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Cross-correlation radial velocity measurements of chromospherically active binaries

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Abstract. We present observational radial velocity curves for 12 chromospherically active binary systems and individual measurements for a further 5 systems. These binaries are LX Per, V471 Tau, EI Eri, OU Gem, GK Hya, TY Pyx, Z Her, MM Her, V772 Her, ER Vul, BD-004234, MY Cyg, AR Lac, KZ And, RT And, SZ Psc and EZ Peg. Six of our target binaries do not agree with published ephemerides or orbital parameters. We also present a rigourous derivation of the resolution limit for binary star cross-correlation radial velocities. Using synthetic spectral data we investigate the errors induced by rotational broadening, signal-to-noise ratio, spectral-type mismatch and luminosity ratio.

Key words: stars:binaries:close-stars:binaries:spectroscopic-techniques: radial velocities.

1. Introduction

Chromospherically active binaries have been the subject of intense research over recent decades in many wave-bands. Such objects display a wide variety of phenomena; flare activity, cool surface star-spots, extended high-temperature coronae and large magnetic structures. Much work has been done in determining the orbital parameters of such systems with light-curve analysis and spectroscopic determinations of velocity variations (see Popper & Jeong 1994 and references therein). For many systems these parameters are poorly known or are still based on old photographic results. We have discovered some discrepancies between predicted velocities and those deduced from high-resolution data and have concluded that some apparently well-studied systems need a radical re-assessment of their properties. In addition to the problem of poor solutions for some of these systems, actual variations in the apparent orbital periods of RS CVn and related binaries are well-documented although these are usually very small (Kreiner 1971; Guarnieri et al. 1975; Hall & Kreiner 1980; Hall et al. 1980; Kalimeris et al. 1995).

Cross-correlation techniques are widespread in spectroscopy, particularly for radial velocity determinations (eg. Philip & Latham 1985). Simkin (1974) discussed the technique in relation to Fourier transforms of photographic spectra. Tonry & Davis (1979) derived a relationship between the velocity dispersions in target and template galaxy spectra and the width of the peak feature in the cross-correlation function (hereafter referred to as the CCF). Simkin (1974) described the use of the cross-variance function, Weiss, Jenkner & Wood (1978) explored the squared difference function while Furenlid & Furenlid (1990) presented an analysis of the use of delta functions as a representation of the template spectrum which unfortunately requires good knowledge of stellar line positions. Although the CCF method has in recent years become refined and widespread due to more advanced electronics and more efficient Fourier algorithms, no analytical treatment of the method has been given for the case of binary stellar data. We present in this paper our preliminary analysis of the CCF technique and derive the relationship between the rotation rates in the binary components and the template spectrum and the width of the features in the CCF. This is used to derive an *ad hoc* resolution criterion which is helpful in showing at which phases of a binary orbit a particular template spectrum will be able to resolve the components of a given binary. We investigate this resolution limit and other effects on the errors in CCF radial velocities by examining synthetic data. We plan to supplement this analysis in the future with an analysis of a large sample of well-studied binaries and template spectra.

In this paper we present radial velocity measurements throughout consecutive orbital cycles for a small sample of chromospherically active binary systems. Our motivation for this work was to (a) determine the suitability of our medium-resolution spectrograph for the determination of stellar radial velocities, (b) assess both quantitatively and analytically the basis of cross-correlation techniques for binary orbit radial velocity determinations and (c) confirm or refute (within our error limits) the accepted ephemerides of a small sample of chromospherically active binary stars for which we require good knowledge of radial velocities. Below we discuss in detail our observational data, the analysis methods used, the results obtained and give a rigourous assessment of the cross-correlation procedure. We wish to emphasize the preliminary nature of this report for both our observational data and analytical assessment.

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Table 1. The properties of our program binaries. For each system we list the CABS catalogue number (in parentheses: Strassmeier et al. 1993), the HD number, the visual magnitude m_v, the spectral types of the hot and cool components, the orbital period P_{orb}, the Julian date of primary conjunction T_{conj}, the rotational velocities for the hot and cool components ($V_h \sin i$ and $V_c \sin i$ respectively), the systemic velocity V_o, the orbital velocity semi-amplitudes for the hot and cool components of the system (K_h and K_c respectively), the orbital eccentricity e and the longitude of periastron passage Ω. Parameters not taken from the CABS catalogue are indicated below.

Name	HD	m_v	Spectral Types hot/cool	P_{orb} (days)	JD T_{conj} -2400000	$V_h \sin i$	$V_c \sin i$	V_o (km s^{-1})	K_h	K_c	e	Ω°
LX Per (25)	-	8.14	G0IV/K0IV	8.0382	27033.120	9	19	+28.0	75.0	70.3	-	-
V471 Tau (32)	-	9.71	WD/K2V	0.5212	41913.024	-	-	+40	-	147	-	-
EI Eri (35)	26337	6.96	G5IV/-	1.9472	46091.539	50	-	+17.6	27.4	-	-	-
OU Gem (62)	45088	6.79	K3V/K5V	6.9919	40202.663	5.6	5.6	-8.4	56.5	66.9	0.150	77.6
GK Hya (77)	-	9.32	F8/G8IV	3.5870	14968.860	17	49	+32	100	91	-	-
TY Pyx (81)	77137	6.84	G5IV/G5IV	3.1986	43548.666	23	23	+63.2	96.2	97.5	-	-
Z Her (147)	163930	7.23	F4V-IV/K0IV	3.9928	13086.348	17	34	-45.0	85.5	105	-	-
MM Her (148)	341465	9.51	G2/K0IV	7.9603	44500.667	10	18	-50.8	70.6	72.3	-	-
V772 Her (149)	165590	7.02	(G0V/M1V)/G5V	0.8795	47372.568	65	11	-22.8	94.7	-	0.045	104
ER Vul (179)	200391	7.27	G0V/G5V	0.6981	40182.259	85	85	-24.6	139.5	145.8	-	-
BD-004234 (184)	-	9.87	K3Ve/K7Ve	3.7569	44064.847	5	5	-109.6	43.2	53.7	-	-
MY Cyg	-	8.40	-	4.0052	41561.598	-	-	-	-	-	-	-
AR Lac (191)	210334	6.09	G2IV/K0IV	1.9832	44977.022	46	81	-33.7	116.1	115.6	-	-
KZ And (200)	218738	7.98	dK2/dK2	3.0329	42371.641	12.3	11.6	-6.9	67.6	71.2	0.034	339
RT And (201)	-	8.95	F8V/K0V	0.6289	47803.509[a]	109	-	+5.0	131.4	168.4[b]	0.089	282.2
SZ Psc (202)	219113	7.20	F8IV/K1IV	3.9659	42308.946	9	70	+12	104	82	-	-
EZ Peg (203)	-	9.53	G5V-IV/K0IV	11.659	45736.665	9	7	-27.2	24.5	24.3	-	-

Notes:
(a) ephemeris from Popper & Jeong (1994), (b) velocities from Wang & Lu (1993).

