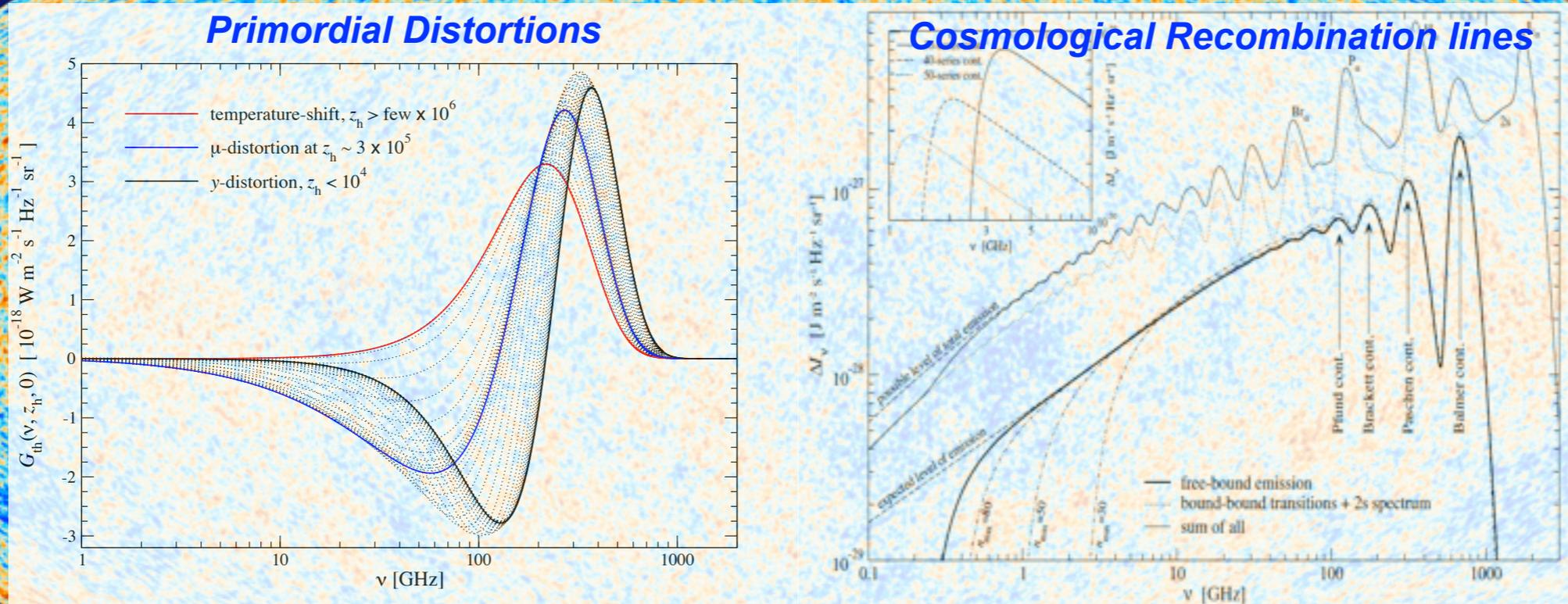


Science with Spectral Distortions of the CMB - III



JOHNS HOPKINS
UNIVERSITY

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CUSO Doctoral Program in Physics

Lausanne, October 30th, 2014



Structure of the Lectures *(at least in theory)*

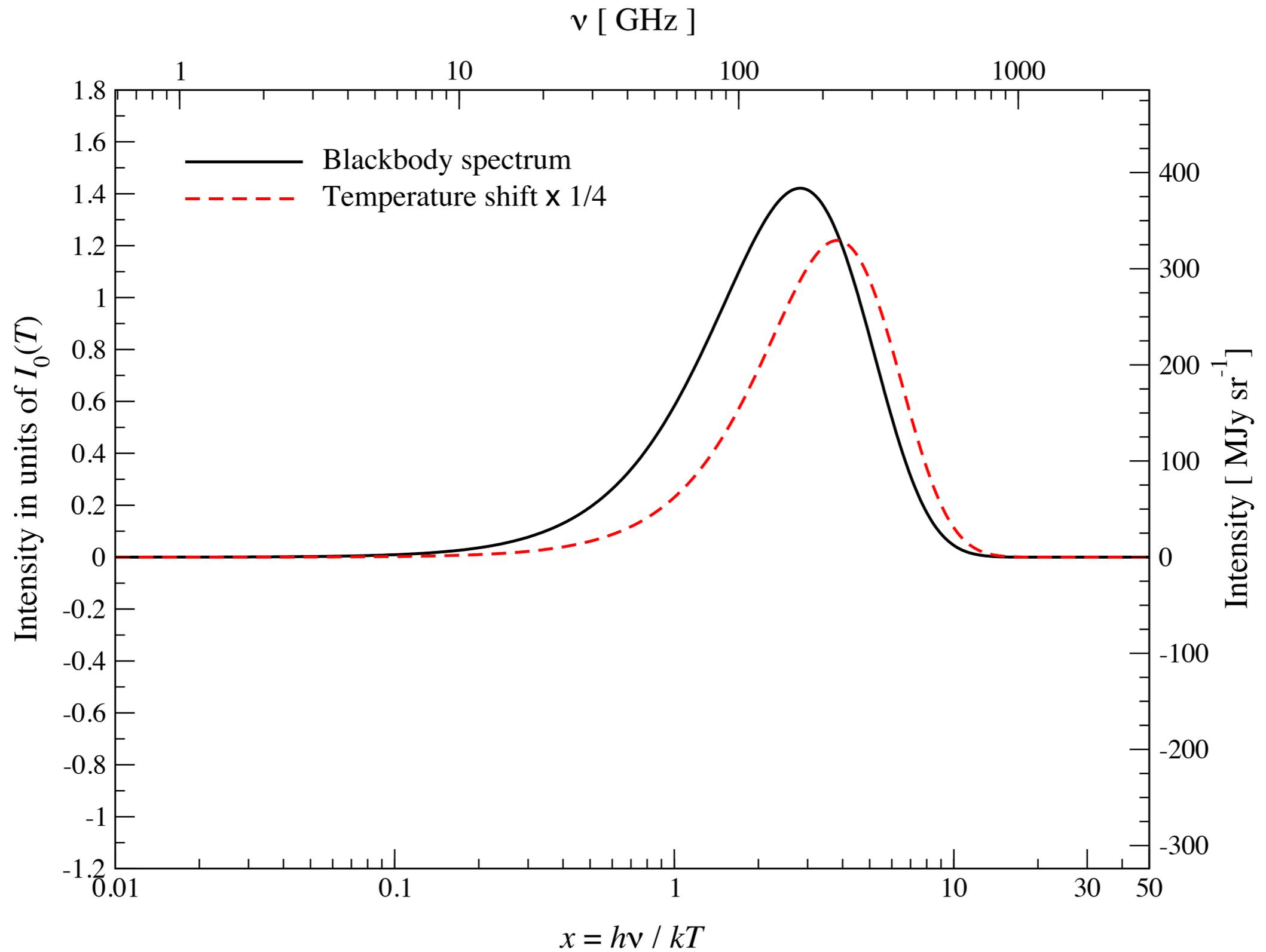
Lecture I:

- Overview and motivation
- Simple blackbody radiation warm-ups
- Formulation of the thermalization problem

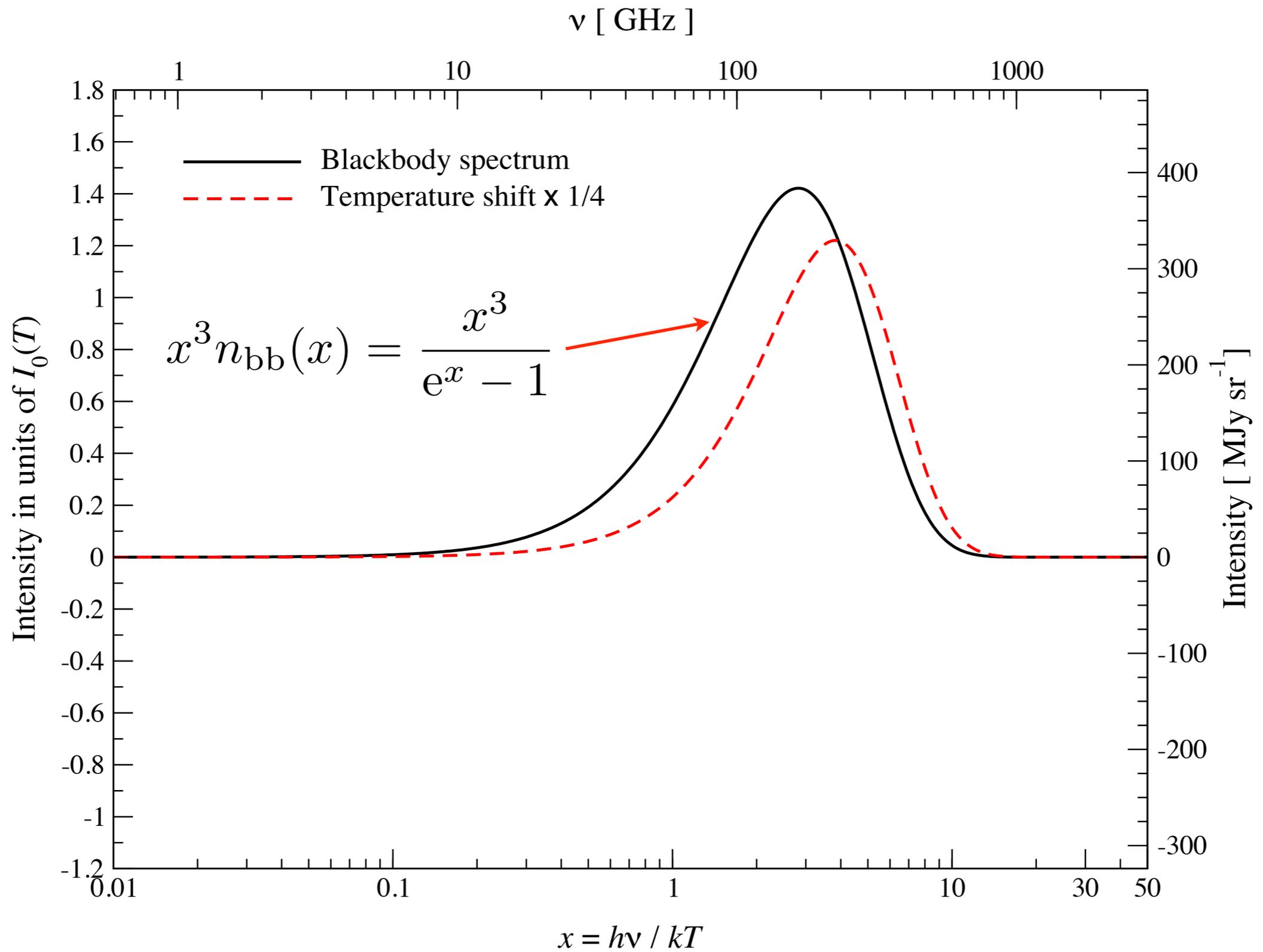
Lecture II:

- Analytic description of the distortions
- Distortion visibility function
- Fast computation of the distortions

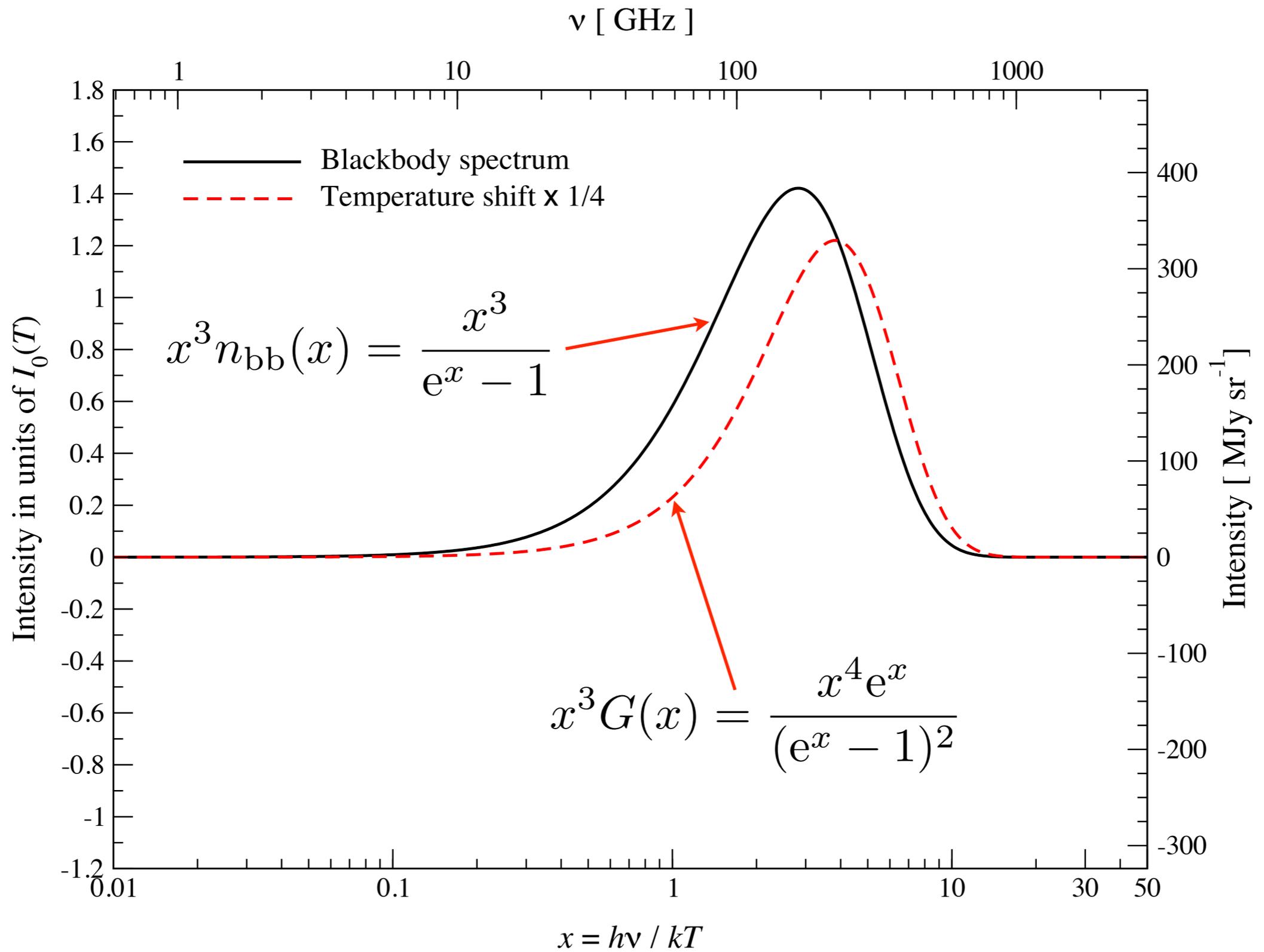
Spectral Signals that we already understand



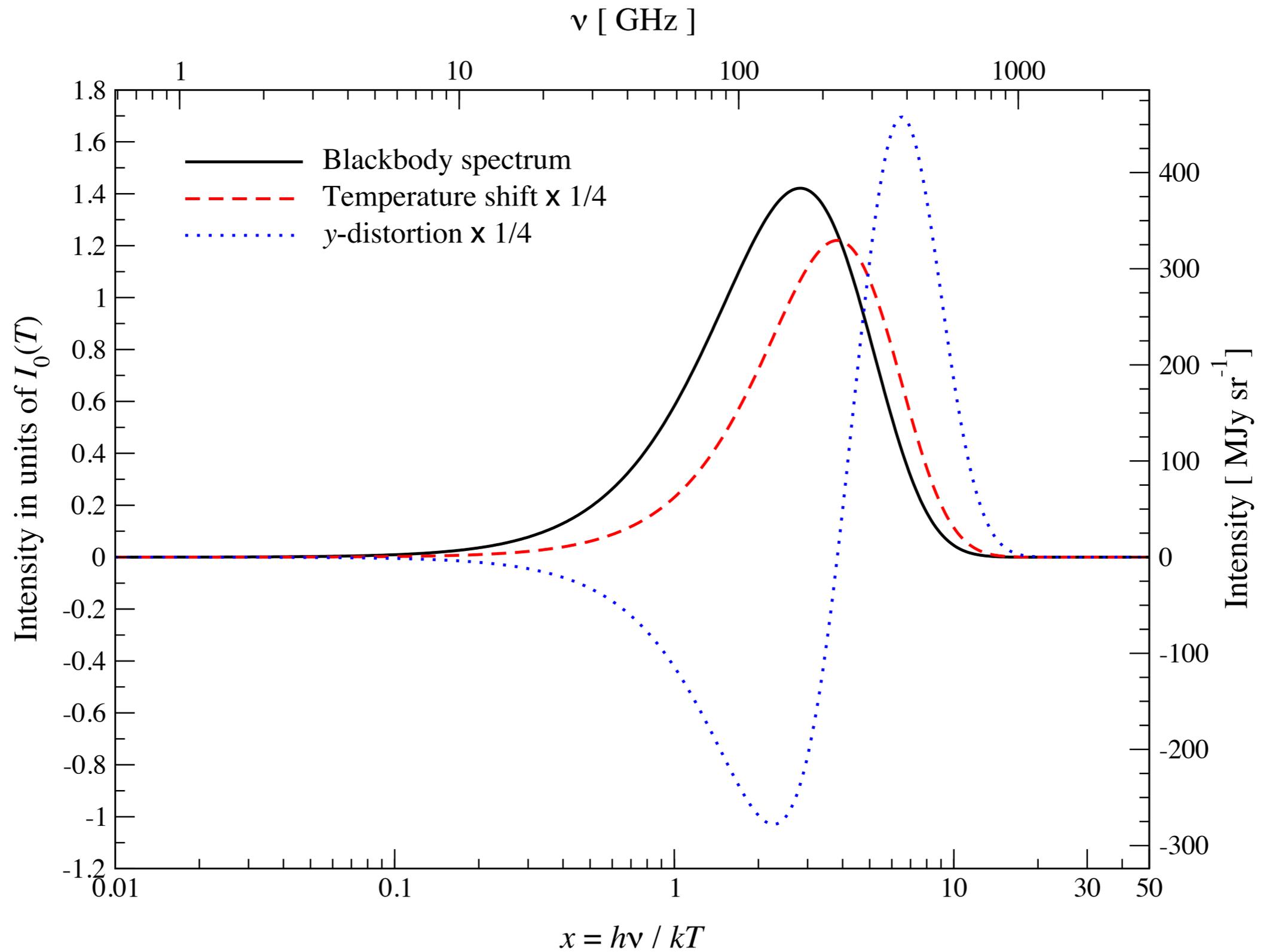
$$I_0 = (2h/c^2)(kT_0/h)^3 \approx 270 \text{ MJy sr}^{-1}$$



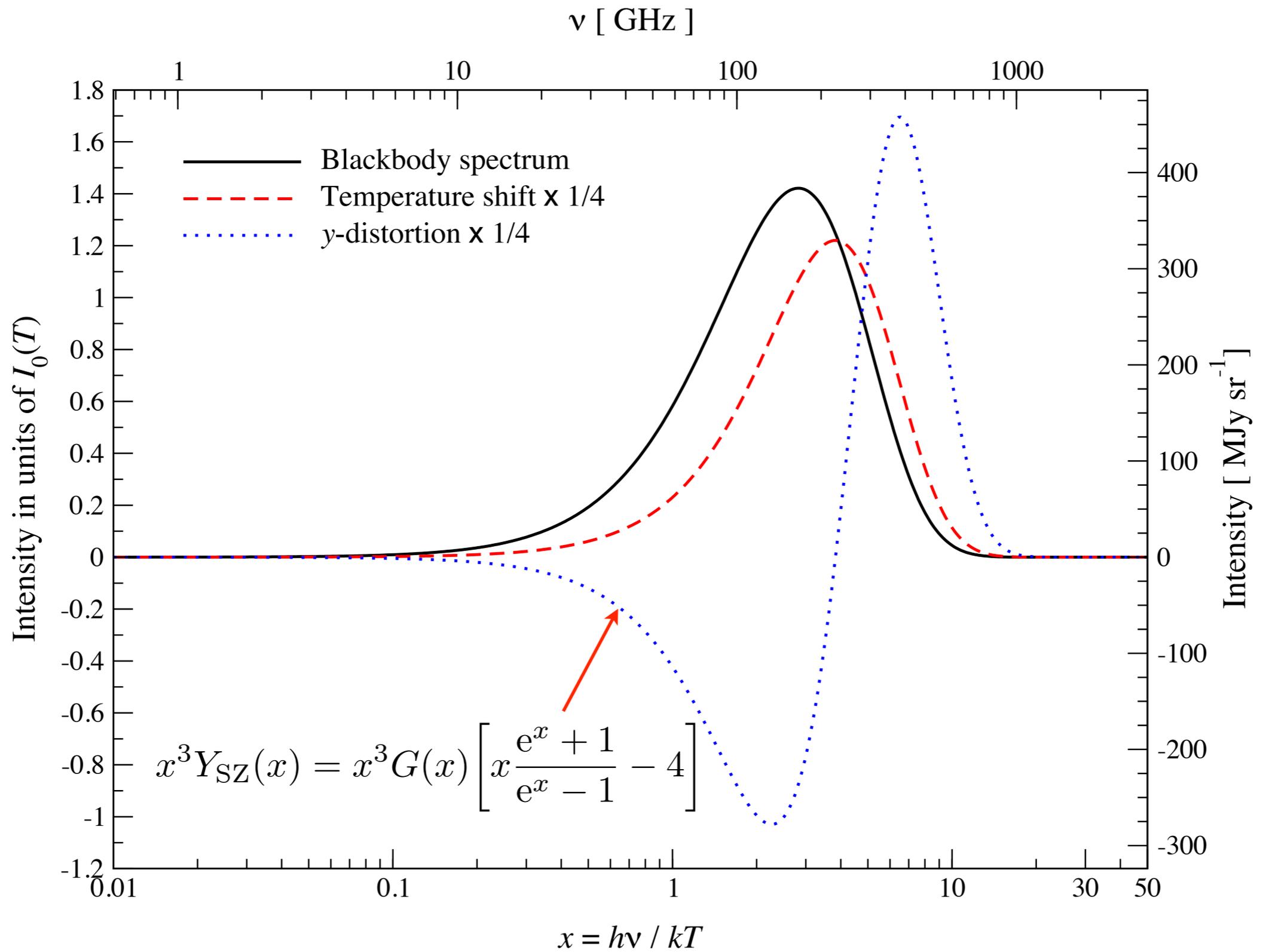
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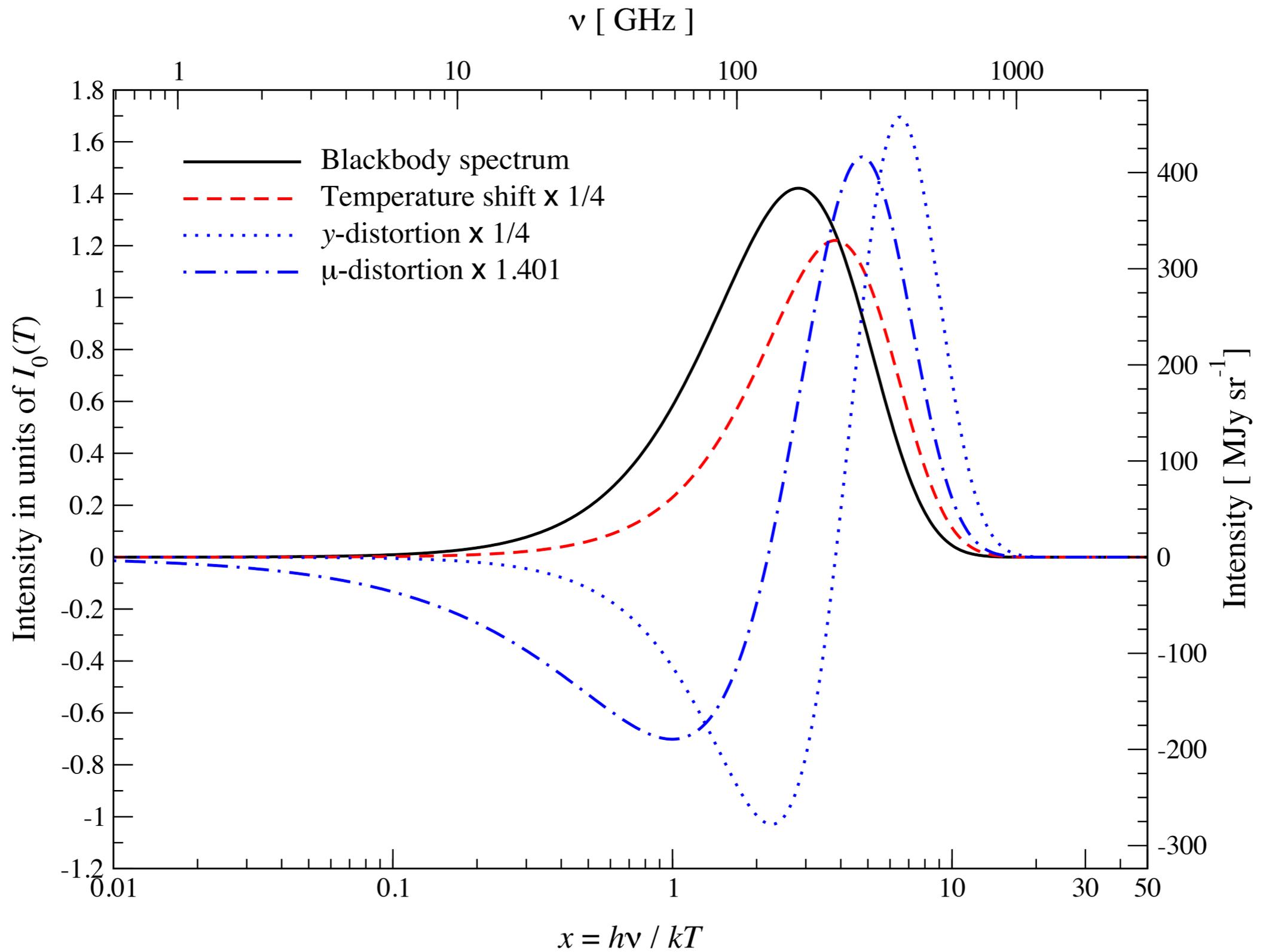
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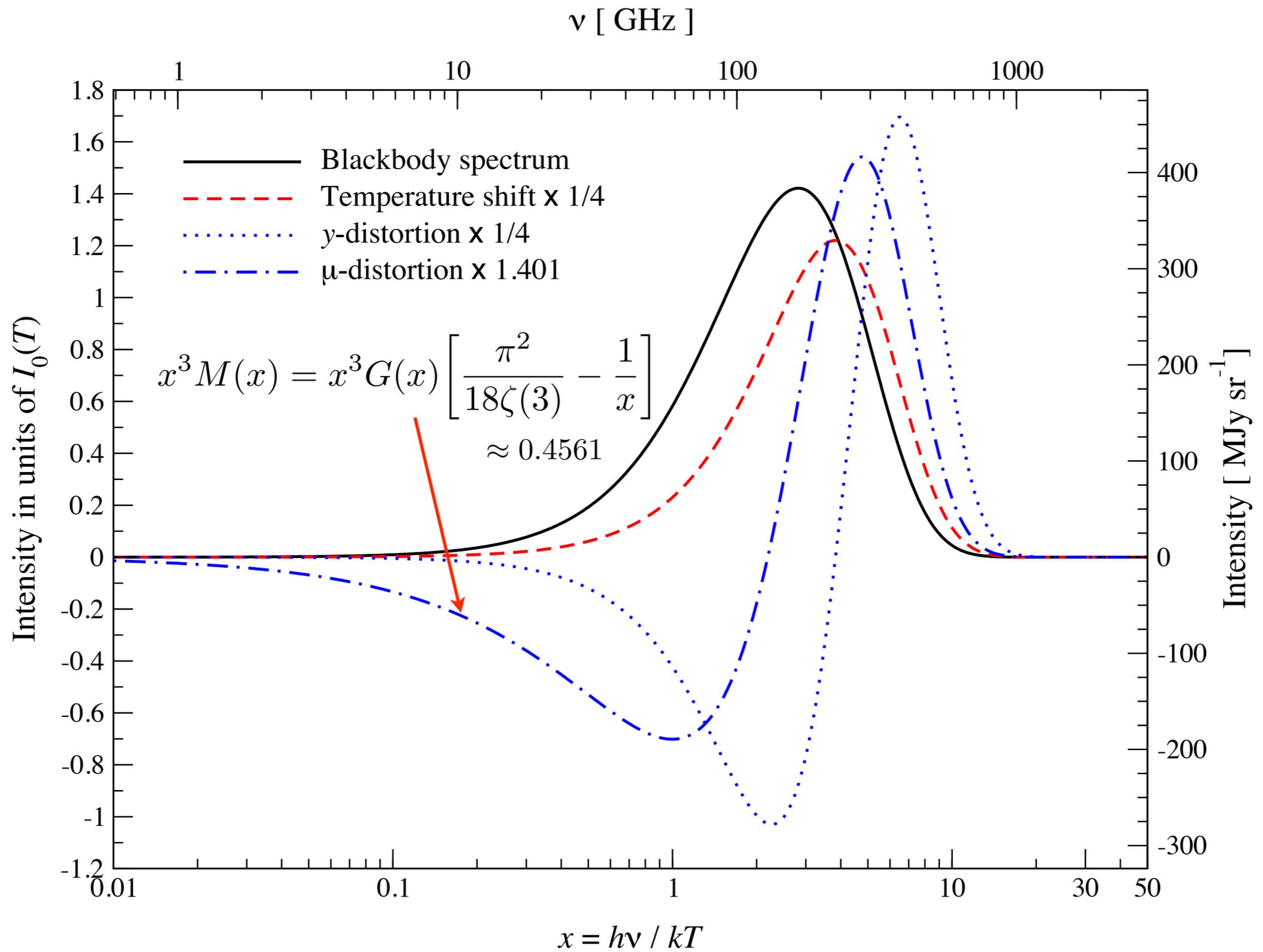
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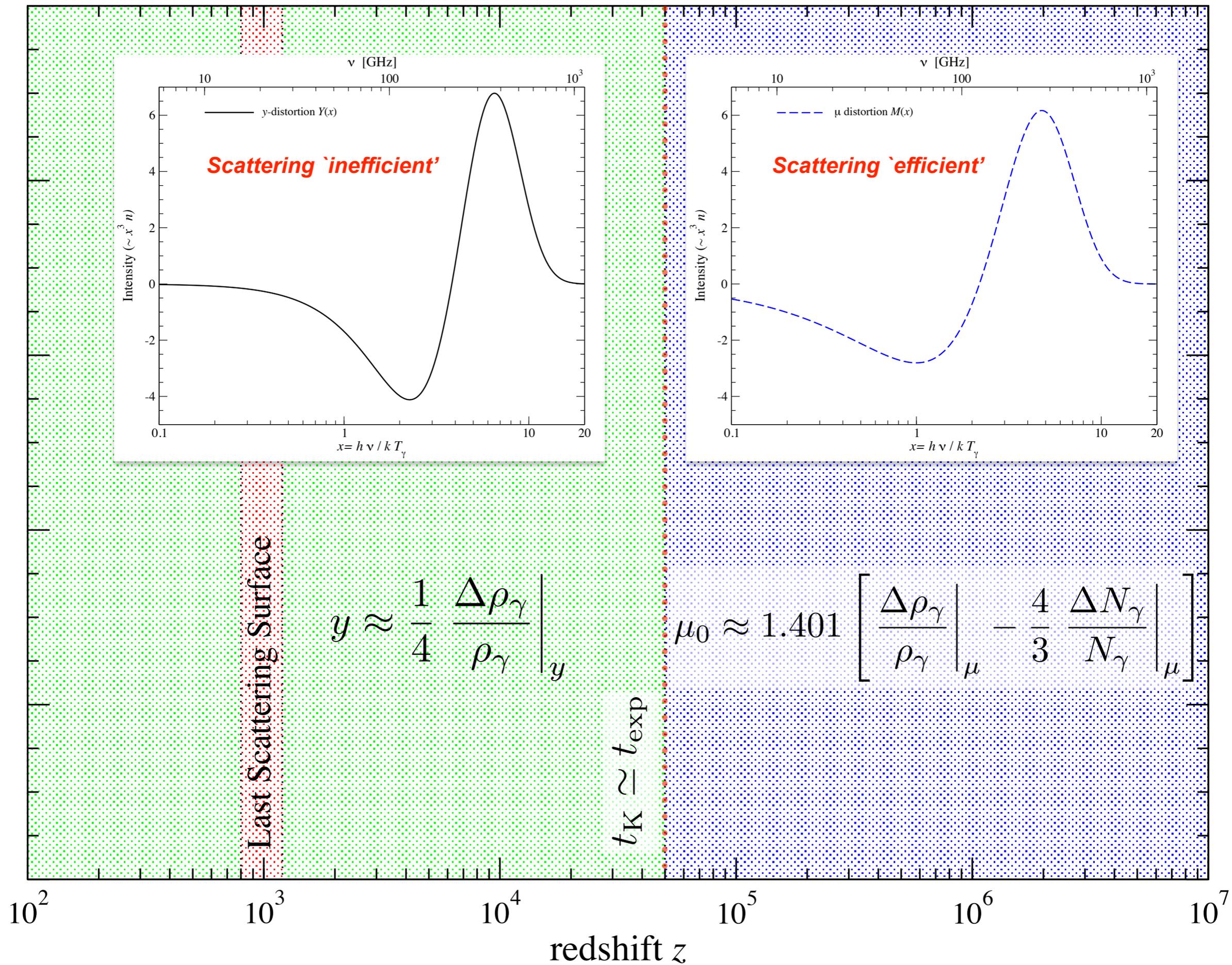
$$I_0 = (2h/c^2)(kT_0/h)^3 \approx 270 \text{ MJy sr}^{-1}$$

Simplest picture for the formation of distortions

y - distortion

μ -y transition

μ - distortion



Approximate analytic solutions for photon injection

(useful for computations in the y -era)

Doppler-dominated scattering

$$\left. \frac{\partial f}{\partial \tau} \right|_{\text{CS}} \approx \frac{\theta_e}{x^2} \frac{\partial}{\partial x} x^4 \frac{\partial}{\partial x} f$$

Zeldovich & Sunyaev, 1969

$$y_e = \int \theta_e d\tau$$

$$f(y_e, x) = \int f(0, x') G_D(y_e, x' \rightarrow x) dx'$$

$$G_D(y_e, x' \rightarrow x) = \frac{x'^3}{x^3} \frac{e^{-\frac{(\ln[x/x'] - 3y_e)^2}{4y_e}}}{\sqrt{4\pi y_e x'}}$$

Recoil-dominated scattering

$$\left. \frac{\partial f}{\partial \tau} \right|_{\text{CS}} \approx \frac{1}{\omega^2} \frac{\partial}{\partial \omega} \omega^4 f$$

$$f(\tau, \omega) = \frac{f\left(0, \frac{\omega}{1 - \omega\tau}\right)}{[1 - \omega\tau]^4} \quad \omega = h\nu/m_e c^2$$

$$\omega_0(\tau) = \omega_0 / (1 + \omega_0\tau)$$

Blackbody-induced scattering

$$\left. \frac{\partial \Delta f}{\partial \tau} \right|_{\text{CS}} \approx \frac{\theta_\gamma}{x^2} \frac{\partial}{\partial x} x^4 \left[\frac{\partial}{\partial x} \Delta f + \frac{2}{x} \Delta f \right]$$

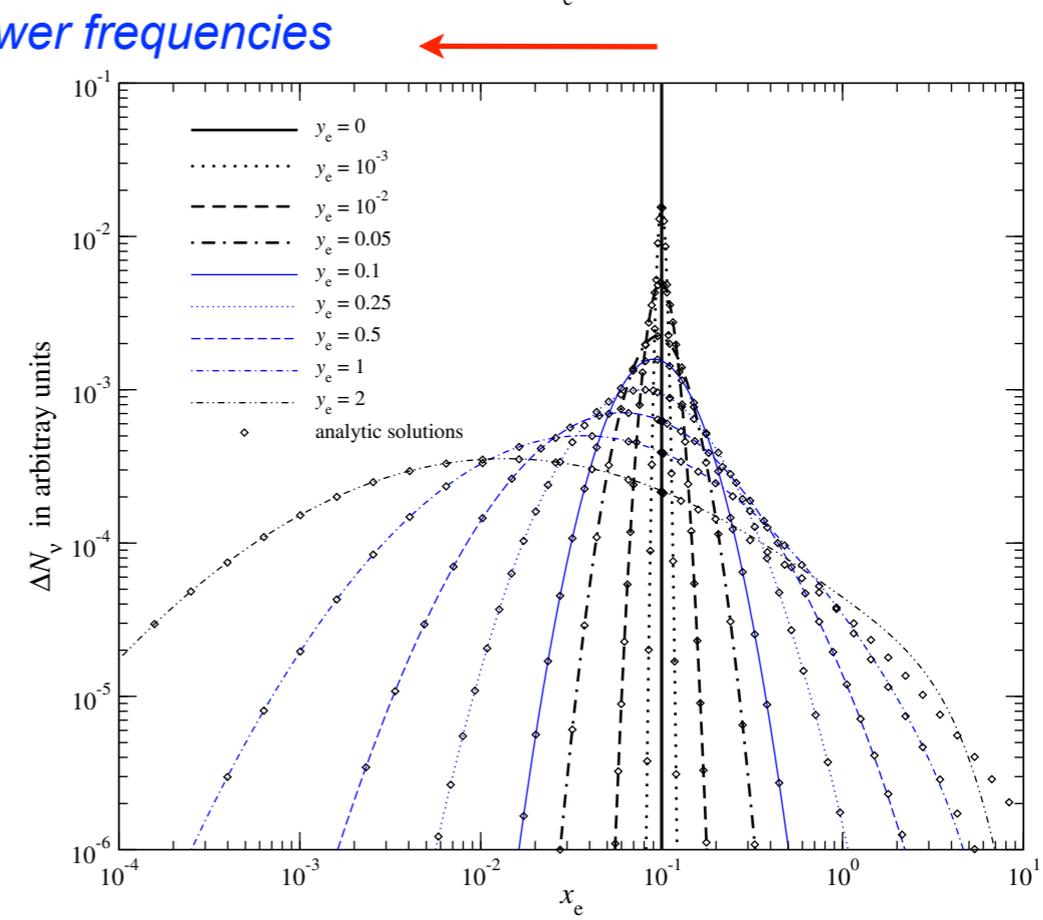
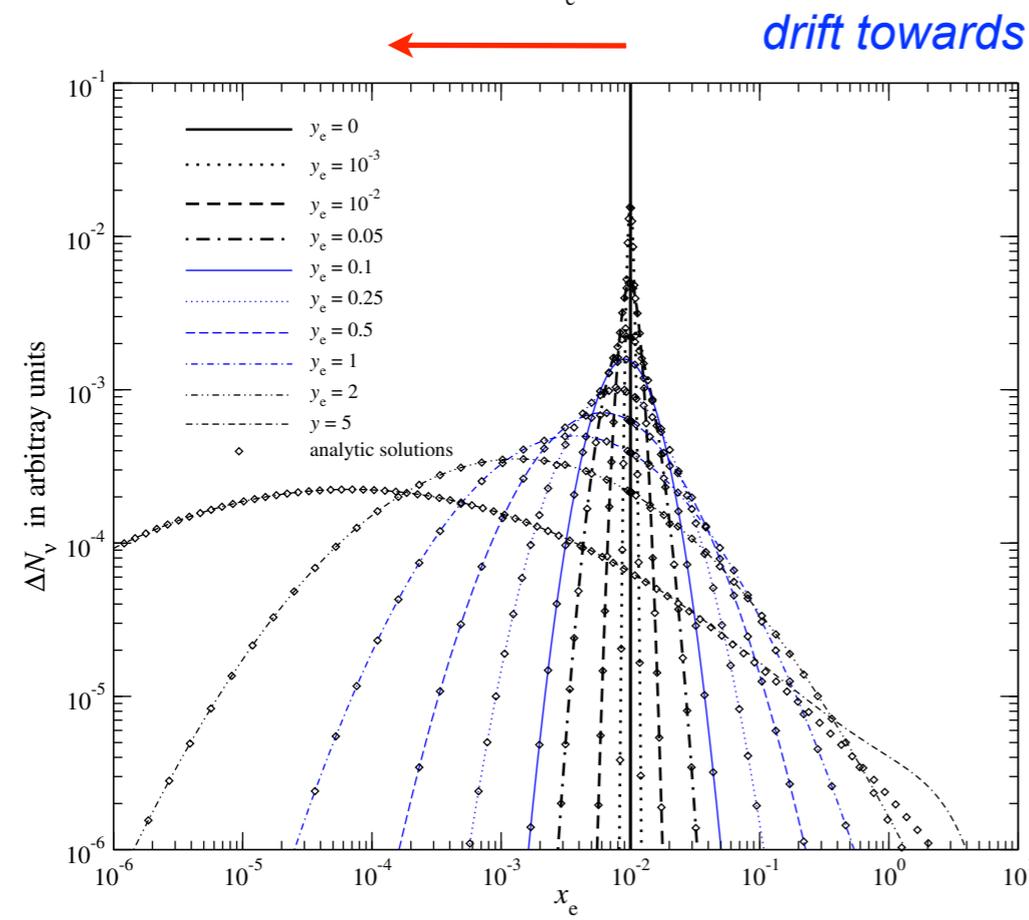
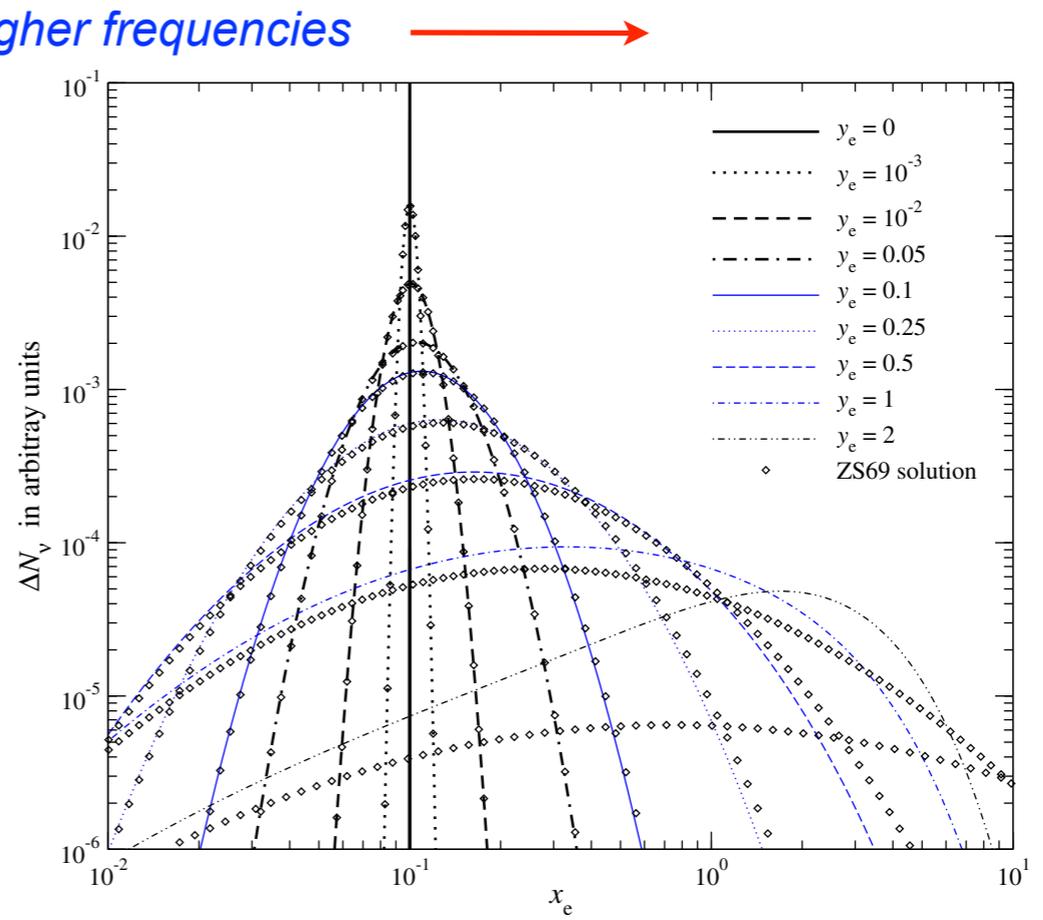
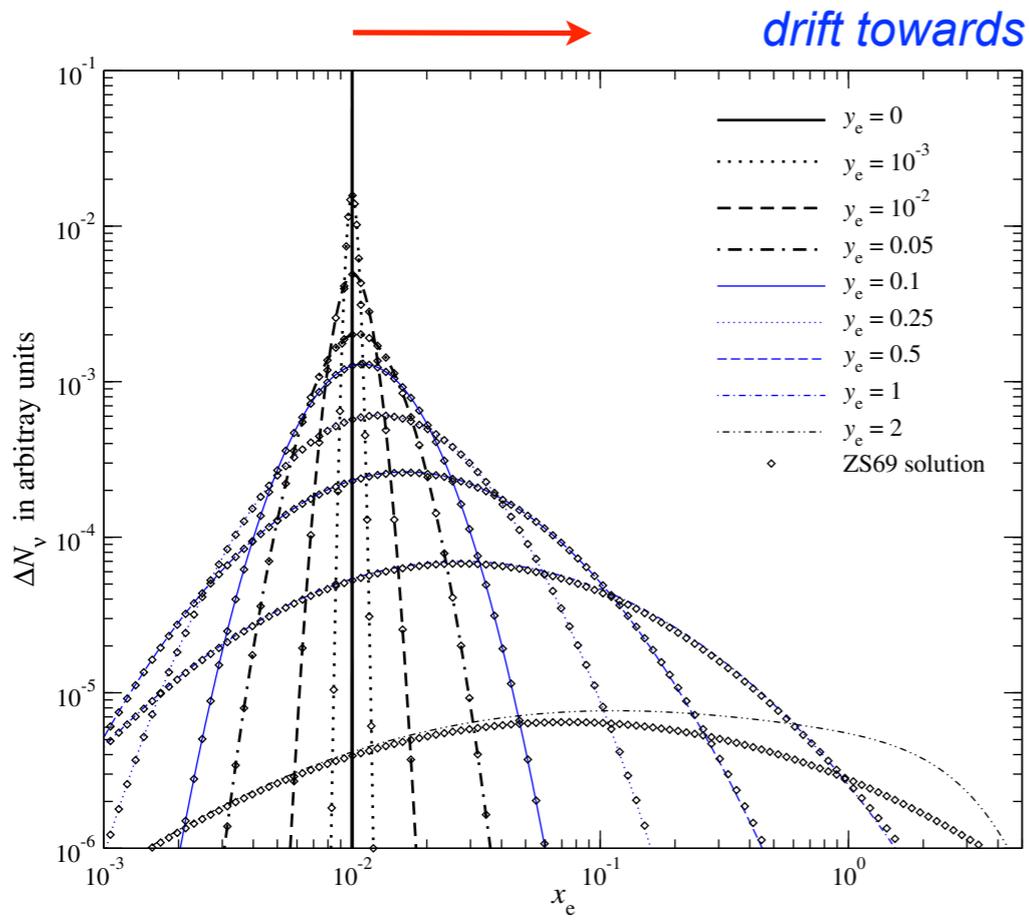
JC & Sunyaev, 2008

$$y_\gamma = \int \theta_\gamma d\tau$$

$$f(y_\gamma, x) = \int f(0, x') G_B(y_\gamma, x' \rightarrow x) dx'$$

$$G_B(y_\gamma, x' \rightarrow x) = \frac{x'^3}{x^3} \frac{e^{-\frac{(\ln[x/x'] - y_\gamma)^2}{4y_\gamma}}}{\sqrt{4\pi y_\gamma x'}}$$

Difference between ZS69 and CS08 solutions



Effect of photon production by DC and BR

Evolution of chemical potential amplitude

$$\mu_0 \approx 1.401 \left[\frac{\Delta \rho_\gamma}{\rho_\gamma} \Big|_\mu - \frac{4}{3} \frac{\Delta N_\gamma}{N_\gamma} \Big|_\mu \right]$$

'Take time-derivative'

Evolution of chemical potential amplitude

$$\Rightarrow \frac{d\mu_0}{d\tau} \approx \frac{3}{\kappa^c} \frac{d \ln a^4 \rho_\gamma}{d\tau} - \frac{4}{\kappa^c} \frac{d \ln a^3 N_\gamma}{d\tau} \quad \kappa^c \approx 2.1419$$

heating of electrons / photon injection

DC & BR process / photon injection

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- *Need solution for low frequency spectrum to compute DC & BR production term!*

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- Need solution for low frequency spectrum to compute DC & BR production term!

Sunyaev & Zeldovich, 1970:

$$0 \approx \partial_x x^2 \partial_x \mu - \frac{x_c^2}{x^2} \mu \quad \Lambda(t, x) \approx \Lambda(t, x_c) \approx \theta_\gamma x_c^2 \quad \Lambda(x, T_\gamma) = K_{\text{BR}}(x, T_\gamma) + K_{\text{DC}}(x, T_\gamma) e^{-x}$$

critical frequency

Evolution of chemical potential amplitude

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$$\Rightarrow \mu(t, x) \approx \mu_0(t) e^{-x_c(t)/x}$$

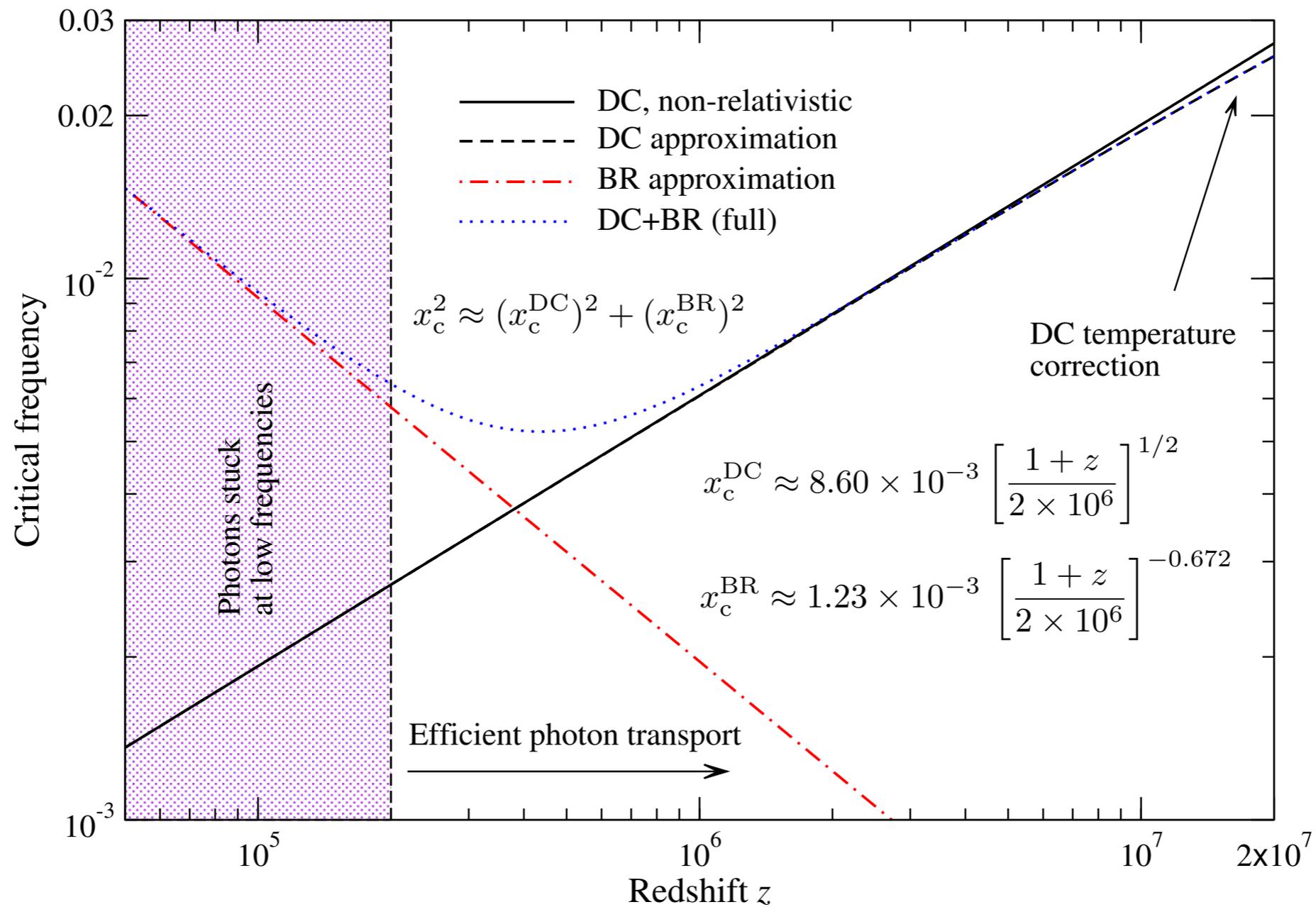


Figure 4.8: Critical frequency, x_c as a function of z . Photon transport is inefficient below $z \simeq 2 \times 10^5$ so that the distortion visibility function quickly approaches unity. DC temperature corrections become noticeable at $z \gtrsim 10^6$. The approximations are from Eq. (4.38) and (4.39). The figure is taken from Chluba [10].

