RE-INVESTIGATION OF ROTATION-ACTIVITY RELATIONS USING RADIO LUMINOSITIES

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ABSTRACT. A re-analysis of a comprehensive survey of radio fluxes for chromospherically active binary systems is given. The correlation between activity and a convection-dependent Rossby number is found to be marginally better than correlations with simple rotation parameters. It is suggested that the evolved state of RS CVn binaries and the tidal effects on convection dynamics requires an improved convection-dependent parameter for use in rotation-activity studies.

1. INTRODUCTION

A possible correlation between rotation and chromospheric activity was first suggested by Kraft (1967). The hypothesis suggested that late-type stars have magnetic fields generated by a dynamo mechanism, that field generation depends partly on the forces produced by rotation and that stellar activity seen in many wavebands is directly influenced by the presence of emerging magnetic flux. As well as rotation other parameters expected to influence magnetic activity include stellar mass, luminosity class and spectral type which dictate the properties of the convection zone where the dynamo is rooted. In particular, the depth of this zone, or its convective turnover time, is thought to play a role in the field generation. It is expected that stars of later spectral type (that is, with deeper convection relative to their radius) will show stronger dynamo behaviour than stars of earlier spectral type, for a given rotation period.

The theory is at least partly demonstrated for a number of stellar classes. Relations between coronal emissions and rotation have been reported by Pallavicini *et al.* (1981) and Schrijver, Mewe & Walter (1984) for mostly single stars. Chromospheric relations were also found by Middelkoop (1981) and Noyes *et al.* (1984). X-ray observations (Vaiana *et al.* 1981; Walter 1981) have also demonstrated the link between stellar activity, rotation rate and spectral type. Although the mechanism is only partly understood its existence is now firmly established.

Some of the highest levels of stellar activity are seen in close binary systems consisting of cool stars, i.e. the chromospherically active binaries. The RS CVn binaries (Hall 1976) are a class of objects normally consisting of at least one component of spectral type F-G V-IV and which show the Ca II H & K lines in emission outside of eclipse. Common features include strong coronal X-ray and radio emission, variable Hα emission, strong UV chromospheric and transition region line emission, frequent flaring activity and orbital period changes. Popper & Ulrich (1977) discussed the evolutionary status of these stars and concluded that the emission characteristics develop as they enter the Hertzsprung gap. More recently Barrado *et al.* (1994) confirmed the evolved state of RS CVn components. In this part of the HR diagram hydrogen shells can ignite in the stellar interior and envelope expansion can occur. Hence the convection zone properties can be significantly altered from main-sequence stars. Metallicity is also important because the temperature of stars at the base of the giant branch is very sensitive to this parameter. Since RS CVn binaries are tidally locked causing rapid rotation and are evolved stars the convection zone dynamics can be significantly altered from similar main-sequence stars. Consequently the activity levels may undergo some quite rapid variations over a narrow range in temperature. It has also been suggested that a relevant parameter for activity studies in binary systems is the ratio of the stellar to the Roche radius (Young & Koniges 1977).

Many authors have studied the correlation of RS CVn flux measurements (over a wide range of wavelengths) with rotational parameters; namely the rotational period or rotational surface velocity. The dynamo theory of activity production suggests that correlations should be more readily apparent using a parameter which involves the convective properties of the star and its interaction with the magnetic field. Drake *et al.* (1989; hereafter D89) analyzed a comprehensive survey of radio flux measurements for a large sample of chromospherically active binaries. These data were originally discussed in terms of simple rotational parameters and not a convection-related parameter. The incentive for this work was to re-analyze these data using the standard Rossby number. If the correlations are not improved by the use of such a parameter then either the reported dependences are unreal or the simple dynamo theory is not sufficient to explain the observations. In this paper a brief summary of dynamo related parameters is given, the use of empirical turnover times is discussed, the statistical technique is presented and the results discussed in terms of the dynamo generated activity in binary stars.

2. CONVECTION, ROTATION AND ACTIVITY

In this analysis the assumption is made that RS CVn binaries are tidally locked causing orbital and rotational synchronicity (i.e. $P_{rot} = P_{orb}$). The most common quantity used in dynamo theory investigations of stellar activity is the Rossby number R_o defined as;

$$R_o = P_{rot}/\tau_{conv} \quad (1)$$

where τ_{conv} is the convective turnover timescale. R_o is an important indicator in hydromagnetic dynamo theory and measures the extent to which rotation can induce both helicity and differential rotation required for dynamo activity in the

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convection zone. Since τ_{conv} is a function of spectral type R_o characterizes the level of dynamo activity for stars of different rotation periods. It is expected that magnetic activity should increase with decreasing Rossby number. Empirically this assertion has been demonstrated by Noyes (1983) who found that Ca II emission strength, when normalized to bolometric flux, was related to the product of angular rotation rate and an additional function of spectral type. That function is identified with the convective turnover timescale.

To form an estimate of R_o for a given star it is necessary to calculate the convective turnover timescale τ_{conv}. The usual method involves modelling the stellar convection zone using standard mixing length theory and stellar structure codes (e.g. Eggleton 1972). In these models the efficiency of convection is described by the parameter l, the characteristic mixing length of the convection flow. In the usual formulation the convective eddies are assumed to form with length scales of:

$$l = \alpha H \tag{2}$$

where H has been variously defined as a density scale height or a pressure scale height and α is the product of mean helicity of convection and τ_{conv}. It should be noted that R_o scales with $\alpha^{\frac{5}{2}}$ and is therefore heavily dependent on the choice of α. Calibration of α is possible only with helioseimological studies of the Sun which directly measure the depth of the convection zone (Gough 1986). Rucinski & Vandenberg (1986) used a value of α=1.6 in computing their stellar convective envelopes because computed isochrones based on this value best reproduce the observed colour-magnitude diagrams for globular clusters with wide-ranging age and composition (Vandenberg 1983;1984). Noyes *et al.* (1984; hereafter N84) used convective turnover timescales computed from an empirical fit to the models of Gilman (1980). Although Gilman's work involved models for various stellar masses N84 used a standard mass-colour relationship to find a mean relation between t_{conv} and the (B-V) colour index. An iterated fitting procedure found that the observational data were best described with an extrapolated Gilman model of α=1.9.

Although the results of N84 favoured α=1.9, there are inherent differences in the formulation of the mixing length theory for various convective models and these themselves create an uncertainty in the choice of α. Even so, some progress has been made in this field by Gilliland (1985) who demonstrated the variability of convective properties for evolved stars of different masses and showed that different colour-temperature relations require quite different values of α necessary to reproduce the N84 relationship.

Hence the precise way in which Rossby number enters the physics of dynamo field generation and the uncertainty in the formulation of convection zone properties and its calibration complicates the rotation-activity correlation. There are also problems in using published stellar models. A consistent set of determinations of τ_{conv} however should yield a qualitative correlation with activity indicators.

3. EMPIRICAL TURNOVER TIMESCALES

As a first step in the re-analysis of the radio flux measurements presented by Drake *et al.* (1989; D89) calculations based on the empirical relationship between τ_{conv} and the spectral type defined by the (B-V) index can be used. Figure 1 shows several of these relationships taken mainly from studies of main-sequence stars of given mass and composition. Although the basic form of these relations is the same differences in log (τ_{conv}) of greater than 0.2 are possible for a single spectral type. As mentioned above, the curve given by N84 is an interpolated fit which matches models derived by Gilman (1980) to empirical data and therefore ignores the range of masses possible within a main-sequence spectral type. Indeed, the relationship should only be considered appropriate as a mean description of a small sample of main-sequence stars. However in the absence of consistent theoretical relationships the polynomial form presented by N84 can be used to improve the correlations given by D89. The relationship shows that for (B-V) < 0.9, τ_{conv} increases with advancing spectral type, steeply at earlier spectral types and less steeply at later spectral types. For (B-V) > 0.9, τ_{conv} varies little.

Fig. 1. The relationship between convective turnover timescale τ_{conv} and (B-V) magnitude. The functions plotted are polynomial fits to either model calculations (Rucinski & Vandenberg (1986); Stepien (1989); Gilliland (1985)) or interpolated fits using observational data (Noyes *et al.* (1984; N84)).