2. Observations and data reduction

The spectroscopic data presented in this paper were obtained during a 13-night observing run in October/November 1994 carried out with the Solar-Stellar Spectrograph (SSS) located at the John Hall 1.1-meter telescope at the Lowell Observatory's dark-sky site near Flagstaff, Arizona. The SSS is a dual spectrograph consisting of an echelle covering wavelengths from 5100 Å to 9000 Å and a Littrow instrument covering the Ca II H and K lines. The resolution of the SSS is 12000, and exposure times for the stars in our program ranged from 15 minutes (for 6th magnitude stars) to 60 minutes (for 9th or 10th magnitude stars). After dispersion the data are recorded on two TEK 512 × 512 CCDs. Data are then stored on disk as a grafted image containing 20 spectral orders.

In Table 1 we list the important parameters of our program binaries. We scheduled approximately one observation per night of the primary targets on our list and observed the other targets when possible. Observations were timed where possible to guarantee at least two spectra of each quadrature and one spectrum of each conjunction, given clear weather throughout. Over our 13 nights we obtained 11 whole nights and 2 half nights of data, so our phase coverage at the end of the run was complete for most of our targets. We also observed a set of radial velocity standard stars each night; these were γ Aql (HD 186791), ι Psc (HD 222368) and α Tau (HD 29139). The parameters of our RV standard stars are given in Table 2.

The data were reduced and analyzed on-line using an IDL (Interactive Data Language) echelle data reduction package called REDUCE 2.0 developed in-house at Lowell and described by Hall et al. (1994). REDUCE allows the user to rapidly obtain wavelength-calibrated spectra normalised to unity by the continuum, which can then be analysed using the spectrum-analysis tool also included with the package. The data were first de-biased and flat-field corrected using suitable frames taken throughout each night's observations. The SSS wavelength calibrations were obtained with a Thorium-Argon (Th-Ar) hollow cathode, which with a 55-second exposure produces about 150 well-exposed lines over the echelle frame and about 10 across the Ca II H and K order. Using the calibration method described in detail by Hall et al. (1994) we obtained wavelength solutions with RMS residuals of about 0.02 Å over the echelle frame and less than 0.01 Å at Ca II H and K. For radial velocity measurement purposes these residuals translate to about 0.9 kms^{-1} at Hα.

Table 2. Details of the radial velocity standard stars used in the cross-correlation procedure. For each system we list the HD number, the spectral type, the visual magnitude m_v, the radial velocity (RV) and the rotational velocity ($v \sin i$).

Name	HD	Spectral Type	m_v	RV (km s^{-1})	$v \sin i$ (km s^{-1})
γ Aql	186791	K3 II	2.7	- 2.1 ± 0.2	< 15
ι Psc	222368	F7 V	4.1	+ 5.3 ± 0.2	< 10
α Tau	29139	K1 III	0.9	+54.1 ± 0.1	< 15

Fig. 1. Example of the cross-correlation procedure for the determination of component radial velocities for the TY Pyx system, 1 (a) shows the target (full-line) and RV standard HD 222368 spectrum (dotted-line) for SSS order number 17 for a spectrum observed at 12:22 UT (mid-exposure) on 20 October 1994 (phase 0.265). The resulting CCF (panel (b)) shows two clearly defined peaks which are fitted by a double gaussian function (dotted). The function is fitted between two user supplied base values and provides the best positions for the two component lags measured in pixels. Panel (c) shows a similar plot for the same spectral order for the TY Pyx spectrum observed at 12:56 UT (mid-exposure) on 1 November 1994 (phase 0.023). In this case the binary component spectra are strongly blended resulting in a single CCF peak (panel (d)). A single gaussian function (dotted) has been fitted to the peak of the blended function.

Fig. 2. Example of the cross-correlation procedure for the 9th magnitude target RT And. In this case averaged CCFs have been used to determine the component radial velocities. Panel (a) shows the data for SSS order 17 for the RT And spectrum (full-line) observed at 2:56 UT (mid-exposure) on 30 October 1994 (phase 0.858) and the RV standard HD 186791 (dotted-line). Panel (b) shows the double-peaked CCF formed by averaging the CCFs for five separate orders of the RT And spectrum and the fitted double-gaussian function (dotted). Panel (c) shows a similar plot for the same spectral order for an RT And spectrum observed at 2:40 UT (mid-exposure) on 21 October 1994 (phase 0.530). Once again the CCF has been averaged over five spectral orders (panel d)) and the resulting blended CCF fitted with a single gaussian function (dotted). The broadness of the CCF in comparison to those in Figure 1 is due to the higher rotational velocities of the RT And components.

All subsequent analysis of the data was done with the REDUCE 2.0 routine SPECTOOL, which among other things can perform the cross-correlations and function fitting necessary to extract radial velocities from the data. We used SPECTOOL both to obtain the radial velocities from the spectroscopic data, as well as to perform a similar analysis of the synthetic data used to study the CCF (see §5 and §6). This ensured consistency in the way all data used in this work were treated by the software.