Evolution of chemical potential amplitude

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heating of electrons / photon injection

DC & BR process / photon injection

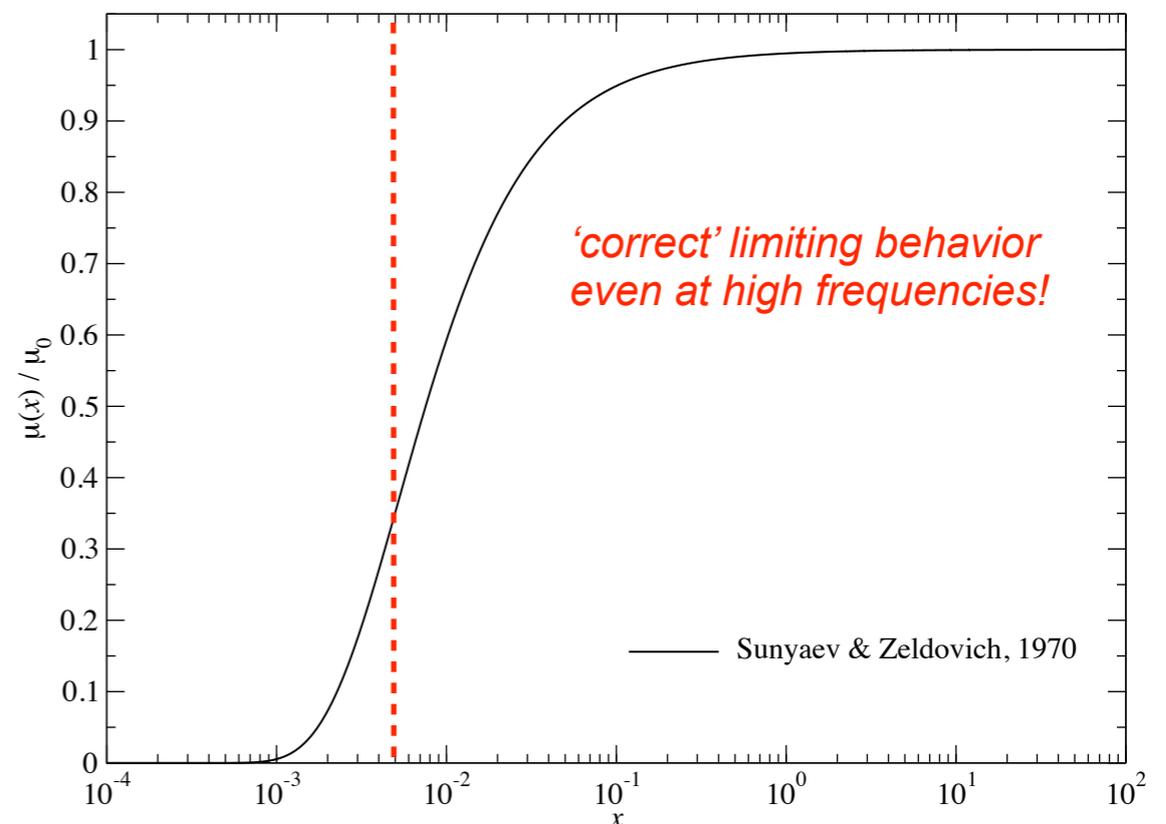
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heating of electrons / photon injection

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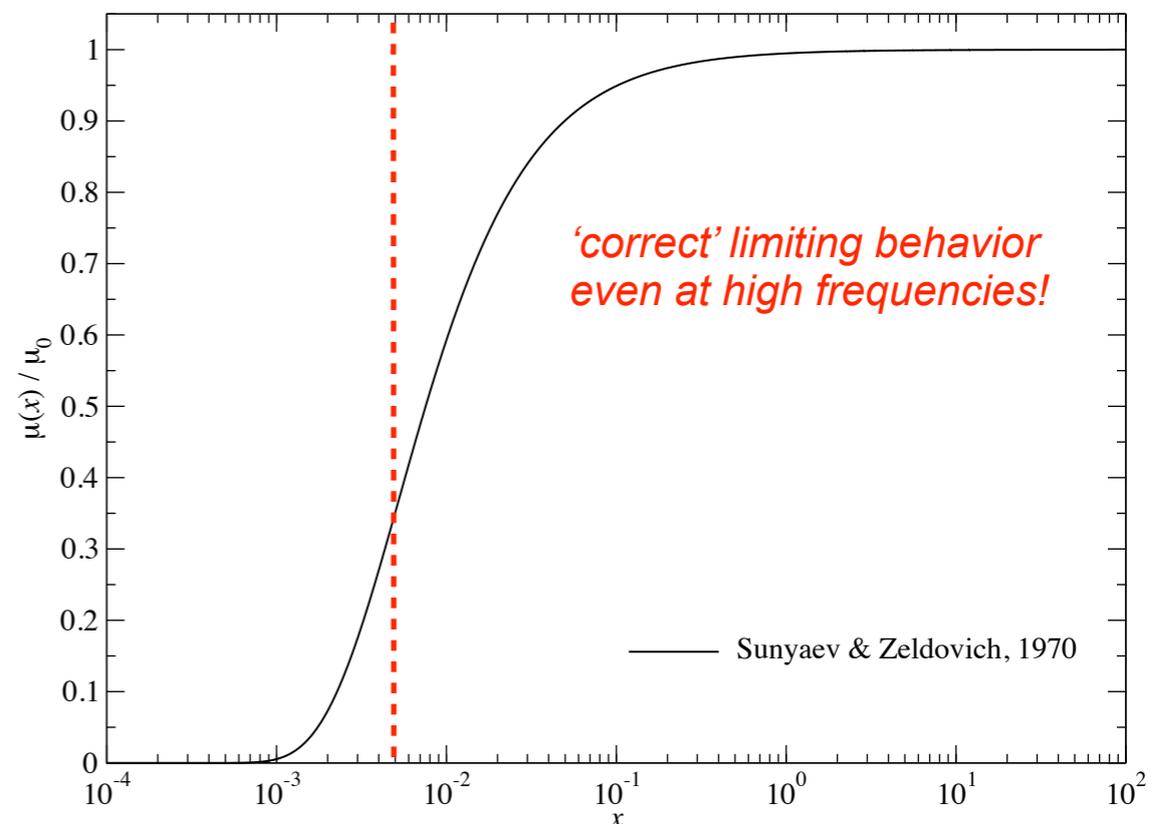
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$$\Rightarrow \frac{d \ln a^3 N_\gamma}{d\tau} \approx \frac{\theta_\gamma x_c}{G_2^{\text{PI}}} \mu_0(\tau)$$



Evolution of chemical potential amplitude

$$\Rightarrow \frac{d\mu_0}{d\tau} \approx \frac{3}{\kappa^c} \frac{d \ln a^4 \rho_\gamma}{d\tau} - \frac{4}{\kappa^c} \frac{d \ln a^3 N_\gamma}{d\tau} \quad \kappa^c \approx 2.1419$$

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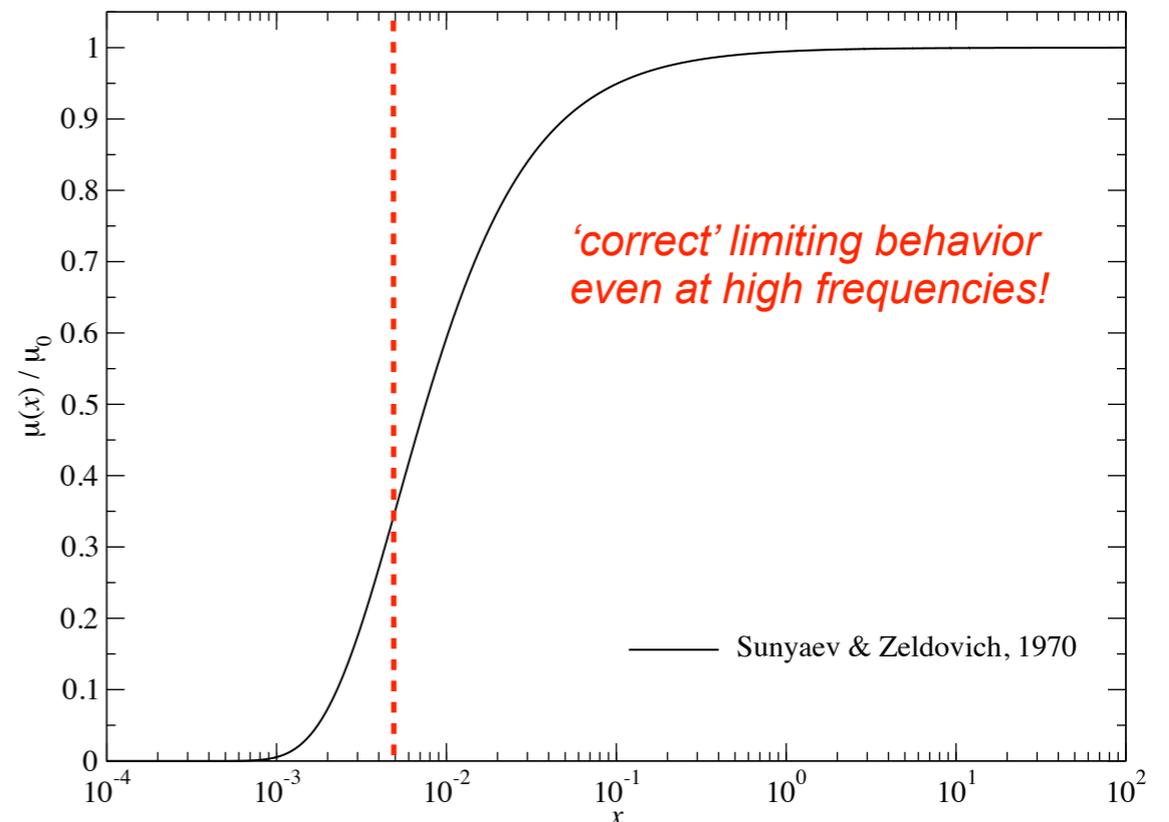
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$$\Rightarrow \frac{d \ln a^3 N_\gamma}{d\tau} \approx \frac{\theta_\gamma x_c}{G_2^{\text{PI}}} \mu_0(\tau)$$

$$\Rightarrow \frac{4}{\kappa^c} \frac{d \ln a^3 N_\gamma}{d\tau} \approx 0.7769 \theta_\gamma x_c \mu_0$$



Solution with photon production & distortion visibility function

Putting things together (no extra photon source term)

$$\Rightarrow \frac{d\mu_0}{d\tau} \approx 1.401 \frac{\dot{Q}_e^*}{\rho_\gamma} - 0.7769 \theta_\gamma x_c \mu_0$$

electron (+matter) heating term

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electron (+matter) heating term

Thermalization optical depth

$$\tau_\mu(z) \approx 0.7769 \int_0^z \theta_\gamma x_c \frac{\sigma_T N_e c dz'}{H(1+z')}$$

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$$\mu_0(z) \approx 1.401 \int_z^\infty \frac{\dot{Q}_e^*}{\rho_\gamma} \frac{e^{-\tau_\mu(z',z)} dz'}{H(1+z')}$$

Thermalization optical depth

$$\tau_\mu(z) \approx 0.7769 \int_0^z \theta_\gamma x_c \frac{\sigma_T N_e c dz'}{H(1+z')}$$

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$$\Rightarrow \mathcal{J}(z, z') = e^{-\tau_\mu(z, z')}$$

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Distortion visibility function between z and z'

Solution with photon production & distortion visibility function

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$$\tau_\mu(z, z') = \tau_\mu(z) - \tau_\mu(z')$$

$$\Rightarrow \mathcal{J}(z, z') = e^{-\tau_\mu(z, z')} \quad \text{Distortion visibility function between } z \text{ and } z'$$

- Shape depends on the photon production process (which defines τ integral)

Solution with photon production & distortion visibility function

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- Shape depends on the photon production process (which defines τ integral)
- Determines what fraction of the released energy remains visible as a distortion

Solution with photon production & distortion visibility function

Putting things together (no extra photon source term)

$$\Rightarrow \frac{d\mu_0}{d\tau} \approx 1.401 \frac{\dot{Q}_e^*}{\rho_\gamma} - 0.7769 \theta_\gamma x_c \mu_0$$

electron (+matter) heating term

Thermalization optical depth

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$$\mu_0(z) \approx 1.401 \int_z^\infty \frac{\dot{Q}_e^*}{\rho_\gamma} \frac{e^{-\tau_\mu(z',z)} dz'}{H(1+z')}$$

$$\tau_\mu(z, z') = \tau_\mu(z) - \tau_\mu(z')$$

$$\Rightarrow \mathcal{J}(z, z') = e^{-\tau_\mu(z, z')} \quad \text{Distortion visibility function between } z \text{ and } z'$$

- Shape depends on the photon production process (which defines τ integral)
- Determines what fraction of the released energy remains visible as a distortion
- Fraction of energy that is fully thermalized:

$$\mathcal{J}_{\text{th}}(z, z') = 1 - \mathcal{J}(z, z')$$

Distortion visibility function for DC and BR only

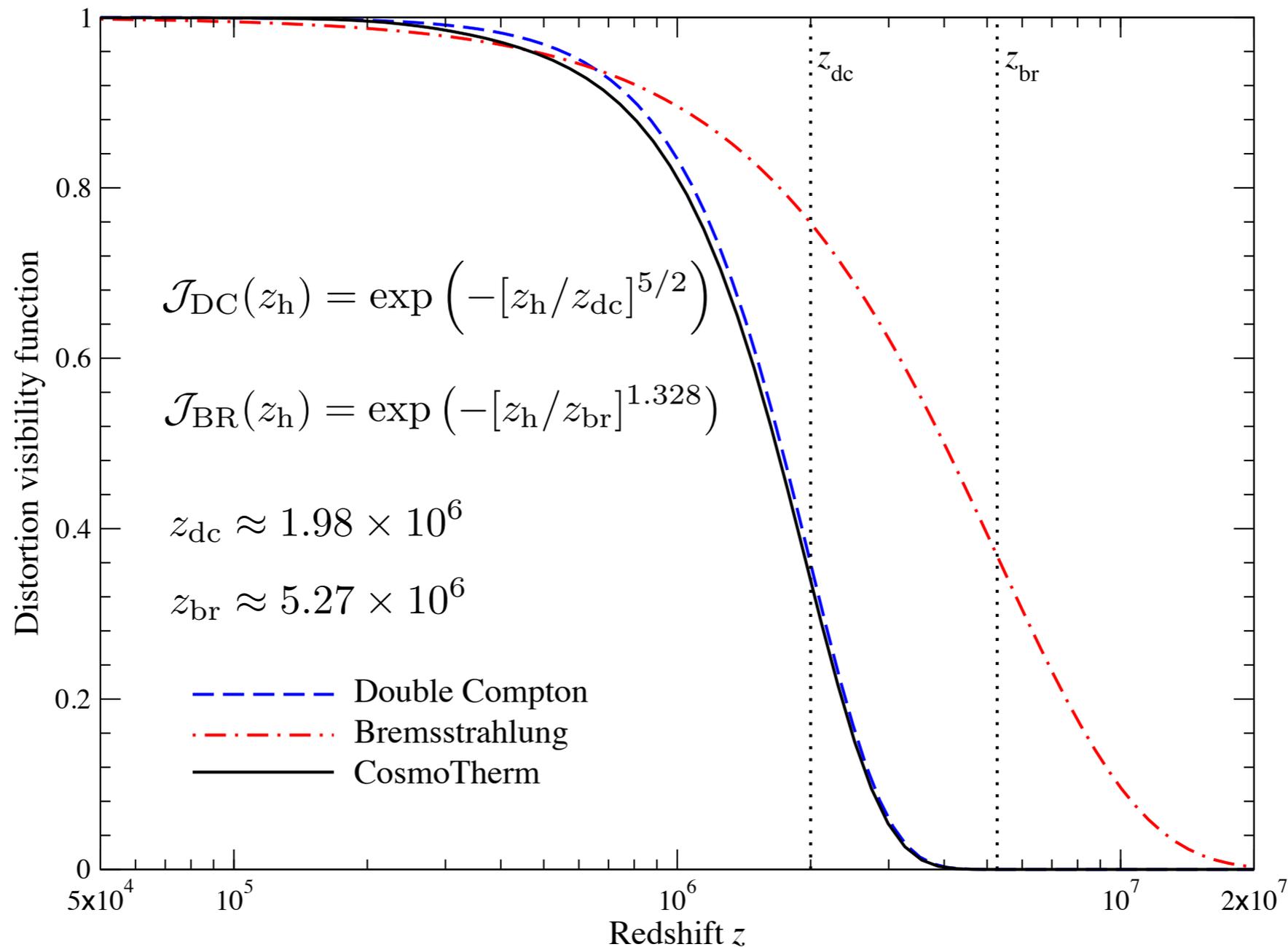


Figure 4.9: Distortion visibility function. We compare $\mathcal{J}_{\text{DC}}(z_h)$, $\mathcal{J}_{\text{BR}}(z_h)$ and the numerical result obtained with CosmoTherm. DC emission significantly change the thermalization efficiency. Deviations from the numerical result can be captured by adding several effects, as discussed in Sect. 4.6.

Improved picture for the formation of distortions

y - distortion

μ -y transition

μ - distortion

Visibility

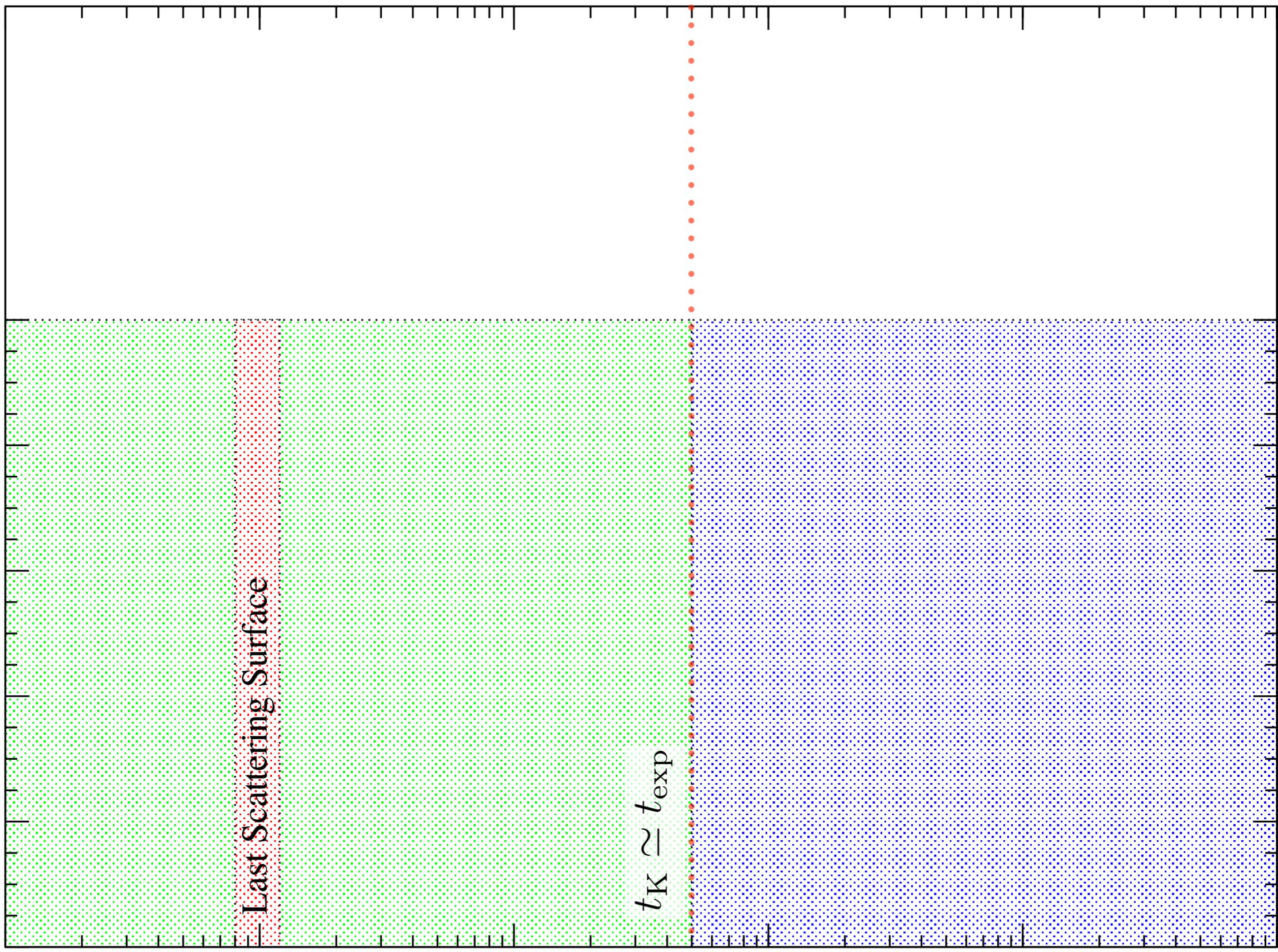
1
0.8
0.6
0.4
0.2
0

Last Scattering Surface

$t_K \simeq t_{\text{exp}}$

redshift z

10^2 10^3 10^4 10^5 10^6 10^7



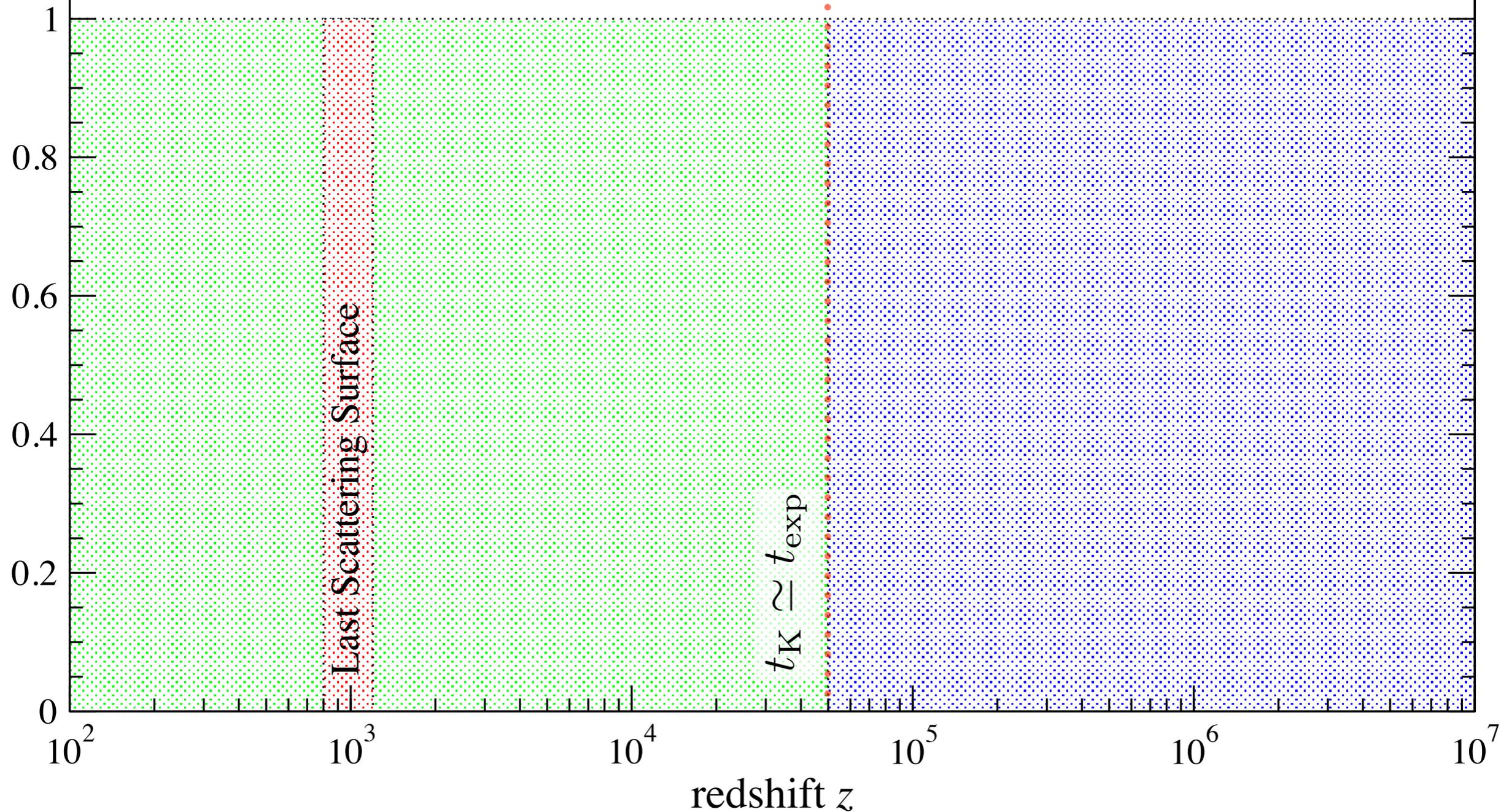
γ - distortion

μ - γ transition

μ - distortion

Photon production in principle works all the time, but photon transport to high frequencies inefficient at $z_K \lesssim 50000$ (Comptonization slow)

Visibility



γ - distortion

μ - γ transition

μ - distortion

Photon production in principle works all the time, but photon transport to high frequencies inefficient at $z_K \approx 50000$ (Comptonization slow)

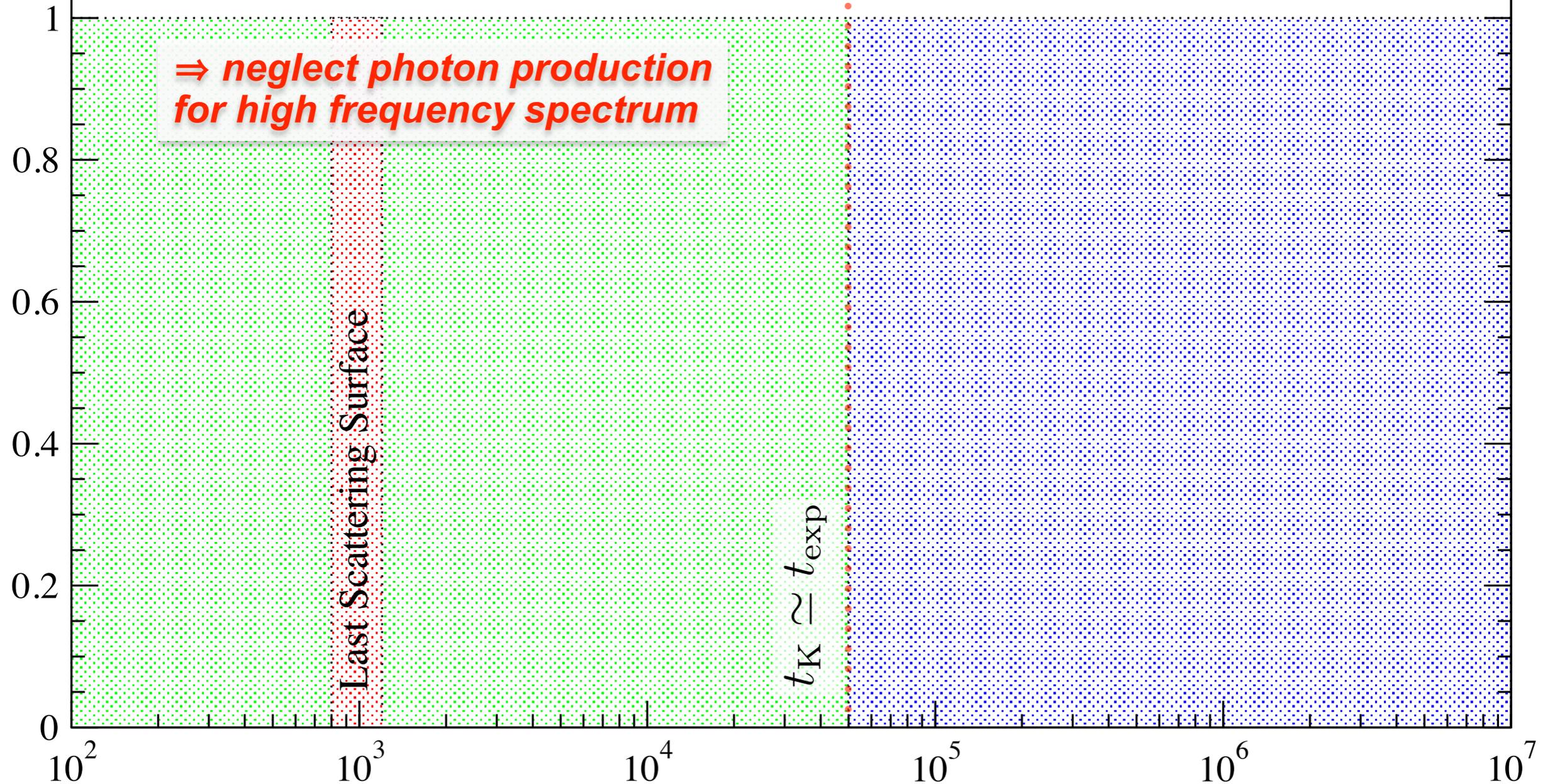
\Rightarrow neglect photon production for high frequency spectrum

Last Scattering Surface

$t_K \simeq t_{\text{exp}}$

Visibility

redshift z



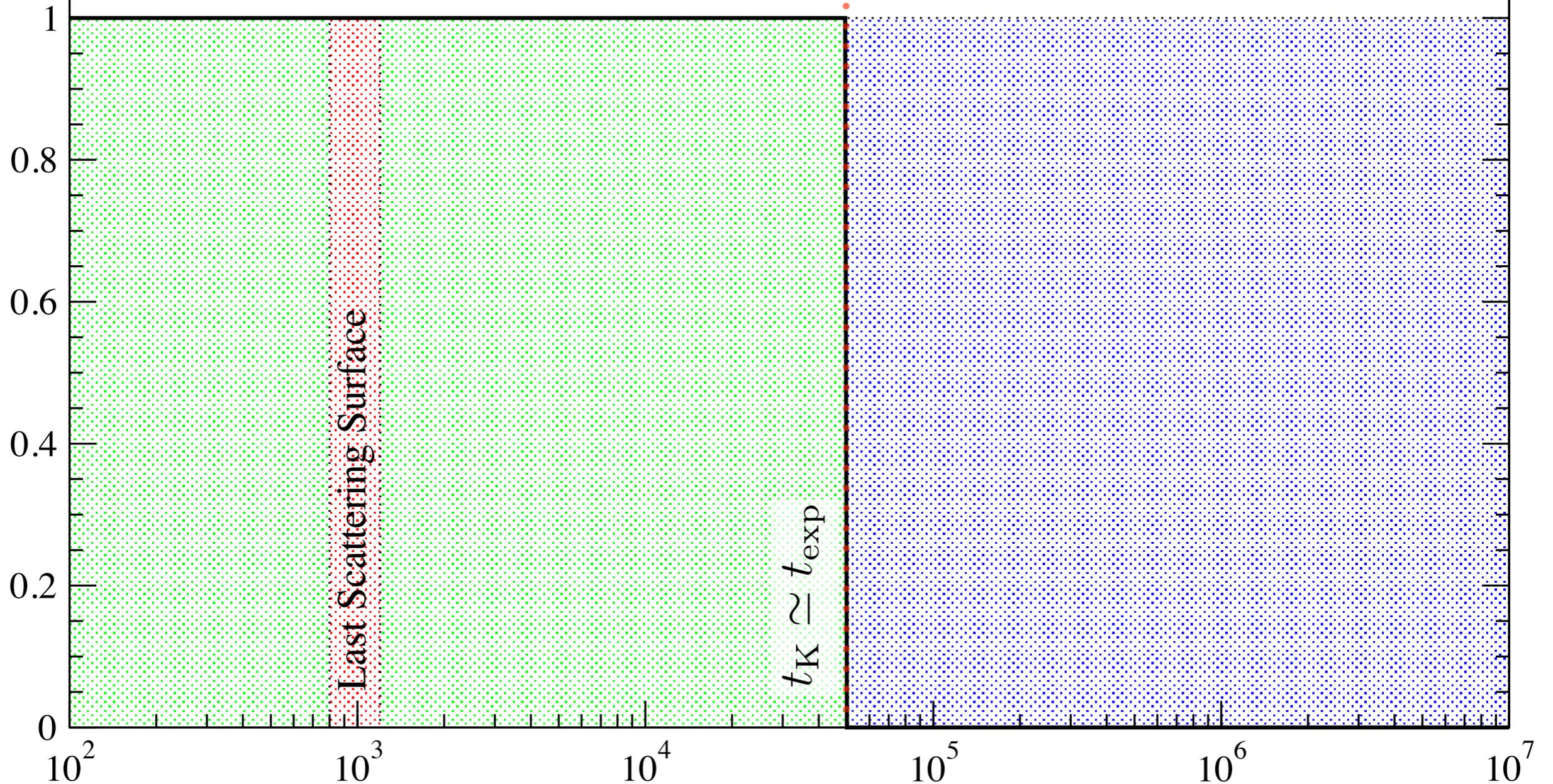
y - distortion

μ -y transition

μ - distortion

$$y \simeq \frac{1}{4} \frac{\Delta\rho_\gamma}{\rho_\gamma} \equiv \frac{1}{4} \int_0^{z_K} \frac{d(Q/\rho_\gamma)}{dz'} dz'$$

Visibility



Last Scattering Surface

$t_K \simeq t_{\text{exp}}$

redshift z

y - distortion

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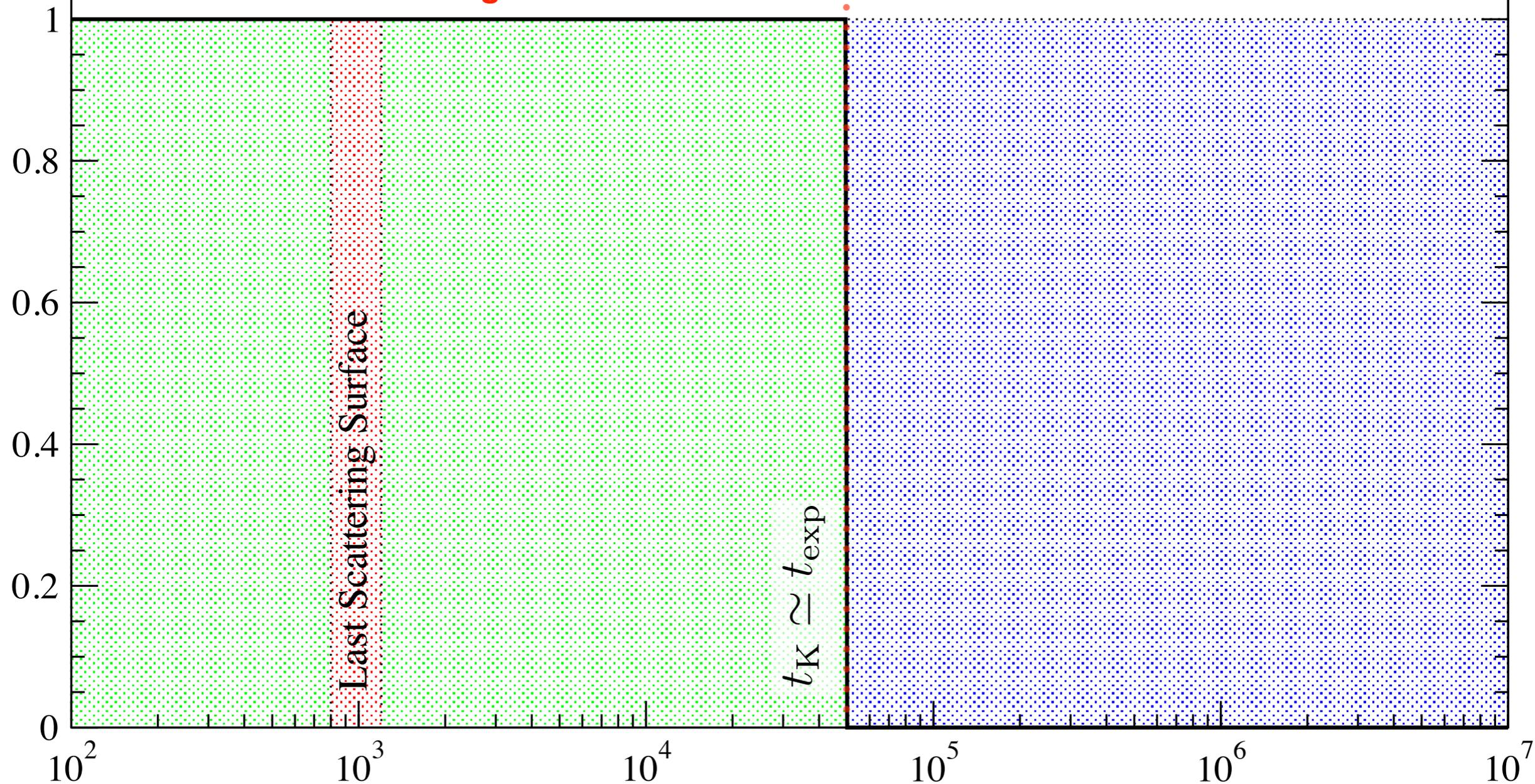
Effective photon heating rate

Visibility

Last Scattering Surface

$t_K \simeq t_{\text{exp}}$

redshift z



y - distortion

μ -y transition

μ - distortion

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Effective photon heating rate

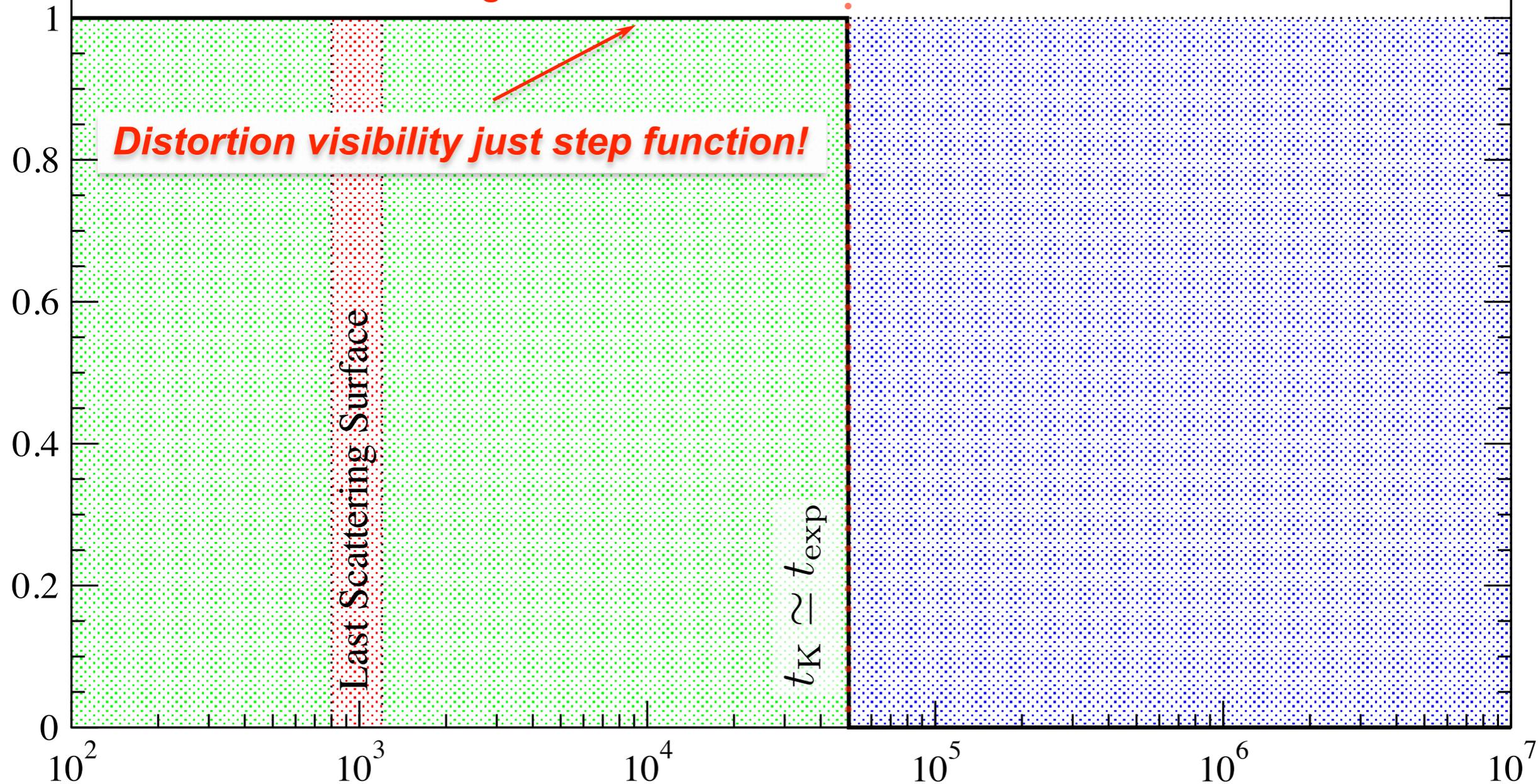
Visibility

Distortion visibility just step function!

Last Scattering Surface

$t_K \simeq t_{\text{exp}}$

redshift z



y - distortion

μ -y transition

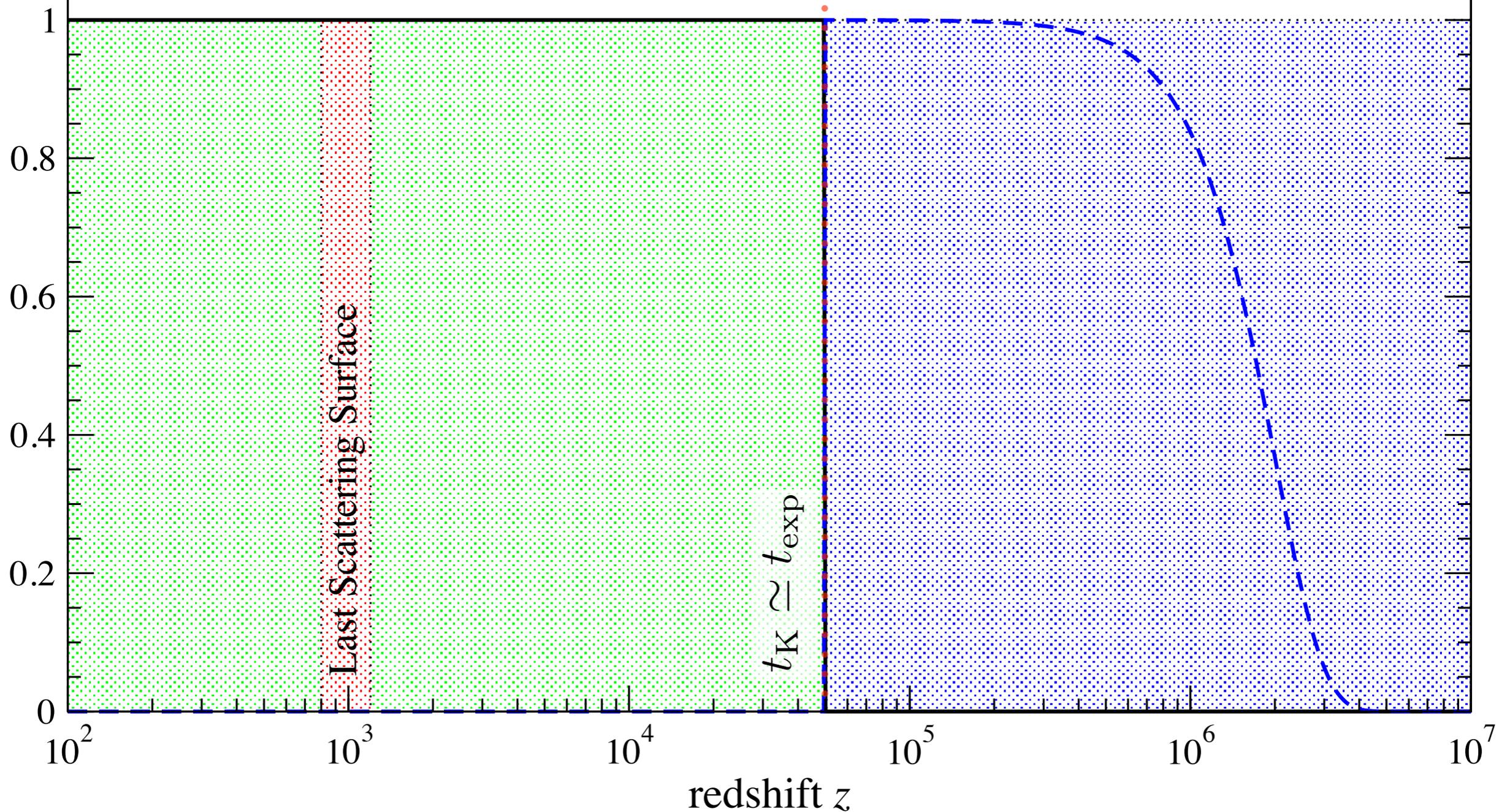
μ - distortion

$$y \simeq \frac{1}{4} \frac{\Delta\rho_\gamma}{\rho_\gamma} \equiv \frac{1}{4} \int_0^{z_K} \frac{d(Q/\rho_\gamma)}{dz'} dz'$$

$$\mu \approx 1.4 \int_{z_K}^{\infty} \frac{d(Q/\rho_\gamma)}{dz'} \mathcal{J}_\mu(z') dz'$$

$$\mathcal{J}_\mu(z) \approx e^{-\left(\frac{z}{2 \times 10^6}\right)^{5/2}}$$

Visibility



y - distortion

μ -y transition

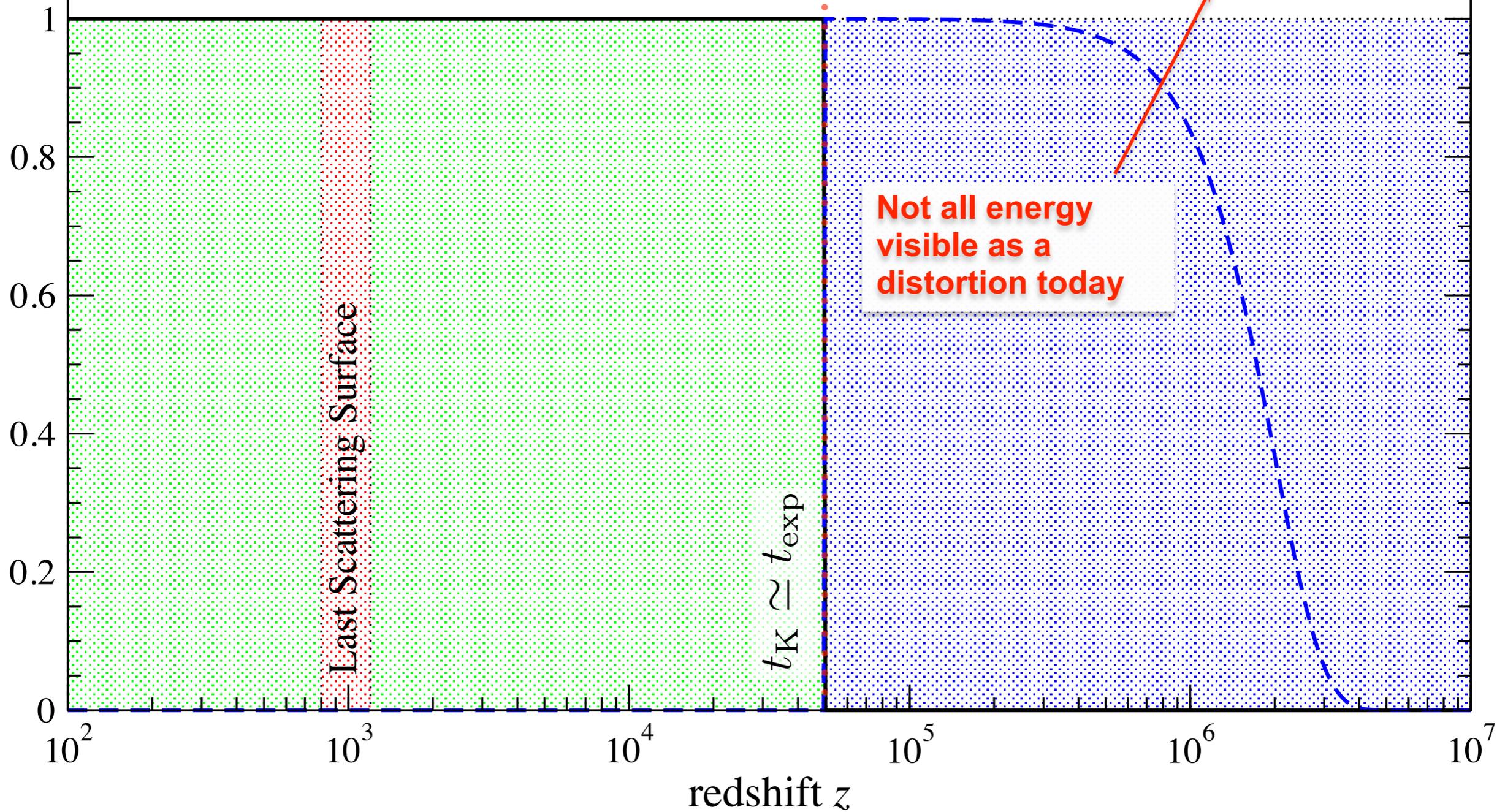
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Visibility



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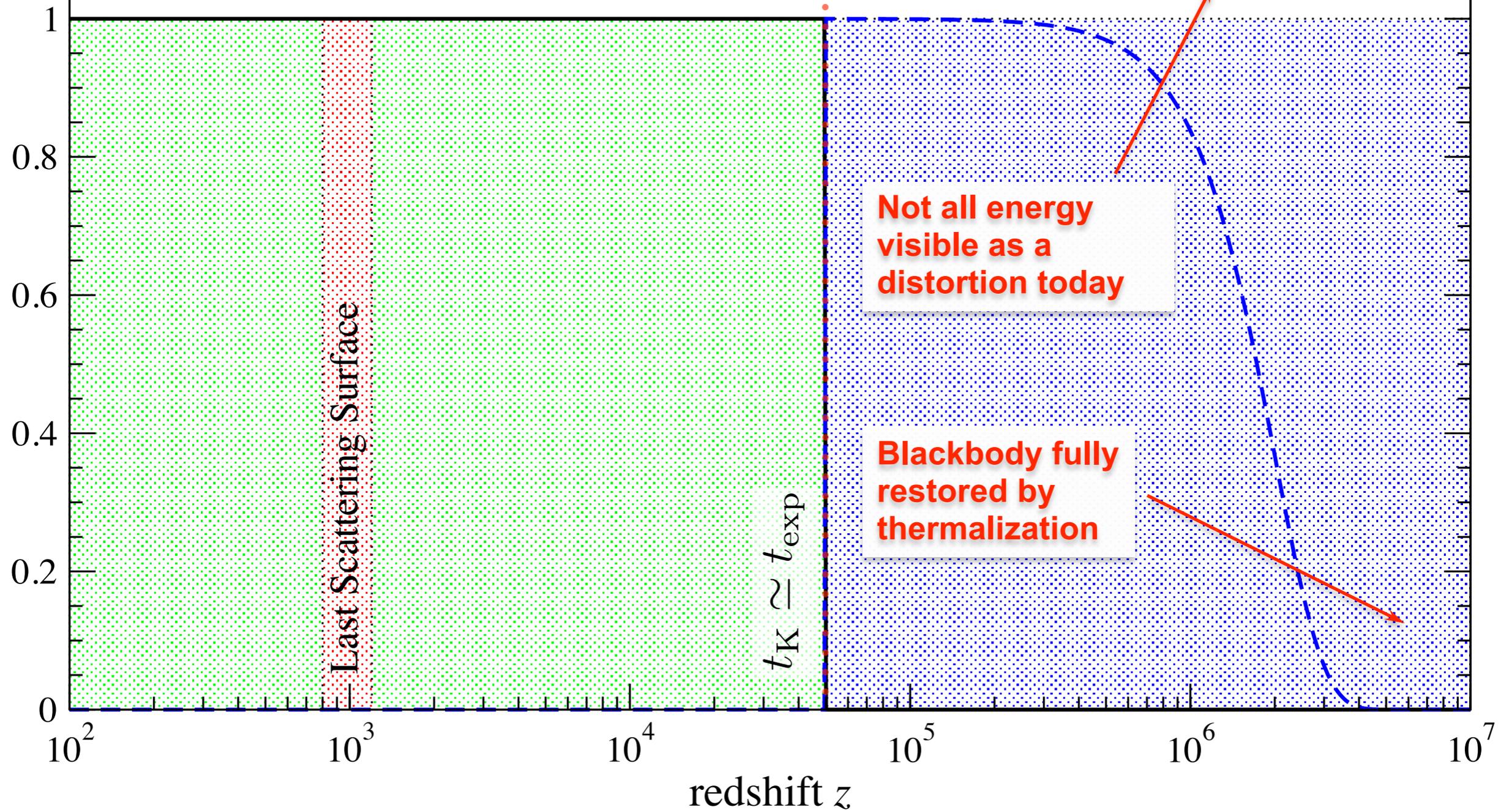
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$$\mathcal{J}_\mu(z) \approx e^{-\left(\frac{z}{2 \times 10^6}\right)^{5/2}}$$

Visibility



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μ -y transition

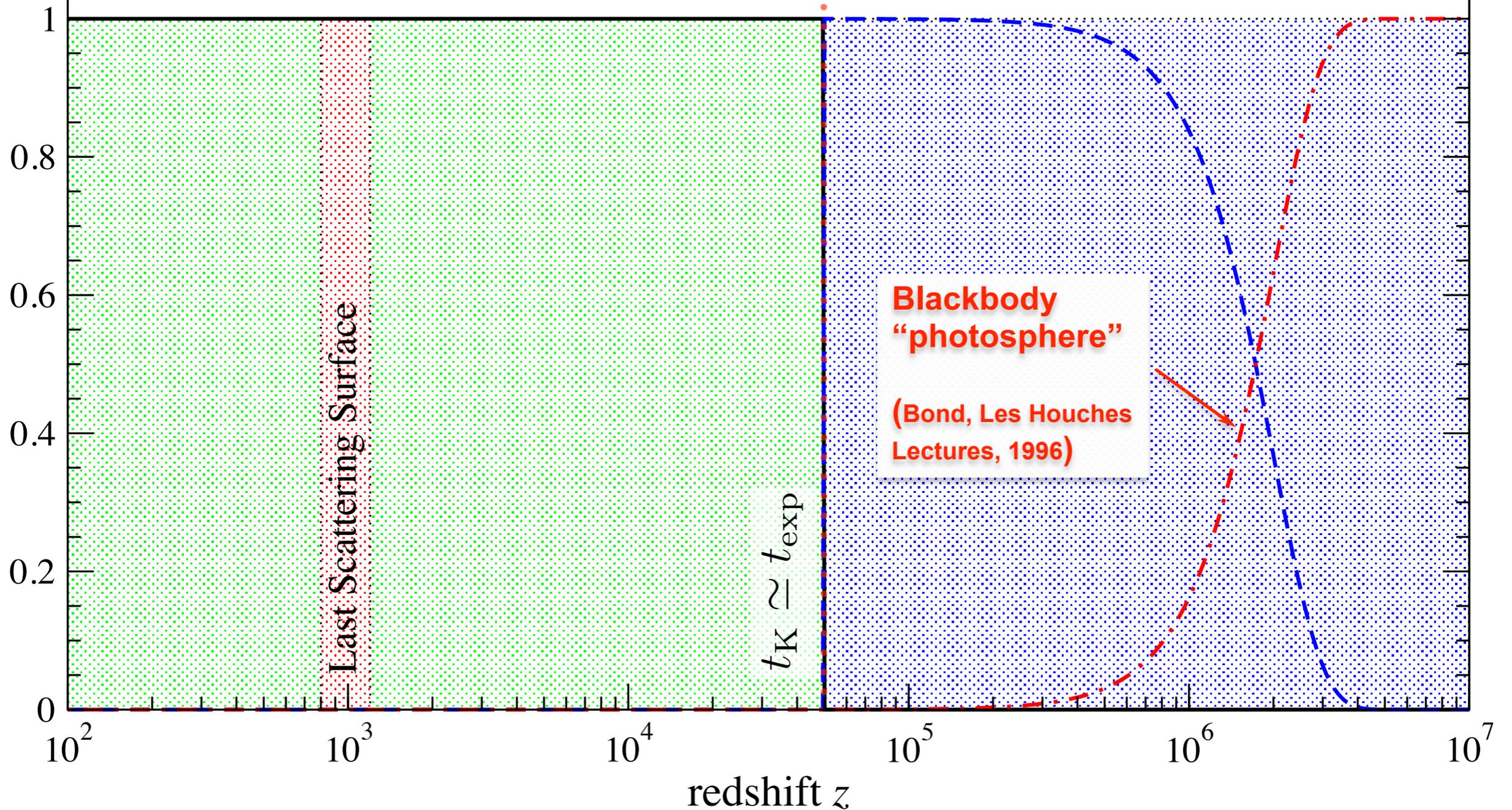
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$$y \simeq \frac{1}{4} \frac{\Delta\rho_\gamma}{\rho_\gamma} \equiv \frac{1}{4} \int_0^{z_K} \frac{d(Q/\rho_\gamma)}{dz'} dz'$$

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$$\mathcal{J}_\mu(z) \approx e^{-\left(\frac{z}{2 \times 10^6}\right)^{5/2}}$$

Visibility



y - distortion

μ -y transition

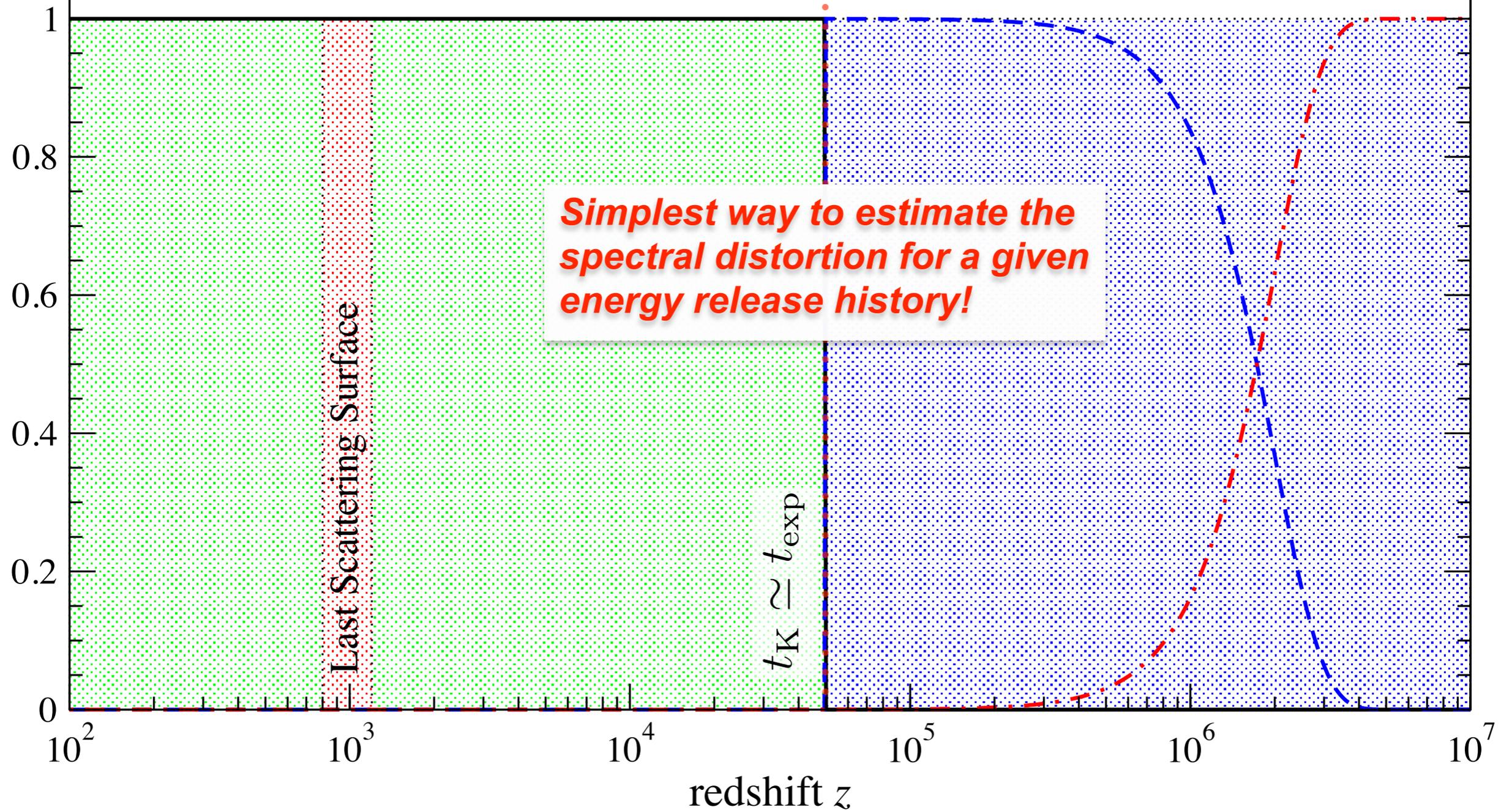
μ - distortion

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$$\mathcal{J}_\mu(z) \approx e^{-\left(\frac{z}{2 \times 10^6}\right)^{5/2}}$$

Visibility



Refined computation of the distortion visibility

(JC 2014, ArXiv:1312.6030)

Can we improve the agreement with CosmoTherm?

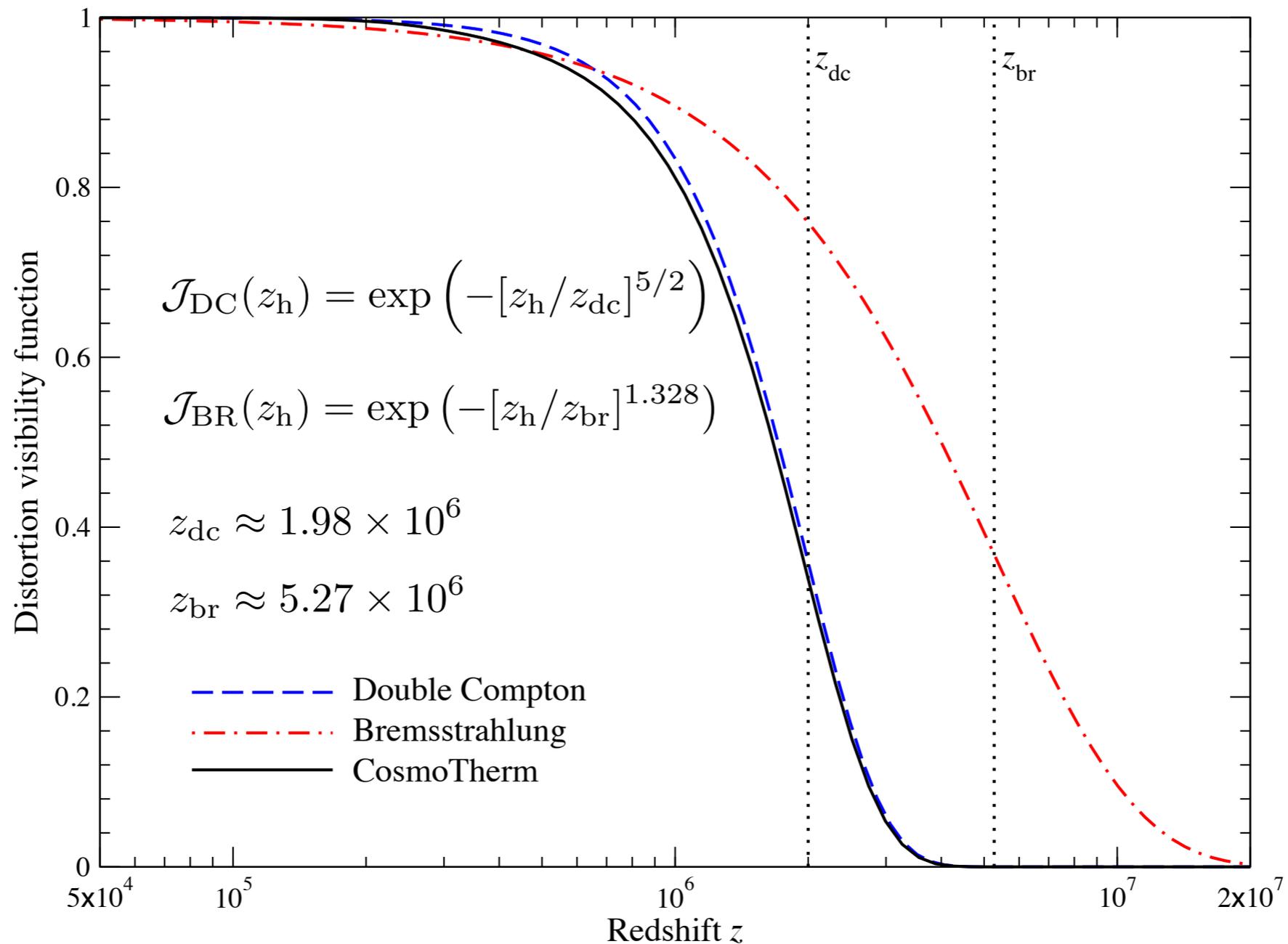


Figure 4.9: Distortion visibility function. We compare $\mathcal{J}_{\text{DC}}(z_h)$, $\mathcal{J}_{\text{BR}}(z_h)$ and the numerical result obtained with CosmoTherm. DC emission significantly change the thermalization efficiency. Deviations from the numerical result can be captured by adding several effects, as discussed in Sect. 4.6.

Can we improve the agreement with CosmoTherm?

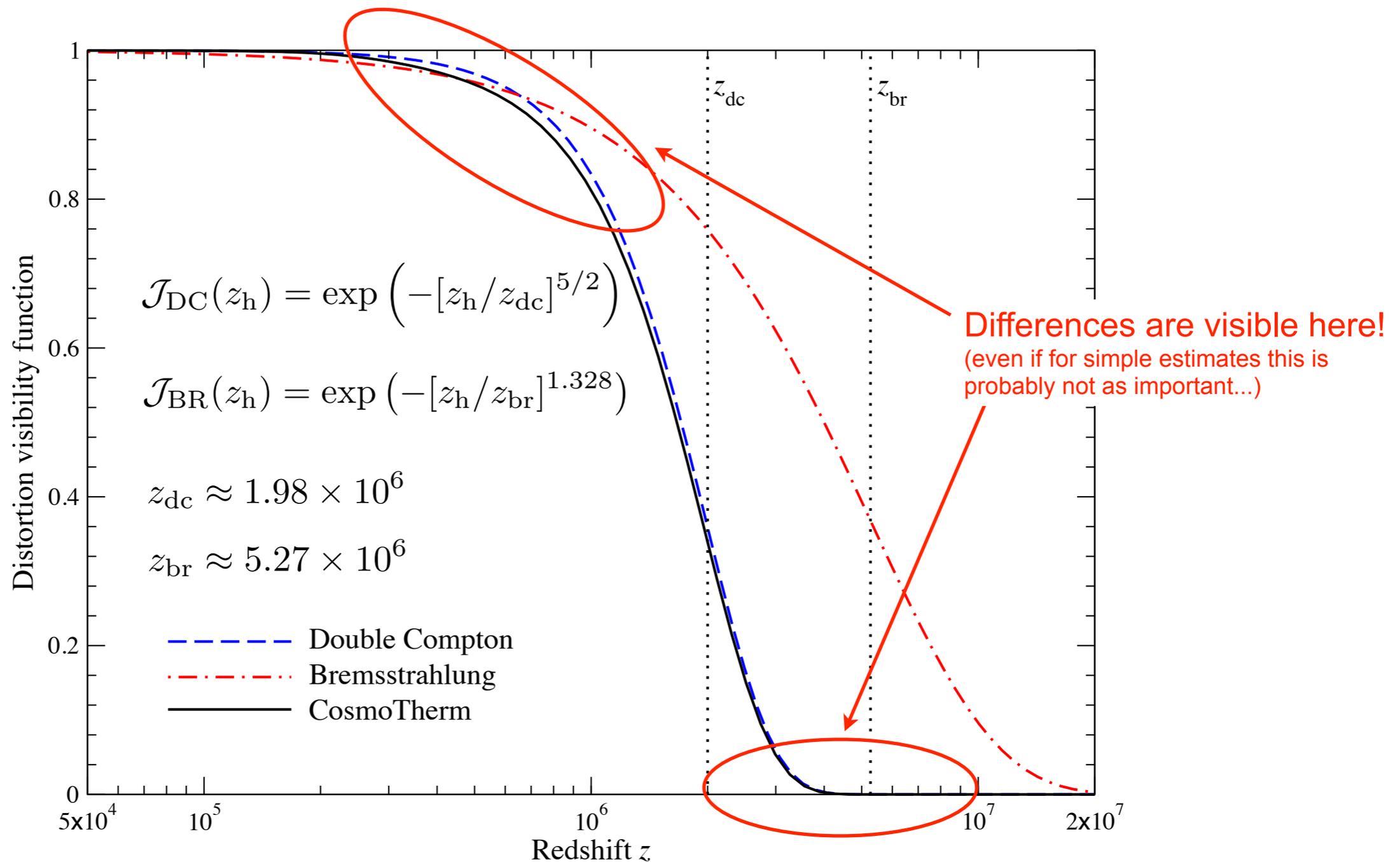


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What do we need to do to improve this?

(i) improve computation of emission integral

- optical depth
- numerical evaluation of integral

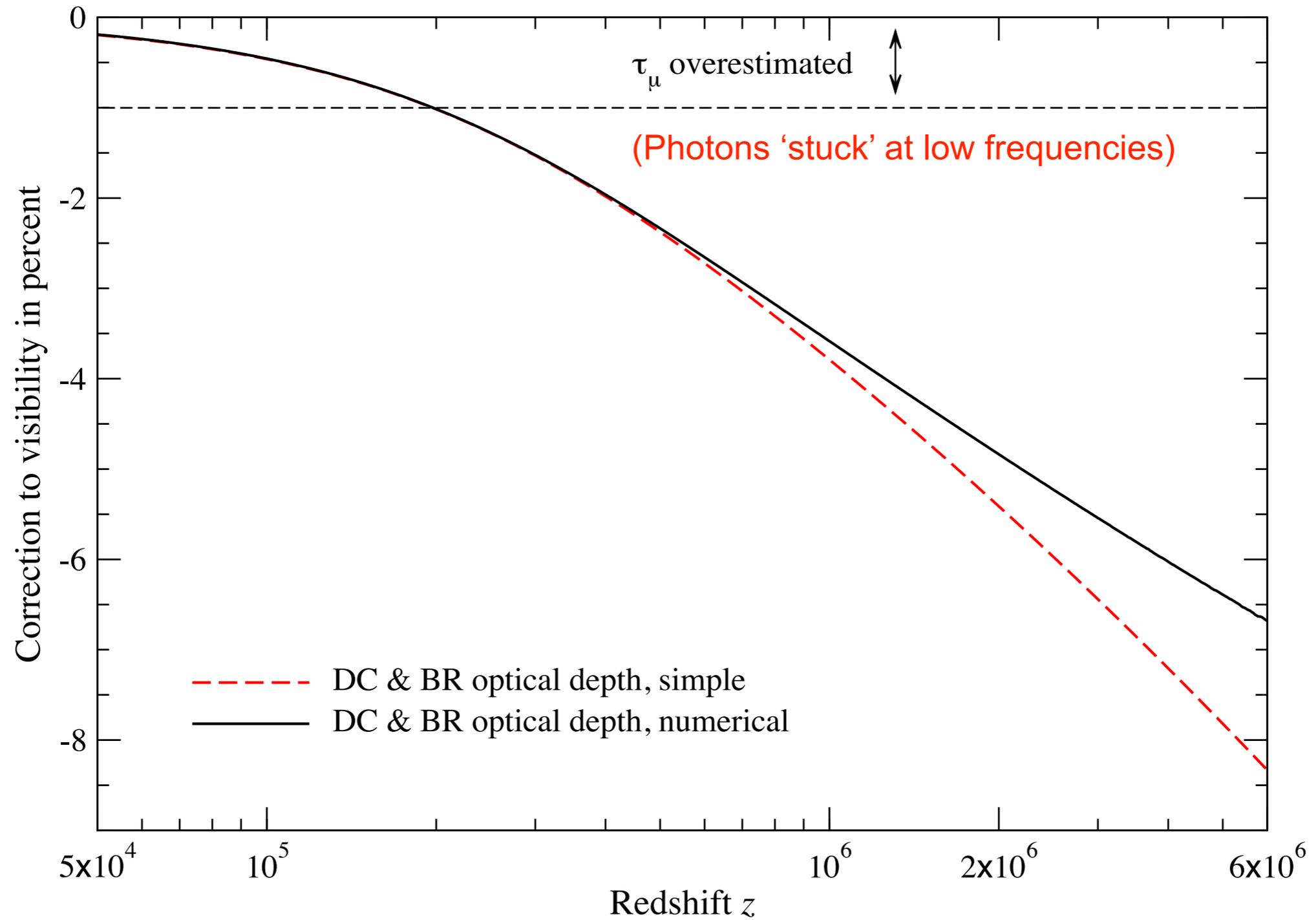
(ii) improve approximation for μ

- beyond the low frequency limit
- add time-dependent corrections
- corrections to emission and scattering terms

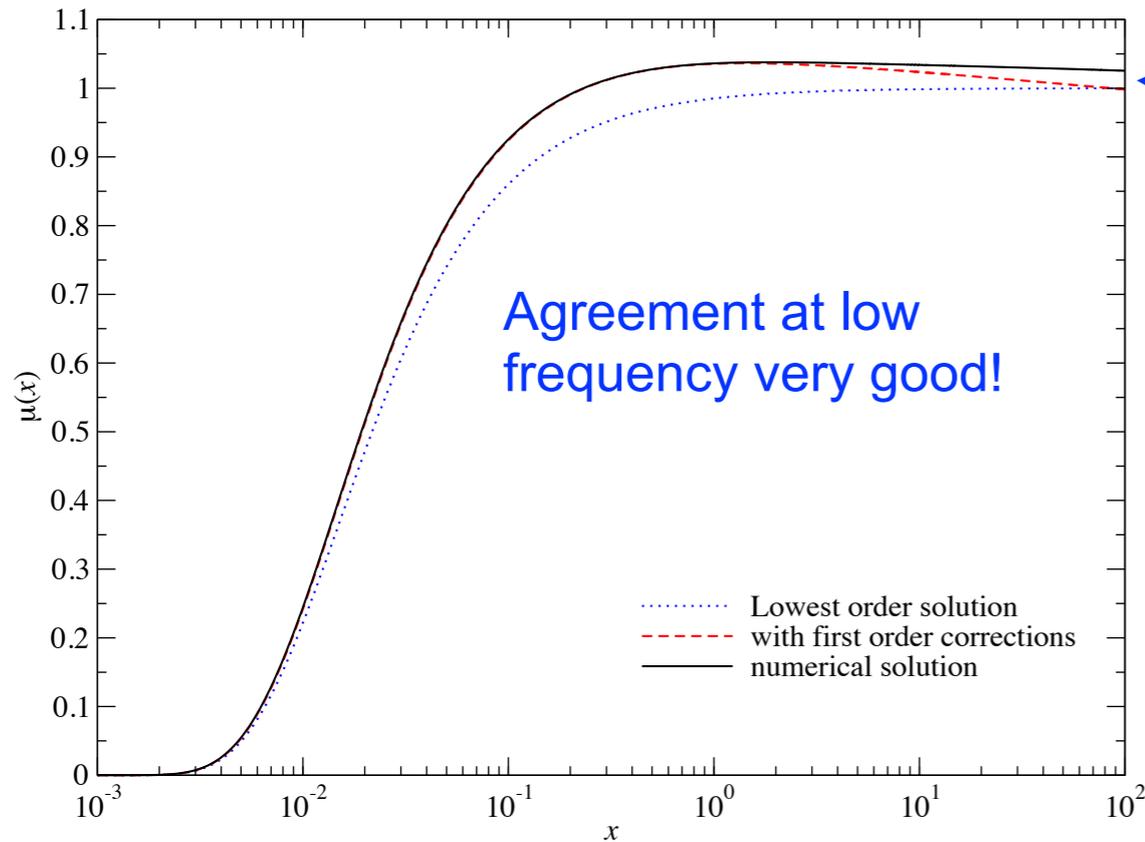
First improvements by Khatri & Sunyaev, 2012

Perturbative study by JC 2014, ArXiv: 1312.6030

Computing the thermalization optical depth better



Correction to the shape of μ



Differences can be avoided by matching with high-frequency solution (see below...)

Determined using energy argument

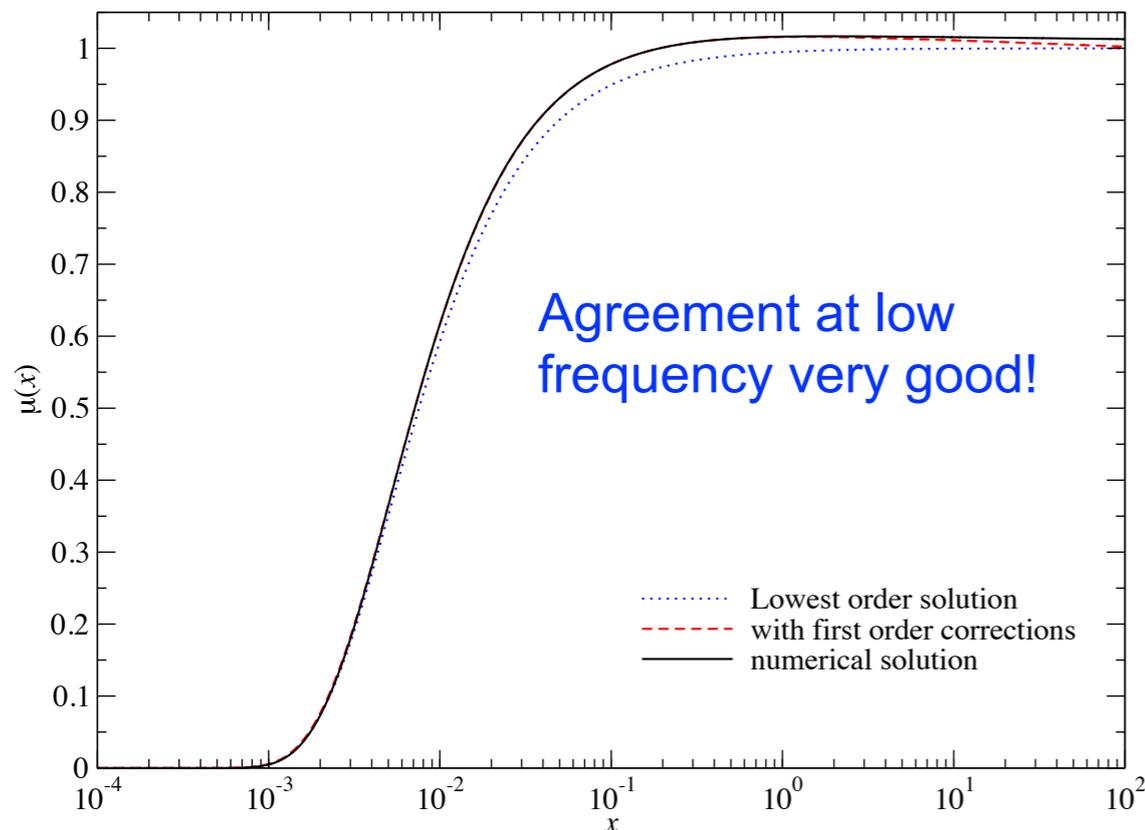
Time-dependence of μ

(similar to Khatri & Sunyaev, 2012)

$$\hat{\mu}^{(1)} \approx C_1 e^{-x_c/x} + \ln(x/x_c) e^{-x_c/x} \partial_y \ln \mu_\infty^{(0)} + D_\mu(x_c/x) \left[\partial_y \ln \mu_\infty^{(0)} + \frac{1}{2} \partial_y \ln x_c \right] + D_{em}(x_c, x_c/x) x_c$$

Time-dependence of μ and x_c

Correction to emission term



- Spectral functions simple expressions

Correction to the photon production integral

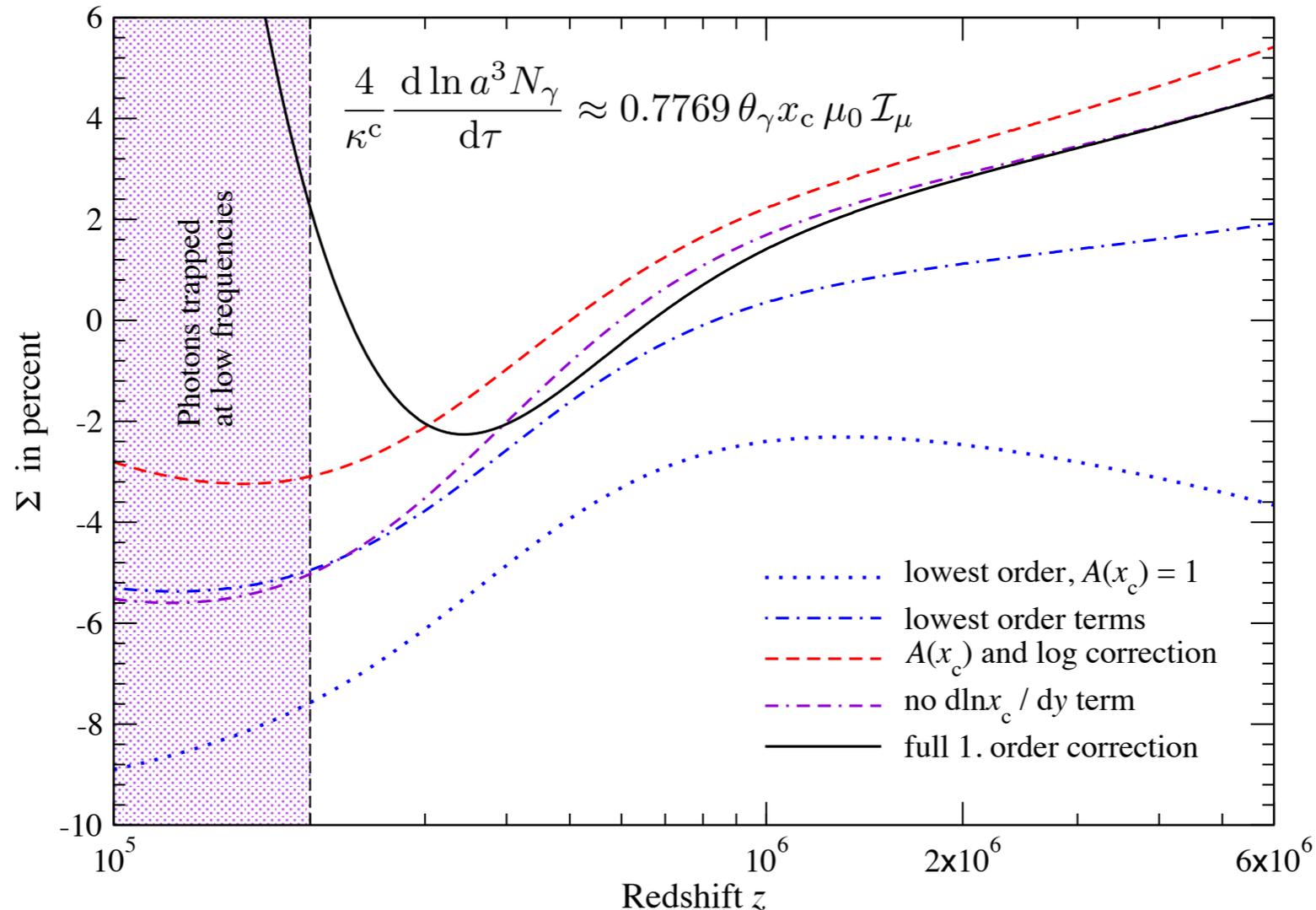


Figure 7. Changes of $\Sigma = \mathcal{I}_{\hat{\mu}} - 1$ for different approximations of $\hat{\mu}$ discussed in Sect. 5.3.2. For the lowest order solution, we used $\hat{\mu} = A(x_c) e^{-x_c/x}$ with $A(x_c) = 1$ and $A(x_c) = A_0(x_c)$ defined by Eq. (57). For the dashed red line, we used $\hat{\mu} = A(x_c) e^{-x_c/x} + \ln(x/x_c) e^{-x_c/x} \partial_y \ln \mu_\infty^{(0)}$, while for the solid black line we used the full first-order expression, Eq. (54), each with their corresponding normalization constants, $A(x_c)$. The double-dash dotted curve also gives the result using Eq. (54), but when neglecting the contribution from $\partial_y \ln x_c$, which becomes large at low redshifts. The shaded region indicates where the high-frequency photon number freezes out.

Correction to the photon production integral

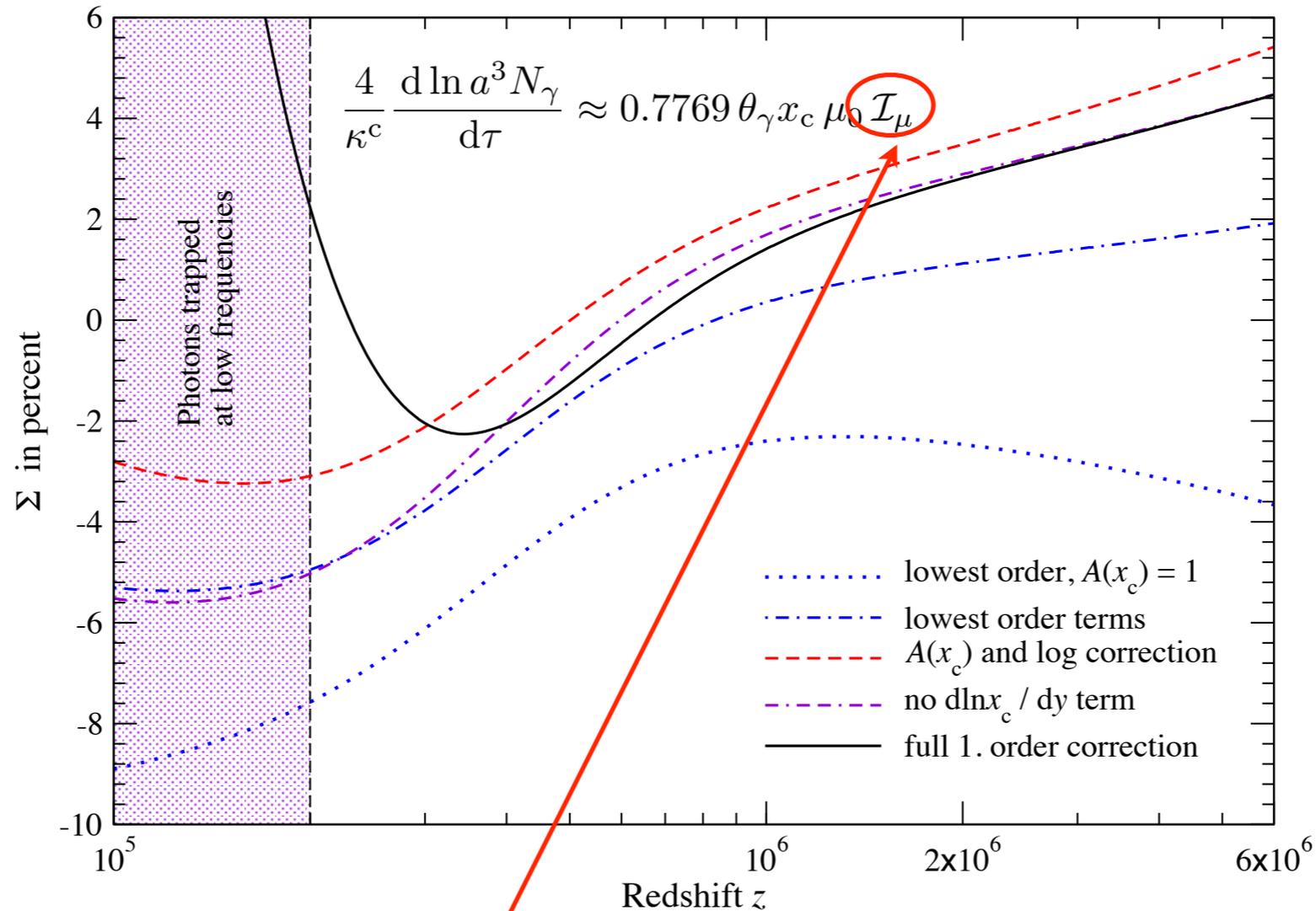


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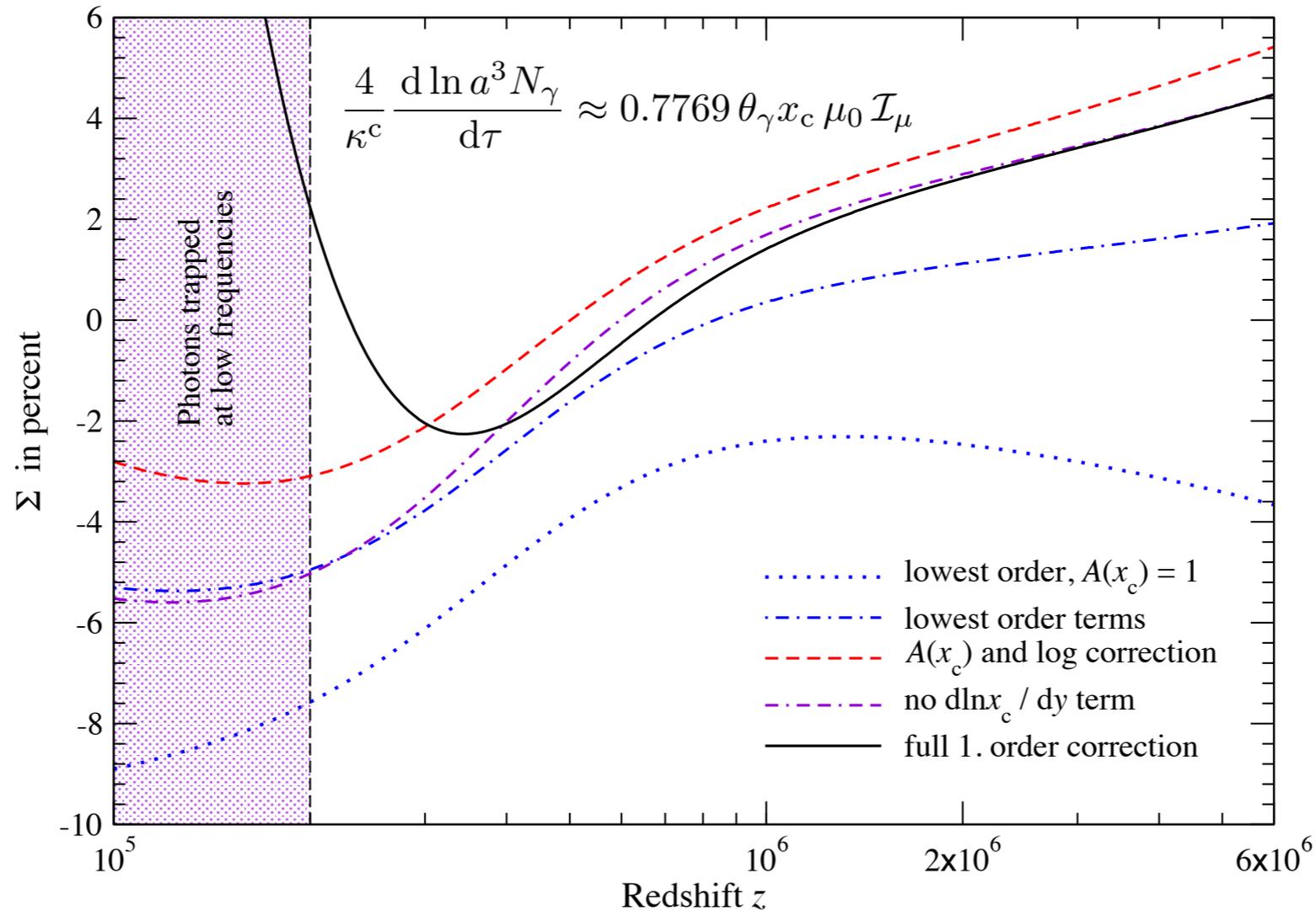
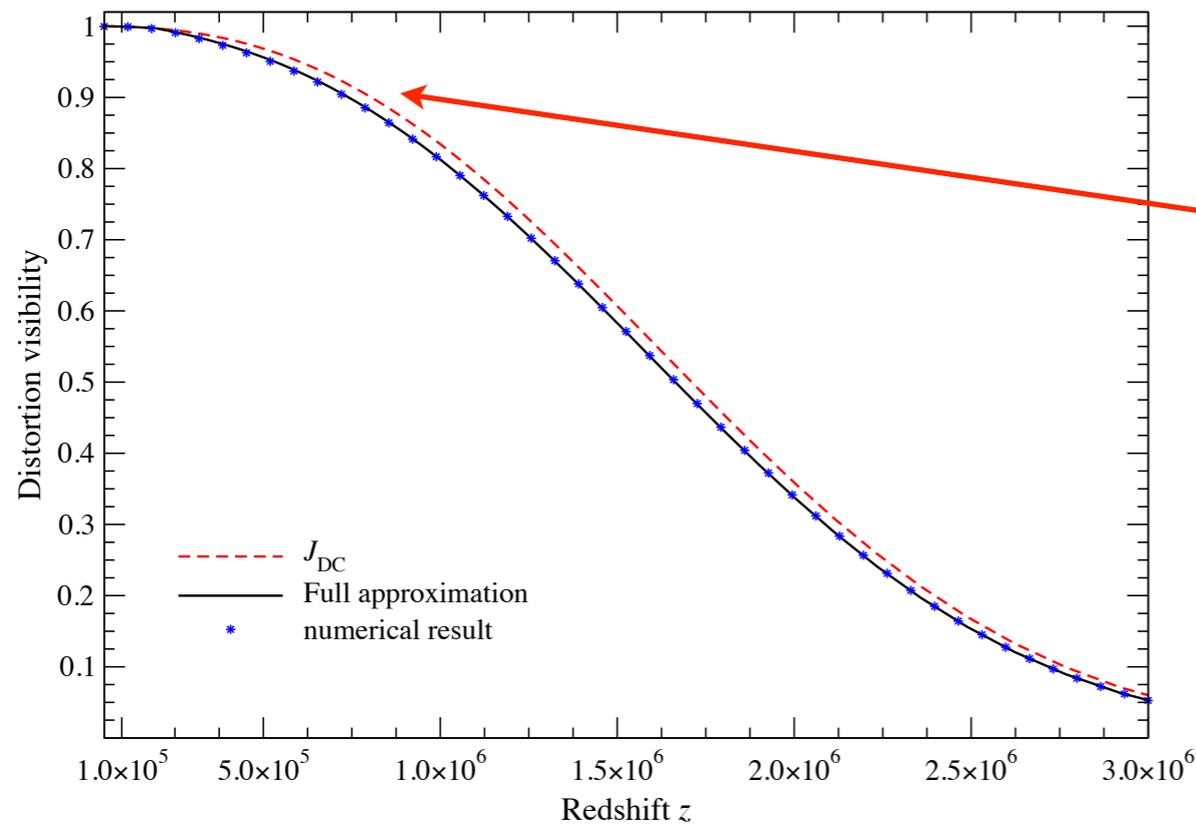


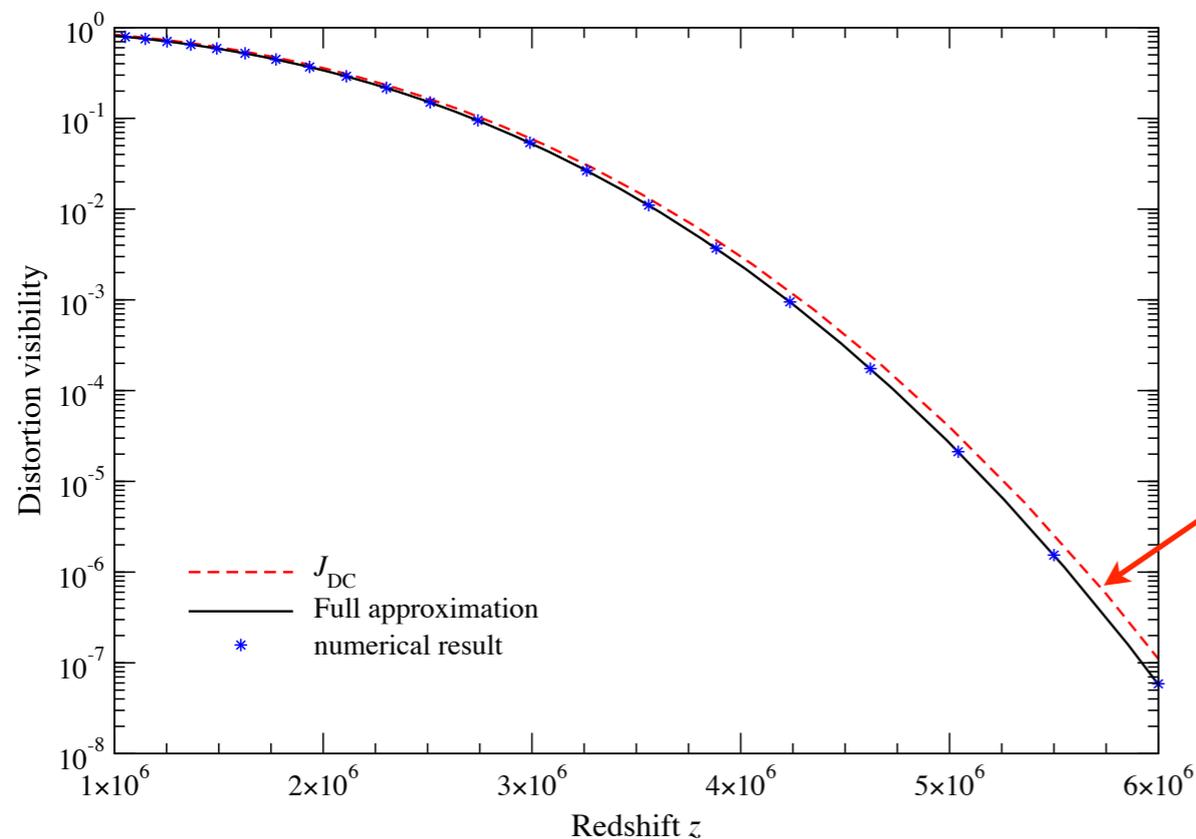
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Final approximation for the visibility function



τ correction from photons being stuck at low frequencies important here
(Not included by KS12)

- Approximation given in terms of simple integrals
- Evaluation very fast and precise
- will be available as part of the *CosmoTherm* package



DC visibility is off by a factor of ~ 2

Contributions from different terms

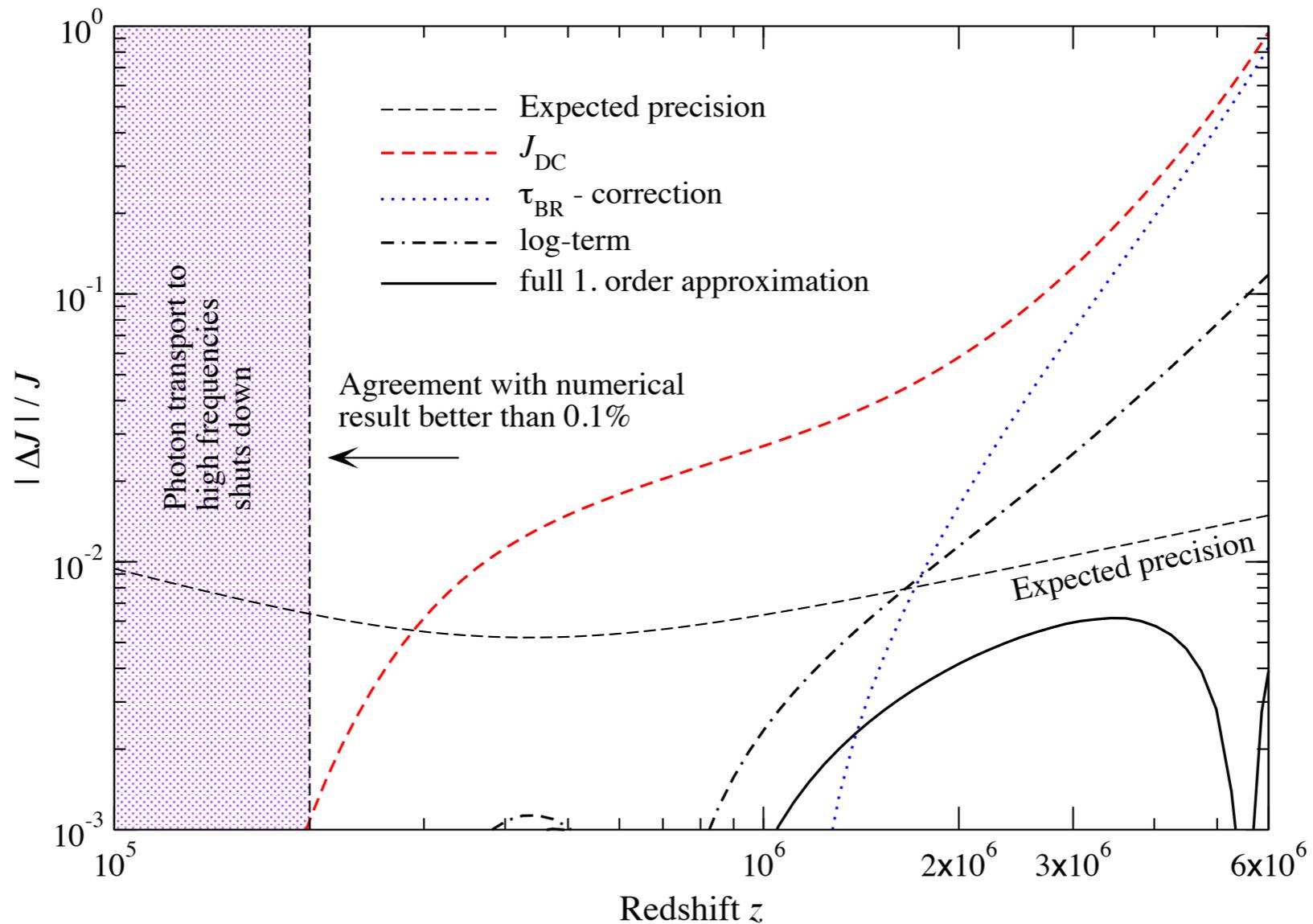


Figure 9. Corrections to the distortion visibility function at different redshifts. For all curves, the numerical result obtained using COSMOTHERM was used as reference. The dashed red line shows $\mathcal{J}_{DC} = e^{-(z/z_{dc})^{5/2}}$ with $z_{dc} = 1.98 \times 10^6$. When only including the BR correction to the optical depth (Sect. 5.2), we obtain the dotted blue line. Only adding the $\ln(x/x_c)e^{-x_c/x}$ term, we improve the agreement at early times. The solid black line gives our approximation based on Eq. (54), with all terms included, showing precision below the level expected in terms of perturbation order $\simeq x_c$.

Comparison with Kharti & Sunyaev, 2012

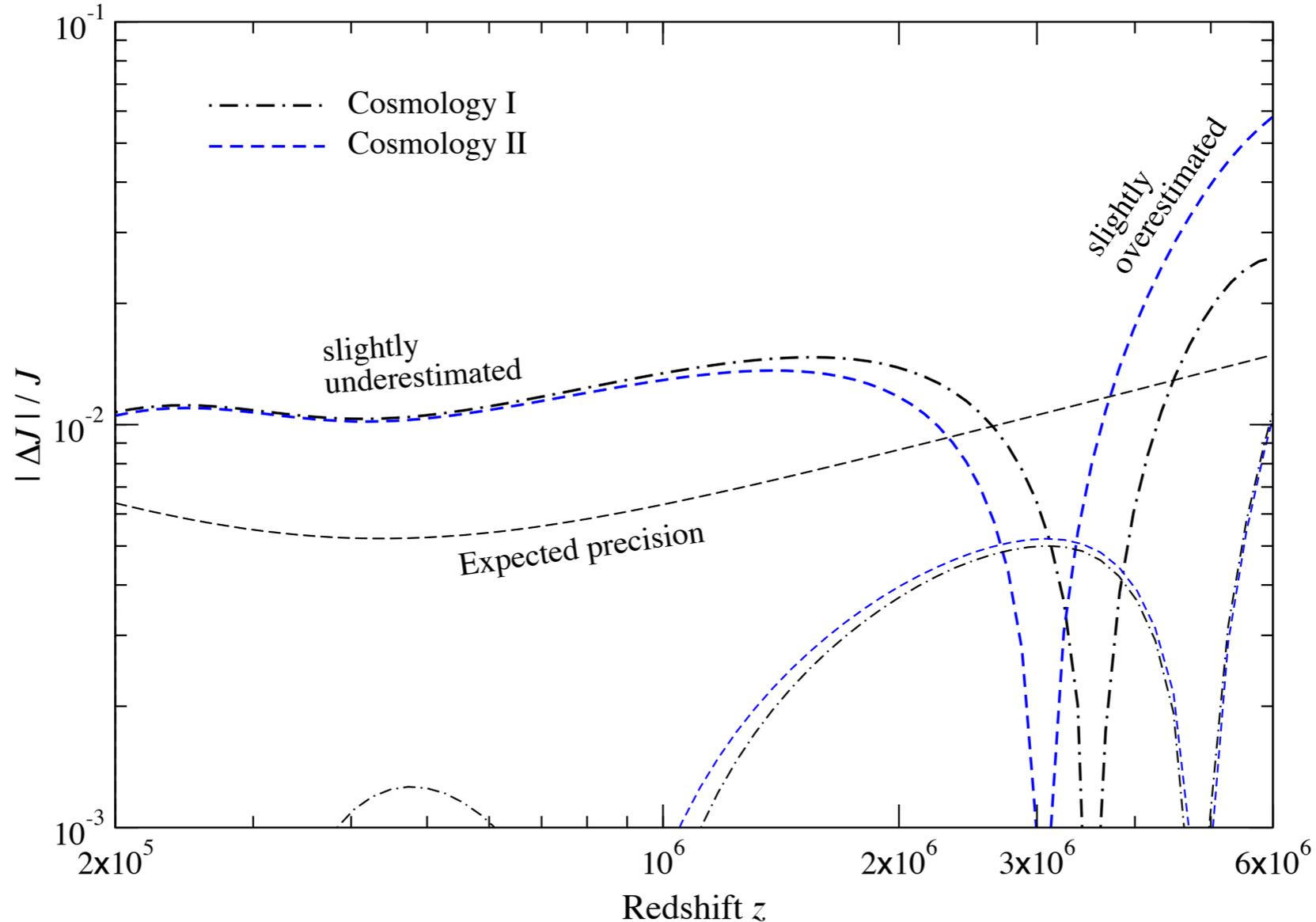
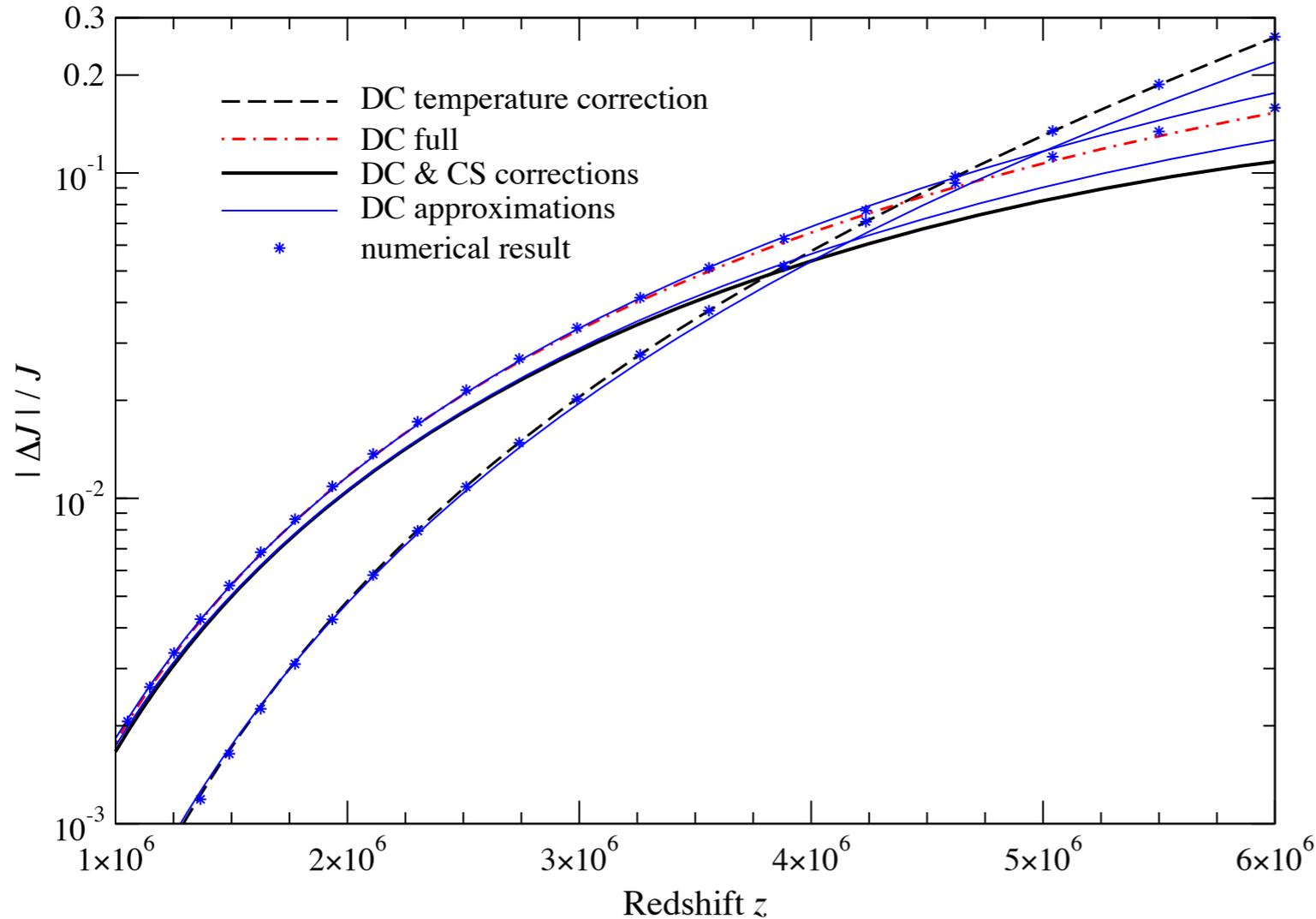


Figure 10. Comparison of the approximation given by KS12 with our numerical result from COSMOTHERM. We compare for the cosmology used in KS12 (blue dashed) and the one used here (black dash-dotted). Their simple expression (heavy lines) works very well overall. Our approximation (thin lines) represents our numerical result below the expected precision $\simeq x_c$ at all redshifts and giving $\lesssim 0.1\%$ precision at $z \lesssim 10^6$. In Sect. 5.4.1, we briefly discuss the possible explanations for difference with KS12.

DC and CS relativistic corrections



DC temperature correction

$$\Delta\tau_{\mu}^{\text{DC}}(z, 0) \approx -5.06 \theta_{\gamma} (z/z_{\text{dc}})^{5/2}$$

DC frequency correction

$$\Delta\tau_{\mu}^{\text{DC}}(z, 0) \approx [3.70 \times 10^{-3} + 1.43 x_c^{\text{DC},0}] (z/z_{\text{dc}})^{5/2}$$

CS correction

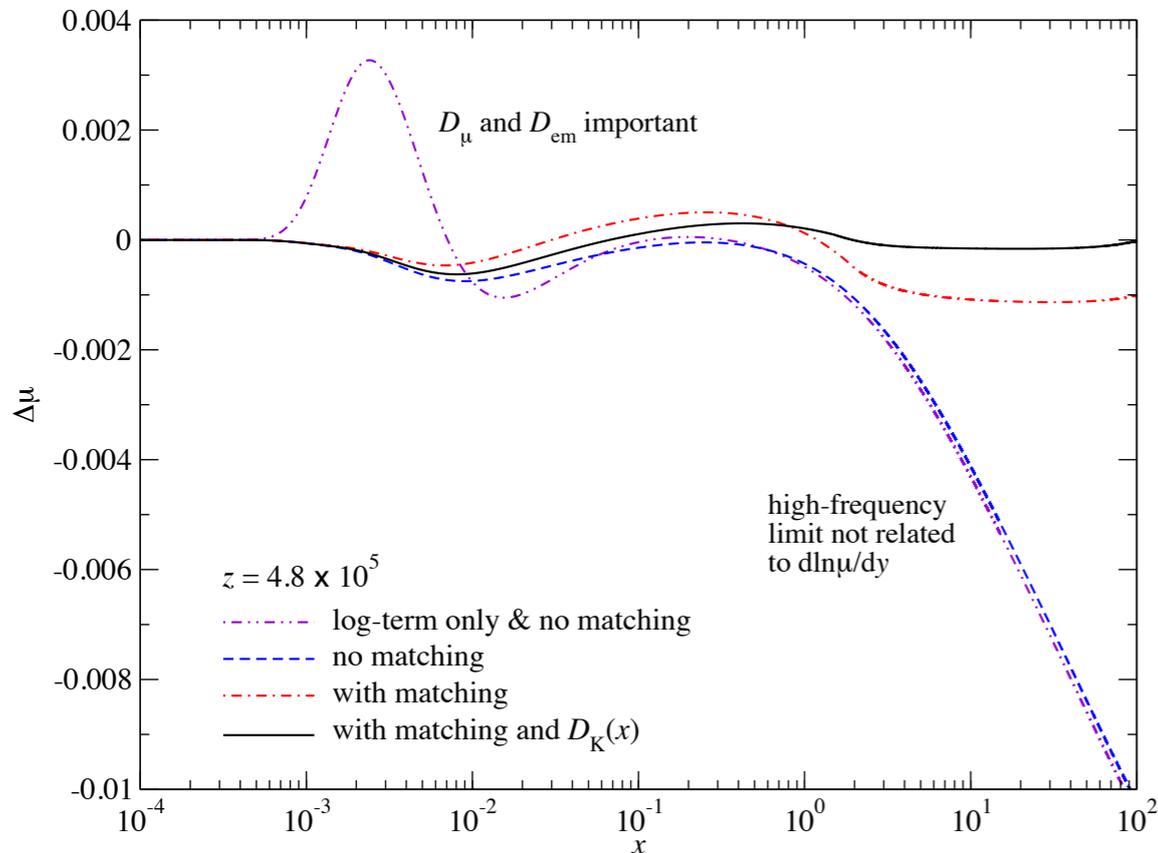
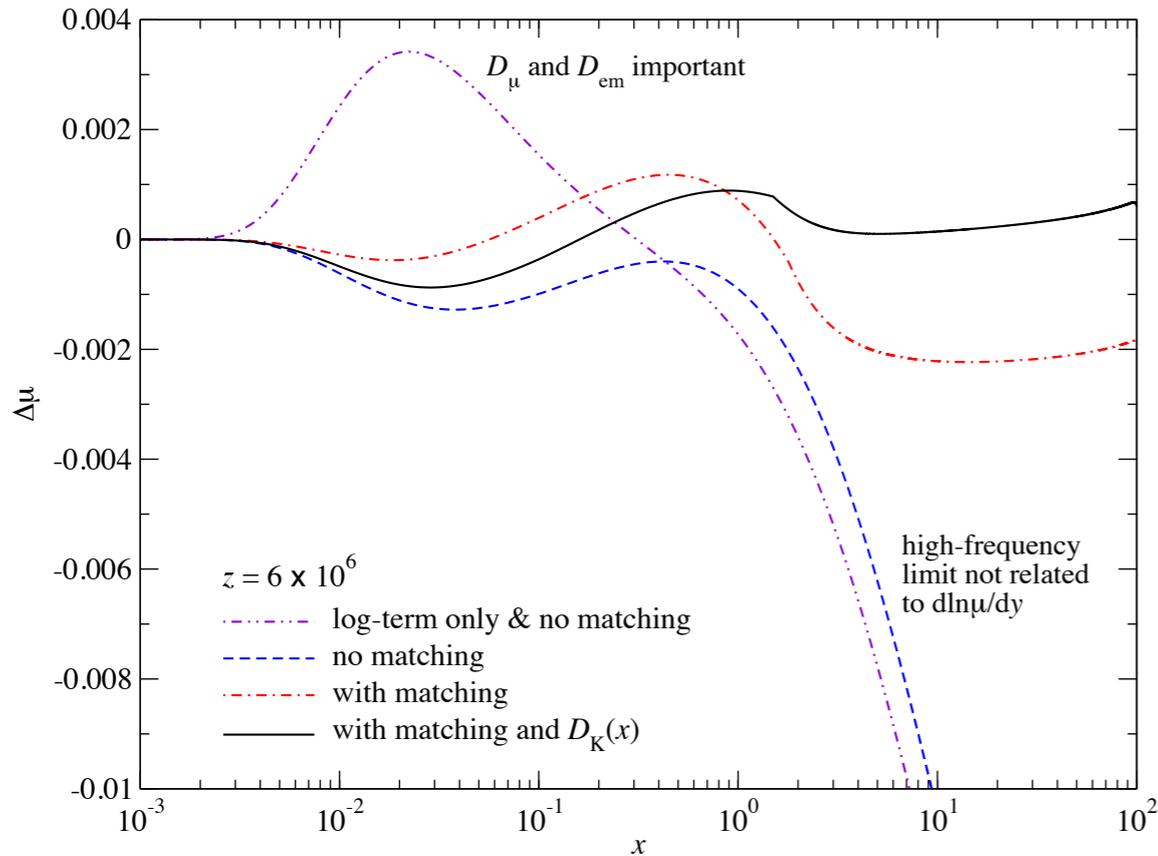
$$\Delta\tau_{\mu}^{\text{CS}}(z, 0) \approx -1.23 \theta_{\gamma} (z/z_{\text{dc}})^{5/2}$$

Figure 13. DC and CS corrections to the distortion visibility function. The full non-relativistic result is used as a reference. The thin solid blue lines give the simple approximations discussed in Sect. 6, while the other lines were obtained by modifying our perturbation expansion, Eq. (54), accordingly. For the DC corrections, we also show the full numerical result for the correction obtained with COSMOTHERM.

- individual effects as large as ~20-30%

- cancelation leaves only a ~10% effect

Solution at high frequencies



Evolution equation at $x \gg 1$

$$\partial_y \mu - x \partial_y (T_e / T_\gamma) \approx x^2 \mu'' + (4 - x) x \mu'$$

Lowest order solution

$$\mu_{\text{high}}(x) \approx C_{\text{high}} + \ln(x) \partial_y (T_e^{(0)} / T_\gamma)$$

$$\implies n(t, x_e) \simeq \alpha(t) x_e^\gamma(t) e^{-x_e}$$

First order correction in x

$$\hat{\mu}_{\text{high}}(x) \approx C_{\text{high}} + \ln(x) \frac{\partial_y (T_e^{(0)} / T_\gamma)}{\mu_\infty^{(0)}} + \left(\partial_y \ln \mu_\infty^{(0)} - 3 \frac{\partial_y (T_e^{(0)} / T_\gamma)}{\mu_\infty^{(0)}} \right) \frac{1 + \frac{1}{x} + \frac{2}{3x}}{x}$$

Compton corrections
↓

(Not captured by Khatri & Sunyaev 2012 solution...)

Numerical solution of the thermalization problem

Final Set of evolution equations

Photon field

$$\frac{\partial f}{\partial \tau} \approx \frac{\theta_e}{x^2} \frac{\partial}{\partial x} x^4 \left[\frac{\partial}{\partial x} f + \frac{T_\gamma}{T_e} f(1+f) \right] + \frac{K_{\text{BR}} e^{-x_e}}{x_e^3} [1 - f(e^{x_e} - 1)] + \frac{K_{\text{DC}} e^{-2x}}{x^3} [1 - f(e^{x_e} - 1)] + S(\tau, x)$$

$$K_{\text{BR}} = \frac{\alpha}{2\pi} \frac{\lambda_e^3}{\sqrt{6\pi} \theta_e^{7/2}} \sum_i Z_i^2 N_i \bar{g}_{\text{ff}}(Z_i, T_e, T_\gamma, x_e), \quad K_{\text{DC}} = \frac{4\alpha}{3\pi} \theta_\gamma^2 I_{\text{dc}} g_{\text{dc}}(T_e, T_\gamma, x)$$

$$\bar{g}_{\text{ff}}(x_e) \approx \begin{cases} \frac{\sqrt{3}}{\pi} \ln\left(\frac{2.25}{x_e}\right) & \text{for } x_e \leq 0.37 \\ 1 & \text{otherwise} \end{cases}, \quad g_{\text{dc}} \approx \frac{1 + \frac{3}{2}x + \frac{29}{24}x^2 + \frac{11}{16}x^3 + \frac{5}{12}x^4}{1 + 19.739\theta_\gamma - 5.5797\theta_e}.$$

$$I_{\text{dc}} = \int x^4 f(1+f) dx \approx 4\pi^4/15$$

Ordinary matter temperature

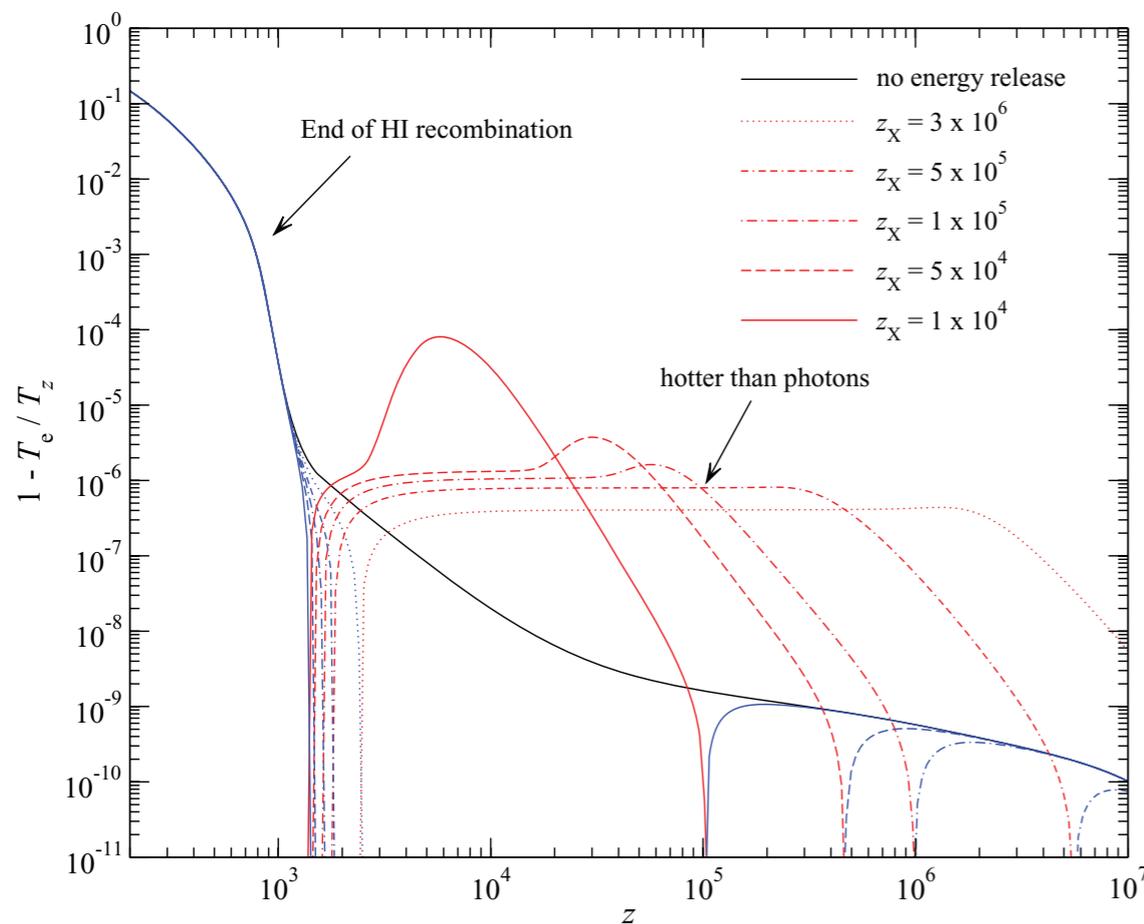
$$\frac{d\rho_e}{d\tau} = \frac{d(T_e/T_\gamma)}{d\tau} = \frac{t_{\text{T}} \dot{Q}}{\alpha_{\text{h}} \theta_\gamma} + \frac{4\tilde{\rho}_\gamma}{\alpha_{\text{h}}} [\rho_e^{\text{eq}} - \rho_e] - \frac{4\tilde{\rho}_\gamma}{\alpha_{\text{h}}} \mathcal{H}_{\text{DC, BR}}(\rho_e) - H t_{\text{T}} \rho_e.$$

$$k\alpha_{\text{h}} = \frac{3}{2}k[N_e + N_{\text{H}} + N_{\text{He}}] = \frac{3}{2}kN_{\text{H}}[1 + f_{\text{He}} + X_e] \quad \rho_e^{\text{eq}} = T_e^{\text{eq}}/T_\gamma$$

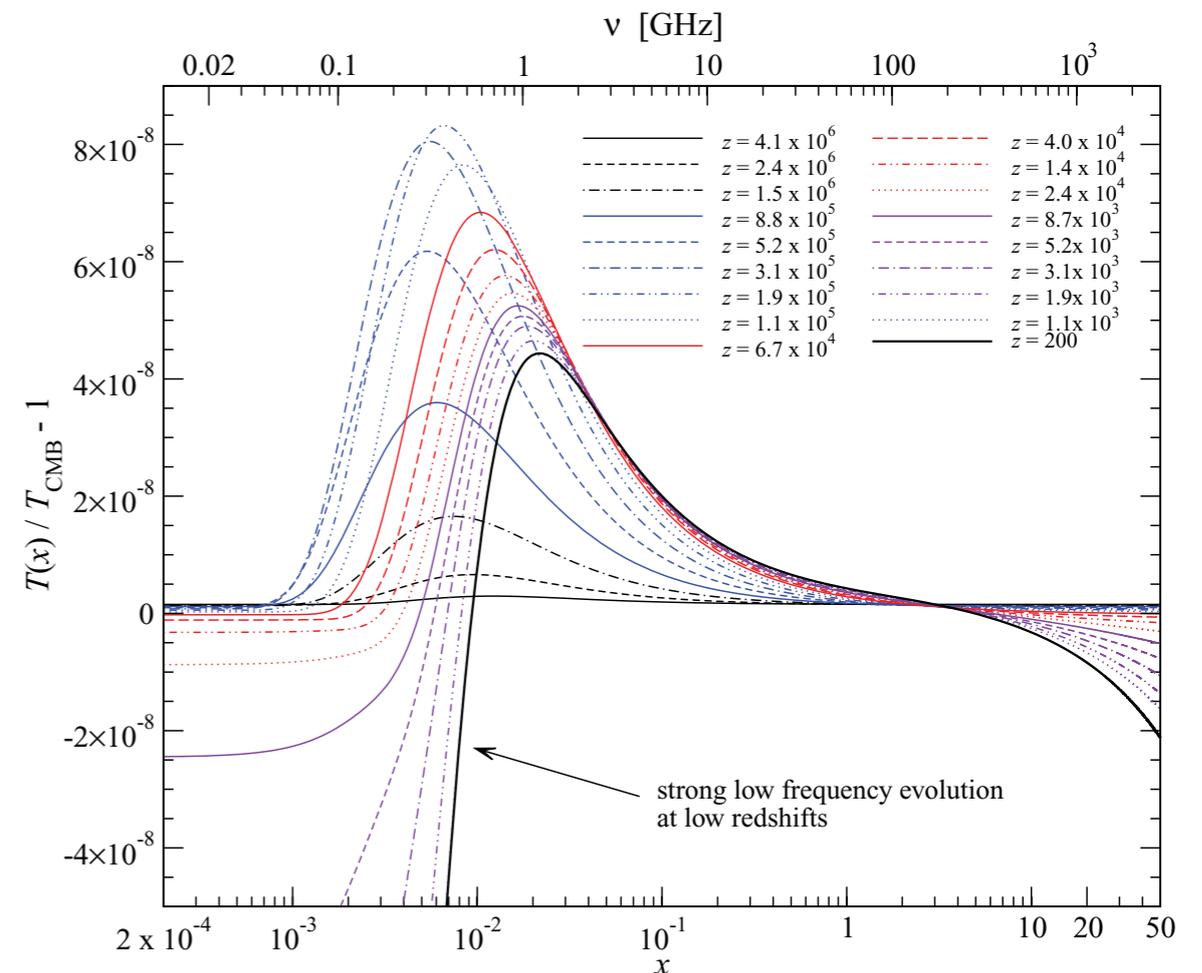
$$\tilde{\rho}_\gamma = \rho_\gamma/m_e c^2 \quad T_e^{\text{eq}} = T_\gamma \frac{\int x^4 f(1+f) dx}{4 \int x^3 f dx} \equiv \frac{h}{k} \frac{\int \nu^4 f(1+f) d\nu}{4 \int \nu^3 f d\nu}$$

CosmoTherm: a new flexible thermalization code

- Solve the thermalization problem for a *wide range* of energy release histories
- several scenarios already implemented (*decaying particles, damping of acoustic modes*)
- first explicit solution of time-dependent energy release scenarios
- open source code
- will be available at www.Chluba.de/CosmoTherm/
- Main reference: JC & Sunyaev, MNRAS, 2012 (arXiv:1109.6552)



Electron temperature evolution



Evolution of distortion

Example: *Energy release by decaying relict particle*

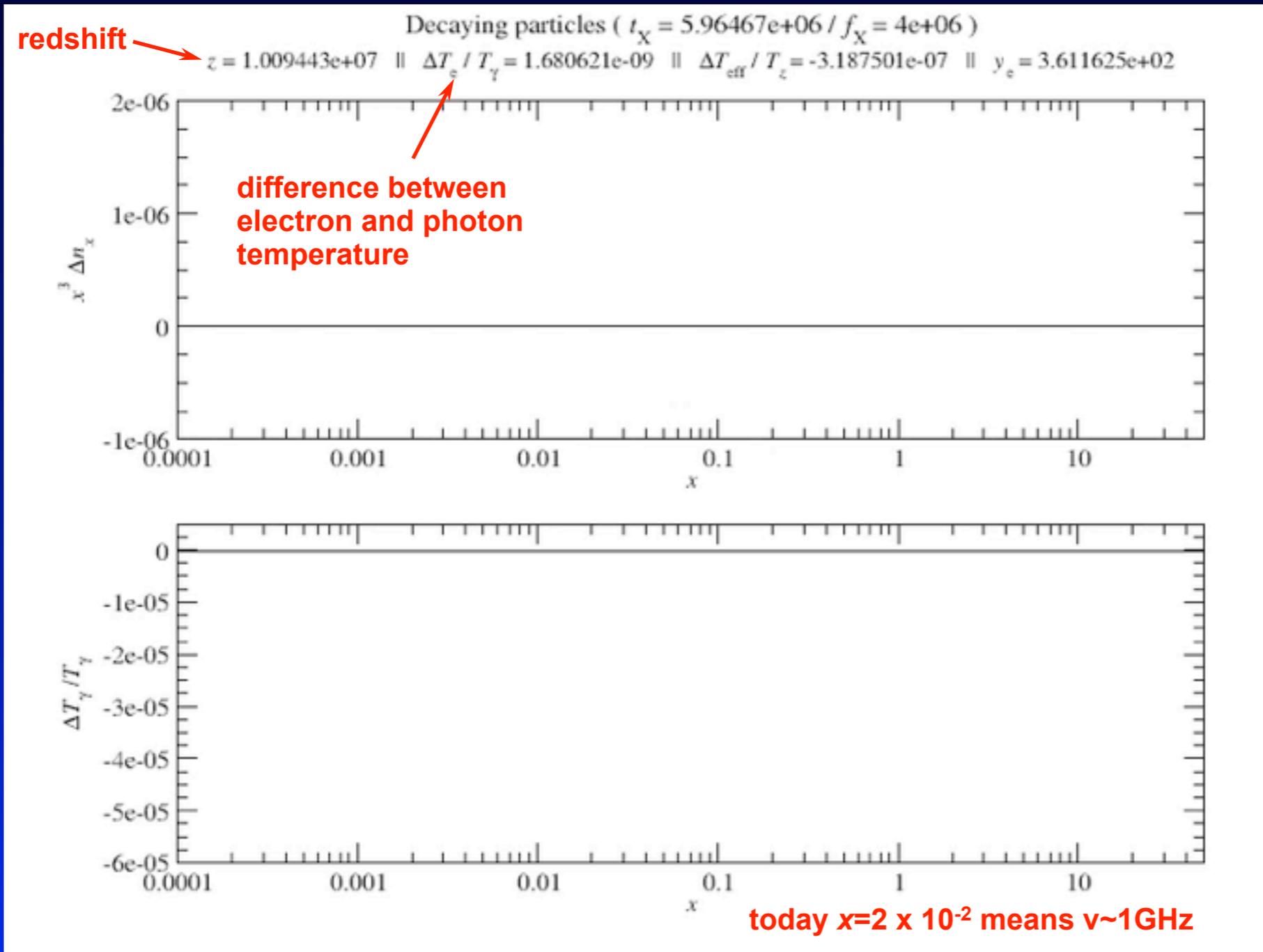
redshift →

↗
difference between
electron and photon
temperature

- initial condition: *full equilibrium*
- total energy release:
 $\Delta\rho/\rho \sim 1.3 \times 10^{-6}$
- most of energy release around:
 $z \sim 2 \times 10^6$
- positive μ -distortion
- high frequency distortion frozen around $z \sim 5 \times 10^5$
- late ($z < 10^3$) free-free absorption at very low frequencies ($T_e < T_\gamma$)

today $x = 2 \times 10^{-2}$ means $\nu \sim 1 \text{ GHz}$

Example: *Energy release by decaying relict particle*



- initial condition: *full equilibrium*
- total energy release: $\Delta\rho/\rho \sim 1.3 \times 10^{-6}$
- most of energy release around: $z_X \sim 2 \times 10^6$
- positive μ -distortion
- high frequency distortion frozen around $z \approx 5 \times 10^5$
- late ($z < 10^3$) free-free absorption at very low frequencies ($T_e < T_\gamma$)

*What about the μ - γ transition regime?
Is the transition really as abrupt?*

y - distortion

μ -y transition

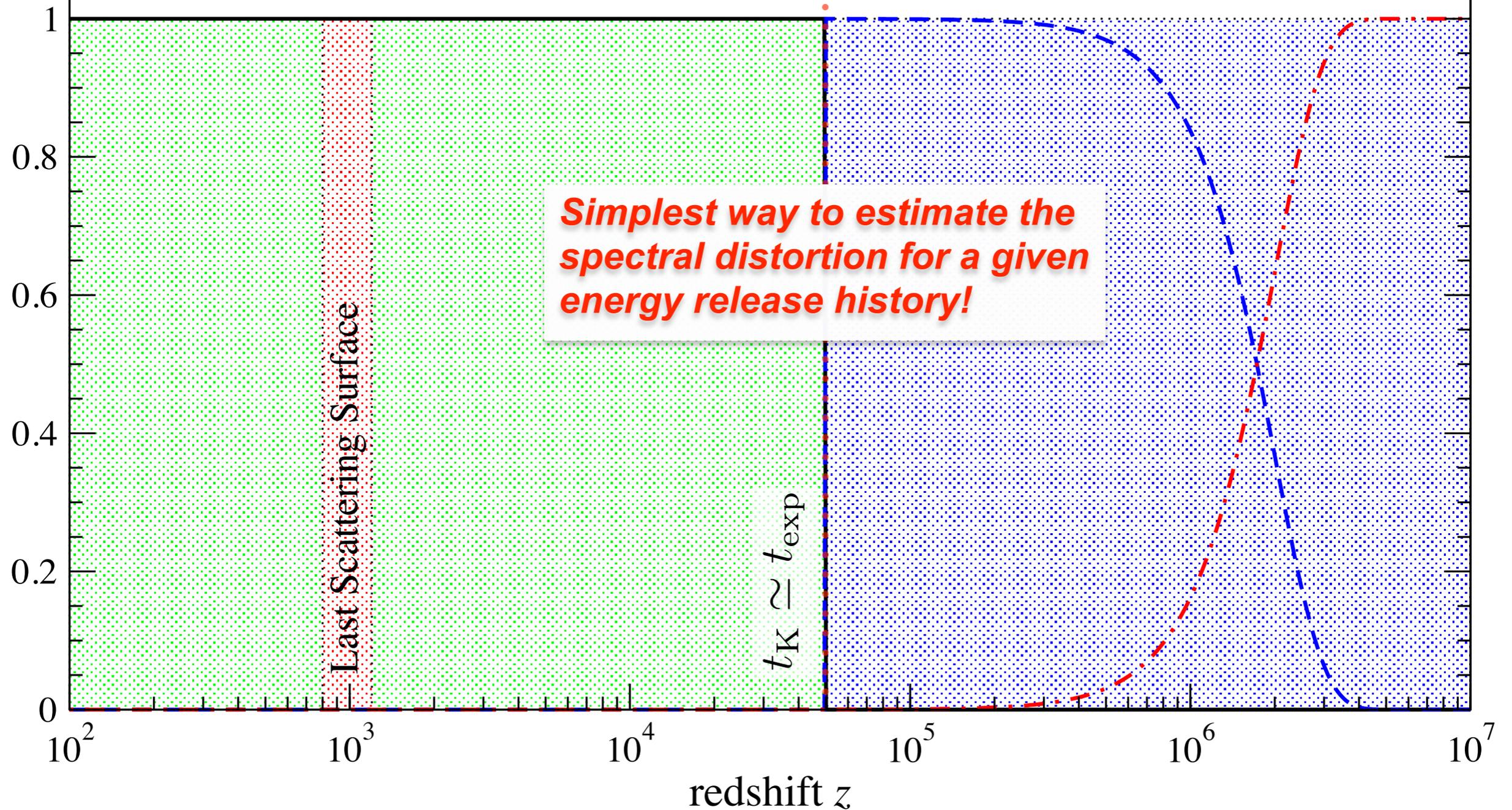
μ - distortion

$$y \simeq \frac{1}{4} \frac{\Delta\rho_\gamma}{\rho_\gamma} \equiv \frac{1}{4} \int_0^{z_K} \frac{d(Q/\rho_\gamma)}{dz'} dz'$$

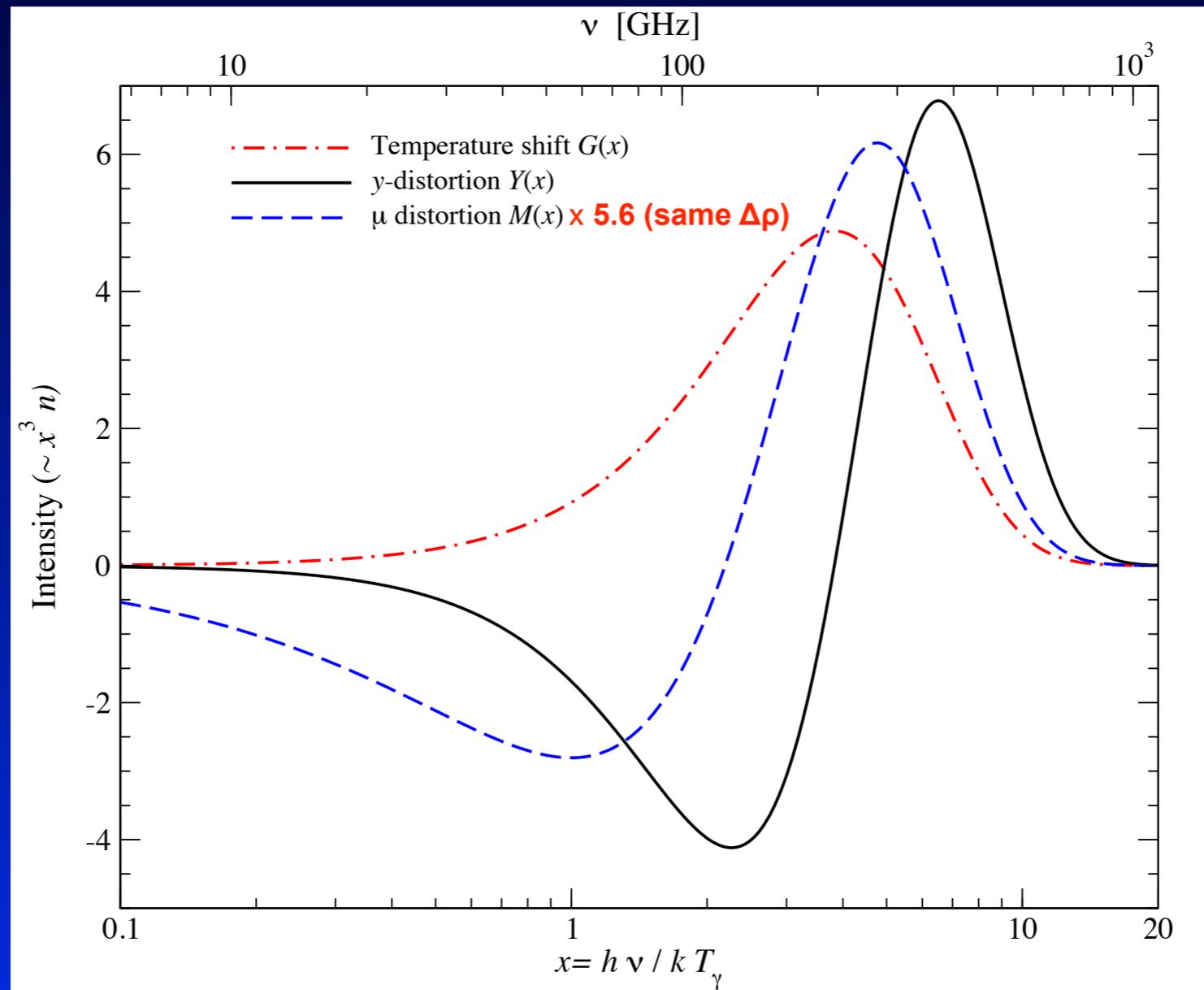
$$\mu \approx 1.4 \int_{z_K}^{\infty} \frac{d(Q/\rho_\gamma)}{dz'} \mathcal{J}_\mu(z') dz'$$

$$\mathcal{J}_\mu(z) \approx e^{-\left(\frac{z}{2 \times 10^6}\right)^{5/2}}$$

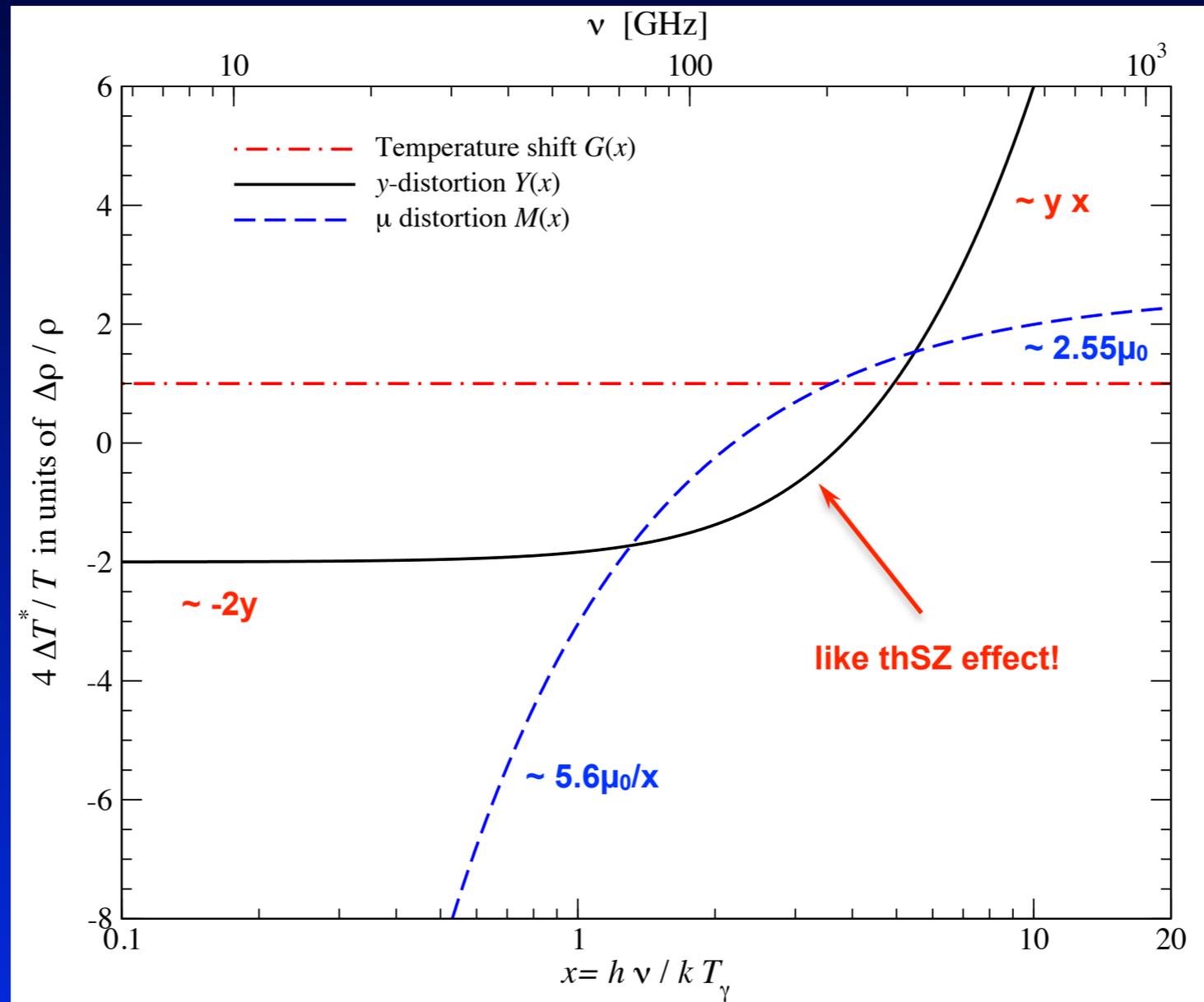
Visibility



Temperature shift \leftrightarrow y-distortion \leftrightarrow μ -distortion



Temperature shift \leftrightarrow y -distortion \leftrightarrow μ -distortion



Same as before but as effective temperature

$$\frac{\Delta T^*}{T} \approx \frac{\Delta n(x)}{G(x)}$$

Transition from y -distortion \rightarrow μ -distortion

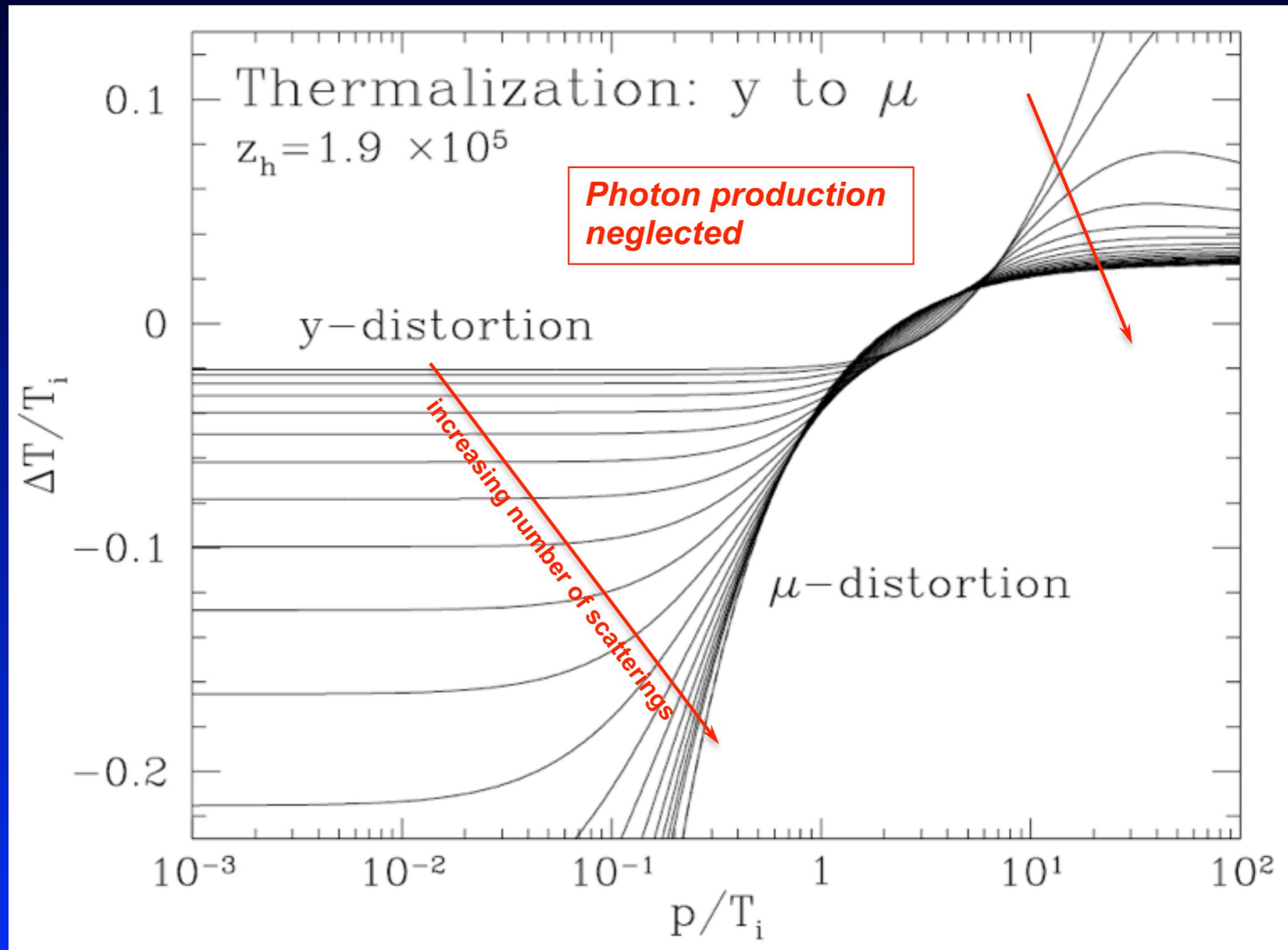


Figure from Wayne Hu's PhD thesis, 1995, but see also discussion in Burigana, 1991

Transition from γ -distortion \rightarrow μ -distortion

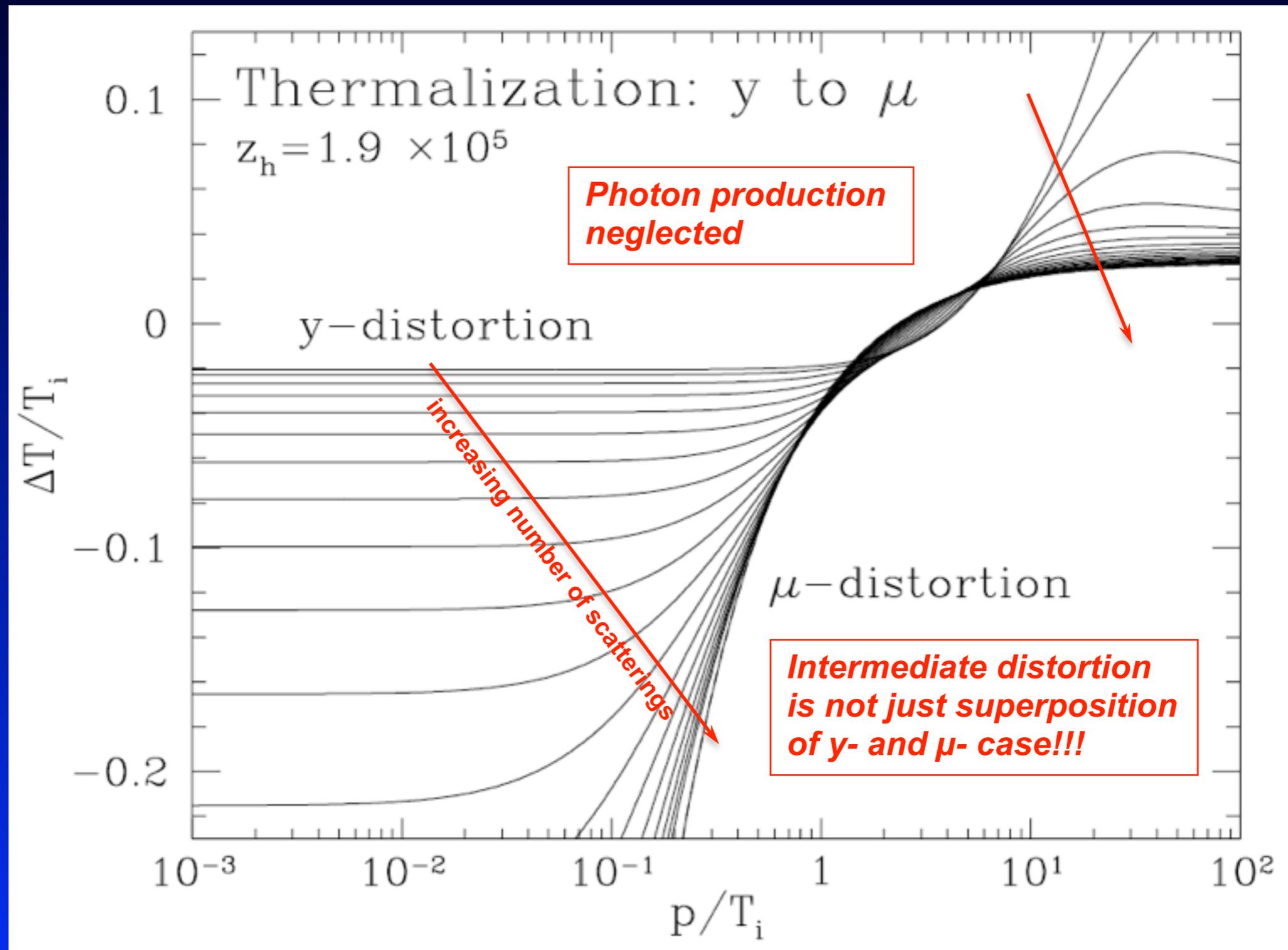
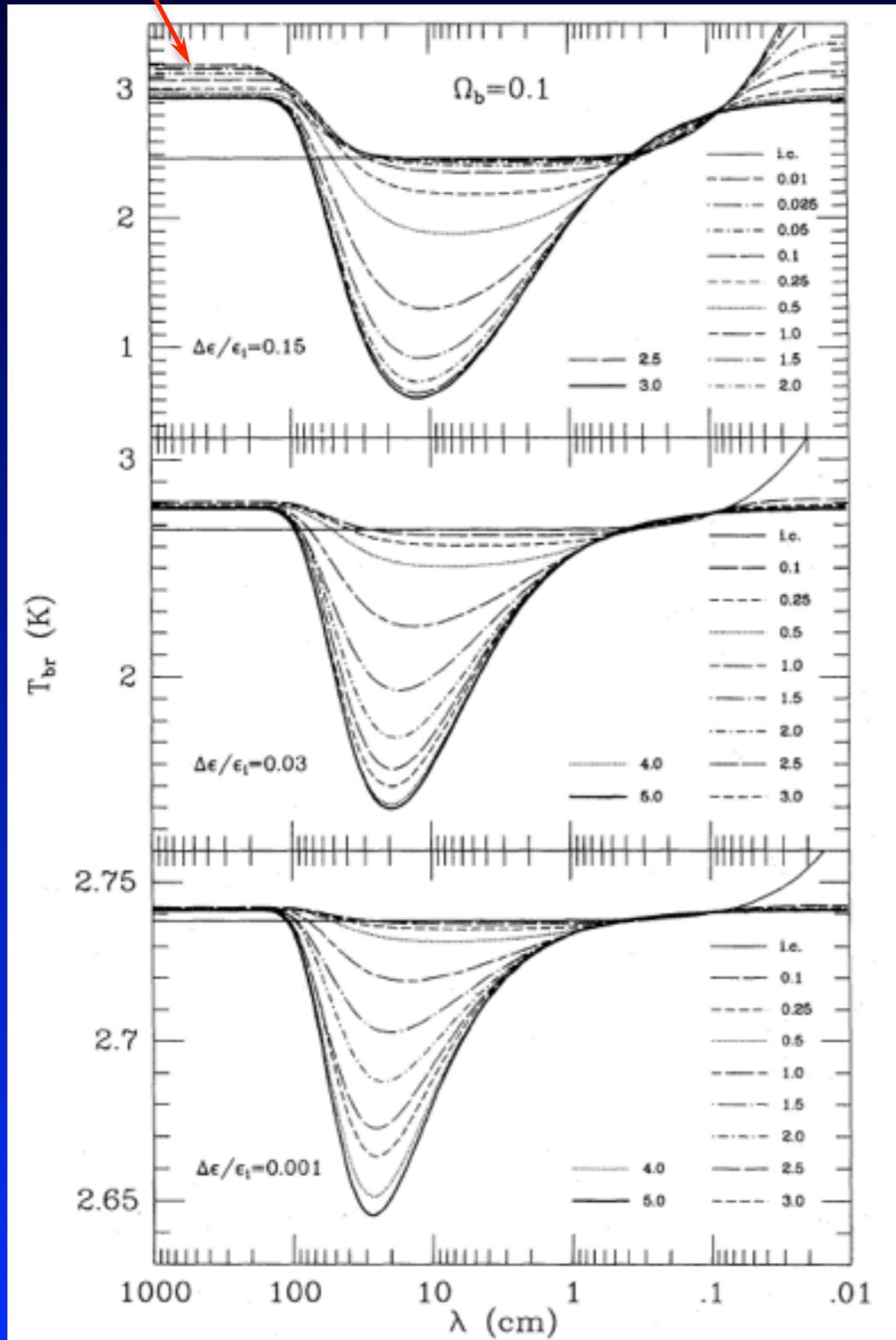


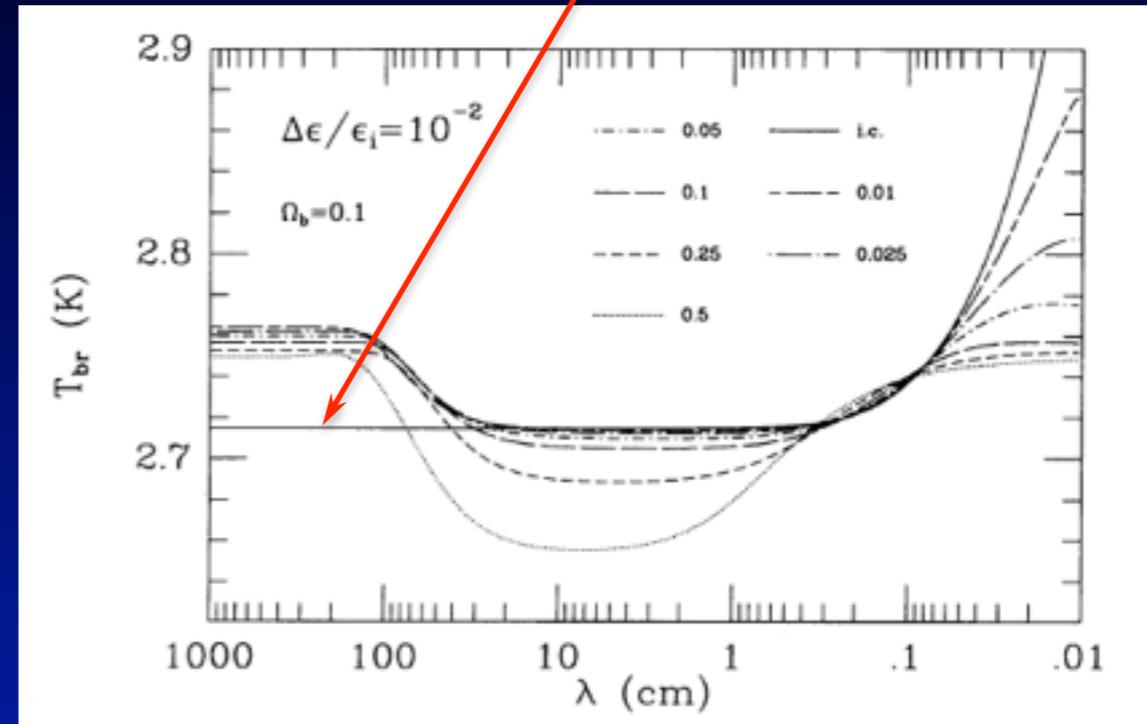
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Thermalization from $\gamma \rightarrow \mu$ at low frequencies

Effect of photon production!

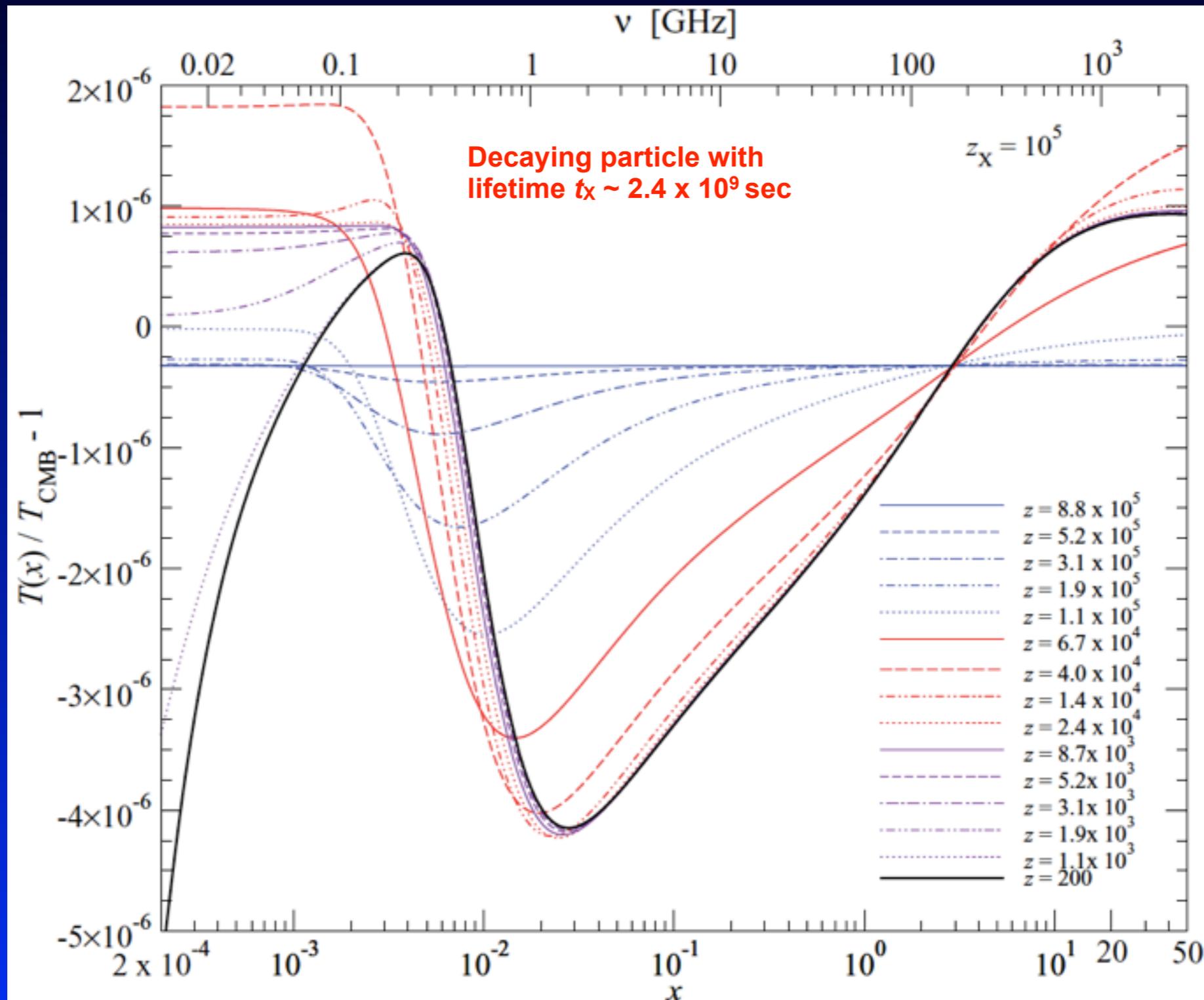


All calculations start with γ -distortion here

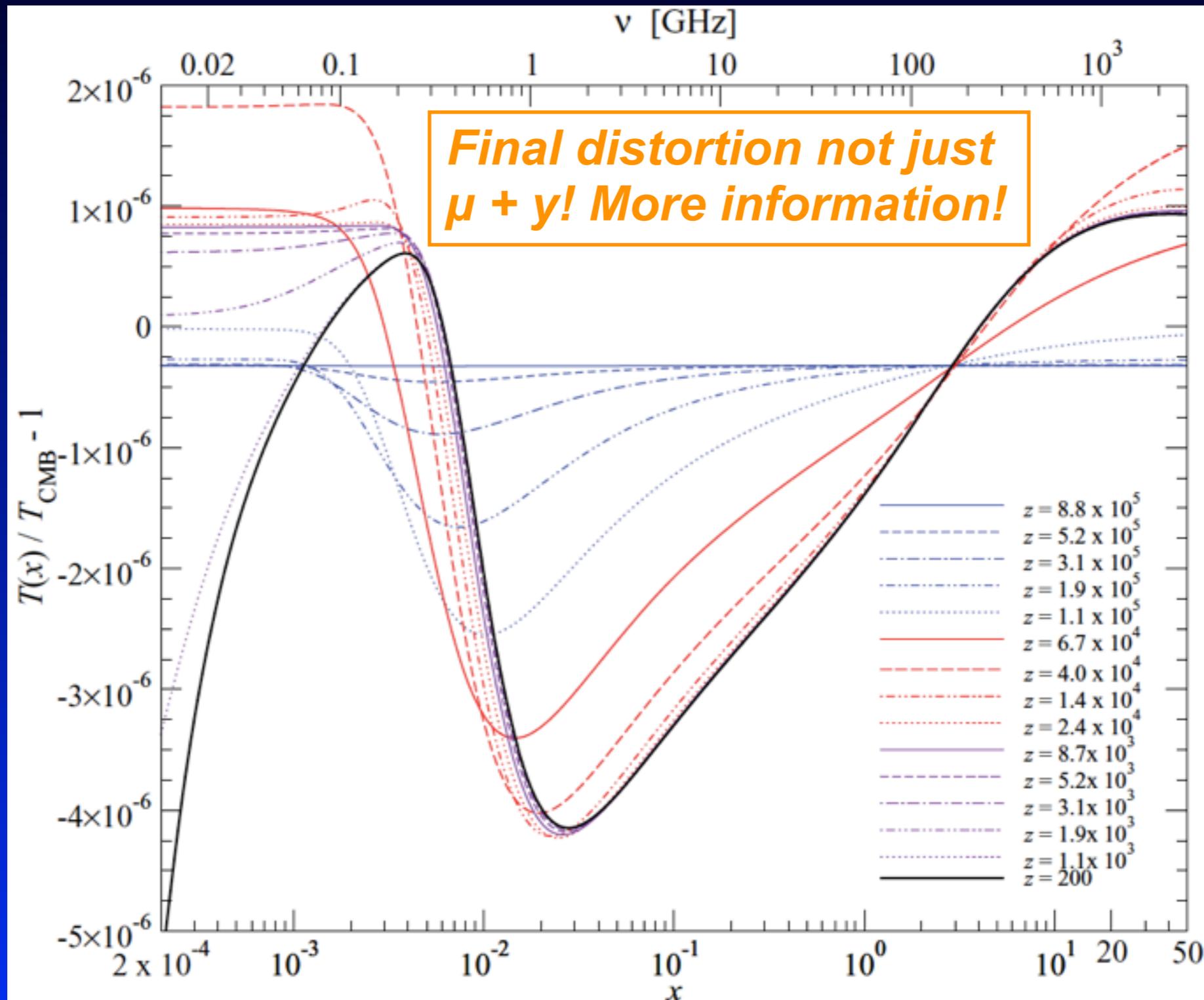


- amount of energy
 \leftrightarrow amplitude of distortion
 \leftrightarrow position of 'dip'
- Intermediate case ($3 \times 10^5 \geq z \geq 10000$)
 \Rightarrow mixture between μ & γ + residual
- details at very low frequencies change

Distortion *not* just superposition of μ and y -distortion!



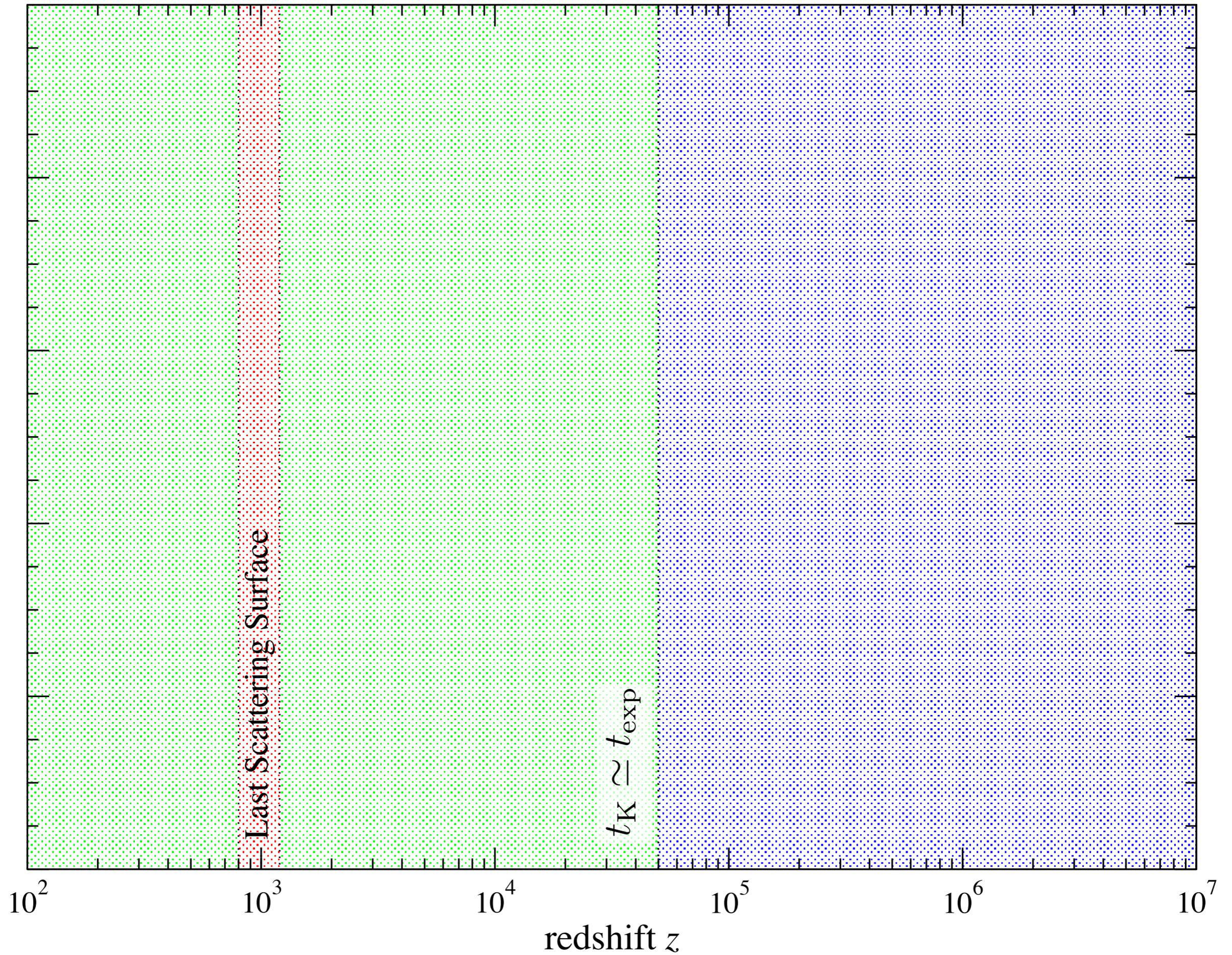
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γ - distortion

μ - γ transition

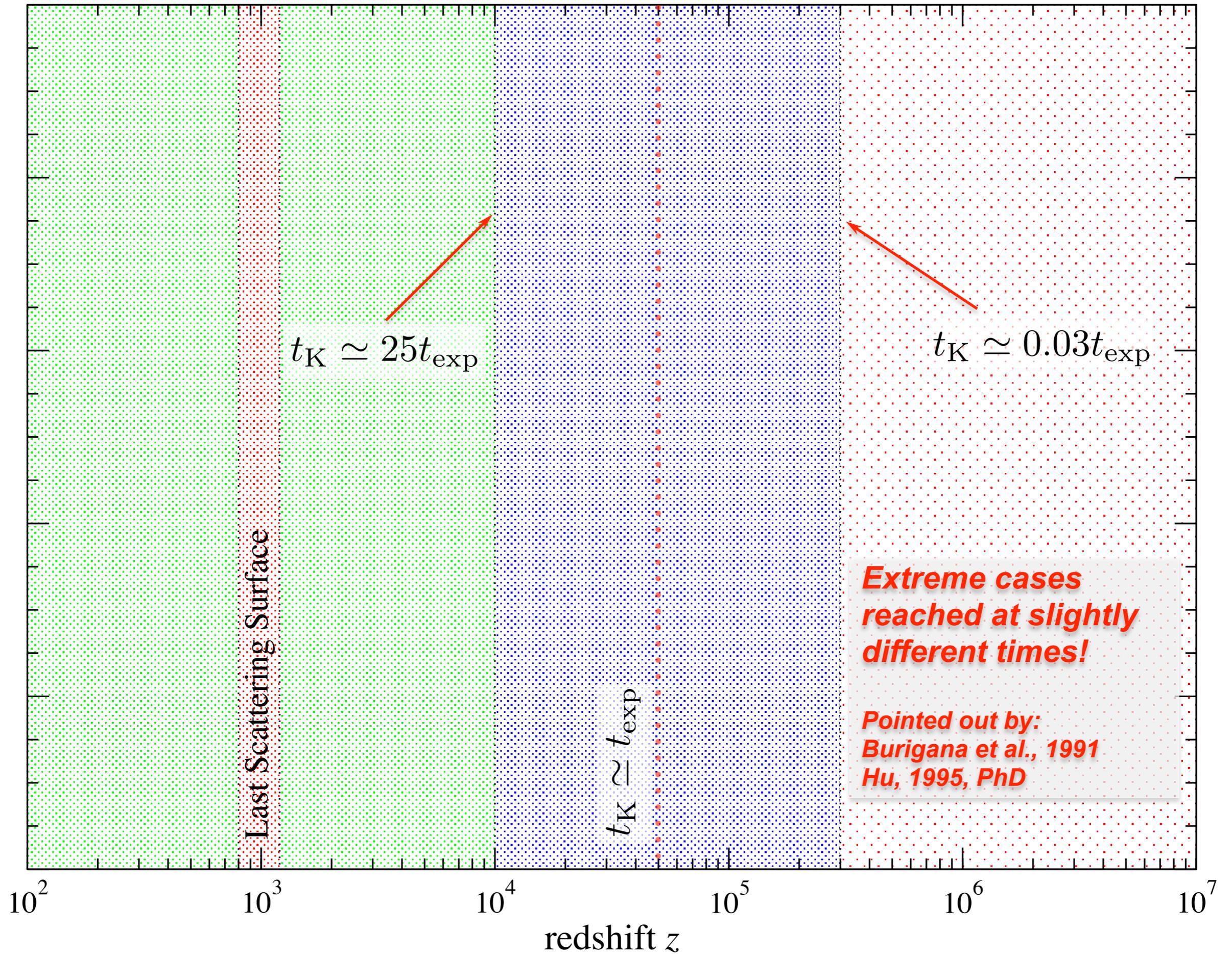
μ - distortion



y - distortion

μ -y transition

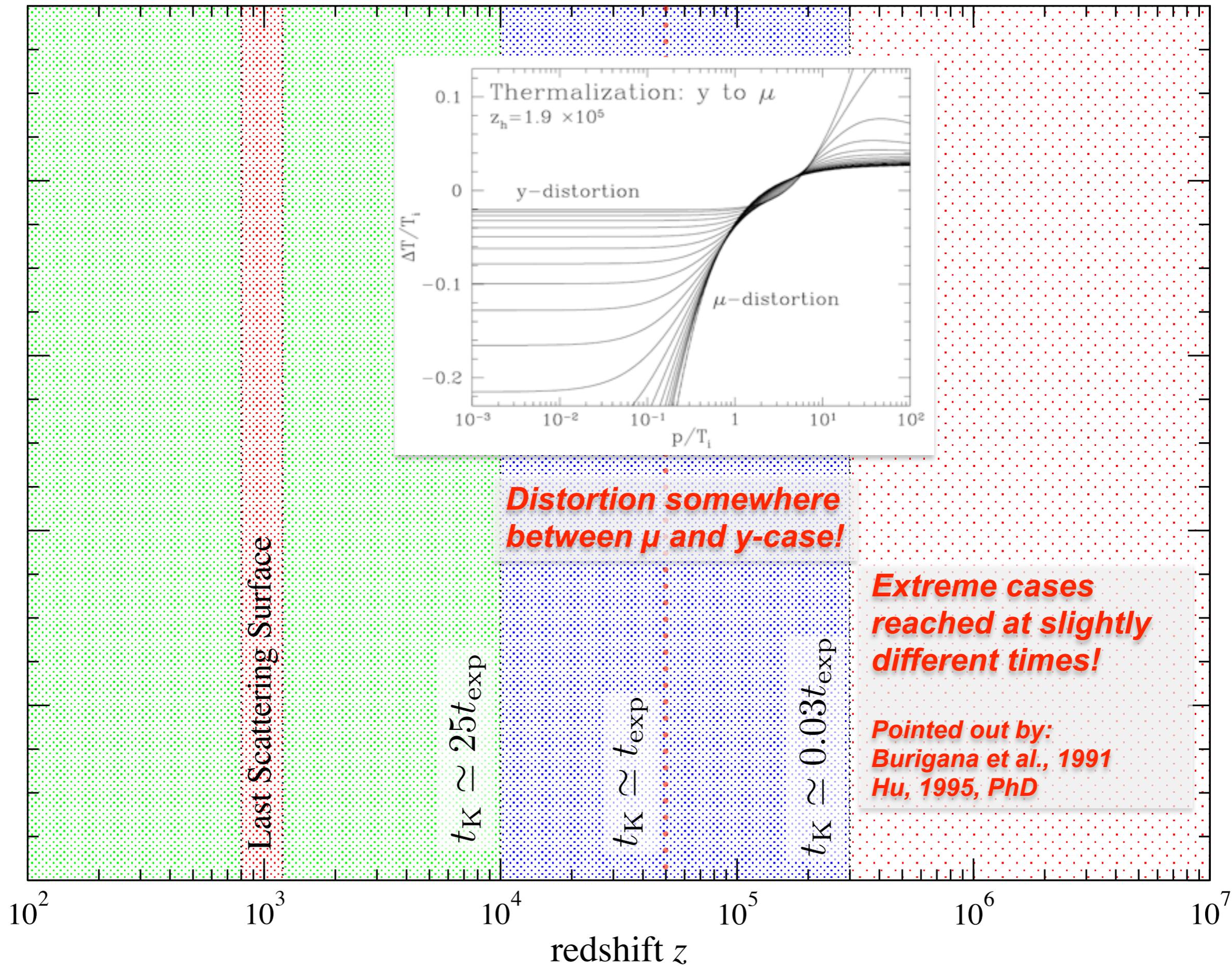
μ - distortion



y - distortion

μ -y transition

μ - distortion



How can we include this efficiently into the picture?

Quasi-Exact Treatment: Thermalization Green's Function

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- *For real forecasts of future prospects a precise & fast method for computing the spectral distortion is needed!*

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- *Final distortion for fixed energy-release history given by*

$$\Delta I_\nu \approx \int_0^\infty G_{\text{th}}(\nu, z') \frac{d(Q/\rho_\gamma)}{dz'} dz'$$

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Thermalization Green's function

Quasi-Exact Treatment: Thermalization Green's Function

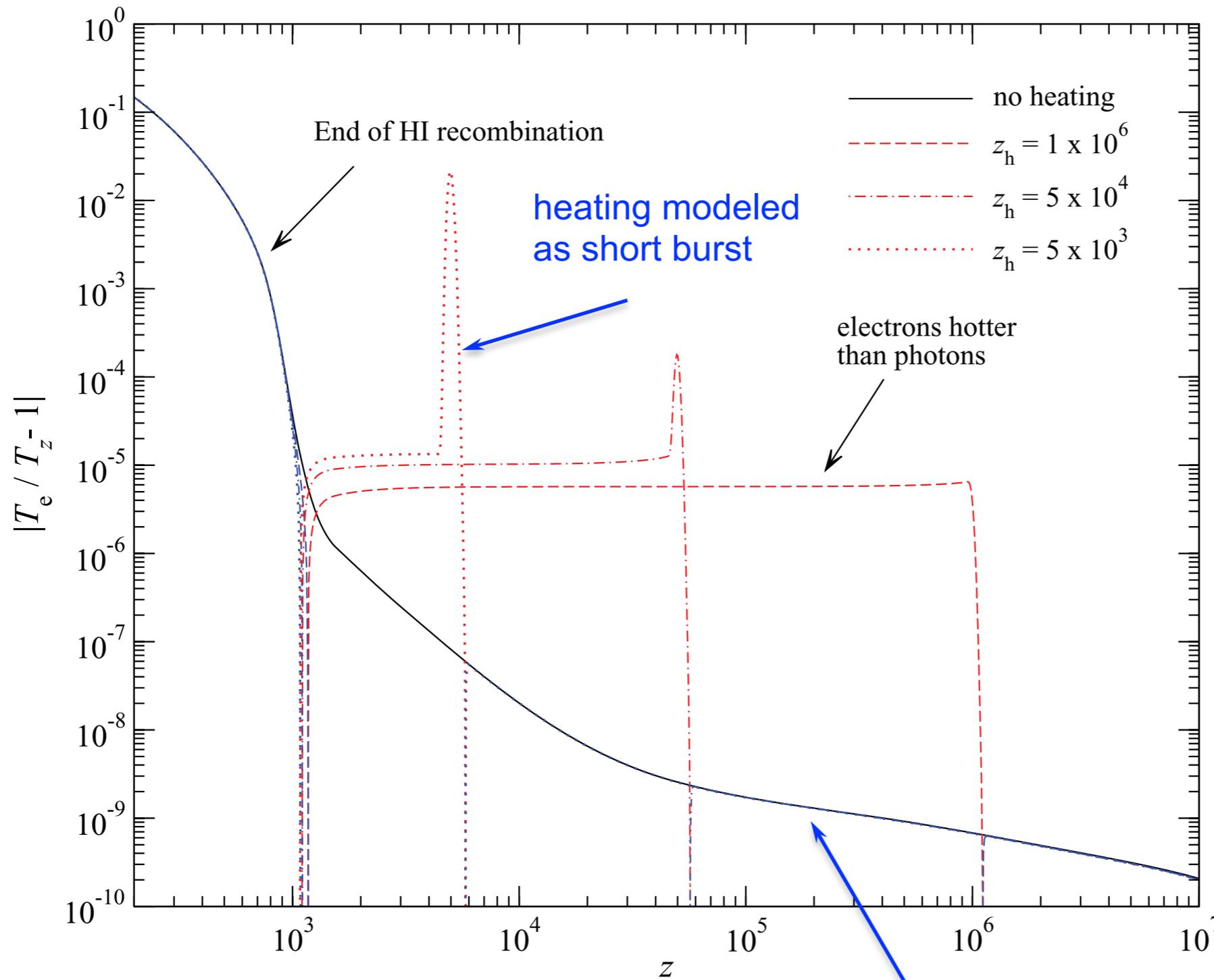
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Thermalization Green's function

- *Fast and quasi-exact! No additional approximations!*

Electron temperature evolution for single energy release

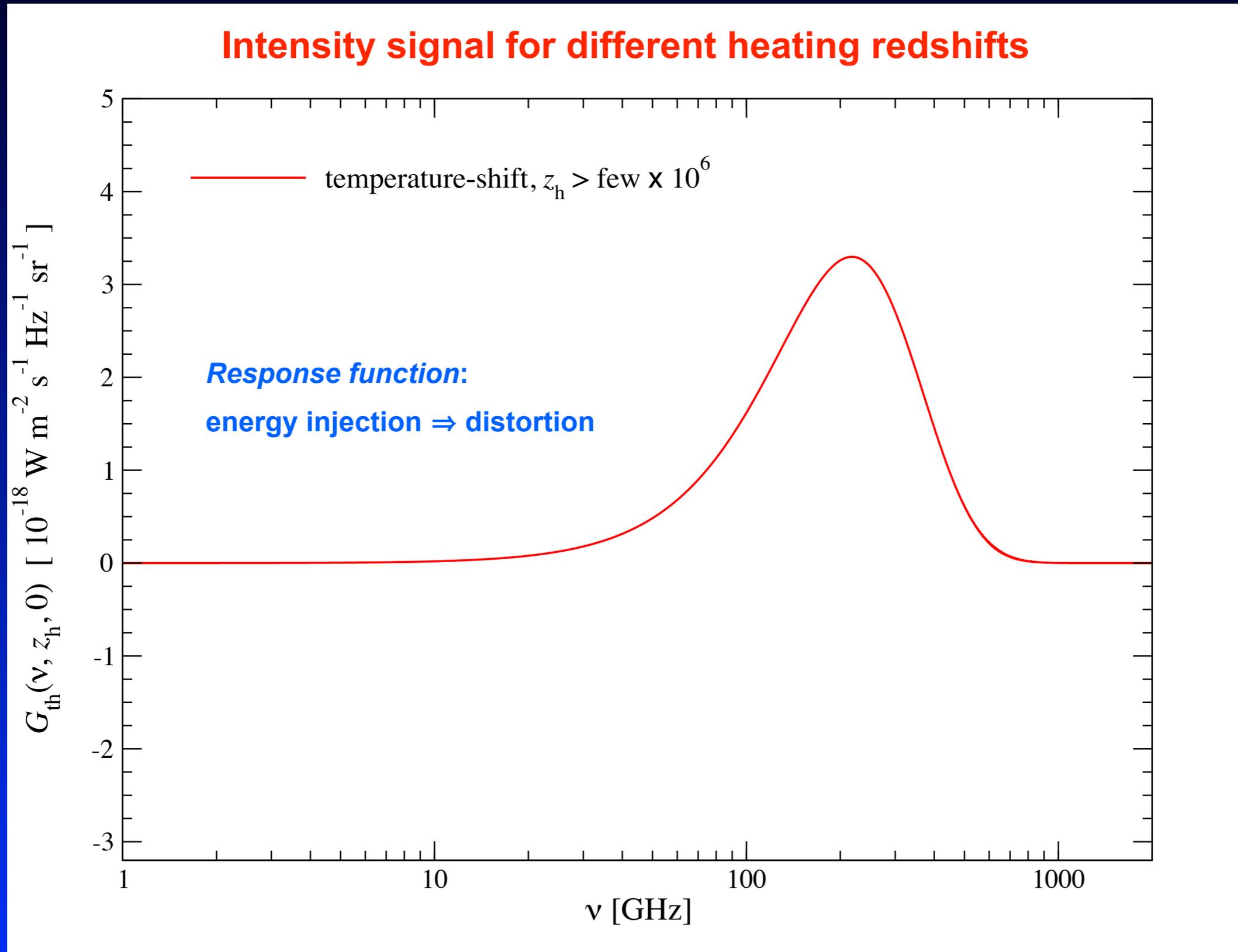


- injected energy

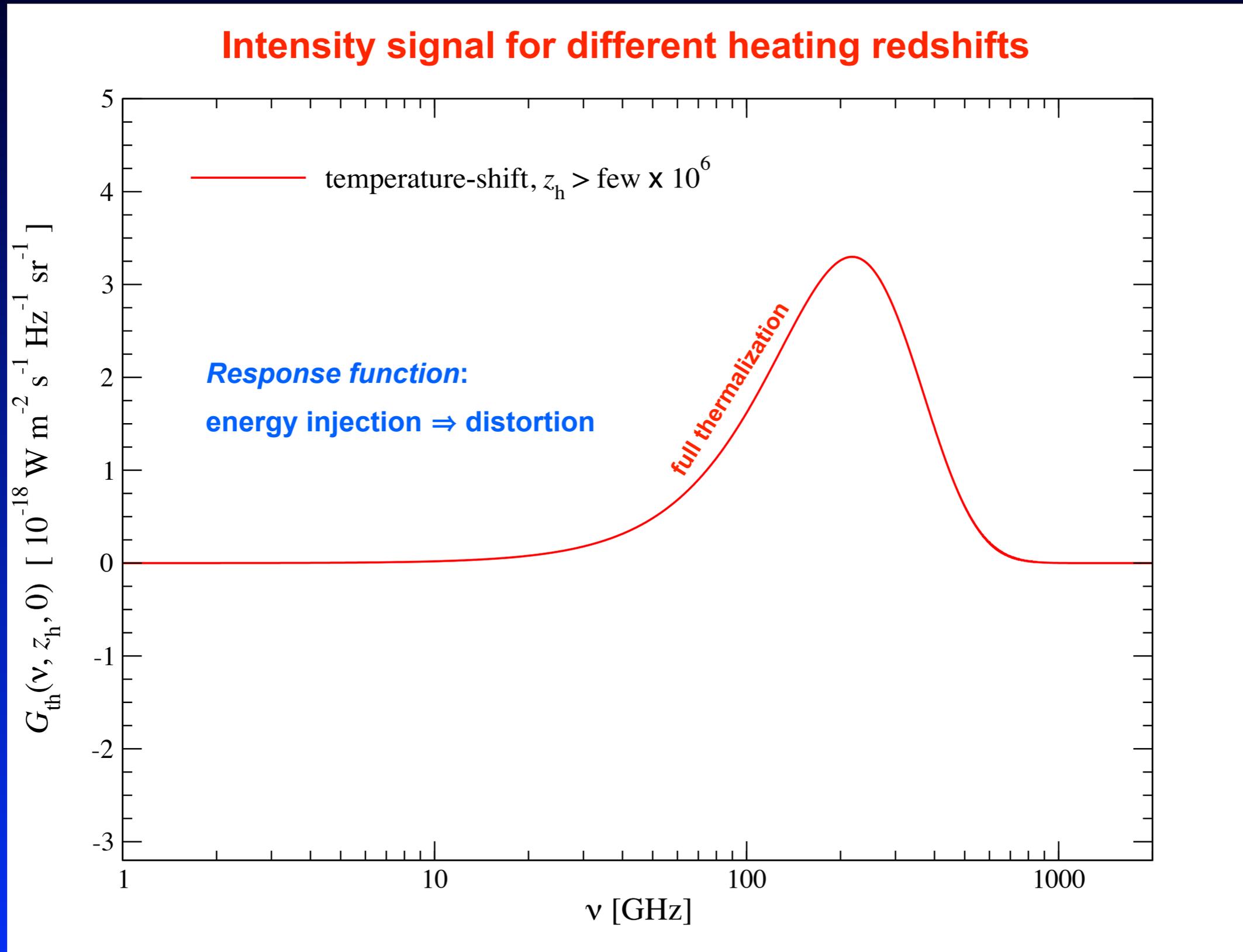
$$\frac{\Delta\rho_\gamma}{\rho_\gamma} = 10^{-5}$$

- need to worry about 'width' of burst
- explicitly done with *CosmoTherm*

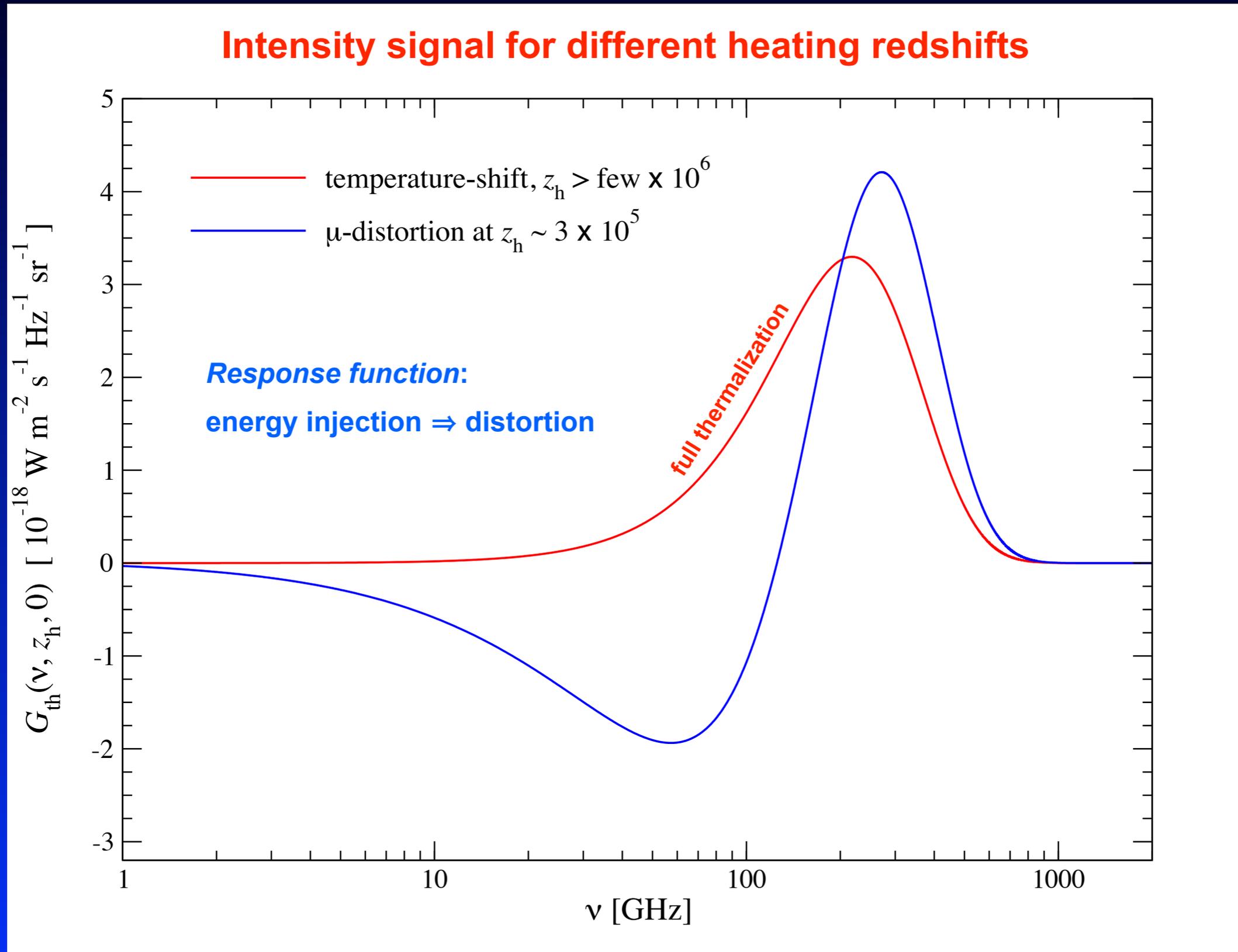
What does the spectrum look like after energy injection?



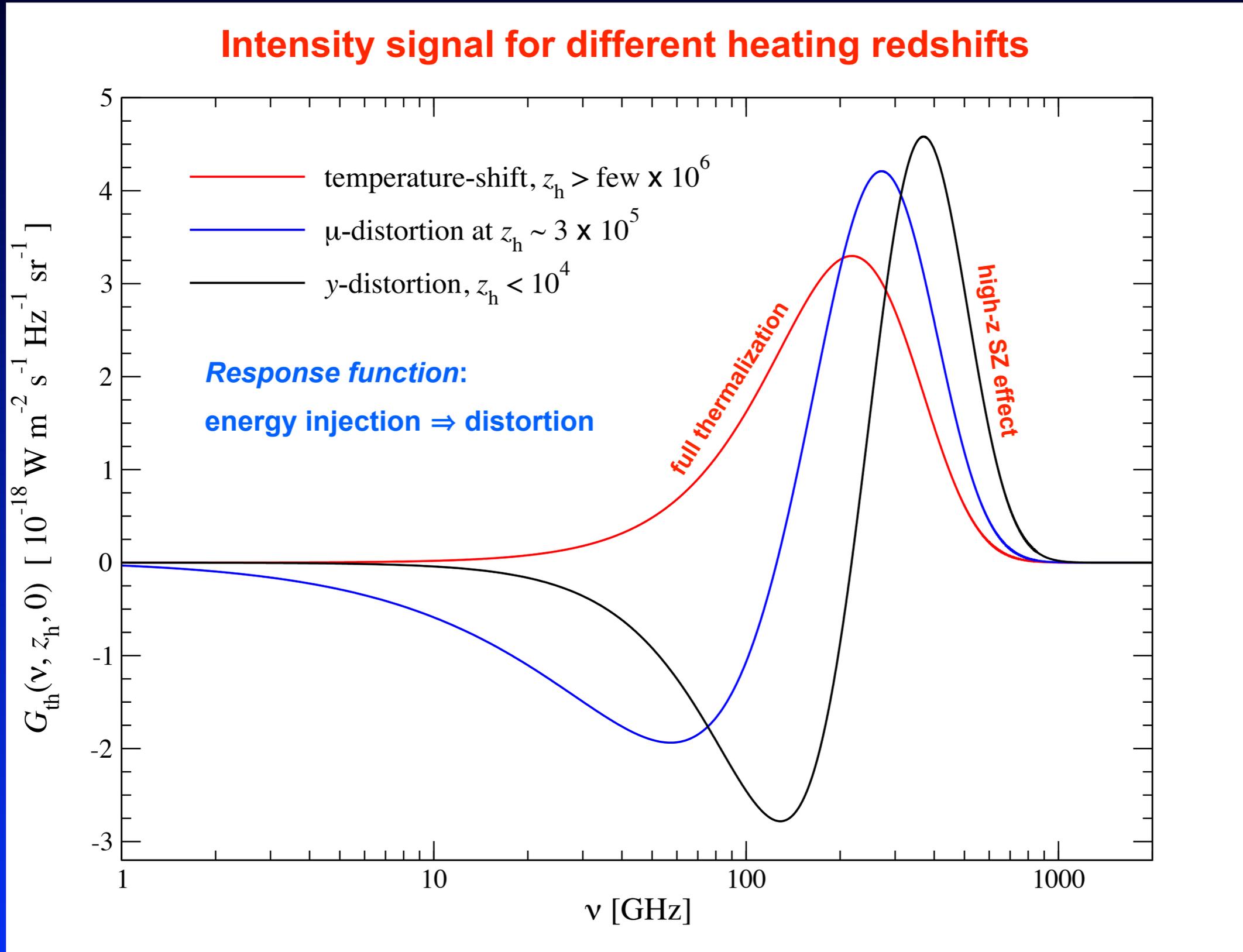
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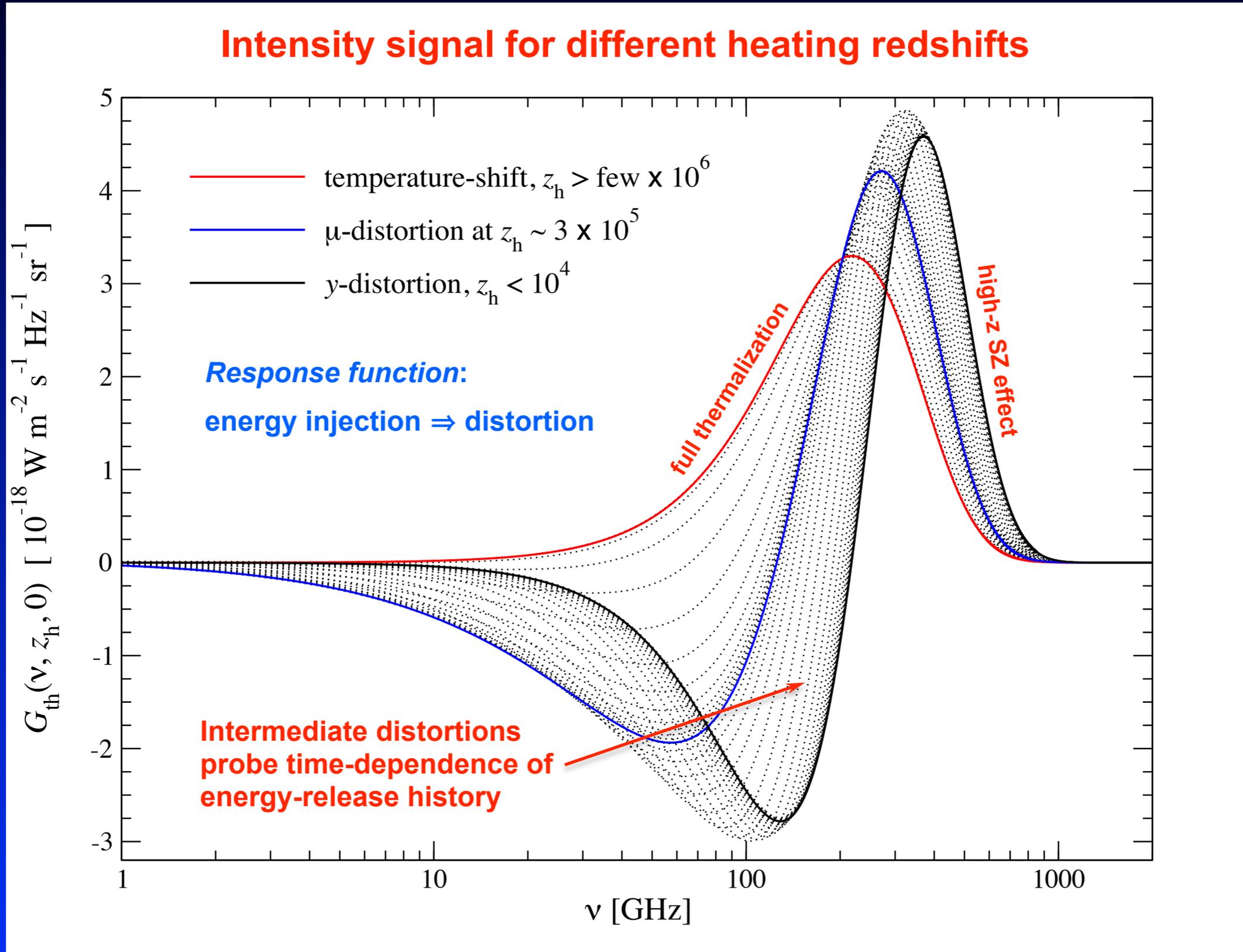
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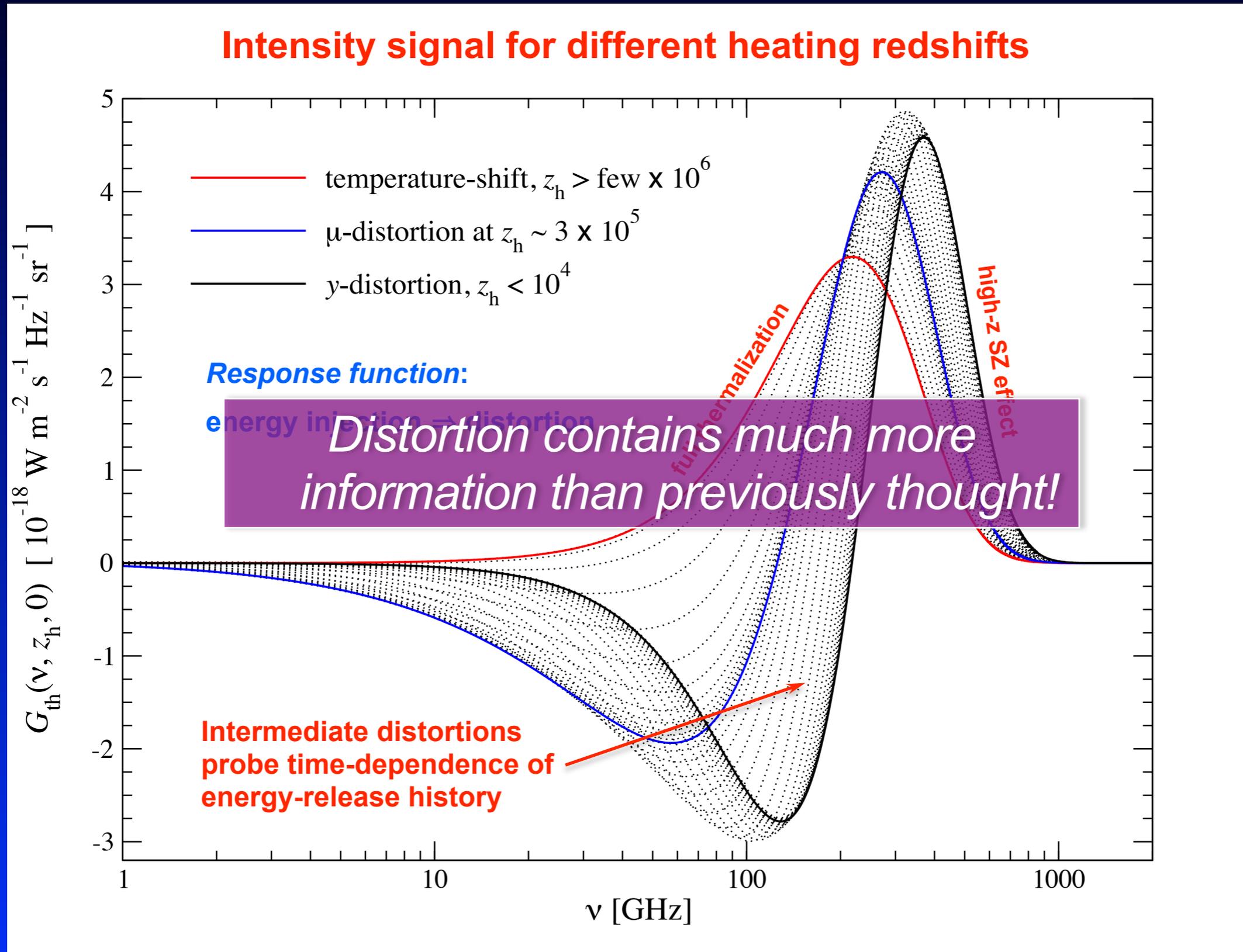
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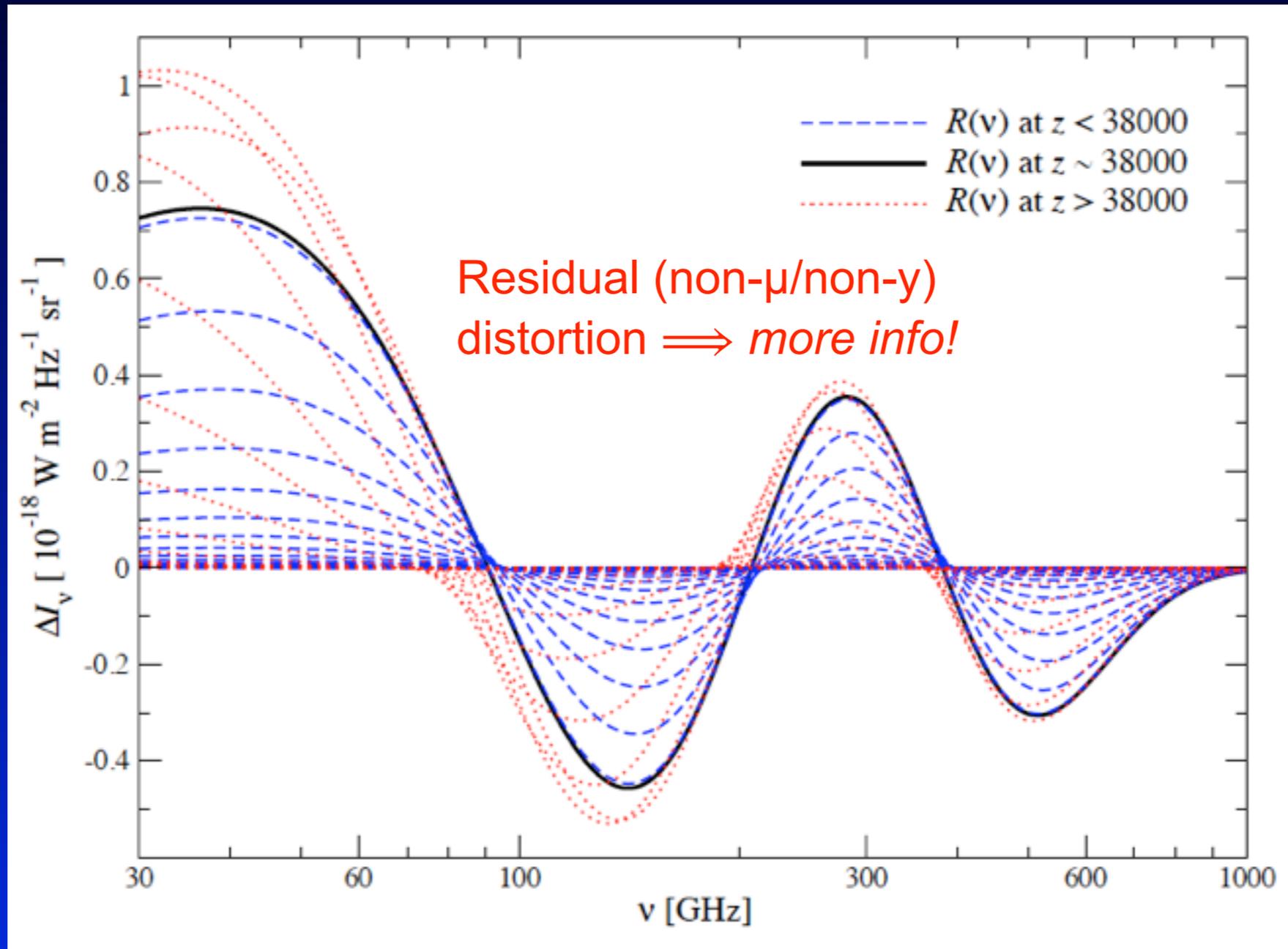
What does the spectrum look like after energy injection?



What does the spectrum look like after energy injection?



Explicitly taking out the superposition of μ & y distortion



- *Allows us to distinguish different energy release scenarios!*

*Approximate modeling of the μ - y transition
(without the residual distortion though...)*

y - distortion

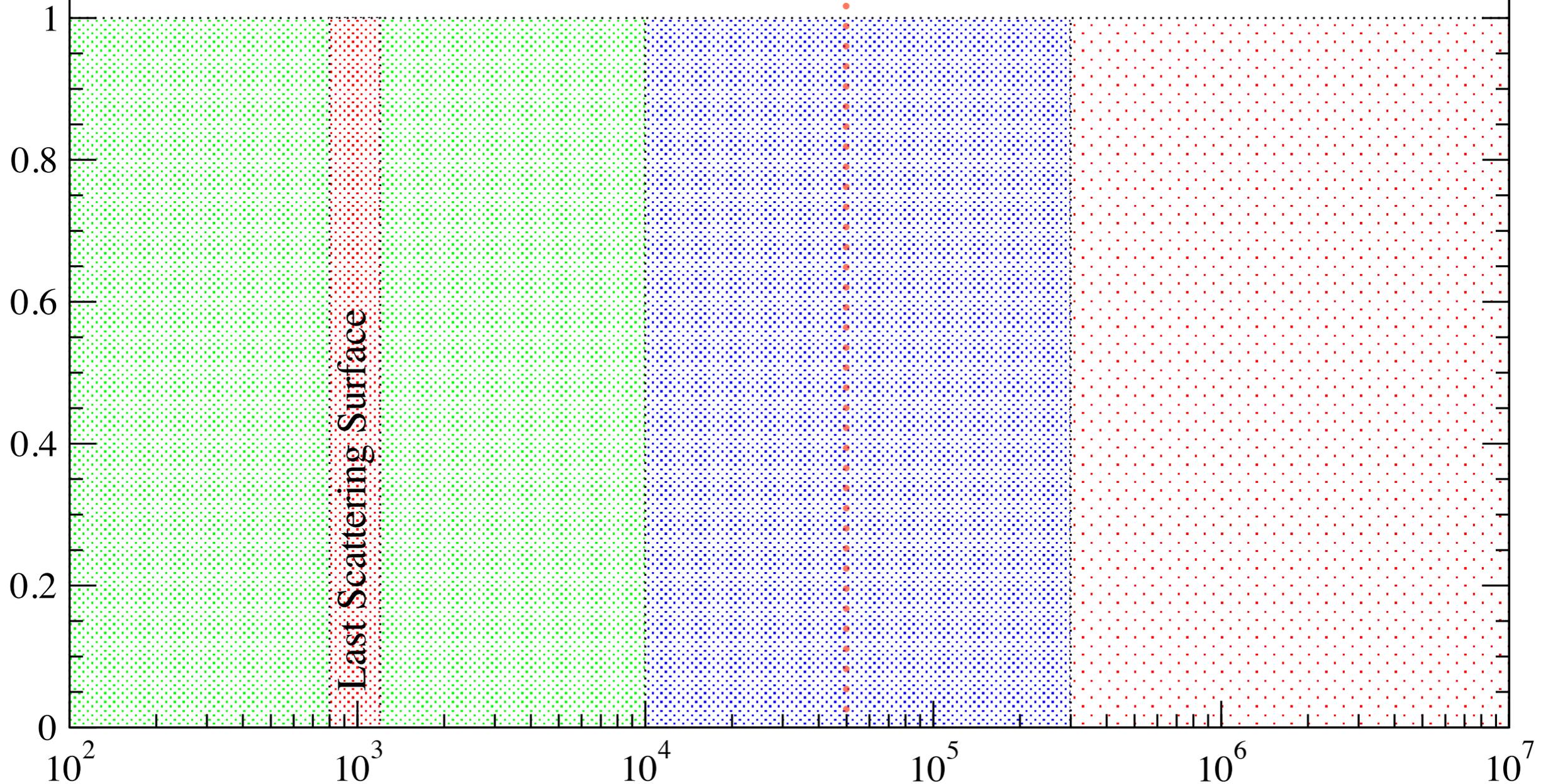
μ -y transition

μ - distortion

Slightly refined estimate that takes mix of μ and y into account (JC, 2013, ArXiv:1304.6120)

Visibility

Last Scattering Surface



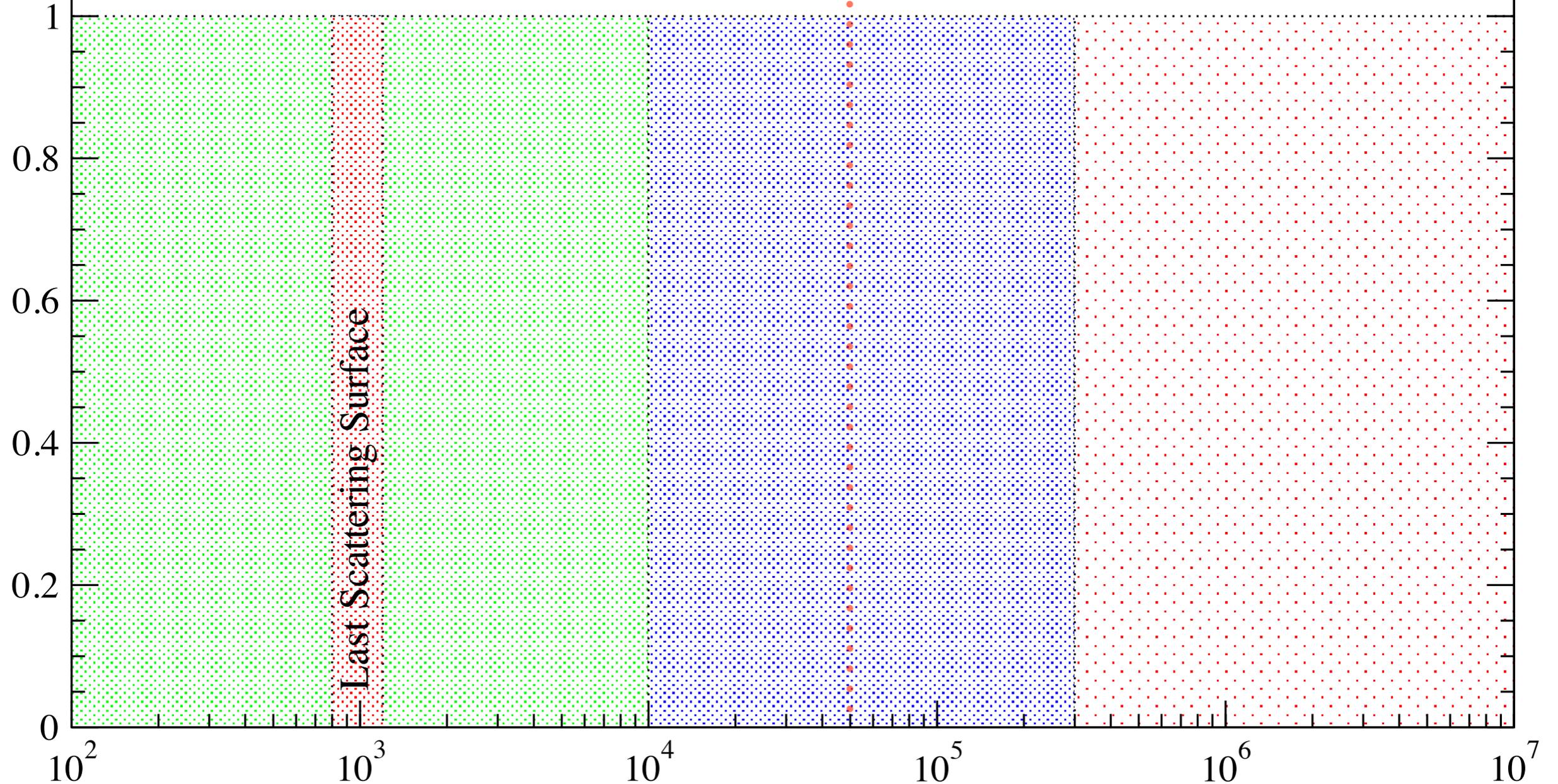
y - distortion

μ -y transition

μ - distortion

Fit numerical result for the Green's function with μ and y distortion
 \Rightarrow define visibility functions

Visibility



y - distortion

μ -y transition

μ - distortion

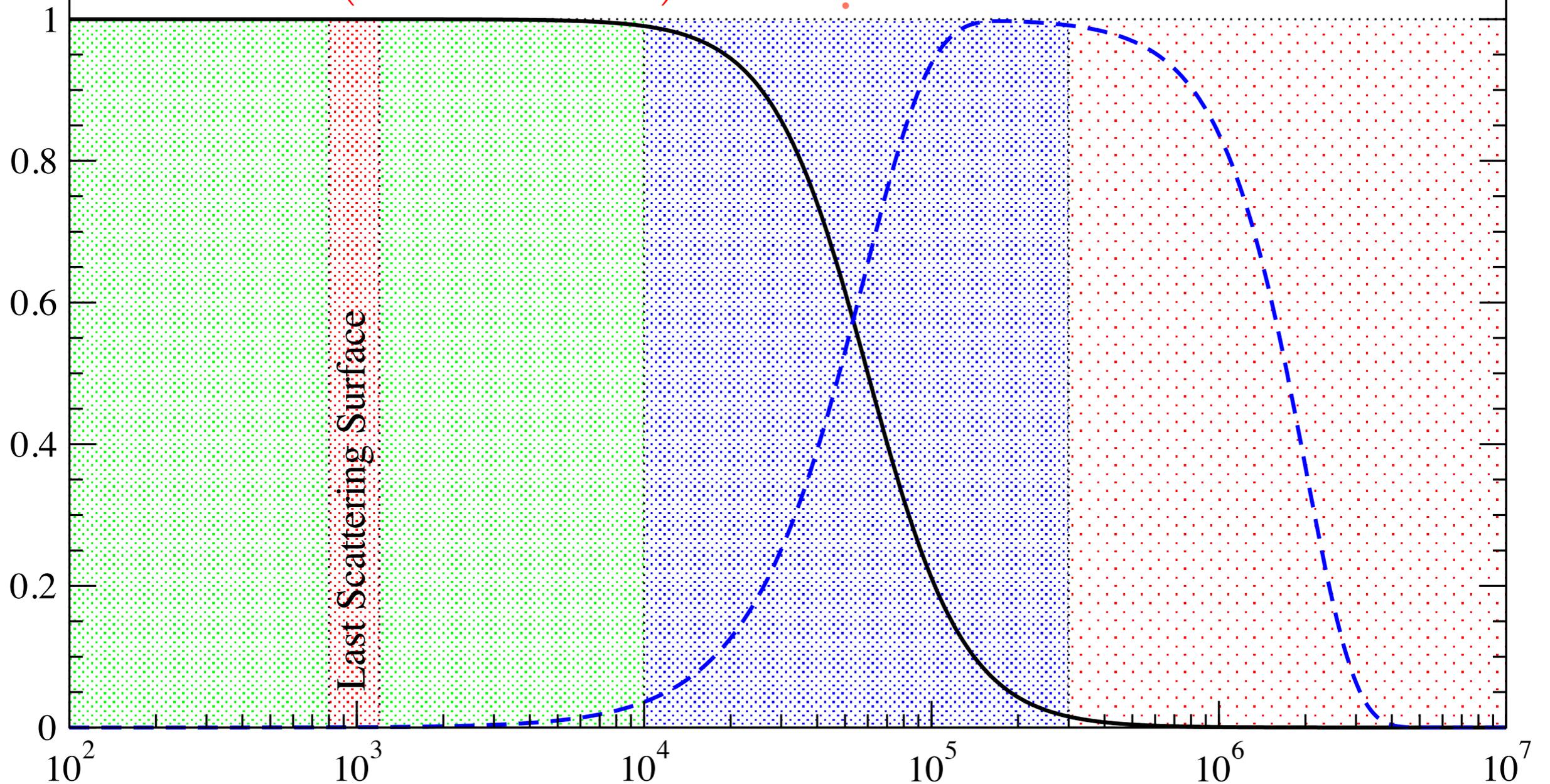
$$y \approx \frac{1}{4} \int_0^\infty \frac{d(Q/\rho_\gamma)}{dz'} \mathcal{J}_y(z') dz'$$

$$\mu \approx 1.4 \int_0^\infty \frac{d(Q/\rho_\gamma)}{dz'} \mathcal{J}_\mu(z') dz'$$

$$\mathcal{J}_y(z) \approx \left(1 + \left[\frac{1+z}{6.0 \times 10^4} \right]^{2.58} \right)^{-1}$$

$$\mathcal{J}_\mu(z) \approx \left[1 - e^{-\left[\frac{1+z}{5.8 \times 10^4} \right]^{1.88}} \right] e^{-\left[\frac{z}{2 \times 10^6} \right]^{2.5}}$$

Visibility



y - distortion

μ -y transition

μ - distortion

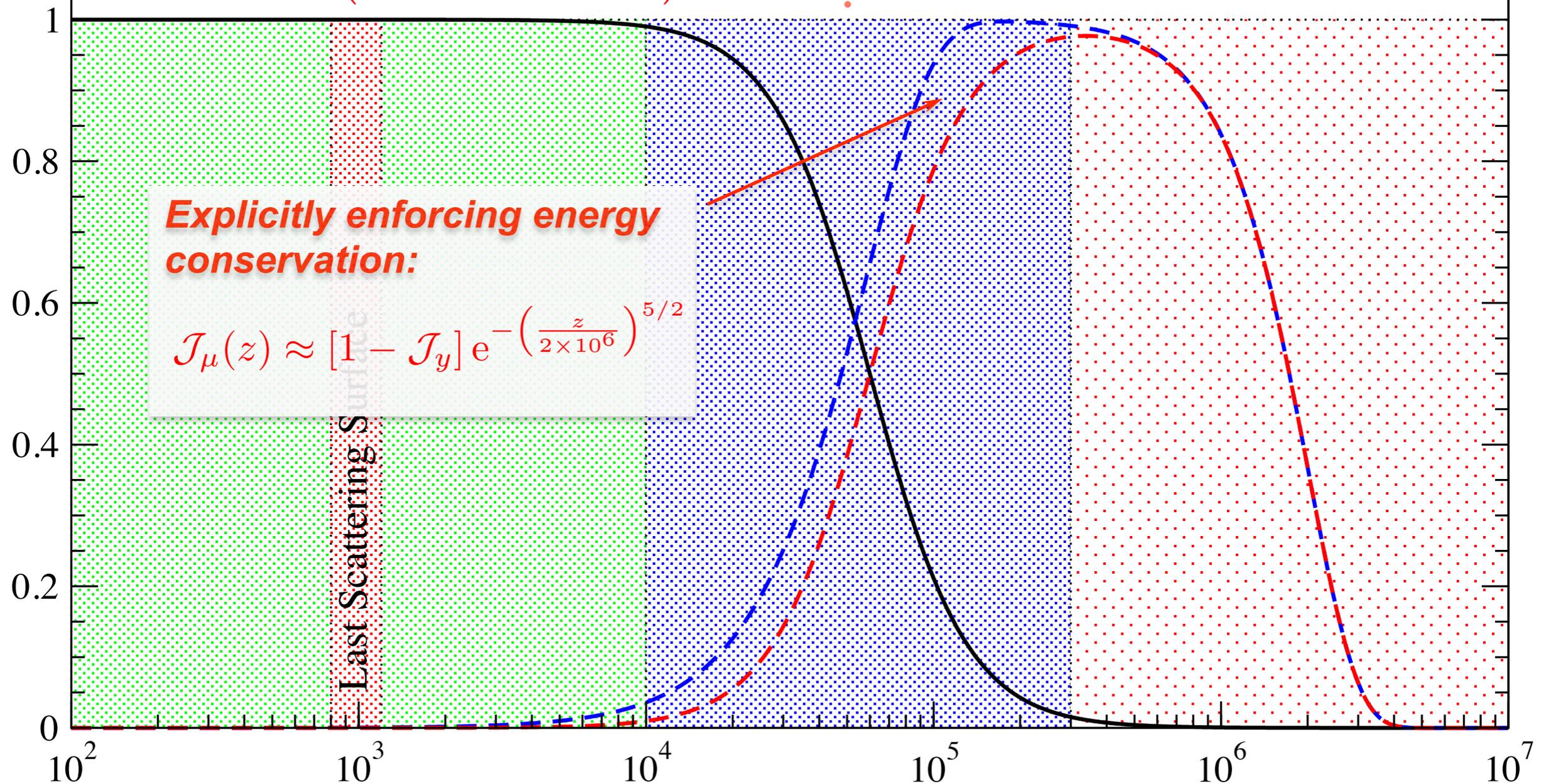
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Visibility



Explicitly enforcing energy conservation:

$$\mathcal{J}_\mu(z) \approx [1 - \mathcal{J}_y] e^{-\left(\frac{z}{2 \times 10^6}\right)^{5/2}}$$

Last Scattering Surface

y - distortion

μ -y transition

μ - distortion

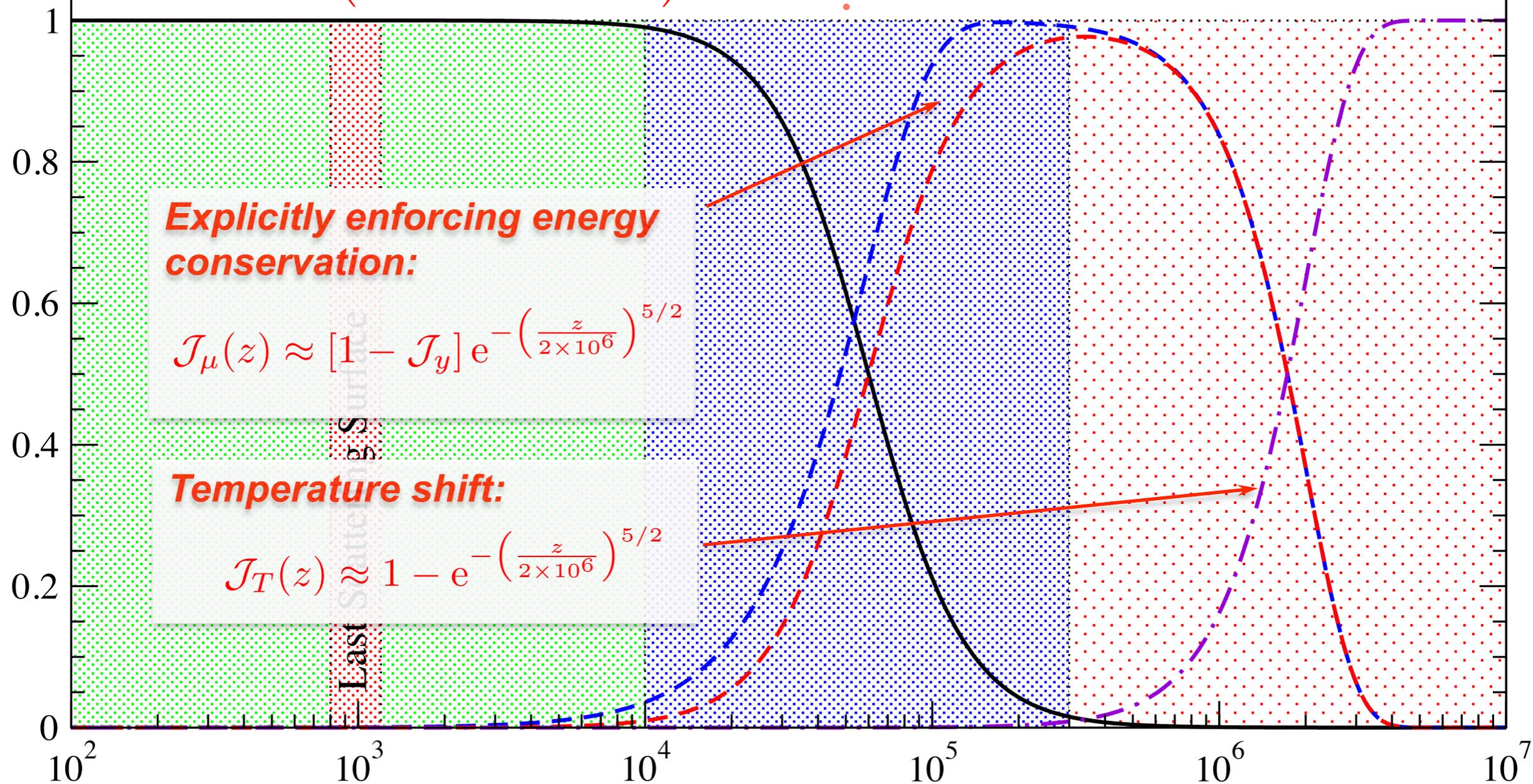
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Visibility



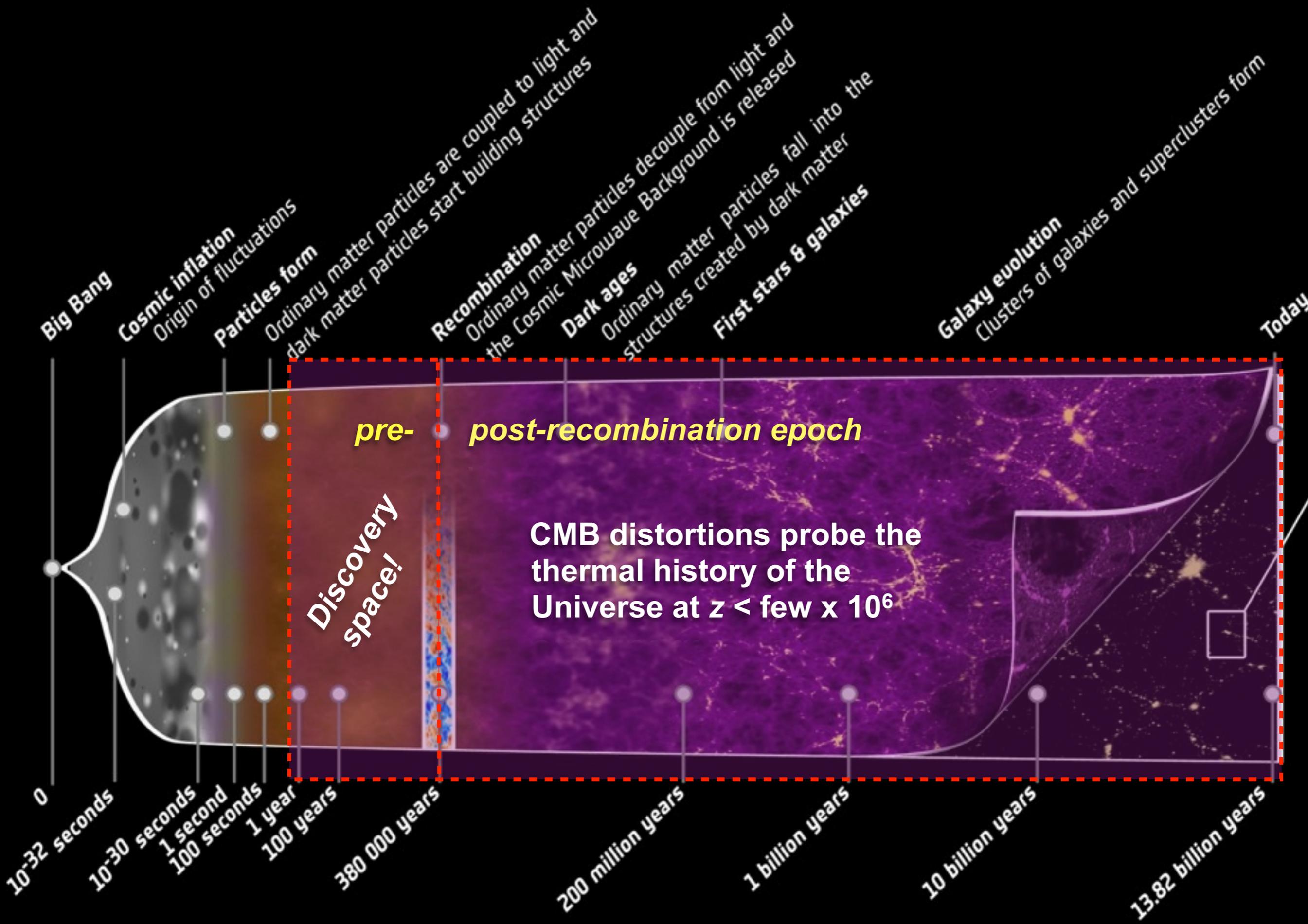
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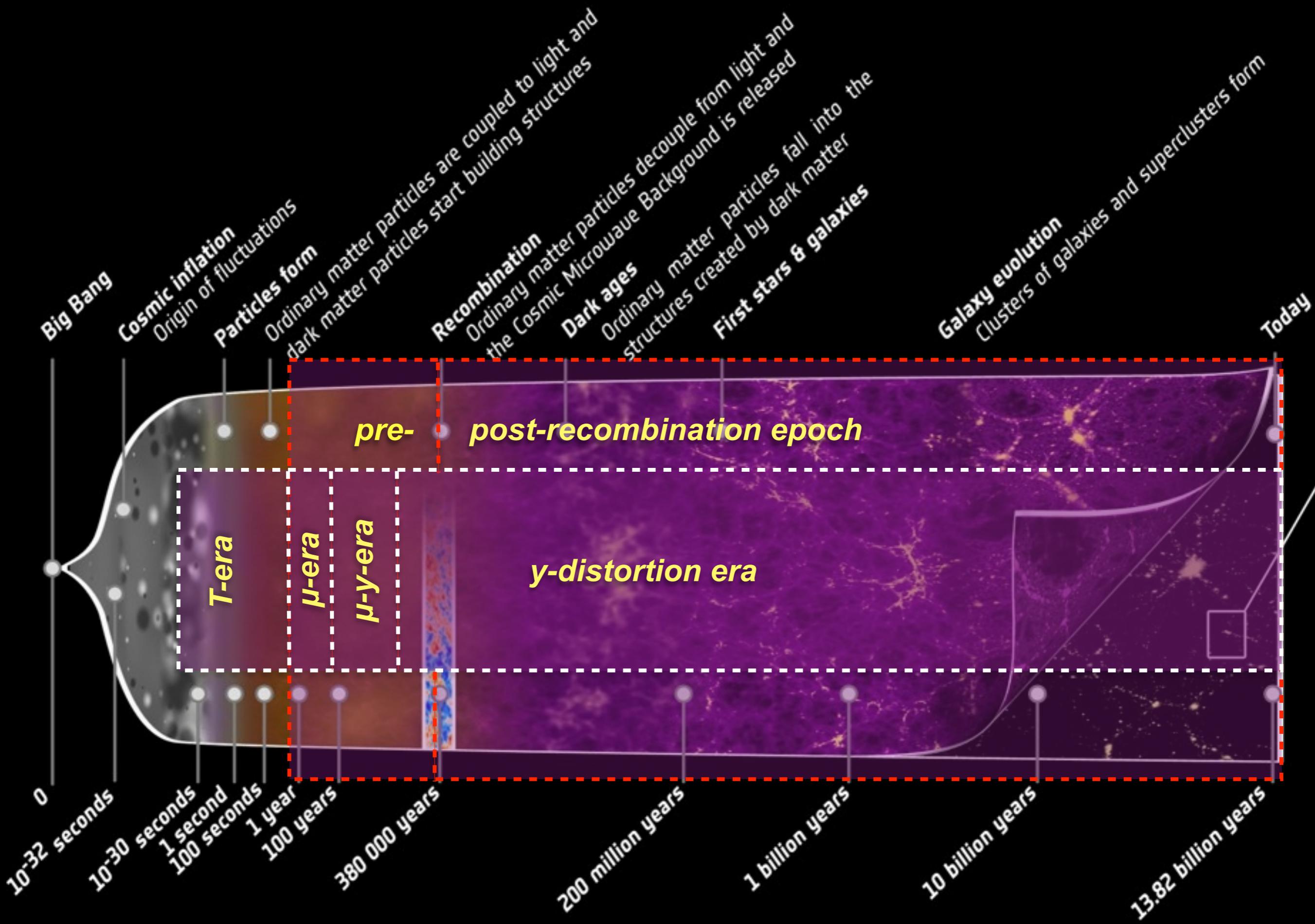
$$\mathcal{J}_\mu(z) \approx [1 - \mathcal{J}_y] e^{-\left(\frac{z}{2 \times 10^6}\right)^{5/2}}$$

Temperature shift:

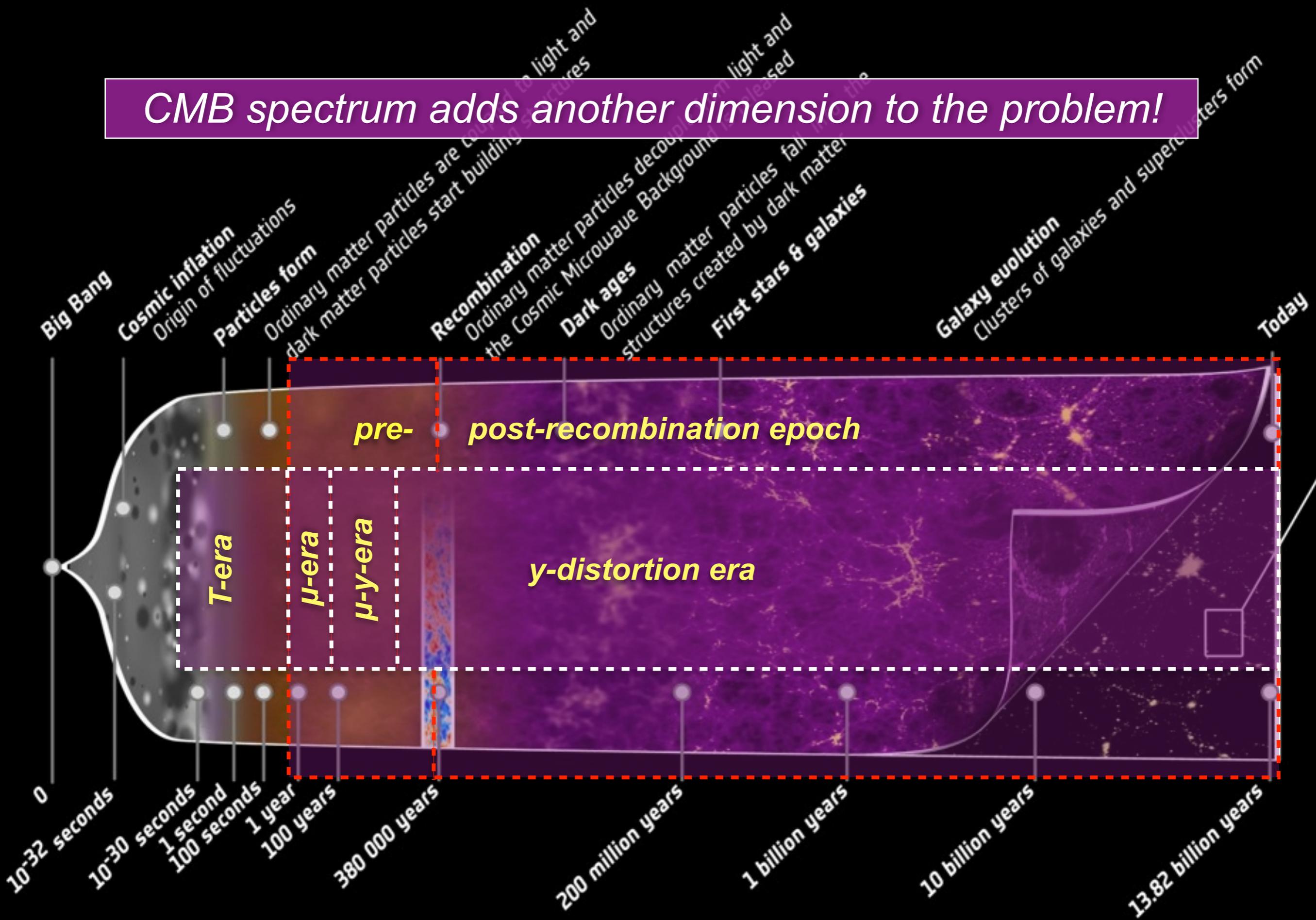
$$\mathcal{J}_T(z) \approx 1 - e^{-\left(\frac{z}{2 \times 10^6}\right)^{5/2}}$$

Last Surface





CMB spectrum adds another dimension to the problem!



Structure of the Lectures (cont.)

Lecture III:

- Overview of different sources of distortions
- Decaying particles
- Dissipation of acoustic modes

Physical mechanisms that lead to spectral distortions

- *Cooling by adiabatically expanding ordinary matter*

(JC, 2005; JC & Sunyaev 2011; Khatri, Sunyaev & JC, 2011)

*Standard sources
of distortions*

- Heating by *decaying* or *annihilating* relic particles

(Kawasaki et al., 1987; Hu & Silk, 1993; McDonald et al., 2001; JC, 2005; JC & Sunyaev, 2011; JC, 2013; JC & Jeong, 2013)

- *Evaporation of primordial black holes & superconducting strings*

(Carr et al. 2010; Ostriker & Thompson, 1987; Tashiro et al. 2012; Pani & Loeb, 2013)

- *Dissipation of primordial acoustic modes & magnetic fields*

(Sunyaev & Zeldovich, 1970; Daly 1991; Hu et al. 1994; JC & Sunyaev, 2011; JC et al. 2012 - Jedamzik et al. 2000; Kunze & Komatsu, 2013)

- *Cosmological recombination radiation*

(Zeldovich et al., 1968; Peebles, 1968; Dubrovich, 1977; Rubino-Martin et al., 2006; JC & Sunyaev, 2006; Sunyaev & JC, 2009)

„high“ redshifts

„low“ redshifts

- *Signatures due to first supernovae and their remnants*

(Oh, Cooray & Kamionkowski, 2003)

- *Shock waves arising due to large-scale structure formation*

(Sunyaev & Zeldovich, 1972; Cen & Ostriker, 1999)

- *SZ-effect from clusters; effects of reionization*

(Refregier et al., 2003; Zhang et al. 2004; Trac et al. 2008)

- *more exotic processes*

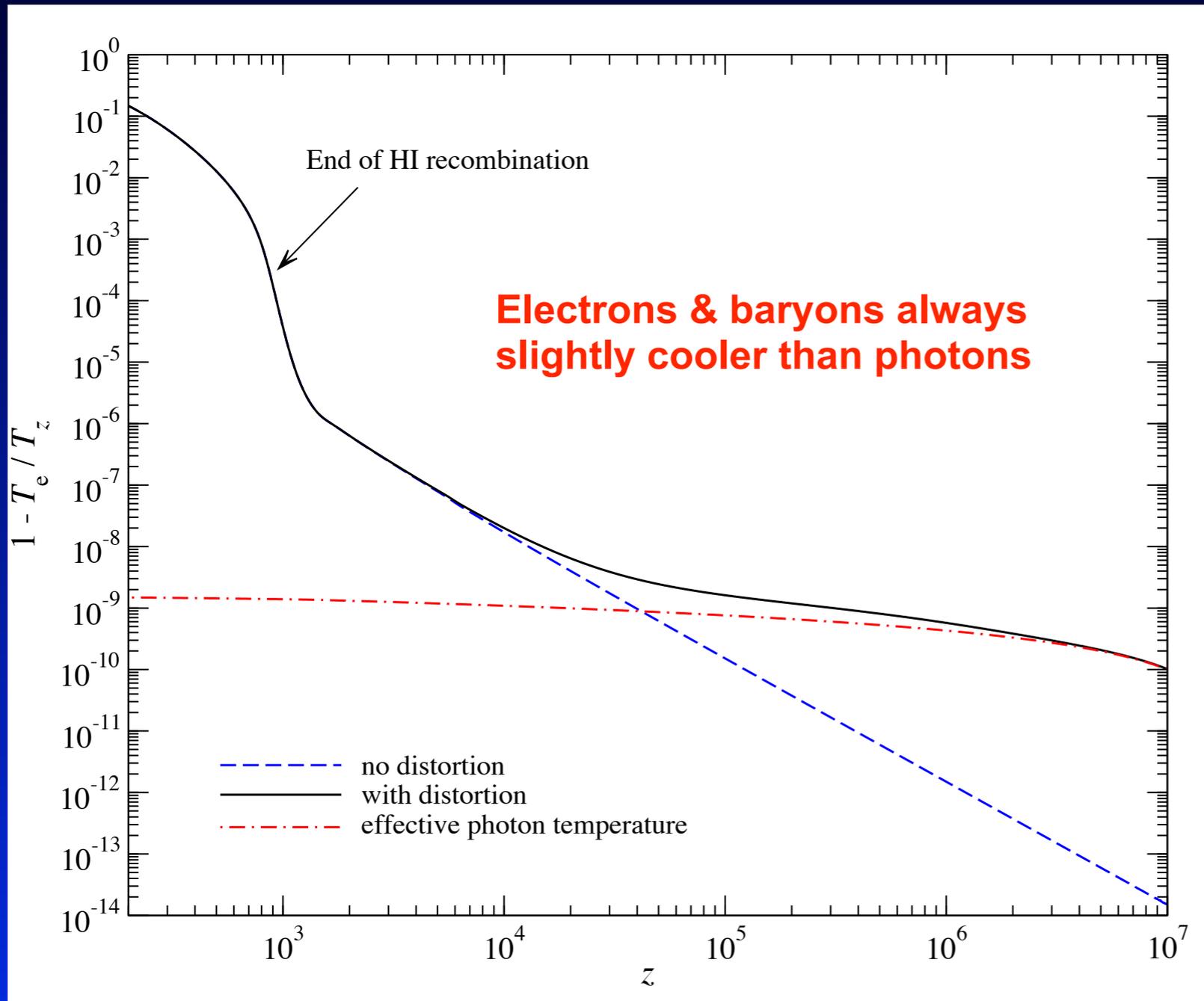
(Lochan et al. 2012; Bull & Kamionkowski, 2013; Brax et al., 2013; Tashiro et al. 2013)

pre-recombination epoch

post-recombination

Adiabatic cooling of ordinary matter

Spectral distortion caused by the cooling of ordinary matter



- adiabatic expansion
 $\Rightarrow T_\gamma \sim (1+z) \leftrightarrow T_m \sim (1+z)^2$
- photons continuously *cooled / down-scattered* since day one of the Universe!
- Compton heating balances adiabatic cooling
 $\Rightarrow \frac{da^4 \rho_\gamma}{a^4 dt} \simeq -Hk\alpha_h T_\gamma \propto (1+z)^6$
- at high redshift same scaling as *annihilation* ($\propto N_X^2$) and *acoustic mode damping*
 \Rightarrow *cancellation* possible

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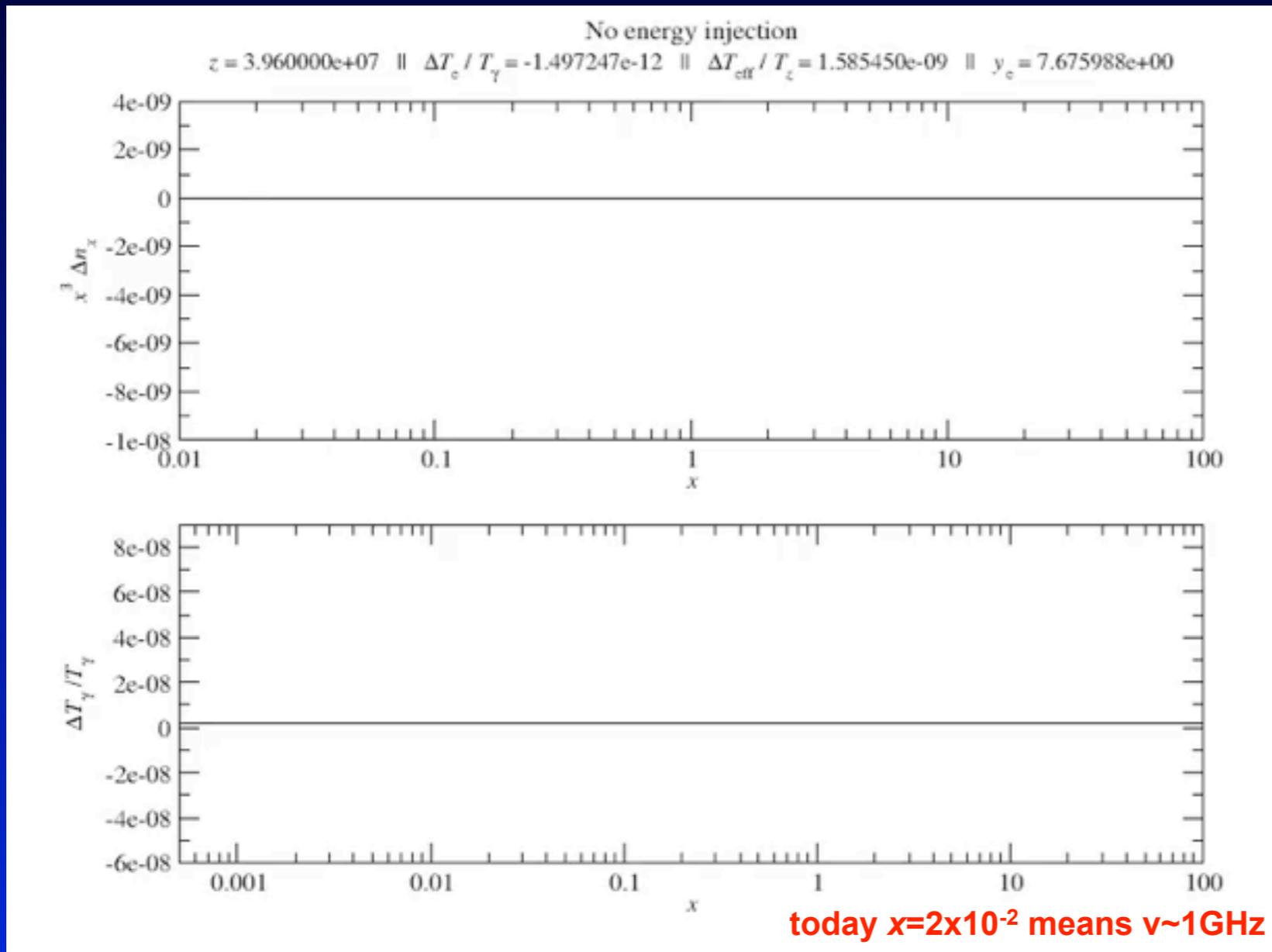
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today $x=2 \times 10^{-2}$ means $\nu \sim 1\text{GHz}$

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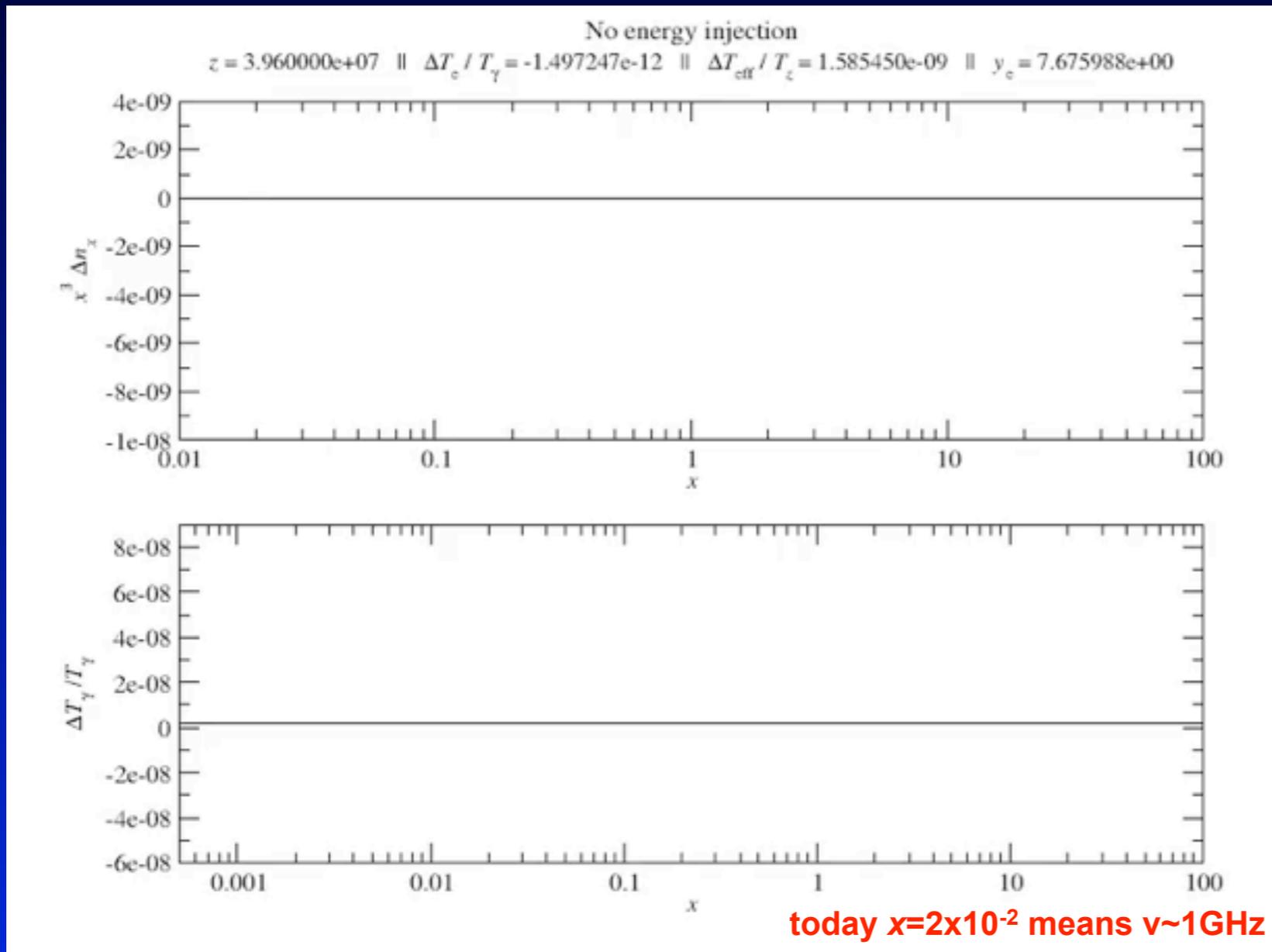
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- *negative* μ and y distortion

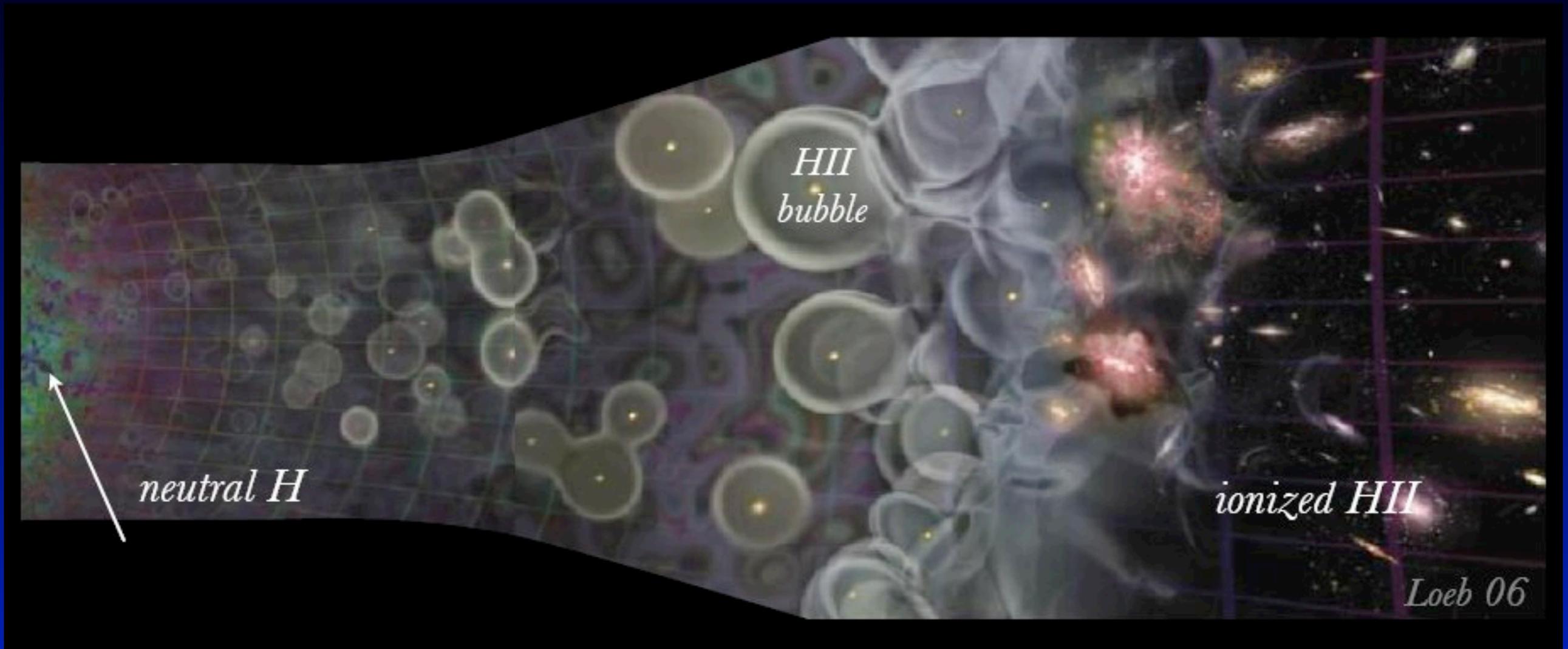
- late free-free absorption at very low frequencies

- Distortion a few times below PIXIE's sensitivity

$$\mu \simeq 1.4 \left. \frac{\Delta \rho_\gamma}{\rho_\gamma} \right|_\mu \approx -3 \times 10^{-9} \quad y \simeq \frac{1}{4} \left. \frac{\Delta \rho_\gamma}{\rho_\gamma} \right|_y \approx -6 \times 10^{-10}$$

Reionization and structure formation

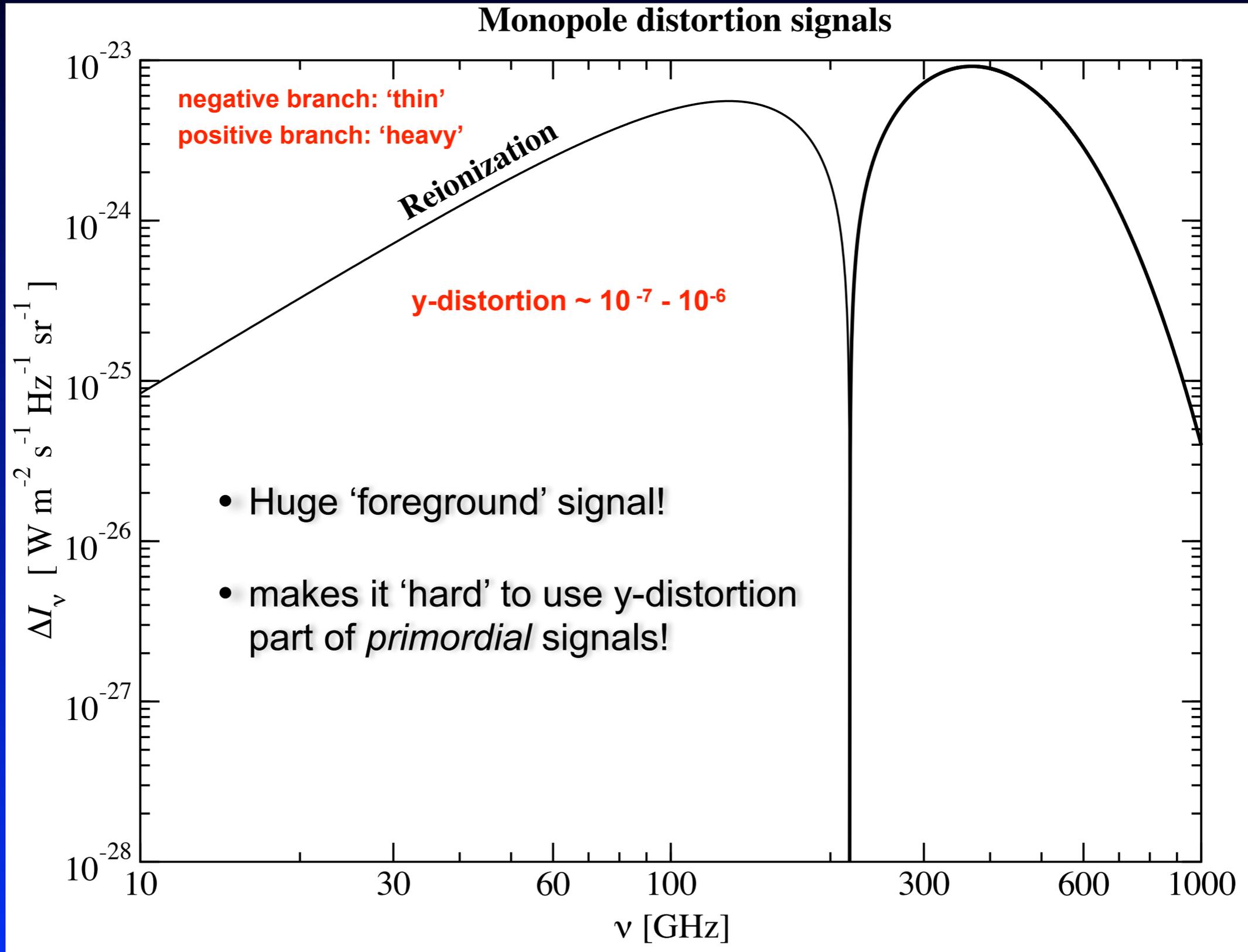
Simple estimates for the distortion



- Gas temperature $T \approx 10^4$ K
 - Thomson optical depth $\tau \approx 0.1$
 - second order Doppler effect $y \approx \text{few} \times 10^{-8}$
 - structure formation / SZ effect (e.g., Refregier et al., 2003) $y \approx \text{few} \times 10^{-7}-10^{-6}$
- $\implies y \approx \frac{kT_e}{m_e c^2} \tau \approx 2 \times 10^{-7}$

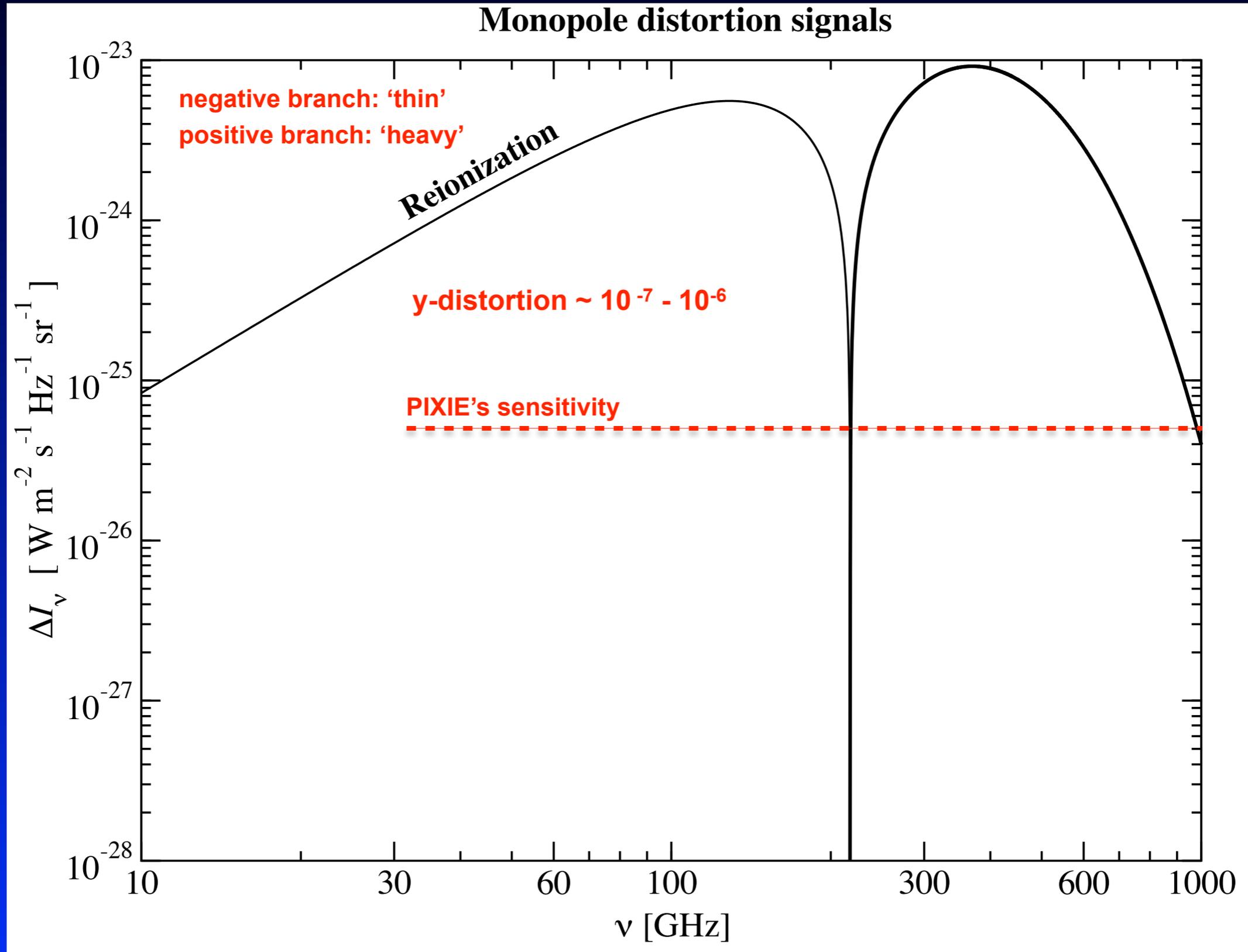
Average CMB spectral distortions

Absolute value of Intensity signal



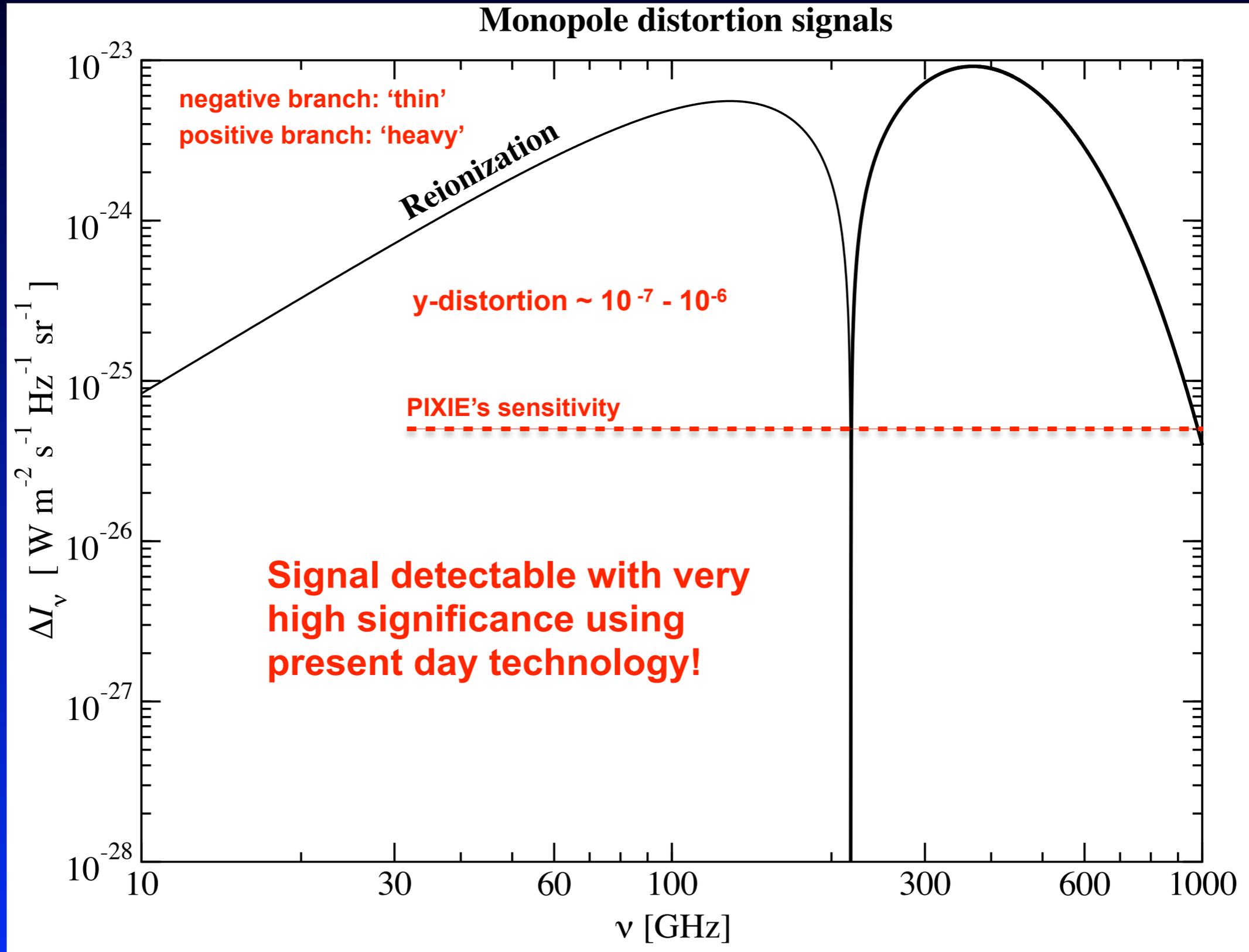
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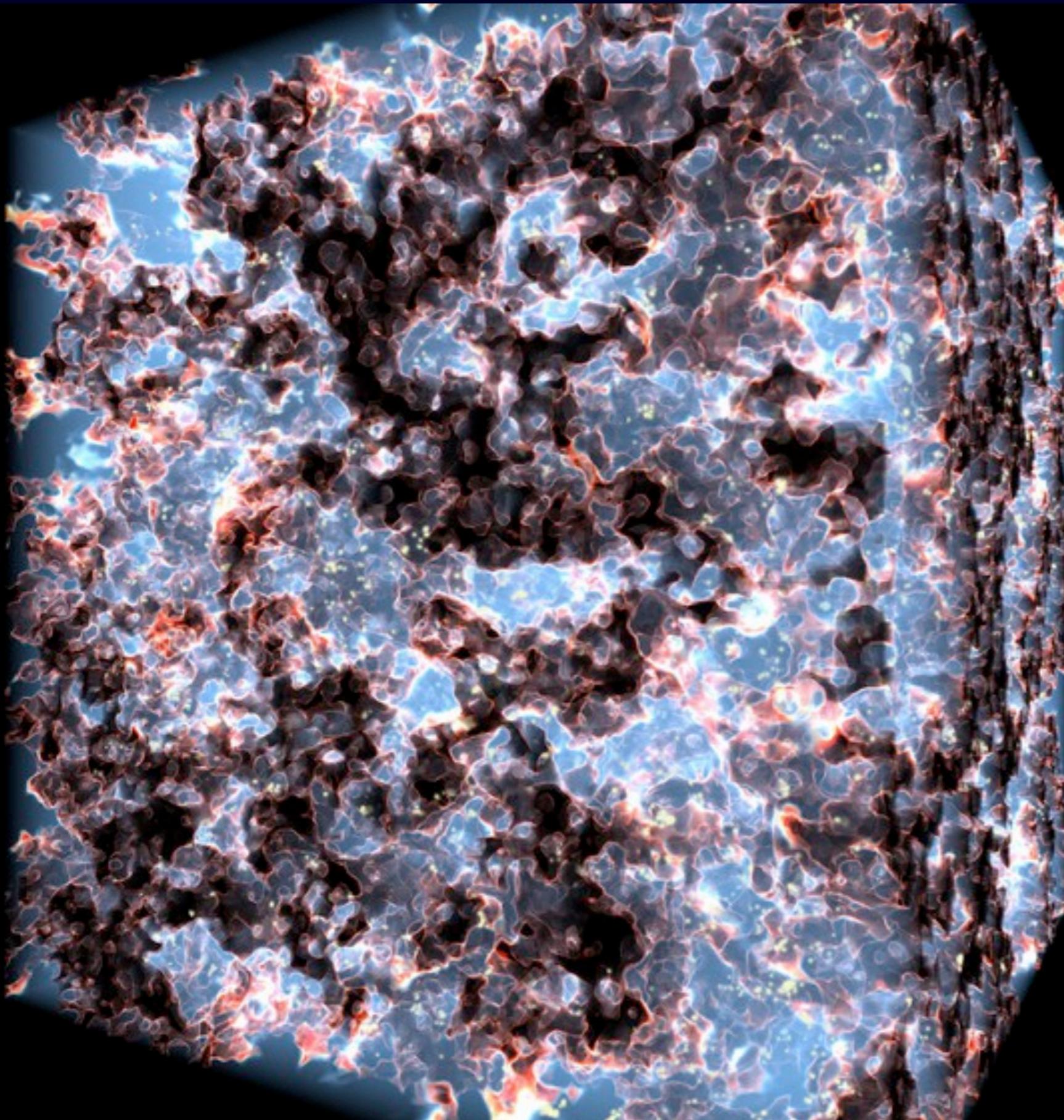


Average CMB spectral distortions

Absolute value of Intensity signal



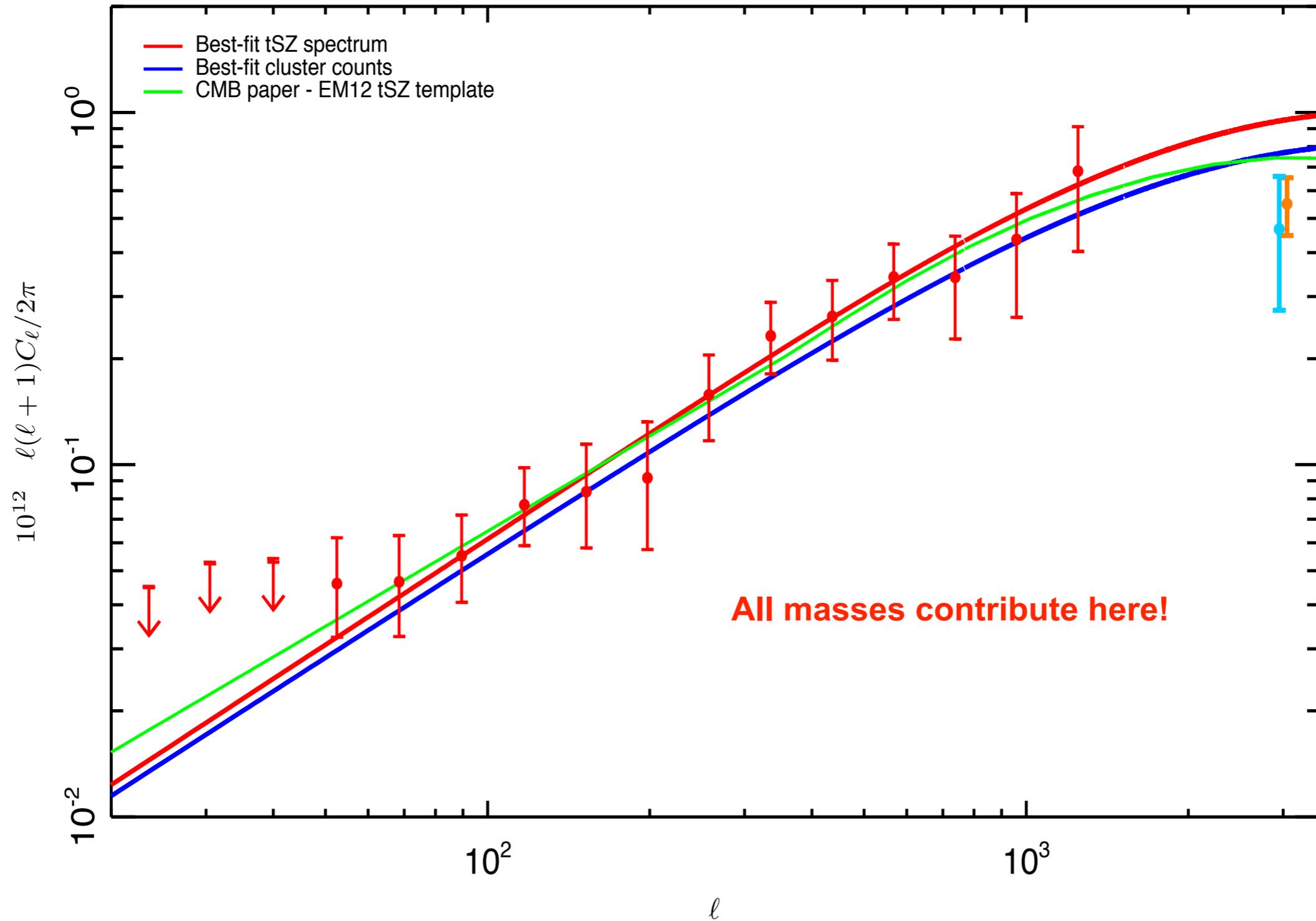
Fluctuations of the γ -parameter at large scales



- spatial variations of the optical depth and temperature cause small-spatial variations of the γ -parameter at different angular scales
- could tell us about the reionization sources and structure formation process
- additional independent piece of information!
- Cross-correlations with other signals

Example:
Simulation of reionization process
(1Gpc/h) by *Alvarez & Abel*

Measured power spectrum for y -parameter



Decaying particles

Why is this interesting?

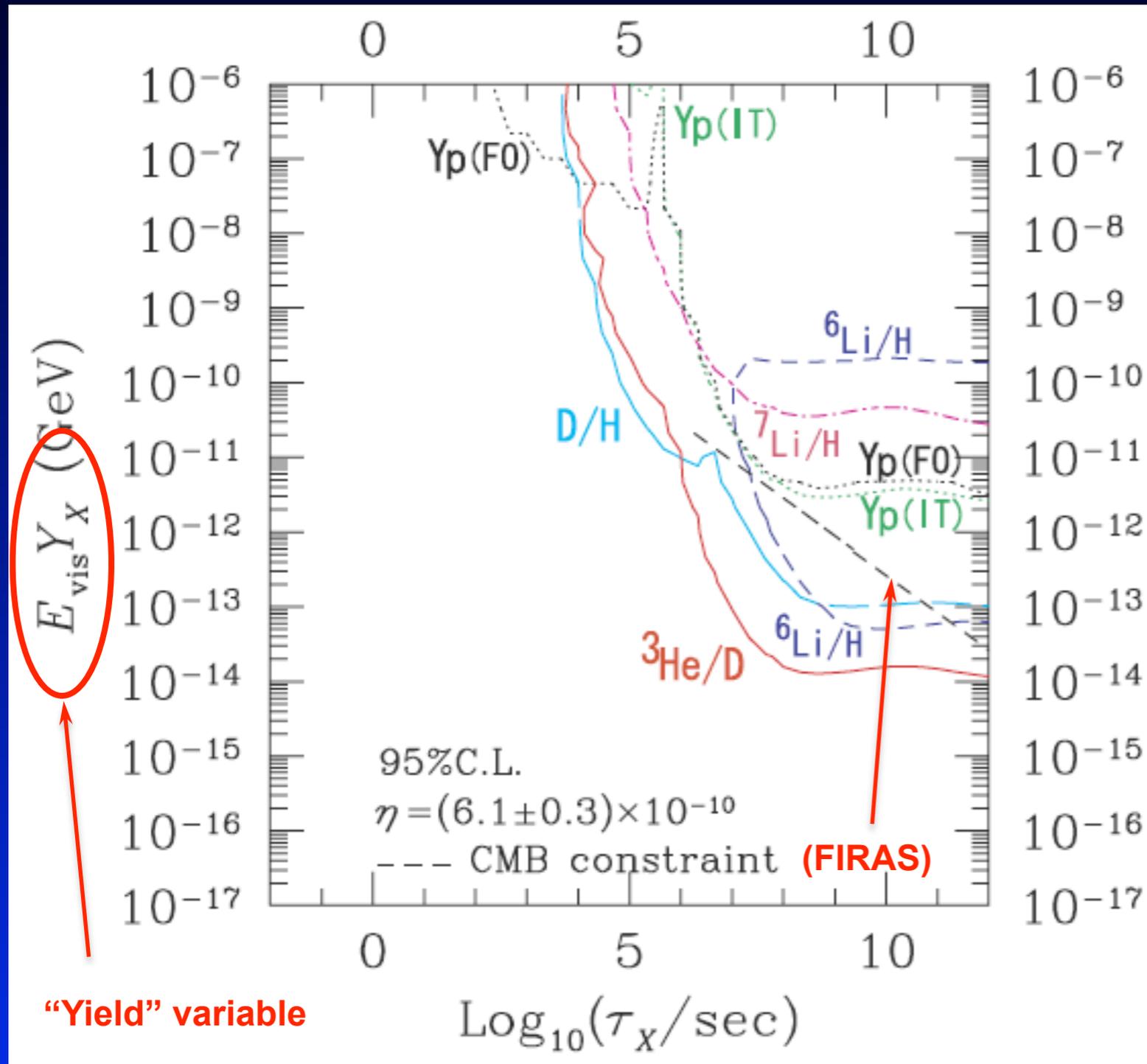
- A priori no specific particle in mind
- *But:* we do not know what dark matter is and where it really came from!
- Was dark matter thermally produced or as a decay product of some heavy particle?
- is dark matter structureless or does it have internal (excited) states?
- sterile neutrinos? moduli? Some other relic particle?
- From the theoretical point of view really no shortage of particles to play with...

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CMB spectral distortions offer a new independent way to constrain these kind of models

Constraints from measurements of light elements

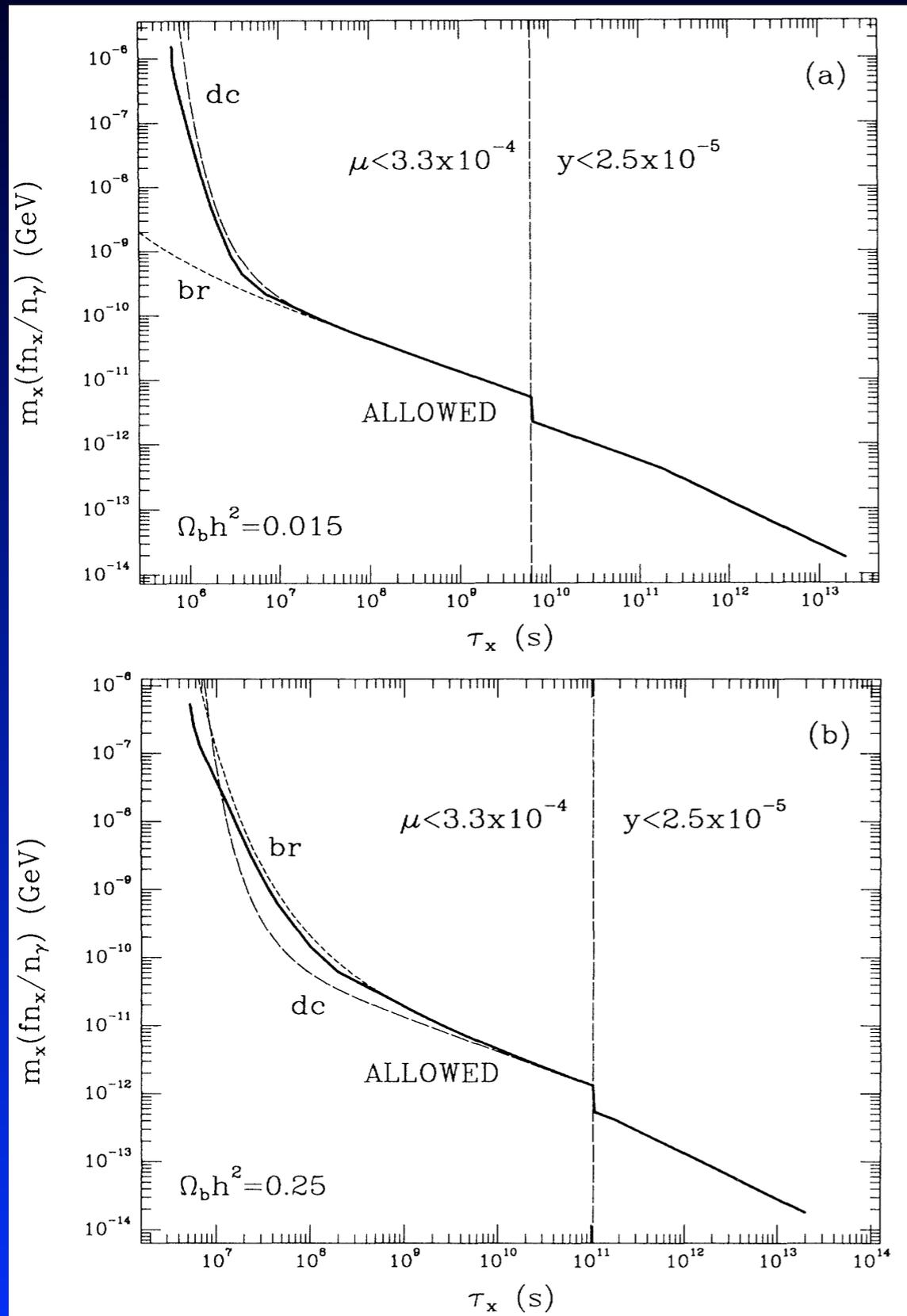


- Yield variable \Rightarrow parametrizes the total energy release relative to total entropy density of the Universe

$$Y_X \simeq N_X/S$$

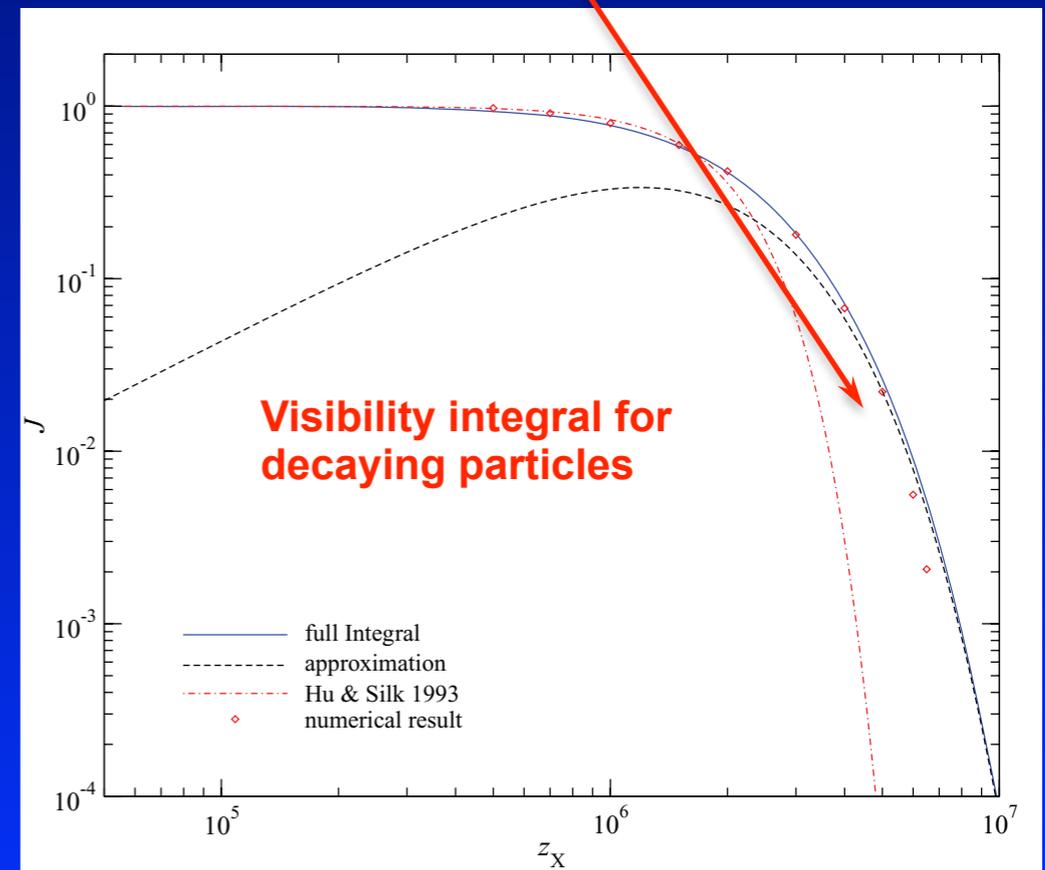
- energy release *destroys* light elements
- E_{vis} hides physics of energy deposition (decay channels, neutrino fraction, etc.)
- current CMB limit rather weak in comparison....

Early constraints from CMB measurements



Hu & Silk, 1993

- Simple estimates for μ and y -distortion from energy arguments just like we discussed above
- Early COBE/FIRAS limits
- constraint a little tighter for short lifetimes than estimated...



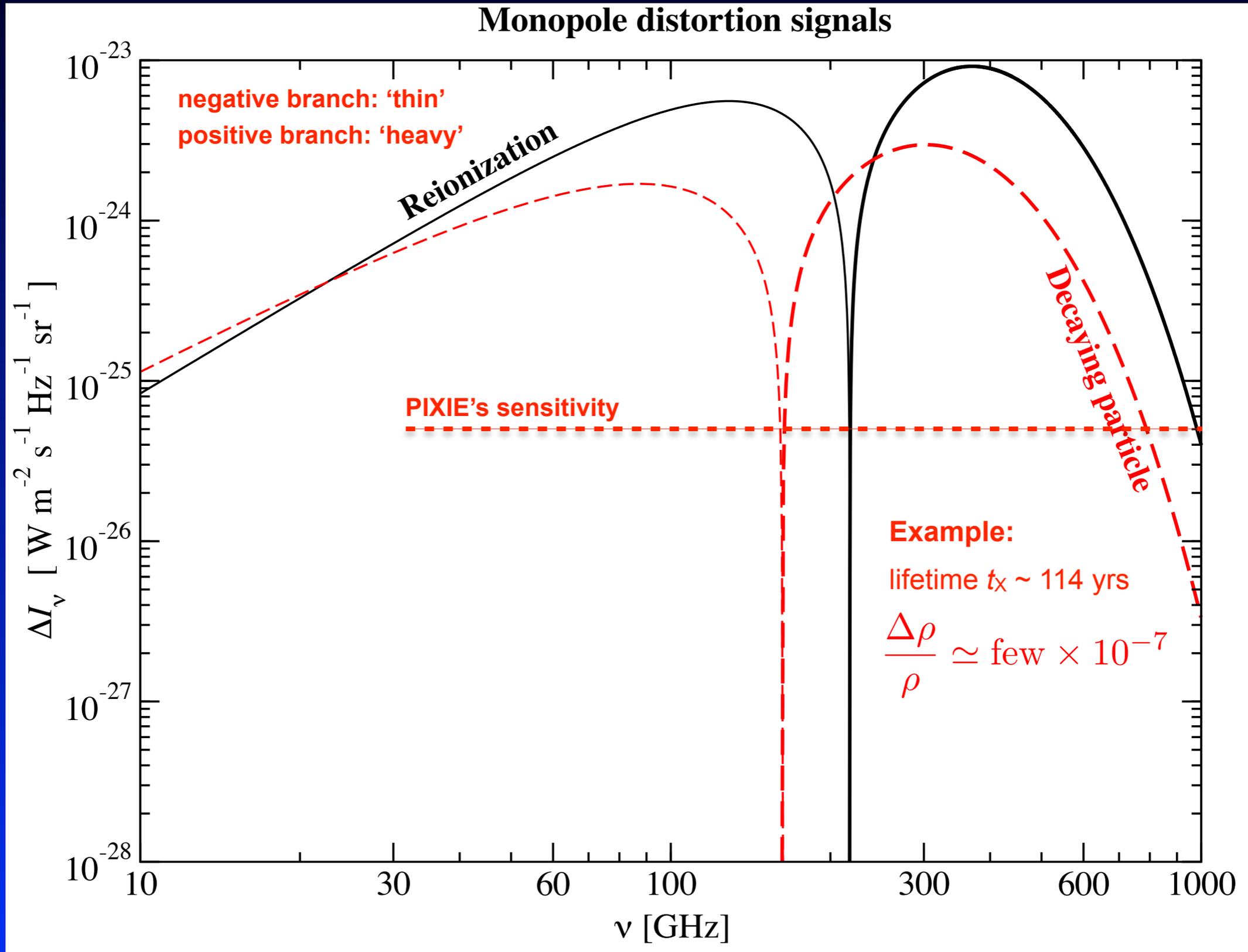
JC, 2005; JC & Sunyaev, 2012

Energy release by decaying particles

- Energy release rate $\frac{d(Q/\rho_\gamma)}{dz} \approx \frac{f^* M_X c^2}{H(z)(1+z)} \frac{N_X(z)}{\rho_\gamma(z)} \Gamma_X e^{-\Gamma_X t}$
- For computations: $f_X = f^* M_X c^2 N_X / N_H$ and $\varepsilon_X = \frac{f_X}{z_X}$
- Efficiency factor f^* contains all the physics describing the cascade of decay products
- At high redshift deposited energy goes all into heat
- Around recombination and after things become more complicated
(Slatyer et al. 2009; Cirelli et al. 2009; Huetsi et al. 2009; Slatyer et al. 2013)
 - \Rightarrow branching ratios into heat, ionizations, and atomic excitation
 - \Rightarrow details would be important for y -distortion part

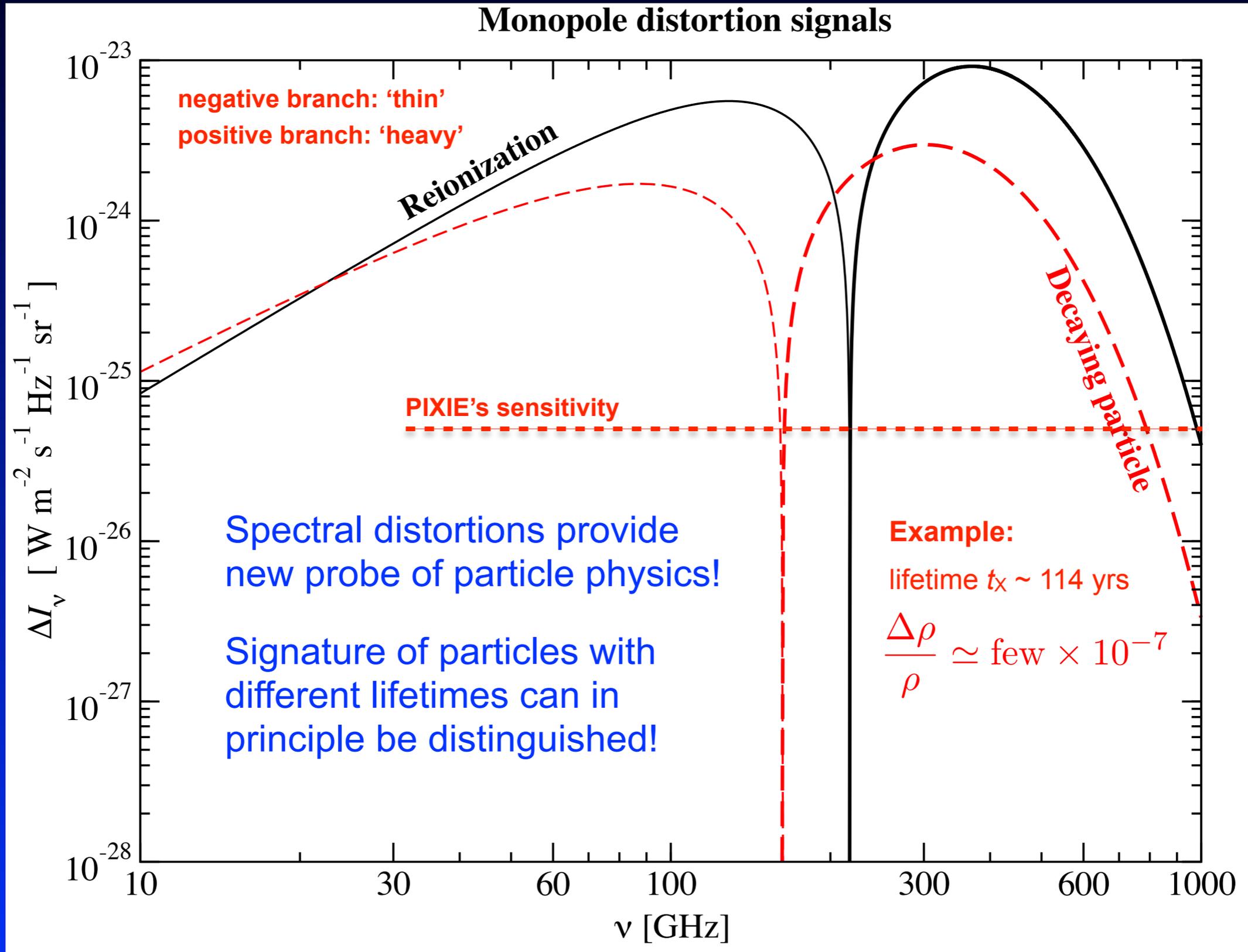
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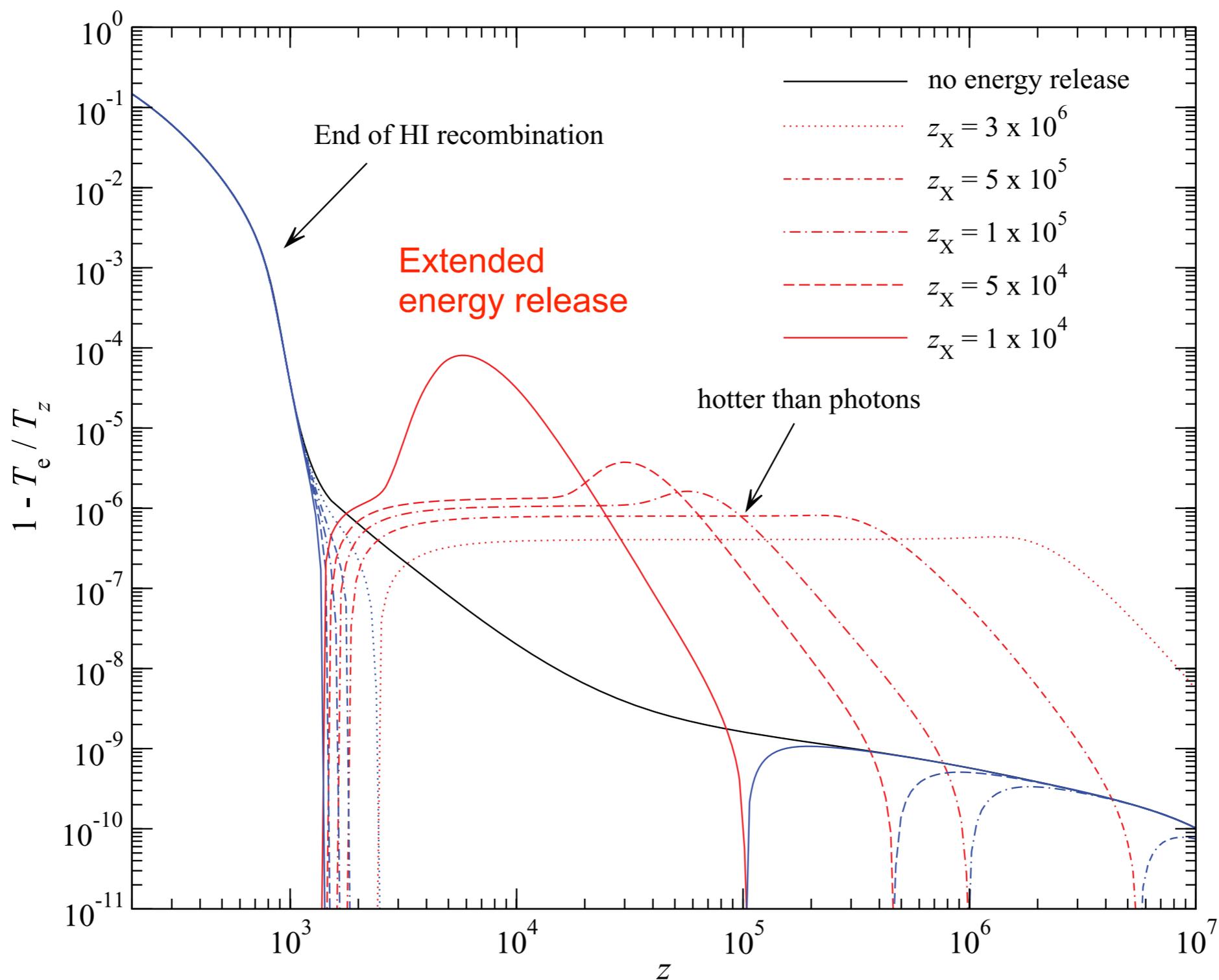


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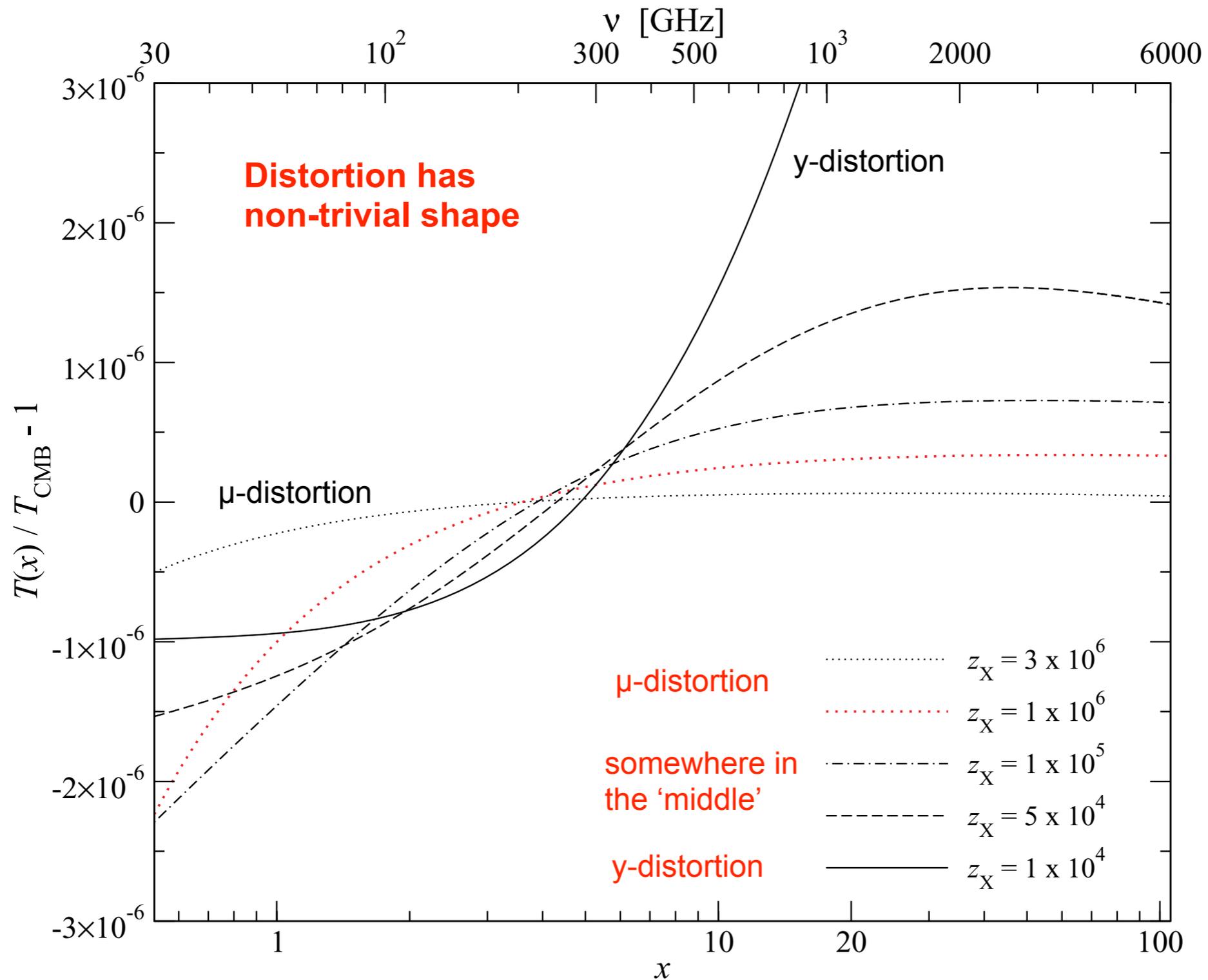
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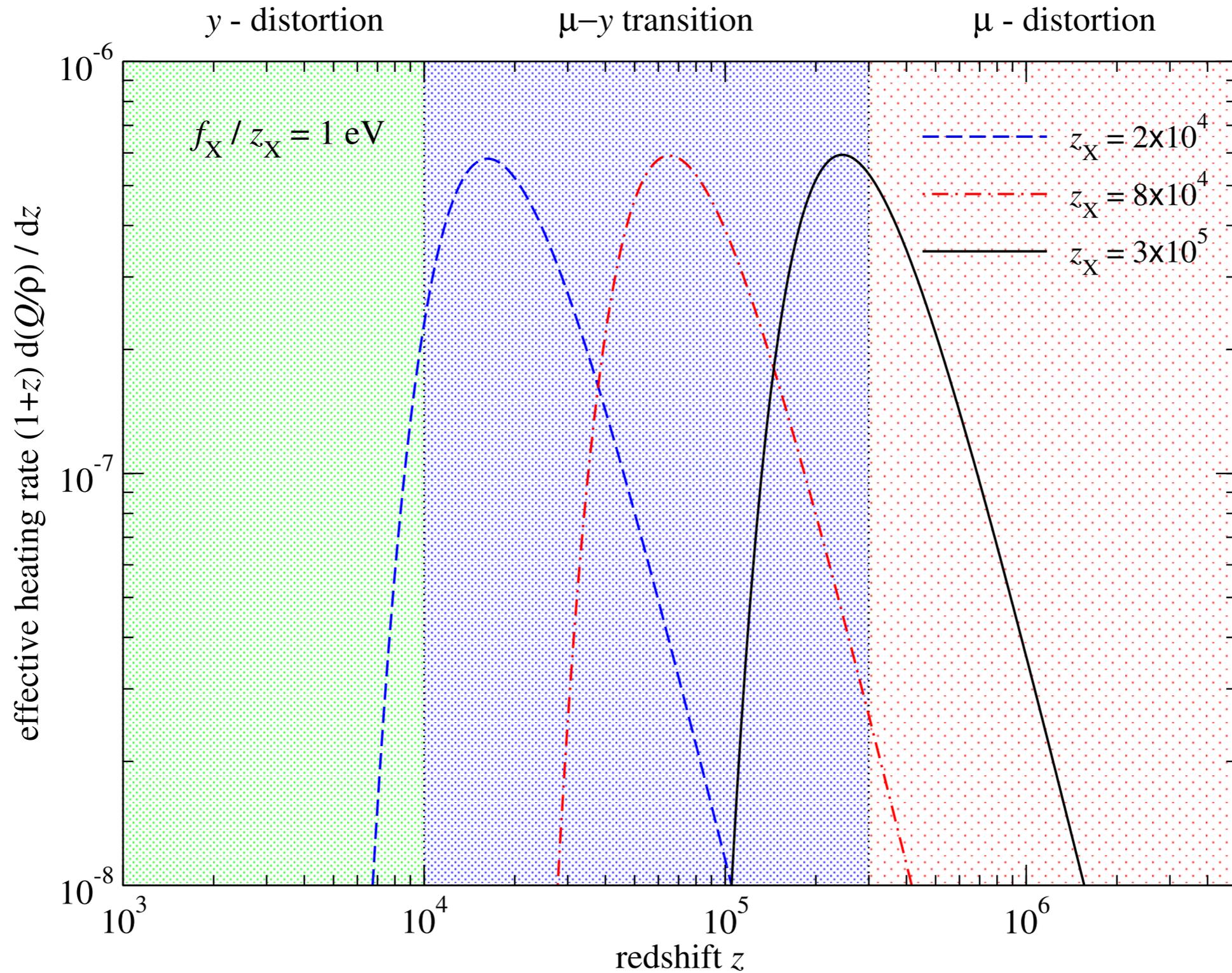
Electron temperature evolution for decaying particle scenarios



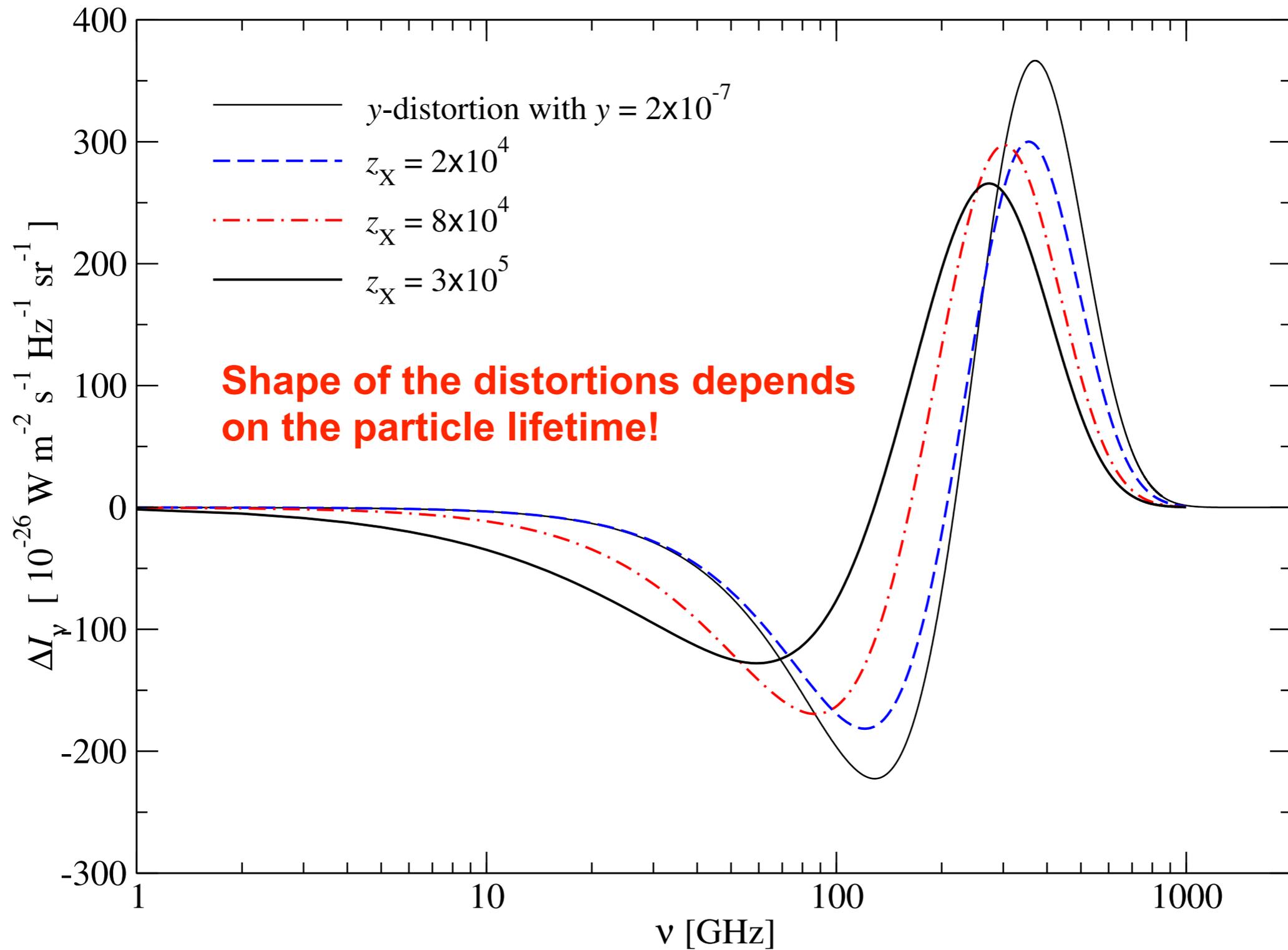
High frequency distortion for decaying particle scenarios



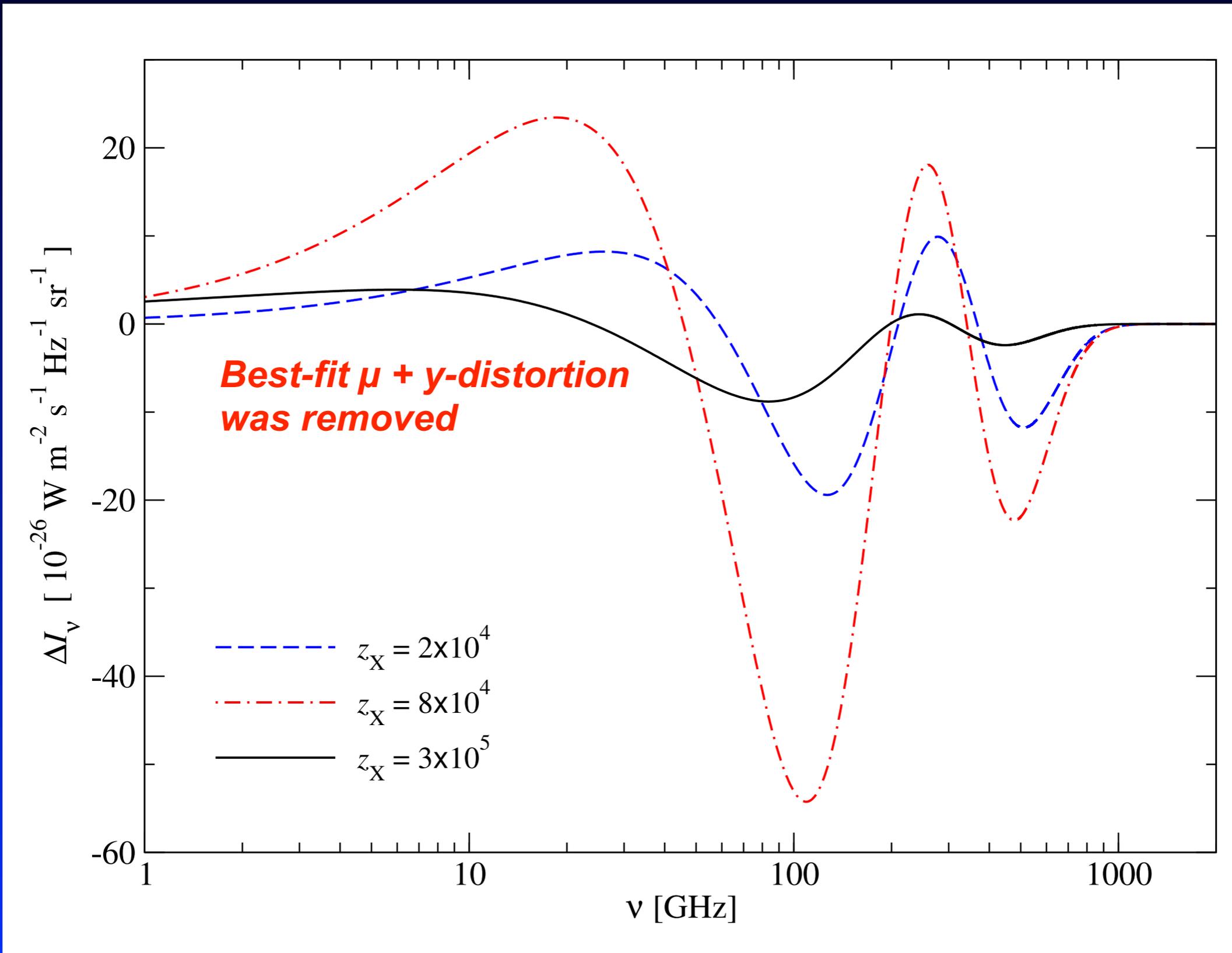
Decaying particle scenarios



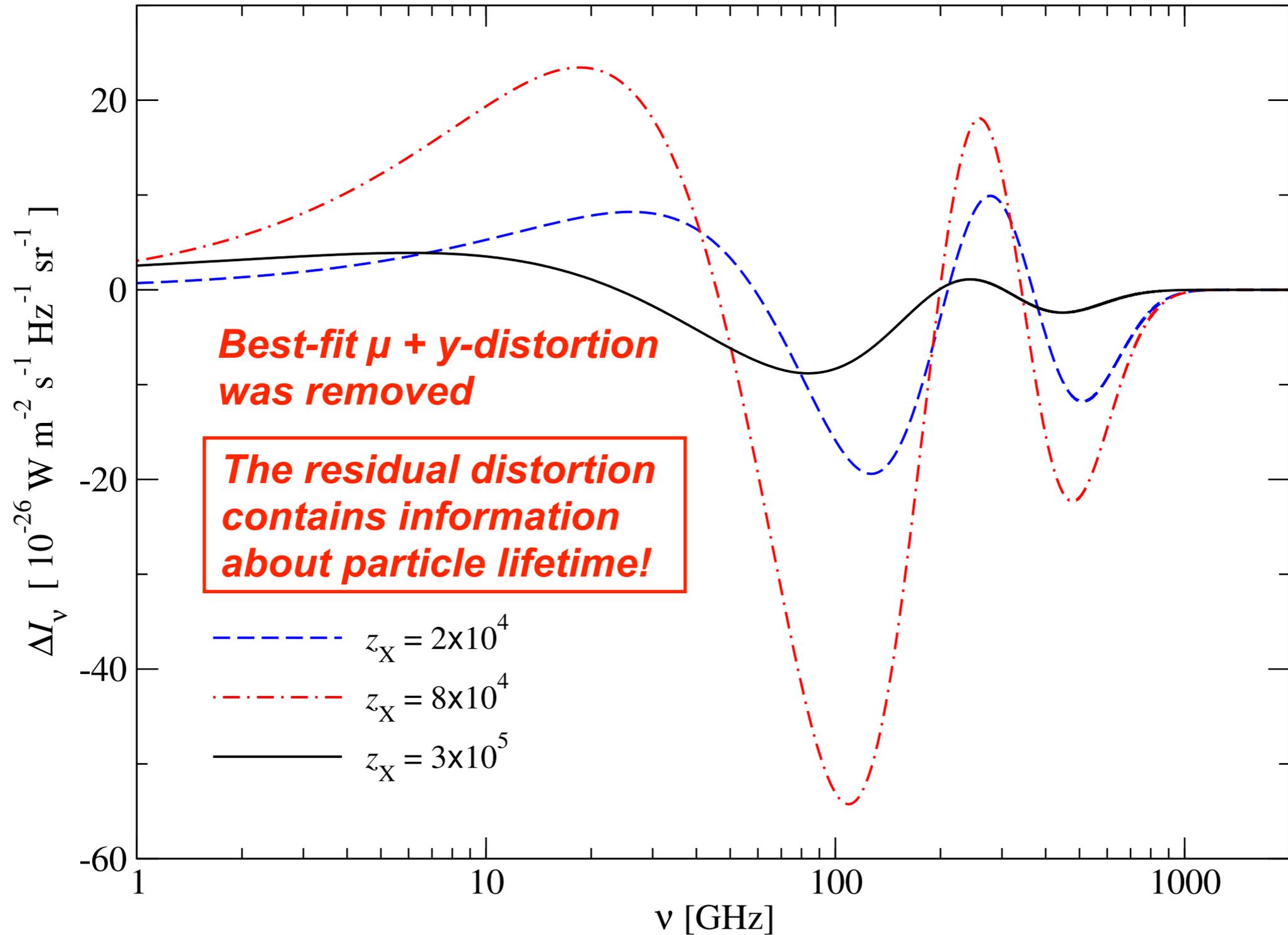
Decaying particle scenarios



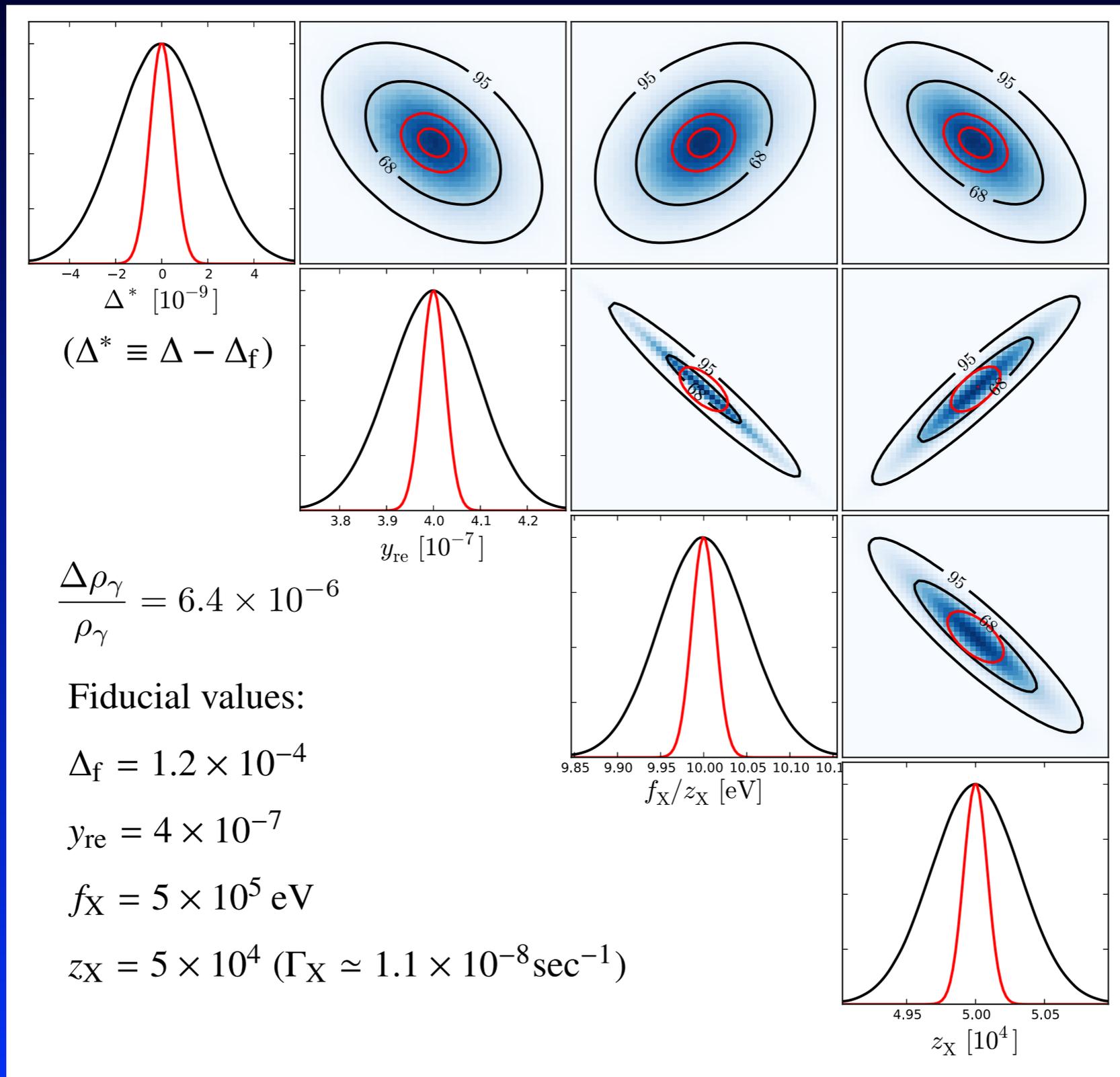
Decaying particle scenarios (information in residual)



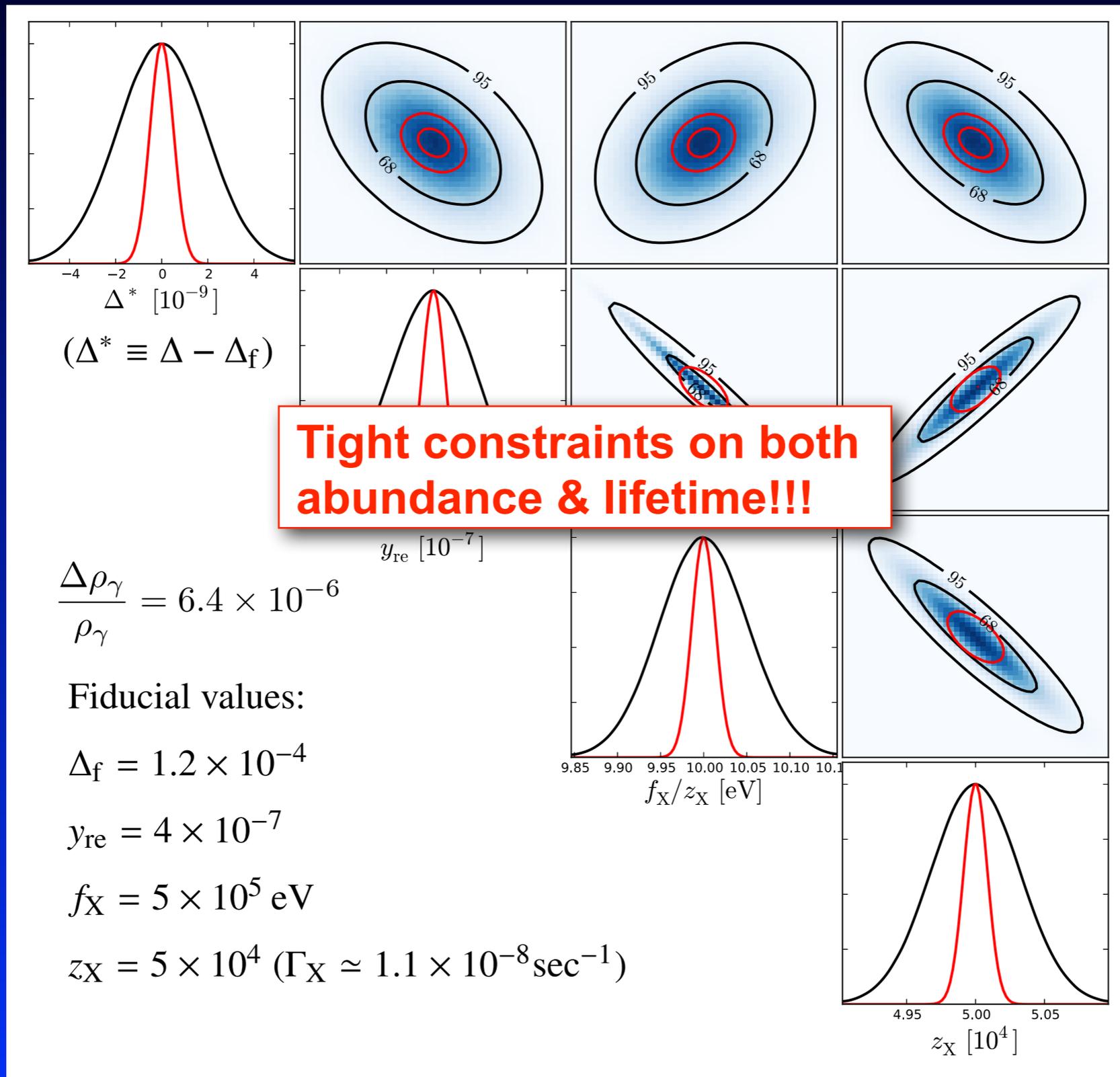
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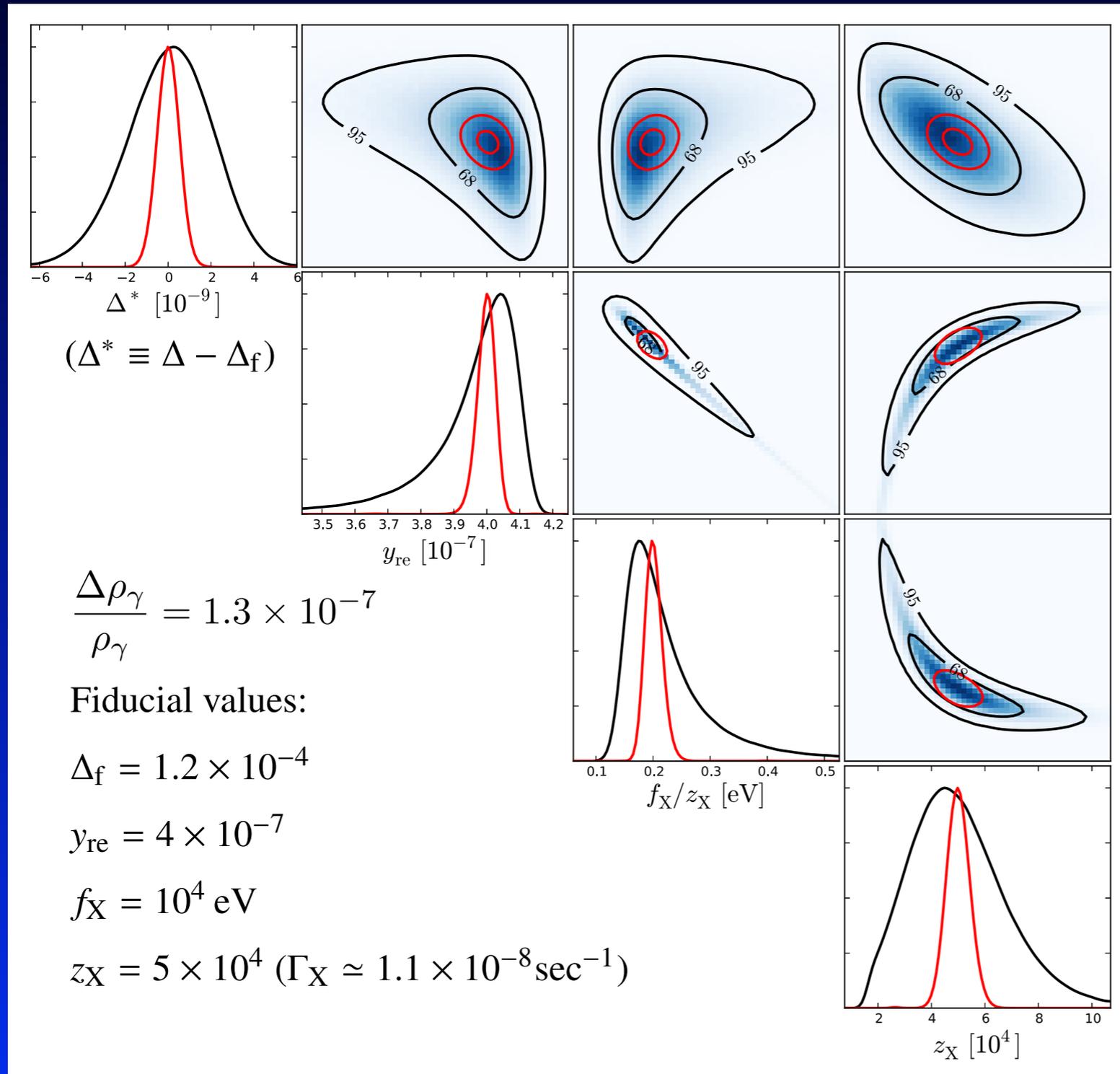
Decaying particle scenarios



Decaying particle scenarios

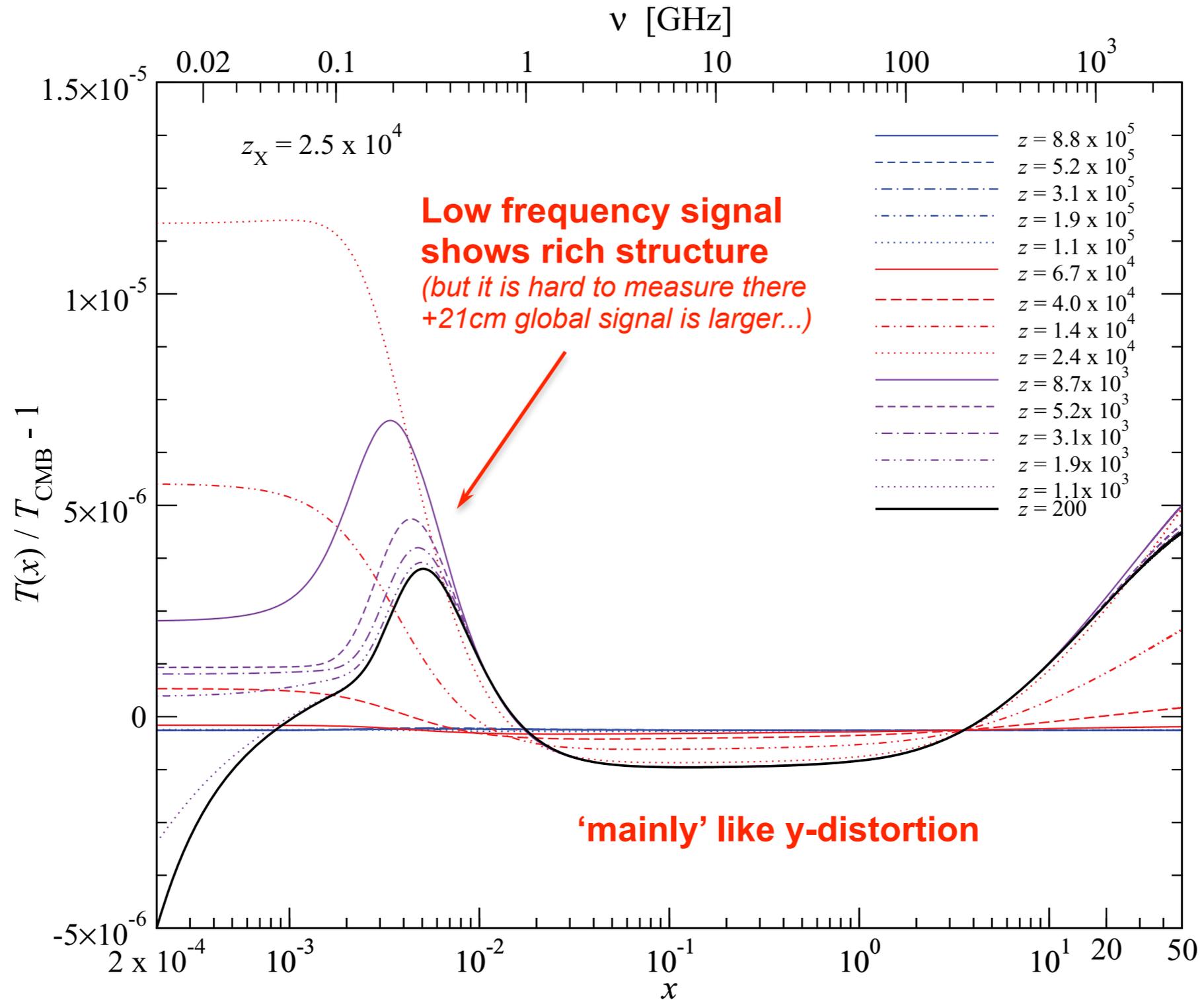


Decaying particle scenario close to detection limit



- Parameters more degenerate close to detection limit
- Non-Gaussian contours
- big improvement with sensitivity

Distortion for late particle decay

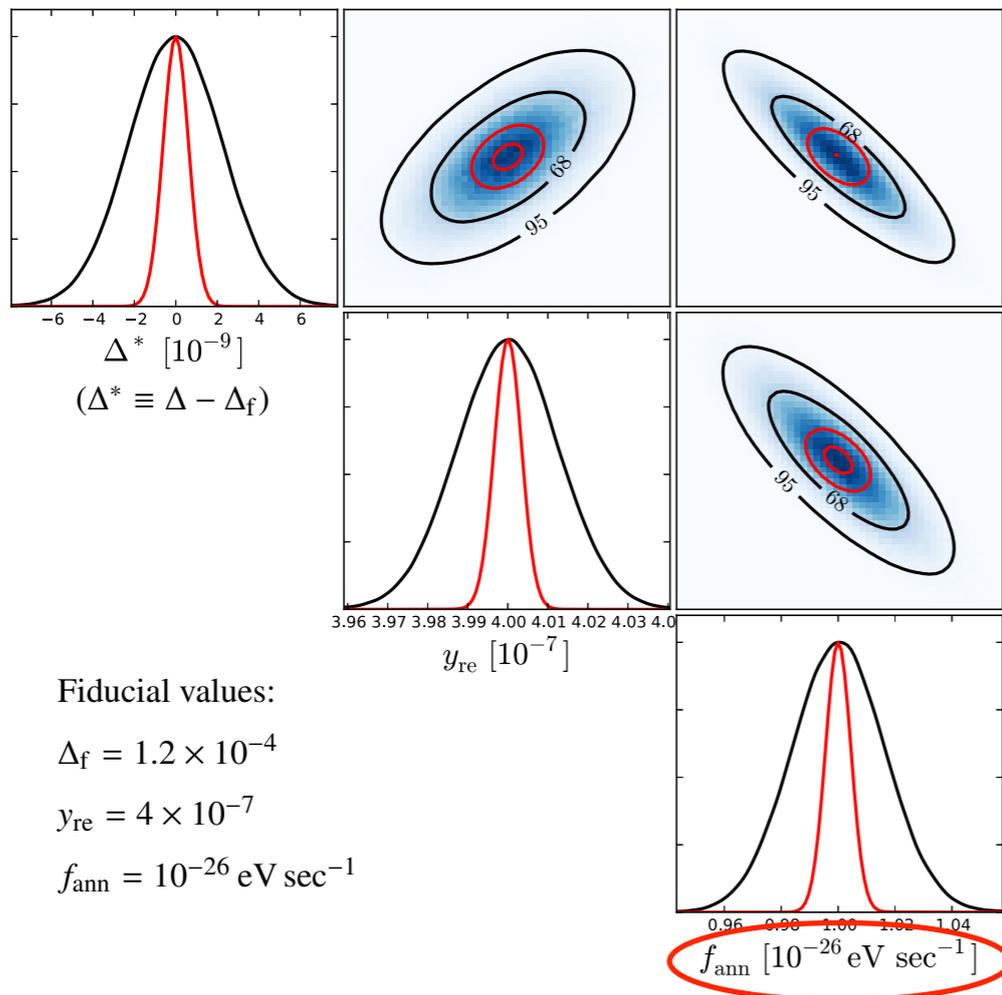


Compressing the information of the distortion signal

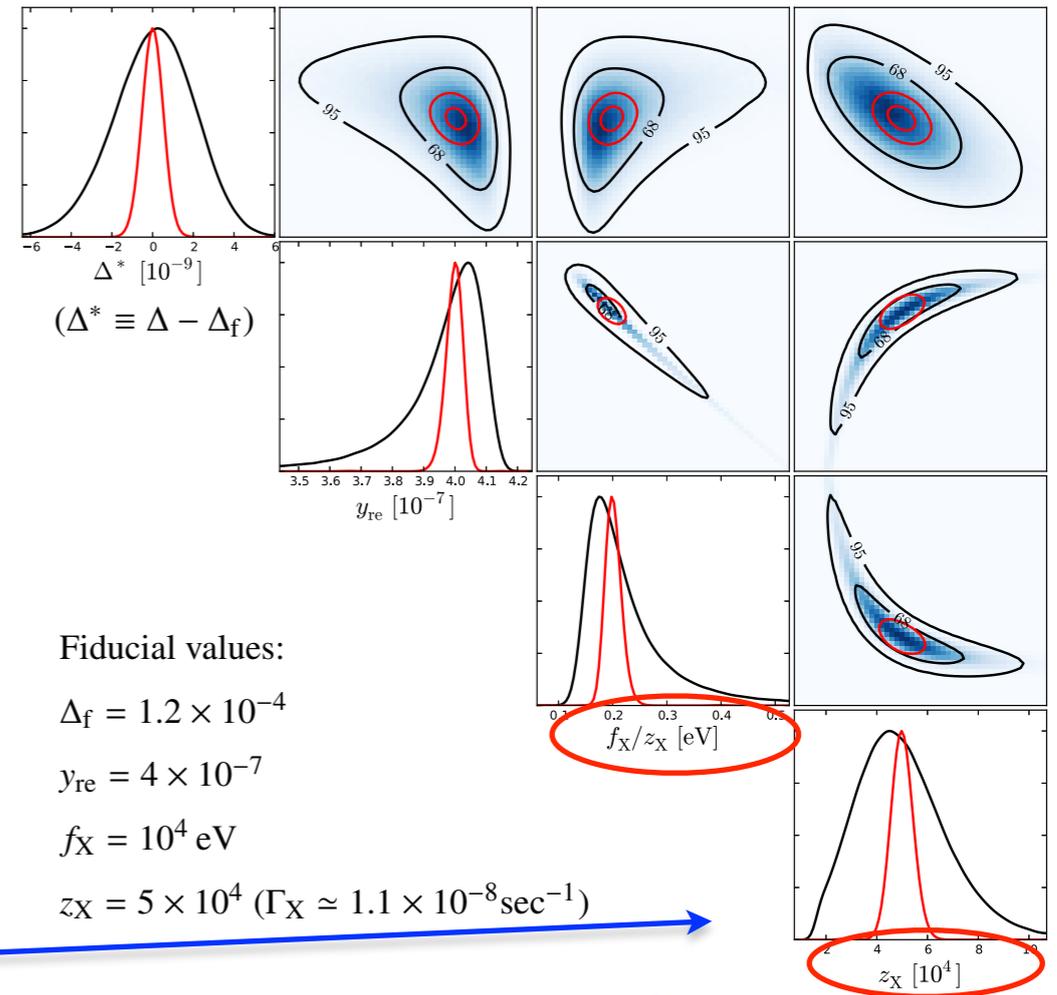
Why model-independent approach to distortion signal

- Model-dependent analysis makes model-selection non-trivial
- Real information in the distortion signal limited by sensitivity and foregrounds
- *Principle Component Analysis (PCA)* can help optimizing this!
- useful for optimizing experimental designs (*frequencies; sensitivities, ...*)!

Annihilation scenario

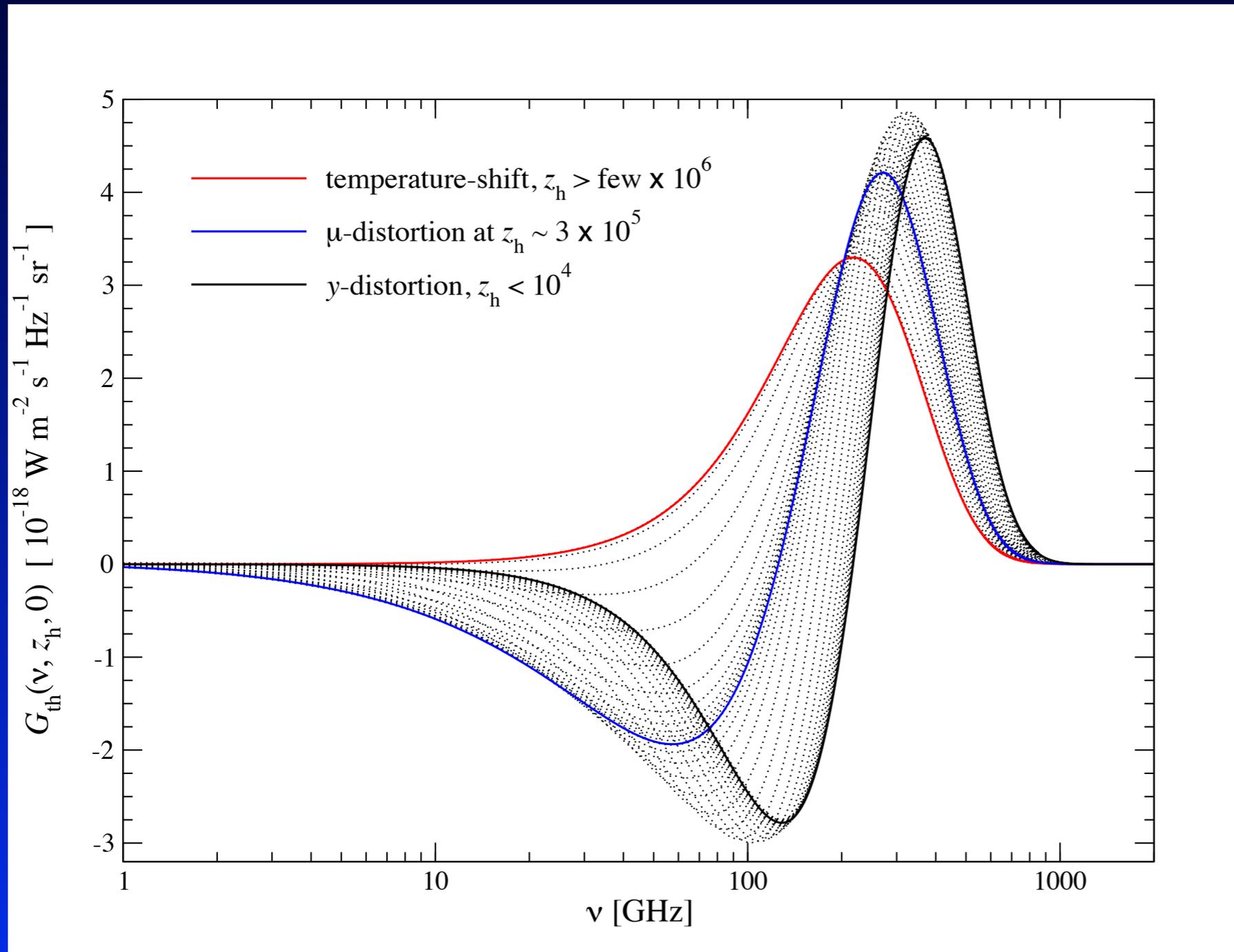


Decaying particle scenario



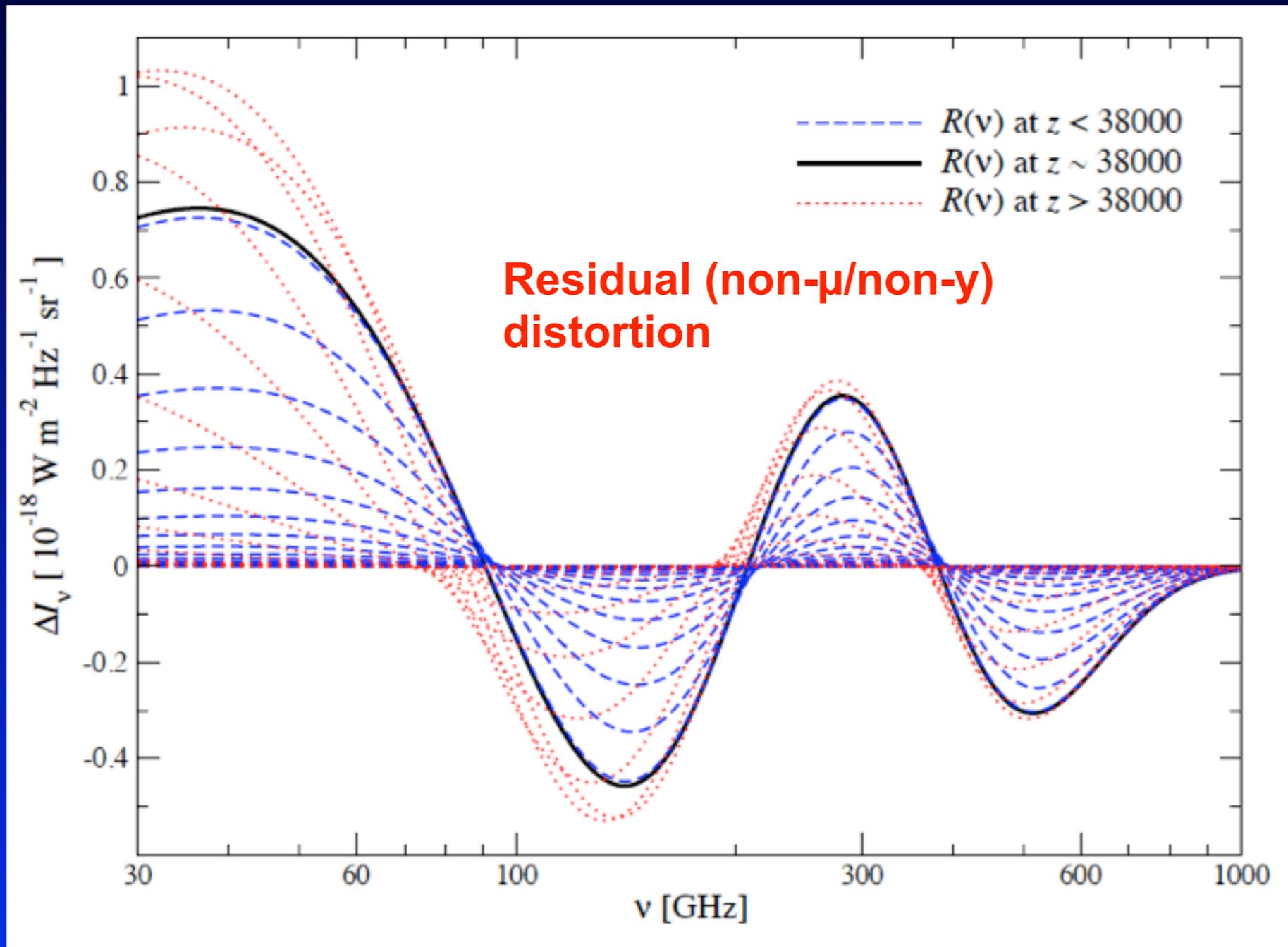
How do we compare these?

Using signal eigenmodes to compress the distortion data



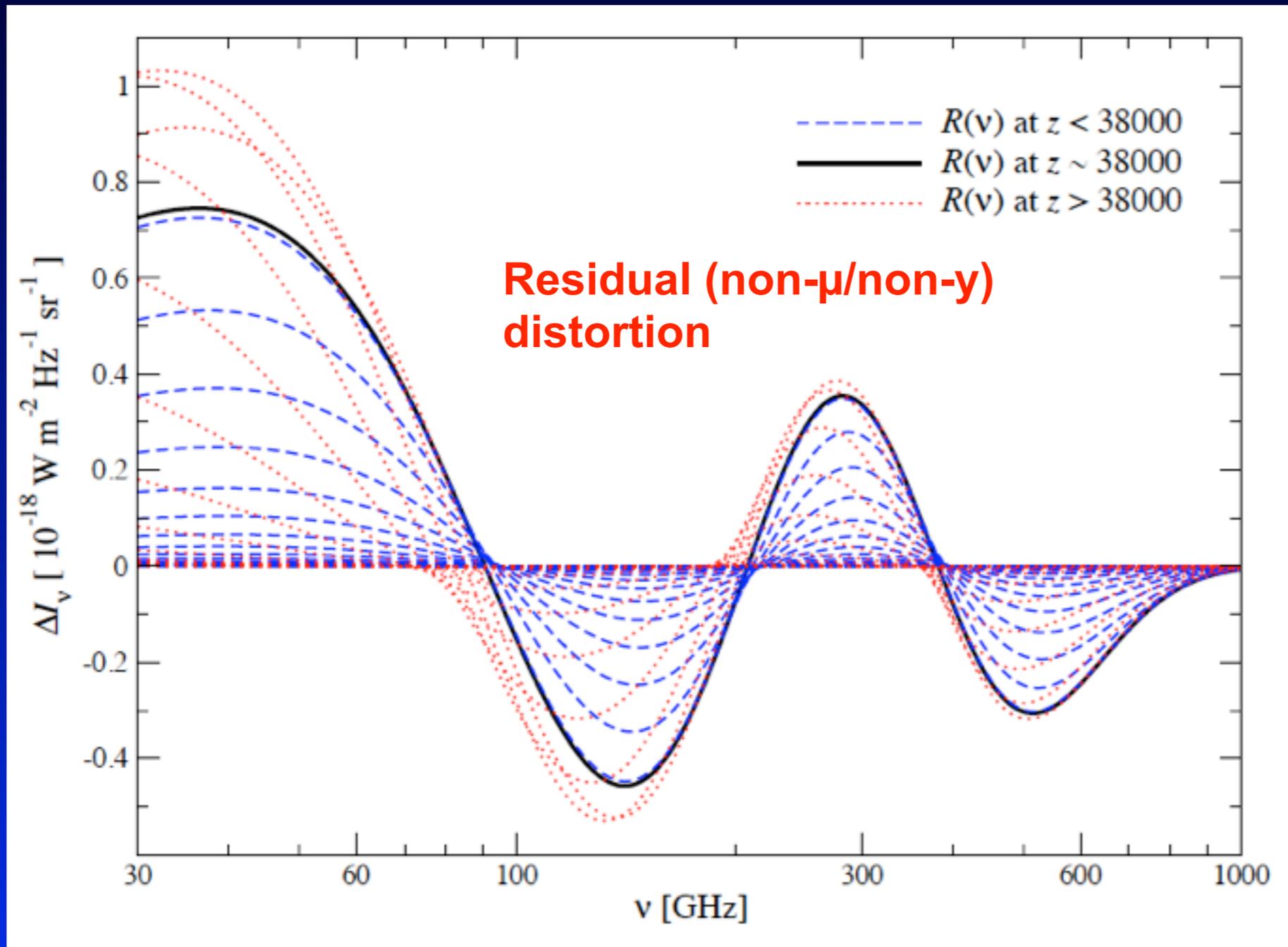
- *Principle component decomposition* of the distortion signal
- compression of the useful information given instrumental settings

Using signal eigenmodes to compress the distortion data



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Using signal eigenmodes to compress the distortion data



- *Principle component decomposition* of the distortion signal
- compression of the useful information given instrumental settings
- new set of observables
 $p = \{y, \mu, \mu_1, \mu_2, \dots\}$
- model-comparison + forecasts of errors very simple!

Residual distortion dependents on settings

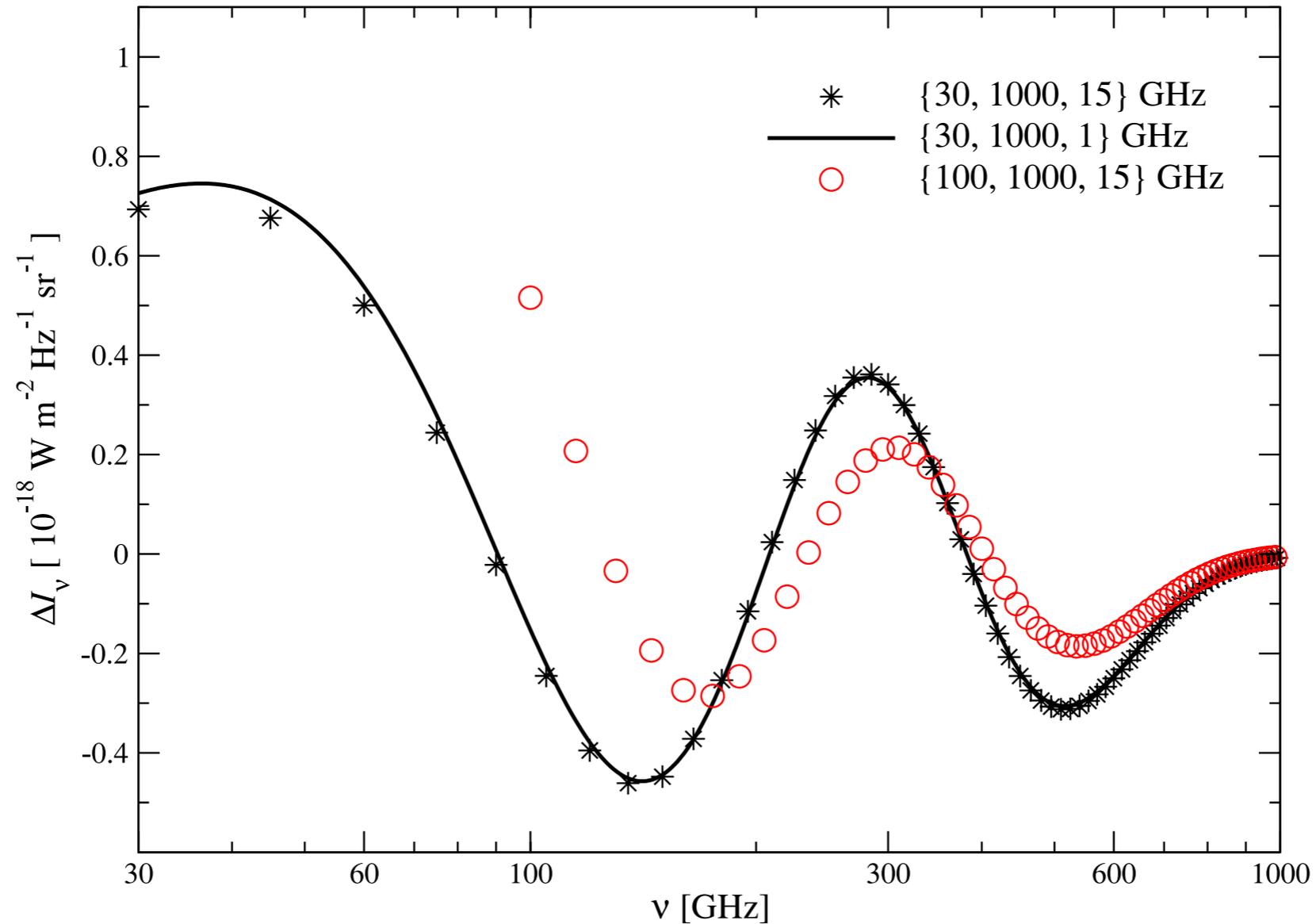


Figure 3. Residual function at redshift $z \simeq 38\,000$ but for different instrumental settings. The annotated values are $\{\nu_{\min}, \nu_{\max}, \Delta\nu_s\}$ and we assumed diagonal noise covariance.

Eigenmodes for a *PIXIE*-type experiment

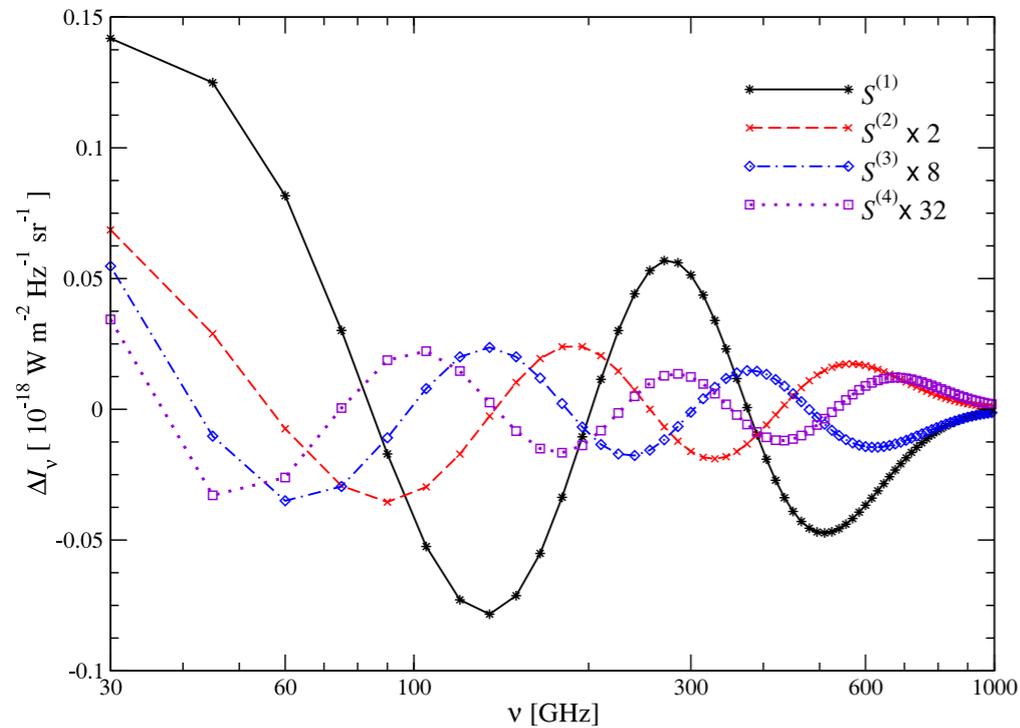
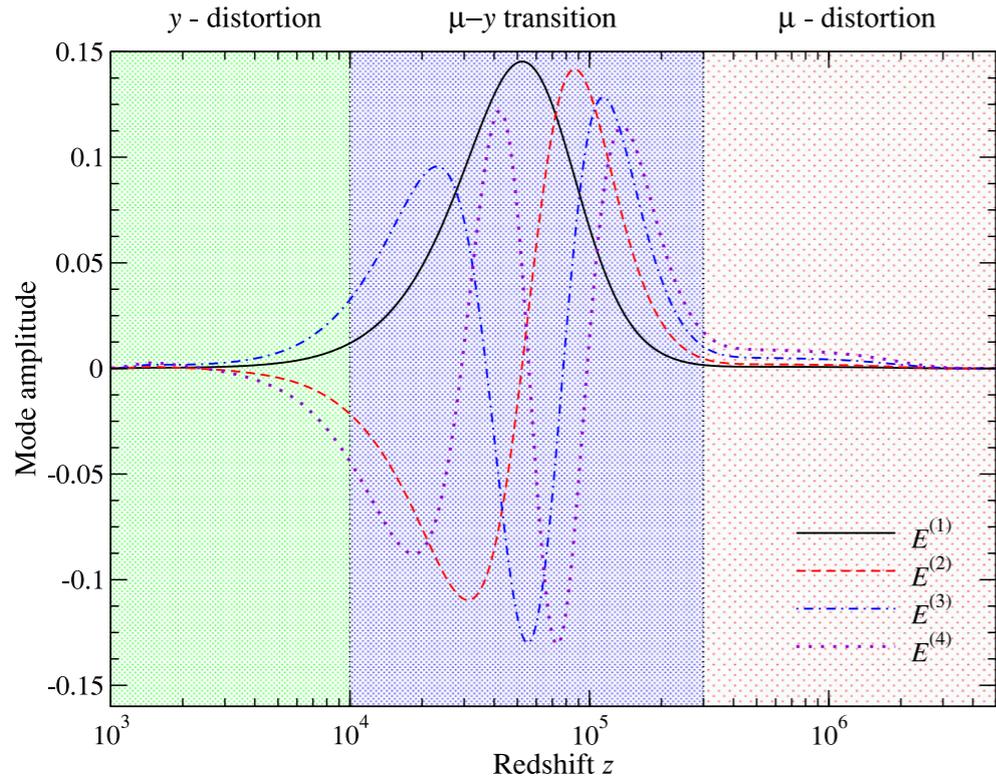


Figure 4. First few eigenmodes $E^{(k)}$ and $S^{(k)}$ for *PIXIE*-type settings ($\nu_{\min} = 30$ GHz, $\nu_{\max} = 1000$ GHz and $\Delta\nu_s = 15$ GHz). In the mode construction, we assumed that energy release only occurred at $10^3 \leq z \leq 5 \times 10^6$.

Estimated error bars

(under idealistic assumptions...)

$$\frac{\Delta T}{T} \simeq 2 \text{ nK} \left(\frac{\Delta I_c}{5 \text{ Jy sr}^{-1}} \right)$$

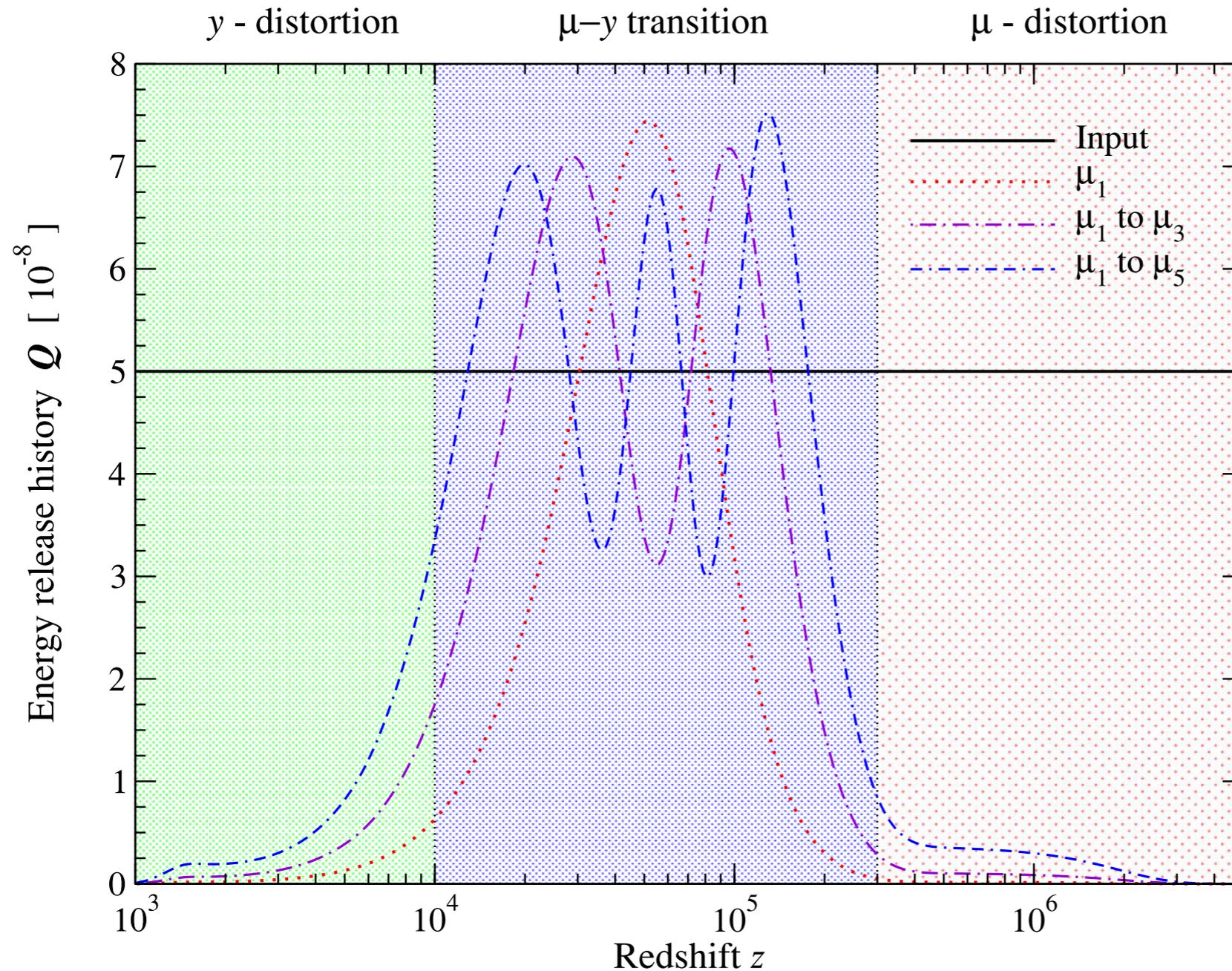
$$\Delta y \simeq 1.2 \times 10^{-9} \left(\frac{\Delta I_c}{5 \text{ Jy sr}^{-1}} \right)$$

$$\Delta \mu \simeq 1.4 \times 10^{-8} \left(\frac{\Delta I_c}{5 \text{ Jy sr}^{-1}} \right)$$

Table 1. Forecasted 1σ errors of the first six eigenmode amplitudes, $E^{(k)}$. We also give $\varepsilon_k = 4 \sum_i S_i^{(k)} / \sum_i G_{i,T}$, and the scalar products $S^{(k)} \cdot S^{(k)}$ (in units of $[10^{-18} \text{ W m}^{-2} \text{ Hz}^{-1} \text{ sr}^{-1}]^2$). The fraction of energy release to the residual distortion and its uncertainty are given by $\varepsilon \approx \sum_k \varepsilon_k \mu_k$ and $\Delta\varepsilon \approx (\sum_k \varepsilon_k^2 \Delta\mu_k^2)^{1/2}$, respectively. For the mode construction we used *PIXIE*-settings ($\{\nu_{\min}, \nu_{\max}, \Delta\nu_s\} = \{30, 1000, 15\}$ GHz and channel sensitivity $\Delta I_c = 5 \times 10^{-26} \text{ W m}^{-2} \text{ Hz}^{-1} \text{ sr}^{-1}$). The errors roughly scale as $\Delta\mu_k \propto \Delta I_c / \sqrt{\Delta\nu_s}$.

k	$\Delta\mu_k$	$\Delta\mu_k / \Delta\mu_1$	ε_k	$S^{(k)} \cdot S^{(k)}$
1	1.48×10^{-7}	1	-6.98×10^{-3}	1.15×10^{-1}
2	7.61×10^{-7}	5.14	2.12×10^{-3}	4.32×10^{-3}
3	3.61×10^{-6}	24.4	-3.71×10^{-4}	1.92×10^{-4}
4	1.74×10^{-5}	1.18×10^2	8.29×10^{-5}	8.29×10^{-6}
5	8.52×10^{-5}	5.76×10^2	-1.55×10^{-5}	3.45×10^{-7}
6	4.24×10^{-4}	2.86×10^3	2.75×10^{-6}	1.39×10^{-8}

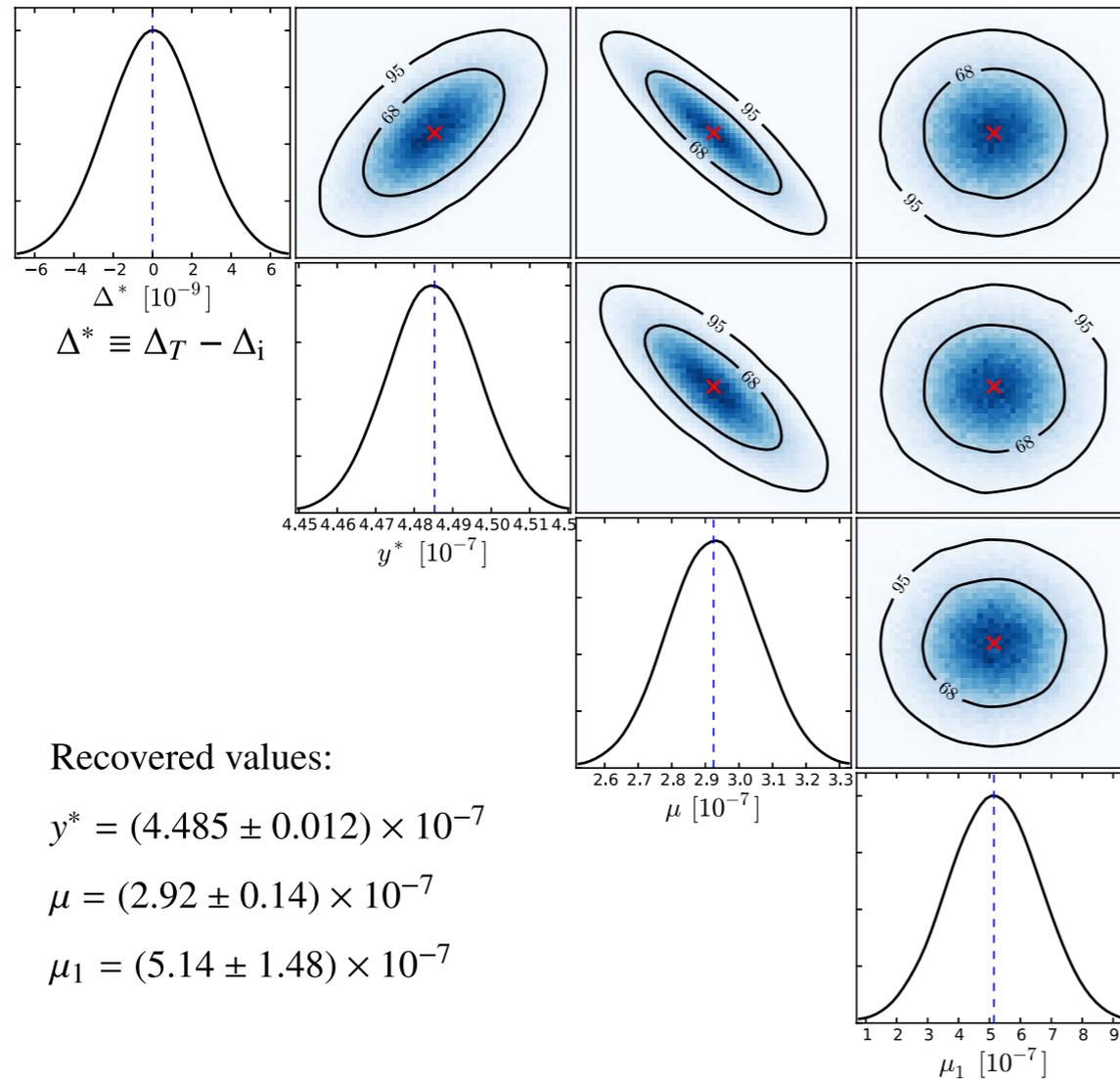
Partial recovery of energy release history



- 'wiggly' recovery of input thermal history possible
- redshift resolution depends on sensitivity and distortion amplitude

Figure 6. Partial recovery of the input energy-release history, $Q = 5 \times 10^{-8}$.

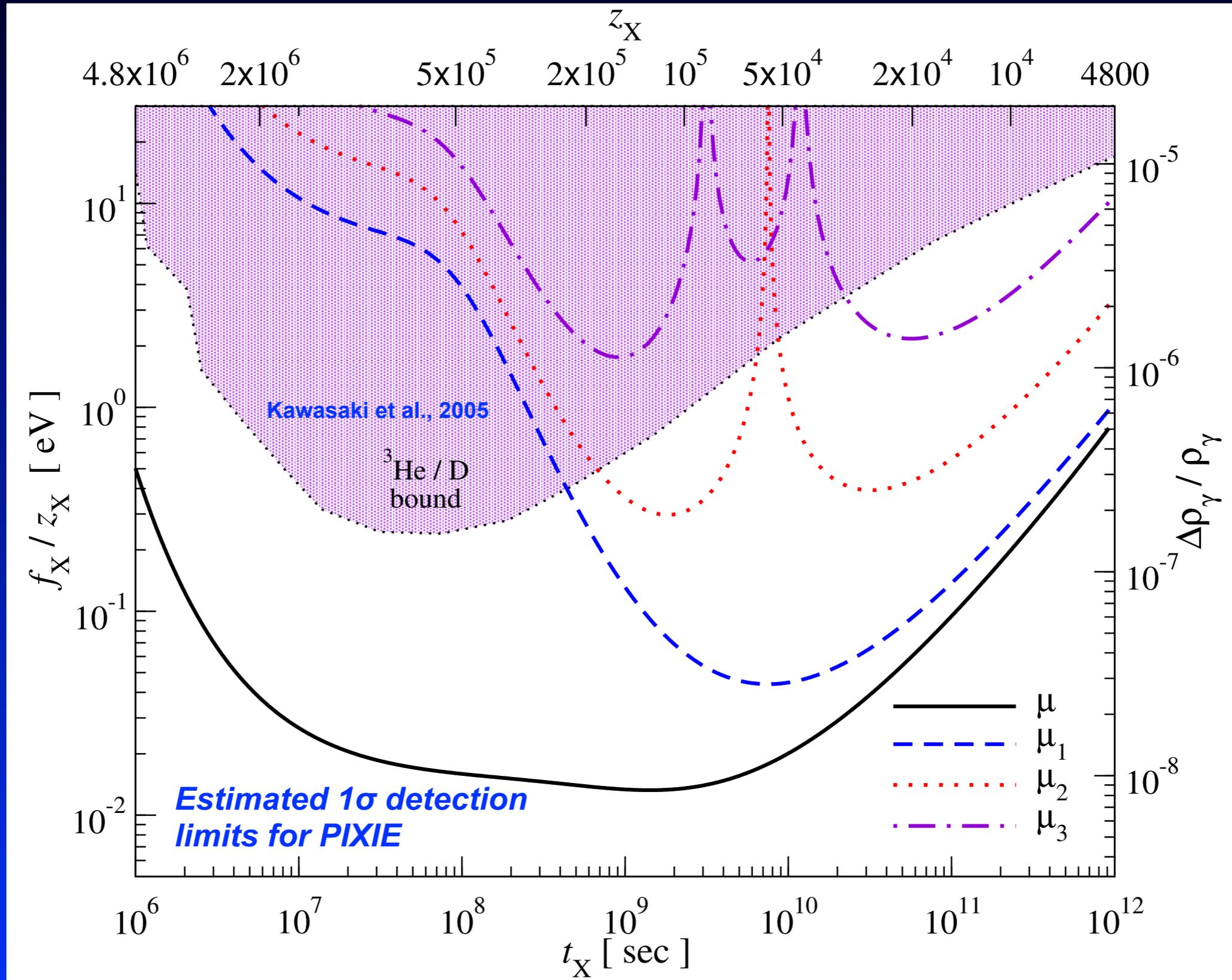
Signal eigenmodes are uncorrelated by construction



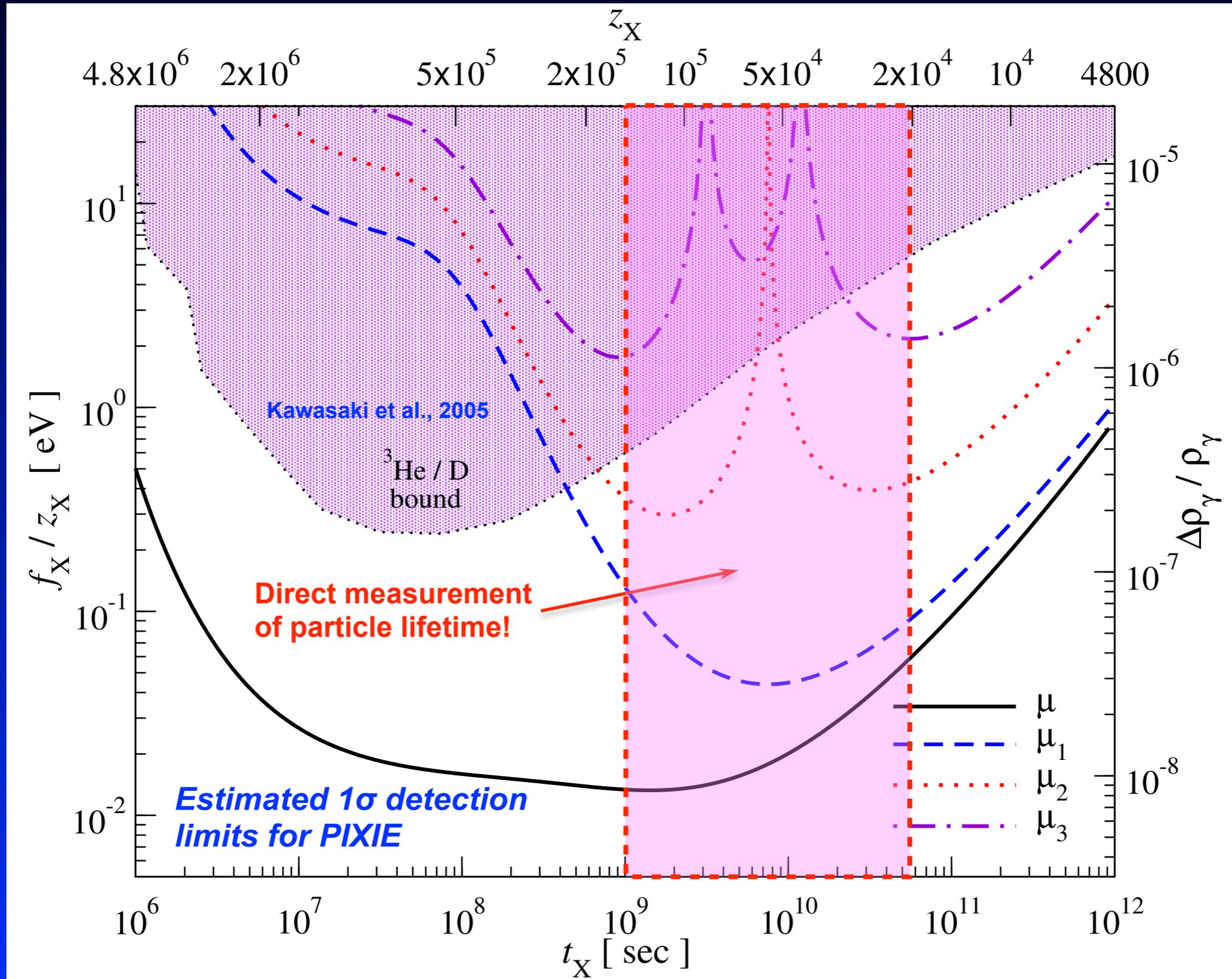
- Adding more modes does not affect error bars
- optimal for given experiment
- *Even non-optimal modes can be used to parametrize distortions!*
- *in this case errors will generally be correlated*

Figure 5. Analysis of energy-release history with $Q(z) = 5 \times 10^{-8}$ in the redshift interval $10^3 < z < 5 \times 10^6$ using signal eigenmode, $\mathcal{S}^{(1)}$ (Fig. 4). We assumed $\{\nu_{\min}, \nu_{\max}, \Delta\nu_s\} = \{30, 1000, 15\}$ GHz and channel sensitivity $\Delta I_c = 5 \times 10^{-26} \text{ W m}^{-2} \text{ Hz}^{-1} \text{ sr}^{-1}$. The dashed blue lines and red crosses indicate the expected recovered values. Contours are for 68 per cent and 95 per cent confidence levels. All errors and recovered values agree with the Fisher estimates. We shifted Δ_T by $\Delta_i = \Delta_f + \Delta_{\text{prim}}$ with $\Delta_f = 1.2 \times 10^{-4}$ and $\Delta_{\text{prim}} \simeq -8.46 \times 10^{-9}$, where Δ_{prim} is the primordial contribution.

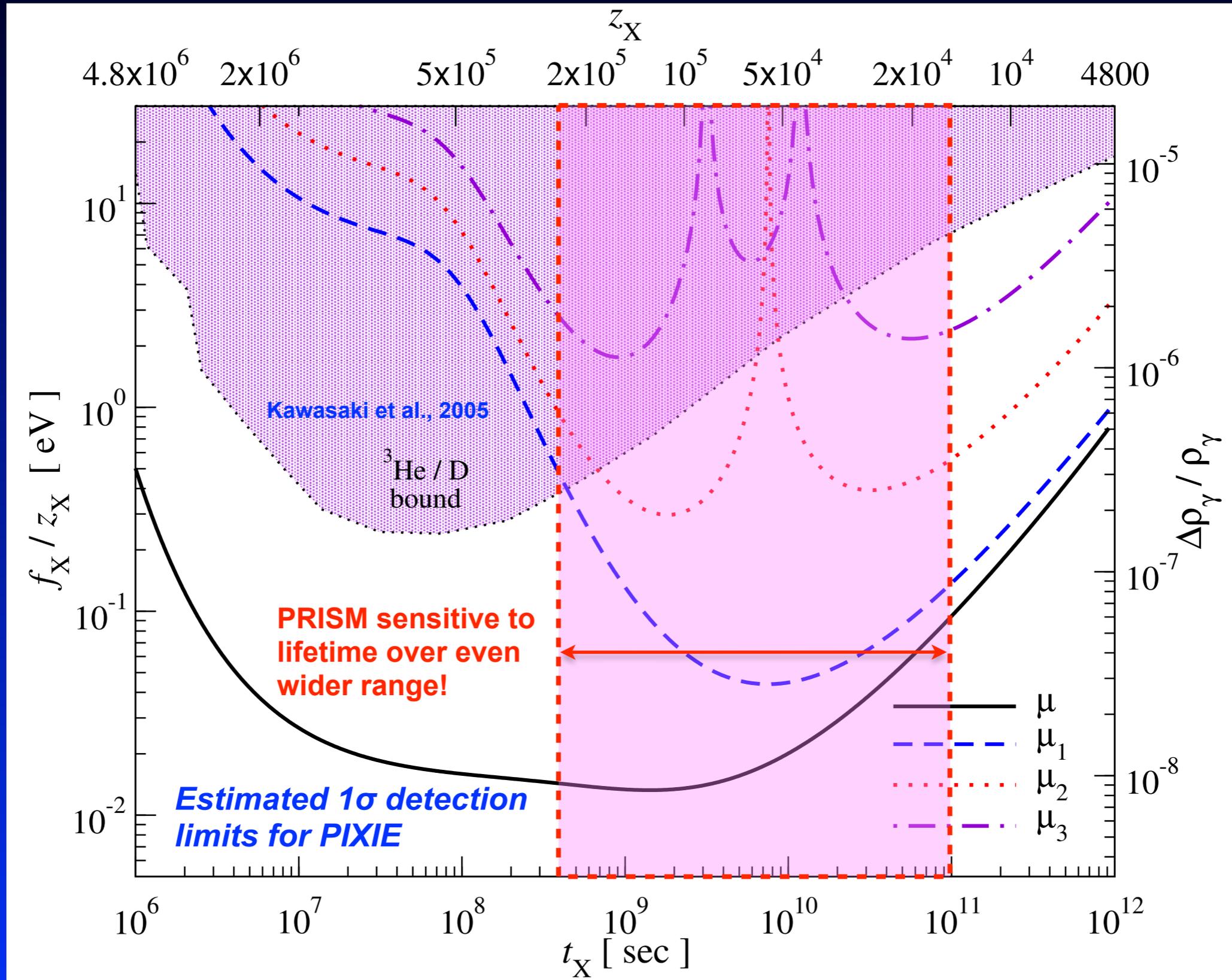
Distortions could shed light on decaying (DM) particles!



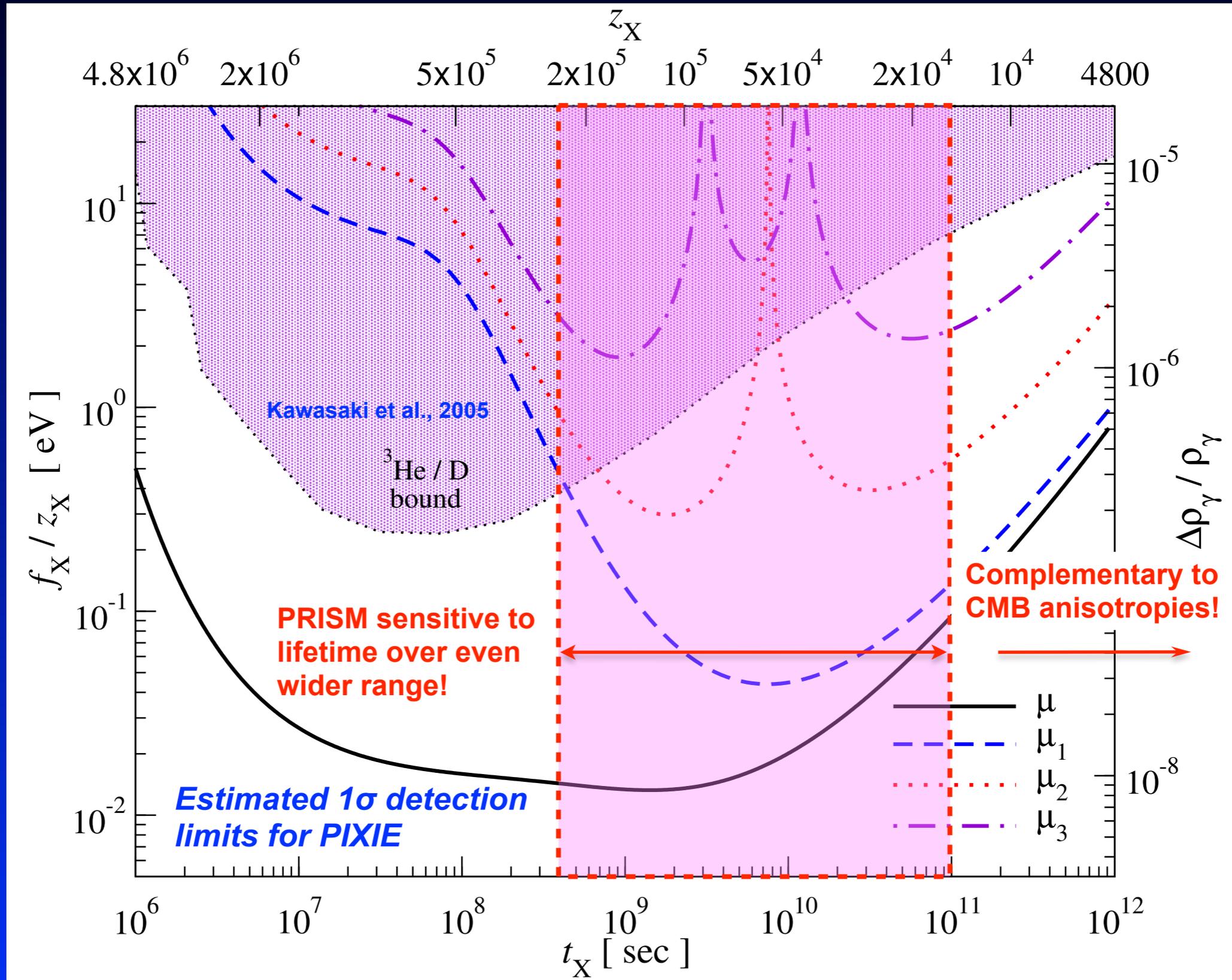
Distortions could shed light on decaying (DM) particles!



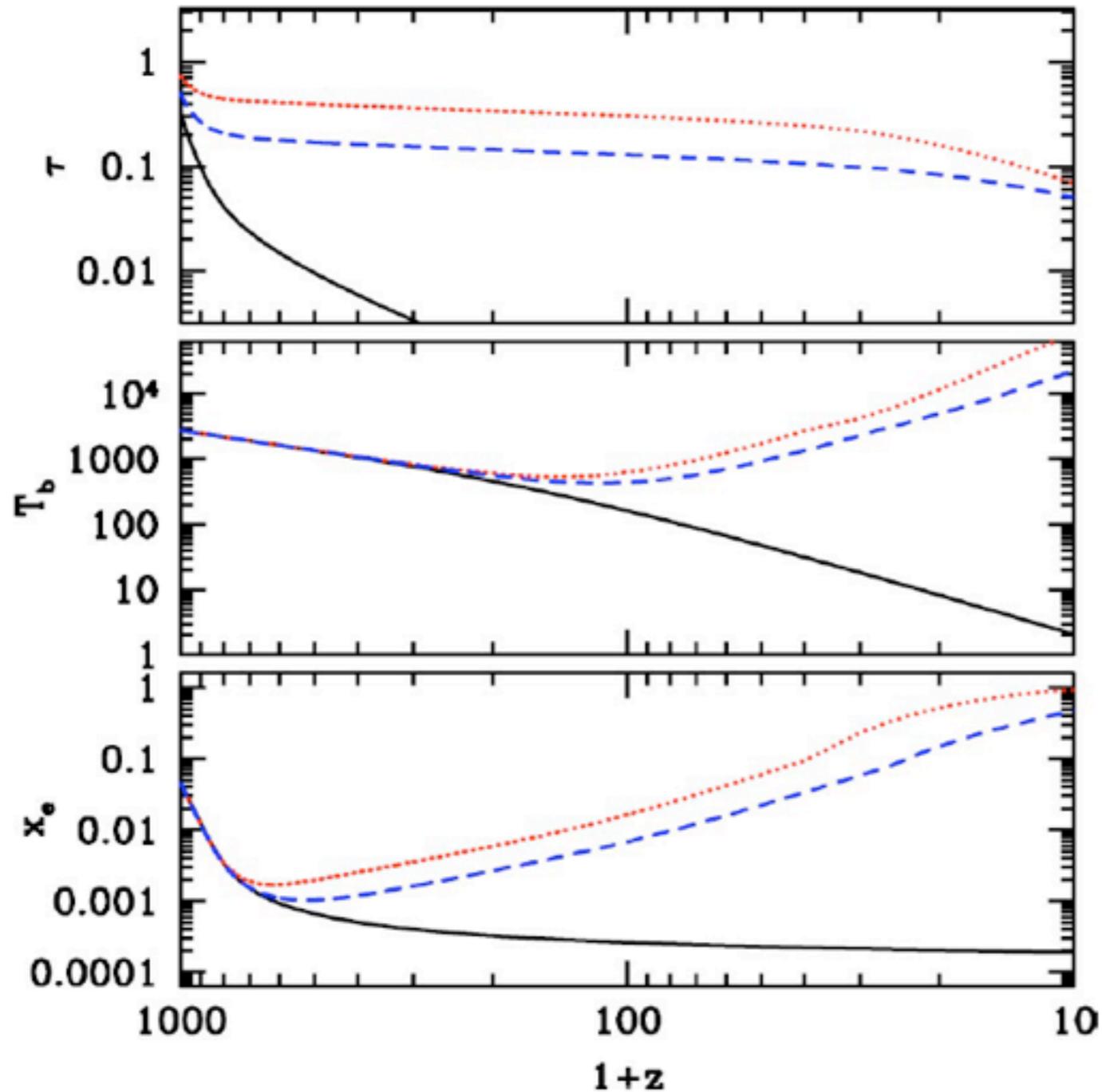
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Distortions could shed light on decaying (DM) particles!

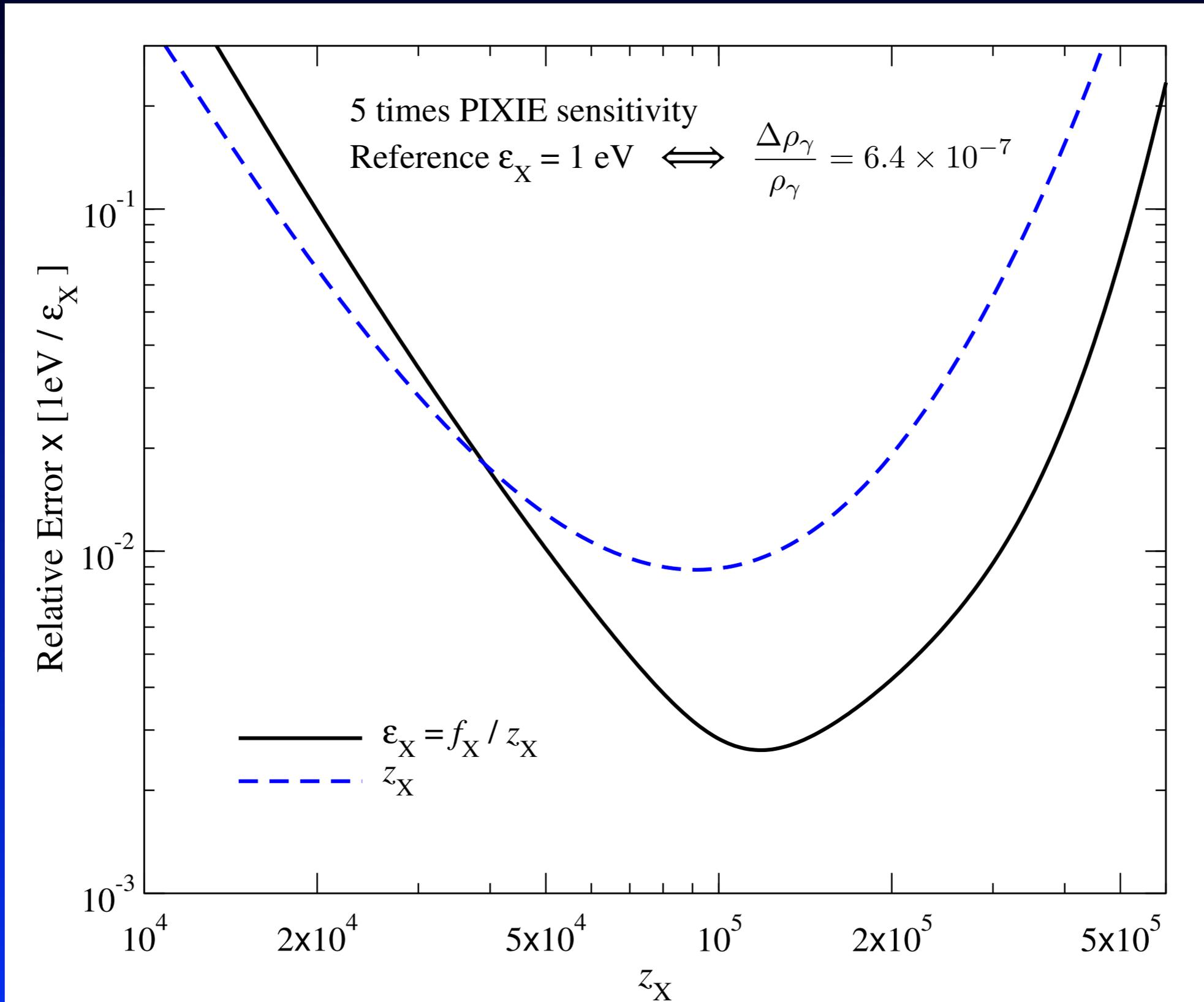


Decaying particle during & after recombination



- Modify recombination history
- this changes Thomson visibility function and thus the CMB temperature and polarization power spectra
- \Rightarrow CMB anisotropies allow probing particles with lifetimes $\gtrsim 10^{12}$ sec
- CMB spectral distortions provide complementary probe!

Decaying particle forecasted error



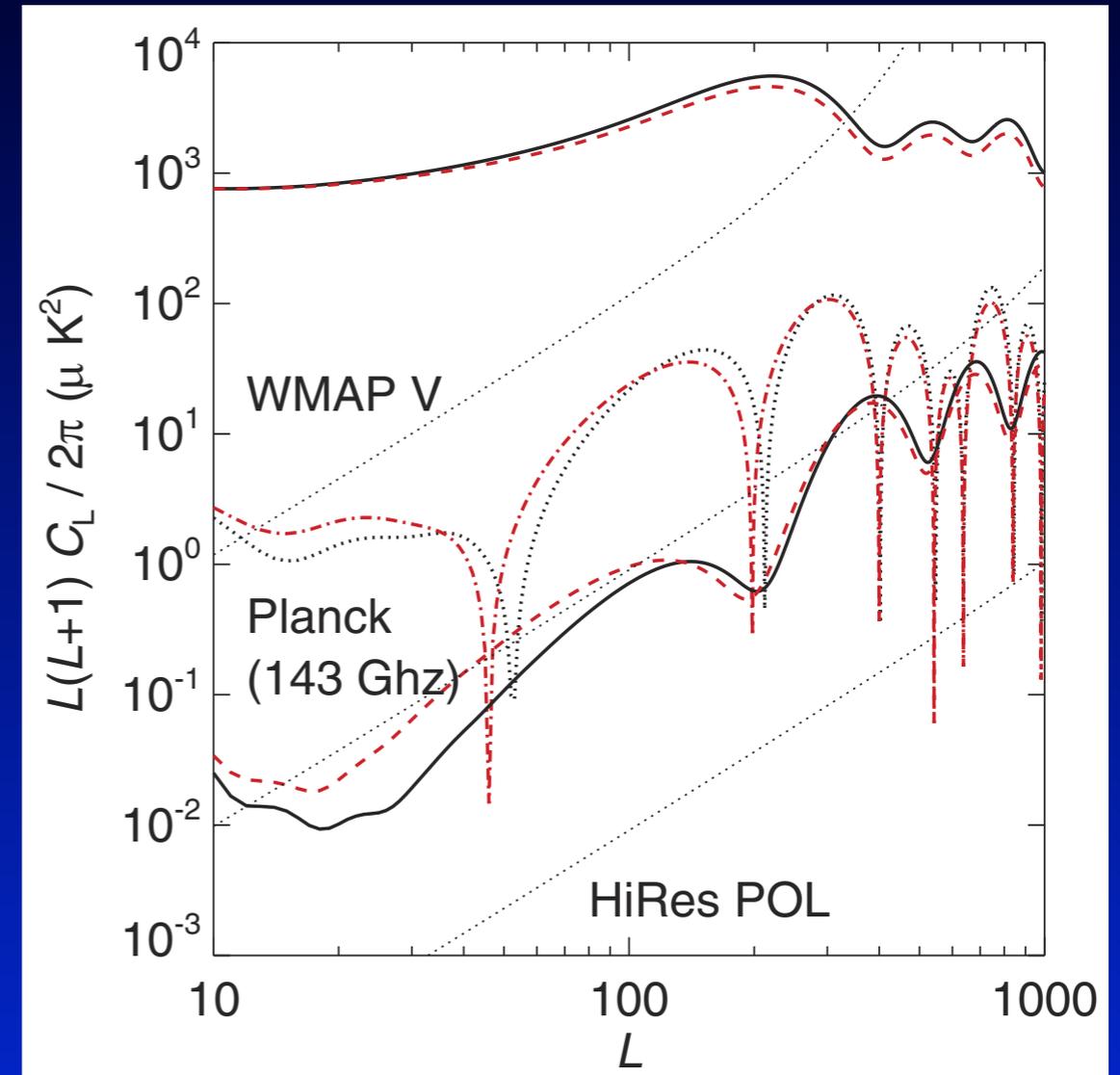
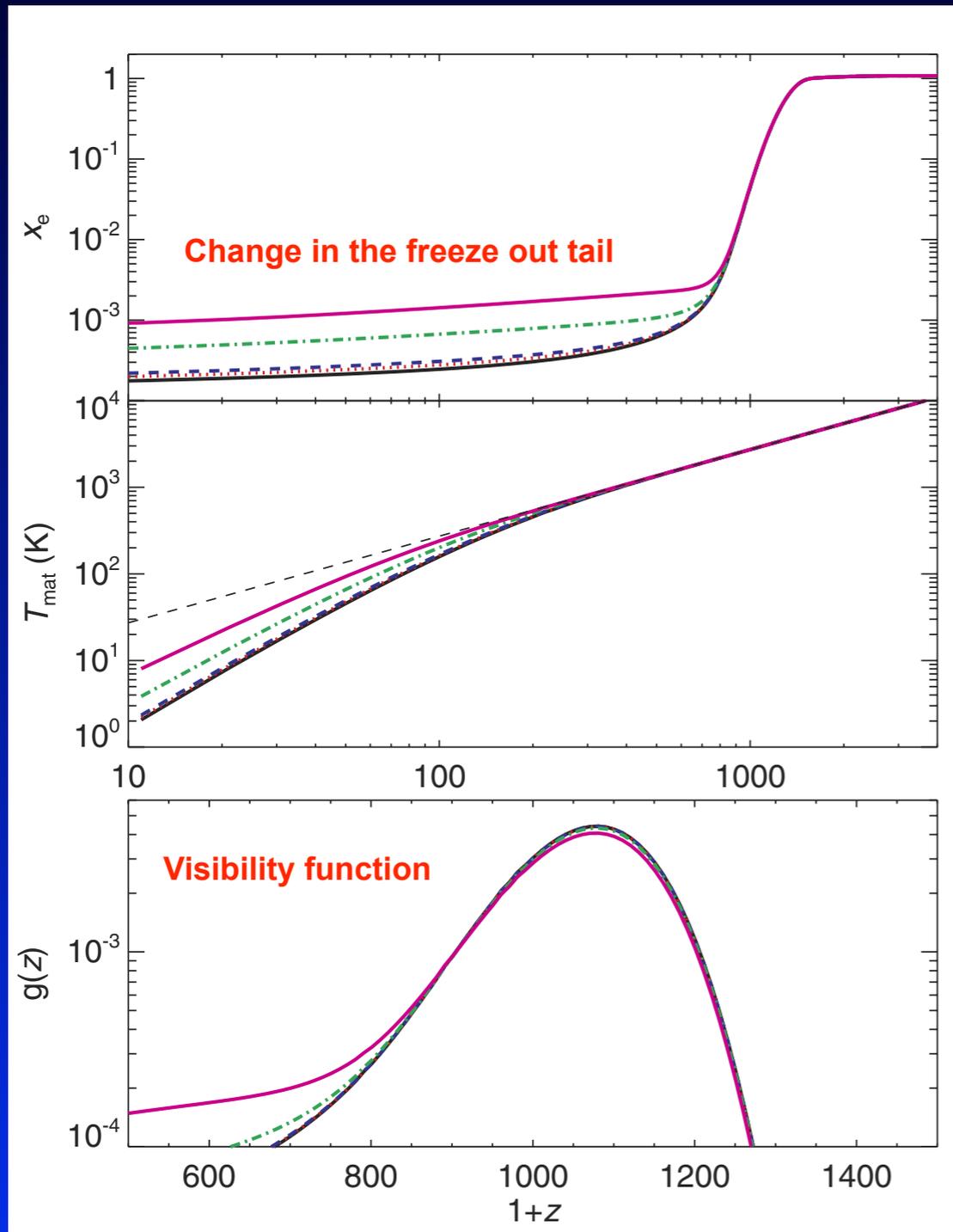
Annihilating particles / dark matter

Why is this interesting and what are the questions

- Thermally produced WIMP should naturally annihilate even today
- CMB anisotropy and X-ray / γ -ray constraints complementary
- s- and p-wave annihilation $\implies \langle \sigma v \rangle \propto T^n$
- Sommerfeld enhanced cross sections
- Boost by structure formation since $\langle N_X \rangle^2 \lesssim \langle N_X^2 \rangle$
- Computation is limited by particle physics (annihilation channels; energy transfer efficiencies, etc.)
- CMB spectral distortions can be used to as a consistency test
- CMB anisotropies insensitive to p-wave annihilation
- Recombination spectrum also affected (see next lecture)

Changes of CMB anisotropies by annihilating particles

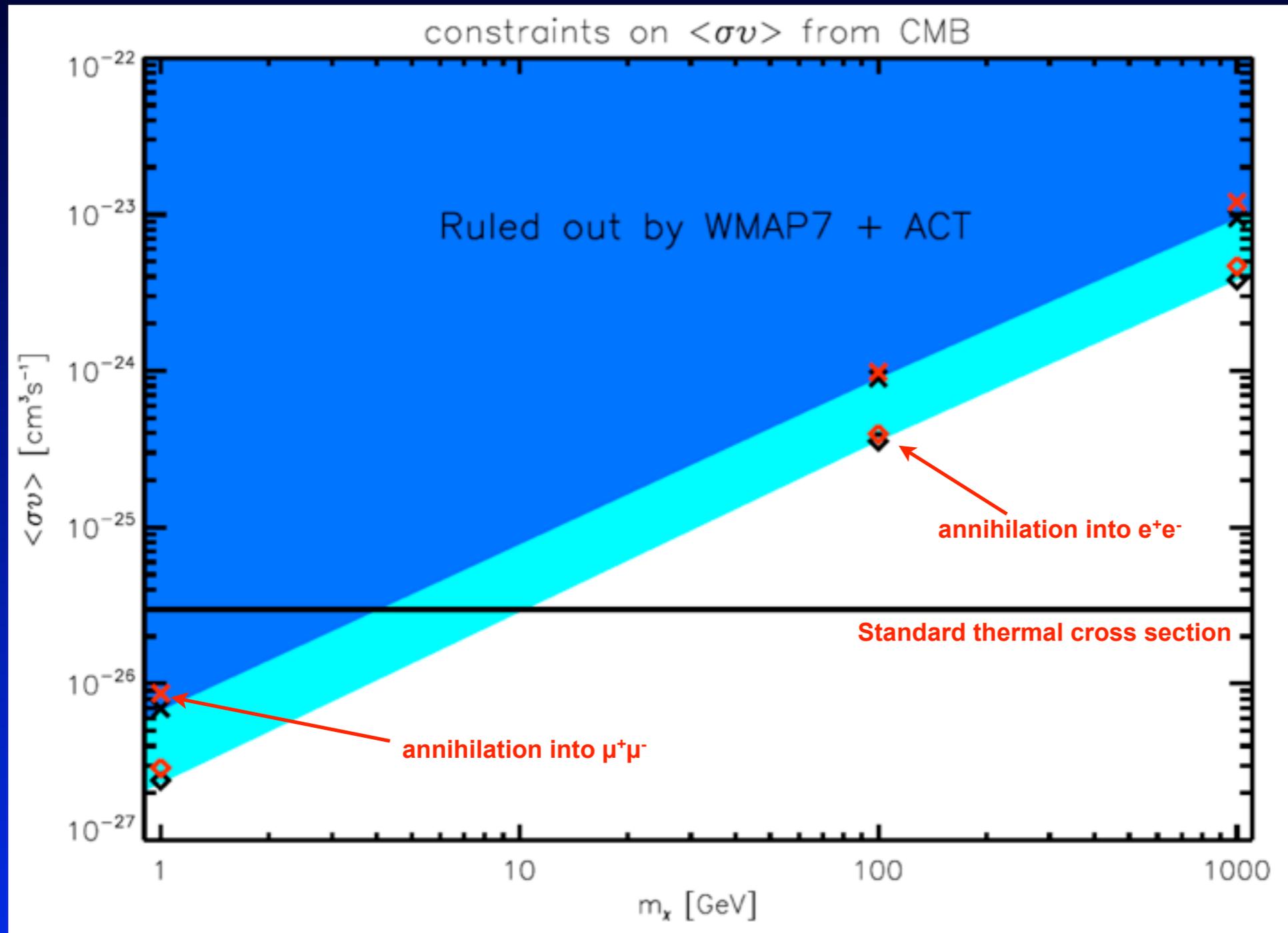
95% c.l.



- more damping because τ increases
- change close to visibility maximum \rightarrow shift in peak positions

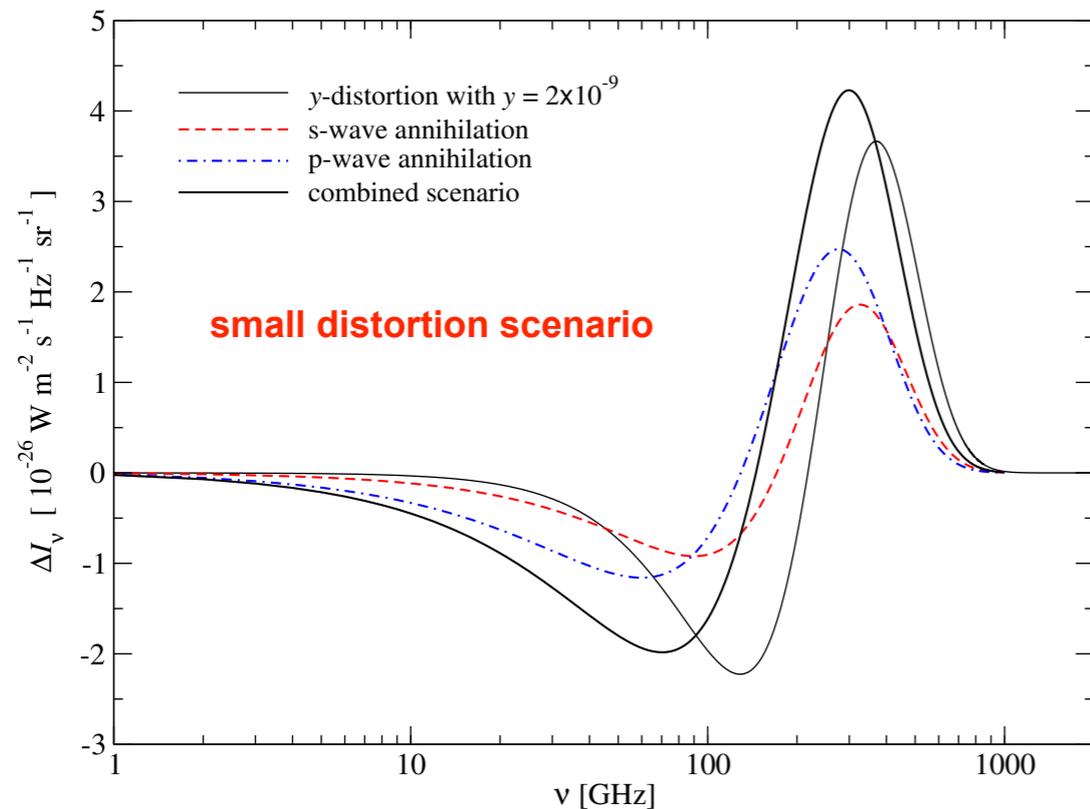
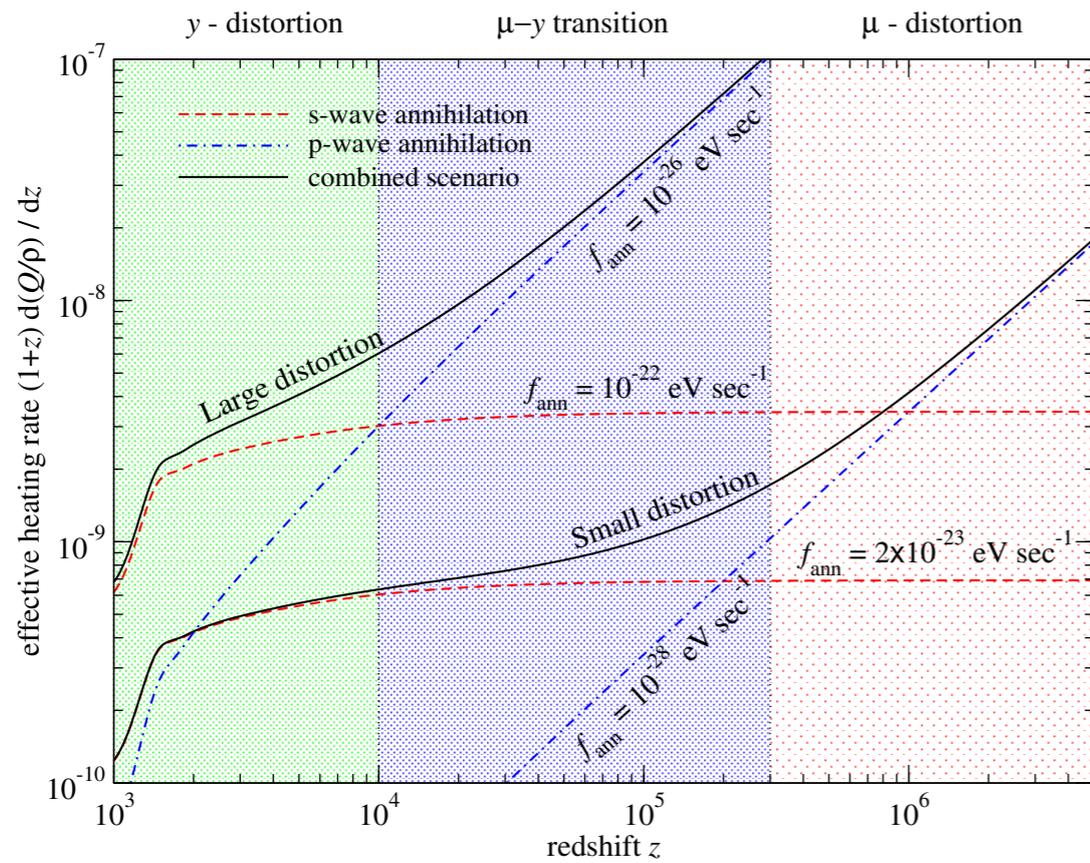
CMB limits on annihilation cross section

95% c.l.



- s-wave annihilation
- Planck limits still not as tight (polarization data)
- in the future factor of $\sim 5-10$ improvement possible
- constraints depend on DM model

Examples of annihilating particle scenarios



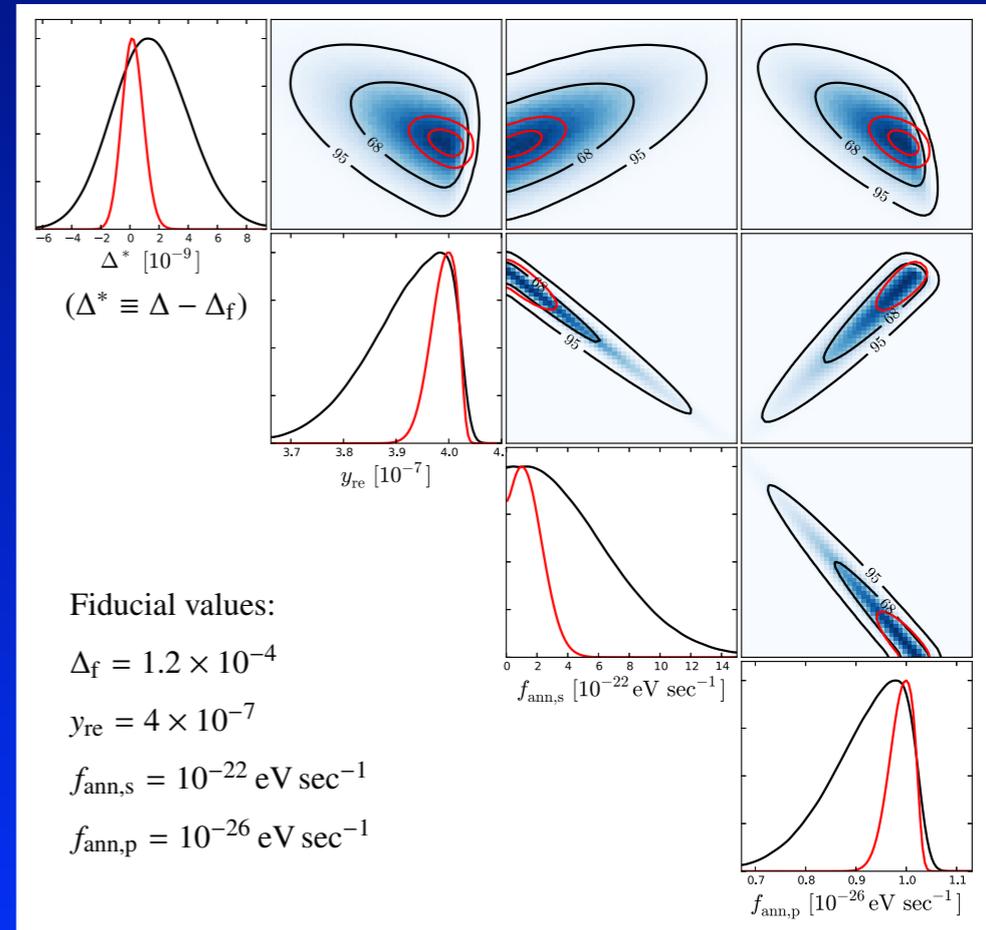
- $f_{\text{ann}} \equiv$ annihilation efficiency (Padmanabhan & Finkbeiner, 2005; JC 2010)

- CMB anisotropy constraint

$$f_{\text{ann}} \lesssim 2 \times 10^{-23} \text{ eV s}^{-1}$$

(Galli et al., 2009; Slatyer et al., 2009; Huetsi et al., 2009, 2011)

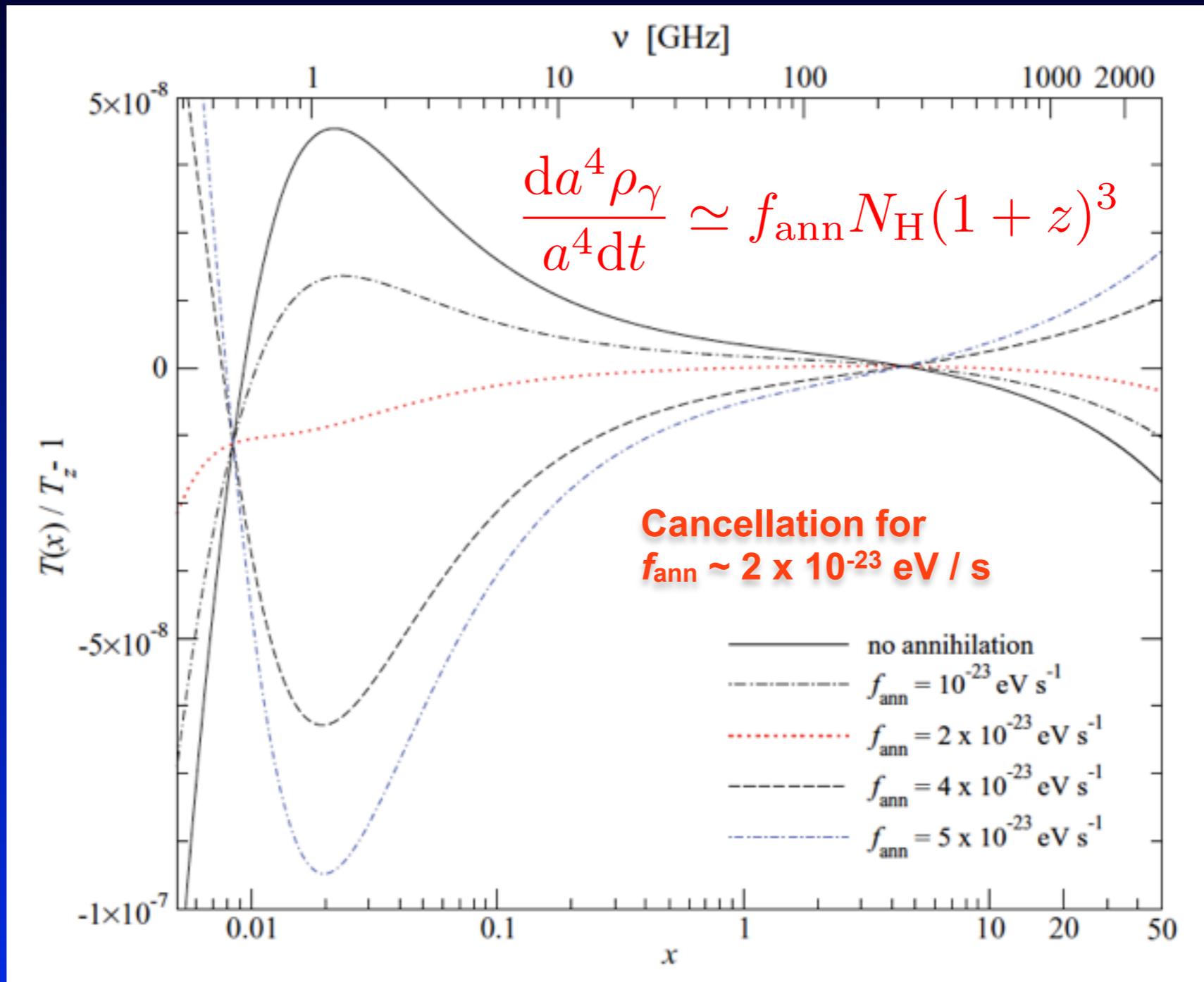
$$\frac{dE}{dt} = f_{\text{ann}} N_{\text{H}} (1+z)^{3+n}$$



JC & Sunyaev, 2011, Arxiv:1109.6552

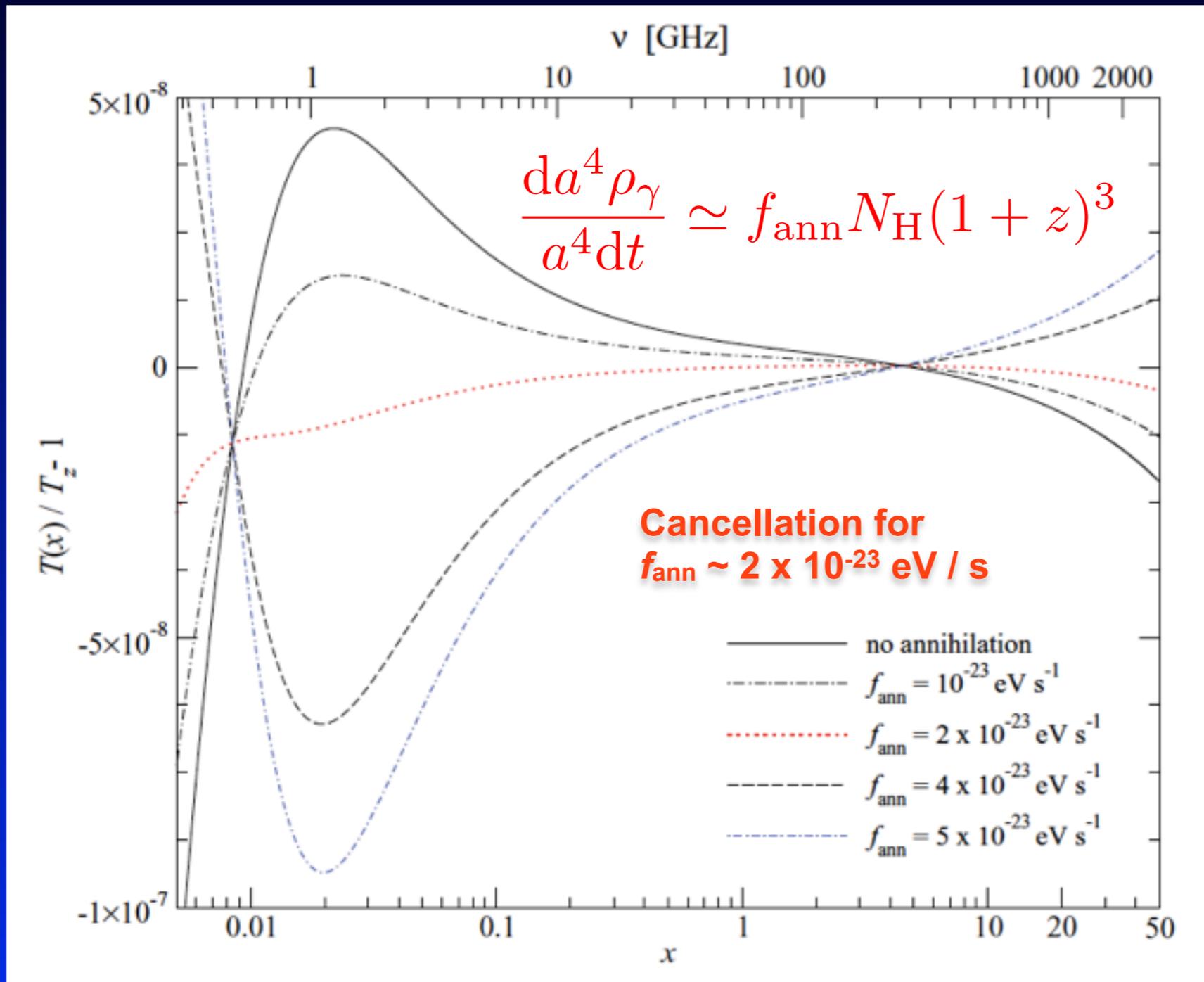
JC, 2013, Arxiv:1304.6120

Cancellation of cooling by heating from annihilation



$$f_{\text{ann}} = 1.1 \times 10^{-24} \frac{100 \text{ GeV}}{M_X c^2} \left[\frac{\Omega_X h^2}{0.11} \right]^2 \frac{\langle \sigma v \rangle}{3 \times 10^{-26} \text{ cm}^3 / \text{s}}$$

Cancellation of cooling by heating from annihilation



- $f_{\text{ann}} \equiv$ annihilation efficiency (Padmanabhan & Finkbeiner, 2005; JC 2010)
- CMB anisotropy constraint

$$f_{\text{ann}} \lesssim 2 \times 10^{-23} \text{ eV s}^{-1}$$
 (Galli et al., 2009; Slatyer et al., 2009; Huetsi et al., 2009, 2011)
- Limit from Planck satellite will be roughly *6 times stronger* → more precise prediction for the distortion will be possible
- uncertainty dominated by particle physics
- limits from PIXIE/PRISM several times weaker, *but* independent

$$f_{\text{ann}} = 1.1 \times 10^{-24} \frac{100 \text{ GeV}}{M_X c^2} \left[\frac{\Omega_X h^2}{0.11} \right]^2 \frac{\langle \sigma v \rangle}{3 \times 10^{-26} \text{ cm}^3 / \text{s}}$$