4. EXPECTATION MAXIMIZATION ANALYSIS

The D89 survey consists of radio flux measurements at 6-cm for a sample of 122 RS CVn and related binary systems with widely varying orbital periods. They searched for correlations of the radio flux of these systems with various systemic and stellar parameters such as rotational periods, surface velocities and X-ray emission. In this paper a re-analysis of these data is performed using Rossby numbers calculated using the N84 relationship.

The basic problem in this analysis is how to handle properly the large percentage ($\sim$ 46%) of cases where the data represents only an upper limit to the radio emission. Such detections should not be ignored because they to provide information about the possible emission from a given system. However upper limits cannot be regarded as detected points since this will introduce an unnecessary bias into the sample. Fortunately, there are advanced statistical techniques applicable to so-called *censored data*, i.e. data containing numbers which are upper or lower limits. The method incoorporates an iterative maximum likelihood (ML) linear regression procedure known as the expectation maximization (EM) algorithm.

Essentially the ML analysis does a linear least-squares fit to the data and includes a statistically correct treatment of the censored points. This field of *survival statistics* is reviewed by Miller (1981) and its application to astronomical data analysis has been presented by Feigelson & Nelson (1985) and Isobe, Feigelson & Nelson (1986).

To study correlations rather than simply regressions a measurement of goodness of fit is required. Even if very weak relations exist between parameters (in that the gradient of the linear relation is small) the data can still be highly correlated. Some of the conclusions made by D89 were based only on regression analysis. For example, they conclude that two parameters show no significant correlation simply when the gradient of the regression line is small. Since these regressions were for log-log relationships the gradients are a measure of the power of the dependent variable in the regression relationship and therefore tells us nothing about how well the *actual* data are correlated. Fortunately, the usual product moment correlation coefficient r is a sufficient measure for censored data since the effect of censoring on this parameter is small. A zero gradient naturally implies that data are completely uncorrelated. The correlation coefficient measures the dispersion of the data around the regression line so r cannot uniquely declare the correspondence between two parameters. For these reasons an improvement in a correlation can be defined as an increase in *both* the modulus of the ML gradient and r. A reversal of sign of either parameter is regarded as no significant improvement.

5. RESULTS AND DISCUSSION

Figure 2 shows the radio luminosities against the Rossby numbers derived using the (B-V) magnitude and the N84 relationship. Symbols indicate the luminosity class of the stars and whether these luminosities are upper limits. Also shown are the maximum likelihood regression lines for all stars in the sample and also for the subsets of giant, subgiant and dwarf stars. Figure 3 shows the radio luminosities normalized to the bolometric luminosities (L_6/L_{bol}) against the Rossby number. From these diagrams it appears that the relation of radio emission to rotation becomes steeper as one goes from dwarfs towards giants. It has been found that the EM algorithm can give significantly different results from the standard LS method which does not take account of censored data. Inspection of these figures reveals that rotation-activity correlations at radio wavelengths are by no means convincing and do not display the often tight relationships associated with chromospheric activity indicators.

D89 found no statistically significant correlation of the 6-cm radio luminosity with either orbital or rotational period but found a fair degree of correlation between L_6 and the surface rotation velocity v_{rot}. The normalized luminosities were correlated with both the orbital period P_{orb} and the surface rotation velocity. The relations presented by D89 can be summarized as follows;

$$L_6 \sim v_{rot}^{1.0\pm0.3} \tag{3}$$

$$L_6/L_{bol} \sim P_{orb}^{-0.9\pm0.2} \tag{4}$$

$$L_6/L_{bol} \sim v_{rot}^{1.4\pm0.3} \tag{5}$$

Separating these systems into three subclasses depending on the luminosity class of the active star in each system results in the dwarfs showing no significant depedence on the rotation parameters while the subgiant and giant systems show reasonable correlations between their radio emission and their rotation.

Fig. 2. Plot of log R_o against log L_6 for stars in the D89 survey. R_o values are derived from the (B-V) magnitude using the relationship of Noyes *et al.* (1984). Maximum likelihood linear regression fits to the data are shown. Symbols are squares for giants, circles for subgiants and triangles for dwarfs. Those points which are upper limits are plotted as open symbols.

Table 1 shows the results of the regression and correlation analyses on the data using orbital periods and Rossby numbers. This table lists the parameters used, the number of stars in the sample, the subset of the data used (a, g, s or d), the ML linear regression line gradient a and the product moment correlation coefficient r. From the regression and correlation results there appears to be no significant correlation between radio luminosity and Rossby number or rotation period for dwarf stars. Correlations are more significant for giants and subgiants and these appear to be marginally better with Rossby number than for rotation period. This is to be expected on the basis of the dynamo theory of stellar activity but is by no means confirmed by the small increase in correlation for these data. Evidently the luminosity class is a significant factor in the determination of active levels or at least its dependence on convection. The use of a radio luminosity normalized to the bolometric luminosity also appears to improve the correlations.

This analysis has concerned a single activity indicator which relates to the large scale coronal activity of the binary systems. The lack of significant correlations like those found for lower atmospheric activity indicators may mean other mechanisms may be operating to mask the underlying rotation-activity relation. Also, the use of empirically derived Rossby numbers may be inappropriate since they are based on fits to a limited sample of main-sequence stars whilst RS CVn binaries are known to be slightly evolved. The convective properties of such stars may differ significantly from their main-sequence counterparts and may therefore show widely varying actvity levels for similar rotation periods. It is suggested that the present definition of Rossby number is an inadequate measure of the efficiency of magnetic activity for evolved close binary systems.

Fig. 3. Plot of log R_o against log L_6/L_{bol} for stars in the D89 survey. Maximum likelihood (ML) linear regression fits to the data are shown. Symbols and abbreviations are the same as those in Figure 2.

6. CONCLUSIONS

It has been demonstrated that the use of an empirically derived Rossby number does not significantly improve the rotation-activity correlation for radio luminosity measurements of RS CVn binaries over that found using the orbital period. Such correlations at radio wavelengths remain unconvincing. The results strongly suggest that luminosity class is an important parameter for magnetic activity generation. To improve our understanding of activity generation in evolved close binary stars, additional stellar structure modelling of the convection zone and a study of the changes in convection dynamics as stars evolve off the main-sequence are required. Such theoretical studies may give more reliable estimates of Rossby numbers for evolved stars (thus improving the correlations) or suggest a more appropriate parameter to characterize the activity generation as a function of spectral type, luminosity class and rotation. Investigation of the effects of binarity and tidal interaction on the convection zone dynamics is also required.

In the future it would be instructive to firstly standardize the stellar and systemic parameters for this sample of stars by eliminating or re-assessing poorly known parameters. Further observational work is required to firmly establish the evolutionary status of the RS CVn systems. Once this is done, a re-analysis of activity indicators from all levels of the chromosphere and corona, and comparison with theoretical results, will give a better indication of the mechanisms of activity production in evolved close binary systems.

Table 1. Results of regression and correlation analysis for data in the D89 survey. The table shows the independent and dependent variables used in each analysis, the number of stars in the sample n, the subset of data used ('a' for all, 'g' for giants, 's' for subgiants and 'd' for dwarfs), the maximum likelihood (ML) regression line gradient a and the product moment correlation coefficient r.

X	Y	n	subset	a	r
log L_6	log P_{orb}	108	a	-0.051	+0.106
		39	g	-1.679	-0.452
		43	s	-0.690	-0.336
		26	d	+0.116	-0.055
log L_6	log R_o	108	a	-0.183	+0.009
		39	g	-1.712	-0.514
		43	s	-0.681	-0.376
		26	d	-0.172	-0.201
log L_6/L_{bol}	log P_{orb}	108	a	-0.909	-0.557
		39	g	-1.987	-0.526
		43	s	-0.891	-0.545
		26	d	-0.086	-0.153
log L_6/L_{bol}	log R_o	108	a	-1.041	-0.610
		39	g	-1.984	-0.556
		43	s	-0.906	-0.593
		26	d	-0.421	-0.372

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