3. Radial velocity determinations

Our method of determining radial velocity information from our binary spectra is to cross-correlate the observed target spectrum with the same spectral order of a radial velocity standard. Simkin (1974) gives a good description of the technique of cross-correlation. Many orders of the binary spectrum are not suitable for the CCF technique since they have low signal-to-noise, contain activity-sensitive absorption/emission features or are heavily affected by telluric lines. We first identified those orders which were least affected by such contamination and used these as our subset of orders. Popper & Jeong (1994) have demonstrated that the use of a weighting scheme based on the apparent quality of different spectral orders does not give results more consistent than the mean over selected orders. Ideally the CCF should be evaluated over a log λ scale but over a short order of 512 pixels at our resolution the errors induced by analysing in pixel space are negligible. The CCF of a single order of the binary spectrum with the same order of the radial velocity standard was performed by Fourier transforming each spectrum, forming the conjugate of the standard star FFT, multiplying these together and then transforming back into the spectral domain. The resulting CCF shows the degree of correlation for increasing and decreasing lags and can be more easily visualised as the common area beneath the two superimposed spectra as one is successively moved by a positive or negative lag from the zero-shift position. The positions of peak values in this function occur at velocity differences where there is high correlation between the two spectra.

With our binary spectra it is neccesary to differentiate two scenarios for velocity resolution; firstly that the two component velocities are easily distinguished in the peaks of the CCF or secondly that only one peak is visible. The first corresponds to the complex set of spectral absorption features being widely separated in velocity (for example at quadrature phases), whilst the second means the two components have very similar velocities (for example during conjunction) and therefore have highly blended absorption profiles. We thus distinguish our velocity measurements as *blended* or *resolved*. In §5 we present an analytical derivation of the position of this resolution limit.

To measure the positions of the resolved peaks in the CCF we fit a double-gaussian function across a user-specified section of the CCF. This procedure also requires first-guess positions for the two peaks. For blended velocities we fit a single-gaussian using a similar iterative technique. We have found a double-gaussian fit for resolved features in the CCF gives more consistent results than two single-gaussian functions. An example of the procedure is shown in Figure 1 which displays the target and standard spectra for TY Pyx observed on two separate occasions which produce resolved and blended CCFs.

The highest priority in any radial velocity study, particularly when developing an analysis protocol for a particular instrument, is a proper assessment of the errors involved in the measurements. We have addressed this in several ways. Firstly we perform cross-correlation separately for several orders for each program star, and perform these with all three of our radial velocity standards. We also check the differences in wavelength calibration solutions for the program and target spectra to see if any calibration drift is inducing spurious velocities. Small differences in the calibrations are removed by supplying the appropriate corrections. So as not to place undue importance to our most error-free measurements we first form the standard error from each set of velocities for a particular standard and then form an RMS from the resulting maximum error and the RMS of the set of velocities from each standard. The resulting velocities are then corrected for Earth motion and the motion of the template.

For some fainter stars or spectra with particularly bad signal-to-noise we have found it necessary to adopt a slightly different procedure. Many orders result in CCFs with a somewhat ill-defined peak or double-peak. Since the velocity-pixel translation in our data set is linear we can add up CCFs for many orders to increase the signal-to-noise and emphasize the peak positions in the function. We thus average the CCFs over many orders and proceed to fit a double- or single-gaussian function (using the same SPECTOOL routines) to the average peaks. This method has proved to be helpful in determining the positions of binary components whose profiles are not always clearly visible from a given spectral order. However, a determination of a likely error in this case is more difficult since we are reducing the number of component measurements. In this case we assign an error of 2σ where σ is the RMS of the velocity sample. An example of data treated in this way is given in Figure 2 for observations of the RT And system.

4. Results and discussion

We present the results of our measurements in Table 3. This table shows for each system the derived velocities of the hot and cool components for each observation. The epoch and period of the binary derived using the published ephemerides (Table 1) are also shown. In Figures 3 to 14 we plot the velocities against the phase of observation and show the respective RV curves predicted by the ephemerides. The figures show, where known, the positions of the systemic velocity (solid lines), semi-amplitudes of hot and cool components (dotted lines) and the positions of our resolution limits for each system (dashed lines) derived with Equation 19 (see §5 and §6). Figures are presented only for those systems for which we have four or more measurements of radial velocities.

The primary aim of this program was not to improve upon the accepted ephemerides but to assess the applicability of the SSS instrument for radial velocity determinations. We can however make some qualitative statements concerning the orbital parameters of these systems using our medium-resolution observations. The observations were obtained on a single observing run so our epoch coverage is poor, however we intend to improve our analysis with further observations.

Table 3. Results of the radial velocity measurements for each system. The table shows for each star the epoch and phase of each observation and the derived radial velocities of the cool and hot components (V_c and V_h respectively). An asterix in the V_h column indicates that the V_c velocity is from a blended profile. A dash in any column indicates that the component is not visible in the data.

Star	Epoch	Phase	V_c (km s^{-1})	V_h (km s^{-1})
LX Per	2813	0.288	100.7 ± 7.3	-40.1 ± 9.6
	2814	0.281	96.6 ± 2.5	-44.3 ± 7.6
	2814	0.654	-28.1 ± 2.5	87.8 ± 1.8
V471 Tau	14833	0.265	168.5 ± 12.4	-
	14835	0.207	186.0 ± 5.0	-
	14837	0.116	125.9 ± 9.6	-
	14838	0.992	37.1 ± 13.8	-
	14840	0.913	-27.9 ± 17.4	-
	14846	0.755	-120.9 ± 11.0	-
	14854	0.342	152.4 ± 1.3	-
	14856	0.343	153.3 ± 13.6	-
	14858	0.262	171.1 ± 11.4	-
	14866	0.115	136.6 ± 15.4	-
EI Eri	1824	0.319	-	28.5 ± 6.9
	1825	0.354	-	45.5 ± 2.8
	1826	0.366	-	30.9 ± 3.6
	1828	0.406	-	19.0 ± 10.1
	1830	0.463	-	14.4 ± 2.3
OU Gem	1350	0.330	52.2 ± 9.1	-61.0 ± 4.9
	1350	0.616	-9.2 ± 3.3	⋆
	1350	0.904	-46.2 ± 9.4	46.3 ± 8.8
	1352	0.329	48.3 ± 8.2	-63.5 ± 2.1
GK Hya	9667	0.076	61.3 ± 10.7	⋆
	9667	0.632	-22.6 ± 9.3	121.4 ± 14.8
	9667	0.917	-15.0 ± 6.0	85.9 ± 21.0
	9669	0.031	36.0 ± 9.2	⋆
	9669	0.871	-34.4 ± 8.1	101.4 ± 13.0
	9670	0.146	97.4 ± 10.0	-49.9 ± 15.6
	9670	0.424	69.5 ± 10.8	-11.6 ± 9.3
	9670	0.708	-55.3 ± 12.6	132.0 ± 6.8
TY Pyx	1905	0.640	-10.2 ± 3.1	139.2 ± 7.0
	1905	0.955	48.8 ± 15.4	102.6 ± 17.2
	1906	0.265	159.0 ± 4.2	-33.9 ± 3.5
	1906	0.581	19.2 ± 6.1	113.4 ± 7.1
	1906	0.895	4.1 ± 4.5	121.3 ± 5.2
	1907	0.830	-19.5 ± 3.8	150.0 ± 6.1
	1908	0.144	129.3 ± 11.8	-23.6 ± 10.4
	1909	0.085	112.4 ± 4.4	16.4 ± 5.5
	1909	0.395	121.4 ± 2.9	3.8 ± 6.5
	1909	0.707	-29.6 ± 3.4	156.5 ± 4.8
	1910	0.023	86.1 ± 3.0	⋆
Z Her	9156	0.293	51.1 ± 4.8	-127.9 ± 1.8
	9156	0.795	-135.4 ± 4.1	49.4 ± 4.5
	9158	0.794	-135.5 ± 6.0	47.5 ± 6.2
	9159	0.296	52.4 ± 5.1	-128.2 ± 3.5
MM Her	647	0.766	-116.2 ± 11.9	18.3 ± 8.7
V772 Her	2582	0.216	-31.1 ± 8.1	-120.5 ± 11.9
	2583	0.287	-28.0 ± 2.2	-113.4 ± 7.2
	2584	0.431	-22.1 ± 1.3	-83.7 ± 2.2
	2585	0.559	-19.7 ± 3.8	⋆
	2586	0.710	-9.1 ± 5.8	70.1 ± 6.8
	2594	0.651	-16.2 ± 2.6	53.8 ± 5.0
	2595	0.779	-13.3 ± 5.3	82.0 ± 14.2
	2596	0.923	-17.9 ± 3.6	42.7 ± 6.5
	2598	0.066	-24.1 ± 3.3	⋆
ER Vul	13553	0.297	113.5 ± 11.4	-150.9 ± 18.0
	13554	0.633	-150.7 ± 12.5	81.3 ± 11.0
	13555	0.991	-26.6 ± 2.5	⋆
	13557	0.481	-21.7 ± 7.1	⋆
	13558	0.962	-13.4 ± 6.0	⋆
	13561	0.805	-173.6 ± 7.0	82.3 ± 8.0
	13563	0.239	114.7 ± 13.4	-163.6 ± 14.6
	13564	0.647	-160.0 ± 16.3	73.1 ± 16.3
	13570	0.389	66.2 ± 4.8	-118.5 ± 5.7
	13571	0.760	-167.5 ± 12.4	103.2 ± 8.3
	13573	0.235	113.7 ± 9.8	-162.9 ± 9.2
BD-004234	1485	0.490	-57.6 ± 2.6	-151.1 ± 2.0
	1485	0.761	-118.0 ± 8.8	⋆
	1486	0.036	-169.5 ± 9.3	-57.1 ± 4.5
	1488	0.427	-53.4 ± 13.5	-145.1 ± 4.0
MY Cyg	2018	0.146	47.0 ± 2.4	-131.5 ± 5.7
	2021	0.145	39.0 ± 8.0	-141.0 ± 8.0
AR Lac	2353	0.198	57.4 ± 9.5	-157.0 ± 13.4
	2353	0.692	-132.9 ± 2.4	84.1 ± 3.0
	2354	0.155	55.9 ± 4.0	-140.9 ± 6.0
	2354	0.669	-136.9 ± 5.0	70.6 ± 7.4
	2355	0.190	66.4 ± 8.9	-149.4 ± 9.8
	2356	0.174	56.2 ± 7.2	-151.6 ± 5.5
	2356	0.680	-140.3 ± 12.2	73.5 ± 8.6
	2357	0.172	57.0 ± 8.6	-149.6 ± 11.0
	2358	0.688	-132.9 ± 8.0	75.5 ± 7.3
	2359	0.191	63.5 ± 5.6	-148.8 ± 8.5
	2359	0.691	-142.1 ± 6.0	76.5 ± 4.9
	2360	0.194	64.8 ± 6.3	-151.6 ± 6.9
KZ And	2398	0.423	44.8 ± 4.0	-54.6 ± 4.5
	2401	0.385	53.1 ± 4.9	-61.9 ± 4.2
	2402	0.389	51.2 ± 3.8	-61.2 ± 2.6
RT And	13517	0.883	64.5 ± 14.0	⋆
	13519	0.514	16.2 ± 2.2	⋆
	13522	0.530	19.5 ± 3.0	⋆
	13524	0.171	146.3 ± 11.9	-118.3 ± 7.4
	13529	0.109	127.8 ± 2.0	-93.7 ± 11.6
	13535	0.278	158.0 ± 5.1	-125.9 ± 6.2
	13536	0.858	-136.6 ± 4.4	98.6 ± 6.6
	13540	0.058	98.5 ± 22.2	-60.4 ± 3.8
SZ Psc	1849	0.751	-39.2 ± 2.3	97.6 ± 4.2
	1852	0.775	-43.6 ± 6.5	75.0 ± 6.7
EZ Peg	335	0.093	-29.7 ± 2.0	⋆
	335	0.177	-32.2 ± 4.4	⋆
	335	0.264	-56.5 ± 6.4	⋆
	335	0.349	-29.5 ± 9.8	⋆
	335	0.438	-33.1 ± 5.0	⋆
	335	0.694	-21.8 ± 4.4	⋆
	335	0.778	-22.4 ± 8.1	⋆
	336	0.035	-31.0 ± 5.3	⋆
	336	0.117	-30.6 ± 2.9	⋆
	336	0.207	-38.1 ± 4.4	⋆
	336	0.297	-48.0 ± 5.0	⋆

Fig. 3. Radial velocities derived from the SSS data for the V471 Tau system.

Fig. 4. Radial velocities derived from the SSS data for the EI Eri system.

V471 Tau is a well-studied active binary system consisting of a K2V secondary and a white-dwarf primary which is not detectable in our spectra. The light curve is highly variable, the eclipse being the only constant feature, and displays the typical wave-distortion characteristic of the RS CVn class of binaries. The orbital parameters given by Young (1976) give good agreement with our observations (Figure 3) and there is no indication of any phase-shift or period change in this system.

EI Eri is a single-lined rapidly rotating RS CVn binary with a period of 1.9472 days. All the orbital parameters are from Strassmeier (1990). Our data for the hot G5 IV component are extremely confused. Without information about the secondary component it is not possible to compute a resolution limit (see §5) for this system but in any case this cannot account for the large dispersion of our values around phase 0.4 (Figure 4) or the large errors associated with each value. Further observations are necessary to account for our failure to record reasonable velocities for EI Eri but we believe the very high rotational velocity of the hot component has caused these large discrepancies.

Fig. 5. Radial velocities derived from the SSS data for the OU Gem system.

Fig. 6. Radial velocities derived from the SSS data for the GK Hya system.

OU Gem is an interesting binary consisting of a K3V primary and a K5V secondary. The ephemeris and period for this system come from Griffin & Emerson (1975) and Bopp et al. (1981) respectively. Velocity information can be found in Tomkin (1980). We only managed to observe this system four times (Figure 5) with one blended velocity. Our results seem to agree within the errors quite well with published results.

GK Hya is an RS CVn binary consisting of an F8 primary and a G8 IV secondary which undergo total eclipses. Our observations of this system (shown in Figure 6) have shown excellent agreement (within our errors) with published orbital parameters (Popper 1990).

TY Pyx is an RS CVn system with almost unity mass ratio, luminosity ratio and identical spectral types (G5). Photometry has shown the presence of light-curve distortions characteristic of the RS CVn systems but these are relatively small. Our observations agree remarkably well with the ephemeris of Andersen et al. (1981) and the velocities of Andersen & Popper (1975). This demonstrates, we believe, the efficiency of our medium resolution measurements since TY Pyx has rapidly

Fig. 7. Radial velocities derived from the SSS data for the TY Pyx system.

Fig. 8. Radial velocities derived from the SSS data for the Z Her system.

Fig. 9. Radial velocities derived from the SSS data for the V772 Her system.

Fig. 10. Radial velocities derived from the SSS data for the ER Vul system.

rotating components and does not have a large maximum velocity separation. The derived velocity curve for TY Pyx is shown in Figure 7.

Z Her is a partially eclipsing binary with F4 and K0 components. Popper (1988) gave velocity information on the system and the ephemeris used is usually that of Hall & Kreiner (1980). The orbital period is so close to four days that it is not possible to observe at widely separated phases with a single observing run (Figure 8). Our limited observations however seem to agree extremely well with published results.

V772 Her is a visual binary which is also a double-lined triple system containing a close eclipsing pair. Batten et al. (1979) assigned spectral classifications of G0 V for the hotter star and G5 V for the tertiary and measured the rotational, systemic and semi-amplitude velocities. More recent observations by Strassmeier & Fekel (1990) give the rotational velocities quoted in the CABS catalogue. The spectral type of the secondary is not known and in any case is not visible in our medium-resolution spectra. The period of the visual pair is somewhat over 20 years (Aitken 1923). We have detected both the primary and tertiary in our observations. The primary velocities agree quite well with published values (ephemeris of Reglero et al. 1991) although there is some evidence of a phase shift (Figure 9). The tertiary component which should have an approximately constant velocity over our two weeks of observations shows a small periodic variation in phase with the primary. This is due to the skewing of the gaussian fitting procedure by the CCF peak from the primary since the tertiary component has a low amplitude.

ER Vul (Figure 10) is a double-lined spectroscopic binary of the short-period RS CVn type. A spectroscopic orbit was determined by McLean (1982) using cross-correlation velocities. He also found evidence for the presence of extended material around the system. The components of ER Vul (G0 V and G5 V) are nearly equal in mass and luminosity and undergo shallow eclipses. The ephemeris and velocity components of the system are usually taken from Hill et al. (1990). These predictions agree extremely well with our observations which show no evidence of phase-shifts or period changes.

BD-004234 has proved a very interesting system (Figure 11). Our results appear to suggest a phase shift of approximately 0.25 from that predicted from the ephemeris of Peter-

Fig. 11. Radial velocities derived from the SSS data for the BD-004234 system.

Fig. 12. Radial velocities derived from the SSS data for the AR Lac system.

Fig. 13. Radial velocities derived from the SSS data for the RT And system.

Fig. 14. Radial velocities derived from the SSS data for the EZ Peg system.

son et al. (1980). We checked the numbers from CABS with the original data and found that this shift is real and not due to differences in the phase zero-point. BD-004234 is peculiar in that it has a very high systemic velocity (-109.6 km s^{-1}) with Ca II H & K lines and Hα in emission. Spite et al. (1984) suggested on the basis of their spectroscopic observations that the orbital period had changed from that given by Peterson et al. (1980) but only by about 3 minutes. Our observations strongly suggest that the parameters of BD-004234 have been incorrectly derived or undergone a dramatic change.

AR Lac is an extremely well-studied short-period RS CVn system. Small period variations for this system were reported by Hall et al. (1976), the ephemeris is from Hall & Kreiner (1980) and velocity information from Sanford (1951). Because the period of this system is almost 2 days it is almost impossible to obtain reasonable phase coverage at one site during one observing session. Our velocities therefore cluster around two phase positions (Figure 12) but seem to give good agreement with the published ephemeris and velocity components.

RT And appears to be a highly variable system. This eclipsing binary consists of an F8 V primary and a K0 V secondary. We have taken the ephemeris of Popper (1994) which is based on recent times of minimum. The original velocity data of Payne-Gaposchkin (1946) (given also by the CABS catalogue) are severely in error as first suggested by Gordon (1955). We found the best semi-amplitudes and systemic velocities to be those provided by Wang & Lu (1993). The more recent orbit deduced by Popper (1994) does not fit our measurements as well although it should be noted that both these orbits do not give good agreement. Our results are limited by the magnitude of the star but appear to be consistent. There is an indication of a large period change for this system (Figure 13). Kristenson (1967) and Williamson (1974) have both given evidence for changes in the period of RT And although these are very small.

EZ Peg (Figure 14) is a curious system with highly variable Hα emission. Schlesinger, Barney & Gesler (1934) first classified the star as G5 but Vyssotsky & Balz (1958) later claimed that the spectrum varied from B0 to G5. In fact Alden (1958) showed that the spectrum appeared to be that of a B-star on one occasion in 1943. Howell & Bopp (1985) later presented evidence that this erroneous spectrum was due to poor resolution data. They also suggested that the system be classified as an RS CVn and showed that the (U-B) colour index implied a K0 spectral type for the companion. The ephemeris and orbital parameters were determined by Griffin (1985). Our observations have failed at all phases to resolve both components of the system. Our resolution criterion (see §5) suggests 22.7 km s^{-1} is the minimum necessary velocity difference for this system which is less than half of the maximum separation (48.8 km s^{-1}). Our results however are clearly confused over most phases of this difficult system which, assuming the orbit has not altered, implies that the $v \sin i$ values quoted by Griffin (1985) are under-estimated. Assuming both components have the same $v \sin i$ the lack of resolution in our data implies a value of $v \sin i > 44$ km s^{-1}. Clearly the SSS instrument has failed to give reliable results for this system with the smallest maximum velocity separation in our sample. However the results indicate that some of the orbital parameters may be in error. High-resolution observations are required to verify the situation.

Other systems for which we have radial velocities are LX Per, MM Her, MY Cyg, KZ And and SZ Psc. Two of these systems, LX Per and MM Her, agree extremely well with published parameters, the RMS difference between measured and predicted velocities over all phases and for both components are 3 km s^{-1} and 5 km s^{-1} for these systems respectively. For KZ And and SZ Psc these values are 14 km s^{-1} and 25 km s^{-1} suggesting a substantial discrepancy with accepted values. Kalimeris et al. (1995) recently reviewed the period changes in SZ Psc.

5. Analytical assessment of cross-correlations

Cross-correlation techniques for the determination of radial velocities and velocity dispersions have been employed in a number of applications. Most analytical discussions relate to the measurement of galaxy red-shifts and velocity dispersions (Simkin 1974; Sargent et al. 1977; Tonry & Davis (1979); Ore et al. 1991). General discussions of Fourier transform techniques and the cross-correlation function in particular can be found in Bracewell (1965), Brault & White (1971) and Scargle (1989). Although the technique has been widely used in the determination of stellar radial velocities, stellar rotational velocities (eg. Soderblom et al. 1989) and binary star orbits (Popper & Hill 1991; Popper & Jeong 1994), no analytical treatment has been presented for the form of the cross-correlation function for binary spectra.

The cross-correlation procedure is one based on the Fast Fourier Transform (FFT) common in many forms of signal processing, and as such is subject to the induction of errors due to aliasing, leakage and convolution. In order to increase the efficiency of the technique it is possible to apply corrections to both the measured spectral data and the Fourier frequencies themselves. Firstly, large scale trends in spectra will result in low-frequency spikes in the Fourier domain and can be avoided by careful normalisation. Secondly the use of an optimal filter to cut off the high-frequency Fourier components beyond the Nyquist frequency ($S_{NY} = \frac{1}{2\Delta x}$, where Δx is the sampling frequency) will both reduce the broadband noise and reduce the possibility of aliasing (the overlap or folding of high Fourier components into the lower frequencies). To completely avoid aliasing, a careful consideration of the sampling interval should be made and the data re-sampled if necessary. Spectral data should also be deconvolved with the instrumental response to give a better estimate of the true signal. Deconvolution of the resolution function should also be performed or an artificial smoothing applied to the discontinuities in the data (perhaps by adding a cosine bell or zero-valued padding). This will avoid the mixing of Fourier components known as leakage. The above methods will all ensure that the FFTs of spectral data are not significantly corrupted and do not induce errors in the subsequent manipulation. During the analysis of the observational data presented above and that of the synthetic data described below, FFTs were performed with the above constraints. The instrumental profile was not deconvolved since it equalled the resolution measured from Th-Ar lines. Any remaining errors are certainly less than those induced by noise and the measurement process itself.

The results presented in this paper have demonstrated that a significant source of error in RV measurements with the SSS instrument is the gaussian fitting procedure. However, these errors become secondary when the CCF peaks are blended by the dual effects of rotation and small velocity difference. In order to assess the applicability of the CCF method for individual measurements we present below a derivation of the approximate relation between the widths of features in the binary and standard spectra and the CCF. This analysis allows us to make some statements on the relative importance of various spectral parameters and define a criterion for the resolution of CCF velocities. Our derivation is similar to that of Tonry & Davis (1979), who analysed galactic velocity dispersions, but with two important differences. We have generalised the target spectrum to be a weighted combination of spectra that represents the binary star spectrum and we have derived the relationship rigorously without the need for minimising a difference equation. Our results reduce to those of Tonry & Davis for the case of a single spectral component.

Let $b(x)$ be the spectrum of a binary star and $t(x)$ the spectrum of a radial velocity standard star of known radial and rotational velocity and similar spectral type as the components of the binary. Let $B(s)$ and $T(s)$ be the corresponding discrete Fourier transforms of $t(x)$ and $b(x)$ where

$$B(s) = \frac{1}{N}\sum_{j=1}^{N} b(x_j)e^{-2\pi x_j i s}; \qquad T(s) = \frac{1}{N}\sum_{j=1}^{N} t(x_j)e^{-2\pi x_j i s} \quad (1)$$

where s is the generalised frequency variable. If $b(x)$ and $t(x)$ have RMS values σ_b and σ_t respectively where

$$\sigma_b^2 = \frac{1}{N}\sum_x b(x)^2; \qquad \sigma_t^2 = \frac{1}{N}\sum_x t(x)^2 \quad (2)$$

then the normalised cross-correlation function $c(x)$ can be written

$$c(x) = b(x) \times t(x) = \frac{1}{N\sigma_b\sigma_t}\sum_{k=1}^{N} b(k)t(k-x) \quad (3)$$

where the $\times$ symbol indicates the cross-correlation product. The Fourier transform of the CCF is therefore

$$C(s) = \frac{1}{N\sigma_b\sigma_t}B(s)T^*(s) \quad (4)$$

where the asterix signifies the complex conjugate. Now let us assume that the binary spectrum is a weighted combination of the primary and secondary spectra ($p(x)$ and $s(x)$ respectively) and that each is some multiple of $t(x)$ which is shifted and broadened by symmetric functions $r_p(x)$ and $r_s(x)$. Then,

$$p(x) \simeq \alpha_p t(x) * r_p(x-\delta_p) \tag{5}$$

$$s(x) \simeq \alpha_s t(x) * r_s(x-\delta_s) \tag{6}$$

where the asterix signifies the convolution, α_p and α_s are the multiple factors and δ_p and δ_s are the respective shifts of primary and secondary components. We can therefore represent the binary spectrum by

$$b(x) = \alpha_p\beta_p t(x) * r_p(x-\delta_p) + \alpha_s\beta_s t(x) * r_s(x-\delta_s) \tag{7}$$

where β_p and β_s are the intensity weights of the primary and secondary components respectively. Using the linearity of Fourier transforms and the shift theorem we can write the Fourier transform of $b(x)$ as

$$B(s) = \left\{\alpha_p\beta_p R_p(s) e^{\frac{2\pi i s\delta_p}{N}} + \alpha_s\beta_s R_s(s) e^{\frac{2\pi i s\delta_s}{N}}\right\} T(s). \tag{8}$$

The Fourier transform of the cross-correlation function can thus be written

$$C(s) = \frac{\mid T(s)\mid^2}{N\sigma_b\sigma_t}\left\{\alpha_p\beta_p R_p(s) e^{\frac{2\pi i s\delta_p}{N}} + \alpha_s\beta_s R_s(s) e^{\frac{2\pi i s\delta_s}{N}}\right\} \tag{9}$$

where $\mid T(s)\mid^2 = T(s)T^*(s)$ is the power spectrum of $t(x)$. We now proceed by making some assumptions about the form of $t(x)$, $r_p(x)$ and $r_s(x)$. Firstly let $t(x)$ have a Fourier transform that is gaussian such that a typical spectral feature in $t(x)$ has a width equal to τ, then we can write the power spectrum of $t(x)$ as

$$\mid T(s)\mid^2 = 2\pi N\tau\sigma_t^2 \exp\left\{-\frac{(2\pi\tau s)^2}{N^2}\right\}. \tag{10}$$

Now suppose that $r_p(x)$ and $r_s(x)$ are gaussian broadening functions of dispersion σ_p and σ_s respectively where

$$r_p = \frac{1}{\sqrt{2\pi}\sigma_p}\exp\left\{\frac{-x^2}{2\sigma_p^2}\right\}; \qquad r_s = \frac{1}{\sqrt{2\pi}\sigma_s}\exp\left\{\frac{-x^2}{2\sigma_s^2}\right\} \tag{11}$$

then we can write the Fourier transforms of $r_p(x)$ and $r_s(x)$ as

$$R_p(s) = \exp\left\{-\frac{(2\pi\sigma_p s)^2}{2N^2}\right\}; \qquad R_s(s) = \exp\left\{-\frac{(2\pi\sigma_s s)^2}{2N^2}\right\}. \tag{12}$$

The assumption that $r_p(x)$ and $r_s(x)$ are gaussian is justified only if we are considering a maxwellian distribution of velocities which is not true for axial rotation, particularly for high rotation rates. We have retained this assumption in our analysis firstly to simplify the mathematics and because any symmetric function representing the degree of broadening is adequate providing an appropriate interpretation of the broadening parameter is made. In subsequent work we will introduce the standard axial broadening function into the procedure.

Substituting for $\mid T(s)\mid^2$, $R_p(s)$ and $R_s(s)$ in Equation (9) and simplifying gives

$$\begin{aligned} C(s) &= \frac{2\sqrt{\pi}\tau\sigma_t}{\sigma_b}\left\{\alpha_p\beta_p\exp\left\{-\frac{2\pi^2 s^2(2\tau^2+\sigma_p^2)}{N^2}\right\}e^{\frac{2\pi i s\delta_p}{N}} + \right. \\ &\left. \alpha_s\beta_s\exp\left\{-\frac{2\pi^2 s^2(2\tau^2+\sigma_s^2)}{N^2}\right\}e^{\frac{2\pi i s\delta_s}{N}}\right\}. \end{aligned} \tag{13}$$

We now assume that the CCF is the simple sum of two gaussian functions centred at δ_p and δ_s with dispersions μ_p and μ_s corresponding to the primary and secondary components respectively, then

$$c(x) = A_p\exp\left\{-\frac{(x-\delta_p)^2}{2\mu_p^2}\right\} + A_s\exp\left\{-\frac{(x-\delta_s)^2}{2\mu_s^2}\right\} \tag{14}$$

where A_p and A_s are the amplitudes of the two peaks in the cross-correlation function. We can therefore write the Fourier transform of $c(x)$ as

$$\begin{aligned} C(s) &= \sqrt{2\pi}\mu_p A_p\exp\left\{-\frac{(2\pi\mu_p s)^2}{2N^2}\right\}e^{-\frac{2\pi i s\delta_p}{N}} + \\ &\sqrt{2\pi}\mu_s A_s\exp\left\{-\frac{(2\pi\mu_s s)^2}{2N^2}\right\}e^{-\frac{2\pi i s\delta_s}{N}}. \end{aligned} \tag{15}$$

Rewriting Equations (13) and (15) in Eulerian form, equating their real parts and simplifying leads to the approximate equality

$$\begin{aligned} &\sqrt{2\pi}\mu_p A_p\exp\left\{-2s^2\left[\frac{\pi\mu_p}{N}\right]^2\right\}\cos\left\{\frac{2\pi\delta_p s}{N}\right\} + \\ &\sqrt{2\pi}\mu_s A_s\exp\left\{-2s^2\left[\frac{\pi\mu_s}{N}\right]^2\right\}\cos\left\{\frac{2\pi\delta_s s}{N}\right\} = \\ &\frac{2\sqrt{\pi}\tau\sigma_t\alpha_p\beta_p}{\sigma_b}\exp\left\{-2s^2\left[\frac{\pi\sqrt{2\tau^2+\sigma_p^2}}{N}\right]^2\right\}\cos\left\{\frac{2\pi\delta_p s}{N}\right\} + \\ &\frac{2\sqrt{\pi}\tau\sigma_t\alpha_s\beta_s}{\sigma_b}\exp\left\{-2s^2\left[\frac{\pi\sqrt{2\tau^2+\sigma_s^2}}{N}\right]^2\right\}\cos\left\{\frac{2\pi\delta_s s}{N}\right\}. \end{aligned} \tag{16}$$

Since this relationship must hold for all values of the generalised frequency variable s we can write the following approximate relationships

$$\mu_p A_p \simeq \frac{\sqrt{2}\tau\sigma_t\alpha_p\beta_p}{\sigma_b}; \qquad \mu_s A_s \simeq \frac{\sqrt{2}\tau\sigma_t\alpha_s\beta_s}{\sigma_b} \tag{17}$$

$$\mu_p^2 \simeq 2\tau^2+\sigma_p^2; \qquad \mu_s^2 \simeq 2\tau^2+\sigma_s^2. \tag{18}$$

Equations (17) and (18) relate the dispersions, multiple factors, intensity weights, rms values and dispersions in the binary components and the RV standard to the width of features in the cross-correlation function. Equation (18) is identical to the relation derived by Tonry & Davis (1979) using a minimisation method for the analysis of galaxy red-shift survey data. Using reasonable assumptions about the form of spectra and cross-correlation functions we have shown that in the case of stellar binary spectra the individual cross-correlation components also follow this relation. In addition we have derived a relationship between the peak values of these components and their widths

Fig. 15. Example of the formation of synthetic binary spectral data. The top panel (a) shows synthetic data for a primary and secondary spectrum and the weighted combination of these to produce a binary spectrum with noise added. The RV standard spectrum is also shown for comparison. The intensity values have been scaled arbitrarily. The lower panel (b) shows the resulting CCF formed between the synthetic binary and RV standard and clearly displays the two binary components. The parameters related by Equation (17) and (18) are also shown schematically. The inset shows the width parameters associated with the binary and RV standard spectra.

in terms of the rms of the spectra, the width of features in the RV standard spectrum, the multiple factors and intensity weights for the binary components. The multiple factors can be regarded as a measurement of the mismatch in spectral type between the binary components and the RV standard star. This will apply only across narrow ranges of spectral type since over such a range the absorption features become generally deeper with advancing type but little change to the sequence of features is found (Popper & Jeong 1994). We have regarded the multiple factors α_p and α_s as constants representing the variation of line-depths across a narrow range of spectral types. The significance of these factors requires further observational work. We plan, in a later paper, to investigate the calibration and possible uses of Equation (17).

We can define a criterion for the limit of the velocity resolution for binary orbit analysis as follows. We use the criterion that the velocity difference between primary and secondary components $(\delta_s - \delta_p)$ is greater than or equal to the sum of the two half-widths in the cross-correlation function peaks. Hence we can write,

$$\begin{aligned} \delta_s - \delta_p &\geq \frac{\mu_p}{2} + \frac{\mu_s}{2} \qquad (19) \\ &\geq \sqrt{\left\{\tau^2 + \frac{\sigma_p^2 + \sigma_s^2}{4}\right\} + \frac{(2\tau^2 + \sigma_p^2)^{1/2}(2\tau^2 + \sigma_s^2)^{1/2}}{2}} \end{aligned}$$

For reliable measurements of binary star radial velocities to be made the above criterion must be valid. Equation (19) shows how the resolution limit is dependent on the broadening in the RV standard and binary components. Further if we use a set of delta functions as the RV template or the template has rotational broadening much less than the instrumental profile then the resolution limit is the mean of the component rotational velocities and the width of the CCF peaks are equal to these velocities. Undoubtedly other criteria are possible to formulate but we believe this relationship is adequate for our preliminary analysis.

6. Qualitative assessment of cross-correlation techniques

To investigate the effects of various parameters on the determination of CCF velocities we have synthesised data for which all the important parameters are well known. This method involves synthesising the spectrum of a binary star with a given radial velocity difference between primary and secondary components, line strength ratio, rotational velocities and signal-to-noise ratio. We then perform the cross-correlation procedure outlined in §3 using a similar synthetic RV standard star for which we also know the spectral parameters. Variation of the synthesis parameters then gives us a qualitative assessment of the reliability of the CCF results for this particular data set and allows us to make some broad remarks on the efficiency of the technique in general and to demonstrate the use of the equations in §5. Similar analyses of faked data have been performed by Popper & Jeong (1994) and using a series of delta-functions as the standard spectrum by Furenlid & Furenlid (1990).

We synthesise single order spectra of 512 pixels in size. We assume that a linear relationship exists between pixel shift and velocity so that a single pixel anywhere in the order corresponds to the same velocity difference. This is in fact the case for the SSS used in this study where the translation is 10.422 km s^{-1} per pixel. We perform cross-correlation analysis in pixel space as is done with real data and can, if we wish, compare the results to the SSS by using this expansion factor.

The method of constructing the synthetic spectra is directly analogous to the mathematical treatment given in §5. Our approach has been to *represent* a stellar spectrum with well known parameters rather than to *simulate* one and therefore we have not synthesized data which resembles our observations. Subsequent work will make use of real data in the synthesis but our primary aim here is to establish the validity of Equation (19). As a template spectrum, we have thus used a set of 20 gaussian-shaped absorption features at arbitrary pixel positions rather than at the positions of actual stellar lines. Popper & Jeong (1994) used a sky spectrum as their template and Furenlid & