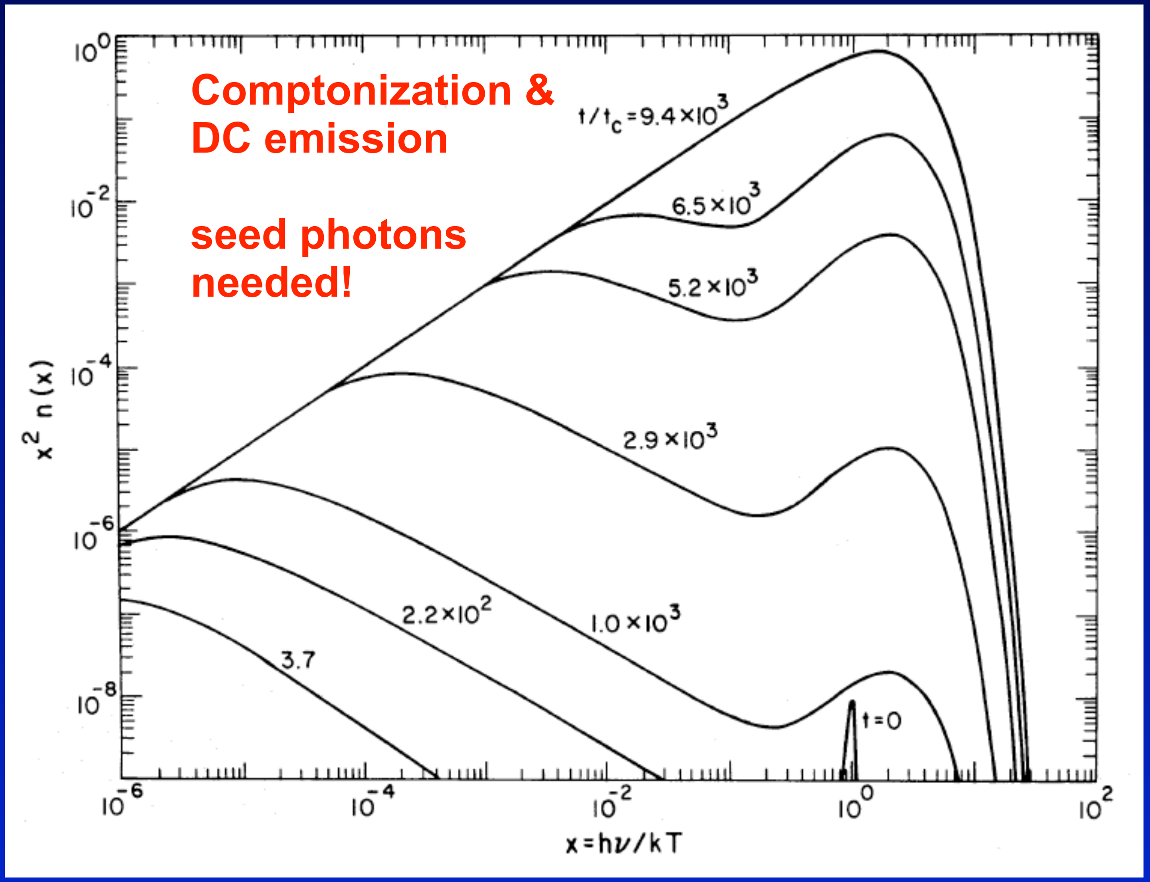
- Reaction: $e + \gamma \leftrightarrow e' + \gamma' + \gamma_2$
  - → 1. order α correction to *Compton* scattering
  - → production of low frequency photons
  - → DC takes over at $z \gtrsim 4 \times 10^5$ (BR contributes to late evolution at low ν)

Important source of photons for photon-dominated plasmas, like our Universe at early times!

Double Compton Emission

- Reaction: $e + \gamma \leftrightarrow e' + \gamma' + \gamma_2$
  - → 1. order α correction to *Compton* scattering
  - → production of low frequency photons
  - → DC takes over at $z \gtrsim 4 \times 10^5$ (BR contributes to late evolution at low ν)
  - → computation in the 'soft' photon limit

Important source of photons for photon-dominated plasmas, like our Universe at early times!

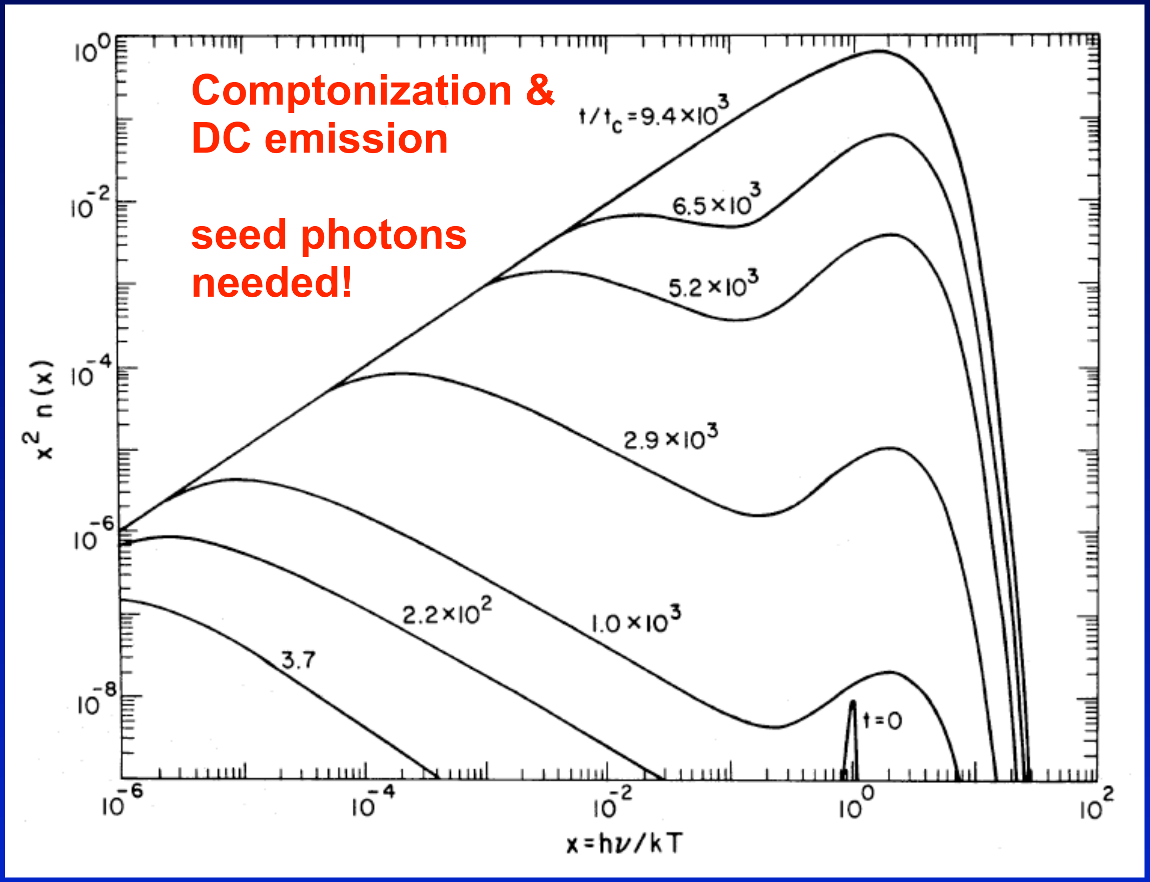


Lightman 1981

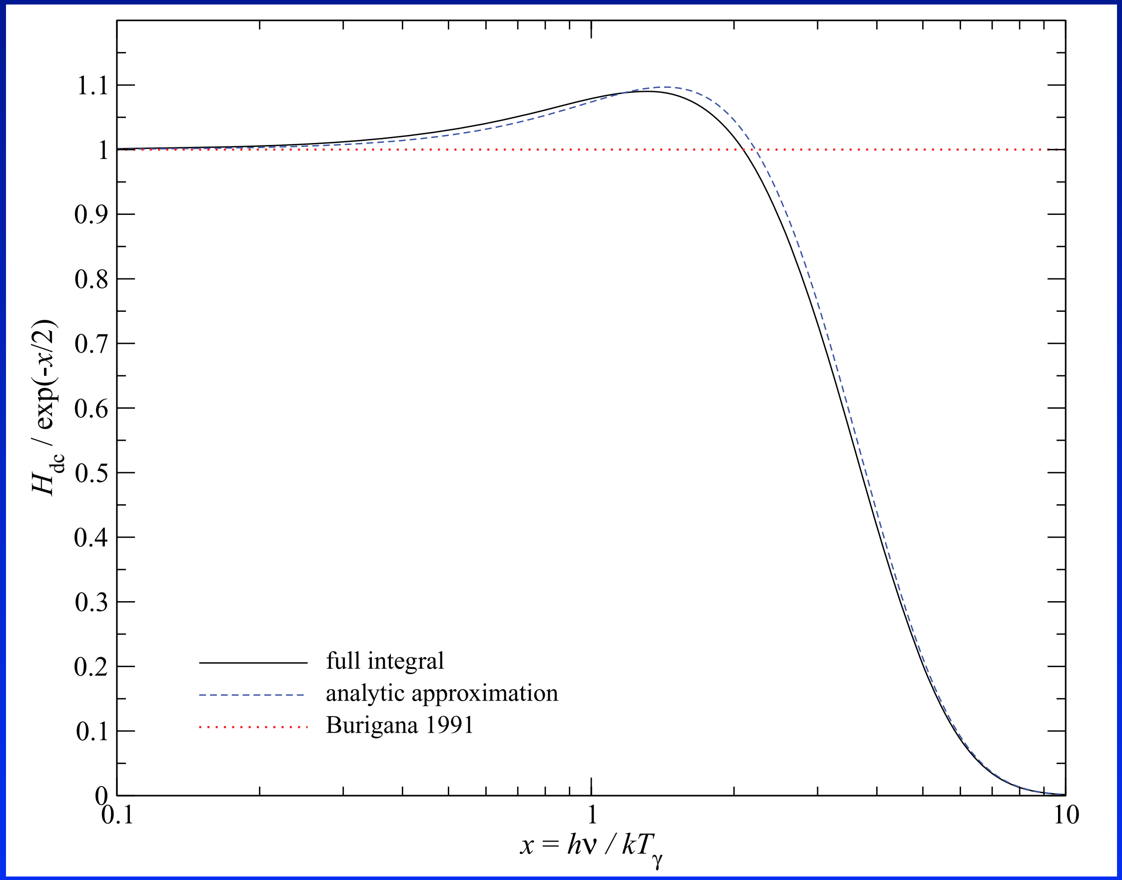
Double Compton Emission

- Reaction: $e+\gamma \leftrightarrow e'+\gamma'+\gamma_2$
  - → 1. order α correction to *Compton* scattering
  - → production of low frequency photons
  - → DC takes over at $z \gtrsim 4 \times 10^5$ (BR contributes to late evolution at low ν)
  - → computation in the 'soft' photon limit

Important source of photons for photon-dominated plasmas, like our Universe at early times!



Lightman 1981



JC, 2005 (PhD Thesis); JC, Sazonov & Sunyaev, 2007; JC & Sunyaev, 2012

- → was only included later **(Danese & De Zotti, 1982)**
- → DC Gaunt-factor and temperature corrections included by latest computations, but the effect is small

Example: *Energy release by decaying relict particle*

redshift

difference between electron and photon temperature

today x=2 x 10^{-2} means ν~1GHz

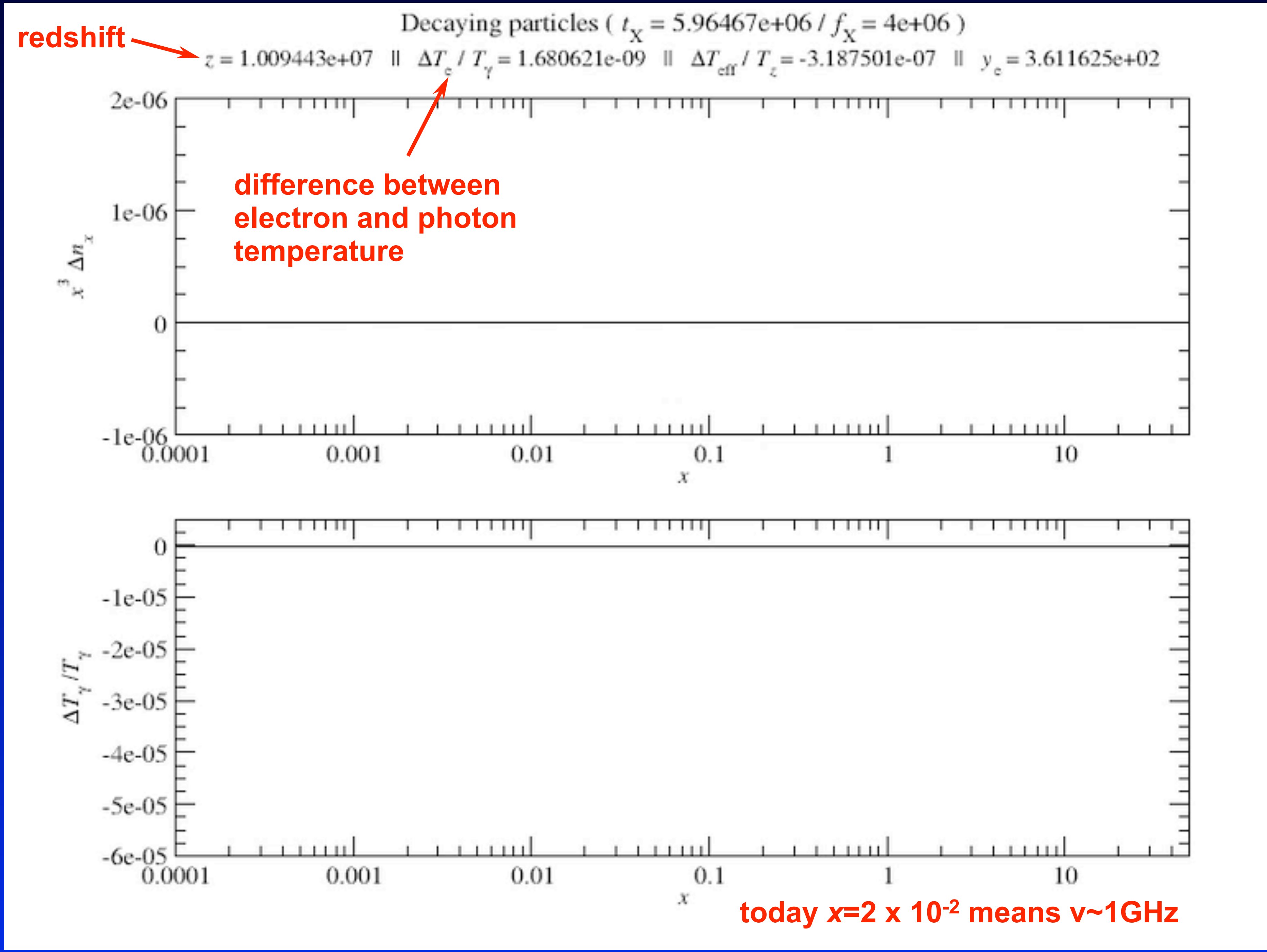
- initial condition: *full equilibrium*
- total energy release:

  $\Delta\rho/\rho \sim 1.3 \times 10^{-6}$
- most of energy release around:

  $z_X \sim 2 \times 10^{6}$
- positive μ-distortion
- high frequency distortion frozen around $z \simeq 5 \times 10^{5}$
- late ($z < 10^{3}$) free-free absorption at very low frequencies ($T_e < T_\gamma$)

Computation carried out with *CosmoTherm*
(JC & Sunyaev 2012)

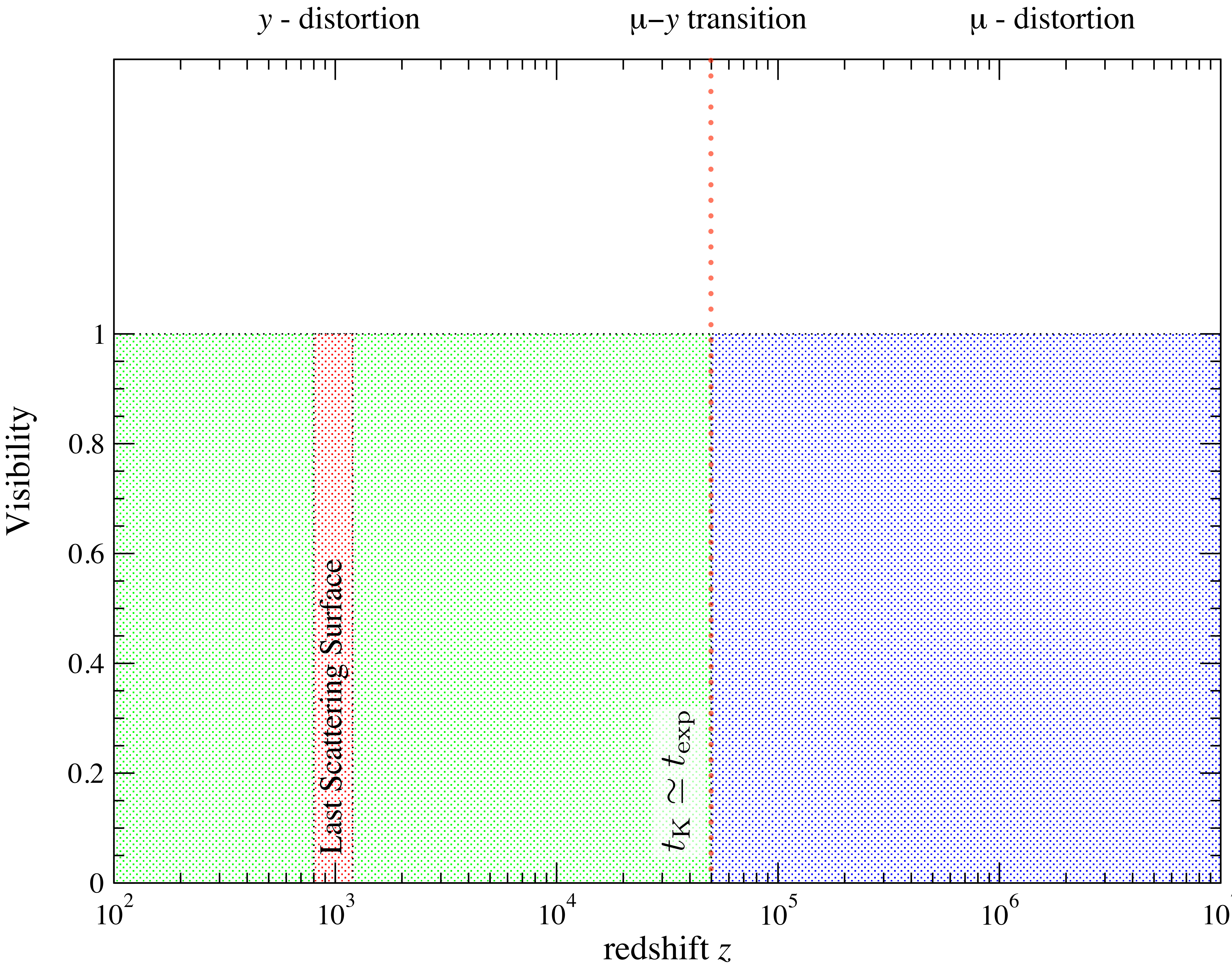
Example: *Energy release by decaying relict particle*



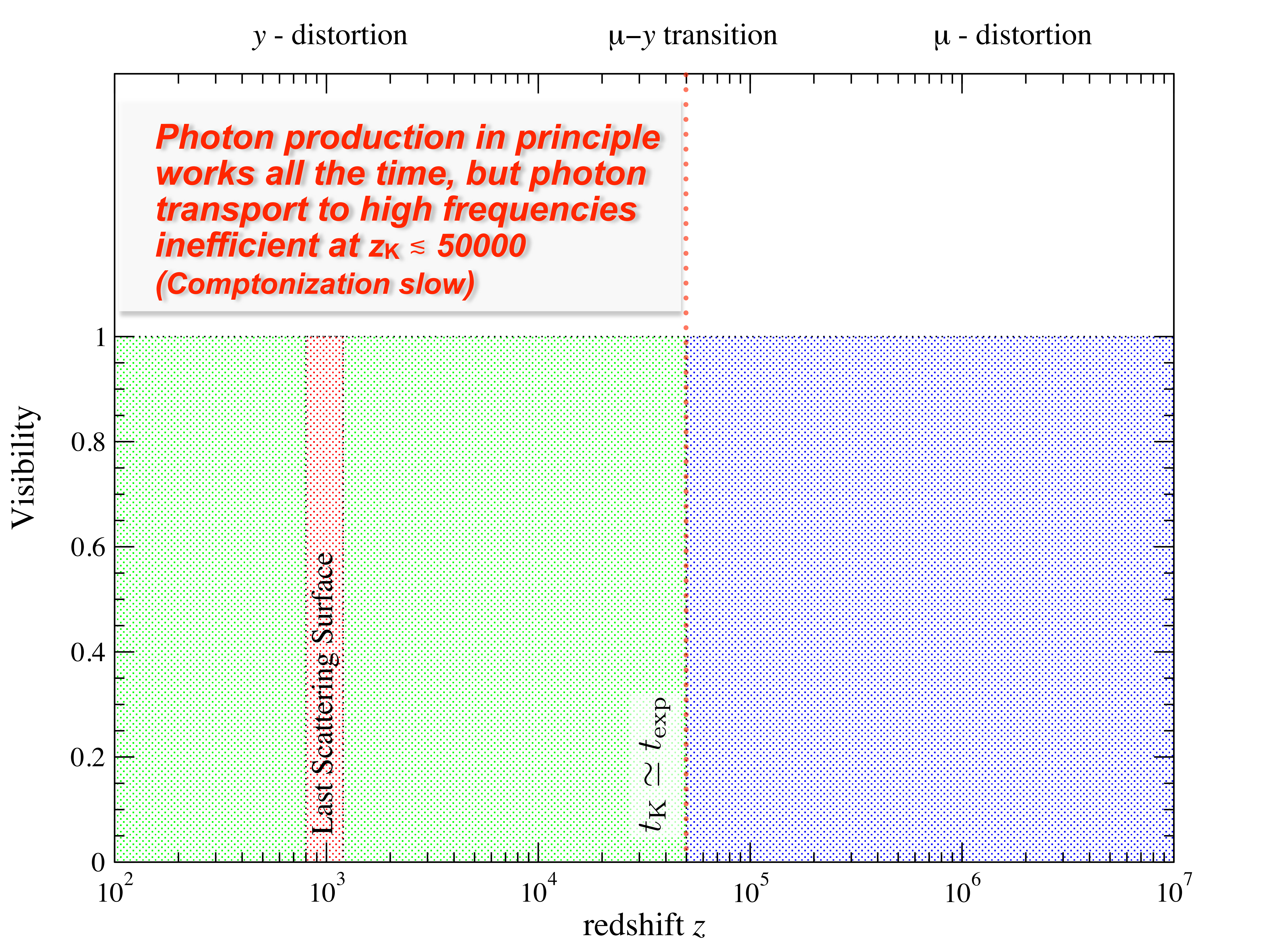
- initial condition: *full equilibrium*
- total energy release: $\Delta\rho/\rho \sim 1.3 \times 10^{-6}$
- most of energy release around: $z_X \sim 2 \times 10^6$
- positive μ-distortion
- high frequency distortion frozen around $z \simeq 5 \times 10^5$
- late ($z < 10^3$) free-free absorption at very low frequencies ($T_e < T_\gamma$)

Computation carried out with *CosmoTherm*
(JC & Sunyaev 2012)

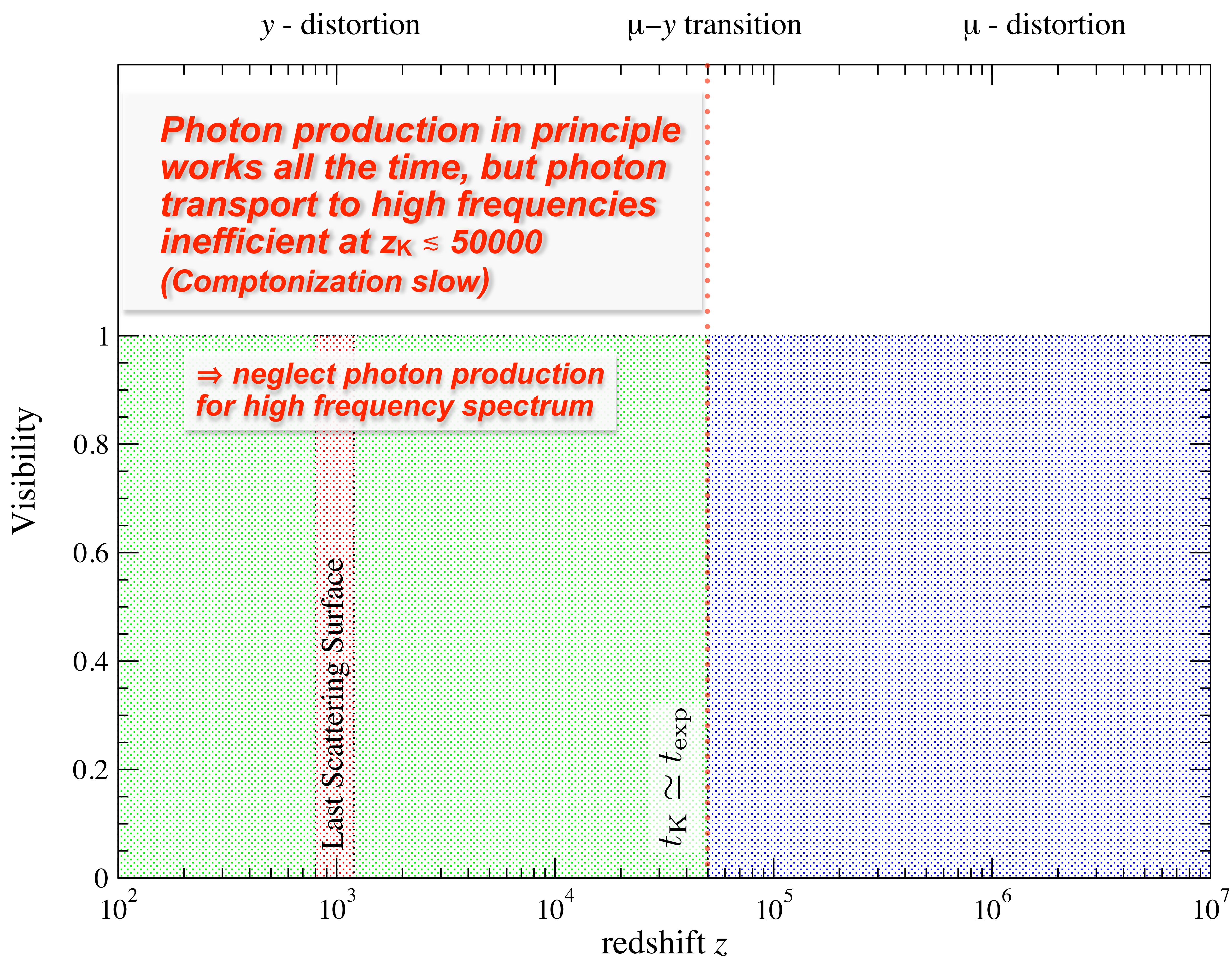
Let’s try to understand the evolution of distortions with photon production analytically!



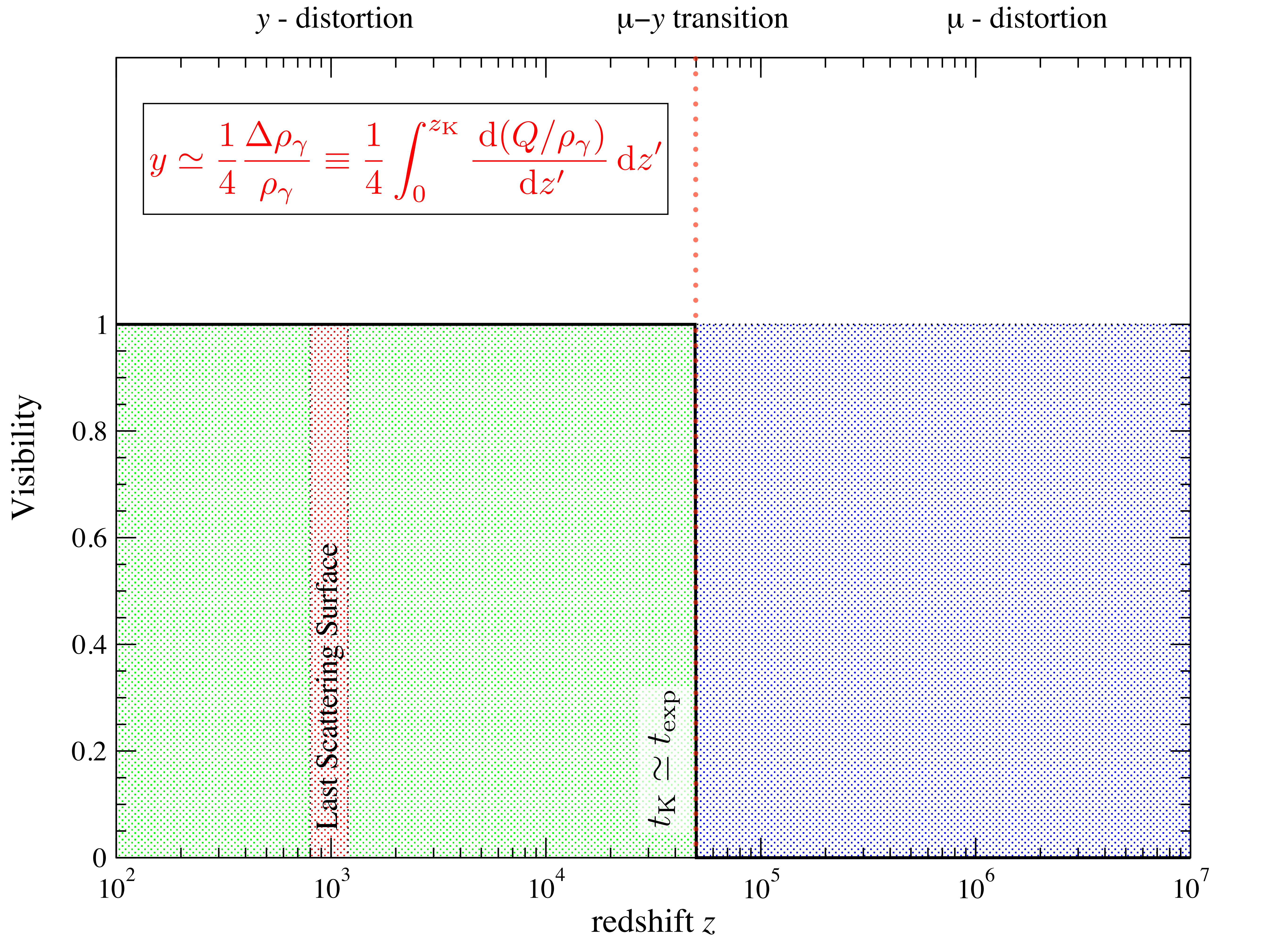
y - distortion
μ−y transition
μ - distortion
Visibility
0
0.2
0.4
0.6
0.8
1
Last Scattering Surface
$t_K \simeq t_{exp}$
10^2
10^3
10^4
10^5
10^6
10^7
redshift z



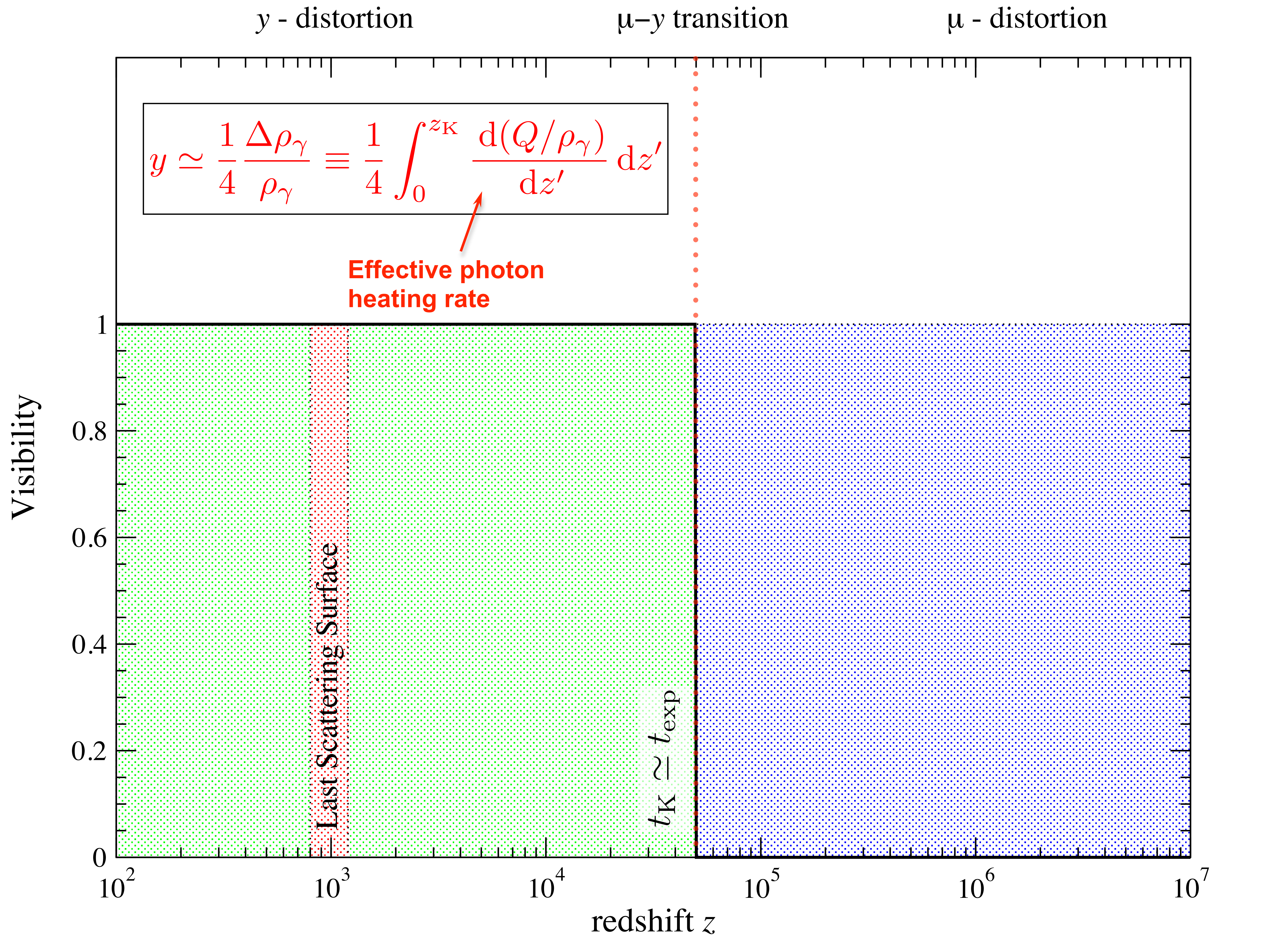
y - distortion
μ–y transition
μ - distortion
Photon production in principle works all the time, but photon transport to high frequencies inefficient at $z_K \lesssim 50000$ (Comptonization slow)
Last Scattering Surface
$t_K \simeq t_{exp}$
Visibility
0
0.2
0.4
0.6
0.8
1
10^2
10^3
10^4
10^5
10^6
10^7
redshift z



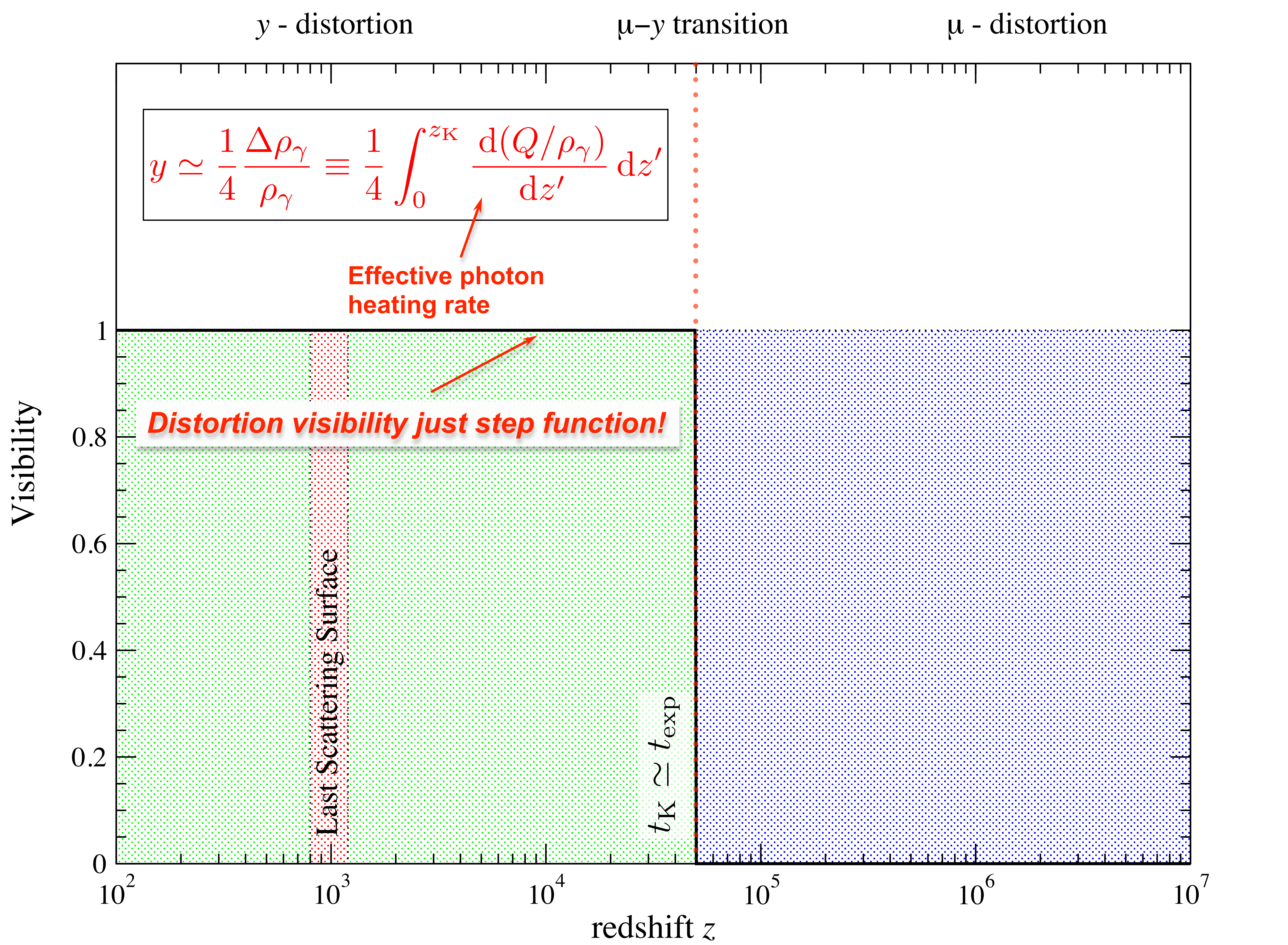
y - distortion
μ–y transition
μ - distortion
Photon production in principle works all the time, but photon transport to high frequencies inefficient at $z_K \lesssim 50000$ (Comptonization slow)
⇒ neglect photon production for high frequency spectrum
Last Scattering Surface
$t_K \simeq t_{exp}$
Visibility
redshift z



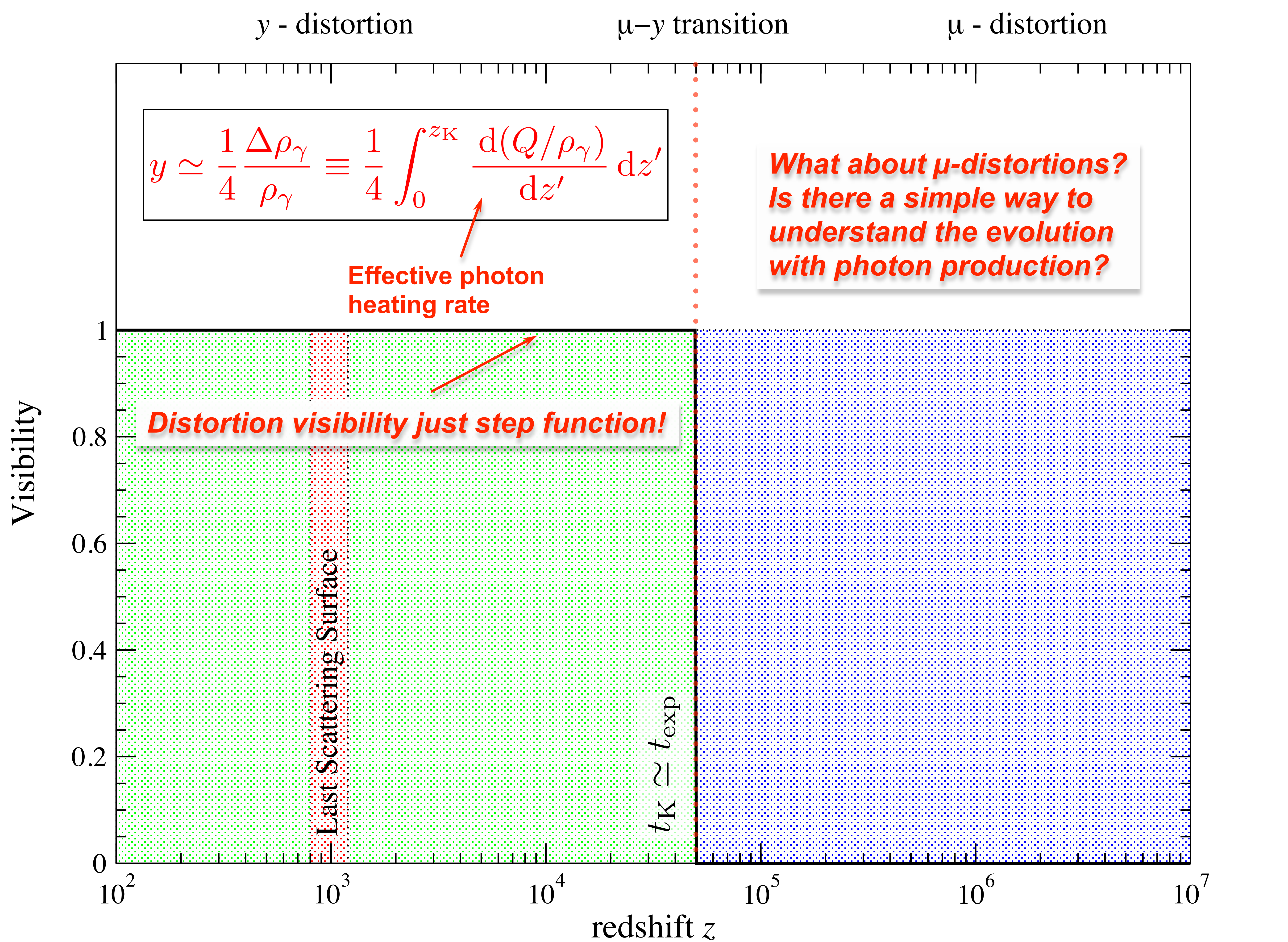
y - distortion
μ−y transition
μ - distortion
$y \simeq \frac{1}{4}\frac{\Delta\rho_\gamma}{\rho_\gamma} \equiv \frac{1}{4}\int_0^{z_K} \frac{d(Q/\rho_\gamma)}{dz'}\,dz'$
Visibility
1
0.8
0.6
0.4
0.2
0
Last Scattering Surface
$t_K \simeq t_{exp}$
10^2
10^3
10^4
10^5
10^6
10^7
redshift z



y - distortion
μ–y transition
μ - distortion
$y \simeq \frac{1}{4}\frac{\Delta\rho_\gamma}{\rho_\gamma} \equiv \frac{1}{4}\int_0^{z_K} \frac{d(Q/\rho_\gamma)}{dz'}\,dz'$
Effective photon heating rate
Visibility
Last Scattering Surface
$t_K \simeq t_{exp}$
redshift z
1
0.8
0.6
0.4
0.2
0
10^2
10^3
10^4
10^5
10^6
10^7



y - distortion
μ–y transition
μ - distortion
$y \simeq \frac{1}{4}\frac{\Delta\rho_\gamma}{\rho_\gamma} \equiv \frac{1}{4}\int_0^{z_K} \frac{d(Q/\rho_\gamma)}{dz'}\,dz'$
Effective photon heating rate
Distortion visibility just step function!
Last Scattering Surface
$t_K \simeq t_{exp}$
Visibility
0
0.2
0.4
0.6
0.8
1
10^2
10^3
10^4
10^5
10^6
10^7
redshift z



y - distortion
μ–y transition
μ - distortion
$y \simeq \frac{1}{4}\frac{\Delta\rho_\gamma}{\rho_\gamma} \equiv \frac{1}{4}\int_0^{z_K} \frac{d(Q/\rho_\gamma)}{dz'}\,dz'$
Effective photon heating rate
What about μ-distortions? Is there a simple way to understand the evolution with photon production?
Distortion visibility just step function!
Last Scattering Surface
$t_K \simeq t_{exp}$
Visibility
redshift z

Analytic Approximation for μ-distortion

Analytic Approximation for μ-distortion

- *Comptonization efficient!* $\Longrightarrow$ $\left.\frac{\mathrm{d}n}{\mathrm{d}\tau}\right|_{\mathrm{C}} + \left.\frac{\mathrm{d}n}{\mathrm{d}\tau}\right|_{\mathrm{em/abs}} \approx 0$

Analytic Approximation for μ-distortion

- *Comptonization efficient!* $\Longrightarrow$ $$\left.\frac{\mathrm{d}n}{\mathrm{d}\tau}\right|_{\mathrm{C}} + \left.\frac{\mathrm{d}n}{\mathrm{d}\tau}\right|_{\mathrm{em/abs}} \approx 0$$

- *low frequency limit & small distortion* $\Longrightarrow$ $$\mu(x, z) \approx \mu_0(z)\,\mathrm{e}^{-x_{\mathrm{c}}(z)/x}$$

(e.g., see Sunyaev & Zeldovich, 1970, ApSS, 7, 20; Hu 1995, PhD Thesis)

Analytic Approximation for μ-distortion

- *Comptonization efficient!* $\Longrightarrow$ $\left.\frac{\mathrm{d}n}{\mathrm{d}\tau}\right|_{\mathrm{C}} + \left.\frac{\mathrm{d}n}{\mathrm{d}\tau}\right|_{\mathrm{em/abs}} \approx 0$

- *low frequency limit & small distortion* $\Longrightarrow$ $\mu(x,z) \approx \mu_0(z)\,\mathrm{e}^{-x_{\mathrm{c}}(z)/x}$

(e.g., see Sunyaev & Zeldovich, 1970, ApSS, 7, 20; Hu 1995, PhD Thesis)

chemical potential at high frequencies

Analytic Approximation for μ-distortion

- *Comptonization efficient!* $\Longrightarrow$ $$\left.\frac{\mathrm{d}n}{\mathrm{d}\tau}\right|_{\mathrm{C}} + \left.\frac{\mathrm{d}n}{\mathrm{d}\tau}\right|_{\mathrm{em/abs}} \approx 0$$

- *low frequency limit & small distortion* $\Longrightarrow$ $$\mu(x, z) \approx \mu_0(z)\, \mathrm{e}^{-x_{\mathrm{c}}(z)/x}$$

(e.g., see Sunyaev & Zeldovich, 1970, ApSS, 7, 20; Hu 1995, PhD Thesis)

critical frequency

chemical potential at high frequencies

Analytic Approximation for μ-distortion

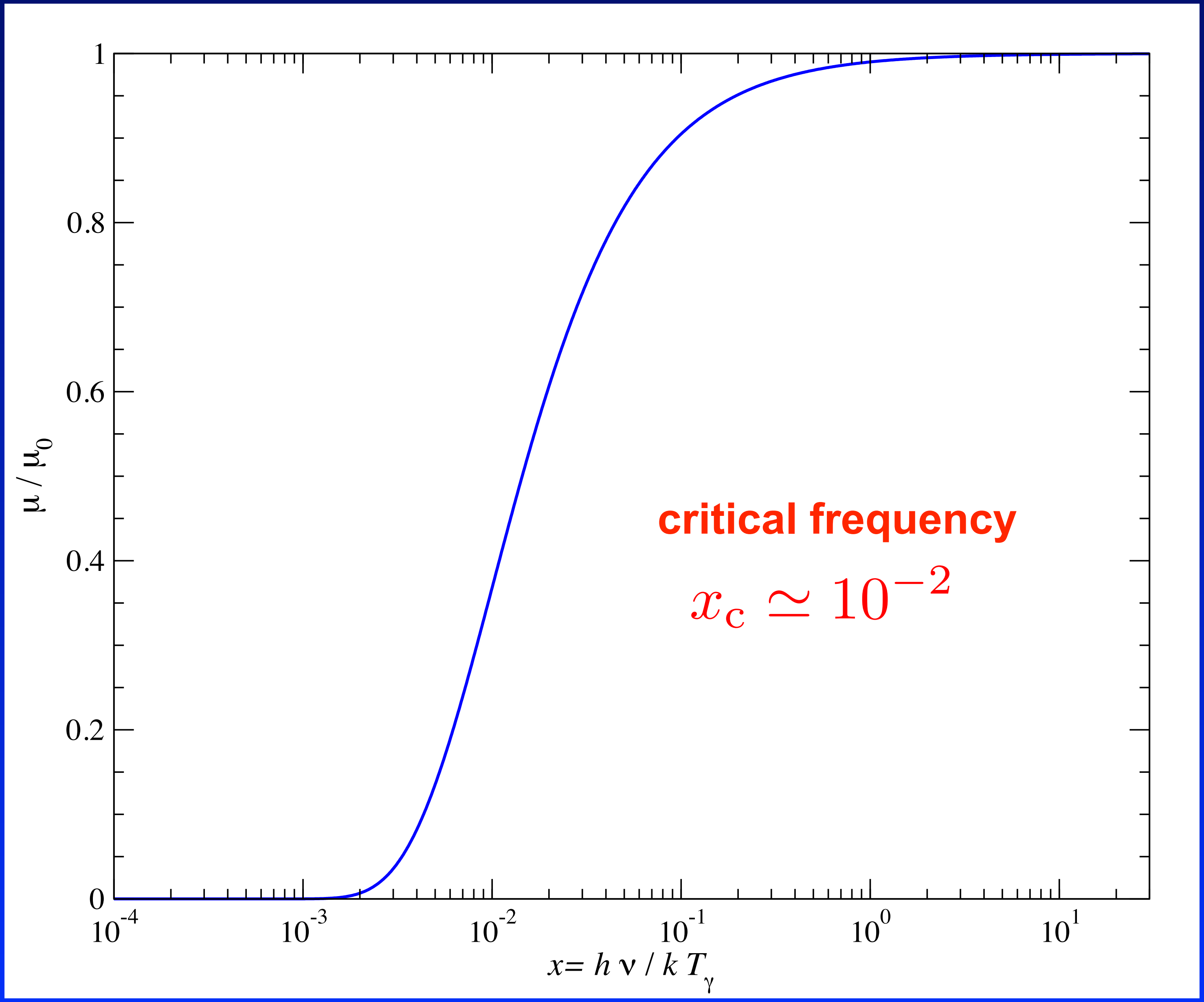
- *Comptonization efficient!* $\Longrightarrow$ $$\left.\frac{\mathrm{d}n}{\mathrm{d}\tau}\right|_{\mathrm{C}} + \left.\frac{\mathrm{d}n}{\mathrm{d}\tau}\right|_{\mathrm{em/abs}} \approx 0$$

- *low frequency limit & small distortion* $\Longrightarrow$ $$\mu(x, z) \approx \mu_0(z)\,\mathrm{e}^{-x_{\mathrm{c}}(z)/x}$$

critical frequency

chemical potential at high frequencies

(e.g., see Sunyaev & Zeldovich, 1970, ApSS, 7, 20; Hu 1995, PhD Thesis)



Analytic Approximation for μ-distortion

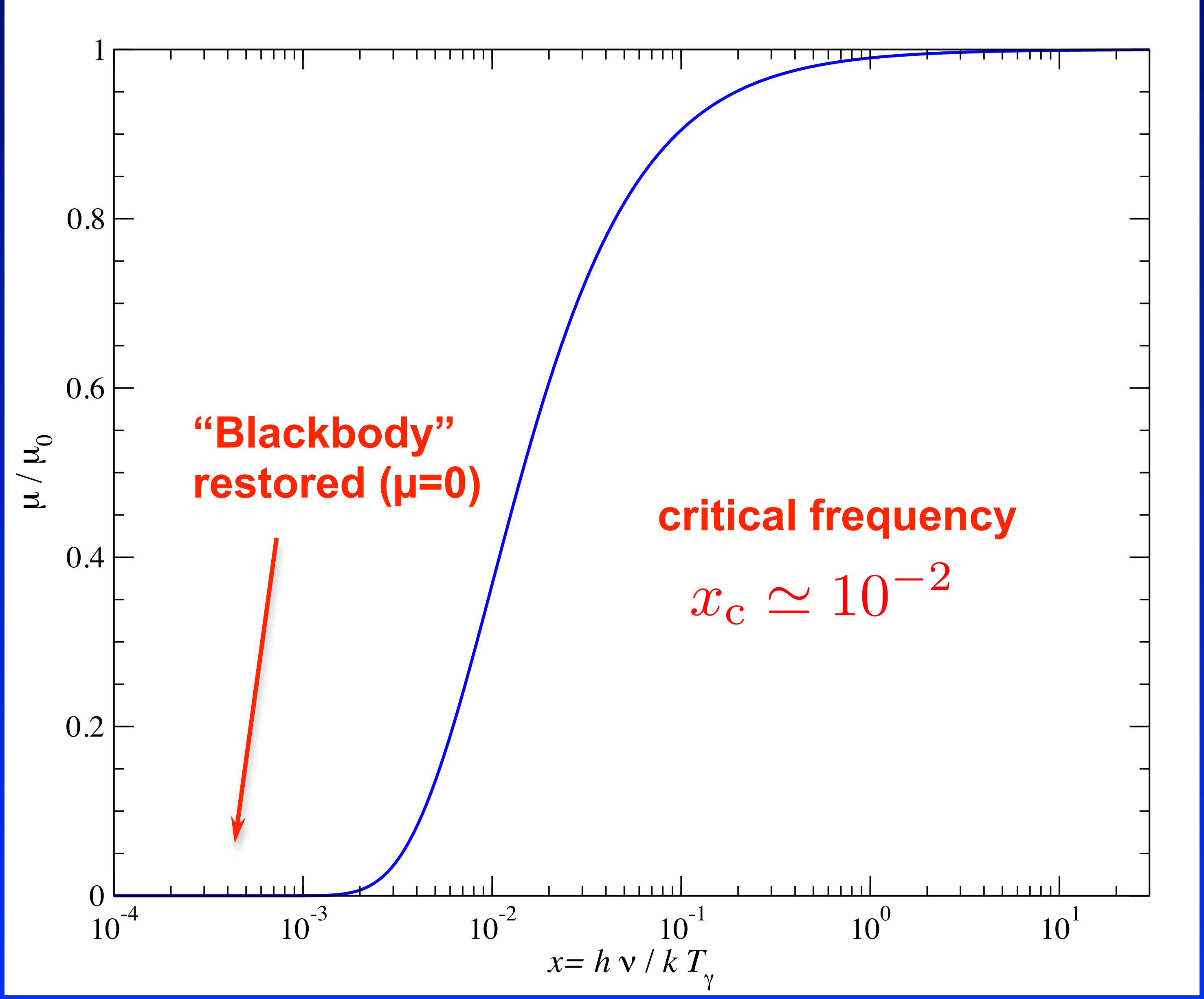
- *Comptonization efficient!* $\Longrightarrow \quad \left.\frac{\mathrm{d}n}{\mathrm{d}\tau}\right|_{\rm C} + \left.\frac{\mathrm{d}n}{\mathrm{d}\tau}\right|_{\rm em/abs} \approx 0$

- *low frequency limit & small distortion* $\Longrightarrow \quad \mu(x,z) \approx \mu_0(z)\,\mathrm{e}^{-x_{\rm c}(z)/x}$

(e.g., see Sunyaev & Zeldovich, 1970, ApSS, 7, 20; Hu 1995, PhD Thesis)

critical frequency

chemical potential at high frequencies



Analytic Approximation for μ-distortion

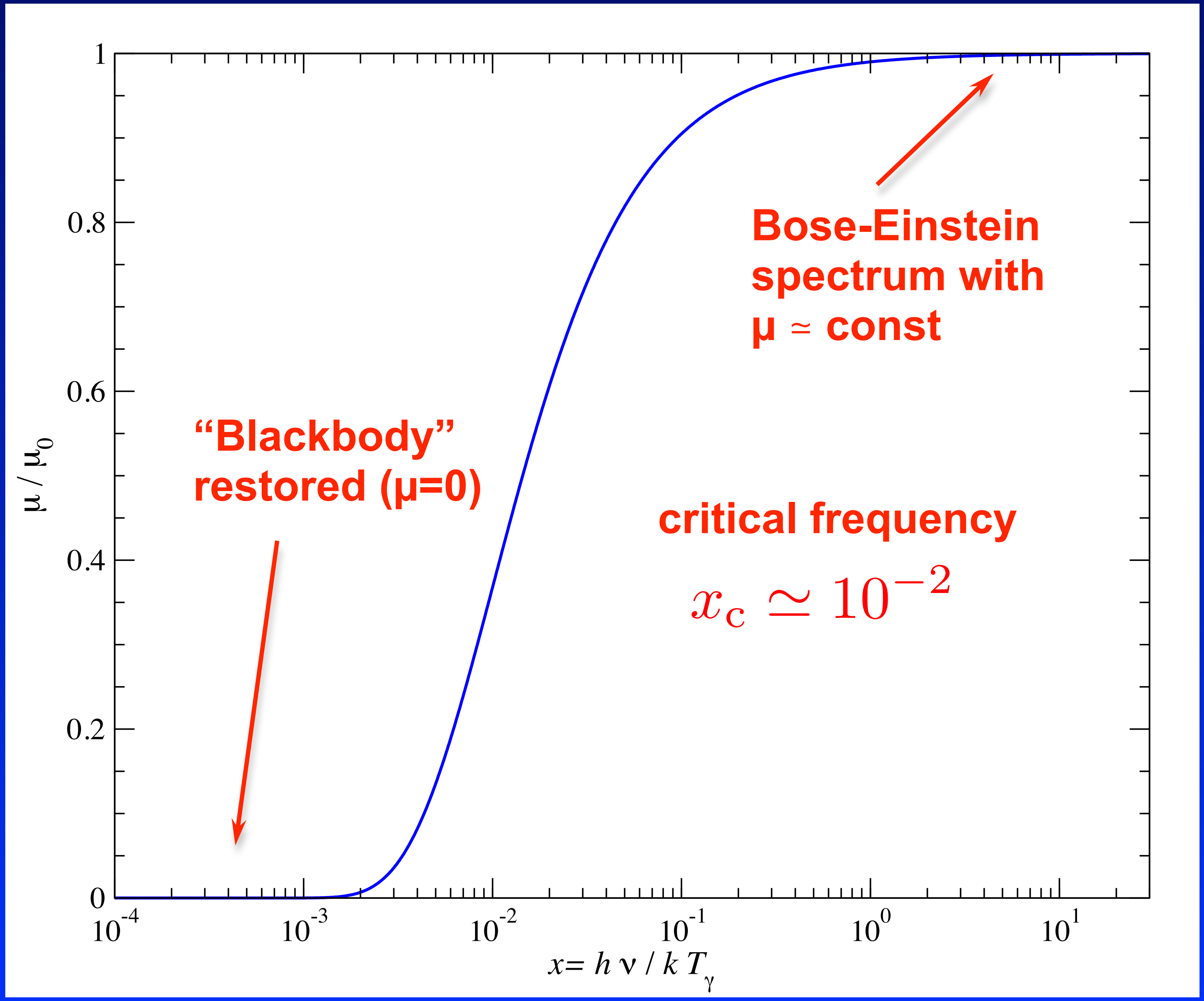
- *Comptonization efficient!* $\Longrightarrow \left.\frac{\mathrm{d}n}{\mathrm{d}\tau}\right|_{\mathrm{C}} + \left.\frac{\mathrm{d}n}{\mathrm{d}\tau}\right|_{\mathrm{em/abs}} \approx 0$

- *low frequency limit & small distortion* $\Longrightarrow \mu(x,z) \approx \mu_0(z)\,\mathrm{e}^{-x_{\mathrm{c}}(z)/x}$

(e.g., see Sunyaev & Zeldovich, 1970, ApSS, 7, 20; Hu 1995, PhD Thesis)

critical frequency (pointing to $x_{\mathrm{c}}(z)$)

chemical potential at high frequencies (pointing to $\mu_0(z)$)



Analytic Approximation for μ-distortion

- *Comptonization efficient!* $\Longrightarrow$ $$\left.\frac{\mathrm{d}n}{\mathrm{d}\tau}\right|_{\mathrm{C}} + \left.\frac{\mathrm{d}n}{\mathrm{d}\tau}\right|_{\mathrm{em/abs}} \approx 0$$

- *low frequency limit & small distortion* $\Longrightarrow$ $$\mu(x, z) \approx \mu_0(z)\, \mathrm{e}^{-x_{\mathrm{c}}(z)/x}$$

(e.g., see Sunyaev & Zeldovich, 1970, ApSS, 7, 20; Hu 1995, PhD Thesis)

Last step: How does $\mu_0(z)$ depend on z?

Analytic Approximation for μ-distortion

- *Comptonization efficient!* $\Longrightarrow$ $$\left.\frac{\mathrm{d}n}{\mathrm{d}\tau}\right|_{\mathrm{C}} + \left.\frac{\mathrm{d}n}{\mathrm{d}\tau}\right|_{\mathrm{em/abs}} \approx 0$$

- *low frequency limit & small distortion* $\Longrightarrow$ $$\mu(x, z) \approx \mu_0(z)\, \mathrm{e}^{-x_{\mathrm{c}}(z)/x}$$

(e.g., see Sunyaev & Zeldovich, 1970, ApSS, 7, 20; Hu 1995, PhD Thesis)

- *Use μ(x, z) to estimate the total photon production rate at low frequencies ⇒ know at which rate the high frequency μ reduces*

Analytic Approximation for μ-distortion

- *Comptonization efficient!* $\Longrightarrow$ $$\left.\frac{\mathrm{d}n}{\mathrm{d}\tau}\right|_{\mathrm{C}} + \left.\frac{\mathrm{d}n}{\mathrm{d}\tau}\right|_{\mathrm{em/abs}} \approx 0$$

- *low frequency limit & small distortion* $\Longrightarrow$ $$\mu(x,z) \approx \mu_0(z)\,\mathrm{e}^{-x_{\mathrm{c}}(z)/x}$$

(e.g., see Sunyaev & Zeldovich, 1970, ApSS, 7, 20; Hu 1995, PhD Thesis)

- *Use μ(x, z) to estimate the total photon production rate at low frequencies ⇒ know at which rate the high frequency μ reduces*

$$\Longrightarrow \quad \mu_0 \approx 1.4 \int_{z_{\mathrm{K}}}^{\infty} \frac{\mathrm{d}(Q/\rho_\gamma)}{\mathrm{d}z'} \mathcal{J}_\mu(z')\mathrm{d}z'$$

Analytic Approximation for μ-distortion

- *Comptonization efficient!* $\Longrightarrow$ $$\left.\frac{\mathrm{d}n}{\mathrm{d}\tau}\right|_{\mathrm{C}} + \left.\frac{\mathrm{d}n}{\mathrm{d}\tau}\right|_{\mathrm{em/abs}} \approx 0$$
- *low frequency limit & small distortion* $\Longrightarrow$ $$\mu(x, z) \approx \mu_0(z)\, \mathrm{e}^{-x_{\mathrm{c}}(z)/x}$$

(e.g., see Sunyaev & Zeldovich, 1970, ApSS, 7, 20; Hu 1995, PhD Thesis)

- *Use μ(x, z) to estimate the total photon production rate at low frequencies ⇒ know at which rate the high frequency μ reduces*

$$\Longrightarrow \quad \mu_0 \approx 1.4 \int_{z_{\mathrm{K}}}^{\infty} \frac{\mathrm{d}(Q/\rho_\gamma)}{\mathrm{d}z'} \mathcal{J}_\mu(z')\mathrm{d}z'$$

- *μ-distortion visibility function:* $\mathcal{J}_\mu(z) \approx \mathrm{e}^{-(z/z_\mu)^{5/2}}$ *with* $z_\mu \approx 2 \times 10^6$

Analytic Approximation for μ-distortion

- *Comptonization efficient!* $\Longrightarrow \quad \left.\frac{\mathrm{d}n}{\mathrm{d}\tau}\right|_{\mathrm{C}} + \left.\frac{\mathrm{d}n}{\mathrm{d}\tau}\right|_{\mathrm{em/abs}} \approx 0$

- *low frequency limit & small distortion* $\Longrightarrow \quad \mu(x, z) \approx \mu_0(z)\,\mathrm{e}^{-x_{\mathrm{c}}(z)/x}$

(e.g., see Sunyaev & Zeldovich, 1970, ApSS, 7, 20; Hu 1995, PhD Thesis)

- *Use μ(x, z) to estimate the total photon production rate at low frequencies ⇒ know at which rate the high frequency μ reduces*

$$\Longrightarrow \quad \mu_0 \approx 1.4 \int_{z_{\mathrm{K}}}^{\infty} \frac{\mathrm{d}(Q/\rho_\gamma)}{\mathrm{d}z'} \mathcal{J}_\mu(z')\mathrm{d}z'$$

- *μ-distortion visibility function:* $\mathcal{J}_\mu(z) \approx \mathrm{e}^{-(z/z_\mu)^{5/2}}$ *with* $z_\mu \approx 2 \times 10^6$

Set by photon DC process

Analytic Approximation for μ-distortion

- *Comptonization efficient!* $\Longrightarrow \quad \left.\frac{\mathrm{d}n}{\mathrm{d}\tau}\right|_{\mathrm{C}} + \left.\frac{\mathrm{d}n}{\mathrm{d}\tau}\right|_{\mathrm{em/abs}} \approx 0$

- *low frequency limit & small distortion* $\Longrightarrow \quad \mu(x, z) \approx \mu_0(z)\, \mathrm{e}^{-x_{\mathrm{c}}(z)/x}$

(e.g., see Sunyaev & Zeldovich, 1970, ApSS, 7, 20; Hu 1995, PhD Thesis)

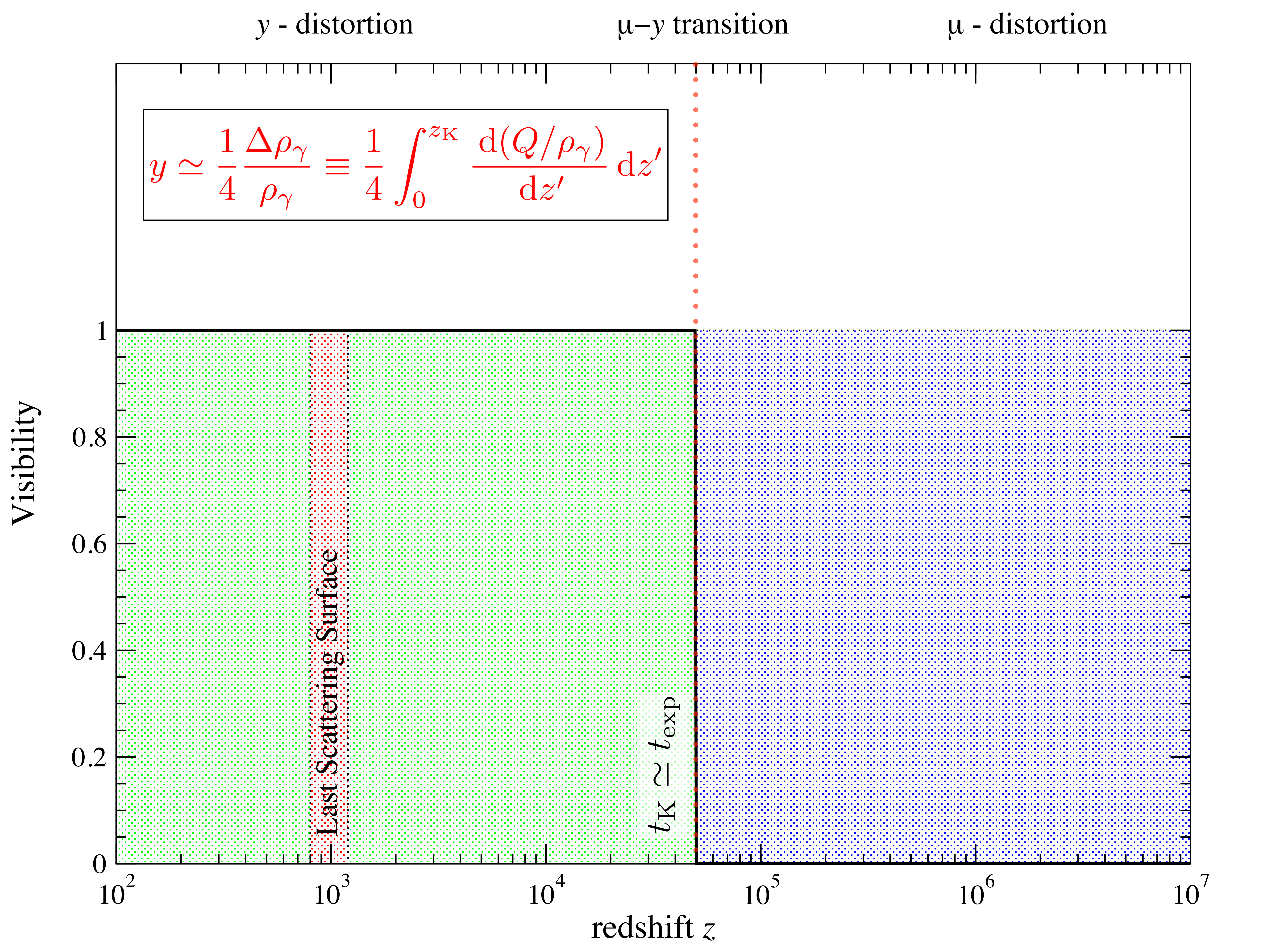
- *Use μ(x, z) to estimate the total photon production rate at low frequencies ⇒ know at which rate the high frequency μ reduces*

$$\Longrightarrow \quad \mu_0 \approx 1.4 \int_{z_{\mathrm{K}}}^{\infty} \frac{\mathrm{d}(Q/\rho_\gamma)}{\mathrm{d}z'} \mathcal{J}_\mu(z') \mathrm{d}z'$$

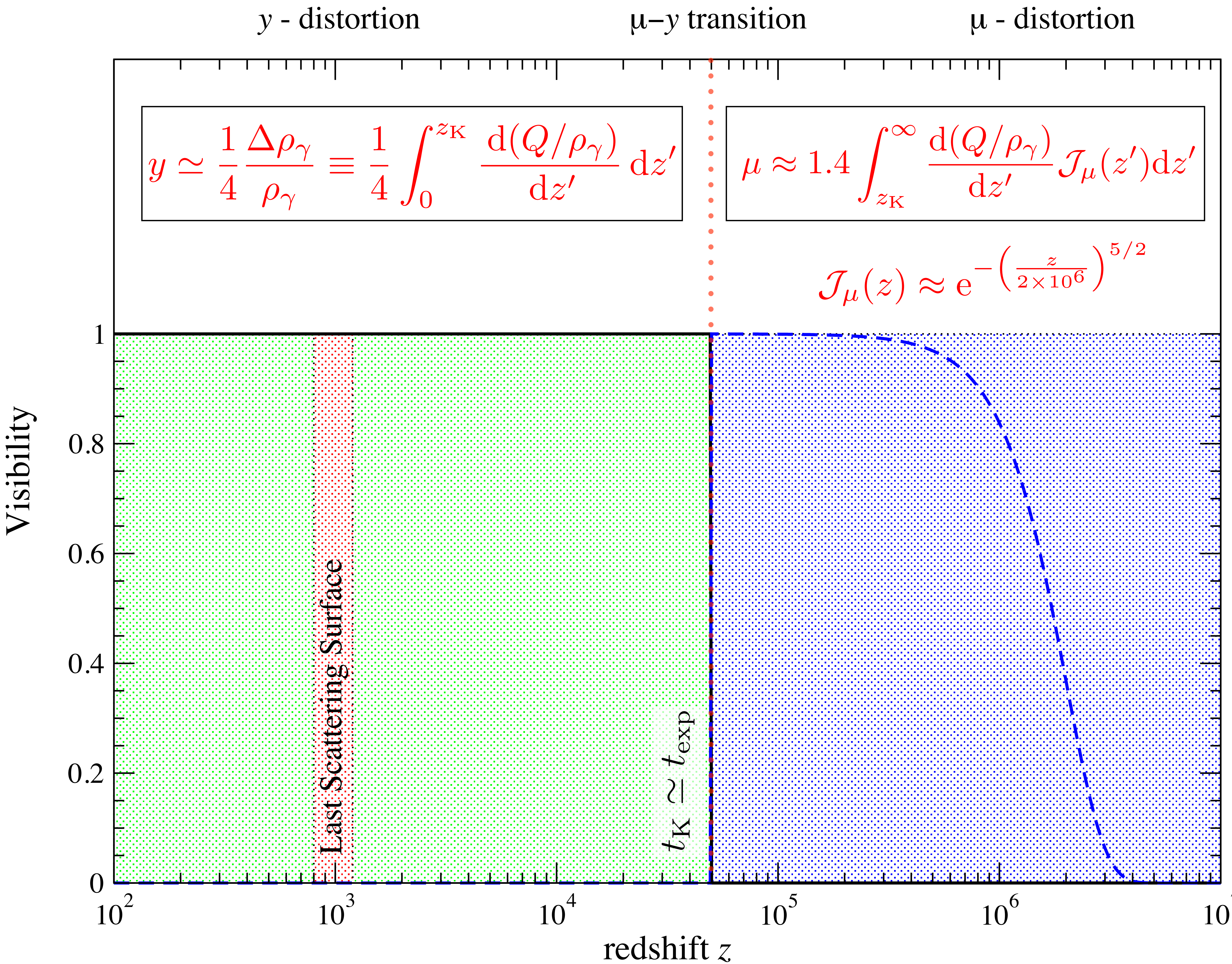
Set by photon DC process

- *μ-distortion visibility function:* $\mathcal{J}_\mu(z) \approx \mathrm{e}^{-(z/z_\mu)^{5/2}}$ *with* $z_\mu \approx 2 \times 10^6$

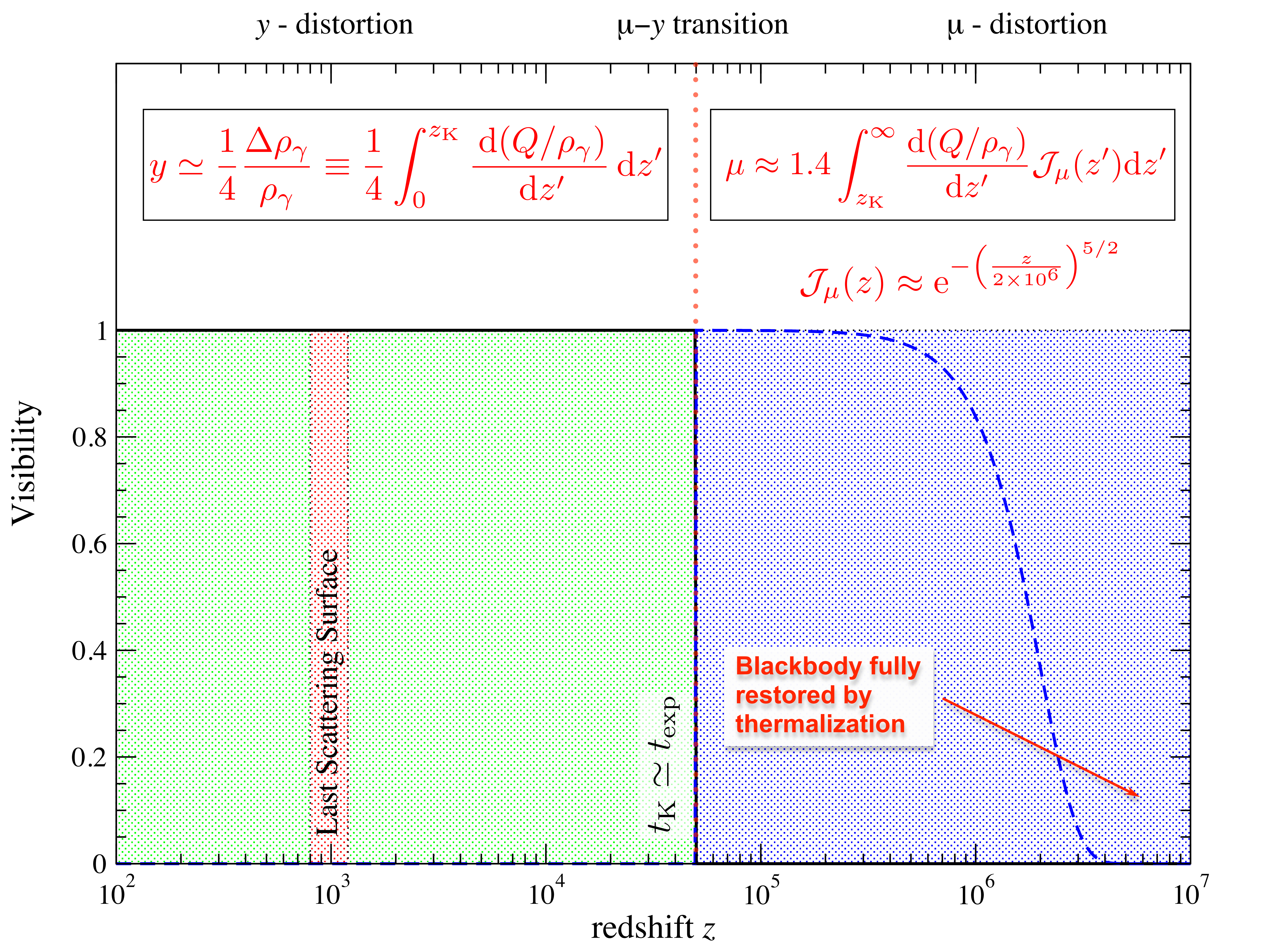
- *Transition between μ and y modeled as simple step function*



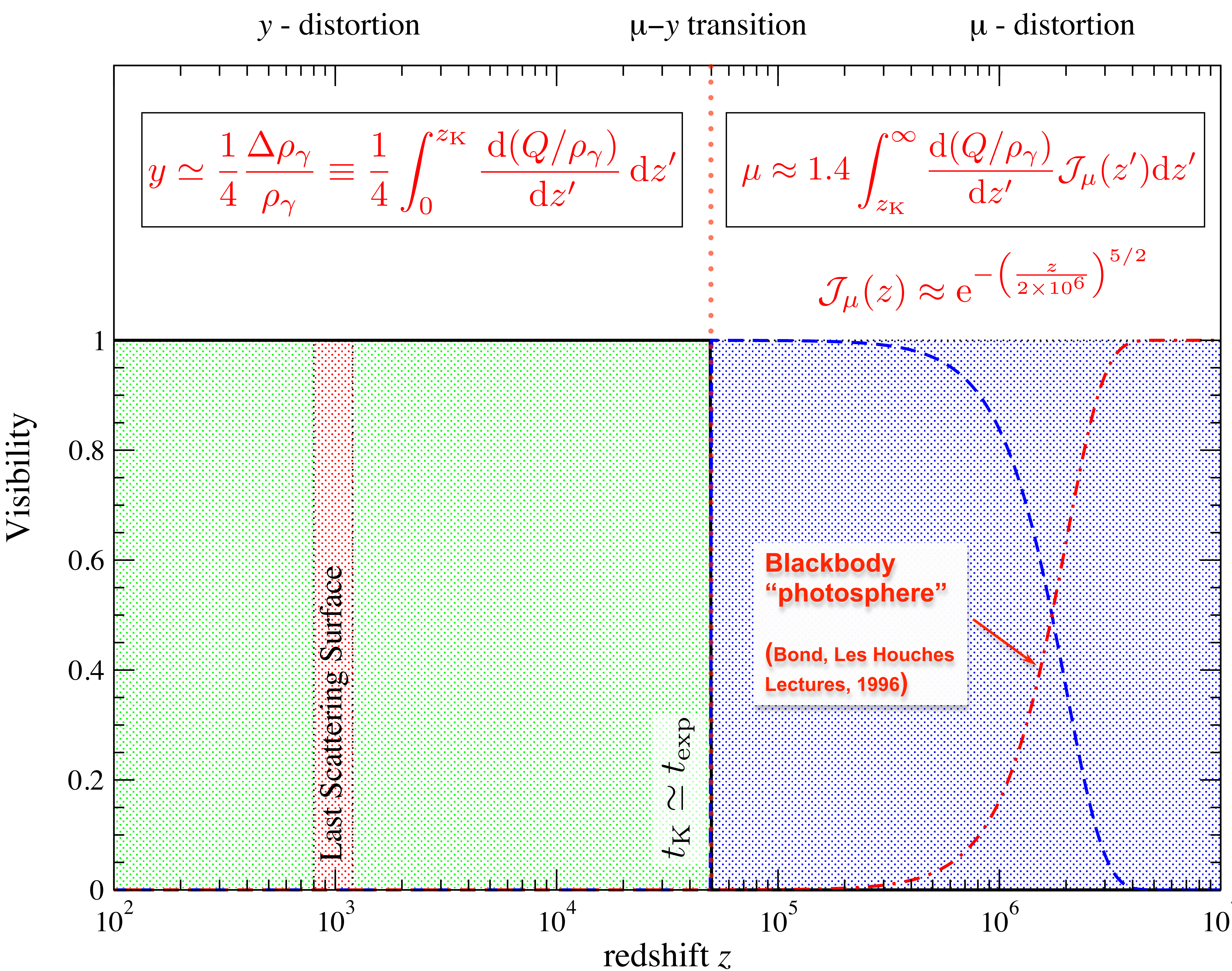
y - distortion
μ–y transition
μ - distortion
$y \simeq \frac{1}{4}\frac{\Delta\rho_\gamma}{\rho_\gamma} \equiv \frac{1}{4}\int_0^{z_K} \frac{d(Q/\rho_\gamma)}{dz'}\,dz'$
Visibility
Last Scattering Surface
$t_K \simeq t_{exp}$
redshift z
0
0.2
0.4
0.6
0.8
1
10^2
10^3
10^4
10^5
10^6
10^7



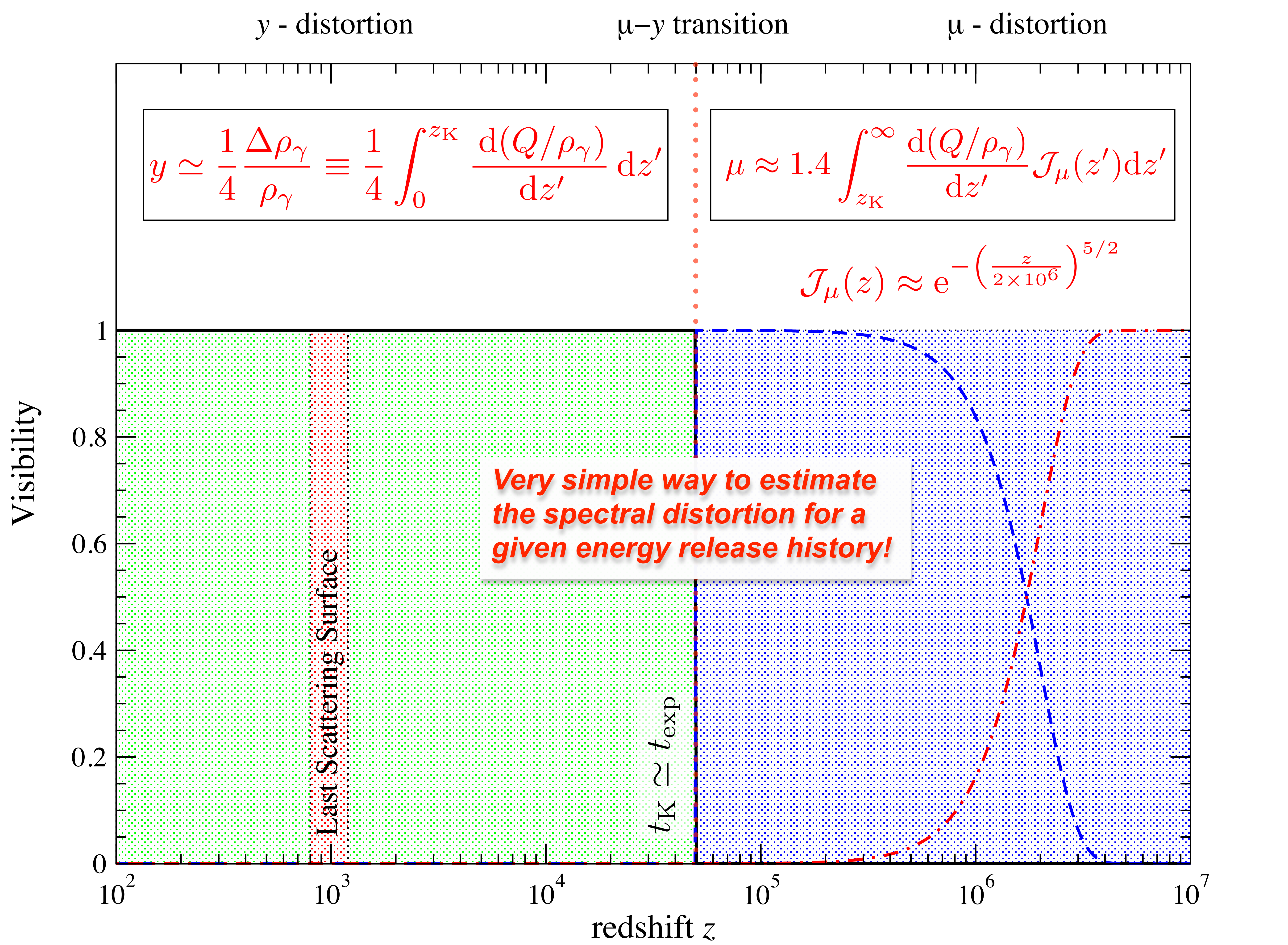
y - distortion
μ–y transition
μ - distortion
$y \simeq \frac{1}{4}\frac{\Delta\rho_\gamma}{\rho_\gamma} \equiv \frac{1}{4}\int_0^{z_K} \frac{d(Q/\rho_\gamma)}{dz'}\,dz'$
$\mu \approx 1.4\int_{z_K}^{\infty} \frac{d(Q/\rho_\gamma)}{dz'}\mathcal{J}_\mu(z')dz'$
$\mathcal{J}_\mu(z) \approx e^{-\left(\frac{z}{2\times10^6}\right)^{5/2}}$
Last Scattering Surface
$t_K \simeq t_{exp}$
Visibility
1
0.8
0.6
0.4
0.2
0
10^2
10^3
10^4
10^5
10^6
10^7
redshift z



y - distortion
μ–y transition
μ - distortion
$y \simeq \frac{1}{4}\frac{\Delta\rho_\gamma}{\rho_\gamma} \equiv \frac{1}{4}\int_0^{z_K} \frac{d(Q/\rho_\gamma)}{dz'}\,dz'$
$\mu \approx 1.4 \int_{z_K}^{\infty} \frac{d(Q/\rho_\gamma)}{dz'} \mathcal{J}_\mu(z')dz'$
$\mathcal{J}_\mu(z) \approx e^{-\left(\frac{z}{2\times10^6}\right)^{5/2}}$
Visibility
1
0.8
0.6
0.4
0.2
0
Last Scattering Surface
$t_K \simeq t_{exp}$
Blackbody fully restored by thermalization
10^2
10^3
10^4
10^5
10^6
10^7
redshift z

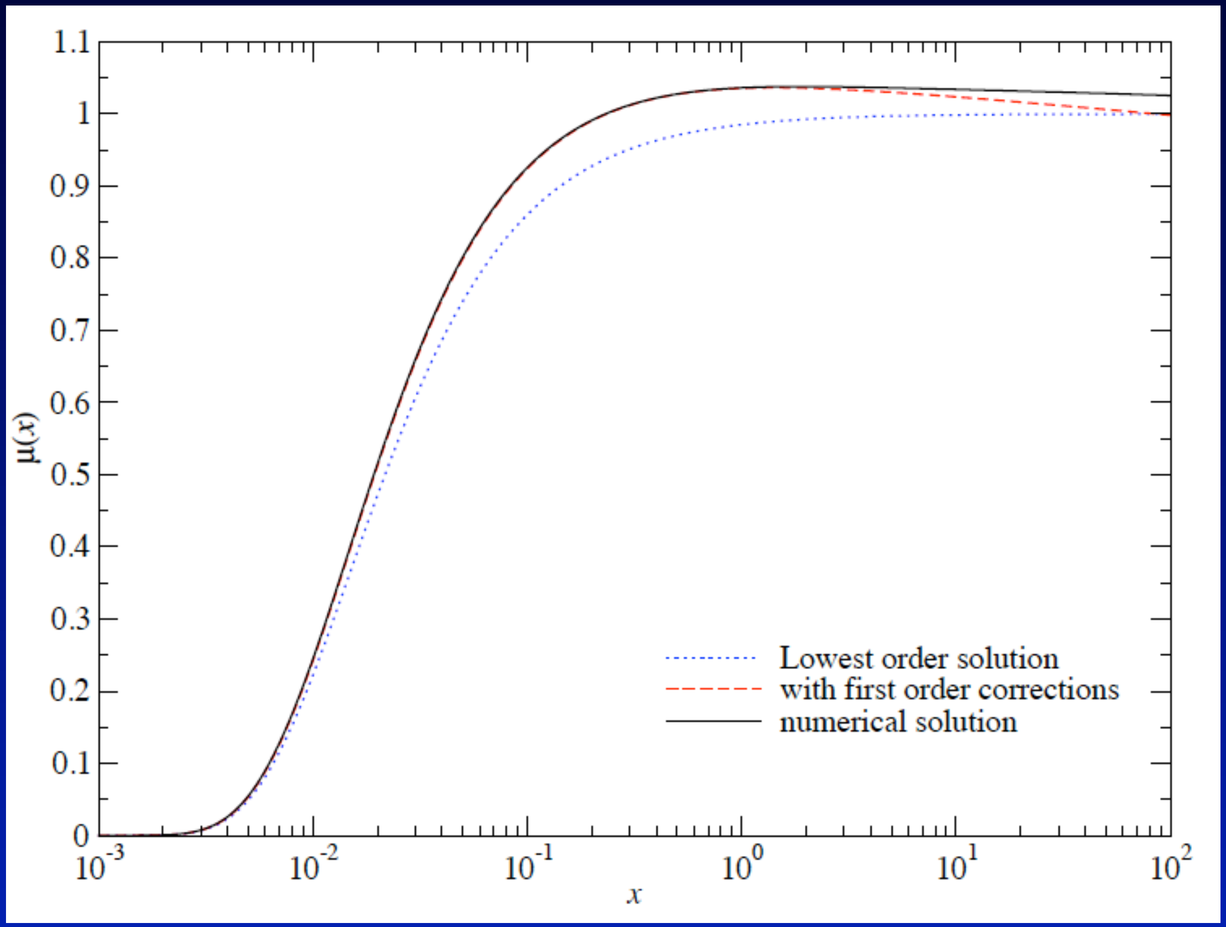


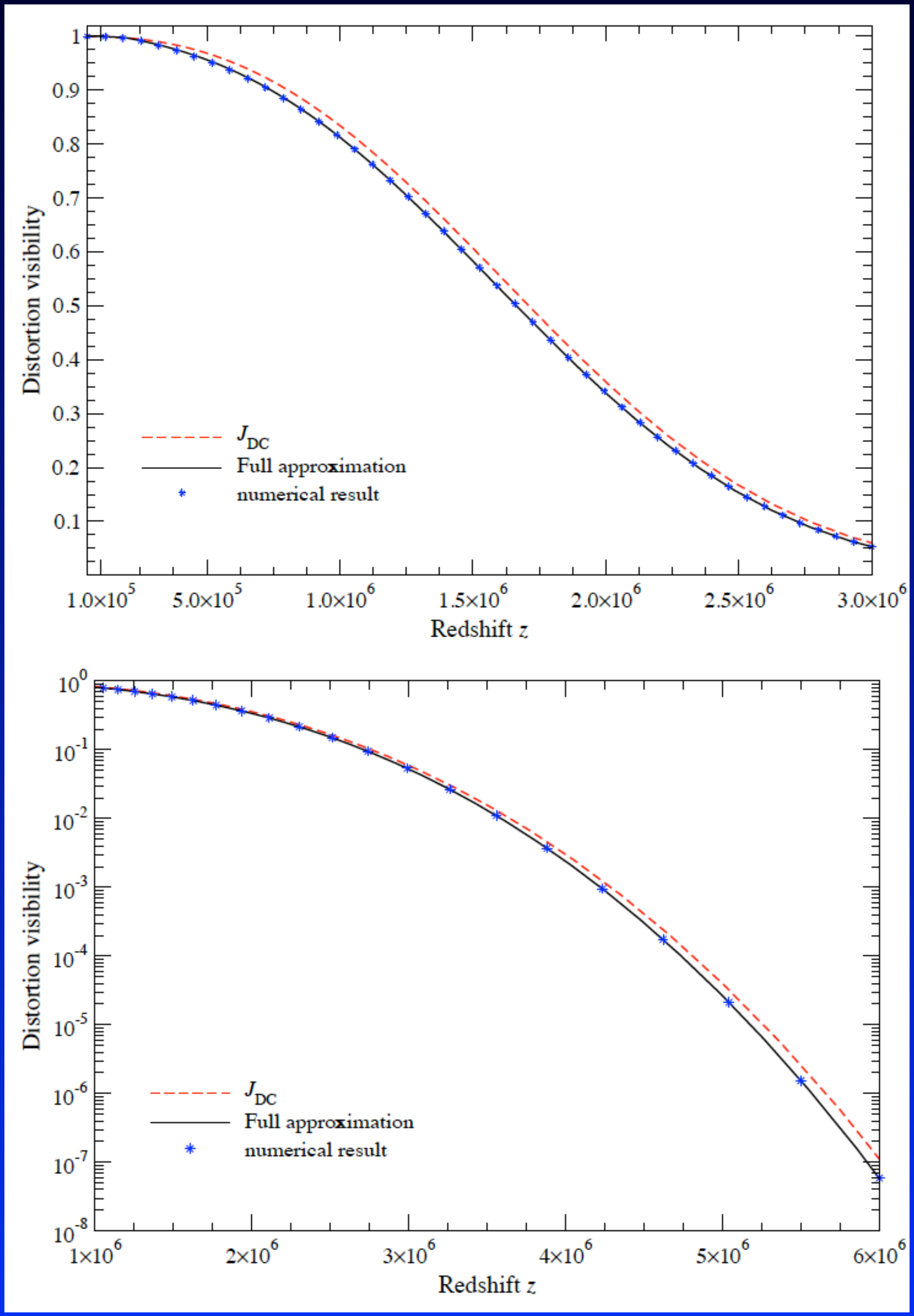
y - distortion
μ–y transition
μ - distortion
$y \simeq \frac{1}{4}\frac{\Delta\rho_\gamma}{\rho_\gamma} \equiv \frac{1}{4}\int_0^{z_K} \frac{d(Q/\rho_\gamma)}{dz'}\,dz'$
$\mu \approx 1.4 \int_{z_K}^{\infty} \frac{d(Q/\rho_\gamma)}{dz'} \mathcal{J}_\mu(z')dz'$
$\mathcal{J}_\mu(z) \approx e^{-\left(\frac{z}{2\times10^6}\right)^{5/2}}$
Visibility
Last Scattering Surface
$t_K \simeq t_{exp}$
Blackbody
"photosphere"
(Bond, Les Houches
Lectures, 1996)
redshift z



y - distortion
μ–y transition
μ - distortion
$y \simeq \frac{1}{4}\frac{\Delta\rho_\gamma}{\rho_\gamma} \equiv \frac{1}{4}\int_0^{z_K} \frac{d(Q/\rho_\gamma)}{dz'}\,dz'$
$\mu \approx 1.4 \int_{z_K}^{\infty} \frac{d(Q/\rho_\gamma)}{dz'} \mathcal{J}_\mu(z')dz'$
$\mathcal{J}_\mu(z) \approx e^{-\left(\frac{z}{2\times10^6}\right)^{5/2}}$
Visibility
1
0.8
0.6
0.4
0.2
0
Last Scattering Surface
$t_K \simeq t_{exp}$
Very simple way to estimate the spectral distortion for a given energy release history!
10^2
10^3
10^4
10^5
10^6
10^7
redshift z

Improved approximations for the distortion visibility function





- high frequency tail not constant μ
- more emission at lower frequency
- faster thermalization ⇒ *visibility lower*
- analytic approximations possible...

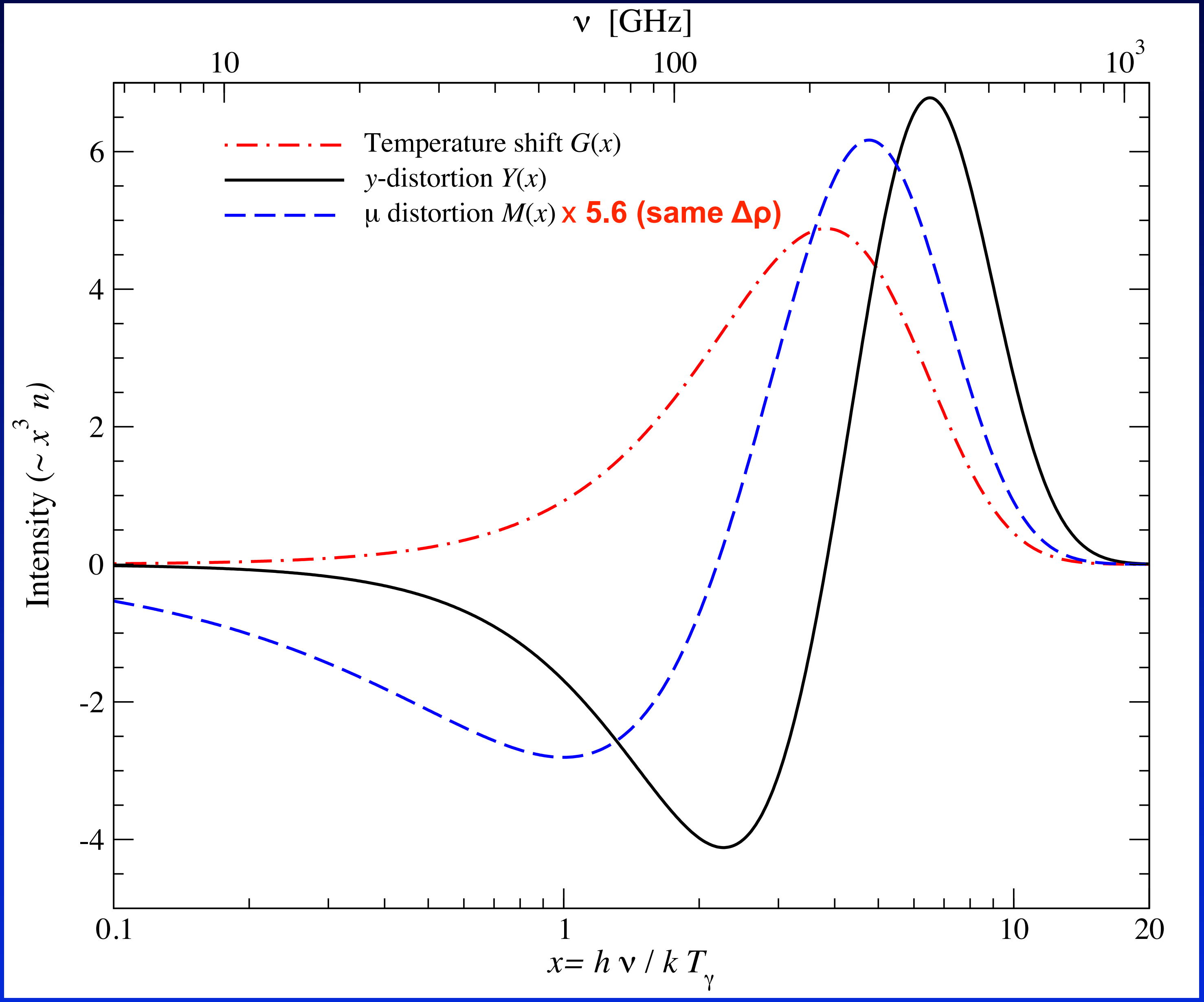
Khatri & Sunyaev, ArXiv:1203.2601
JC, ArXiv:1312.6030

JC, ArXiv:1312.6030

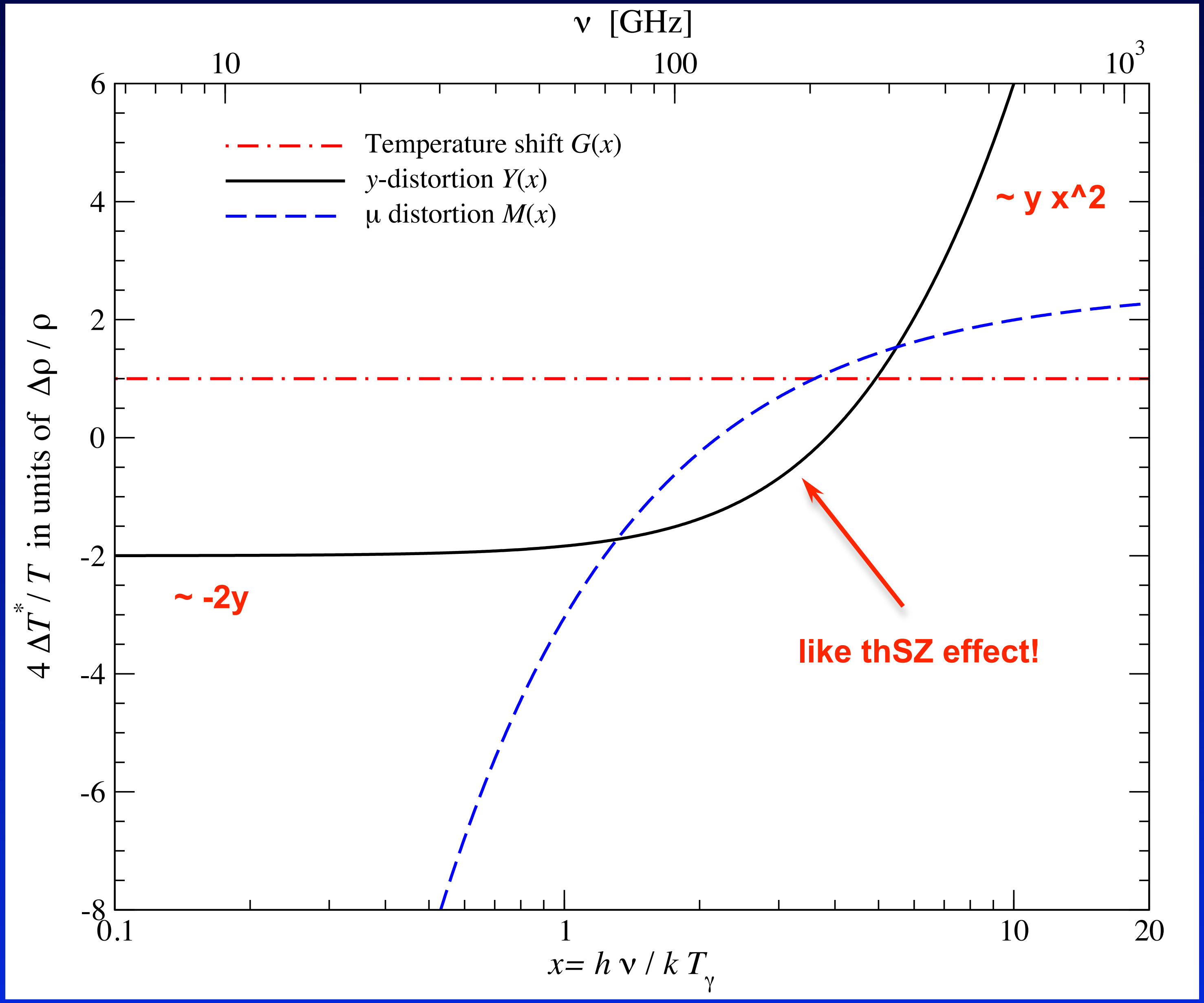
What about the μ-y transition regime?

Is the transition really as abrupt?

Temperature shift ↔ y-distortion ↔ μ-distortion



Temperature shift ↔ y-distortion ↔ μ-distortion



Same as before but as effective temperature

$$\frac{\Delta T^*}{T} \approx \frac{\Delta n(x)}{G(x)}$$

Transition from y-distortion → μ-distortion

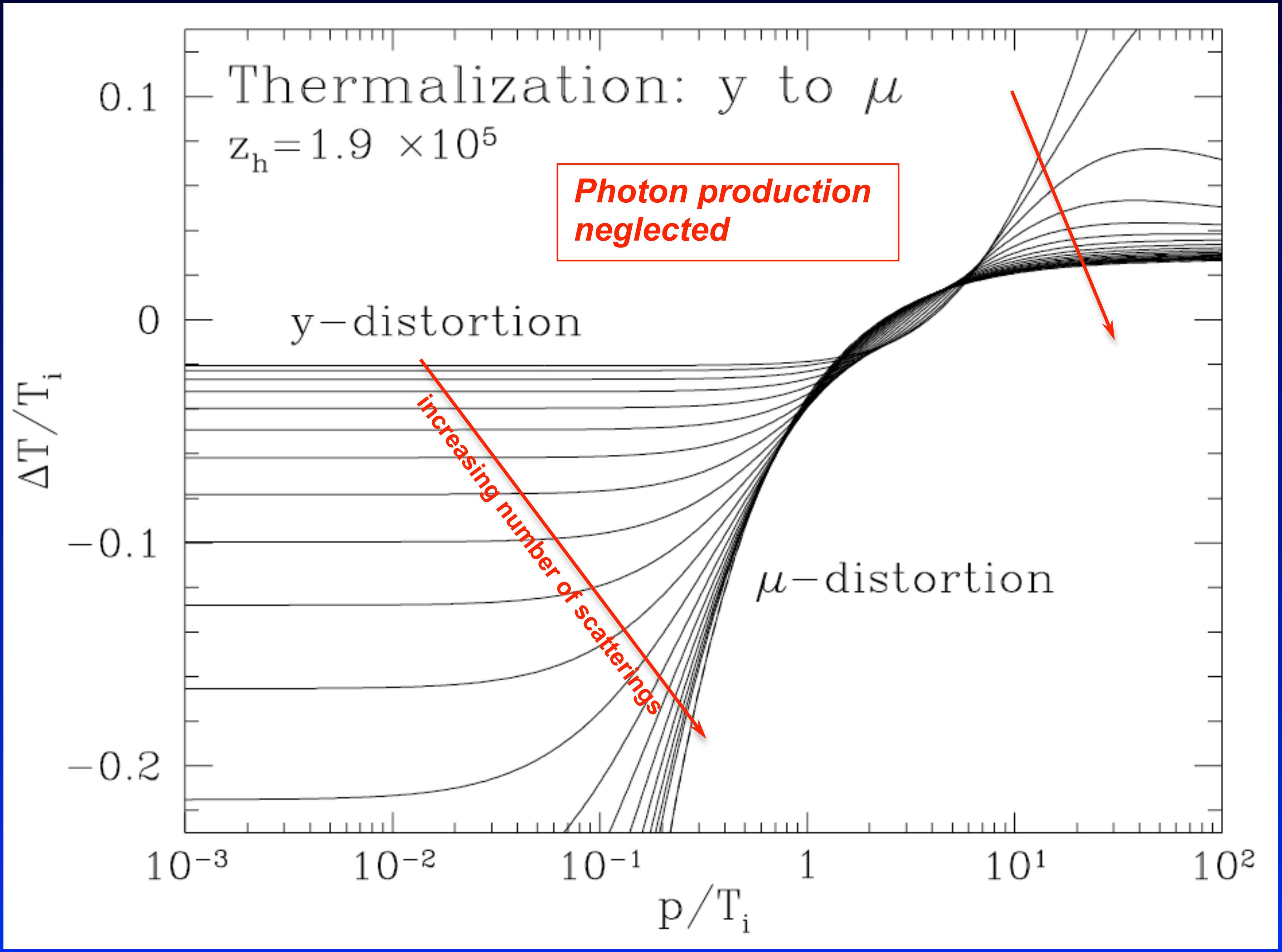


Figure from Wayne Hu's PhD thesis, 1995, but see also discussion in Burigana, 1991

Transition from y-distortion → μ-distortion

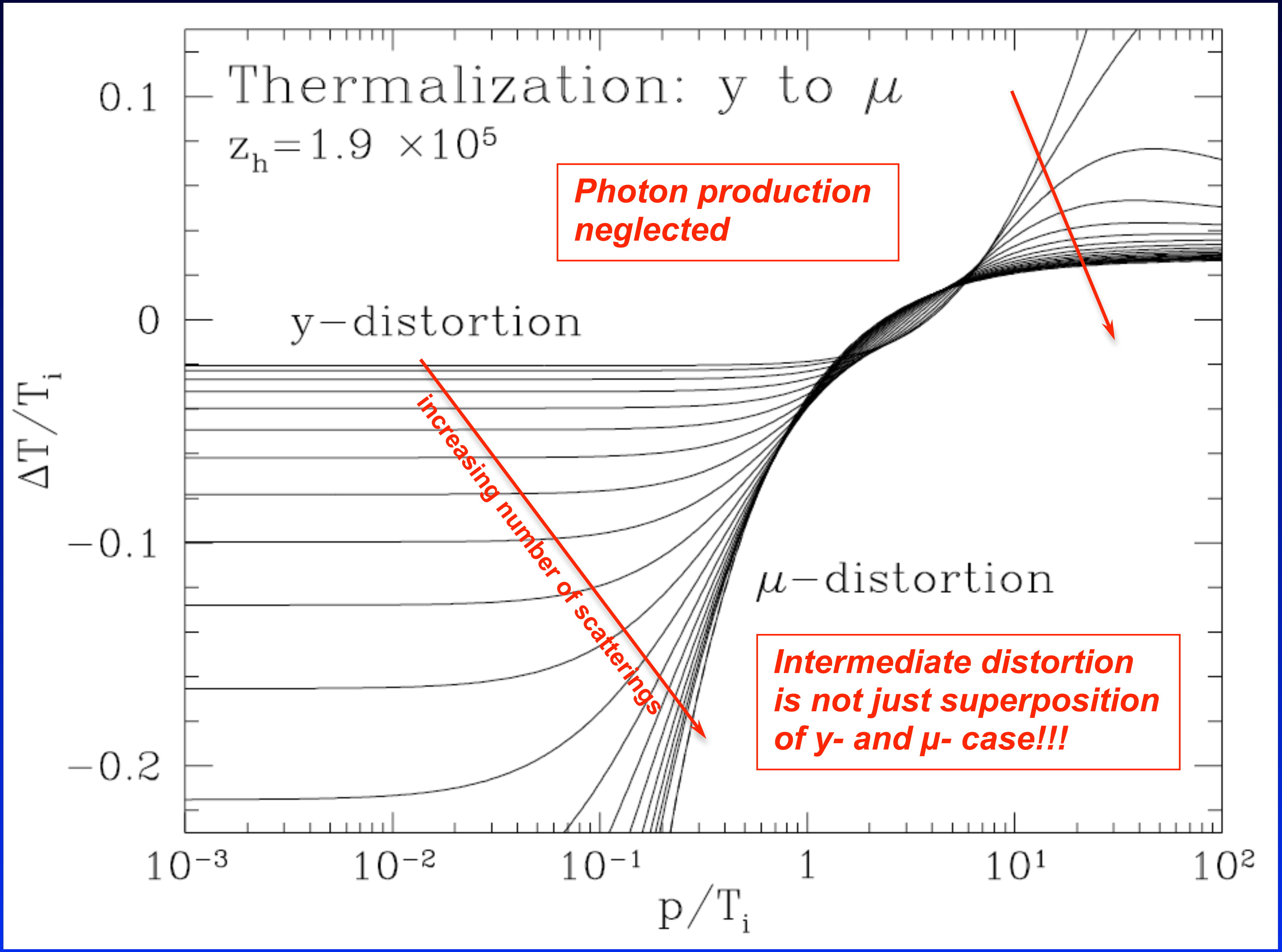
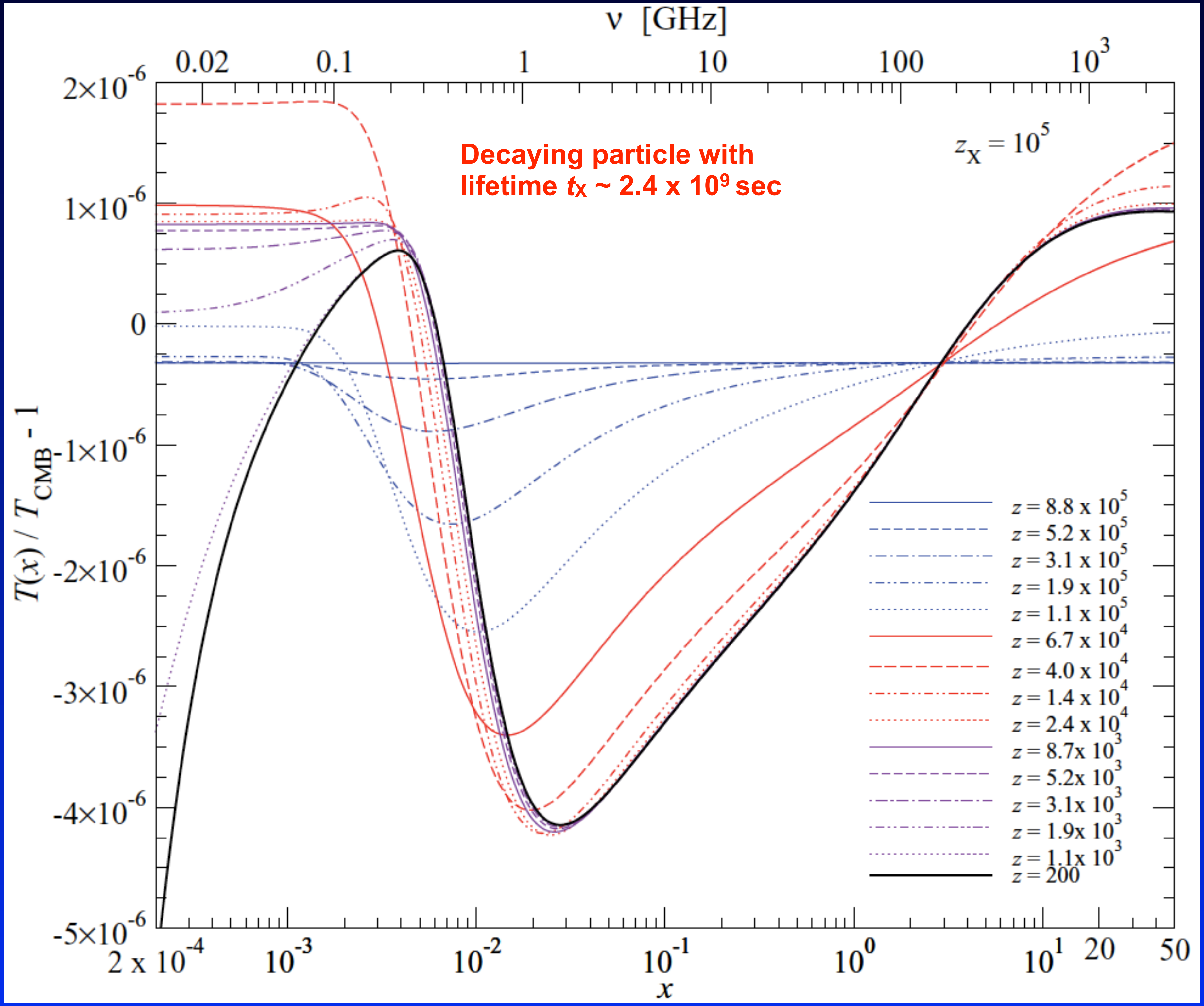


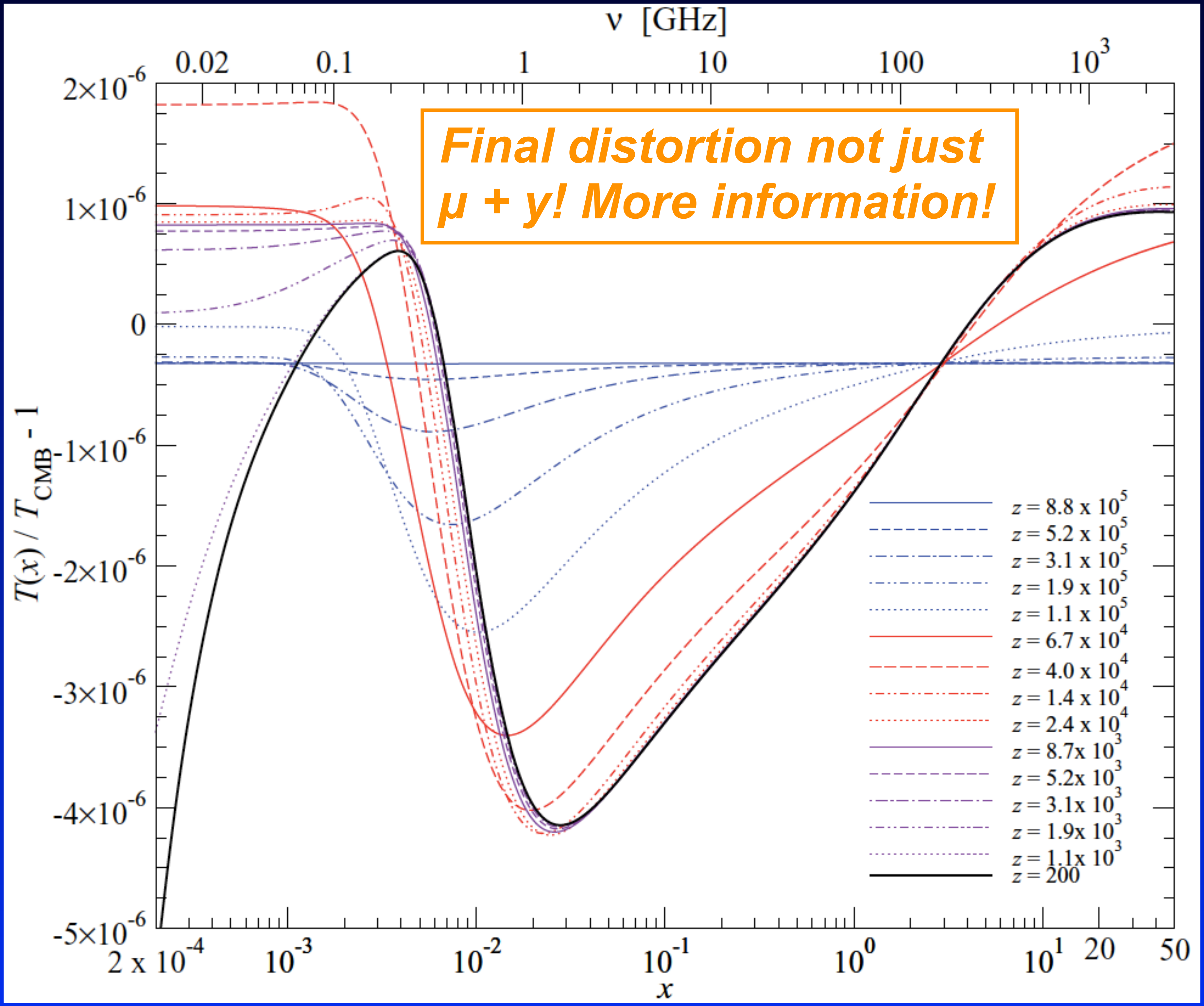
Figure from Wayne Hu's PhD thesis, 1995, but see also discussion in Burigana, 1991

Distortion *not* just superposition of μ and y-distortion!

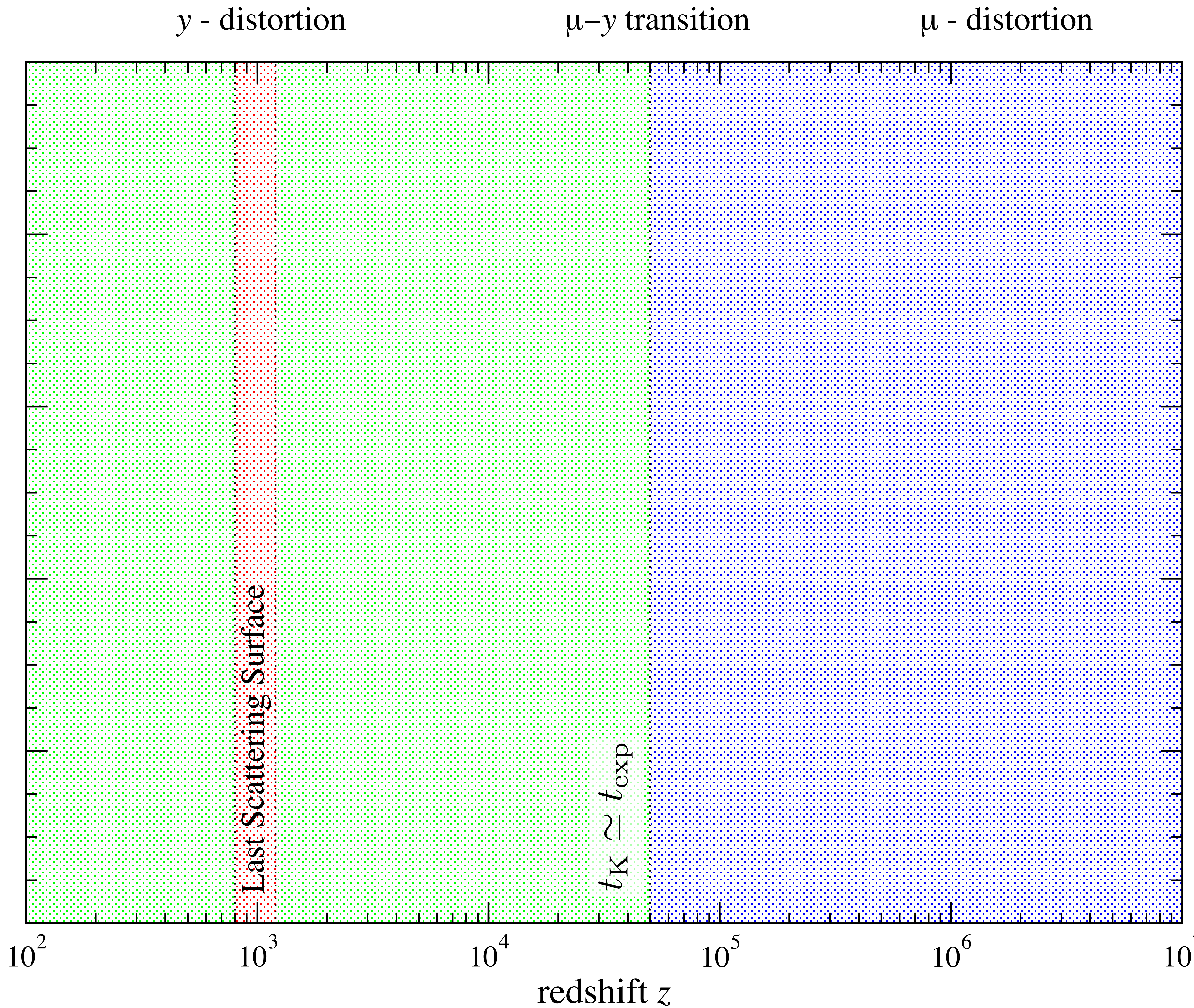


Computation carried out with ***CosmoTherm***
(JC & Sunyaev 2011)

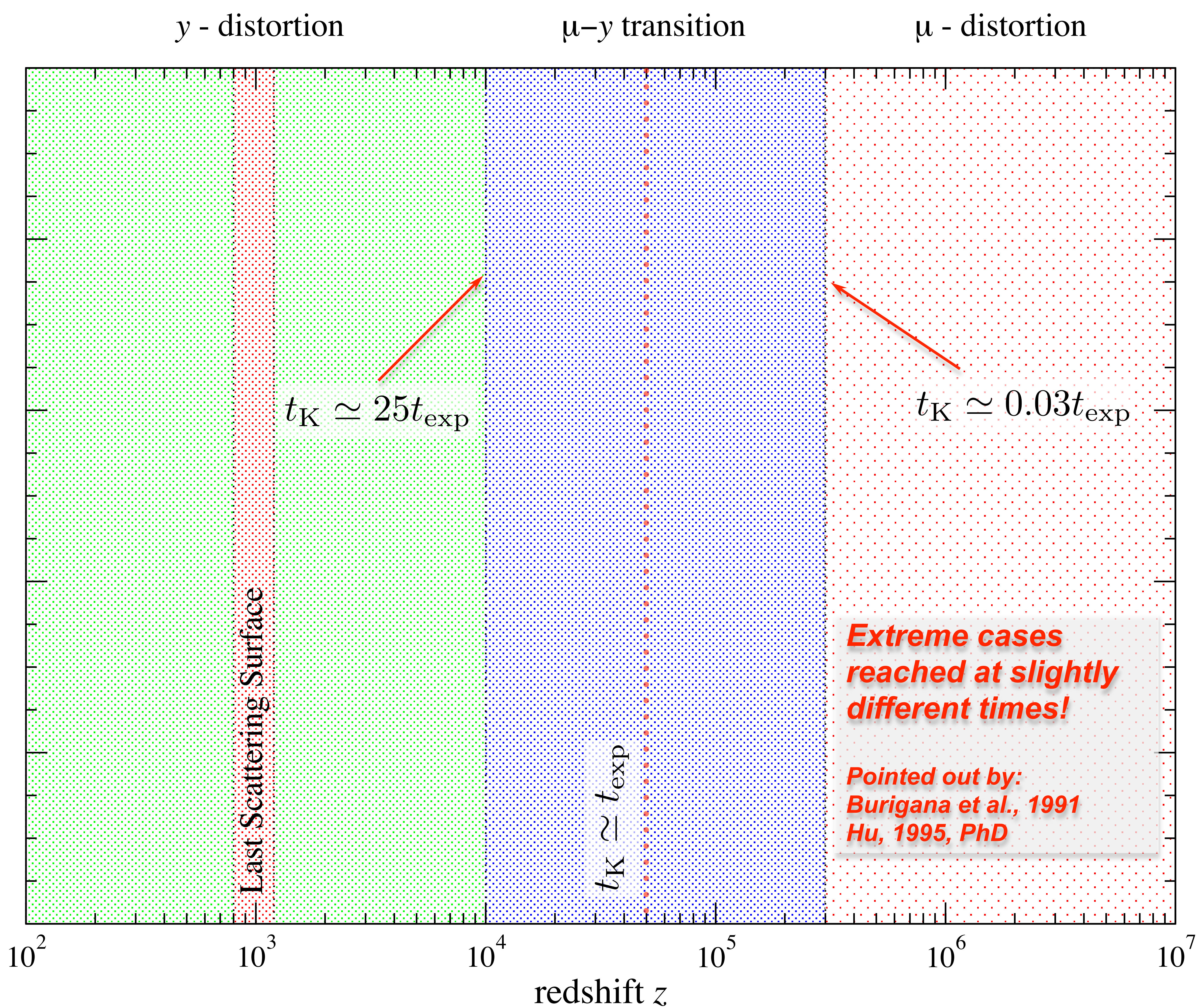
Distortion *not* just superposition of μ and y-distortion!

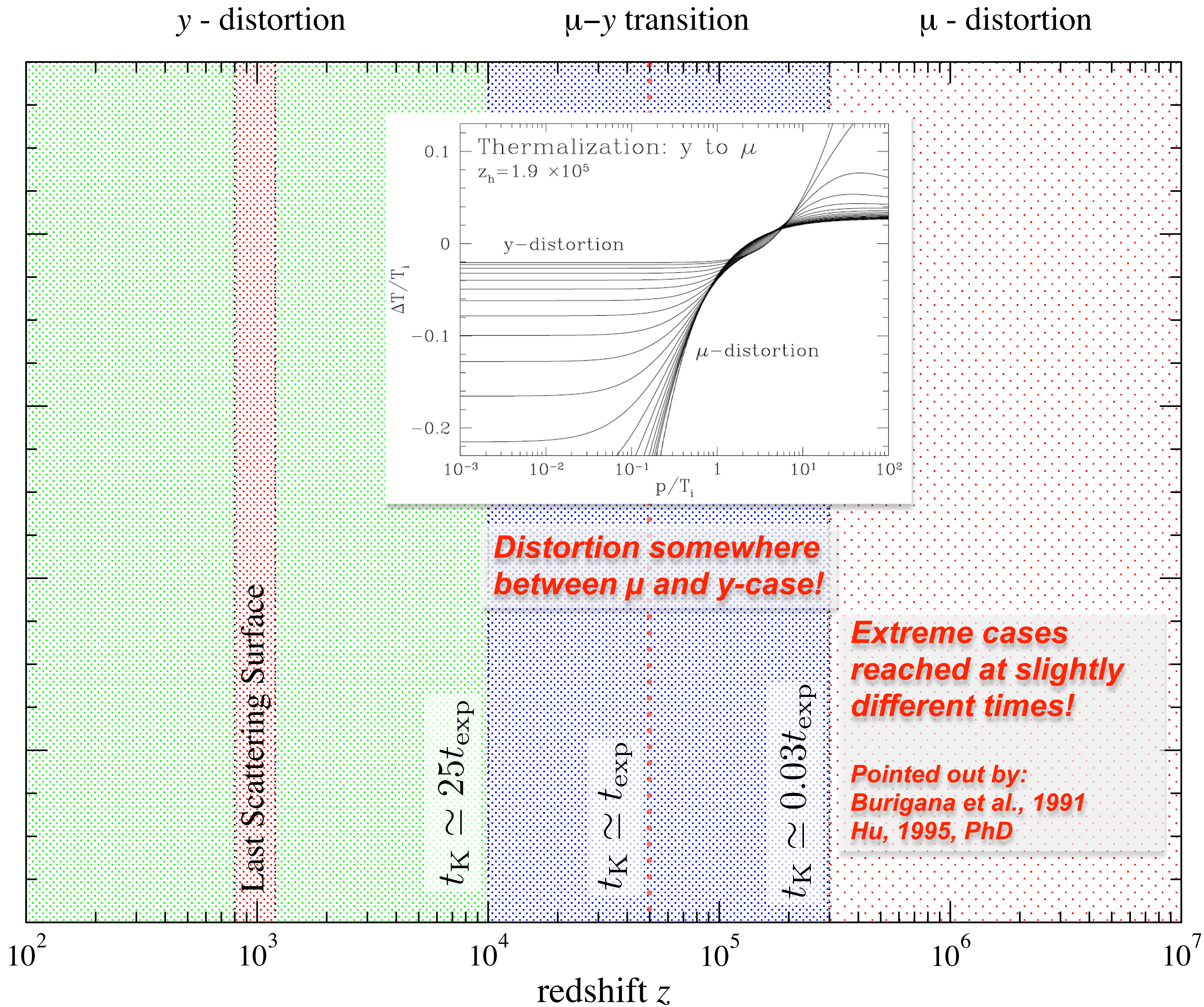


Computation carried out with ***CosmoTherm***
(JC & Sunyaev 2011)



y - distortion
μ–y transition
μ - distortion
Last Scattering Surface
$t_{\rm K} \simeq t_{\rm exp}$
10^2
10^3
10^4
10^5
10^6
10^7
redshift z


y - distortion
μ–y transition
μ - distortion
Last Scattering Surface
$t_{\rm K} \simeq 25 t_{\rm exp}$
$t_{\rm K} \simeq 0.03 t_{\rm exp}$
$t_{\rm K} \simeq t_{\rm exp}$
Extreme cases reached at slightly different times!
Pointed out by:
Burigana et al., 1991
Hu, 1995, PhD
10^2
10^3
10^4
10^5
10^6
10^7
redshift z



y - distortion
μ–y transition
μ - distortion
Thermalization: y to μ
z_h=1.9 ×10^5
y−distortion
μ−distortion
ΔT/T_i
p/T_i
0.1
0
−0.1
−0.2
10^-3
10^-2
10^-1
1
10^1
10^2
Last Scattering Surface
t_K ≃ 25t_exp
t_K ≃ t_exp
t_K ≃ 0.03t_exp
Distortion somewhere between μ and y-case!
Extreme cases reached at slightly different times!
Pointed out by:
Burigana et al., 1991
Hu, 1995, PhD
10^2
10^3
10^4
10^5
10^6
10^7
redshift z

Quasi-Exact Treatment: Thermalization Green's Function

Quasi-Exact Treatment: Thermalization Green's Function

- *For real forecasts of future prospects a precise & fast method for computing the spectral distortion is needed!*

Quasi-Exact Treatment: Thermalization Green's Function

- *For real forecasts of future prospects a precise & fast method for computing the spectral distortion is needed!*
- *Case-by-case computation of the distortion (e.g., with CosmoTherm, JC & Sunyaev, 2012, ArXiv:1109.6552) still rather time-consuming*

Quasi-Exact Treatment: Thermalization Green's Function

- *For real forecasts of future prospects a precise & fast method for computing the spectral distortion is needed!*
- *Case-by-case computation of the distortion (e.g., with CosmoTherm, JC & Sunyaev, 2012, ArXiv:1109.6552) still rather time-consuming*
- *But: distortions are small ⇒ thermalization problem becomes linear!*

Quasi-Exact Treatment: Thermalization Green's Function

- *For real forecasts of future prospects a precise & fast method for computing the spectral distortion is needed!*
- *Case-by-case computation of the distortion (e.g., with CosmoTherm, JC & Sunyaev, 2012, ArXiv:1109.6552) still rather time-consuming*
- *But: distortions are small ⇒ thermalization problem becomes linear!*
- *Simple solution: compute "response function" of the thermalization problem ⇒ Green's function approach (JC, 2013, ArXiv:1304.6120)*

Quasi-Exact Treatment: Thermalization Green's Function

- *For real forecasts of future prospects a precise & fast method for computing the spectral distortion is needed!*
- *Case-by-case computation of the distortion (e.g., with CosmoTherm, JC & Sunyaev, 2012, ArXiv:1109.6552) still rather time-consuming*
- *But: distortions are small ⇒ thermalization problem becomes linear!*
- *Simple solution: compute "response function" of the thermalization problem ⇒ Green's function approach (JC, 2013, ArXiv:1304.6120)*
- *Final distortion for fixed energy-release history given by*

$$\Delta I_\nu \approx \int_0^\infty G_{\rm th}(\nu, z') \frac{\mathrm{d}(Q/\rho_\gamma)}{\mathrm{d}z'} \mathrm{d}z'$$

Quasi-Exact Treatment: Thermalization Green's Function

- *For real forecasts of future prospects a precise & fast method for computing the spectral distortion is needed!*
- *Case-by-case computation of the distortion (e.g., with CosmoTherm, JC & Sunyaev, 2012, ArXiv:1109.6552) still rather time-consuming*
- *But: distortions are small ⇒ thermalization problem becomes linear!*
- *Simple solution: compute "response function" of the thermalization problem ⇒ Green's function approach (JC, 2013, ArXiv:1304.6120)*
- *Final distortion for fixed energy-release history given by*

$$\Delta I_\nu \approx \int_0^\infty G_{\rm th}(\nu, z') \frac{{\rm d}(Q/\rho_\gamma)}{{\rm d}z'} {\rm d}z'$$

Thermalization Green's function

Quasi-Exact Treatment: Thermalization Green's Function

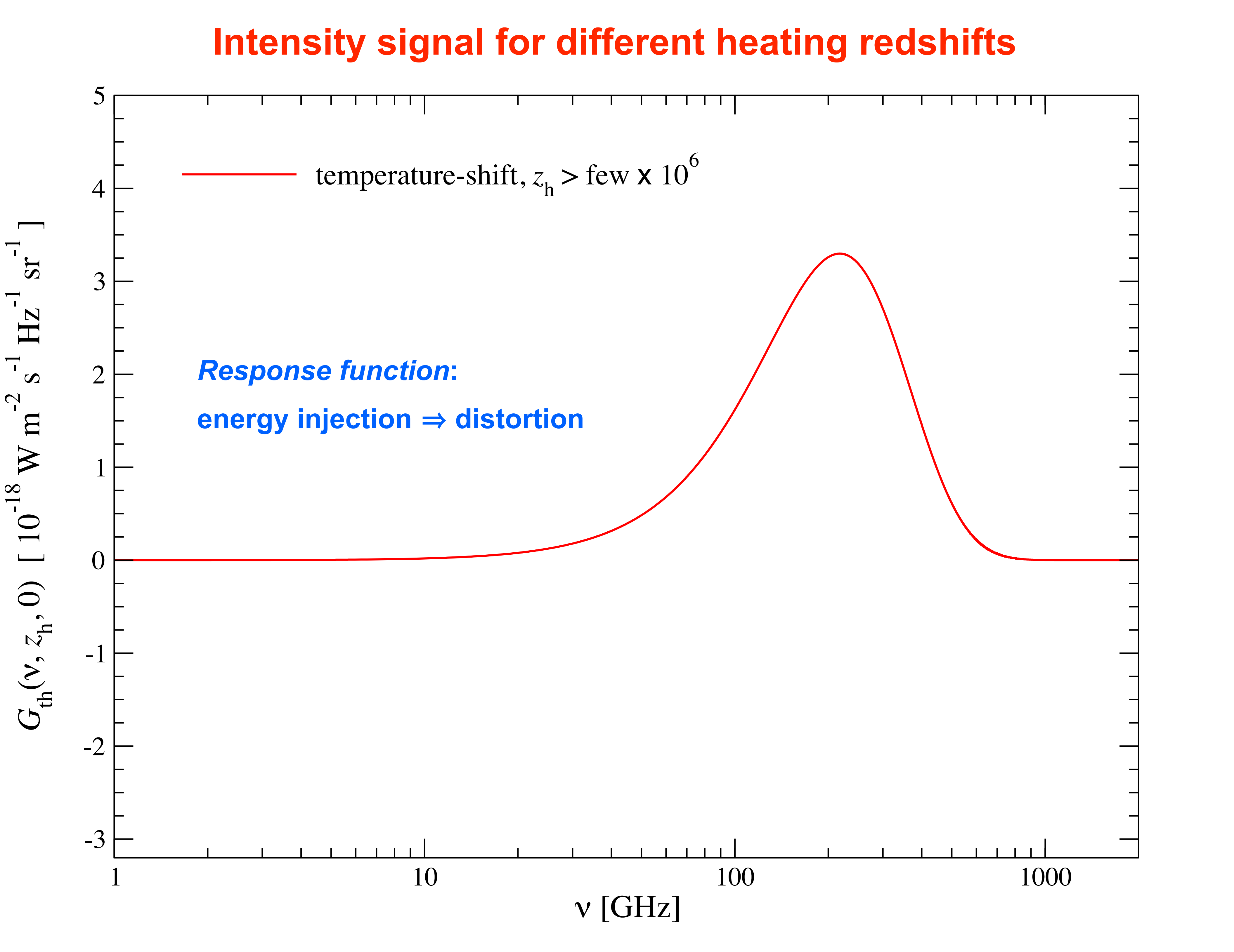
- *For real forecasts of future prospects a precise & fast method for computing the spectral distortion is needed!*
- *Case-by-case computation of the distortion (e.g., with CosmoTherm, JC & Sunyaev, 2012, ArXiv:1109.6552) still rather time-consuming*
- *But: distortions are small ⇒ thermalization problem becomes linear!*
- *Simple solution: compute "response function" of the thermalization problem ⇒ Green's function approach (JC, 2013, ArXiv:1304.6120)*
- *Final distortion for fixed energy-release history given by*

$$\Delta I_\nu \approx \int_0^\infty G_{\rm th}(\nu, z') \frac{{\rm d}(Q/\rho_\gamma)}{{\rm d}z'} {\rm d}z'$$

Thermalization Green's function

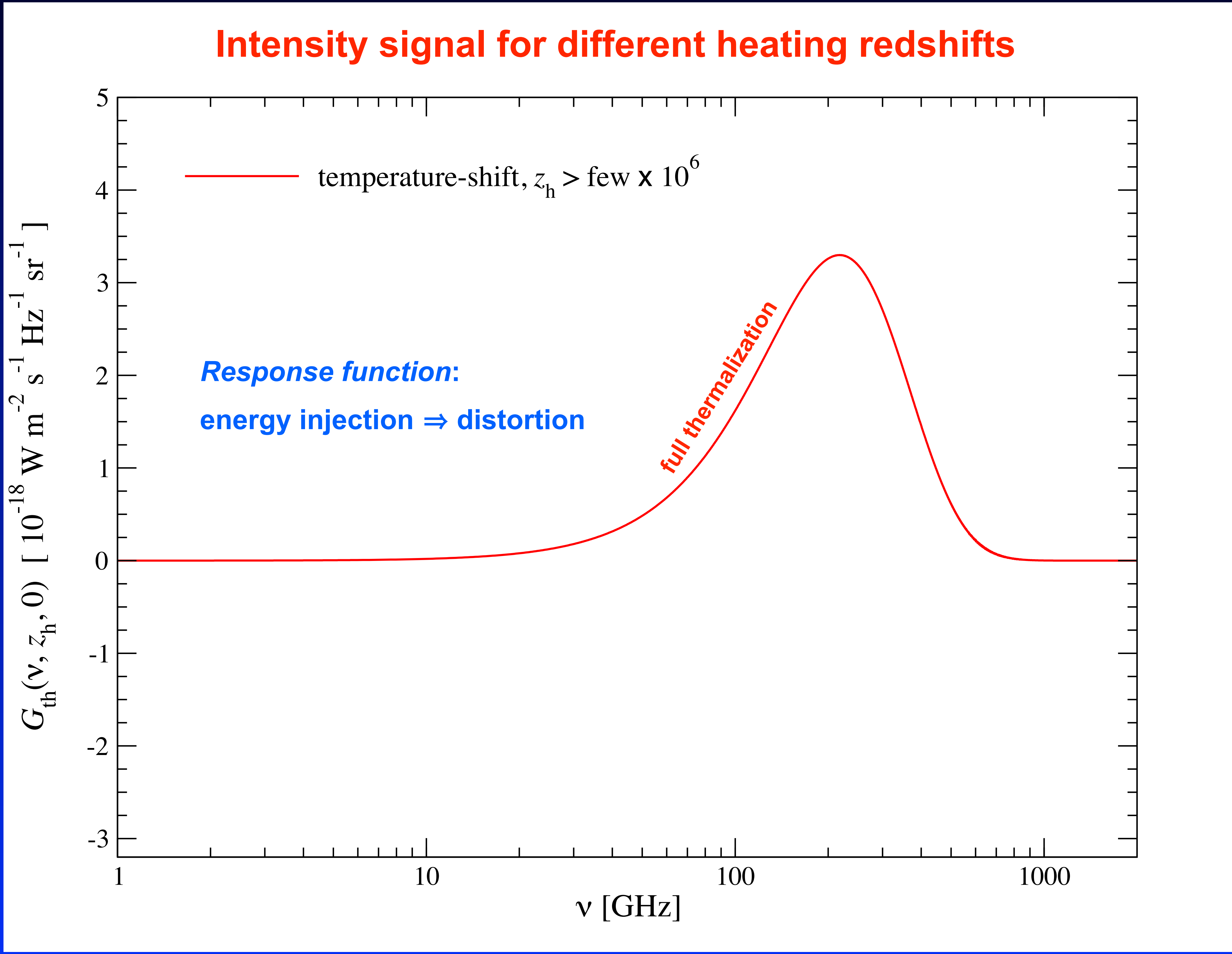
- *Fast and quasi-exact! No additional approximations!*

What does the spectrum look like after energy injection?



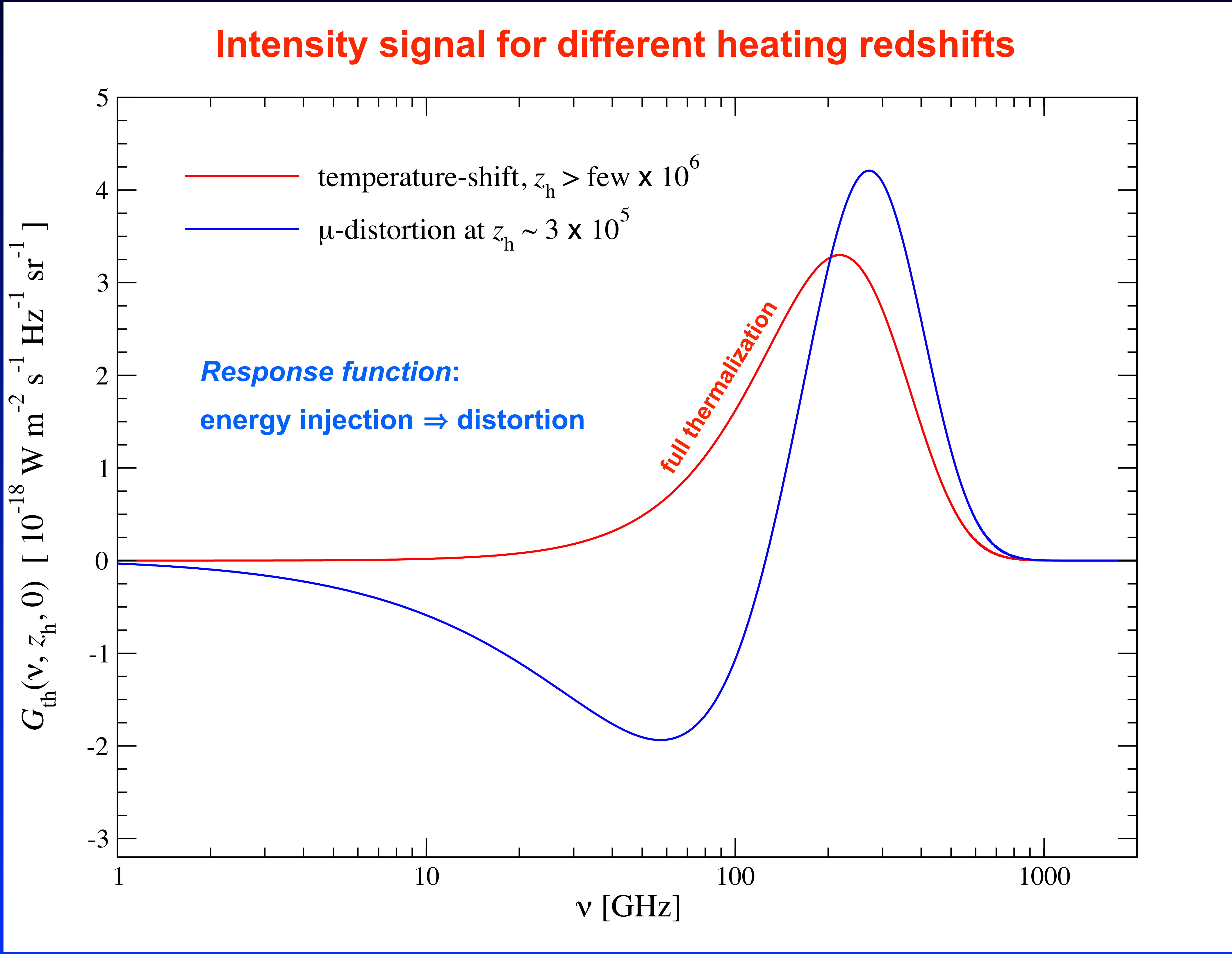
JC & Sunyaev, 2012, ArXiv:1109.6552
JC, 2013, ArXiv:1304.6120

What does the spectrum look like after energy injection?



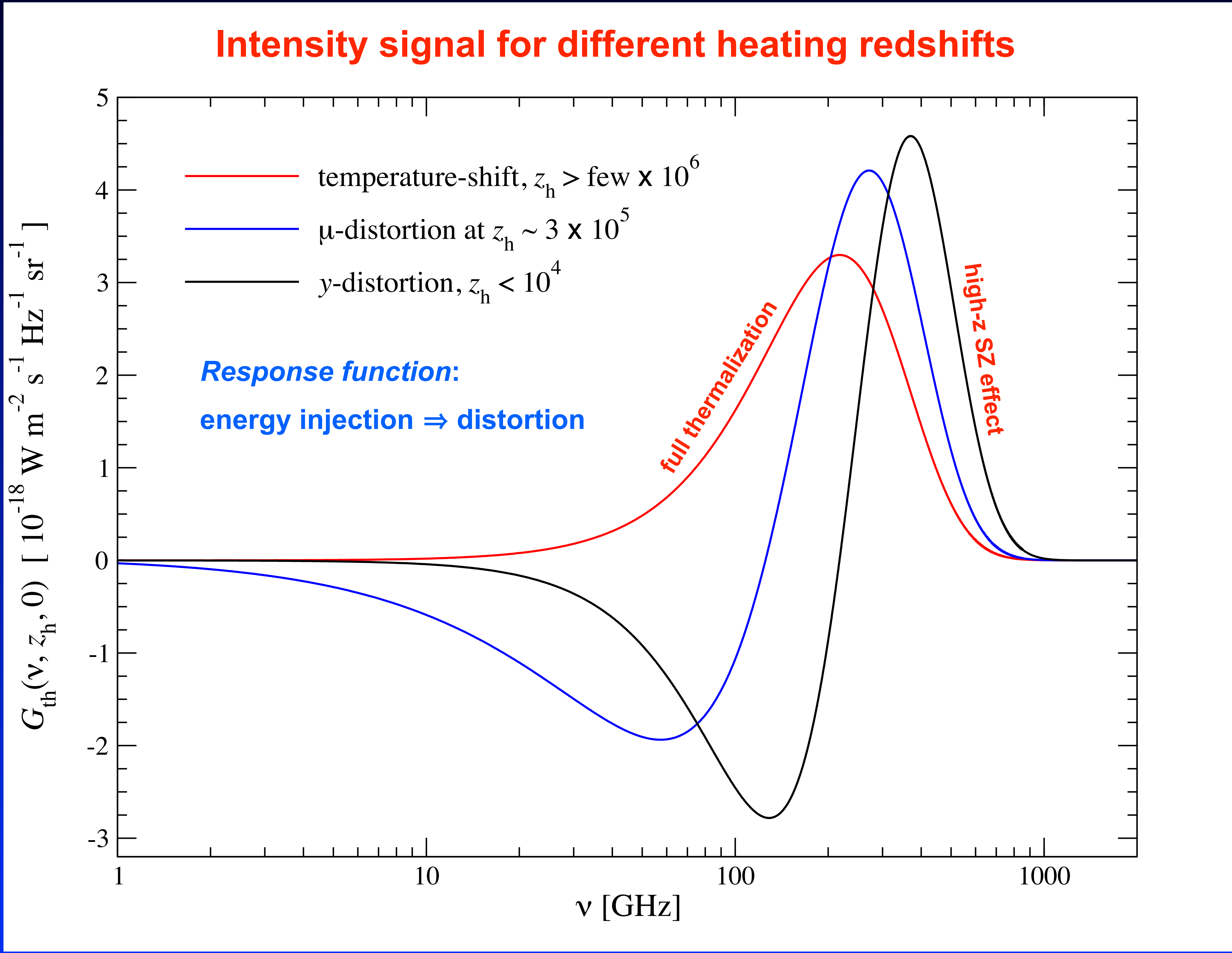
JC & Sunyaev, 2012, ArXiv:1109.6552
JC, 2013, ArXiv:1304.6120

What does the spectrum look like after energy injection?



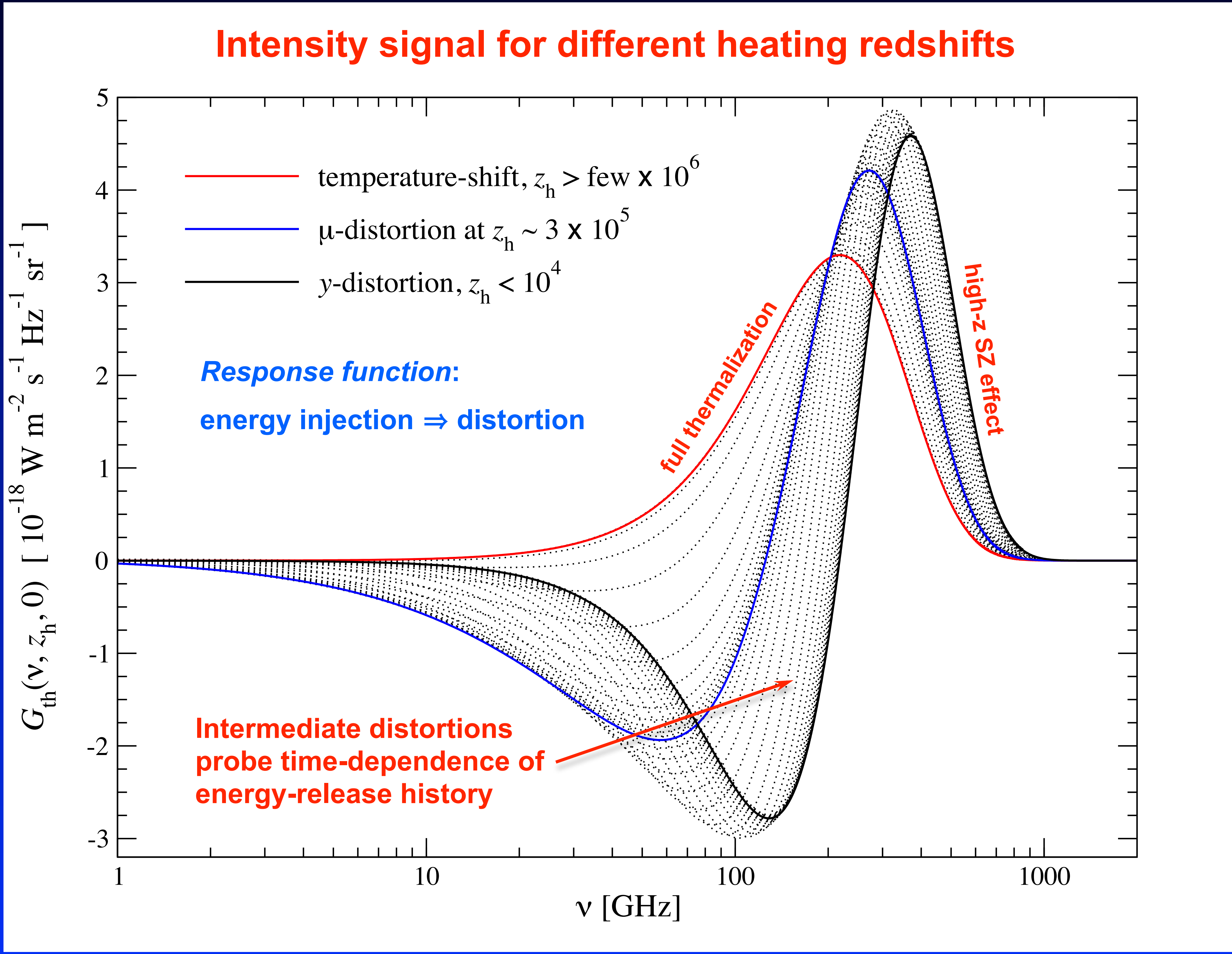
JC & Sunyaev, 2012, ArXiv:1109.6552
JC, 2013, ArXiv:1304.6120

What does the spectrum look like after energy injection?



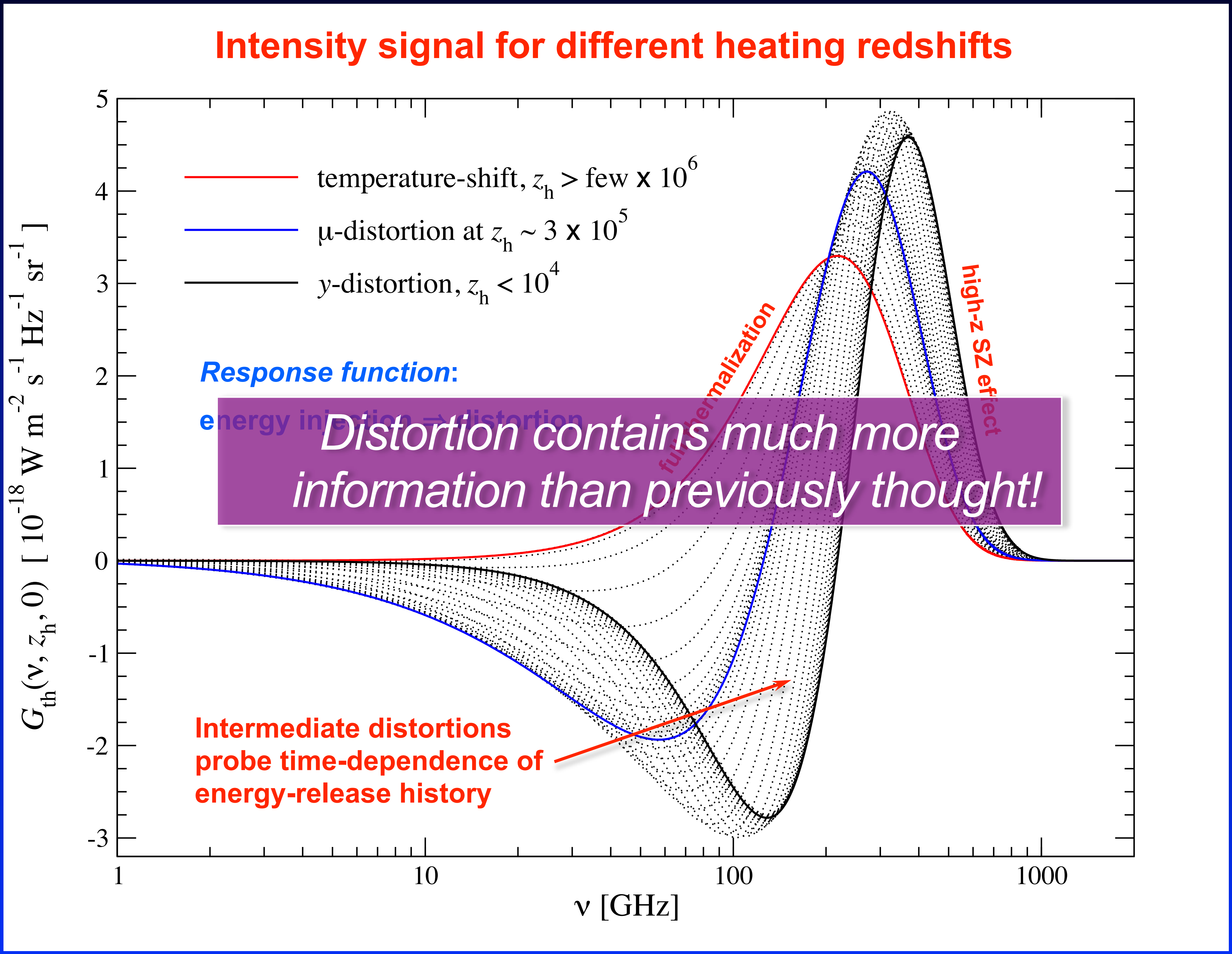
JC & Sunyaev, 2012, ArXiv:1109.6552
JC, 2013, ArXiv:1304.6120

What does the spectrum look like after energy injection?



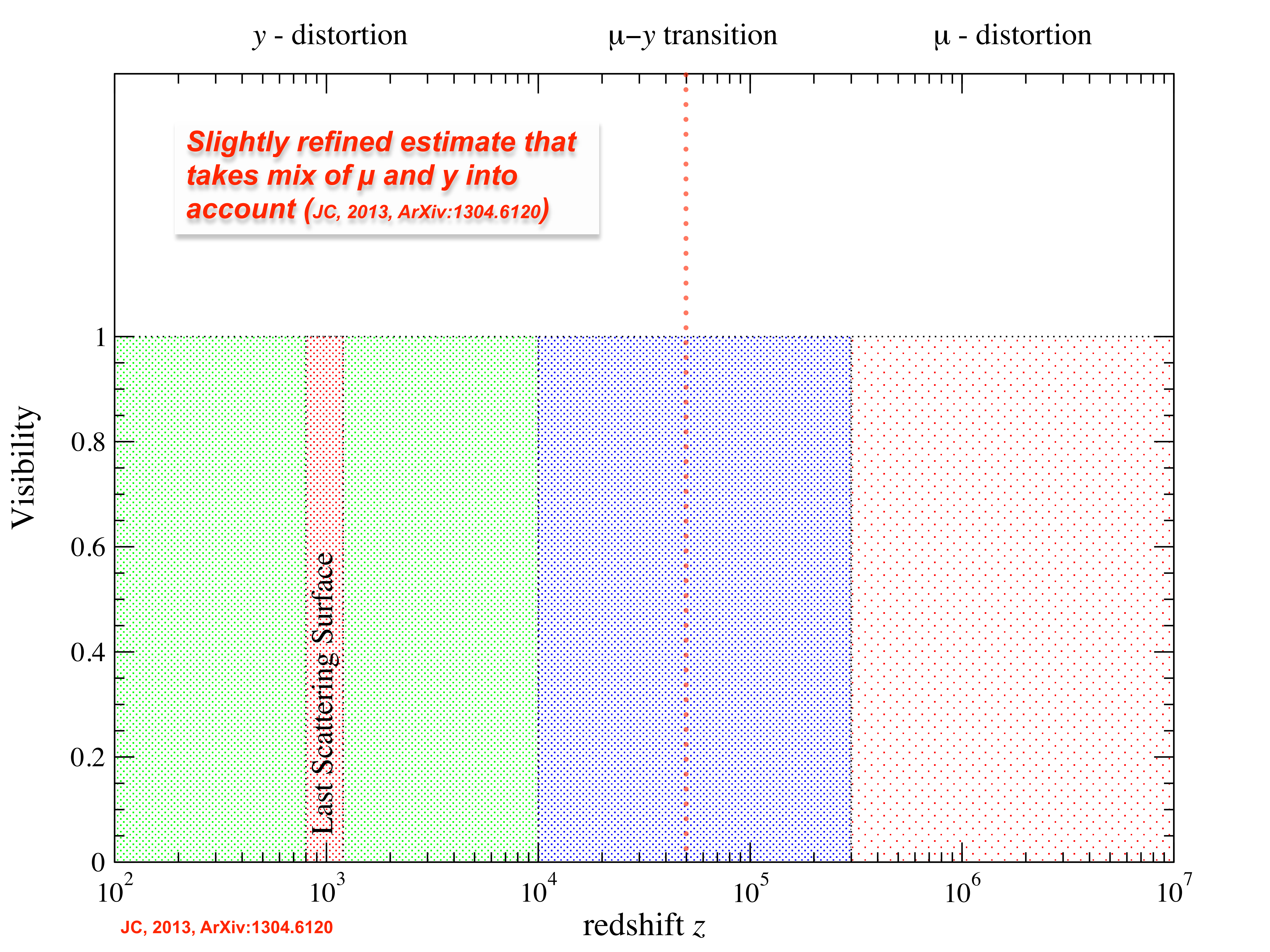
JC & Sunyaev, 2012, ArXiv:1109.6552
JC, 2013, ArXiv:1304.6120

What does the spectrum look like after energy injection?

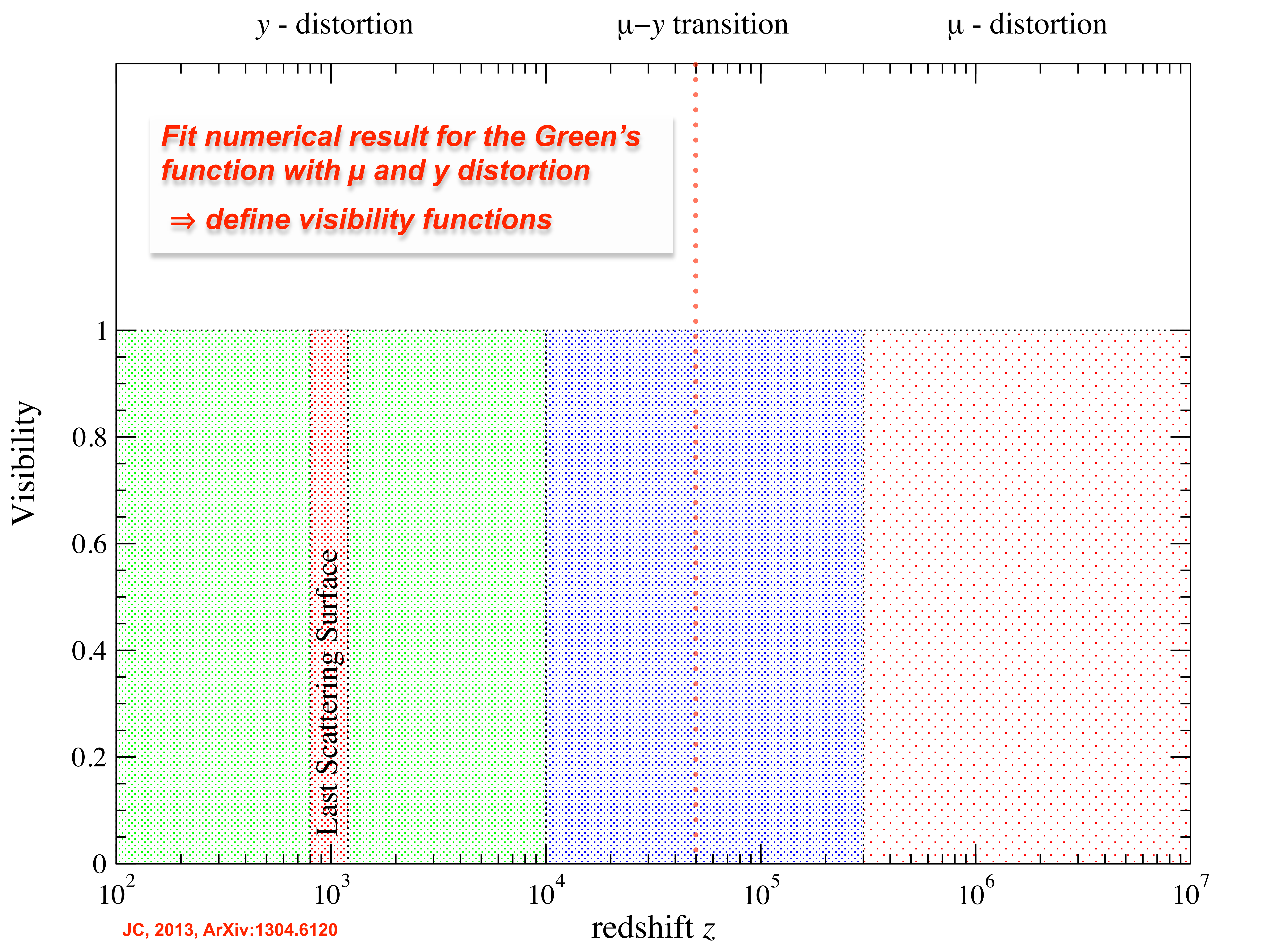


Distortion contains much more information than previously thought!

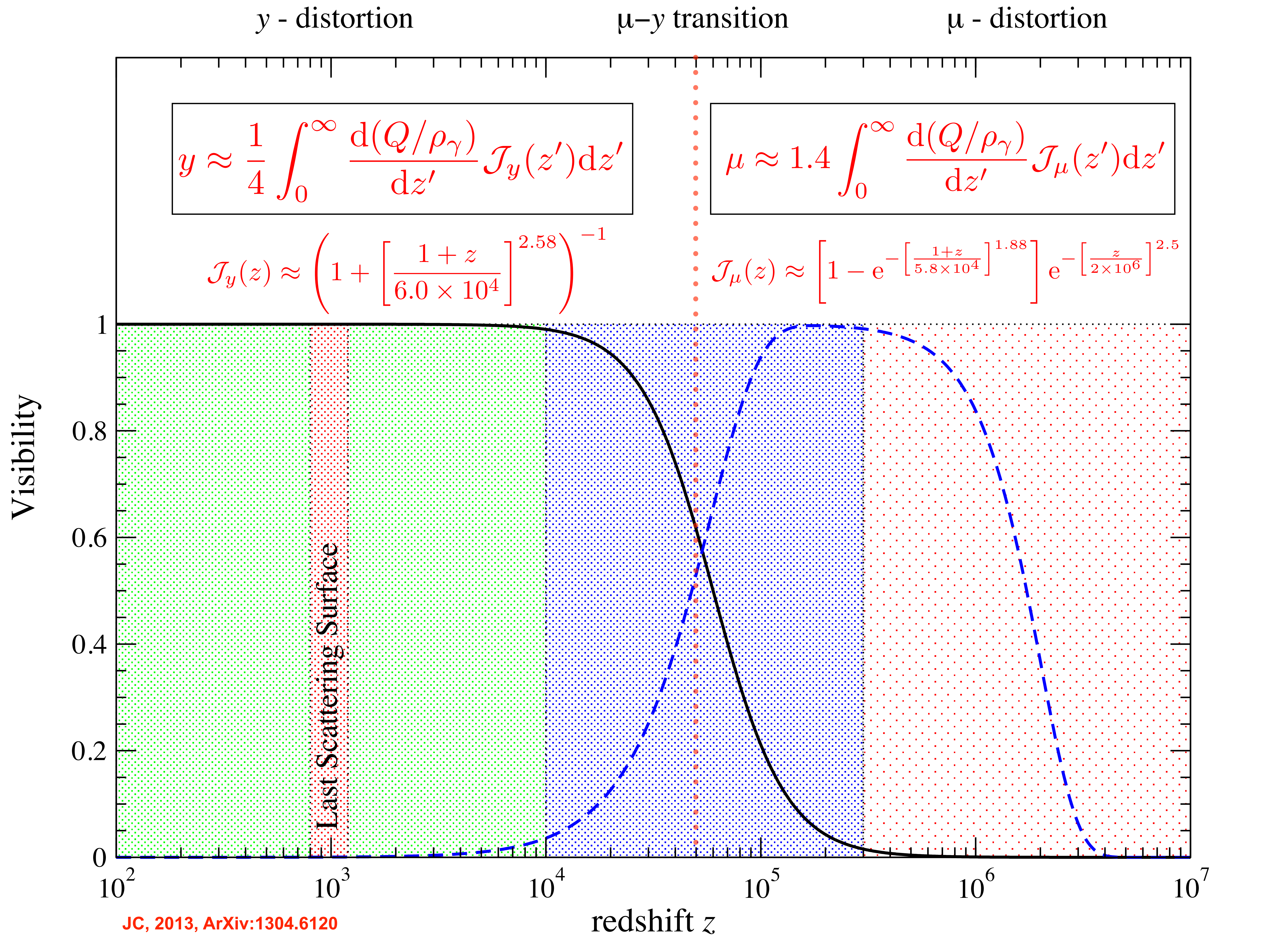
JC & Sunyaev, 2012, ArXiv:1109.6552
JC, 2013, ArXiv:1304.6120



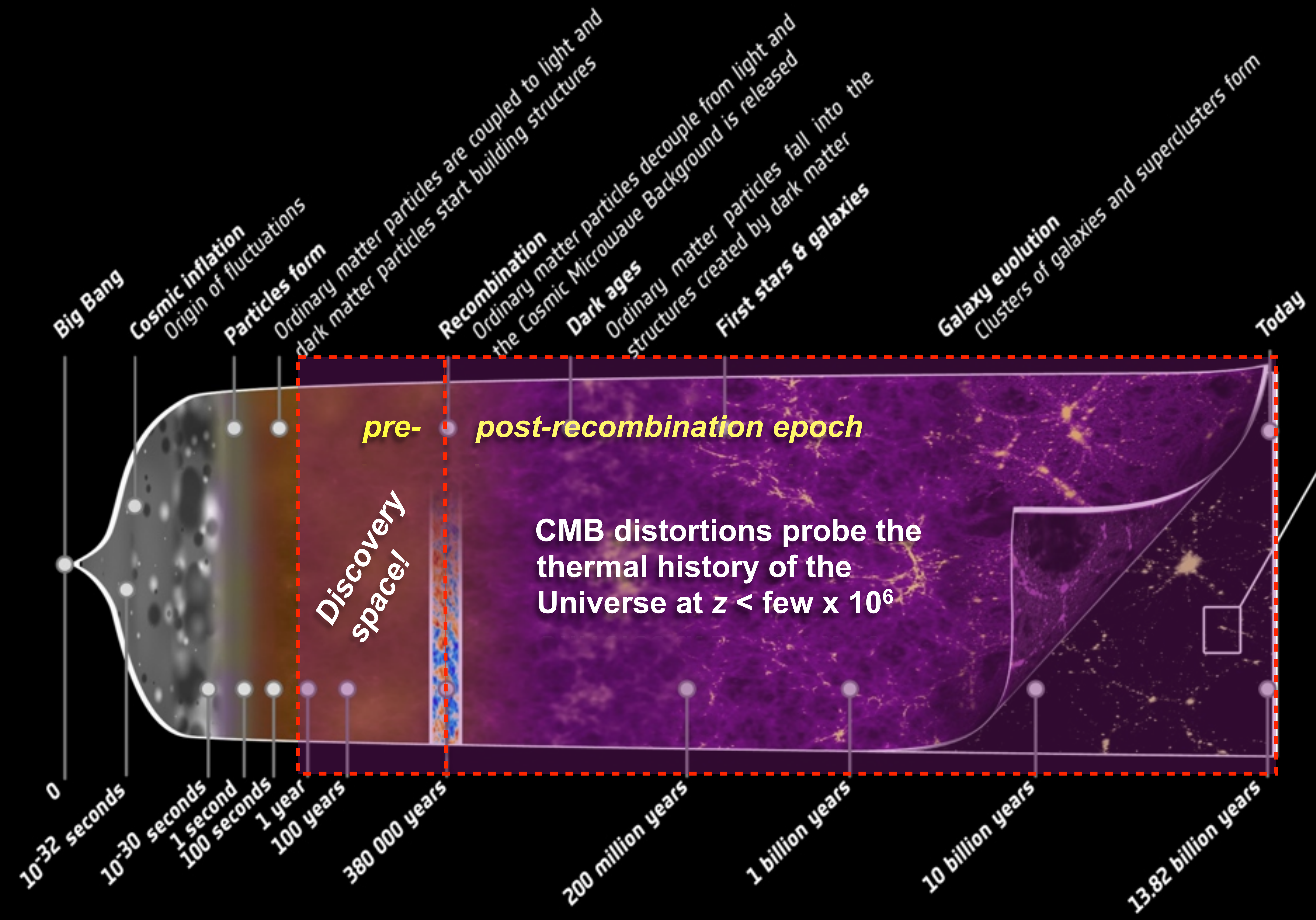
y - distortion
μ−y transition
μ - distortion
Slightly refined estimate that takes mix of μ and y into account (JC, 2013, ArXiv:1304.6120)
Visibility
1
0.8
0.6
0.4
0.2
0
Last Scattering Surface
10^2
10^3
10^4
10^5
10^6
10^7
redshift z
JC, 2013, ArXiv:1304.6120



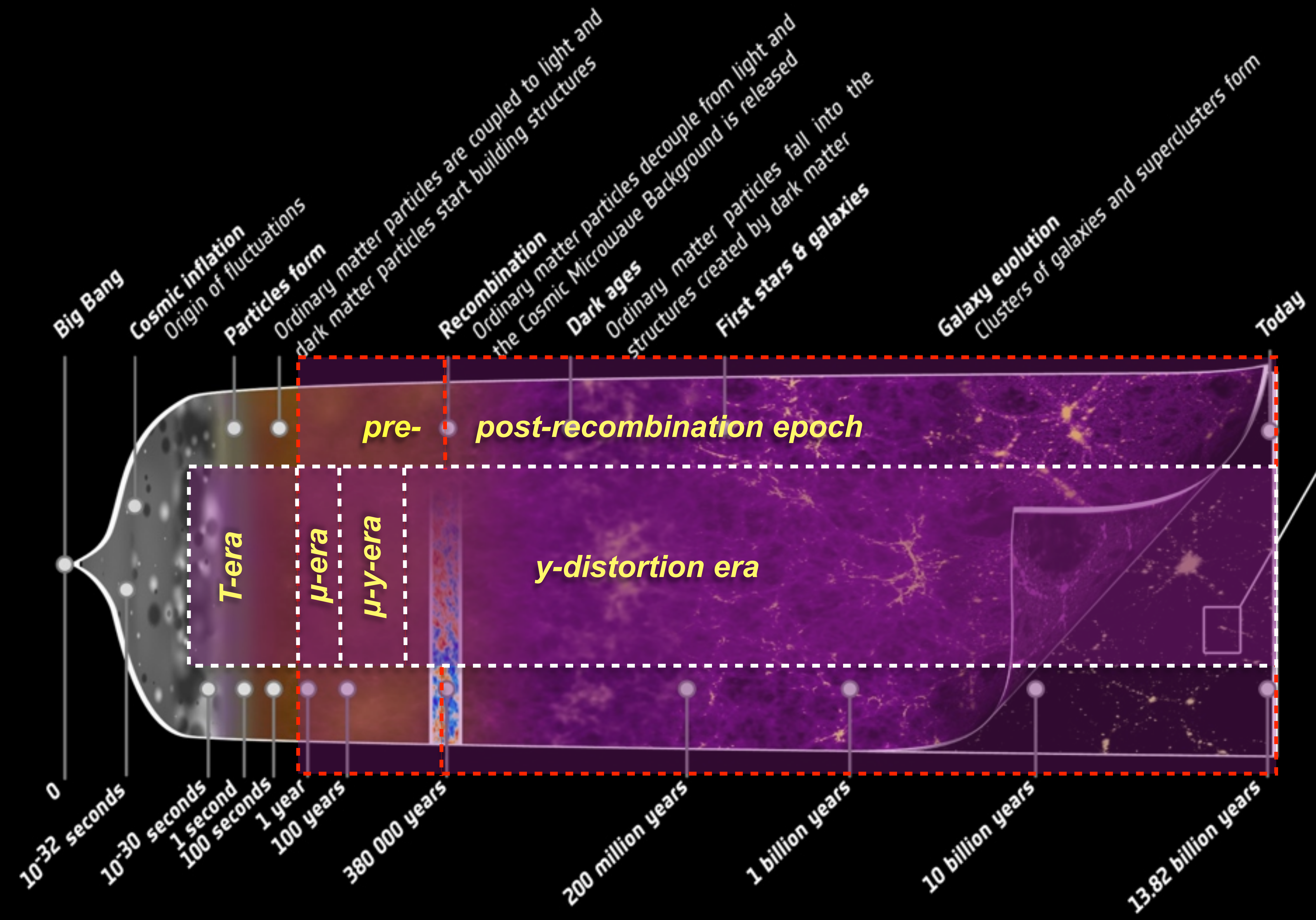
y - distortion
μ–y transition
μ - distortion
Fit numerical result for the Green's function with μ and y distortion
⇒ define visibility functions
Visibility
Last Scattering Surface
1
0.8
0.6
0.4
0.2
0
10^2
10^3
10^4
10^5
10^6
10^7
redshift z
JC, 2013, ArXiv:1304.6120



y - distortion
μ–y transition
μ - distortion
$y \approx \frac{1}{4}\int_0^\infty \frac{d(Q/\rho_\gamma)}{dz'}\mathcal{J}_y(z')dz'$
$\mu \approx 1.4\int_0^\infty \frac{d(Q/\rho_\gamma)}{dz'}\mathcal{J}_\mu(z')dz'$
$\mathcal{J}_y(z) \approx \left(1+\left[\frac{1+z}{6.0\times10^4}\right]^{2.58}\right)^{-1}$
$\mathcal{J}_\mu(z) \approx \left[1-e^{-\left[\frac{1+z}{5.8\times10^4}\right]^{1.88}}\right]e^{-\left[\frac{z}{2\times10^6}\right]^{2.5}}$
Visibility
Last Scattering Surface
redshift z
JC, 2013, ArXiv:1304.6120

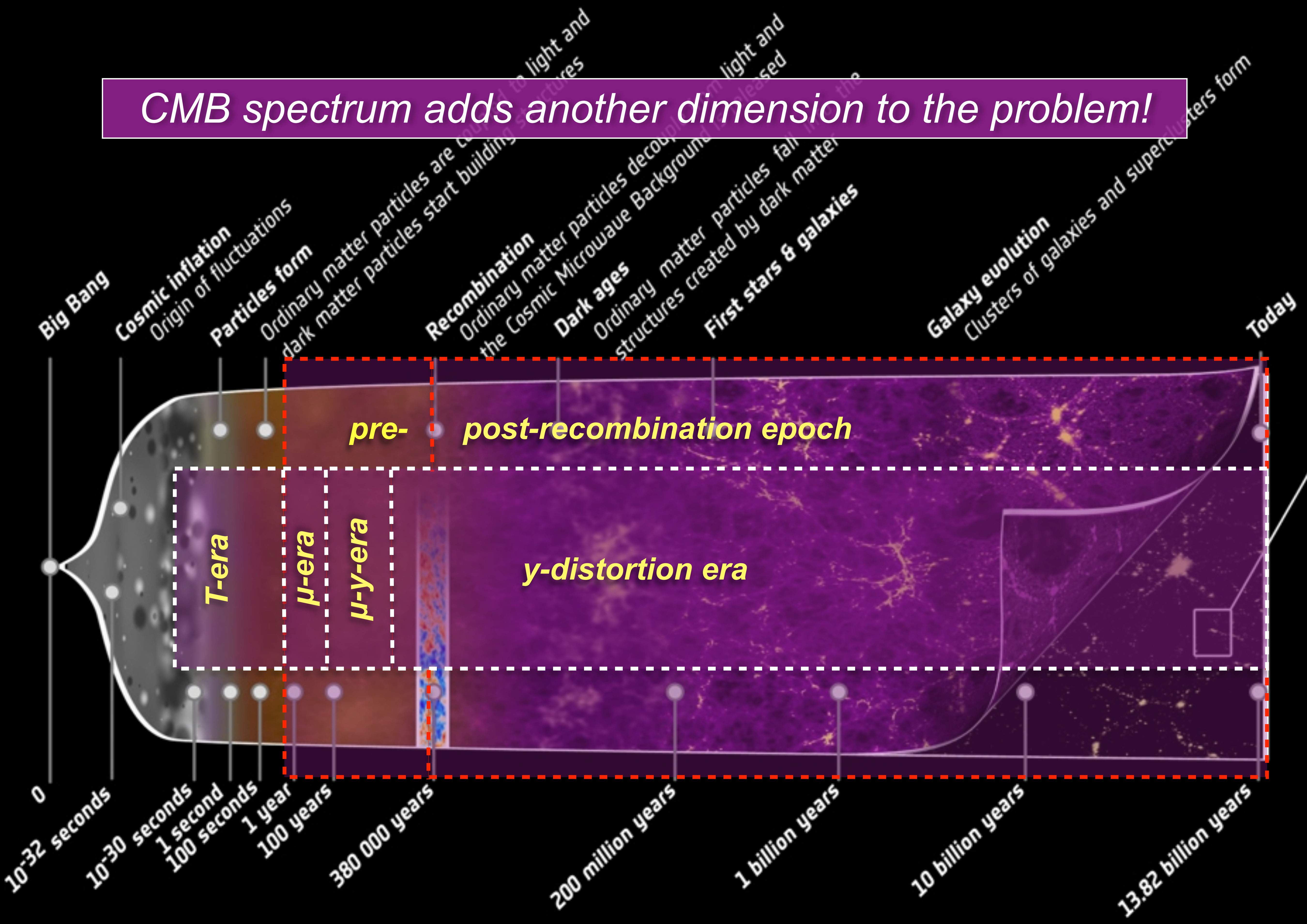


Big Bang
Cosmic inflation
Origin of fluctuations
Particles form
Ordinary matter particles are coupled to light and dark matter particles start building structures
Recombination
Ordinary matter particles decouple from light and the Cosmic Microwave Background is released
Dark ages
Ordinary matter particles fall into the structures created by dark matter
First stars & galaxies
Galaxy evolution
Clusters of galaxies and superclusters form
Today
pre-
post-recombination epoch
Discovery space!
CMB distortions probe the thermal history of the Universe at z < few x 10^6
0
10^-32 seconds
10^-30 seconds
1 second
100 seconds
1 year
100 years
380 000 years
200 million years
1 billion years
10 billion years
13.82 billion years



Big Bang
Cosmic inflation
Origin of fluctuations
Particles form
Ordinary matter particles are coupled to light and dark matter particles start building structures
Recombination
Ordinary matter particles decouple from light and the Cosmic Microwave Background is released
Dark ages
Ordinary matter particles fall into the structures created by dark matter
First stars & galaxies
Galaxy evolution
Clusters of galaxies and superclusters form
Today
pre-
post-recombination epoch
T-era
μ-era
μ-y-era
y-distortion era
0
10-32 seconds
10-30 seconds
1 second
100 seconds
1 year
100 years
380 000 years
200 million years
1 billion years
10 billion years
13.82 billion years

CMB spectrum adds another dimension to the problem!



Physical mechanisms that lead to spectral distortions

Standard sources of distortions

- *Cooling by adiabatically expanding ordinary matter*
  (JC, 2005; JC & Sunyaev 2011; Khatri, Sunyaev & JC, 2011)
- Heating by *decaying* or *annihilating* relic particles
  (Kawasaki et al., 1987; Hu & Silk, 1993; McDonald et al., 2001; JC, 2005; JC & Sunyaev, 2011; JC, 2013; JC & Jeong, 2013)
- *Evaporation of primordial black holes & superconducting strings*
  (Carr et al. 2010; Ostriker & Thompson, 1987; Tashiro et al. 2012; Pani & Loeb, 2013)
- *Dissipation of primordial acoustic modes & magnetic fields*
  (Sunyaev & Zeldovich, 1970; Daly 1991; Hu et al. 1994; JC & Sunyaev, 2011; JC et al. 2012 - Jedamzik et al. 2000; Kunze & Komatsu, 2013)
- *Cosmological recombination radiation*
  (Zeldovich et al., 1968; Peebles, 1968; Dubrovich, 1977; Rubino-Martin et al., 2006; JC & Sunyaev, 2006; Sunyaev & JC, 2009)

pre-recombination epoch

„high“ redshifts

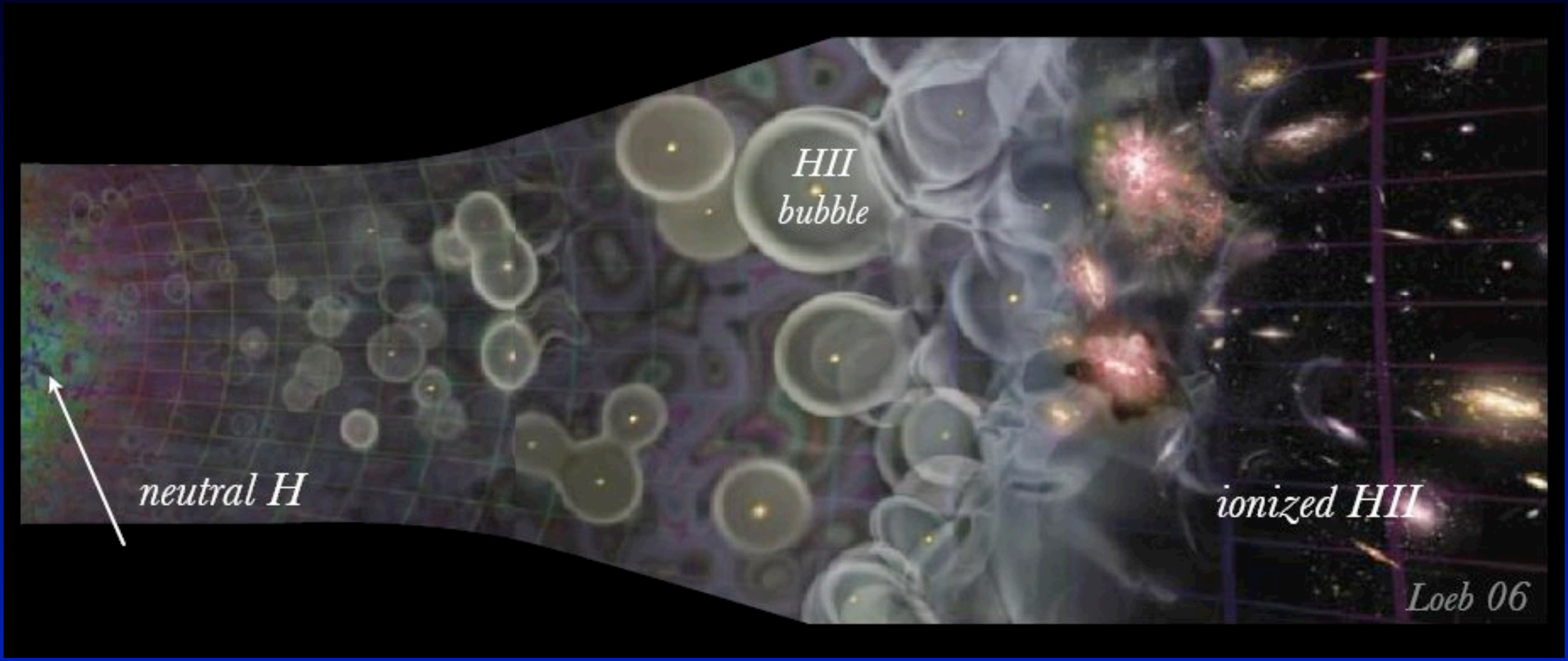
„low“ redshifts

post-recombination

- Signatures due to first supernovae and their remnants
  (Oh, Cooray & Kamionkowski, 2003)
- Shock waves arising due to large-scale structure formation
  (Sunyaev & Zeldovich, 1972; Cen & Ostriker, 1999)
- SZ-effect from clusters; effects of reionization
  (Refregier et al., 2003; Zhang et al. 2004; Trac et al. 2008)
- more exotic processes
  (Lochan et al. 2012; Bull & Kamionkowski, 2013; Brax et al., 2013; Tashiro et al. 2013)

Reionization and structure formation

Simple estimates for the distortion

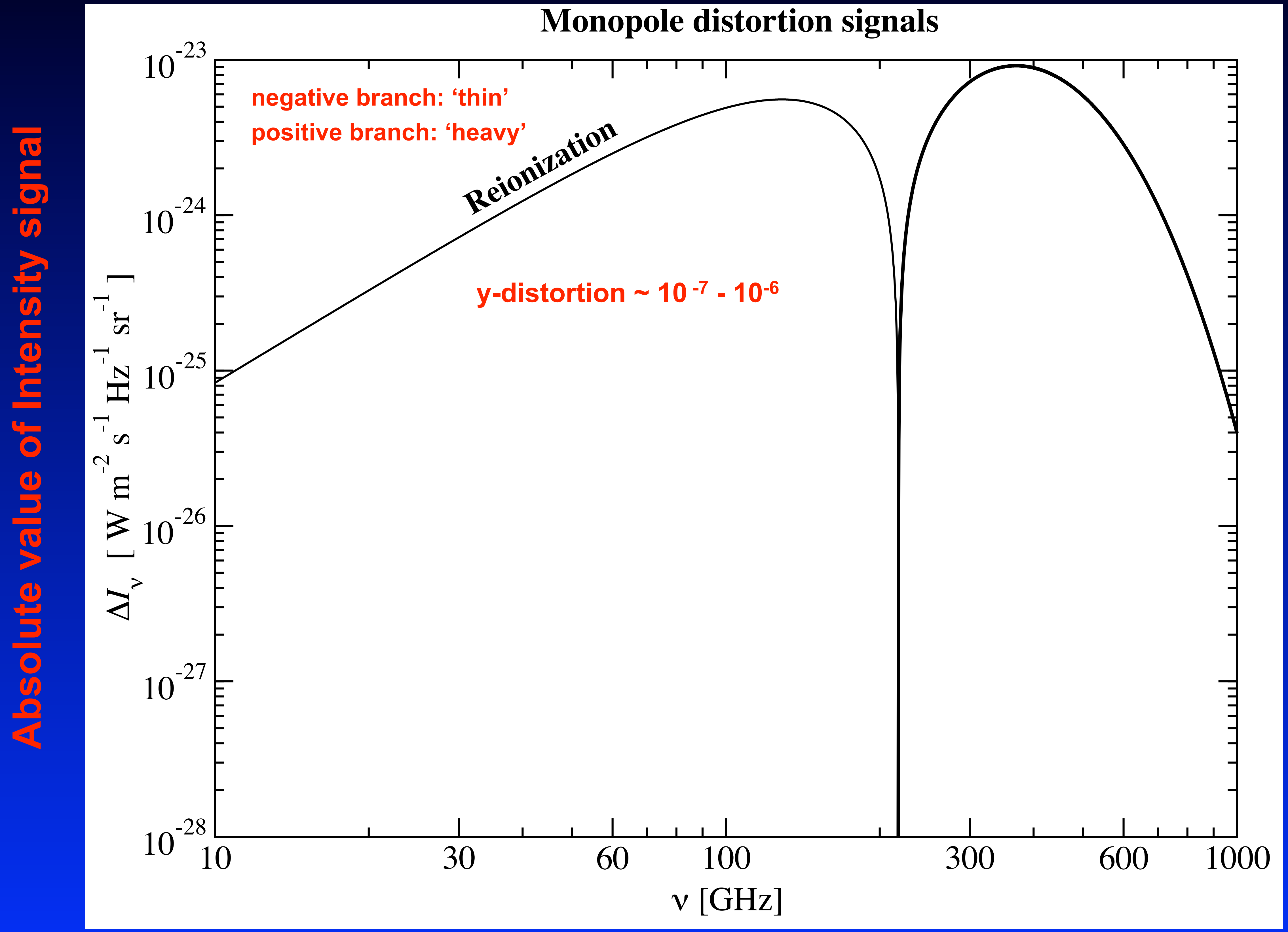


- Gas temperature $T \simeq 10^4$ K
- Thomson optical depth $\tau \simeq 0.1$

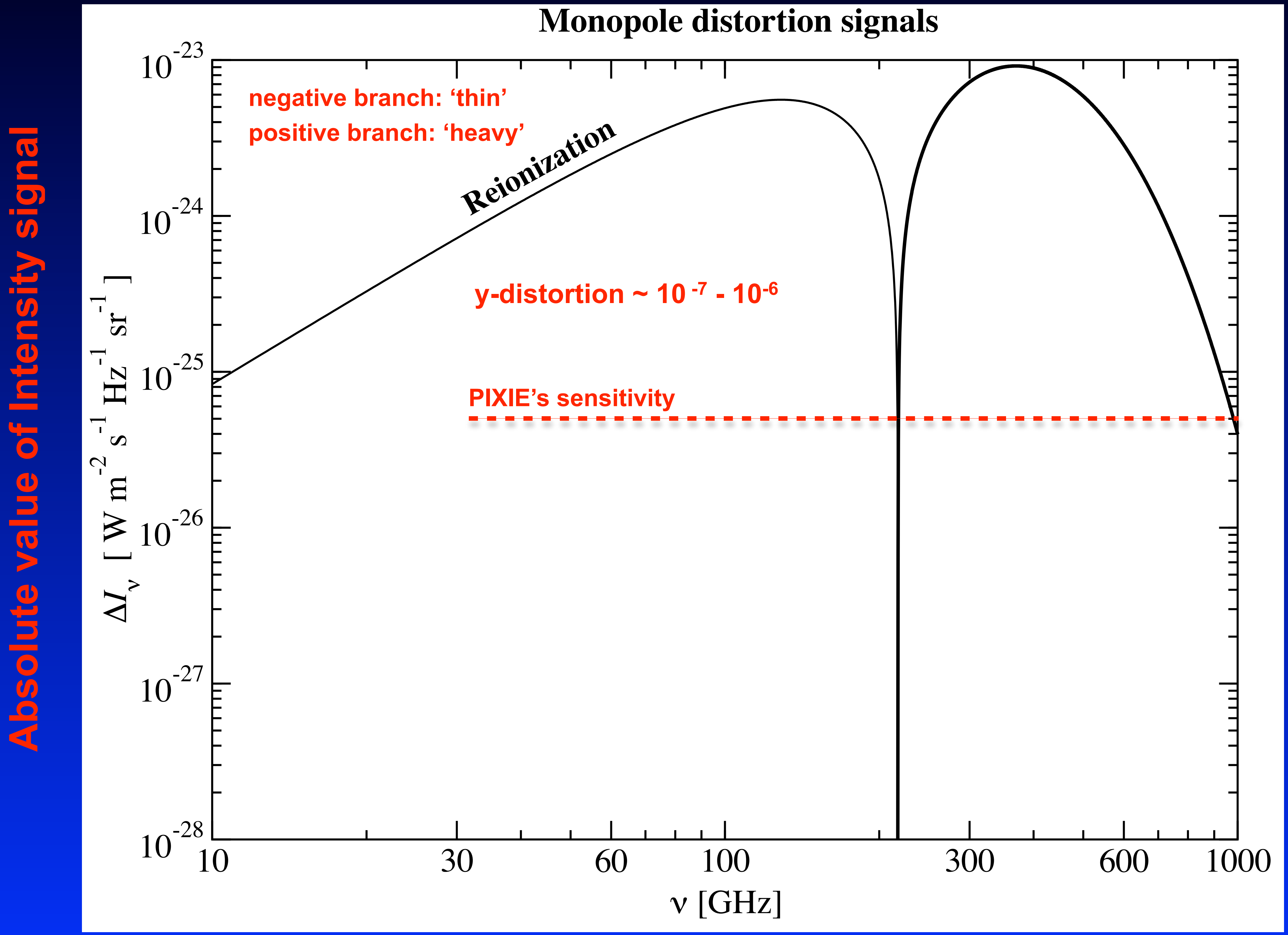
$$\Longrightarrow \quad y \simeq \frac{kT_e}{m_e c^2} \simeq 2 \times 10^{-7}$$

- second order Doppler effect $y \simeq$ few x 10^{-8}
- structure formation / SZ effect (e.g., Refregier et al., 2003) $y \simeq$ few x 10^{-7}-10^{-6}

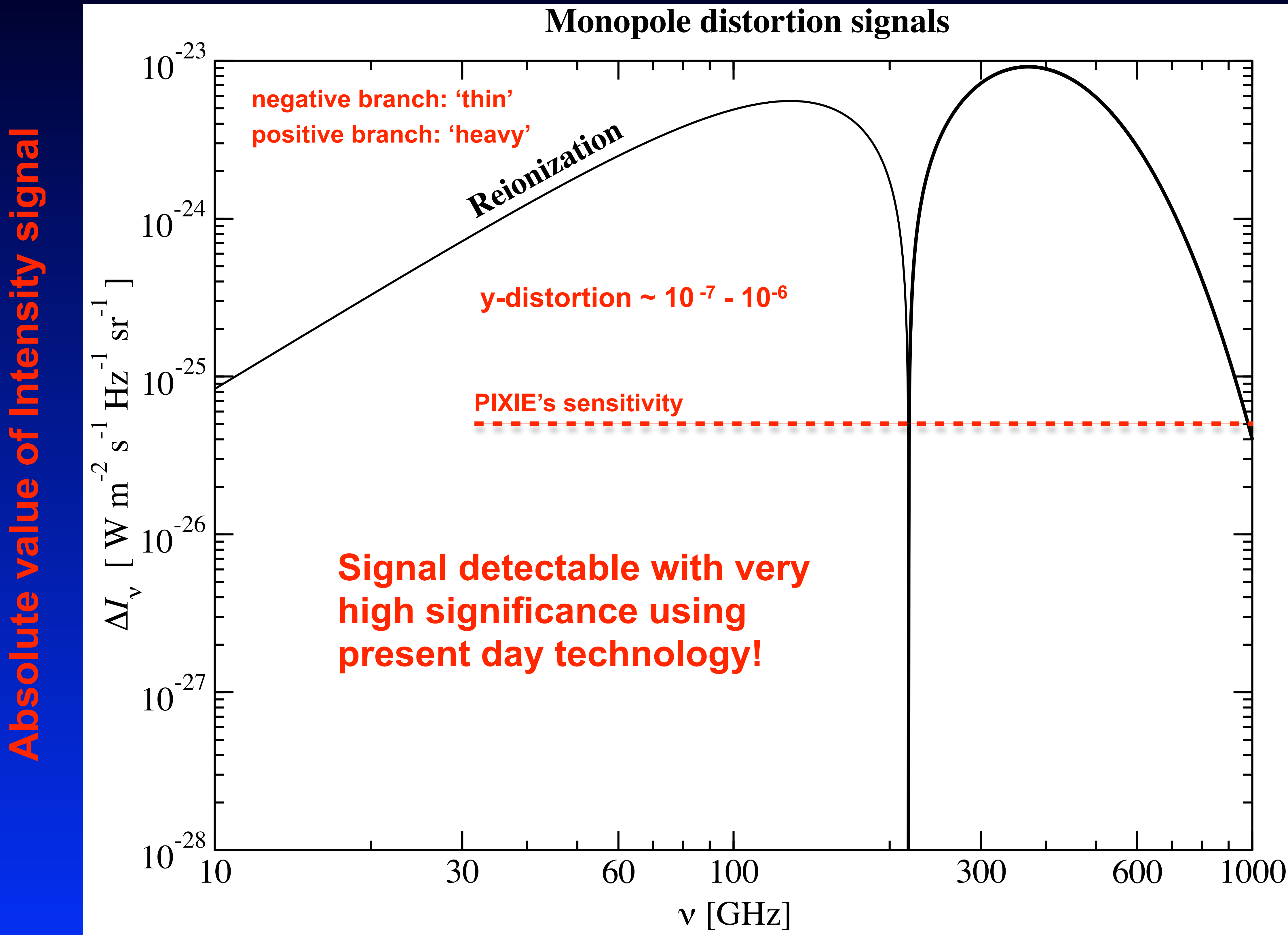
Average CMB spectral distortions



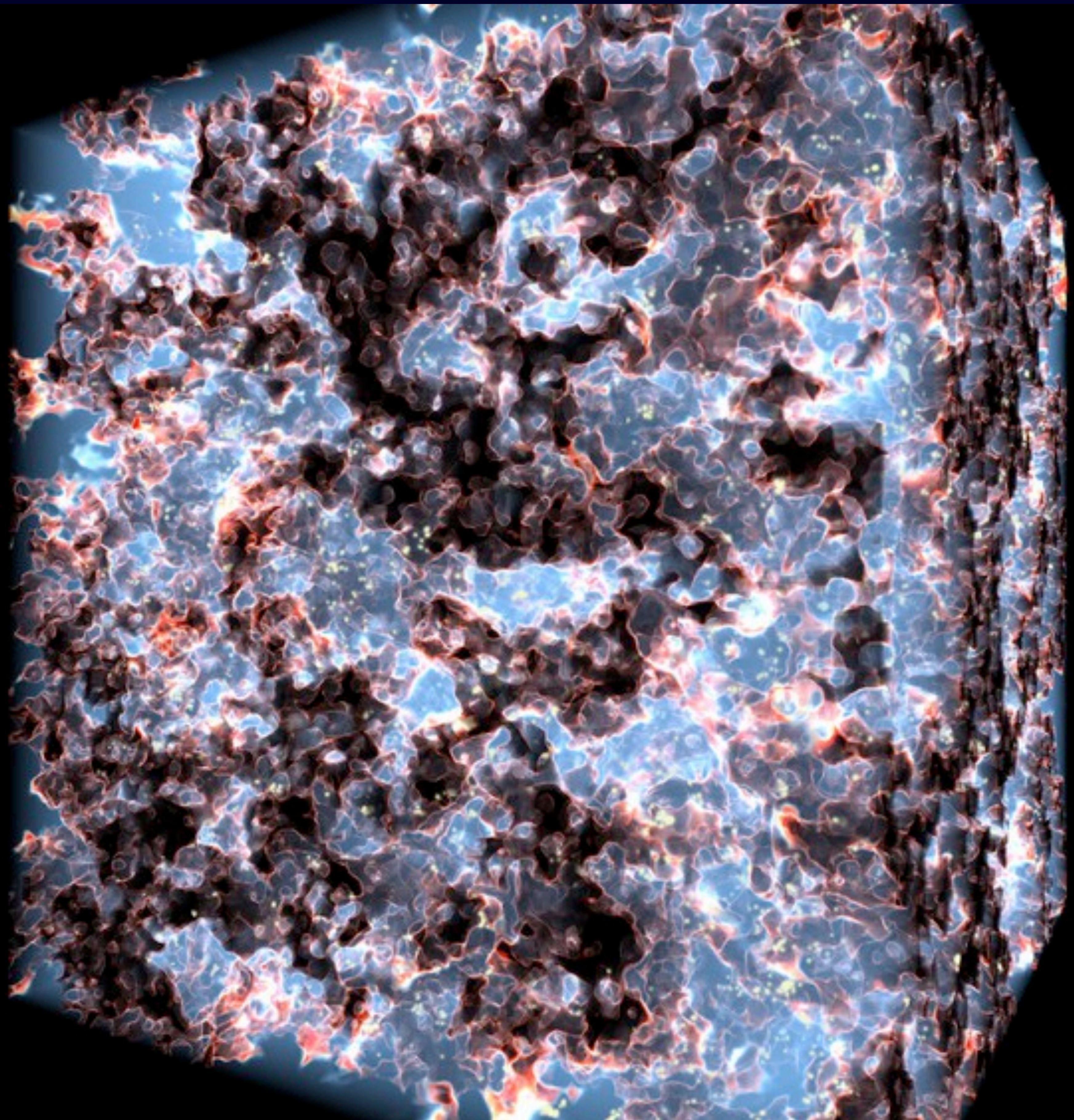
Average CMB spectral distortions



Average CMB spectral distortions



Fluctuations of the y-parameter at large scales

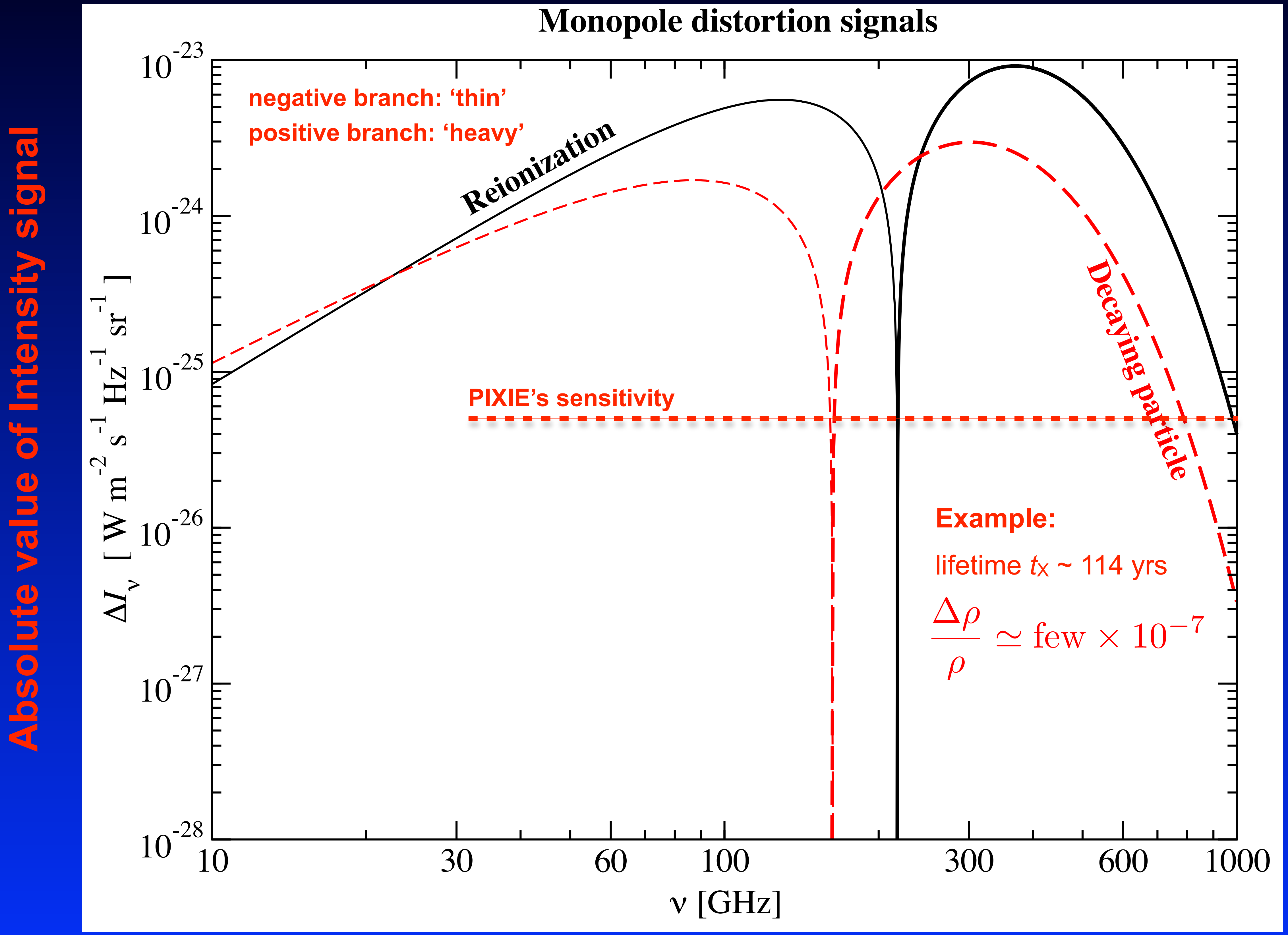


- spatial variations of the optical depth and temperature cause small-spatial variations of the y-parameter at different angular scales
- could tell us about the reionization sources and structure formation process
- additional independent piece of information!
- Cross-correlations with other signals

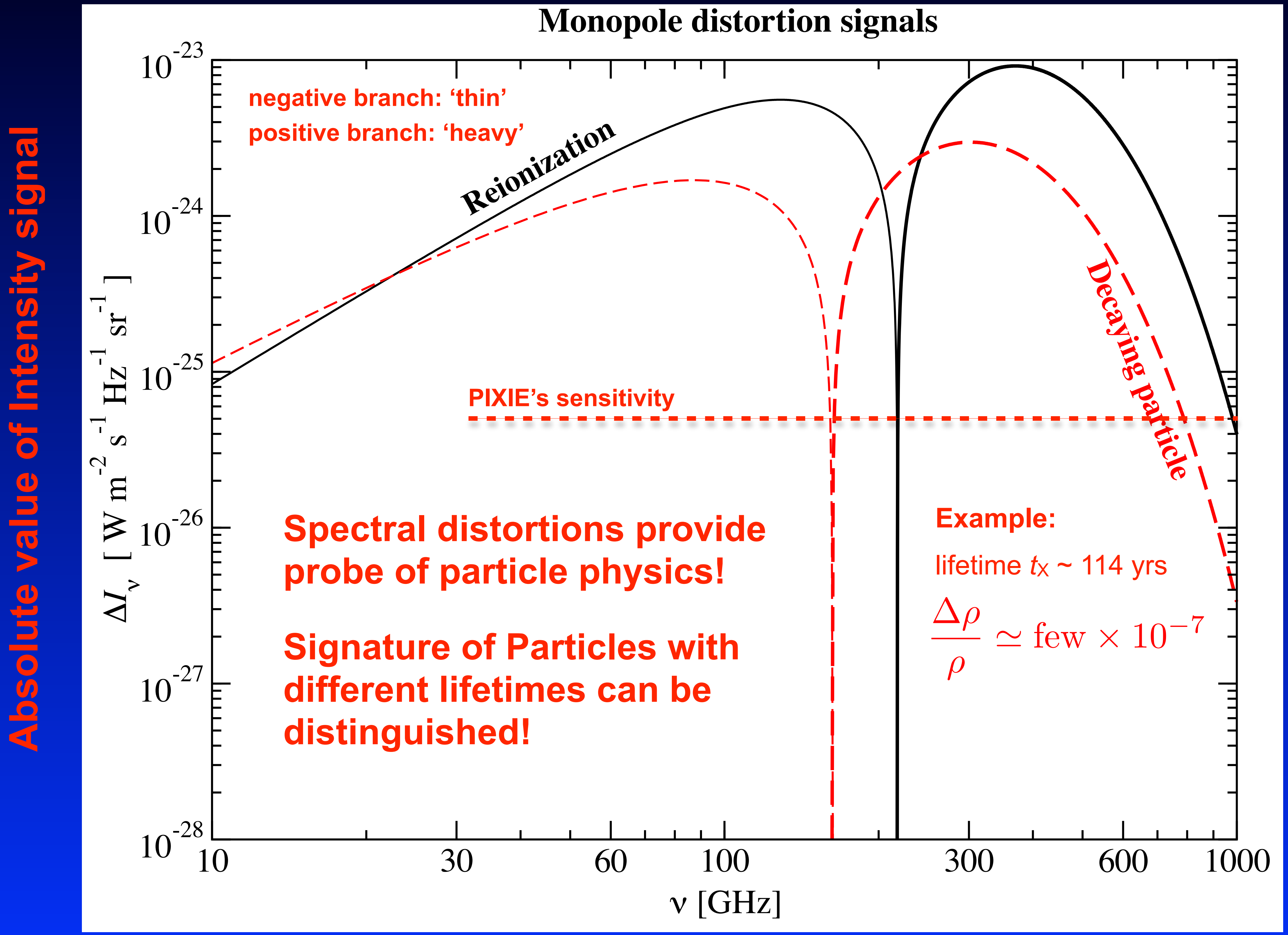
Example:
Simulation of reionization process (1Gpc/h) by *Alvarez* & *Abel*

Decaying particles

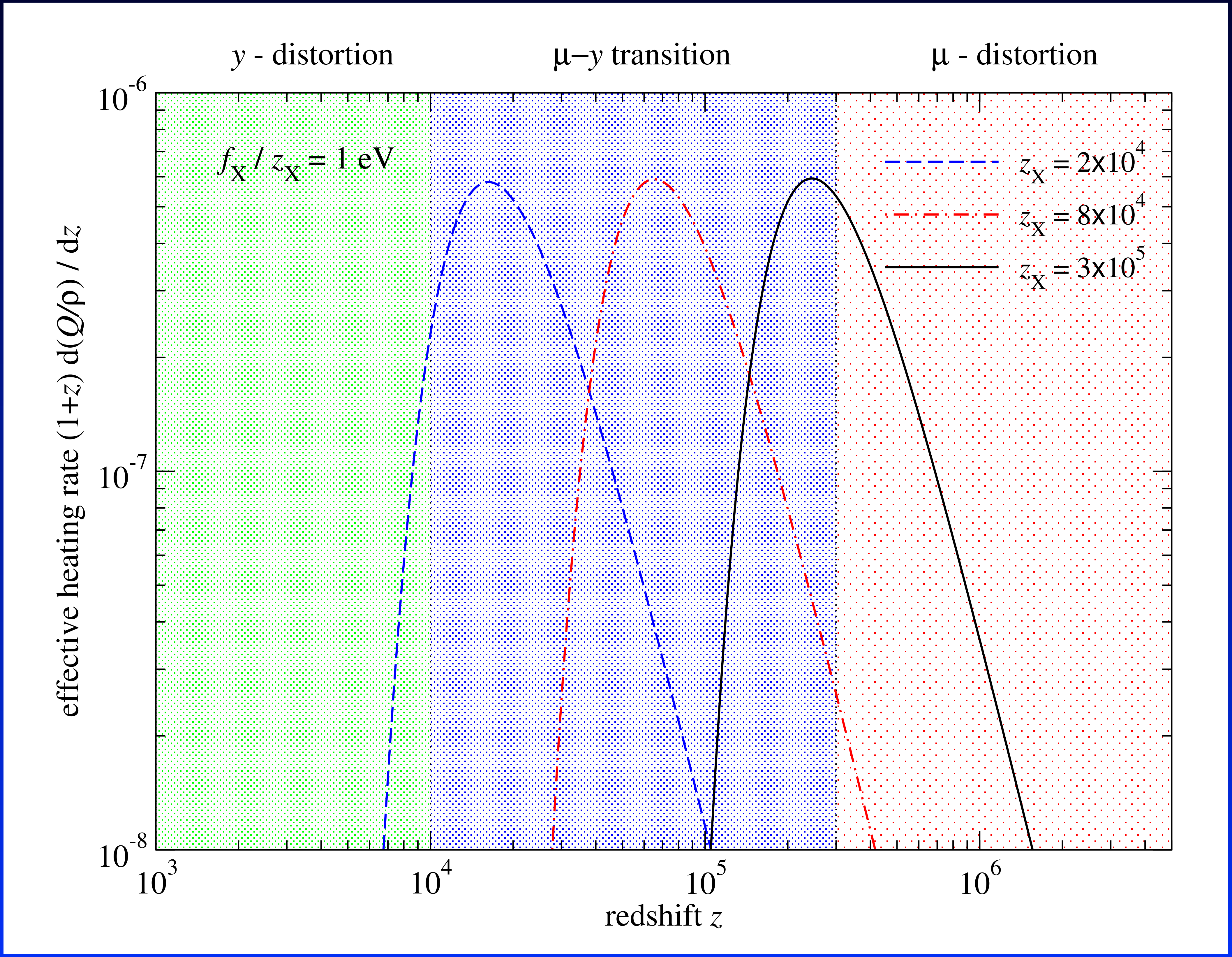
Average CMB spectral distortions



Average CMB spectral distortions

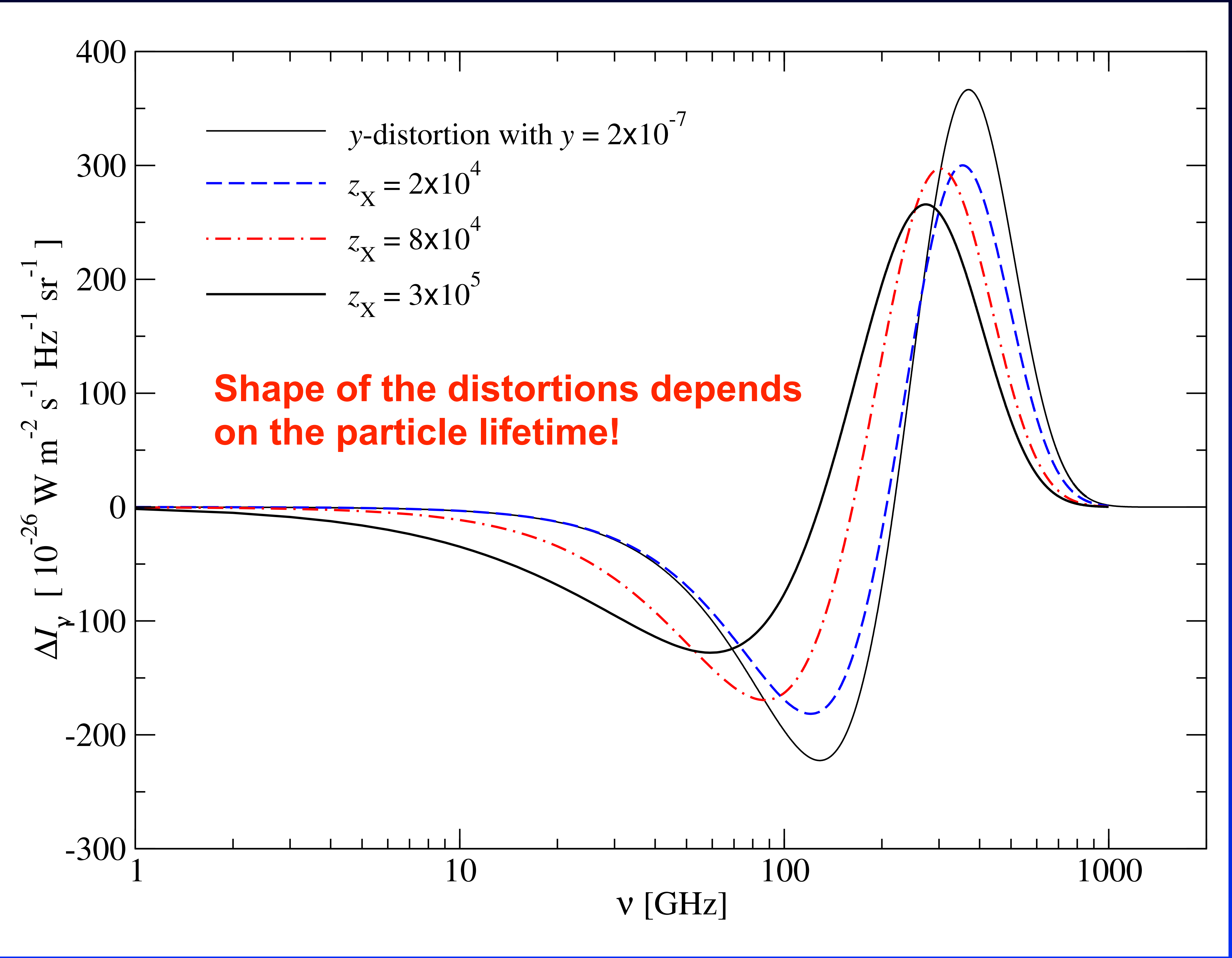


Decaying particle scenarios



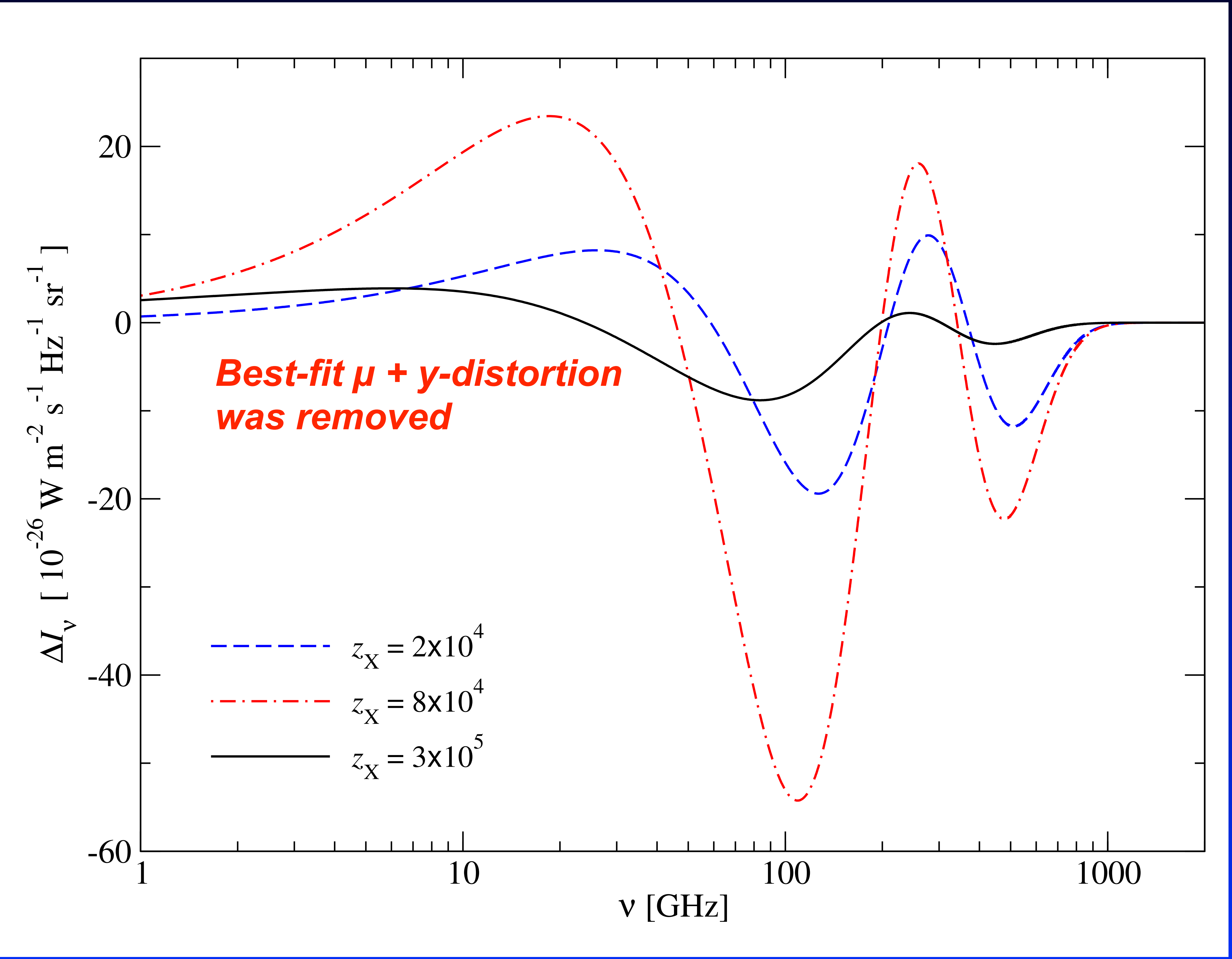
JC & Sunyaev, 2011, Arxiv:1109.6552
JC, 2013, Arxiv:1304.6120

Decaying particle scenarios



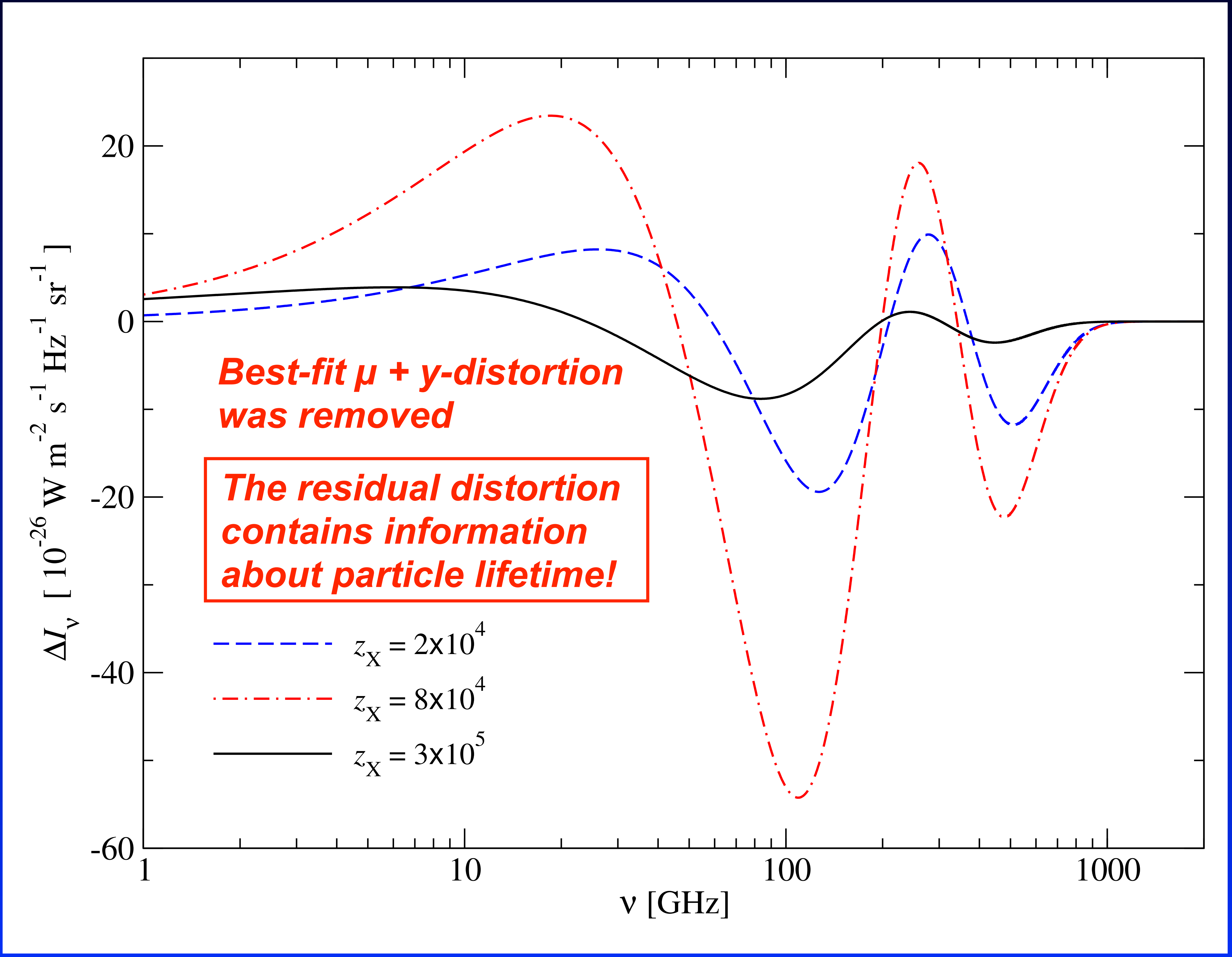
JC & Sunyaev, 2011, Arxiv:1109.6552
JC, 2013, Arxiv:1304.6120

Decaying particle scenarios (information in residual)



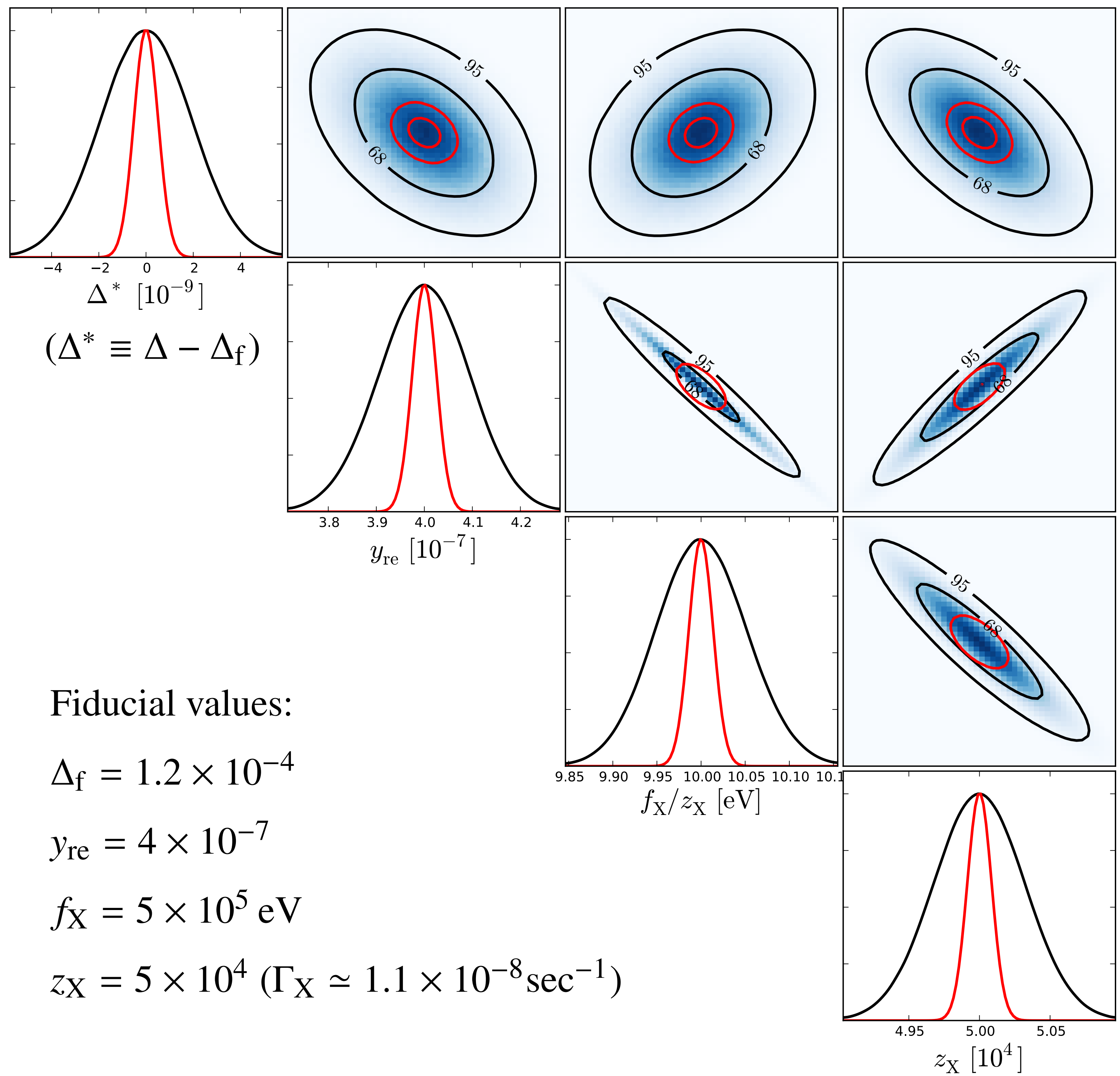
JC & Sunyaev, 2011, Arxiv:1109.6552
JC, 2013, Arxiv:1304.6120

Decaying particle scenarios (information in residual)



JC & Sunyaev, 2011, Arxiv:1109.6552
JC, 2013, Arxiv:1304.6120

Decaying particle scenarios



Fiducial values:

$\Delta_f = 1.2 \times 10^{-4}$

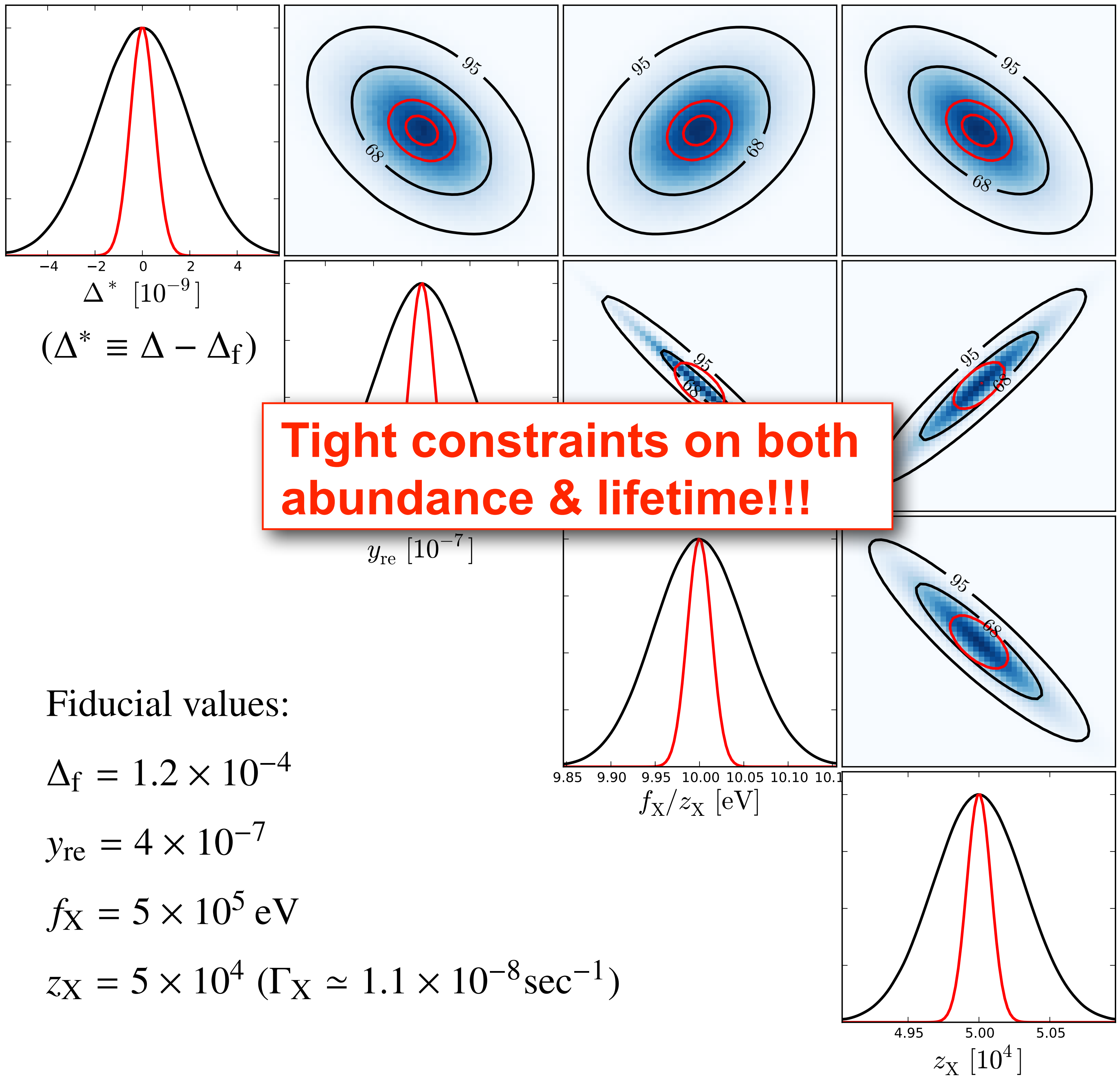
$y_{re} = 4 \times 10^{-7}$

$f_X = 5 \times 10^5$ eV

$z_X = 5 \times 10^4$ ($\Gamma_X \simeq 1.1 \times 10^{-8} \mathrm{sec}^{-1}$)

JC, 2013, ArXiv:1304.6120

Decaying particle scenarios



Fiducial values:

$\Delta_f = 1.2 \times 10^{-4}$

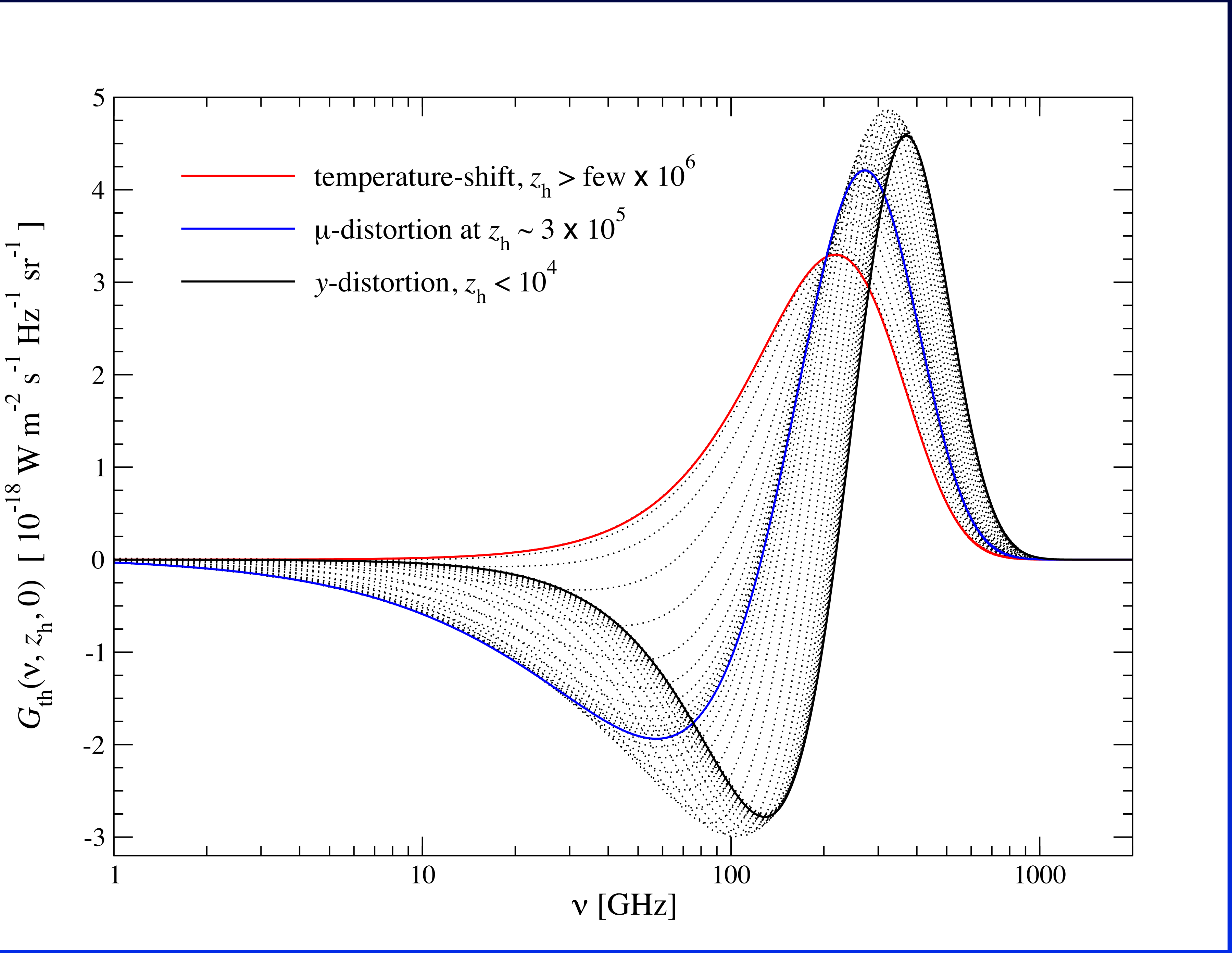
$y_{re} = 4 \times 10^{-7}$

$f_X = 5 \times 10^5$ eV

$z_X = 5 \times 10^4$ ($\Gamma_X \simeq 1.1 \times 10^{-8} \text{sec}^{-1}$)

JC, 2013, ArXiv:1304.6120

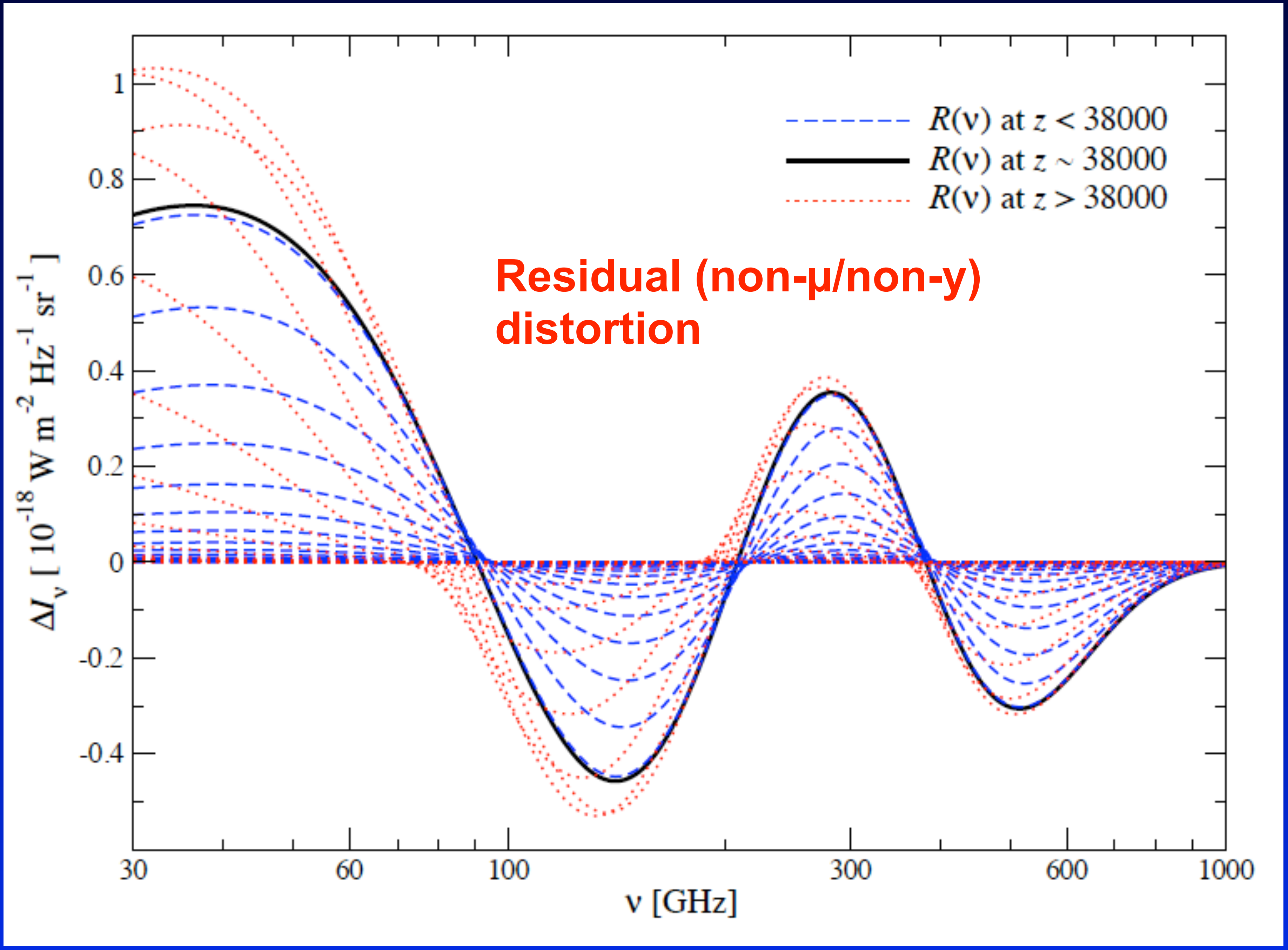
Using signal eigenmodes to compress the distortion data



- *Principle component decomposition* of the distortion signal
- compression of the useful information given instrumental settings

JC & Jeong, 2013

Using signal eigenmodes to compress the distortion data



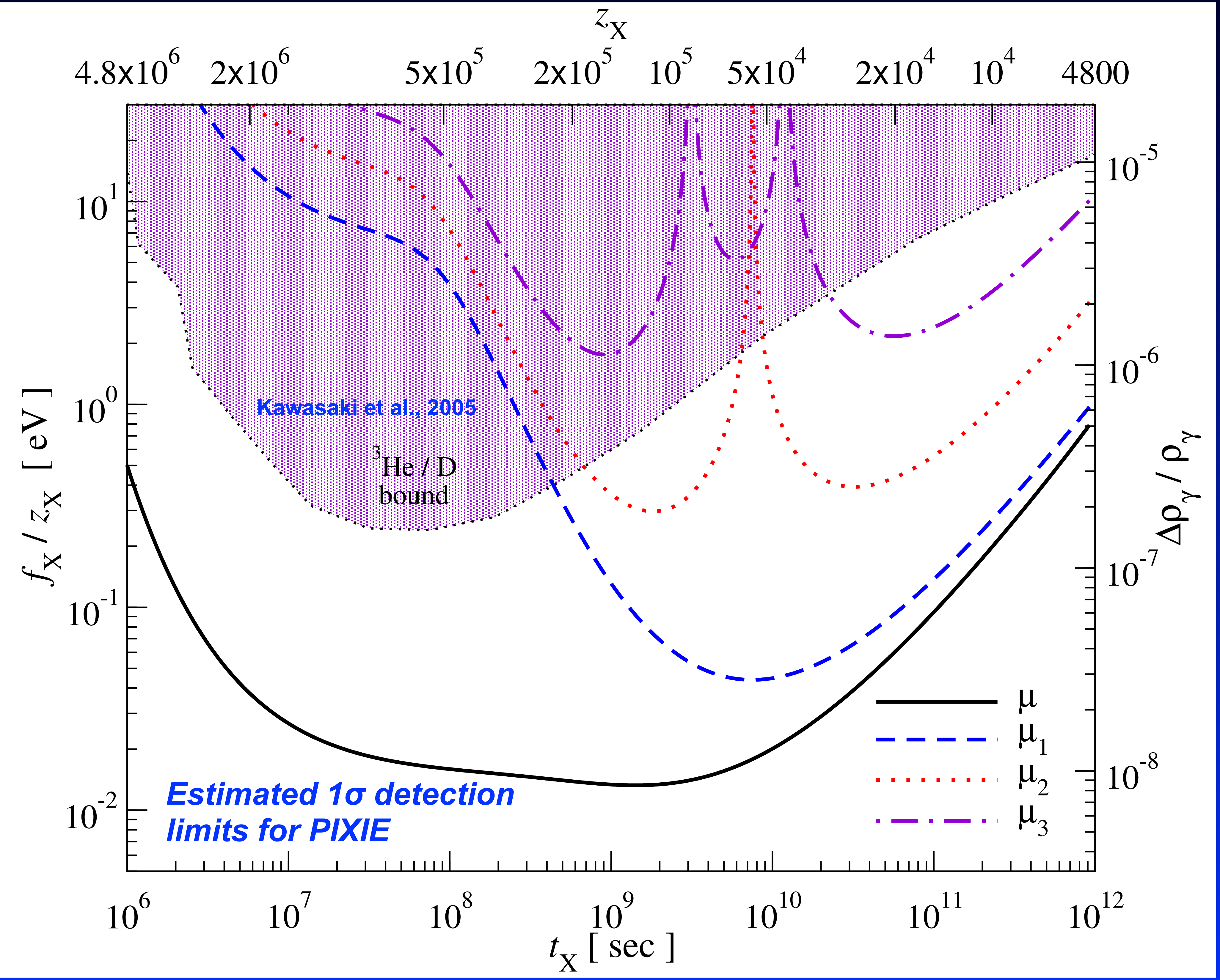
- *Principle component decomposition* of the distortion signal
- compression of the useful information given instrumental settings
- new set of observables

  p={y, μ, μ_1, μ_2, ...}

- model-comparison + forecasts of errors very simple!

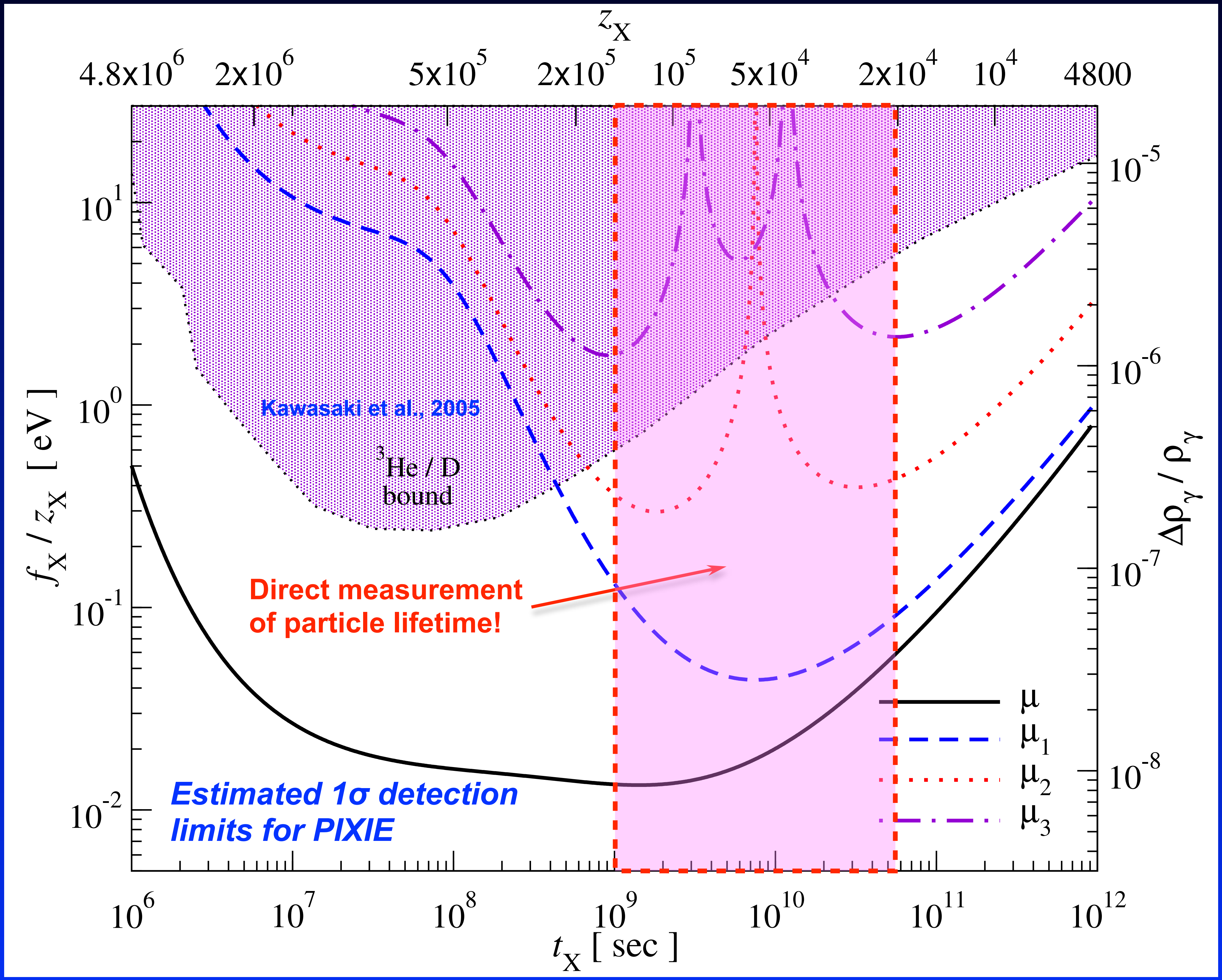
JC & Jeong, 2013

Distortions could shed light on decaying (DM) particles!



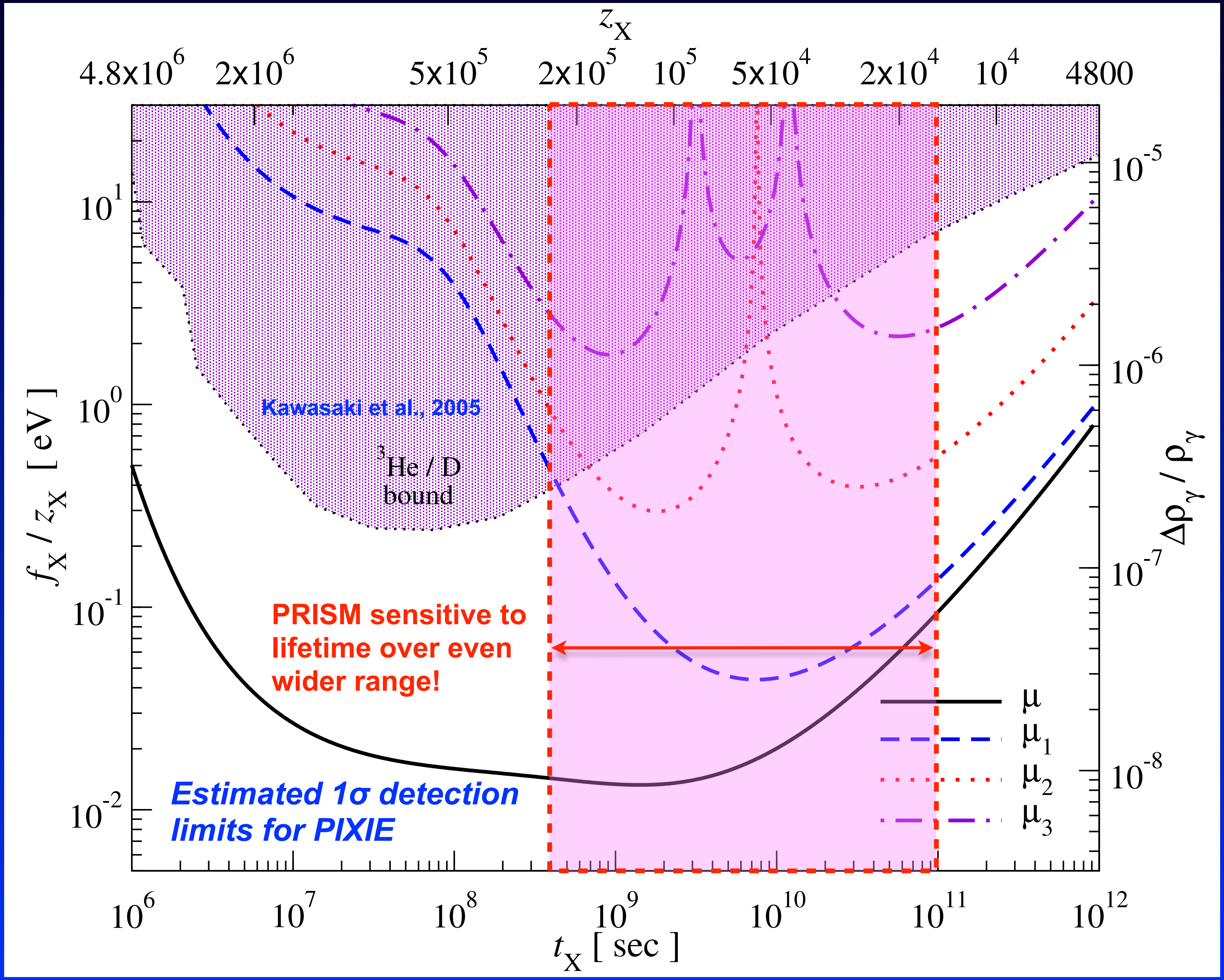
JC & Jeong, 2013

Distortions could shed light on decaying (DM) particles!



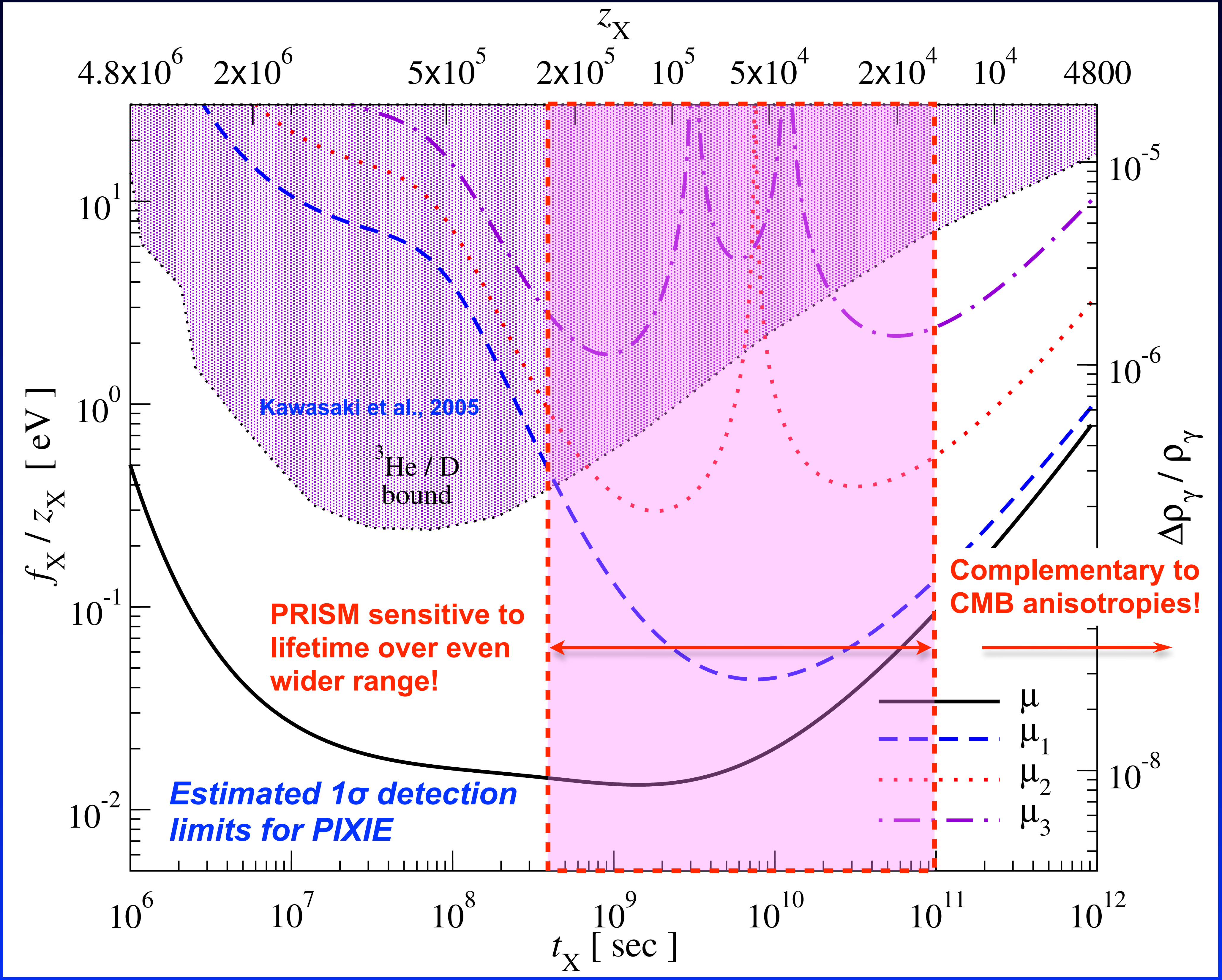
JC & Jeong, 2013

Distortions could shed light on decaying (DM) particles!



JC & Jeong, 2013

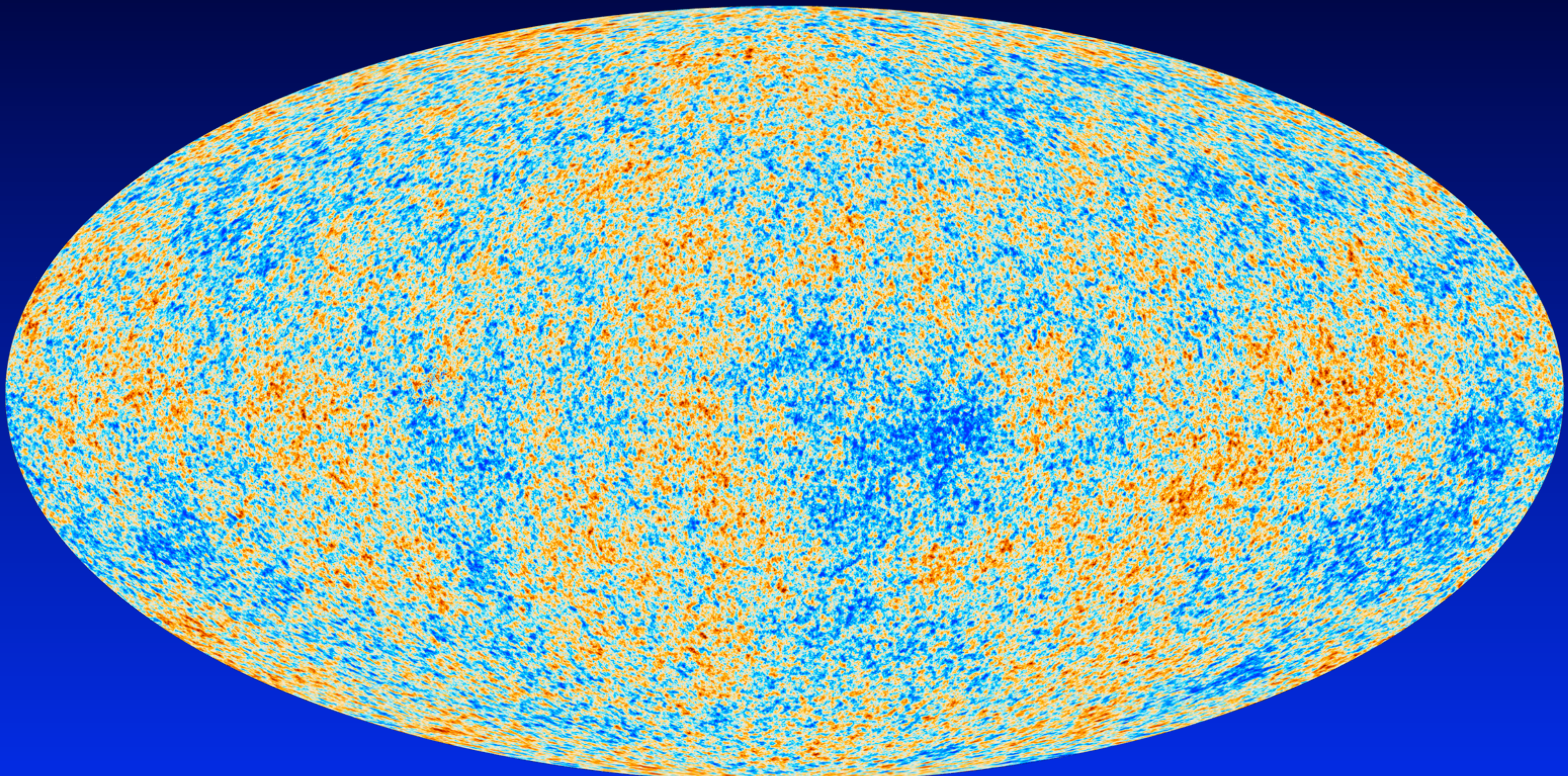
Distortions could shed light on decaying (DM) particles!



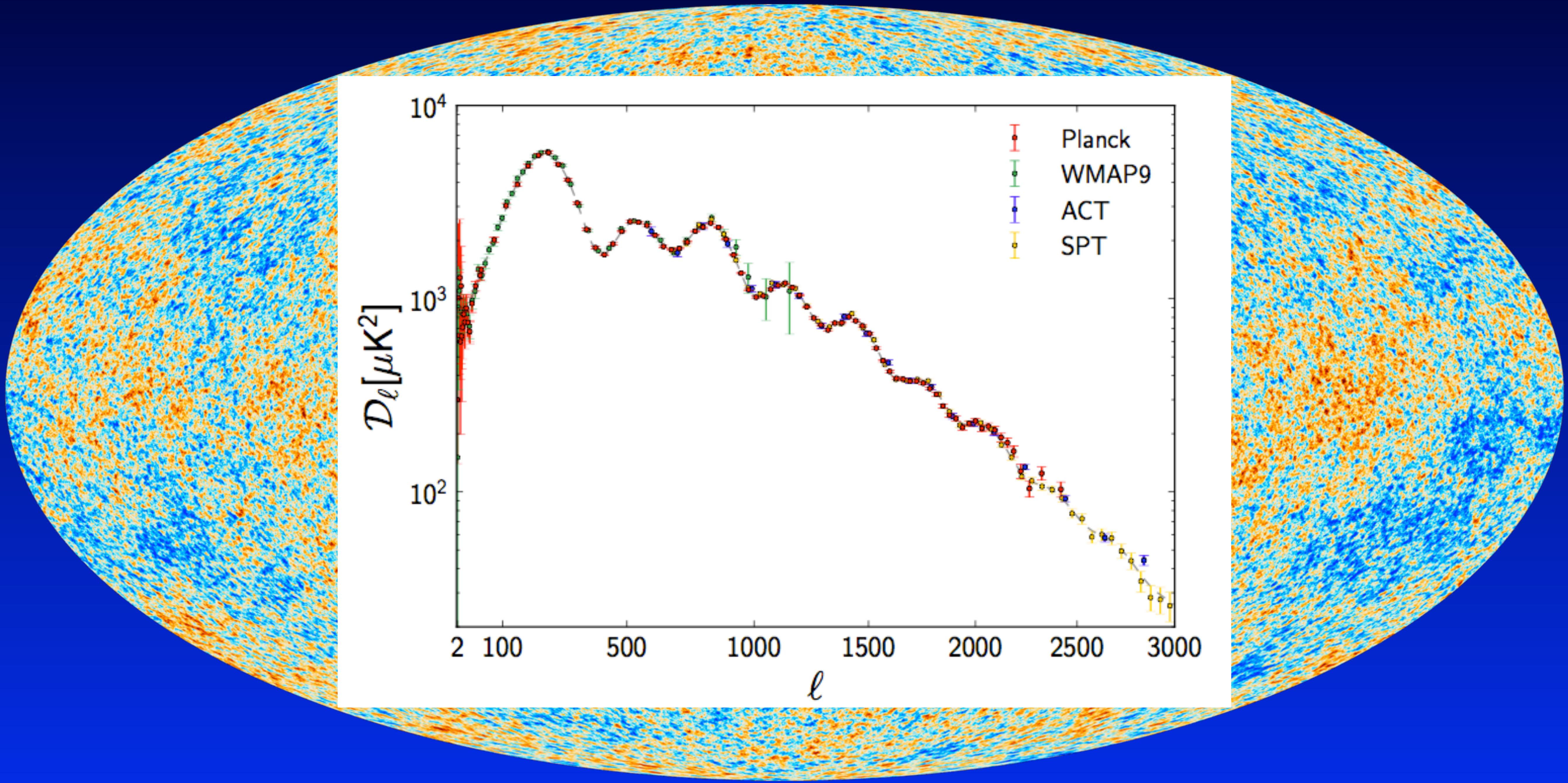
JC & Jeong, 2013

The dissipation of small-scale acoustic modes

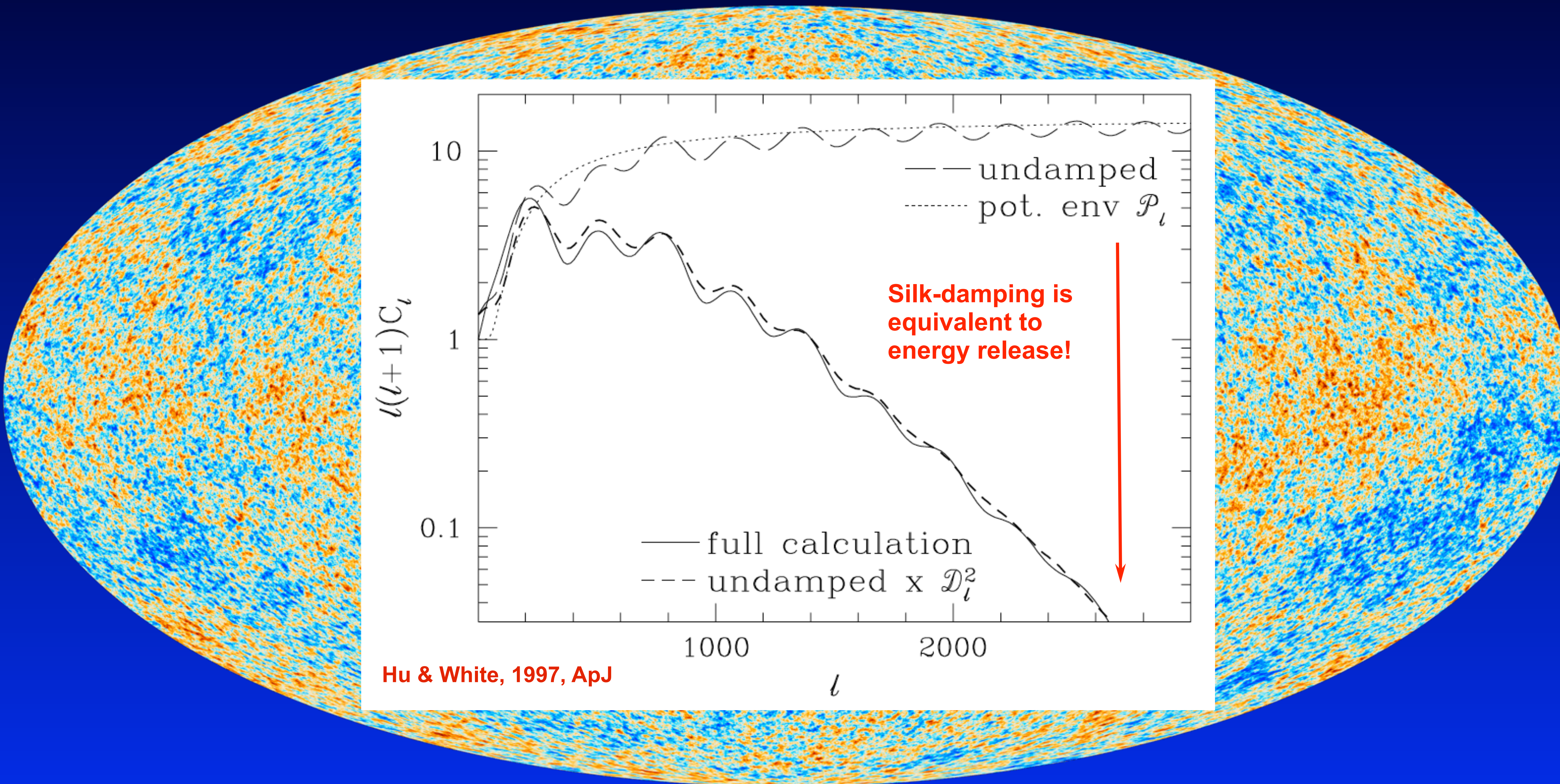
Dissipation of small-scale acoustic modes



Dissipation of small-scale acoustic modes



Dissipation of small-scale acoustic modes



Energy release caused by dissipation process

'Obvious' dependencies:

- *Amplitude* of the small-scale power spectrum
- *Shape* of the small-scale power spectrum
- *Dissipation scale* → $k_D \sim (H_0\, \Omega_{rel}^{1/2}\, N_{e,0})^{1/2}\, (1+z)^{3/2}$ at early times

not so 'obvious' dependencies:

- *primordial non-Gaussianity* in the ultra squeezed limit
  (Pajer & Zaldarriaga, 2012; Ganc & Komatsu, 2012)
- *Type* of the perturbations (adiabatic ↔ isocurvature)
  (Barrow & Coles, 1991; Hu et al., 1994; Dent et al, 2012, JC & Grin, 2012)
- *Neutrinos* (or any extra relativistic degree of freedom)

Energy release caused by dissipation process

'Obvious' dependencies:

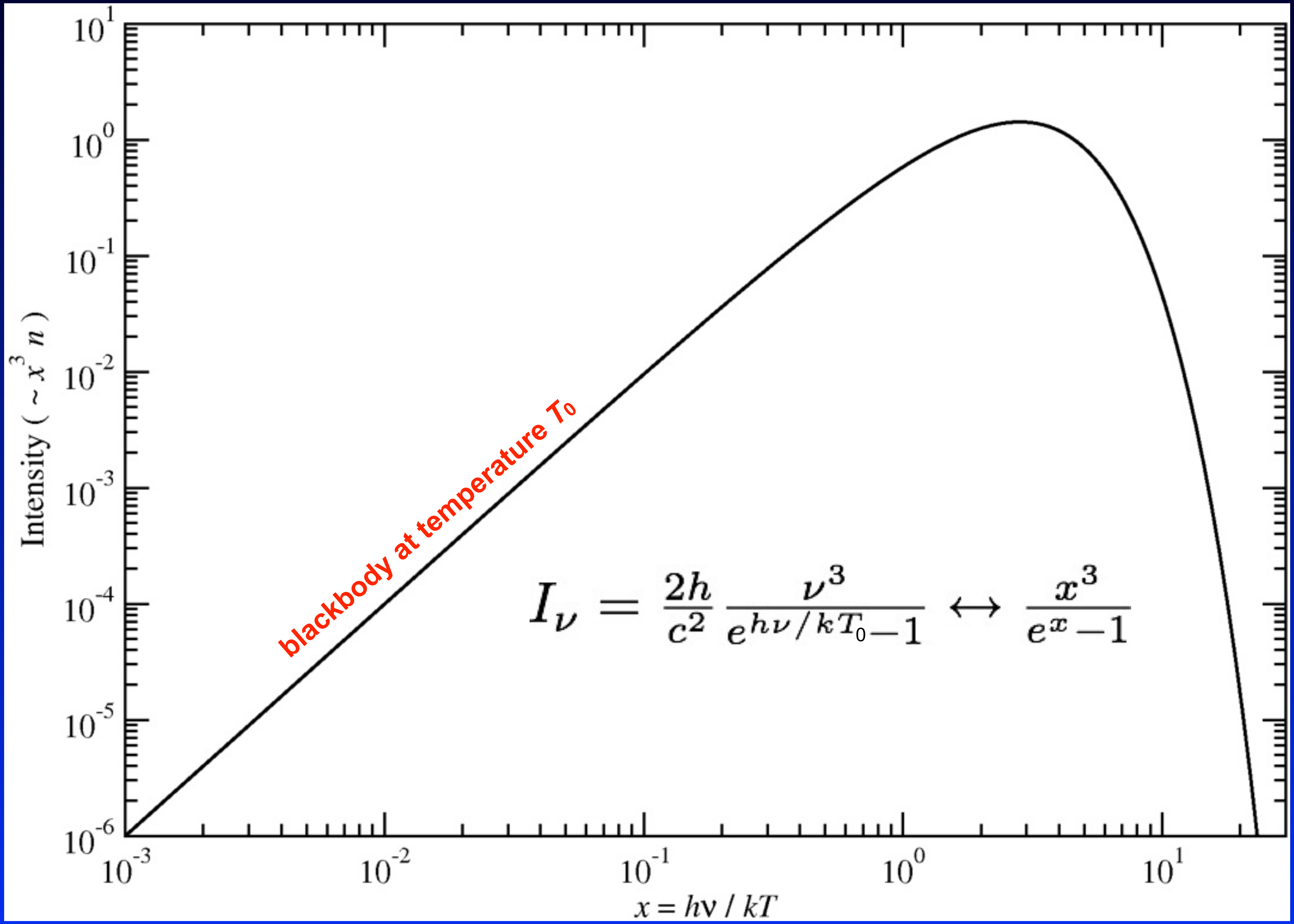
- *Amplitude* of the small-scale power spectrum
- *Shape* of the small-scale power spectrum
- *Dissipation scale* → $k_D \sim (H_0\, \Omega_{rel}^{1/2}\, N_{e,0})^{1/2}\, (1+z)^{3/2}$ at early times

not so 'obvious' dependencies:

- *primordial non-Gaussianity* in the ultra squeezed limit
  (Pajer & Zaldarriaga, 2012; Ganc & Komatsu, 2012)
- *Type* of the perturbations (adiabatic ↔ isocurvature)
  (Barrow & Coles, 1991; Hu et al., 1994; Dent et al, 2012, JC & Grin, 2012)
- *Neutrinos* (or any extra relativistic degree of freedom)

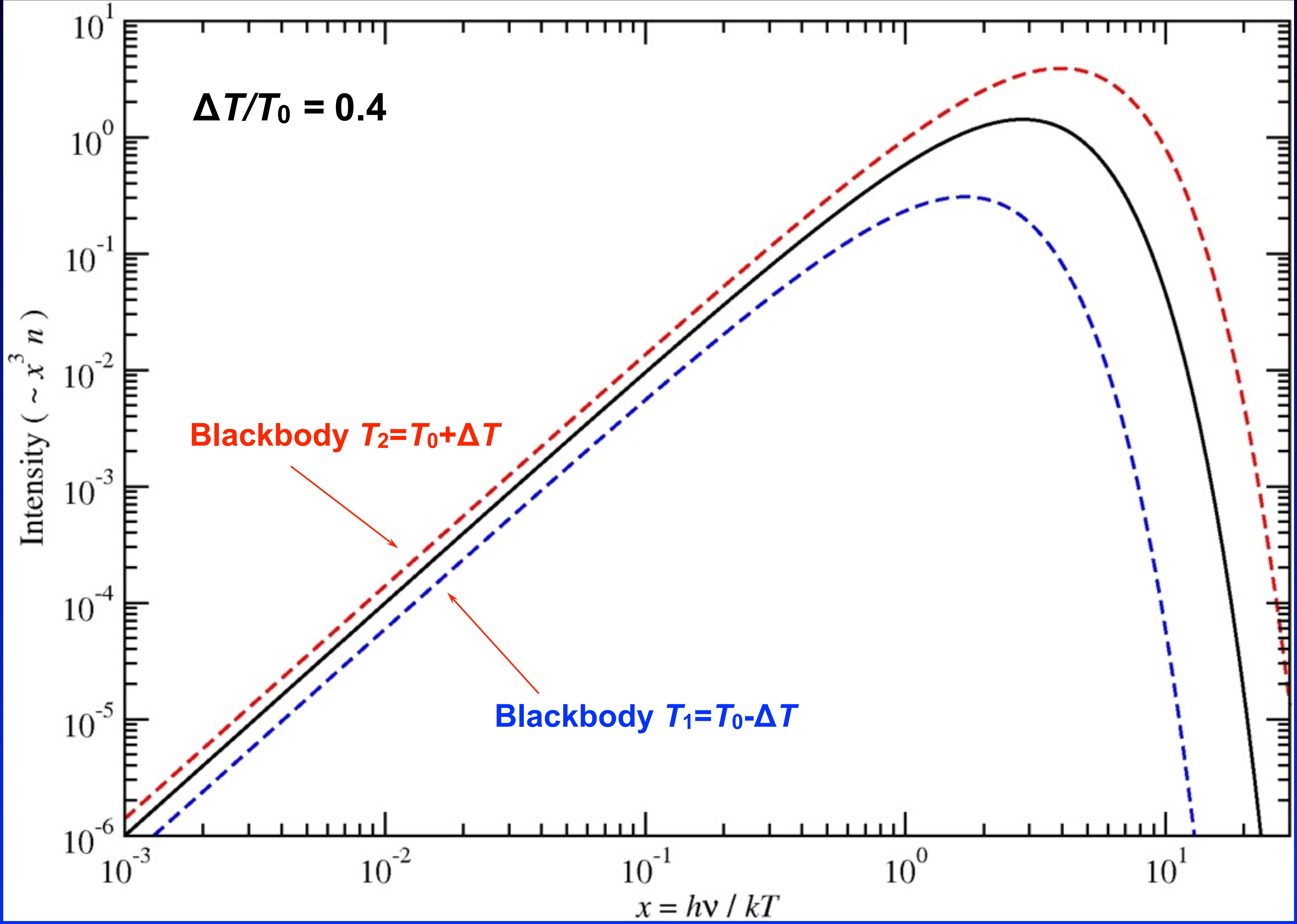
CMB Spectral distortions could add additional numbers beyond 'just' the tensor-to-scalar ratio from B-modes!

Superpositions of blackbody spectra



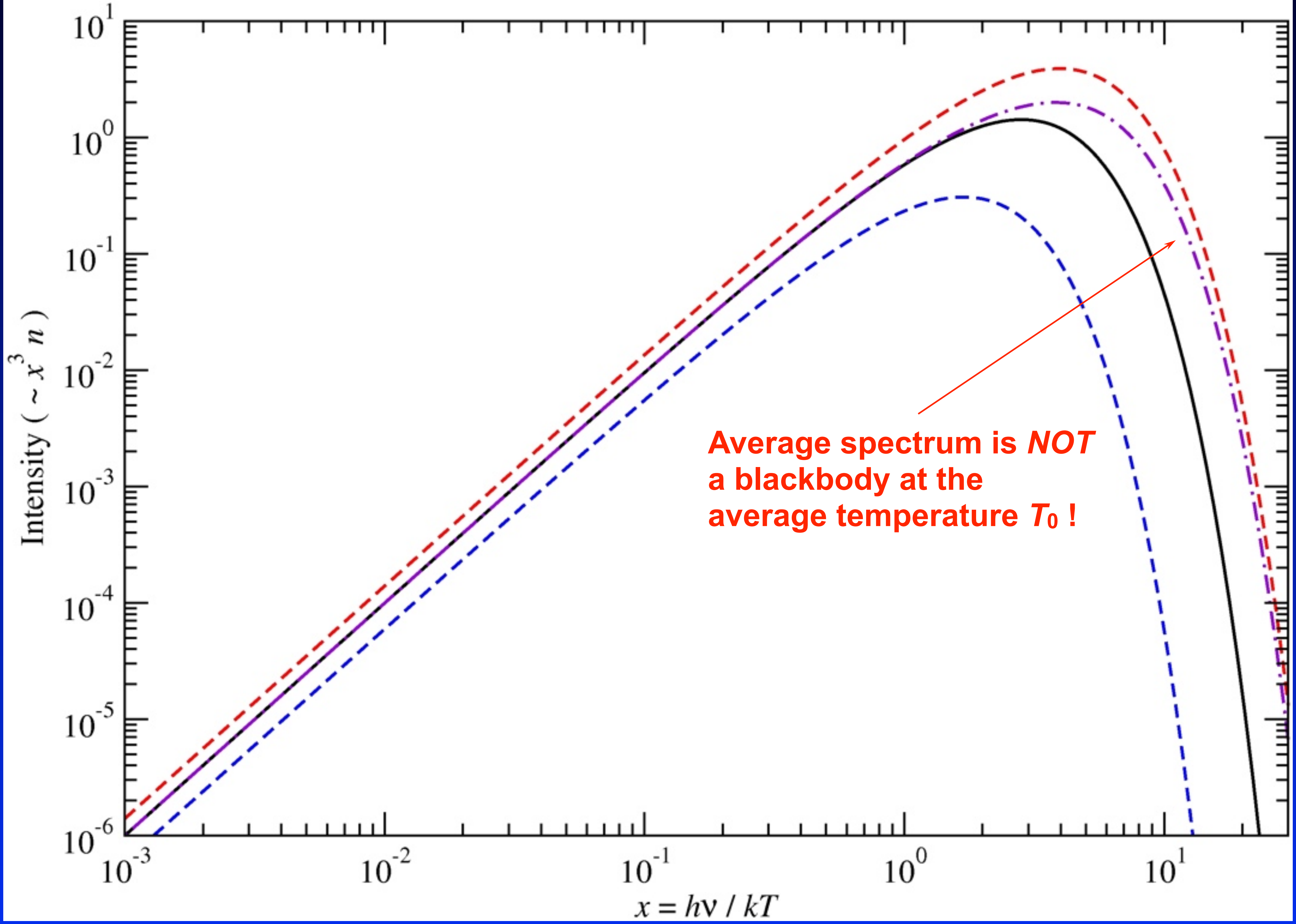
Zeldovich, Illarionov & Sunyaev, 1972
JC & Sunyaev 2004

Superpositions of blackbody spectra



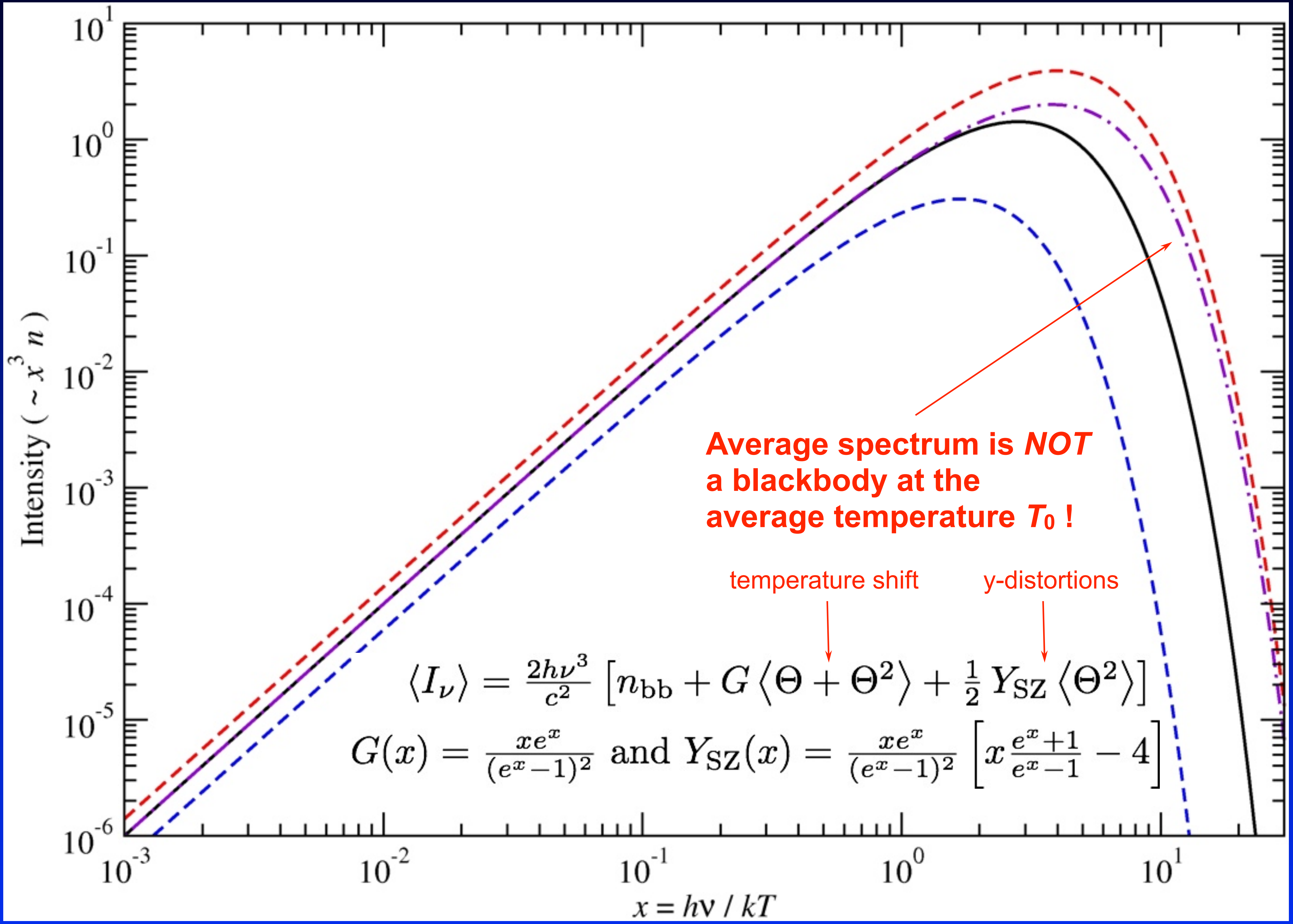
Zeldovich, Illarionov & Sunyaev, 1972
JC & Sunyaev 2004

Superpositions of blackbody spectra



Zeldovich, Illarionov & Sunyaev, 1972
JC & Sunyaev 2004

Superpositions of blackbody spectra

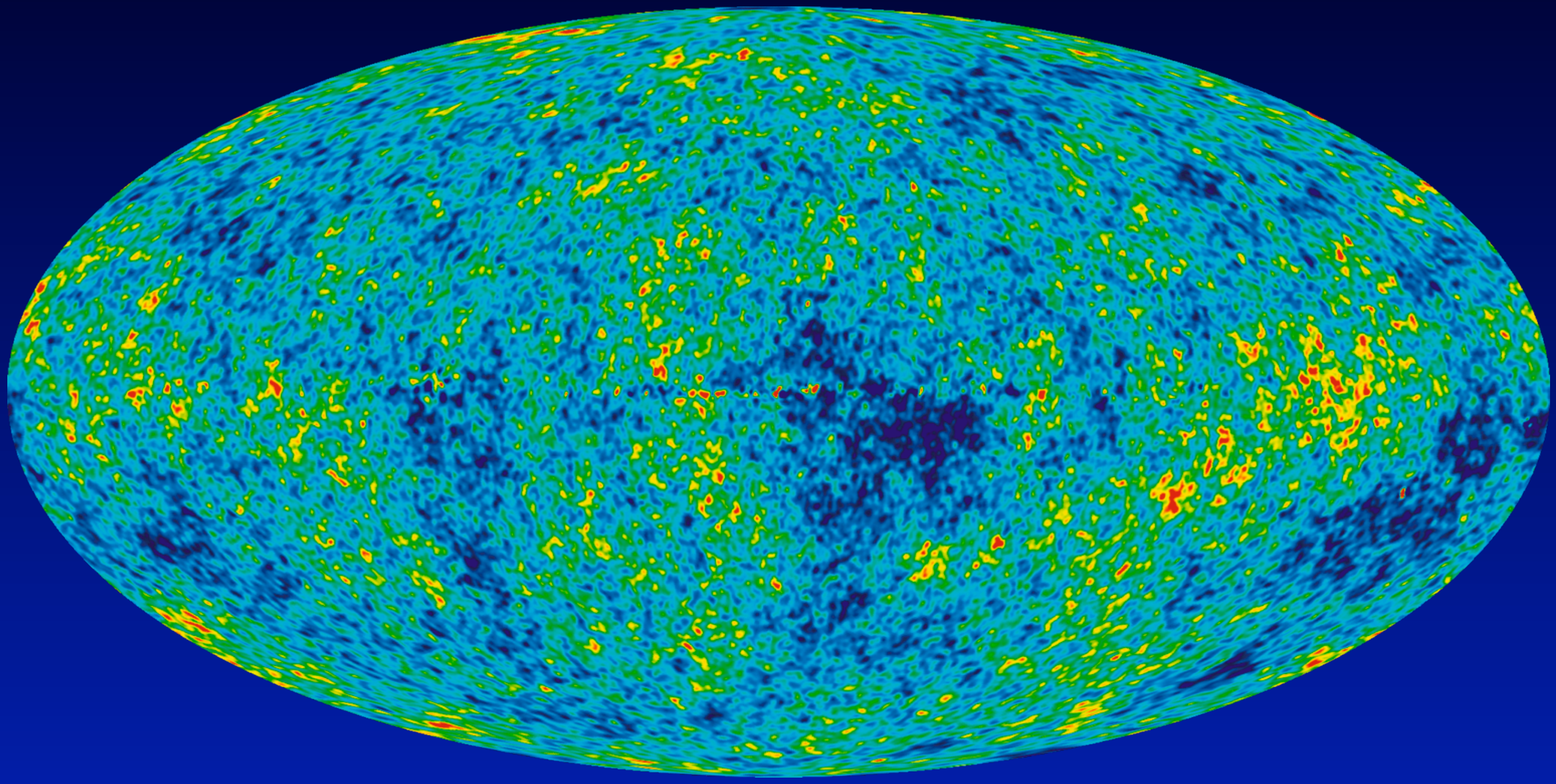


$$\langle I_\nu \rangle = \frac{2h\nu^3}{c^2}\left[n_{\rm bb} + G\left\langle \Theta + \Theta^2 \right\rangle + \frac{1}{2}\, Y_{\rm SZ}\left\langle \Theta^2 \right\rangle\right]$$

$$G(x) = \frac{x e^x}{(e^x - 1)^2} \text{ and } Y_{\rm SZ}(x) = \frac{x e^x}{(e^x - 1)^2}\left[x\,\frac{e^x + 1}{e^x - 1} - 4\right]$$

Zeldovich, Illarionov & Sunyaev, 1972
JC & Sunyaev 2004

Distortion caused by superposition of blackbodies



- average spectrum

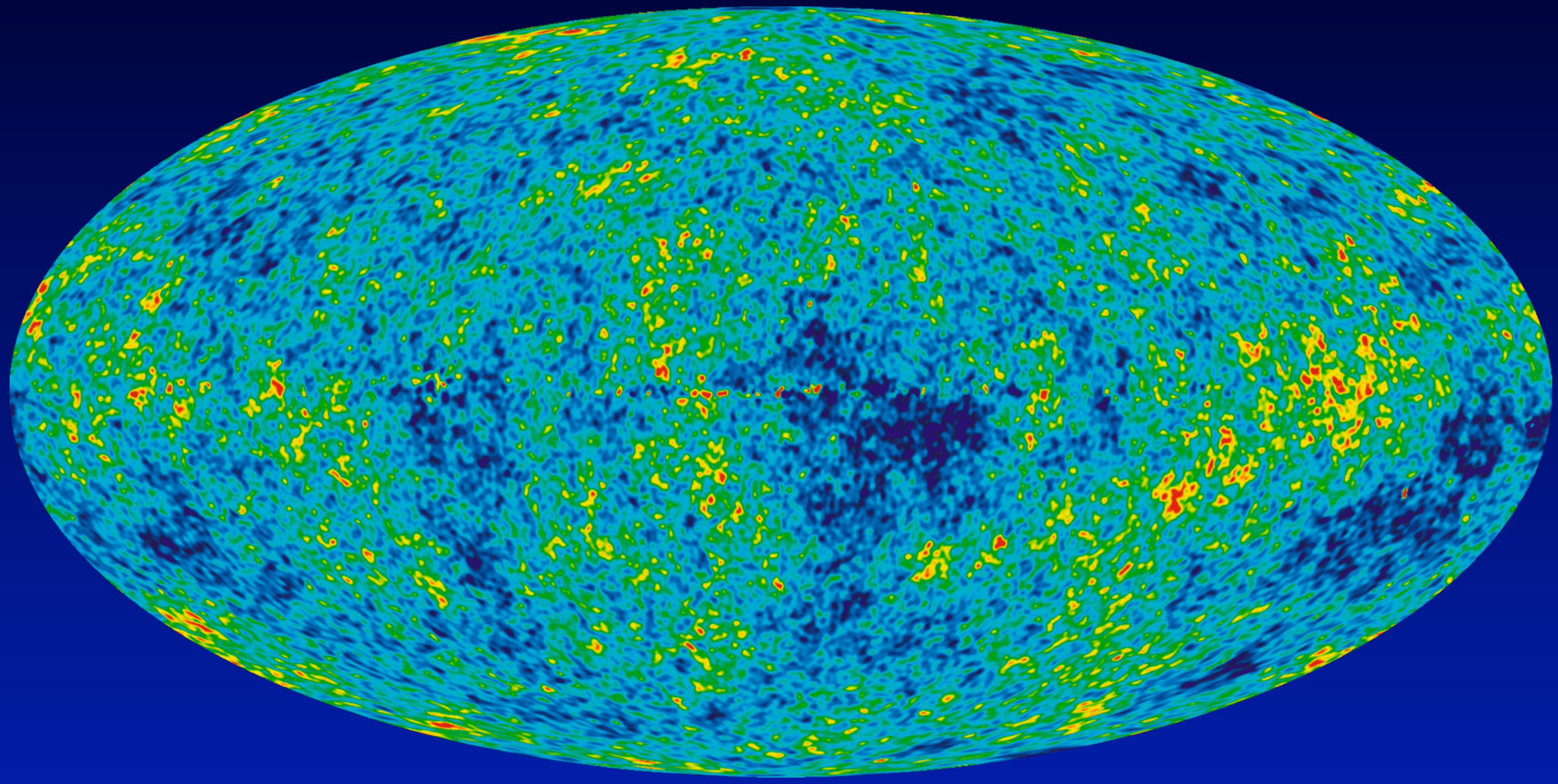
$$\Rightarrow \; y \simeq \frac{1}{2}\left\langle \left(\frac{\Delta T}{T}\right)^2 \right\rangle \approx 8 \times 10^{-10}$$

$$\Delta T_{\rm sup} \simeq T \left\langle \left(\frac{\Delta T}{T}\right)^2 \right\rangle \approx 4.4 {\rm nK}$$

- known with very high precision

JC & Sunyaev, 2004
JC, Khatri & Sunyaev, 2012

Distortion caused by superposition of blackbodies

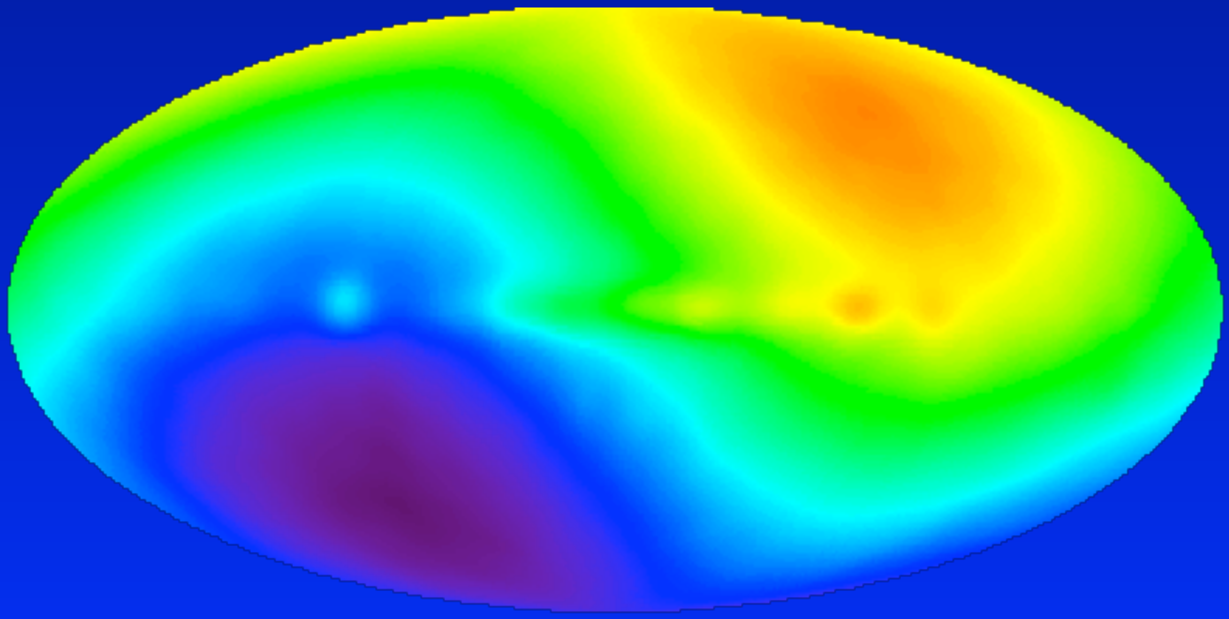


- average spectrum

$$\Rightarrow \quad y \simeq \frac{1}{2}\left\langle\left(\frac{\Delta T}{T}\right)^2\right\rangle \approx 8\times 10^{-10}$$

$$\Delta T_{\rm sup} \simeq T\left\langle\left(\frac{\Delta T}{T}\right)^2\right\rangle \approx 4.4\,{\rm nK}$$

- known with very high precision



- *CMB dipole* ($\beta_c \sim 1.23\times10^{-3}$)

$$\Rightarrow \qquad y \simeq \frac{\beta_c^2}{6} \approx 2.6\times 10^{-7}$$

$$\Delta T_{\rm sup} \simeq T\,\frac{\beta_c^2}{3} \approx 1.4\,\mu{\rm K}$$

- electrons are up-scattered
- can be taken out at the level of ~ 10^{-9}

JC & Sunyaev, 2004
JC, Khatri & Sunyaev, 2012

COBE/DMR: ΔT = 3.353 mK

Effective energy release caused by damping effect

- Effective heating rate from full 2x2 Boltzmann treatment **(JC, Khatri & Sunyaev, 2012)**

$$\frac{1}{a^4\rho_\gamma}\frac{\mathrm{d}a^4 Q_{\rm ac}}{\mathrm{d}t} = 4\sigma_{\rm T} N_{\rm e} c \left\langle \frac{(3\Theta_1-\beta)^2}{3} + \frac{9}{2}\Theta_2^2 - \frac{1}{2}\Theta_2(\Theta_0^{\rm P}+\Theta_2^{\rm P}) + \sum_{l\geq 3}(2l+1)\Theta_\ell^2 \right\rangle$$

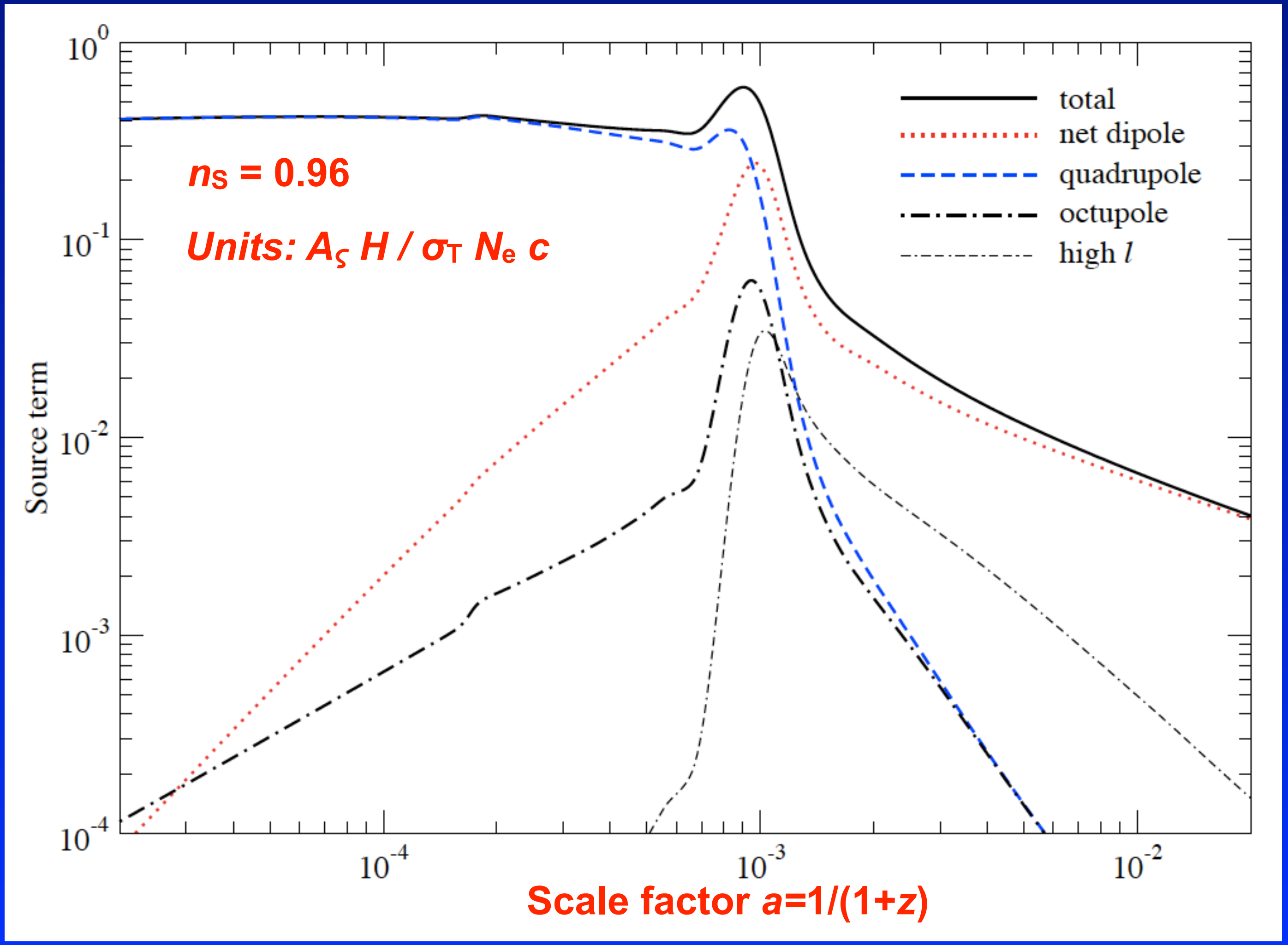
$$\Theta_\ell = \frac{1}{2}\int \Theta(\mu) P_\ell(\mu)\mathrm{d}\mu$$

gauge-independent dipole **effect of polarization** **higher multipoles**

$$\langle XY\rangle = \int \frac{k^2 \mathrm{d}k}{2\pi^2} P(k) X(k) Y(k)$$

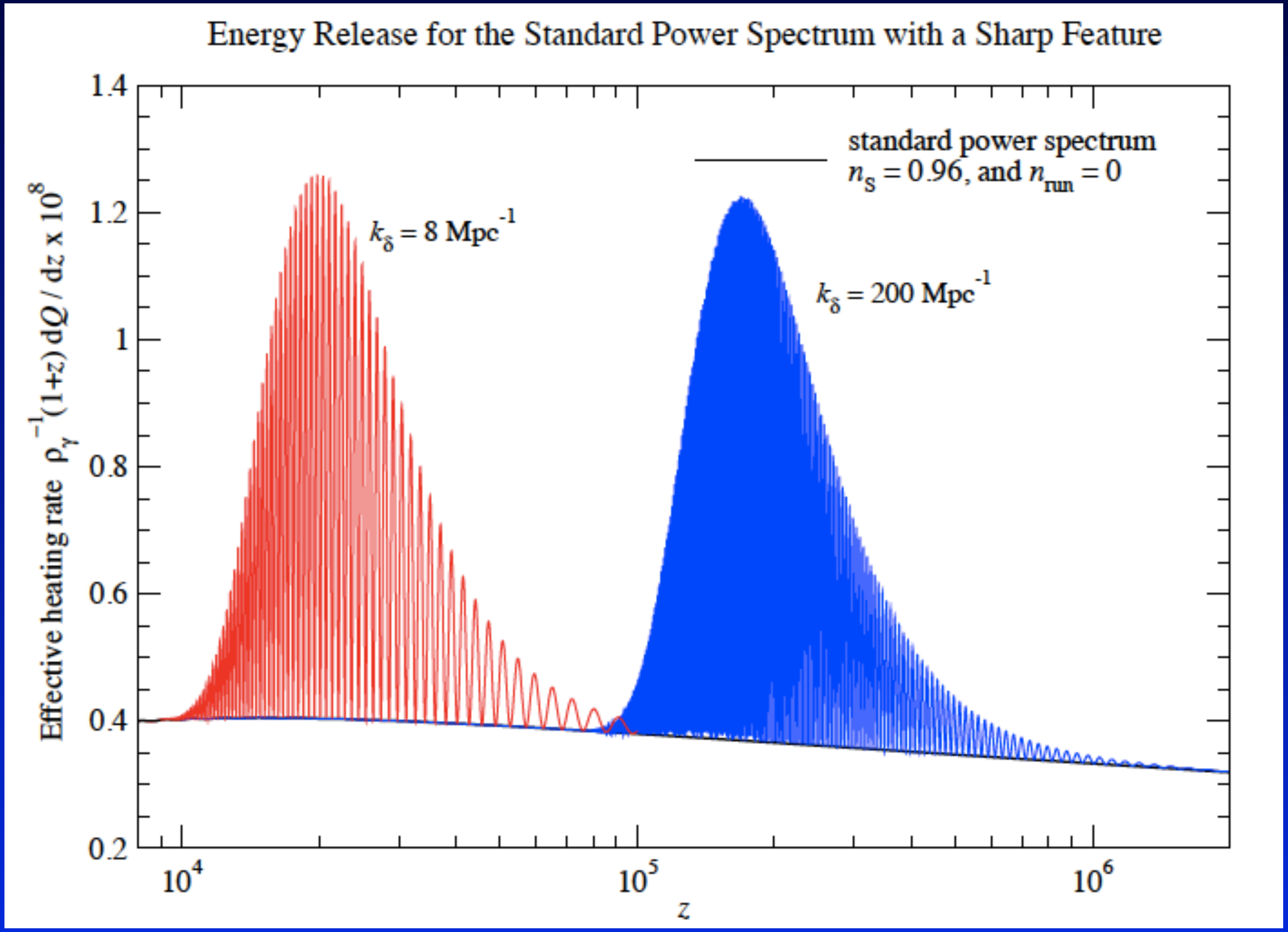
Primordial power spectrum

- *quadrupole* dominant at high *z*
- *net dipole* important only at low redshifts
- polarization ~5% effect
- contribution from higher multipoles rather small



JC, Khatri & Sunyaev, 2012

Which modes dissipate in the μ and y-eras?



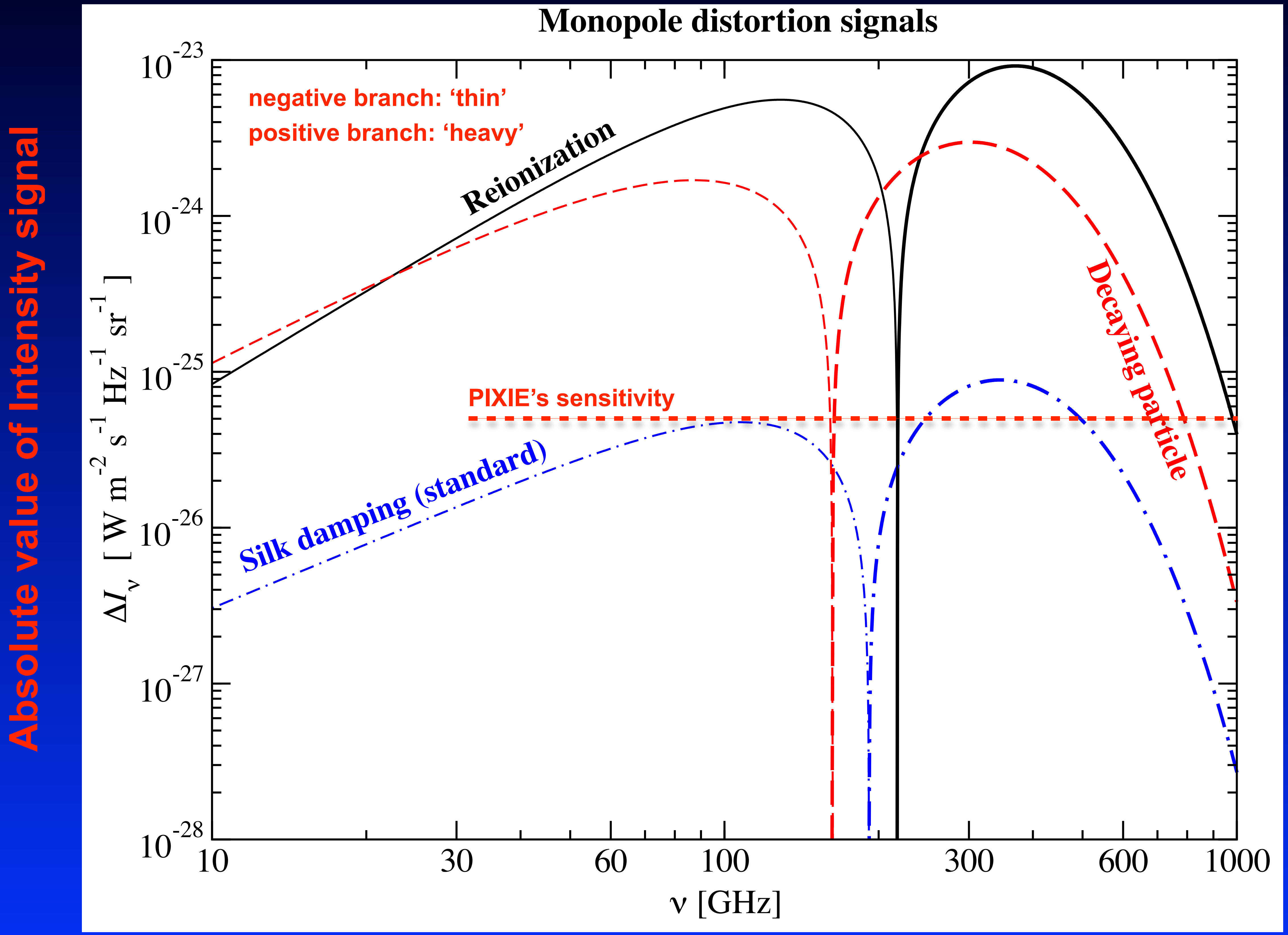
JC, Erickcek & Ben-Dayan, 2012

- Single mode with wavenumber k dissipates its energy at

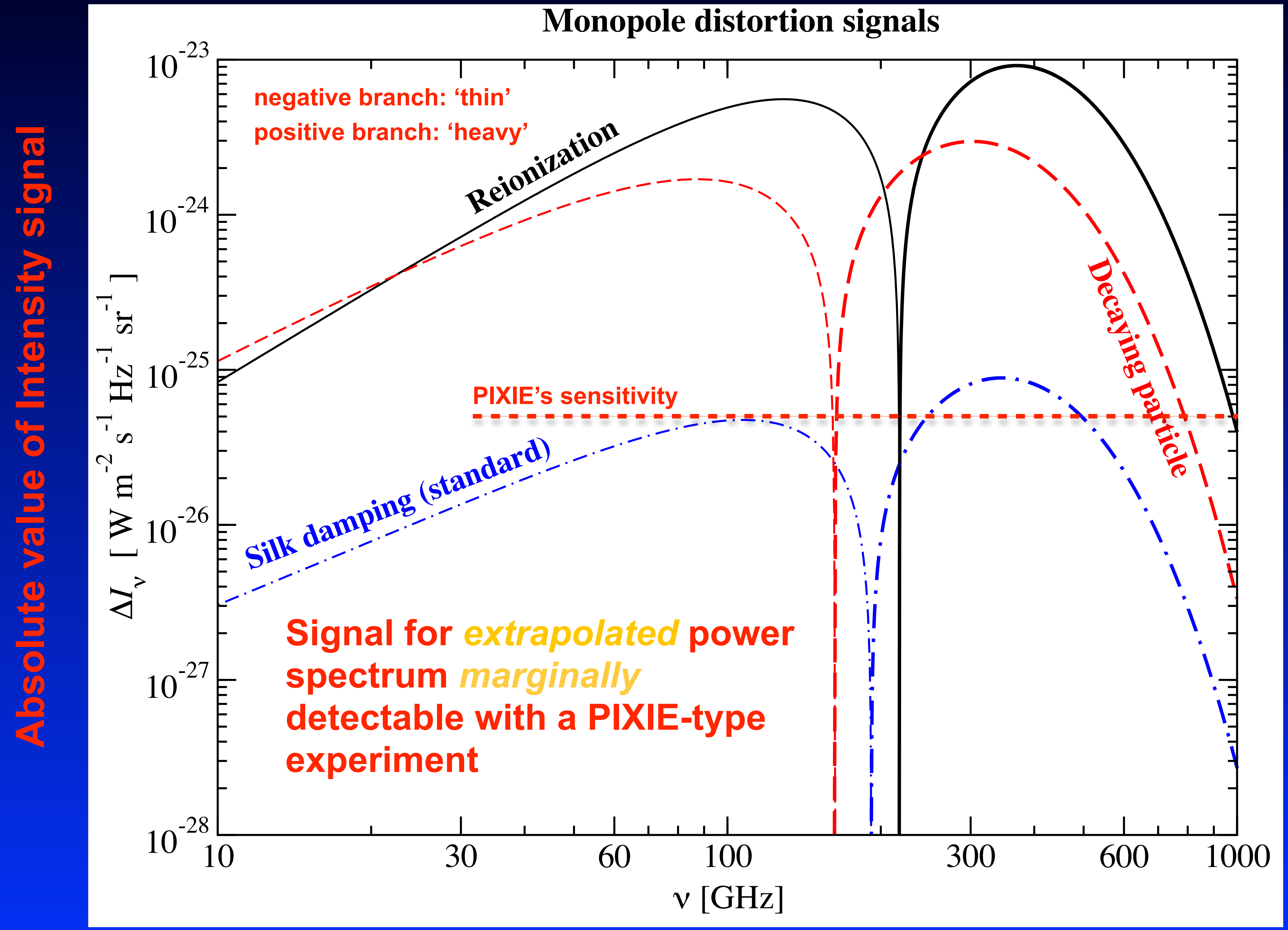
$$z_d \sim 4.5\text{x}10^5 (k \text{ Mpc}/10^3)^{2/3}$$

- Modes with wavenumber $50 \text{ Mpc}^{-1} < k < 10^4 \text{ Mpc}^{-1}$ dissipate their energy during the μ-era
- Modes with $k < 50 \text{ Mpc}^{-1}$ cause y-distortion

Average CMB spectral distortions

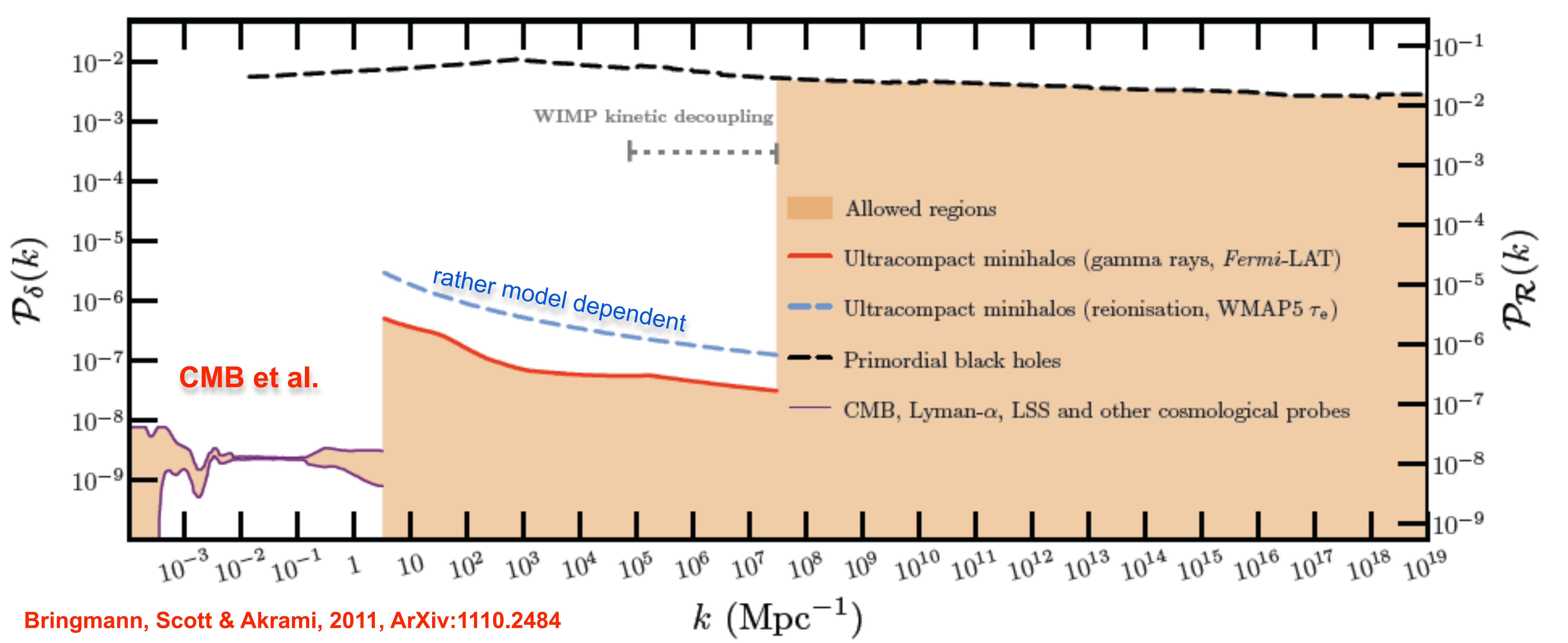


Average CMB spectral distortions



But this is not all that one could look at !!!

Distortions provide additional power spectrum constraints!

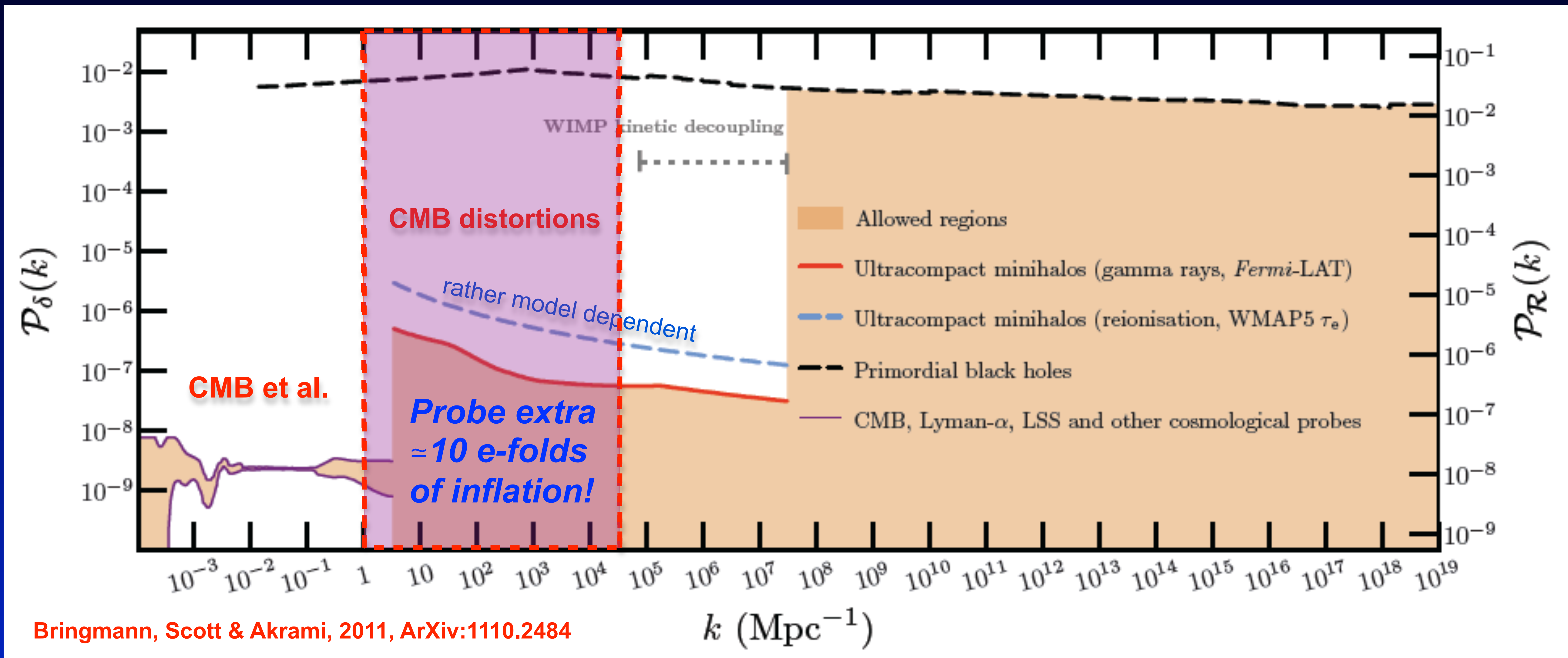


Bringmann, Scott & Akrami, 2011, ArXiv:1110.2484

- Amplitude of power spectrum rather uncertain at $k > 3$ Mpc^{-1}
- improved limits at smaller scales can *rule out* many *inflationary models*

e.g., JC, Khatri & Sunyaev, 2012; JC, Erickcek & Ben-Dayan, 2012; JC & Jeong, 2013

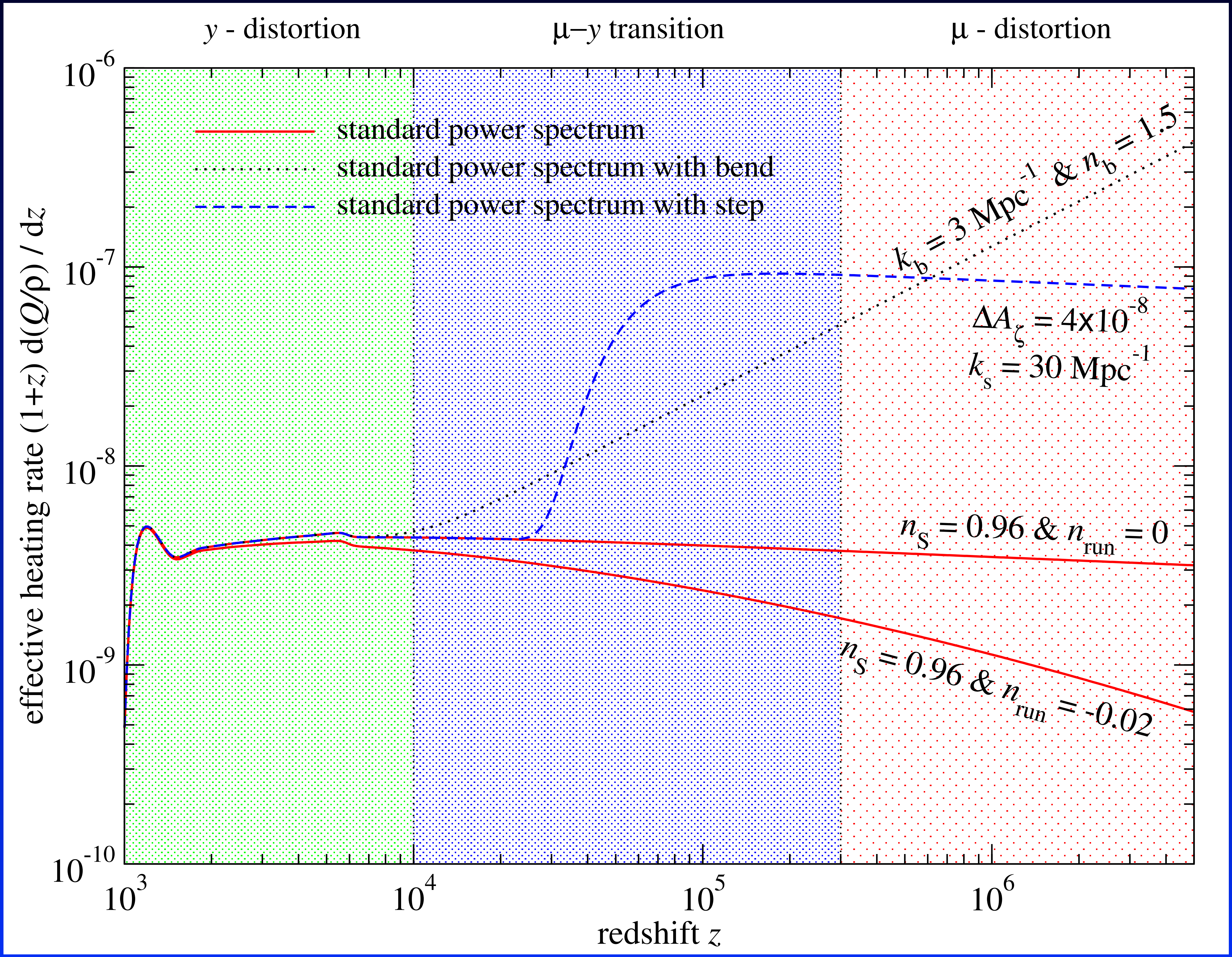
Distortions provide additional power spectrum constraints!



- Amplitude of power spectrum rather uncertain at $k > 3$ Mpc^{-1}
- improved limits at smaller scales can *rule out* many *inflationary models*
- CMB spectral distortions would *extend* our *lever arm* to $k \sim 10^4$ Mpc^{-1}
- very *complementary* piece of information about early-universe physics

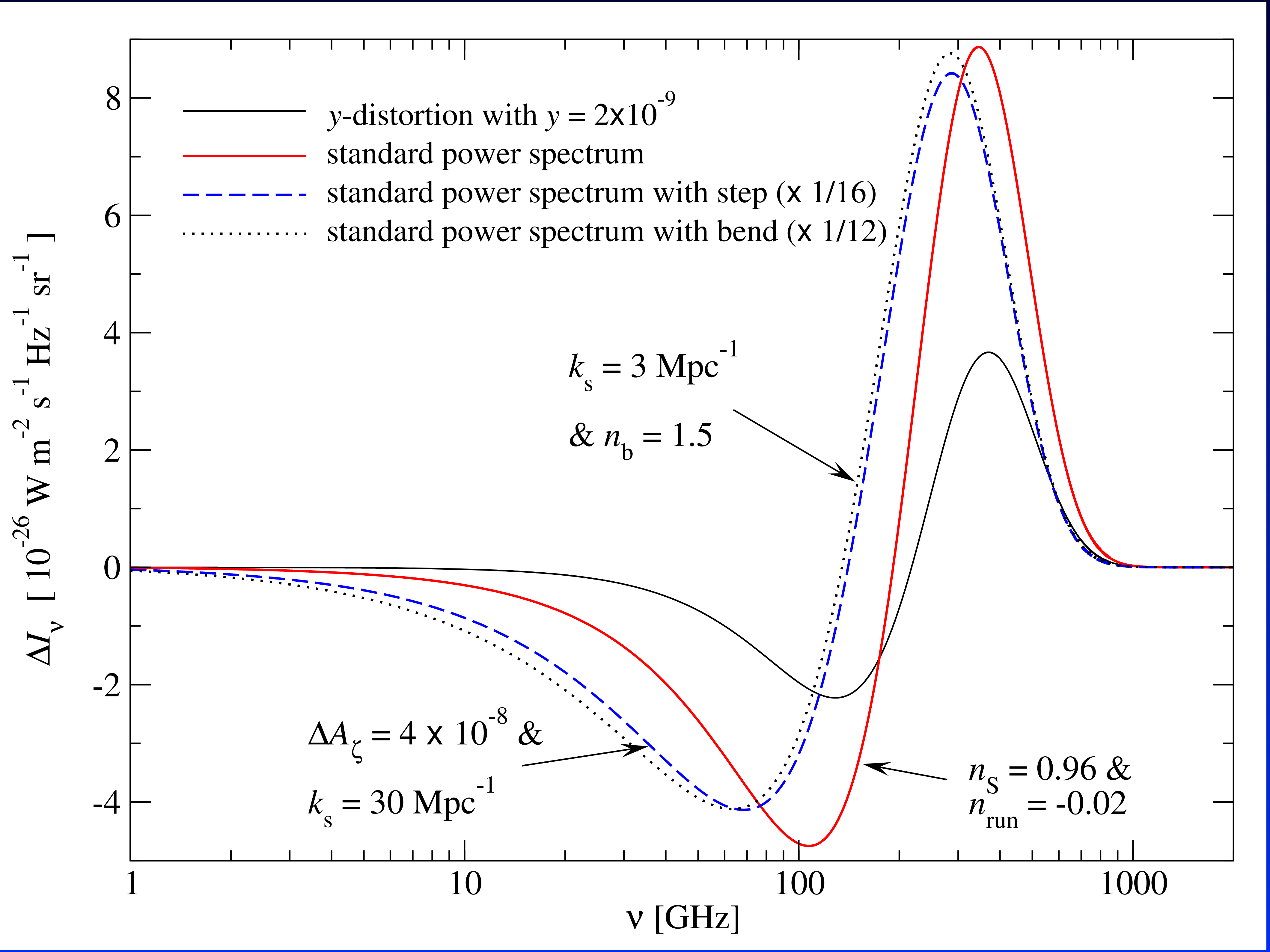
e.g., JC, Khatri & Sunyaev, 2012; JC, Erickcek & Ben-Dayan, 2012; JC & Jeong, 2013

Probing the small-scale power spectrum



JC, 2013, Arxiv:1304.6120

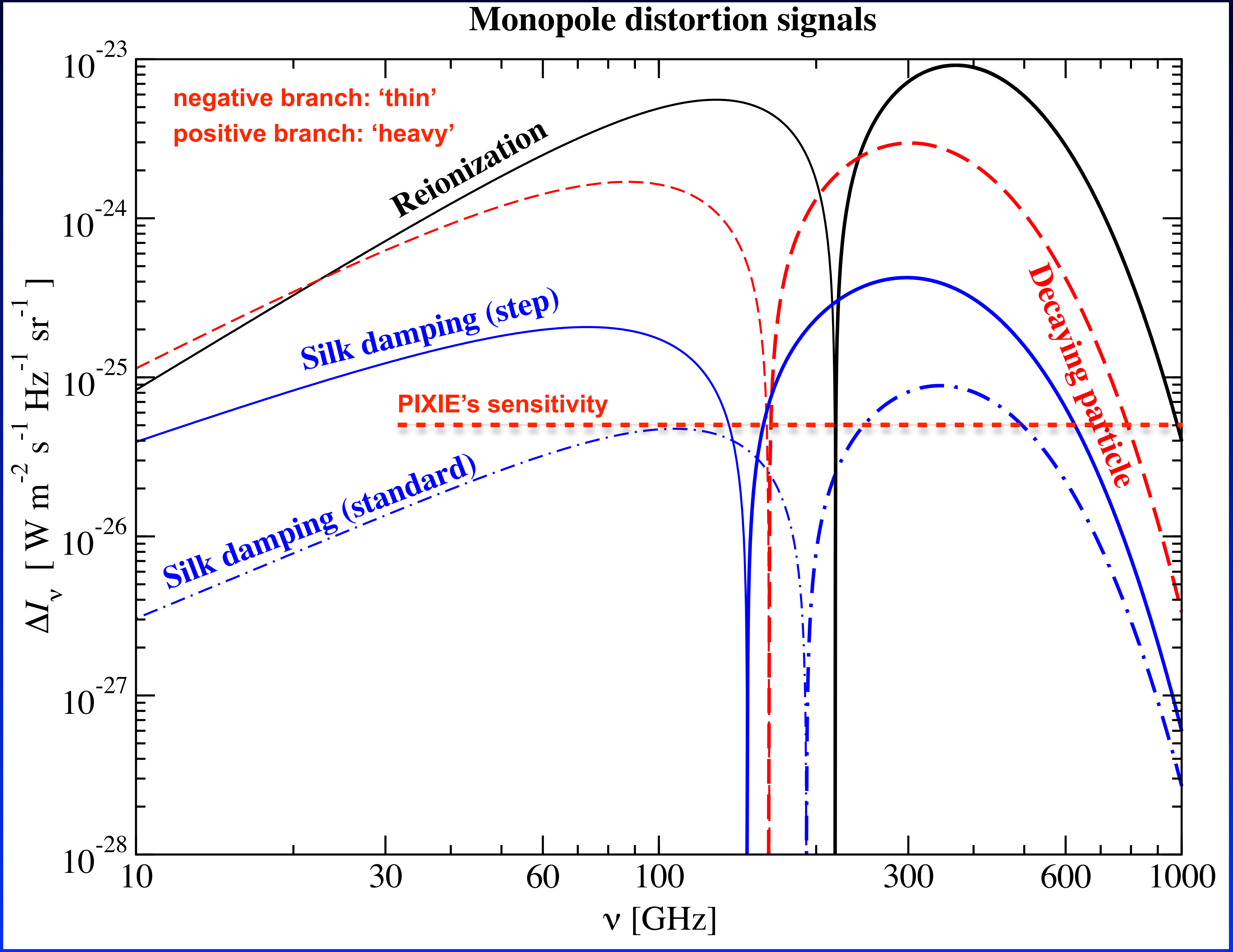
Probing the small-scale power spectrum



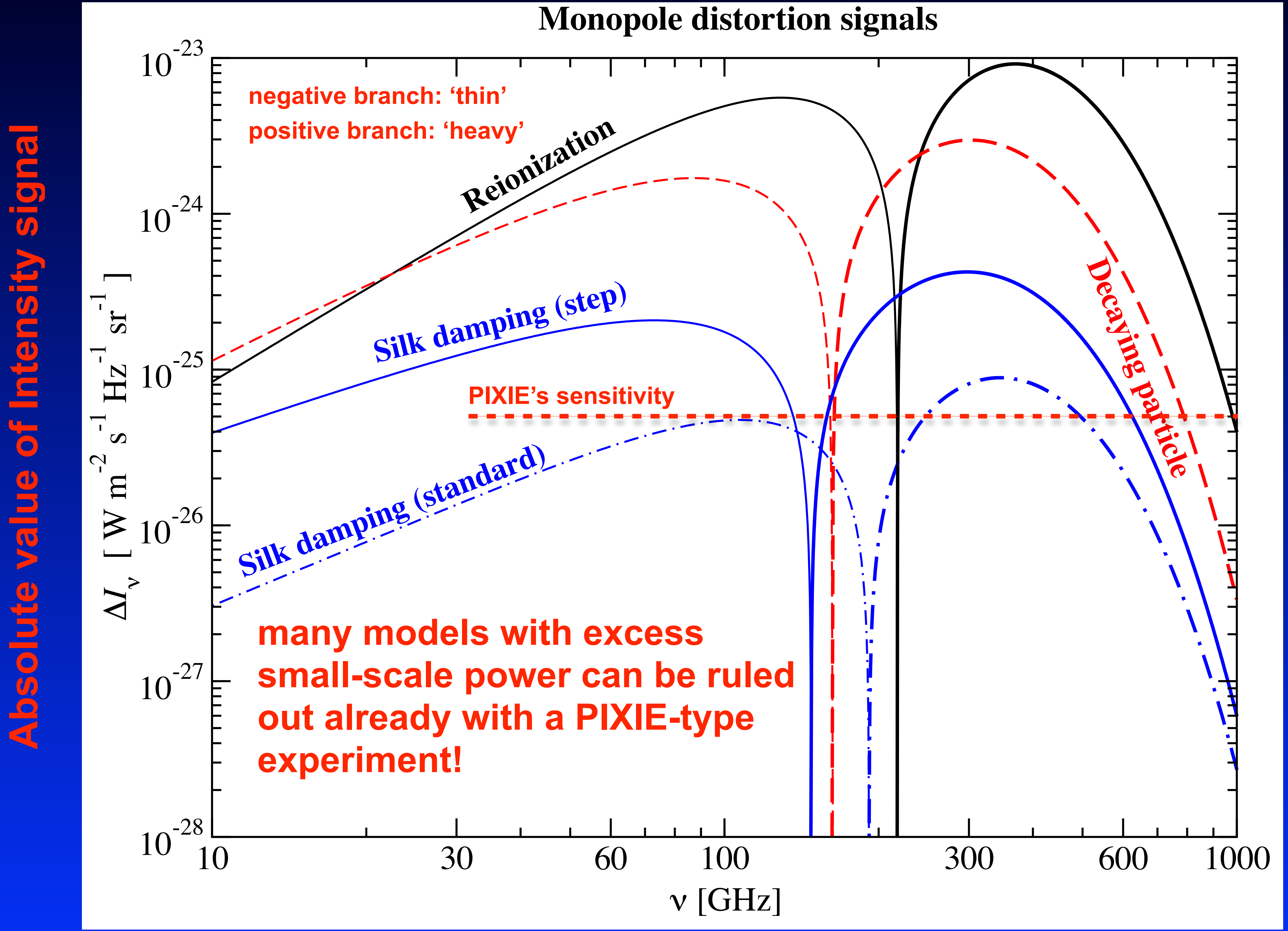
JC, 2013, Arxiv:1304.6120

Average CMB spectral distortions

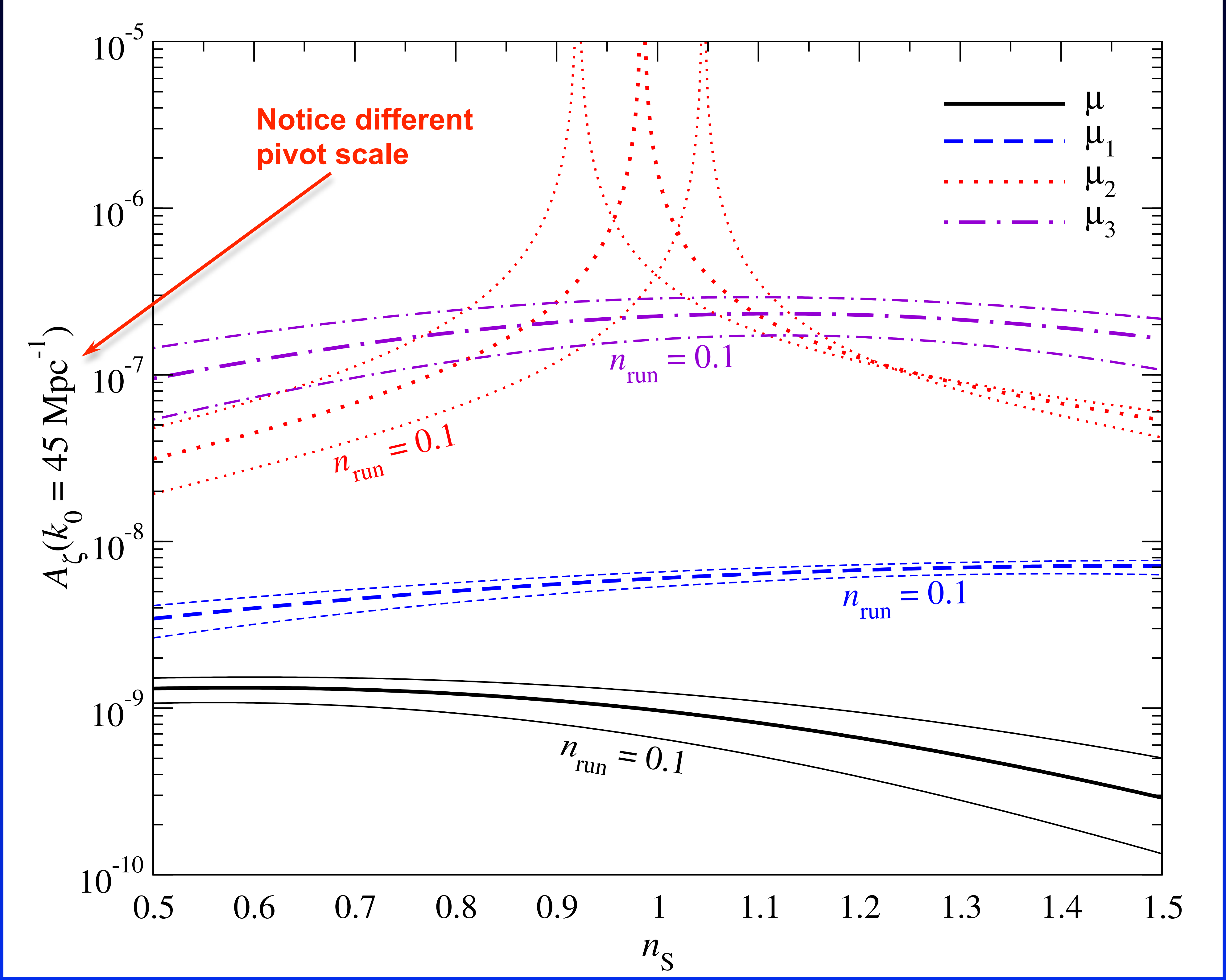
Absolute value of Intensity signal



Average CMB spectral distortions



Dissipation scenario: 1σ-detection limits for PIXIE



JC & Jeong, 2013

$$P_\zeta(k) = 2\pi^2 A_\zeta k^{-3} (k/k_0)^{n_S - 1 + \frac{1}{2} n_{run} \ln(k/k_0)}$$

The cosmological recombination radiation

Simple estimates for hydrogen recombination

Hydrogen recombination:

- per recombined hydrogen atom an energy of ~ 13.6 eV in form of photons is released
- at z ~ 1100 → $\Delta\varepsilon/\varepsilon \sim 13.6\ \text{eV}\ N_b / (N_\gamma\, 2.7kT_r) \sim 10^{-9} - 10^{-8}$

→ recombination occurs at redshifts $z < 10^4$

→ At that time the *thermalization* process doesn't work anymore!

→ There should be some *small* spectral distortion due to additional Ly-α and 2s-1s photons!
(Zeldovich, Kurt & Sunyaev, 1968, ZhETF, 55, 278; Peebles, 1968, ApJ, 153, 1)

→ In 1975 ***Viktor Dubrovich*** emphasized the possibility to observe the recombinational lines from n > 3 and Δn << n!

First recombination computations completed in 1968!

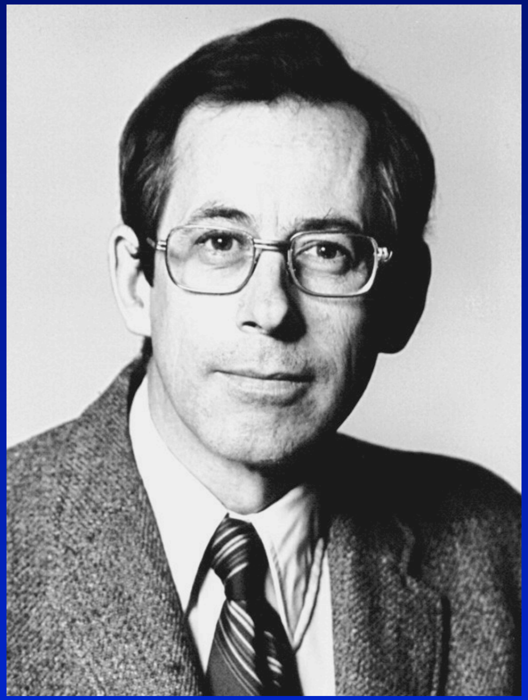
Yakov Zeldovich

Vladimir Kurt
(UV astronomer)

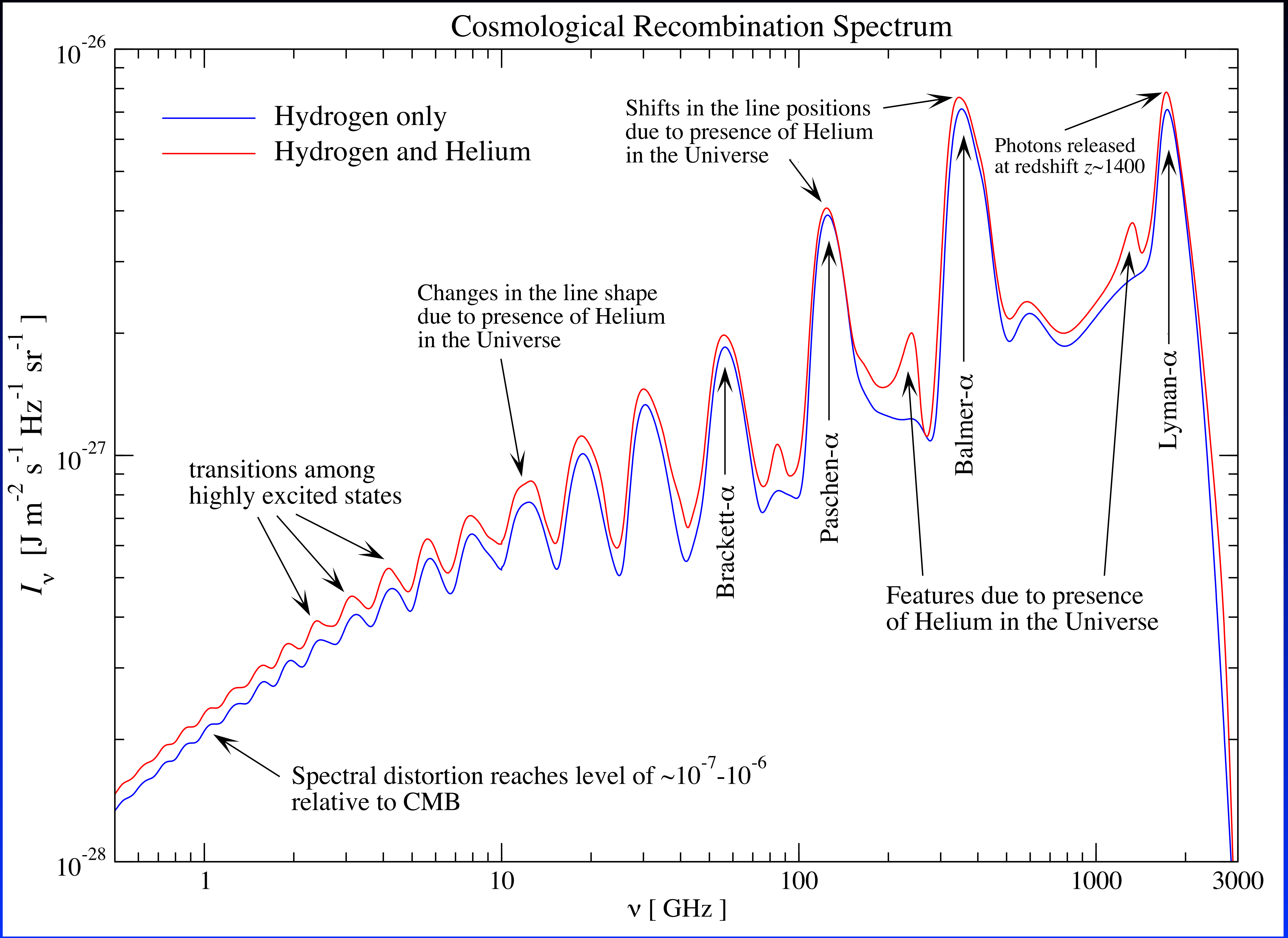
Moscow

Rashid Sunyaev

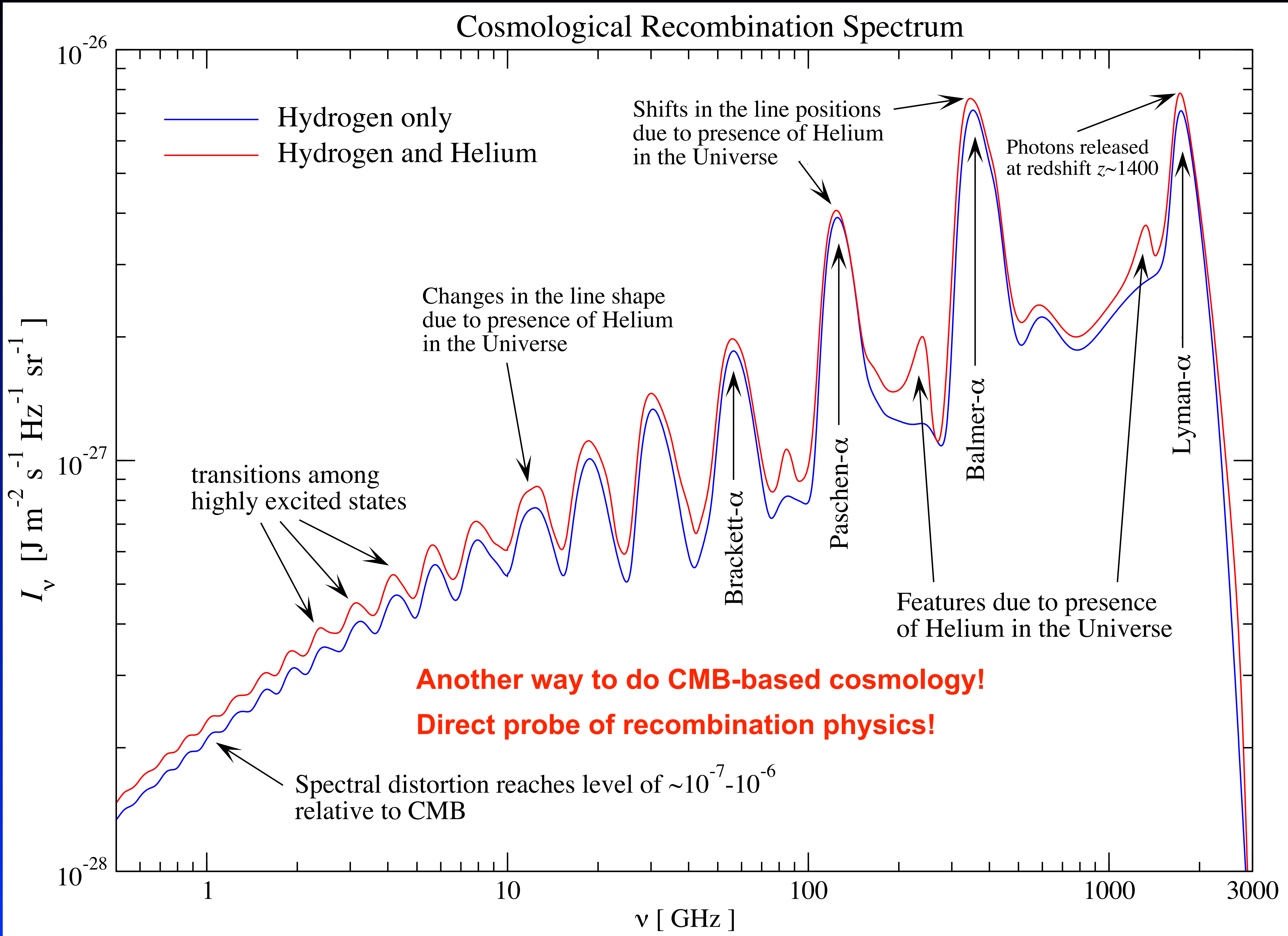
Princeton



Jim Peebles

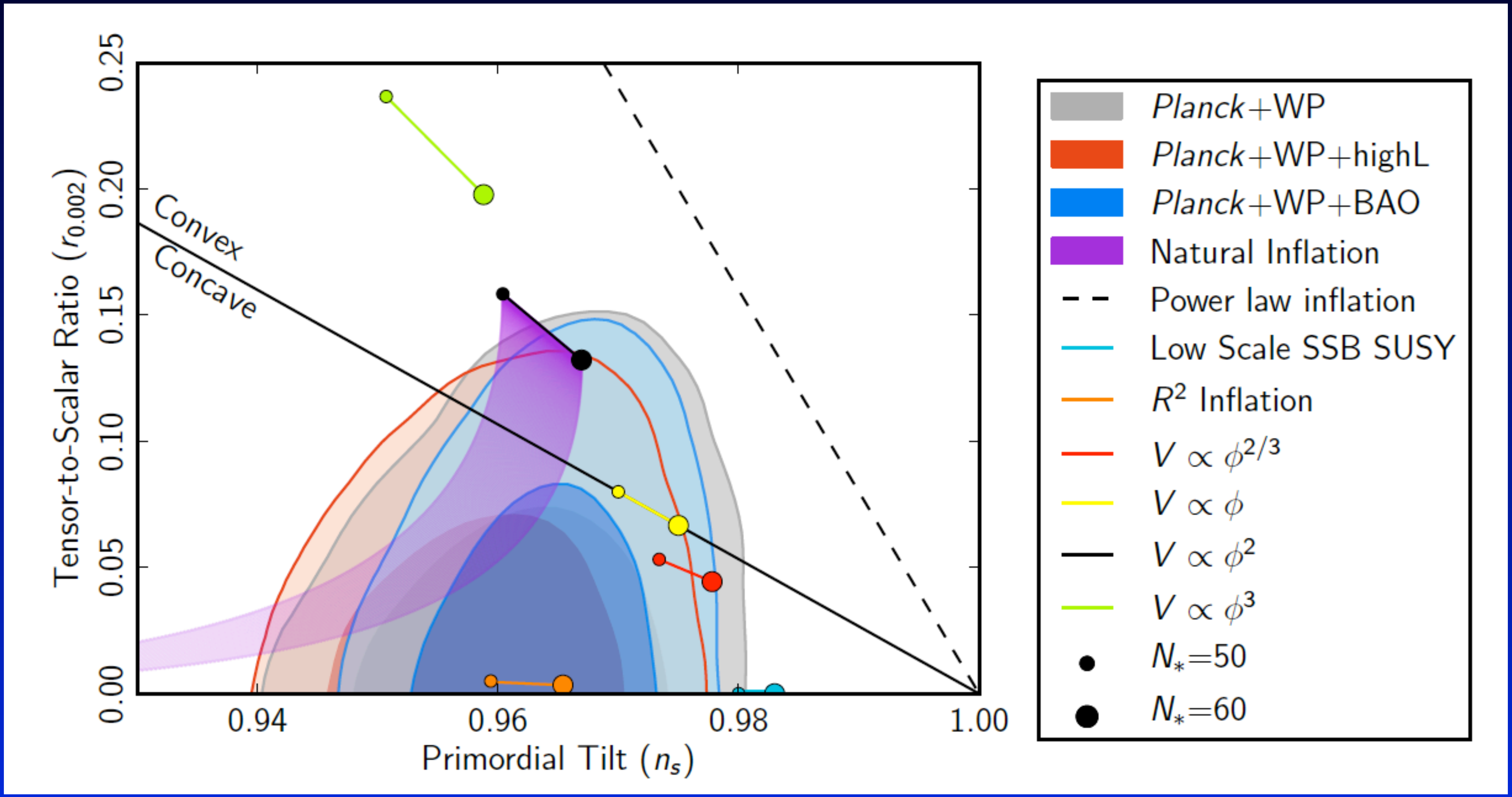


Cosmological Recombination Spectrum
Hydrogen only
Hydrogen and Helium
Shifts in the line positions due to presence of Helium in the Universe
Photons released at redshift z~1400
Changes in the line shape due to presence of Helium in the Universe
transitions among highly excited states
Brackett-α
Paschen-α
Balmer-α
Lyman-α
Features due to presence of Helium in the Universe
Spectral distortion reaches level of ~10^-7-10^-6 relative to CMB
I_ν [J m^-2 s^-1 Hz^-1 sr^-1]
ν [GHz]
10^-26
10^-27
10^-28
1
10
100
1000
3000



Cosmological Recombination Spectrum
Hydrogen only
Hydrogen and Helium
Shifts in the line positions due to presence of Helium in the Universe
Photons released at redshift z~1400
Changes in the line shape due to presence of Helium in the Universe
transitions among highly excited states
Brackett-α
Paschen-α
Balmer-α
Lyman-α
Features due to presence of Helium in the Universe
Another way to do CMB-based cosmology!
Direct probe of recombination physics!
Spectral distortion reaches level of ~10^-7-10^-6 relative to CMB
I_ν [J m^-2 s^-1 Hz^-1 sr^-1]
ν [GHz]
10^-26
10^-27
10^-28
1
10
100
1000
3000

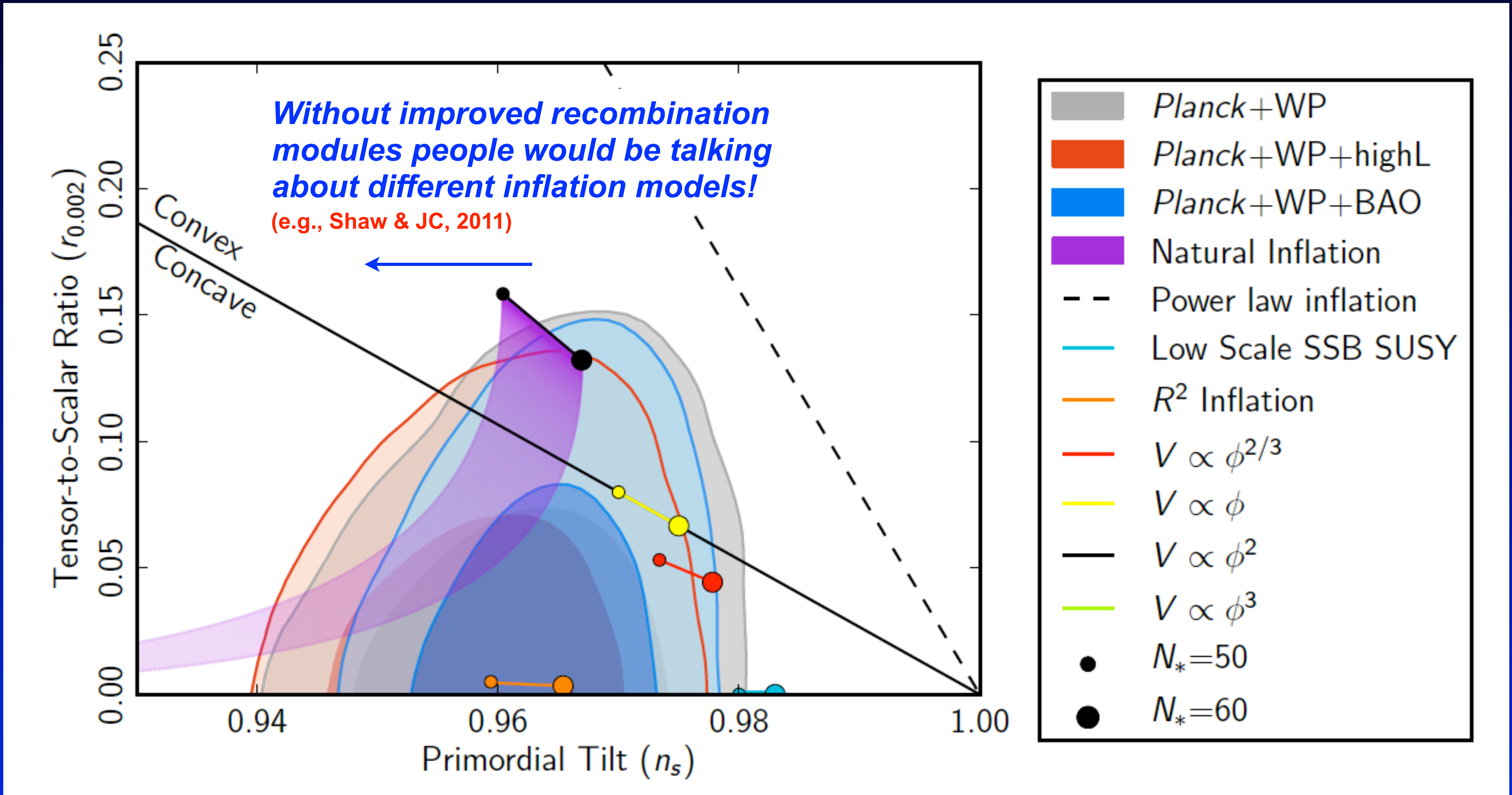
Importance of recombination for inflation constraints



Planck Collaboration, 2013, paper XXII

- Analysis uses refined recombination model (CosmoRec/HyRec)

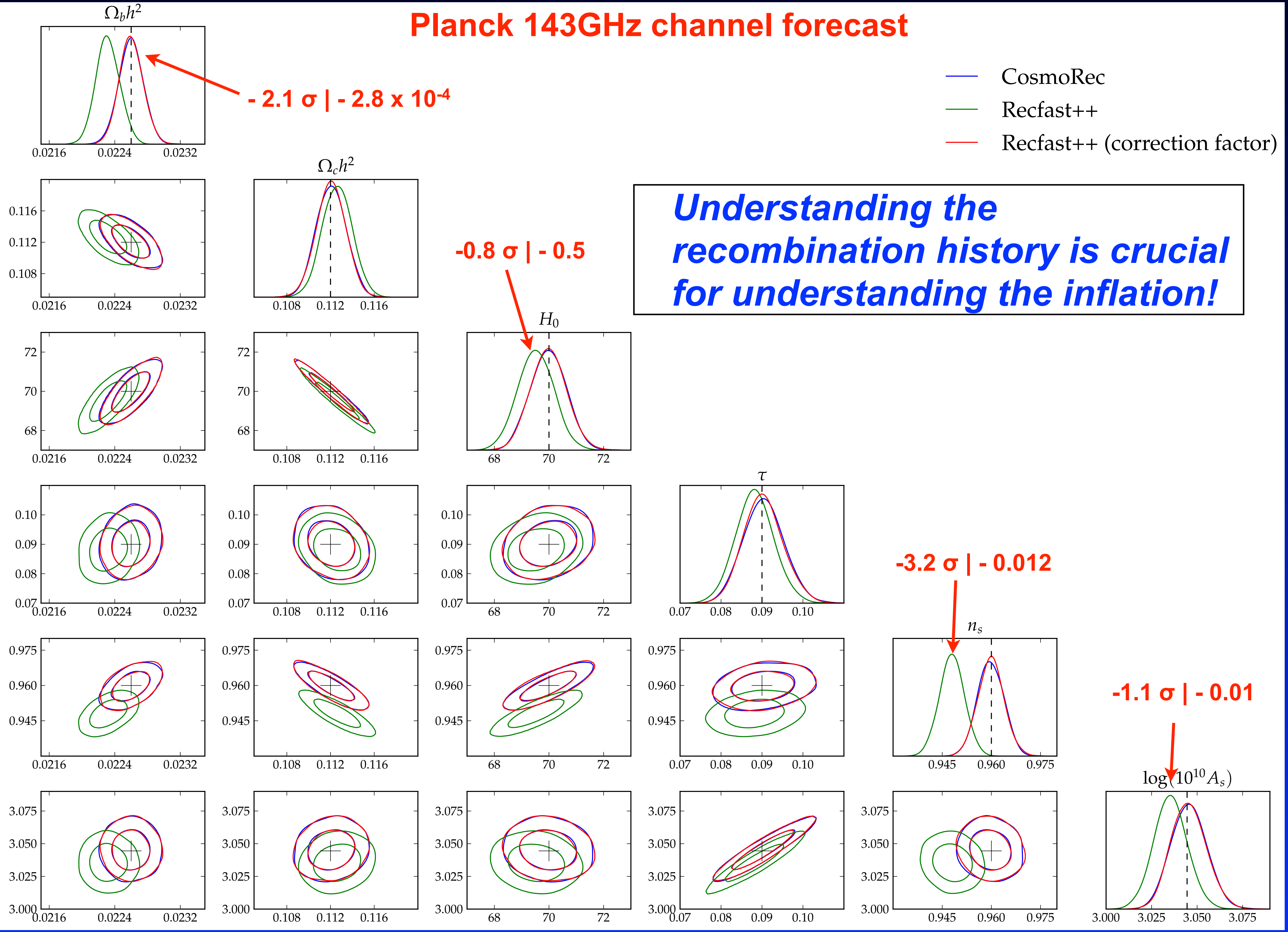
Importance of recombination for inflation constraints



Planck Collaboration, 2013, paper XXII

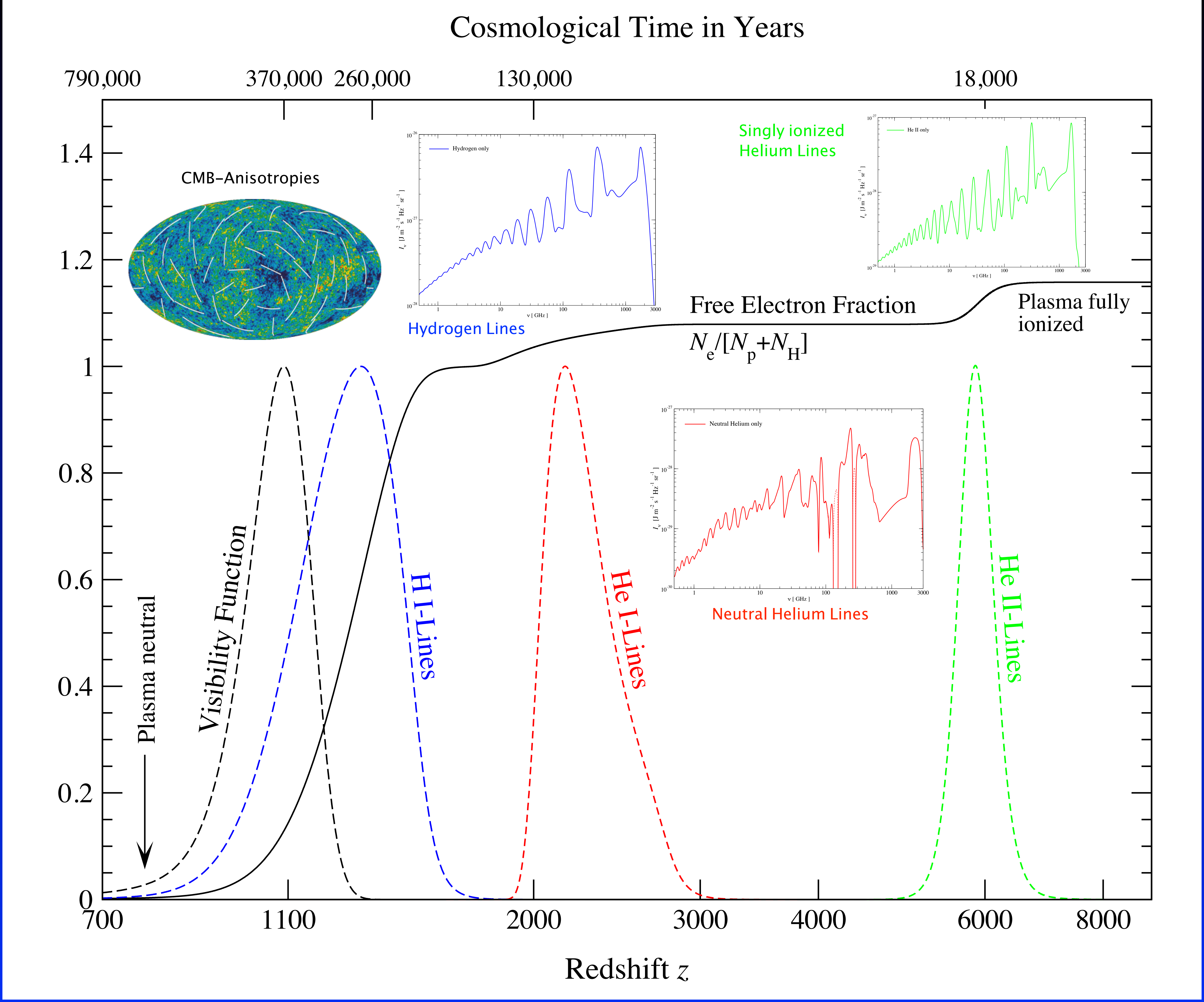
- Analysis uses refined recombination model (CosmoRec/HyRec)

Importance of recombination



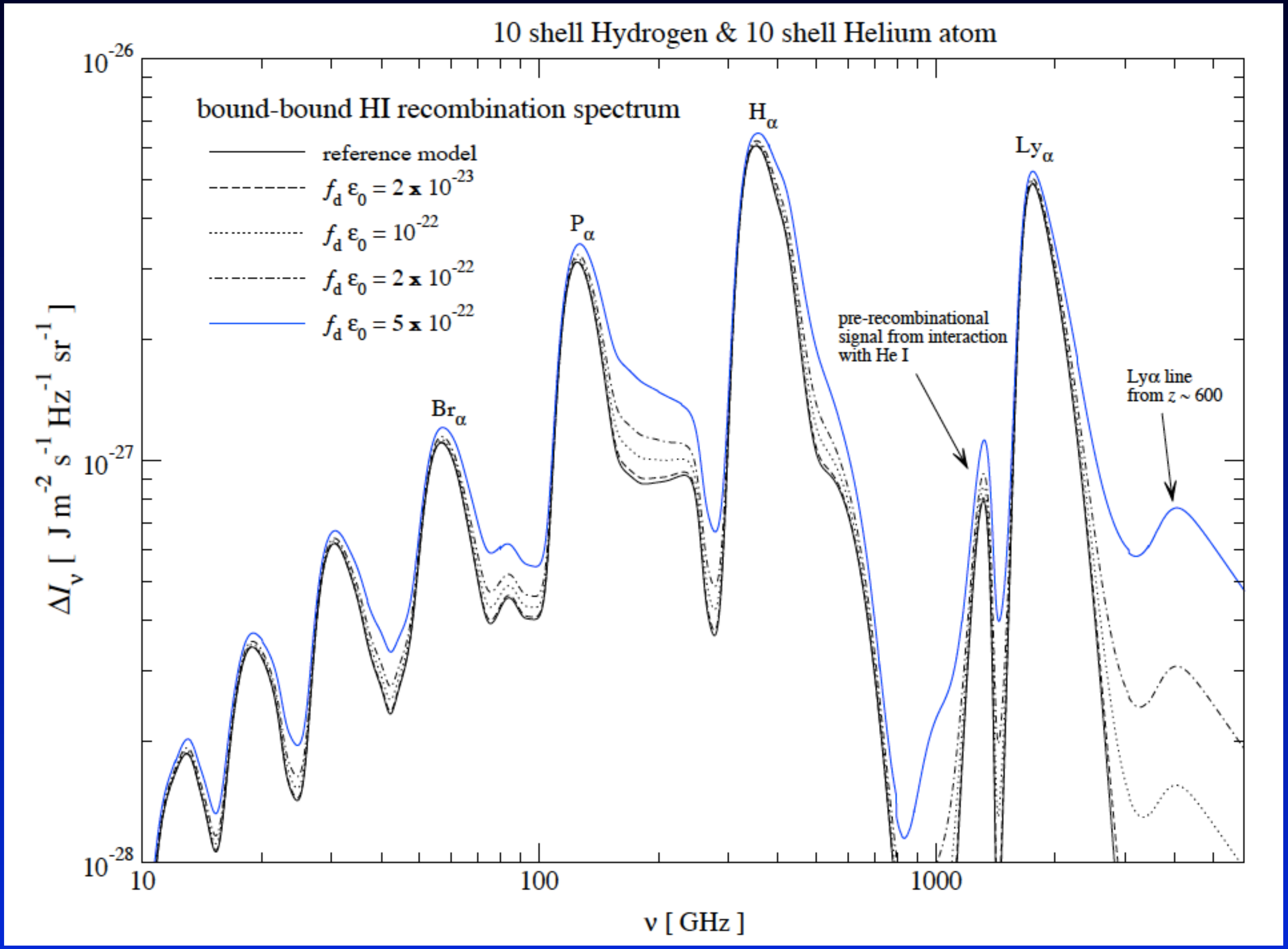
Understanding the recombination history is crucial for understanding the inflation!

Shaw & JC, 2011, and references therein



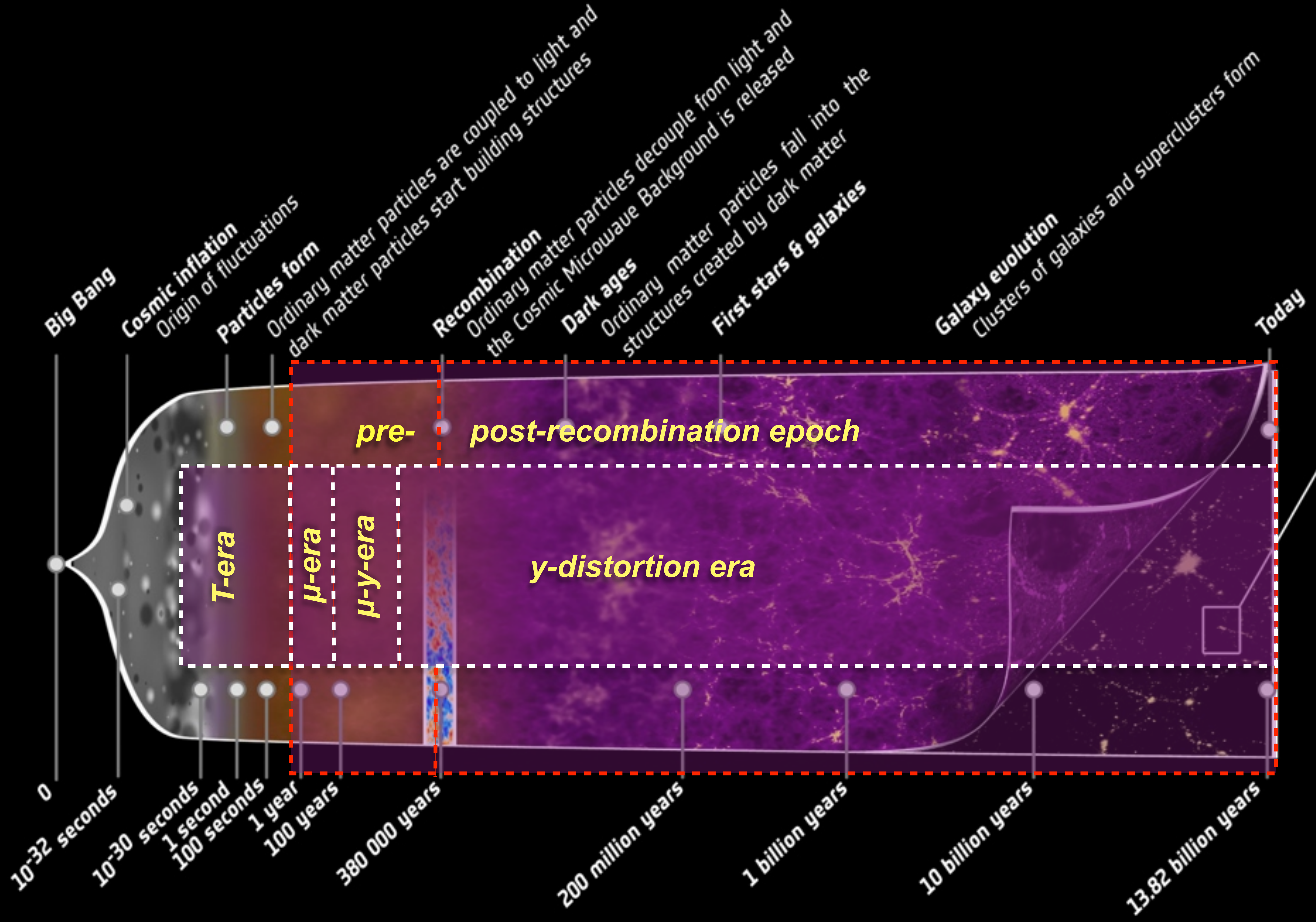
Cosmological Time in Years
790,000
370,000
260,000
130,000
18,000
CMB-Anisotropies
Hydrogen Lines
Singly ionized Helium Lines
Free Electron Fraction
$N_e/[N_p+N_H]$
Plasma fully ionized
Neutral Helium Lines
Visibility Function
Plasma neutral
H I-Lines
He I-Lines
He II-Lines
1.4
1.2
1
0.8
0.6
0.4
0.2
0
700
1100
2000
3000
4000
6000
8000
Redshift z

Dark matter annihilations / decays

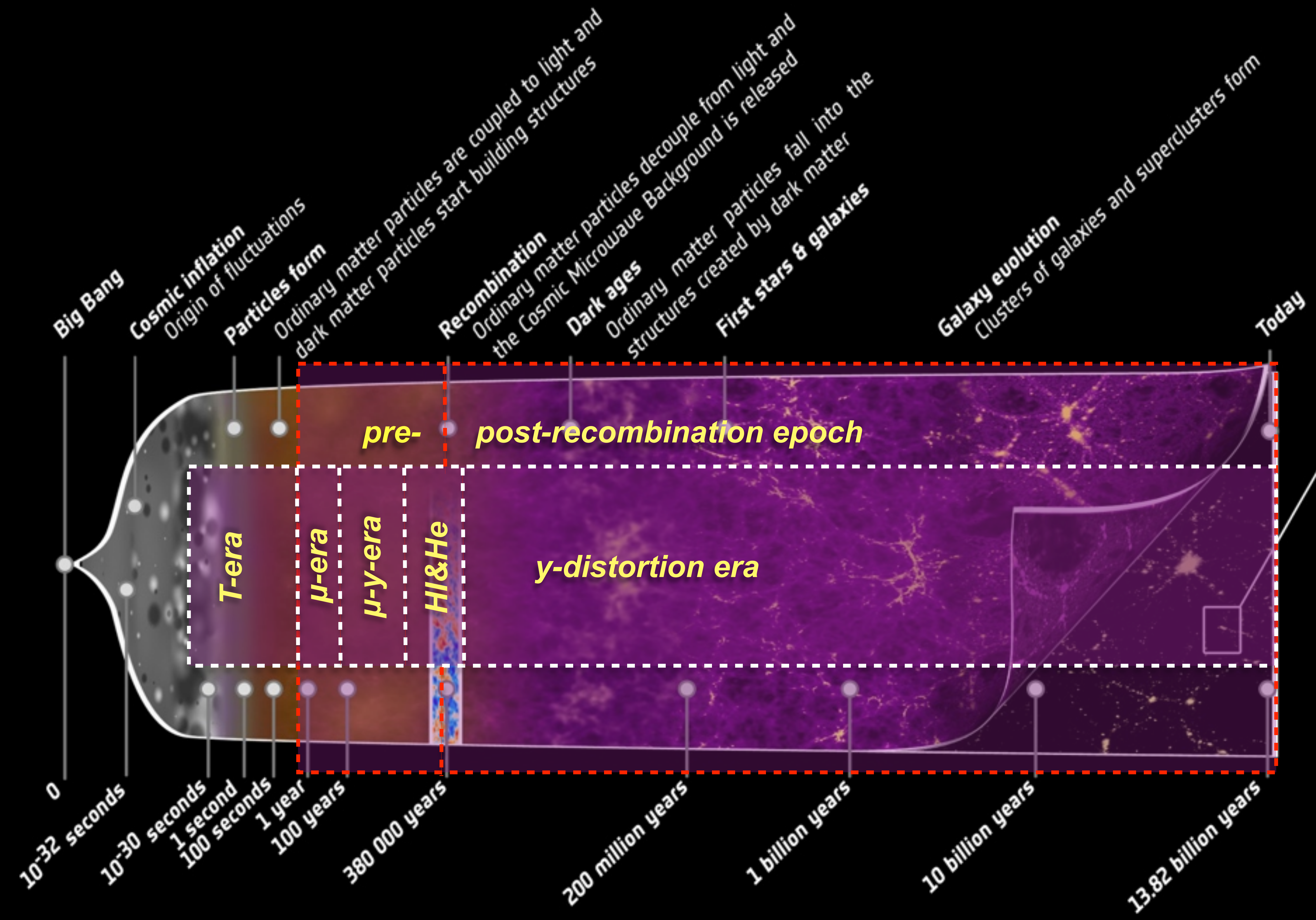


JC, 2009, arXiv:0910.3663

- Additional photons at all frequencies
- Broadening of spectral features
- Shifts in the positions

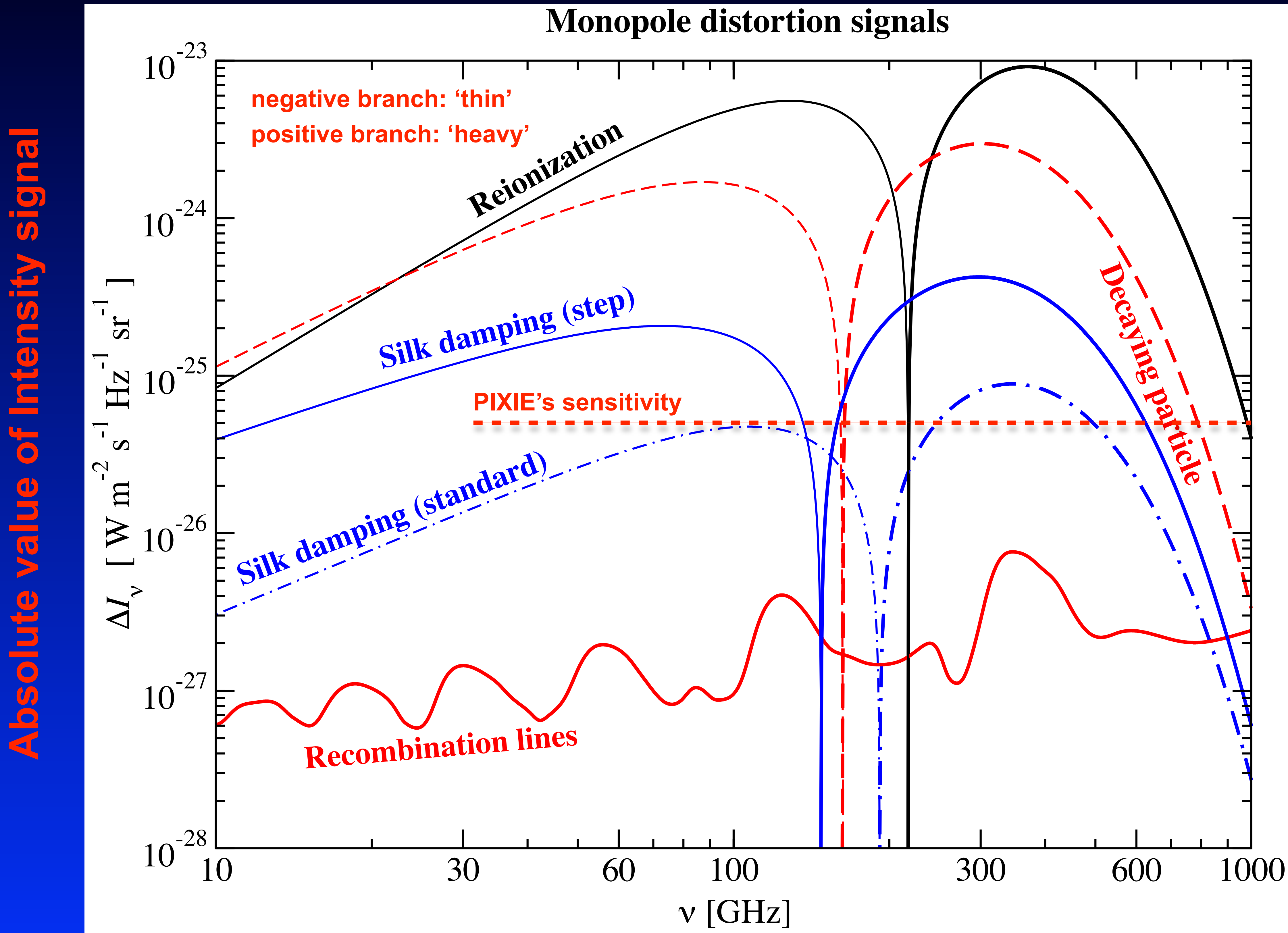


Big Bang
Cosmic inflation
Origin of fluctuations
Particles form
Ordinary matter particles are coupled to light and
dark matter particles start building structures
Recombination
Ordinary matter particles decouple from light and
the Cosmic Microwave Background is released
Dark ages
Ordinary matter particles fall into the
structures created by dark matter
First stars & galaxies
Galaxy evolution
Clusters of galaxies and superclusters form
Today
pre-
post-recombination epoch
T-era
μ-era
μ-y-era
y-distortion era
0
10-32 seconds
10-30 seconds
1 second
100 seconds
1 year
100 years
380 000 years
200 million years
1 billion years
10 billion years
13.82 billion years

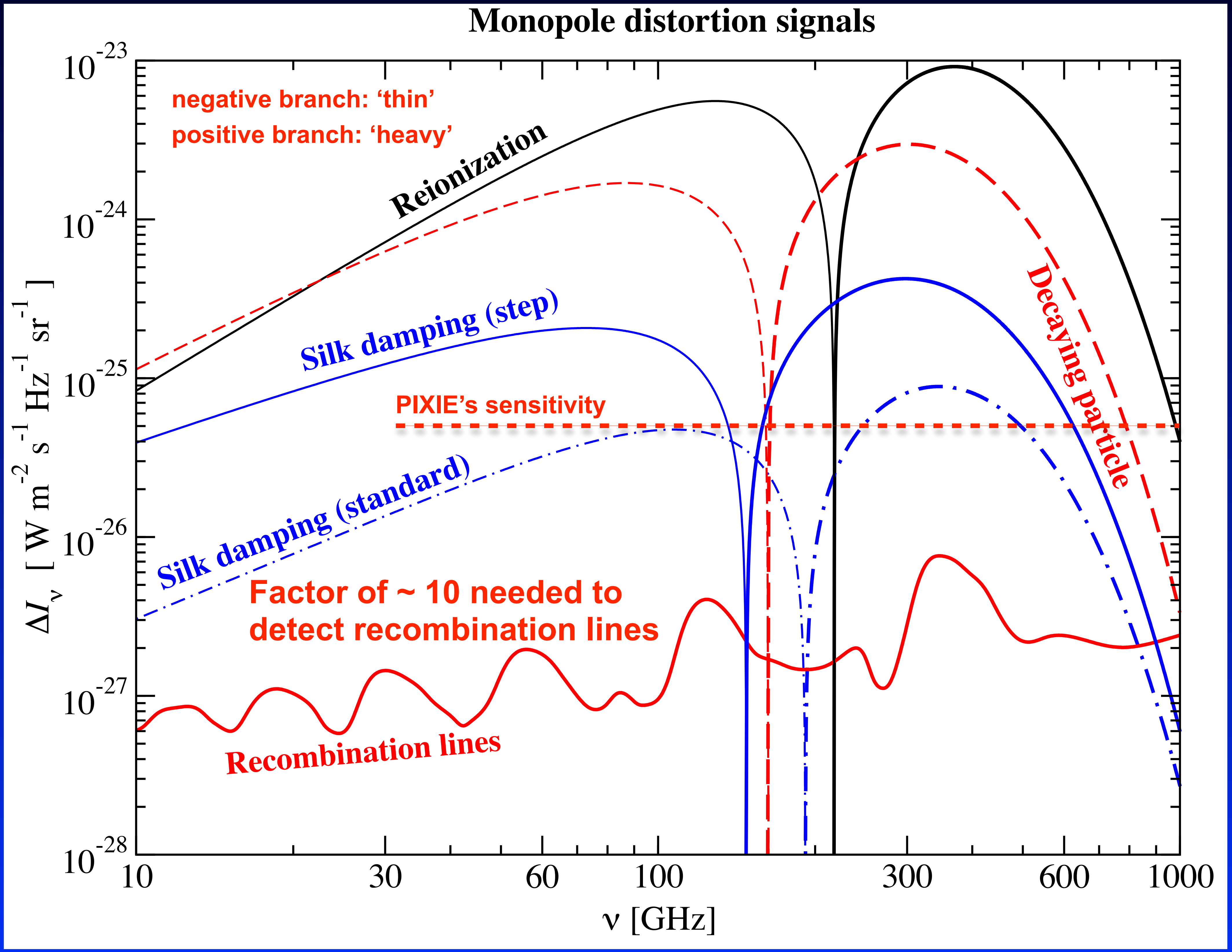


Big Bang
Cosmic inflation
Origin of fluctuations
Particles form
Ordinary matter particles are coupled to light and dark matter particles start building structures
Recombination
Ordinary matter particles decouple from light and the Cosmic Microwave Background is released
Dark ages
Ordinary matter particles fall into the structures created by dark matter
First stars & galaxies
Galaxy evolution
Clusters of galaxies and superclusters form
Today
pre-
post-recombination epoch
T-era
μ-era
μ-y-era
HI&He
y-distortion era
0
10-32 seconds
10-30 seconds
1 second
100 seconds
1 year
100 years
380 000 years
200 million years
1 billion years
10 billion years
13.82 billion years

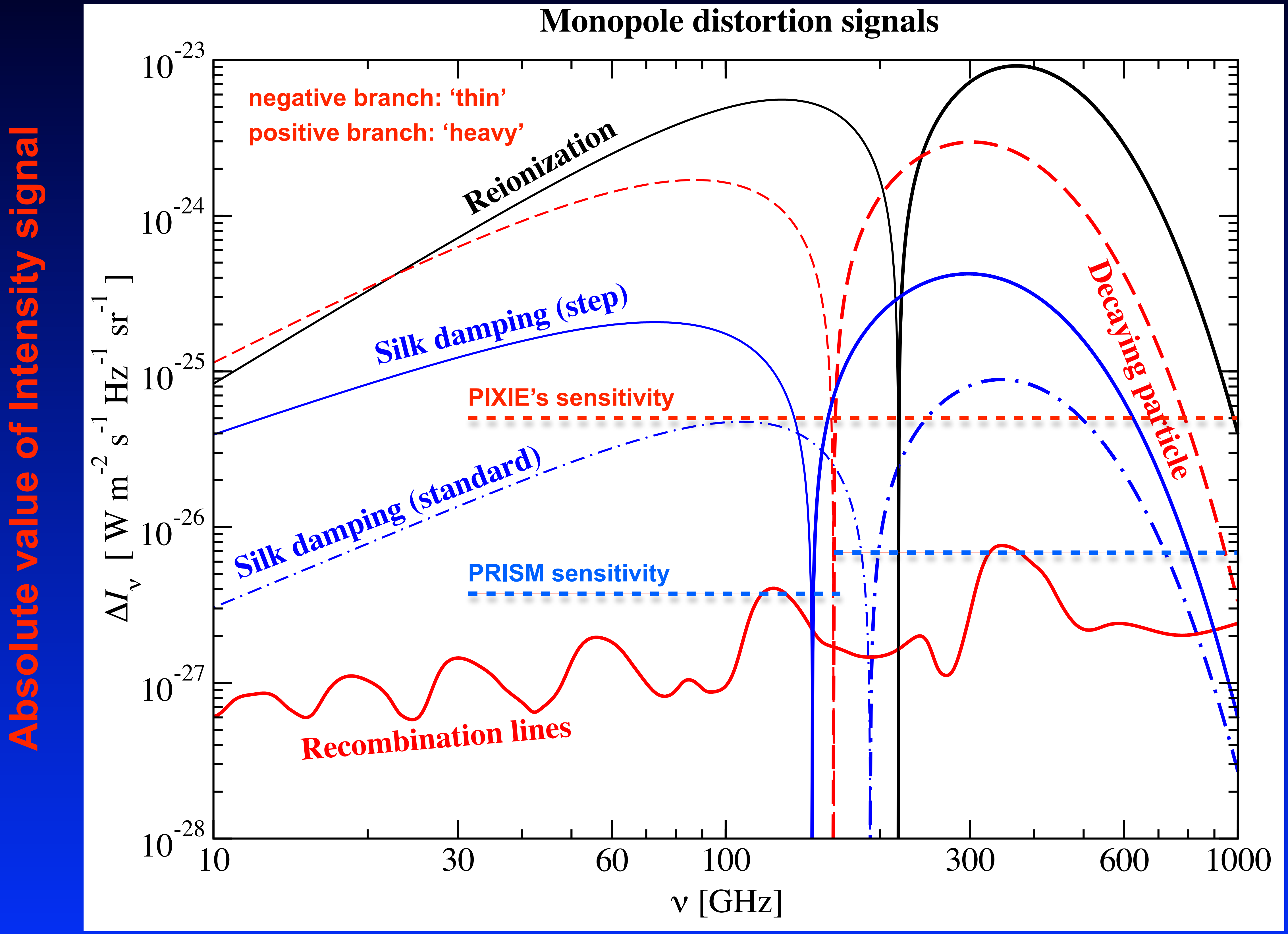
Average CMB spectral distortions



Average CMB spectral distortions



Average CMB spectral distortions



What would we actually learn by doing such hard job?

Cosmological Recombination Spectrum opens a way to measure:

→ **the specific *entropy* of our universe (related to $\Omega_b h^2$)**

→ **the CMB *monopole* temperature T_0**

→ ***the pre-stellar abundance of helium Y_p***

→ ***If recombination occurs as we think it does, then the lines can be predicted with very high accuracy!***

→ ***In principle allows us to directly check our understanding of the standard recombination physics***

If something unexpected or non-standard happened:

→ ***non-standard thermal histories should leave some measurable traces***

→ ***direct way to measure/reconstruct the recombination history!***

→ **possibility to distinguish *pre-* and *post-recombination y*-type distortions**

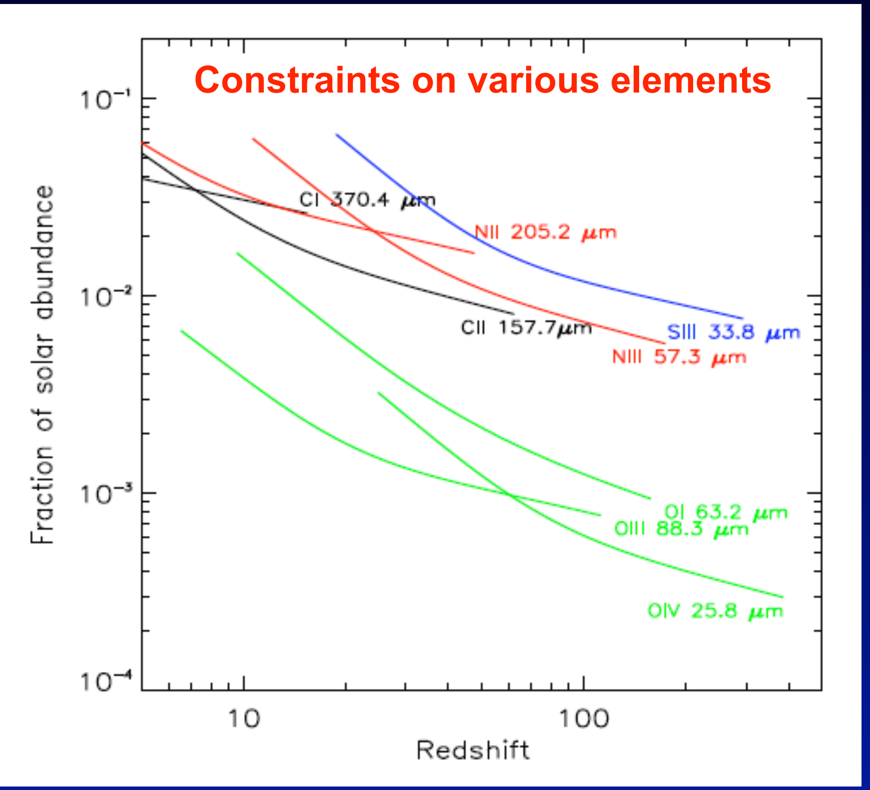
→ ***sensitive to energy release during recombination***

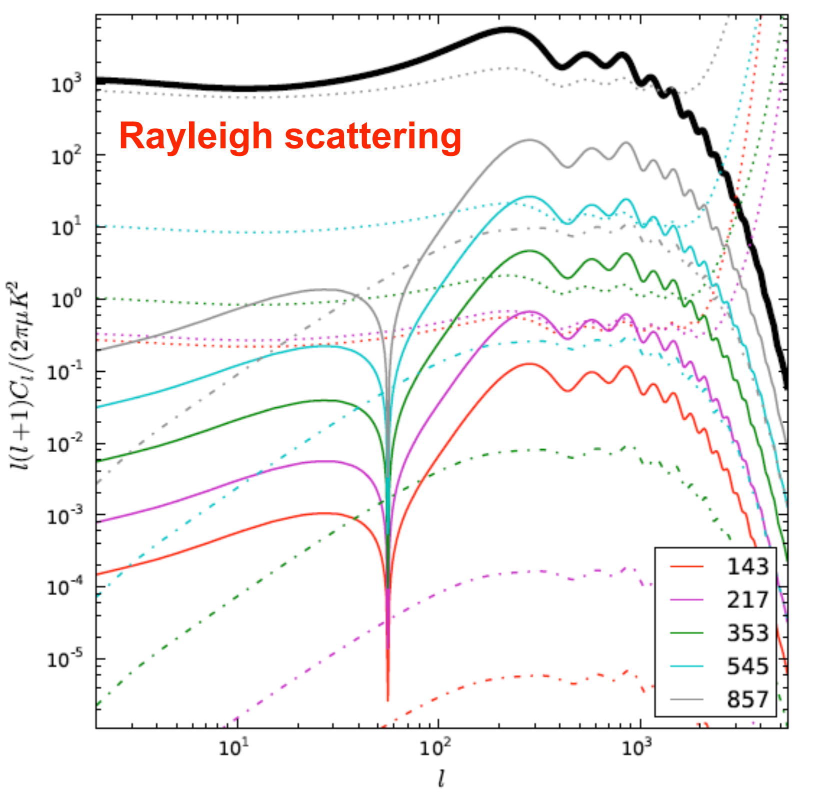
→ **variation of fundamental constants**

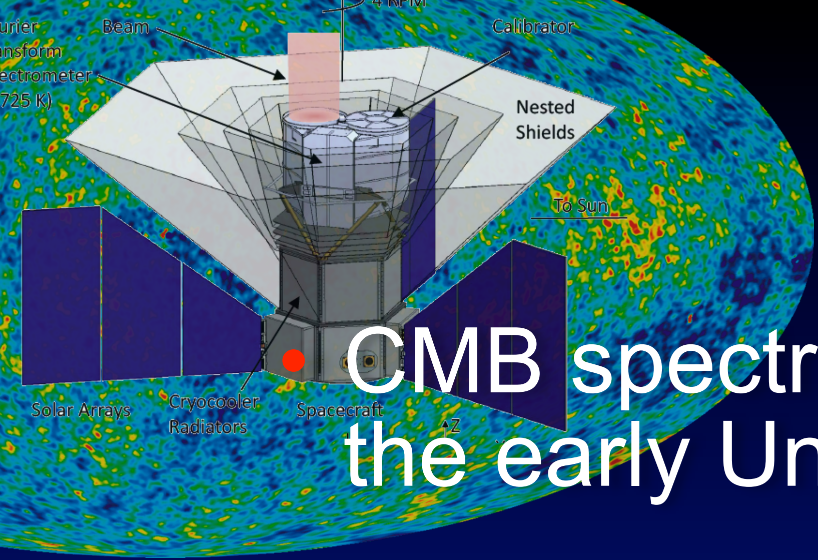
Other extremely interesting new signals

- Scattering signals from the dark ages
  (e.g., Basu et al., 2004; Hernandez-Monteagudo et al., 2007; Schleicher et al., 2009)
  - constrain abundances of chemical elements at high redshift
  - learn about star formation history
- Rayleigh / HI scattering signals
  (e.g., Yu et al., 2001; Rubino-Martin et al., 2005; Lewis 2013)
  - provides way to constrain recombination history
  - important when asking questions about N_{eff} and Y_{p}
- Free-free signals from reionization
  (e.g., Burigana et al. 1995; Trombetti & Burigana, 2013)
  - constrains reionization history
  - depends on clumpiness of the medium

All these effects give spectral-spatial signals, and an absolute spectrometer will help with channel cross calibration!

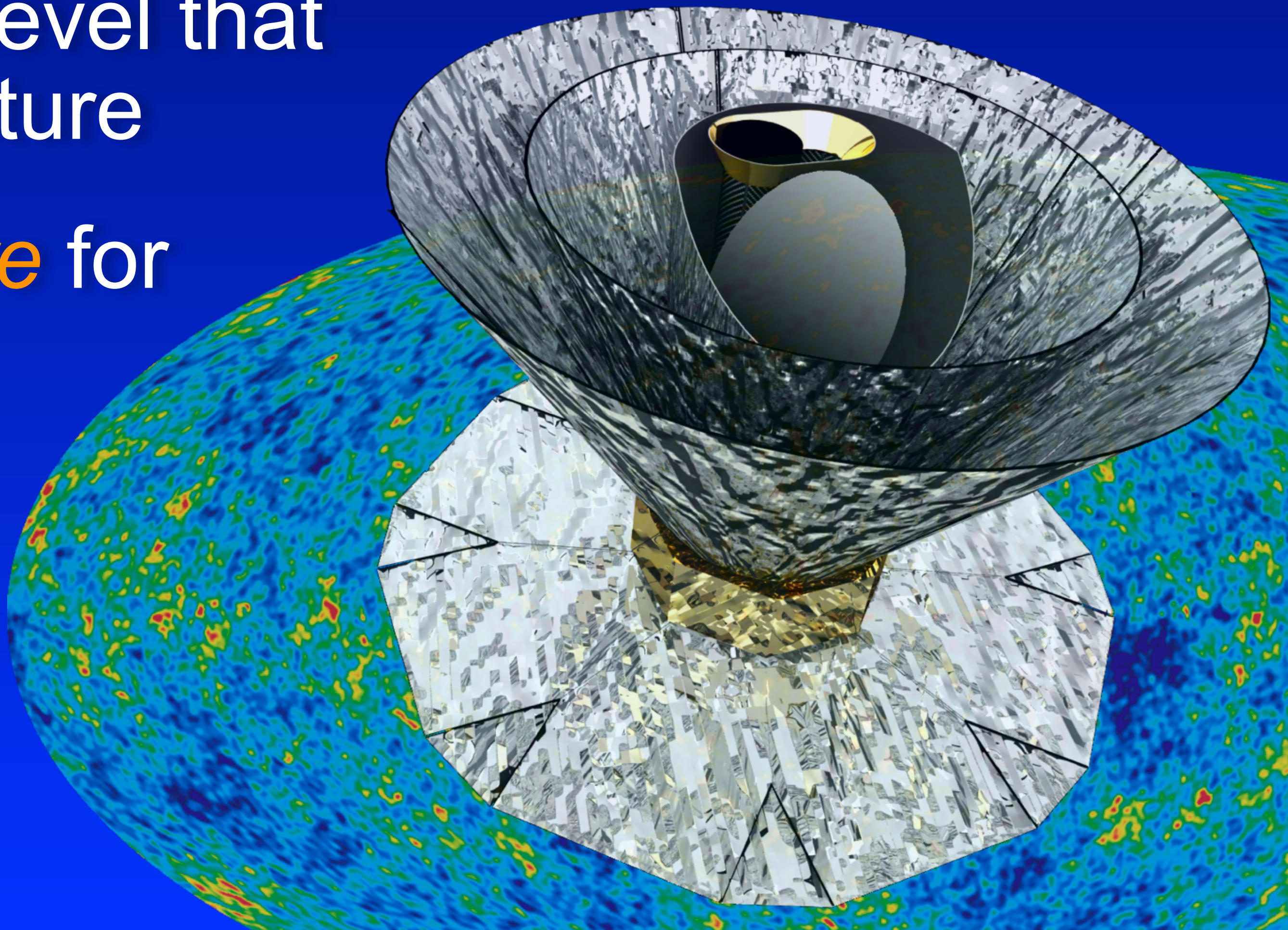


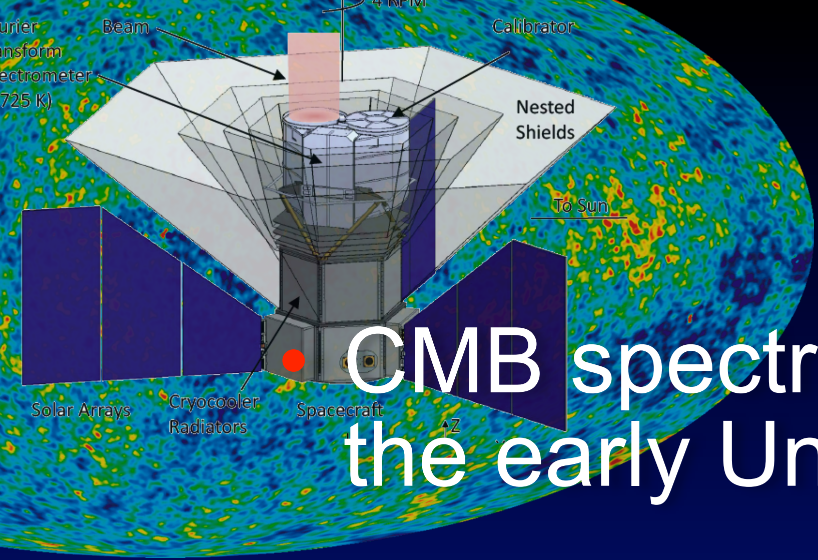




Conclusions

- CMB spectral distortions *will* open a *new window* to the early Universe
- new probe of the *inflation epoch* and *particle physics*
- *complementary* and *independent* source of information *not* just confirmation
- in *standard cosmology* several processes lead to *early energy release* at a level that will be detectable in the future
- extremely interesting *future* for CMB-based science!

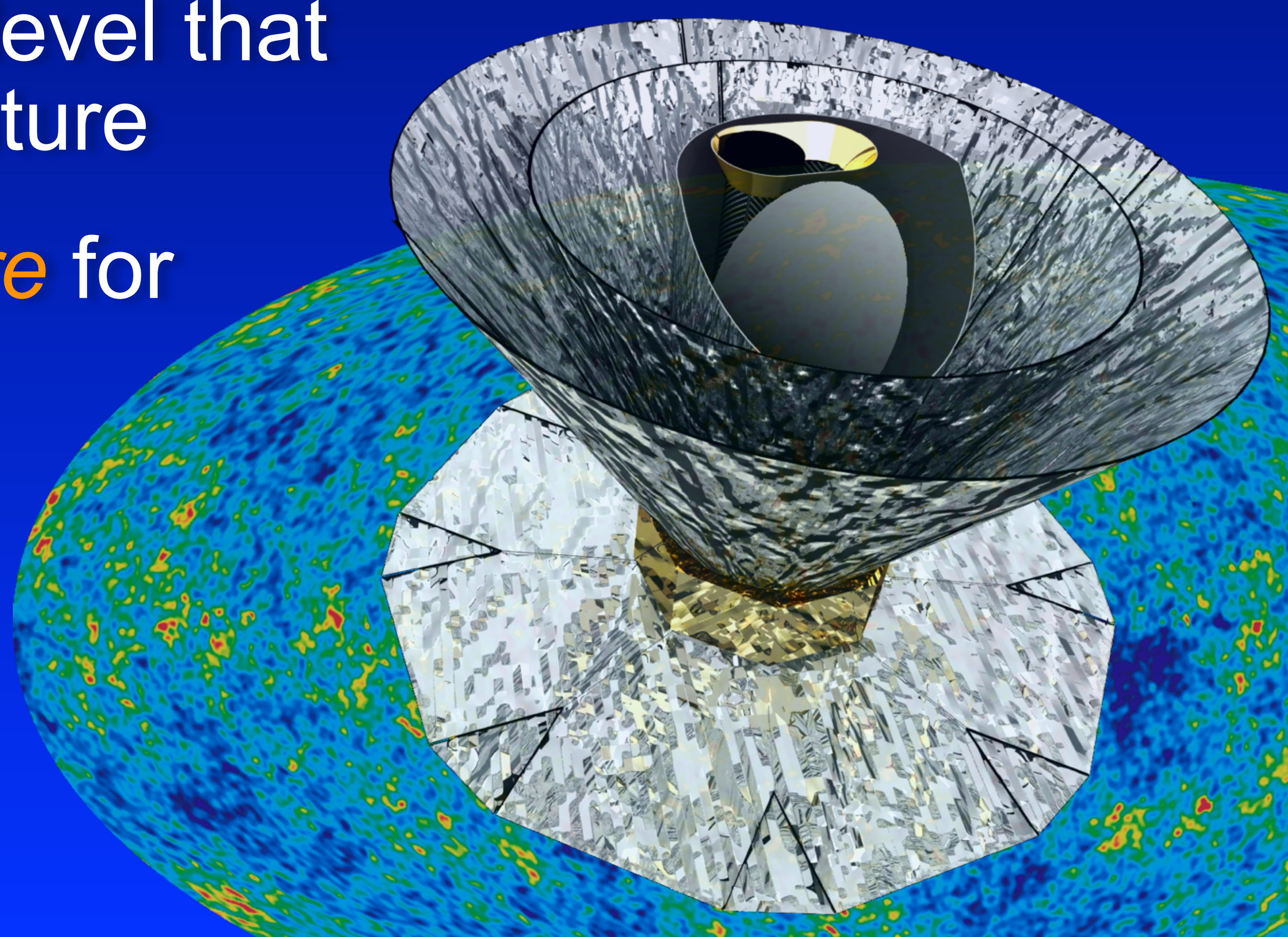


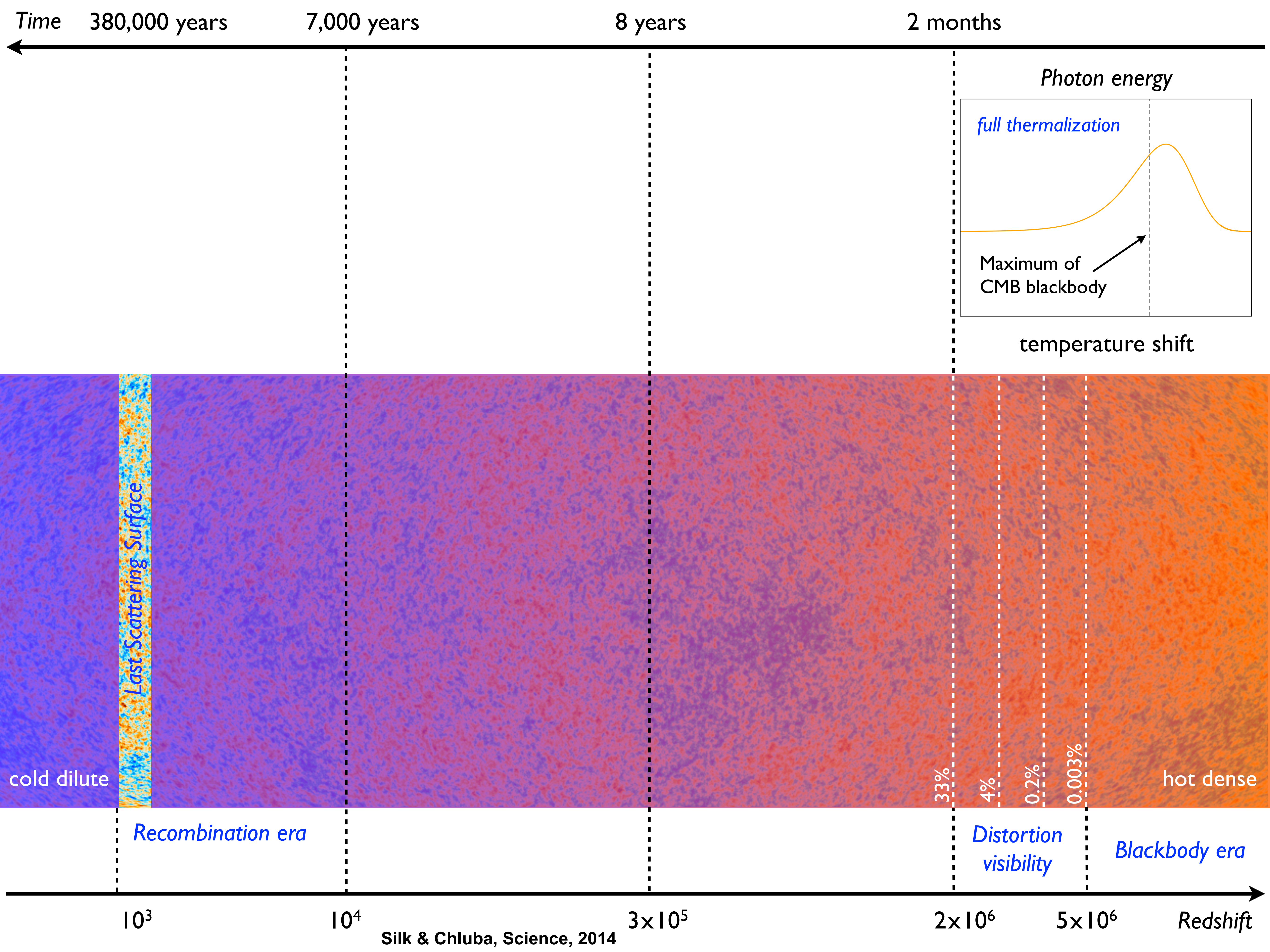


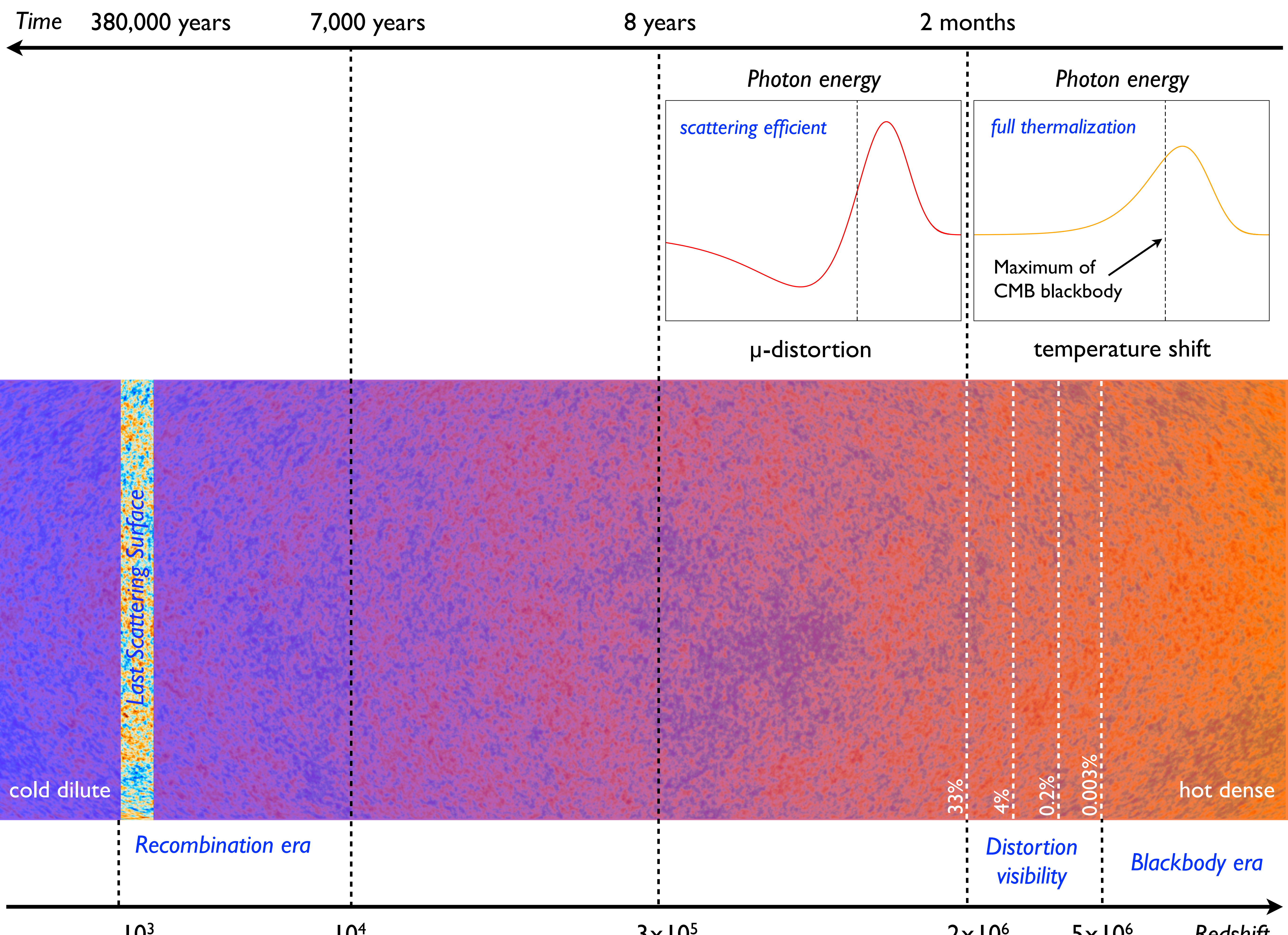
Conclusions

- CMB spectral distortions *will* open a *new window* to the early Universe
- new probe of the *inflation epoch* and *particle physics*
- *complementary* and *independent* source of information *not* just confirmation
- in *standard cosmology* several processes lead to *early energy release* at a level that will be detectable in the future
- extremely interesting *future* for CMB-based science!

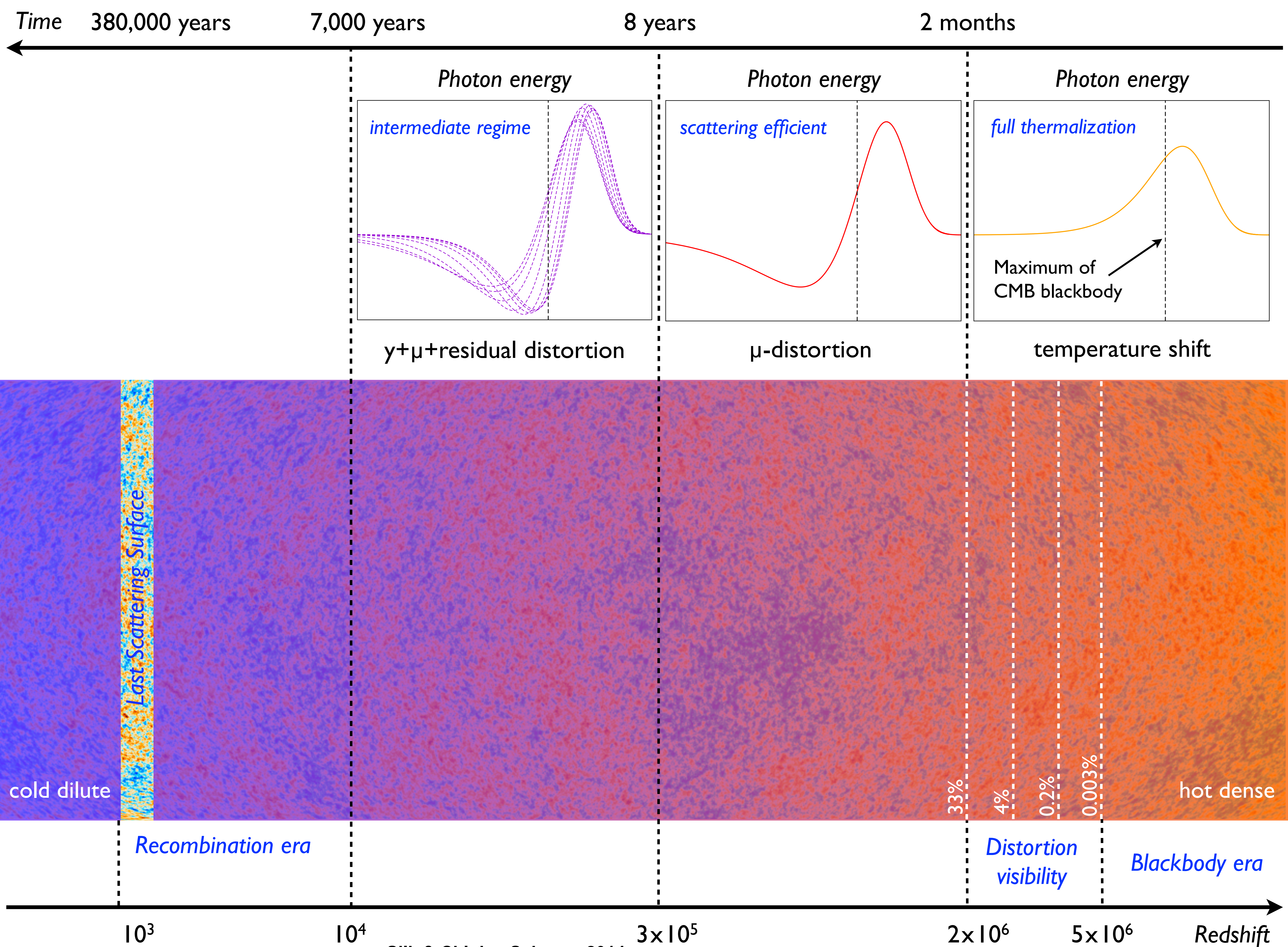
We should make use of all this information!



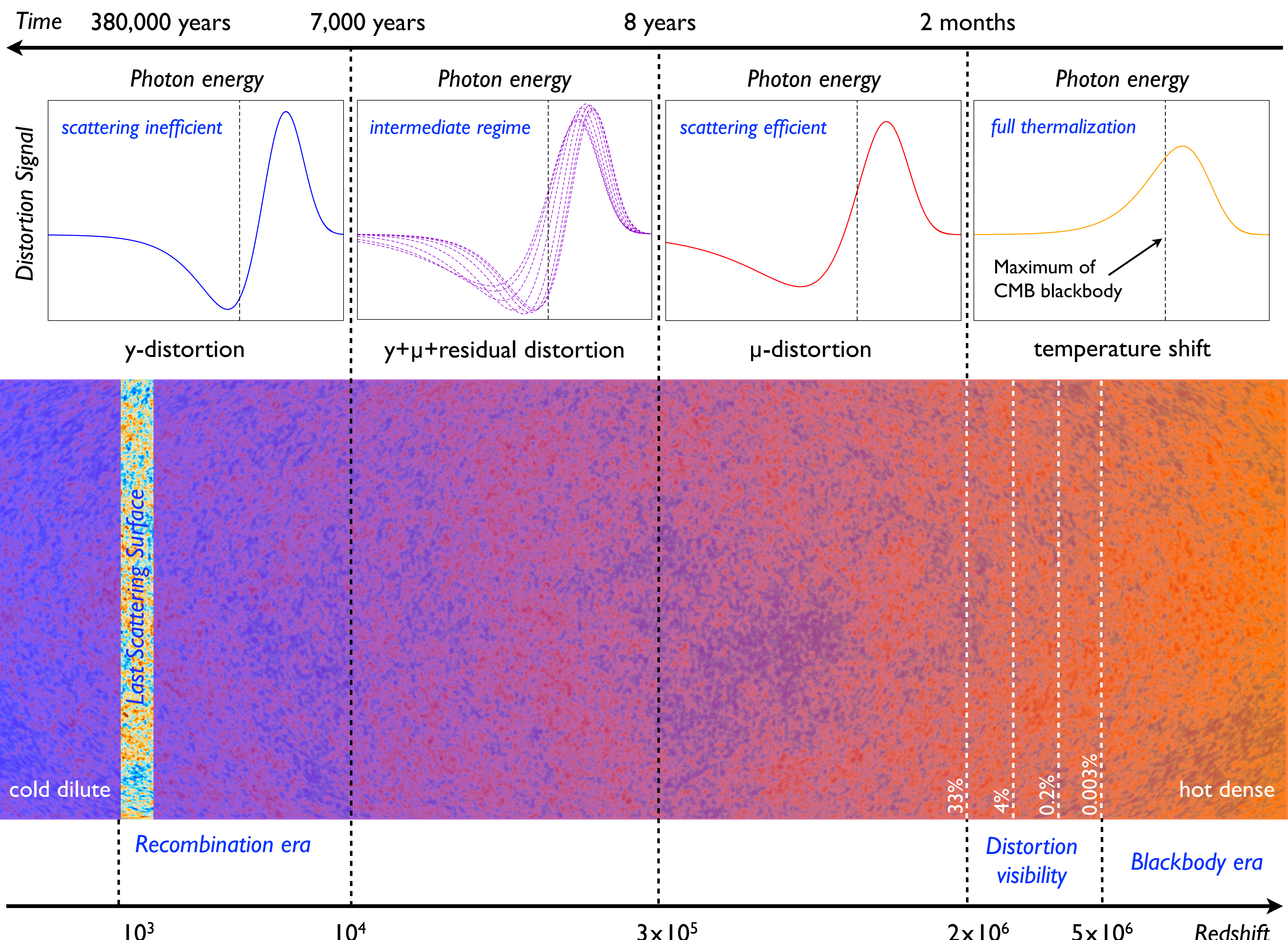

Time
380,000 years
7,000 years
8 years
2 months
Photon energy
full thermalization
Maximum of
CMB blackbody
temperature shift
Last Scattering Surface
cold dilute
33%
4%
0.2%
0.003%
hot dense
Recombination era
Distortion
visibility
Blackbody era
10^3
10^4
3×10^5
2×10^6
5×10^6
Redshift
Silk & Chluba, Science, 2014



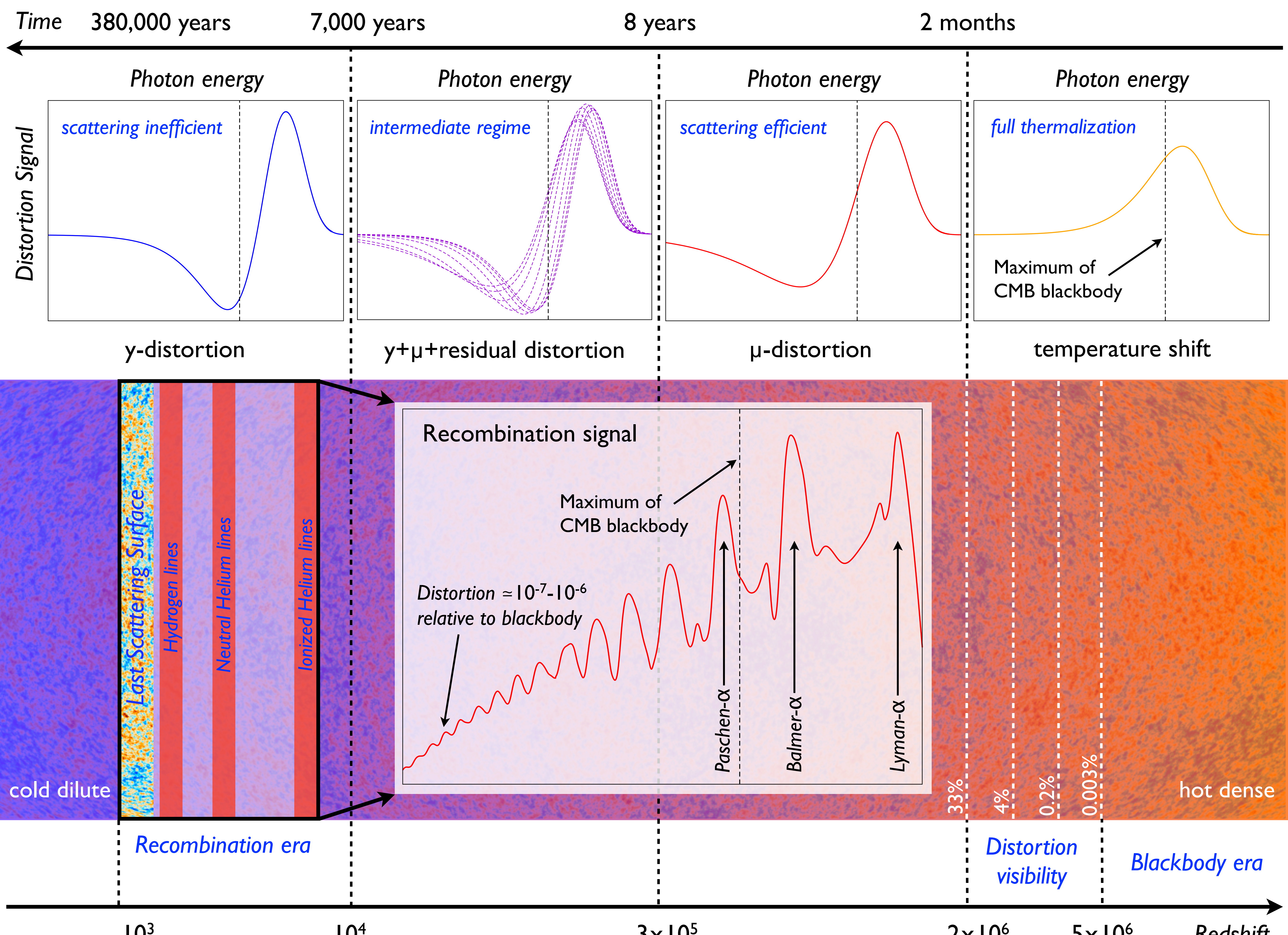
Silk & Chluba, Science, 2014



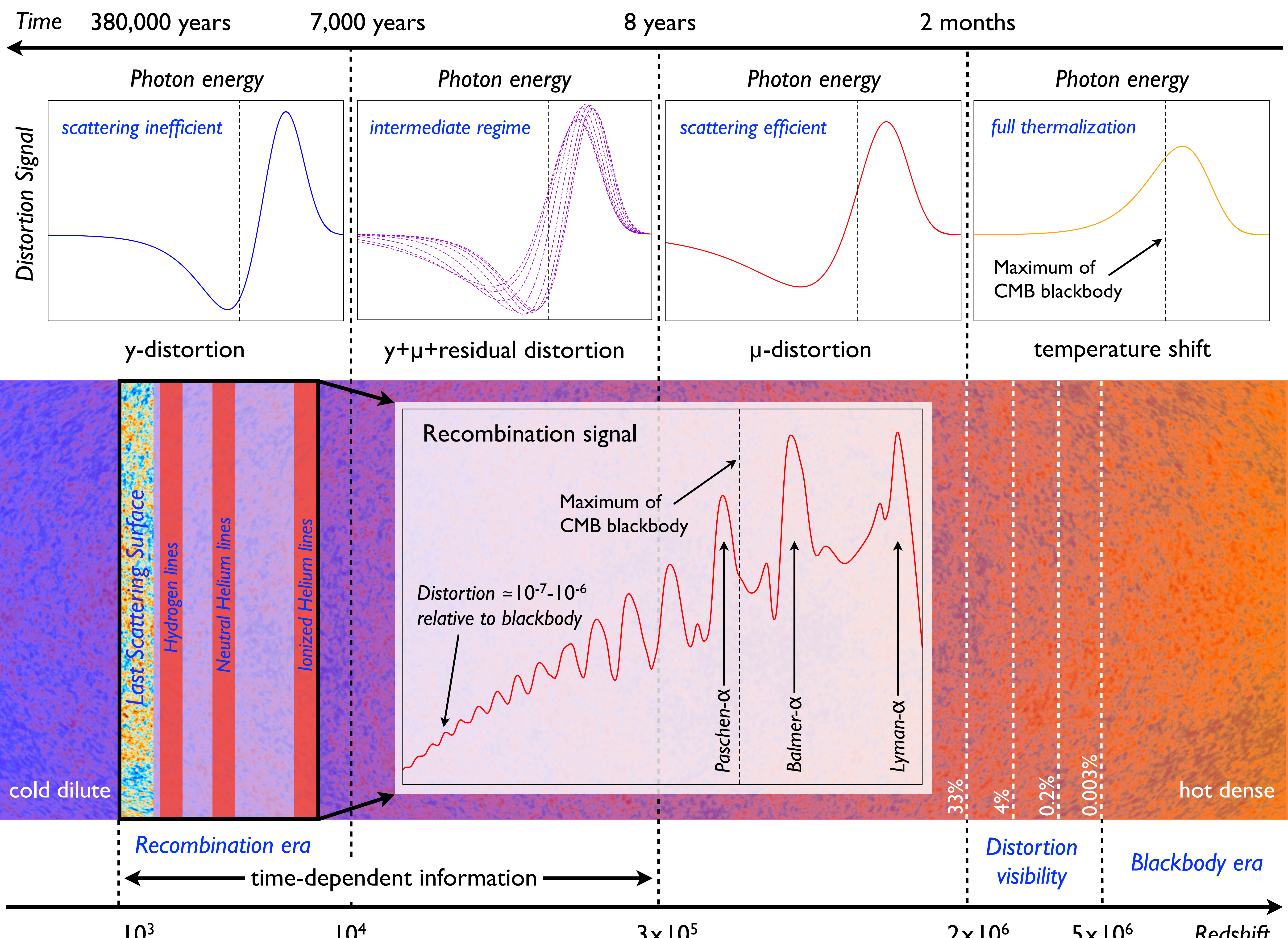
Silk & Chluba, Science, 2014



Silk & Chluba, Science, 2014



Silk & Chluba, Science, 2014



Silk & Chluba, Science, 2014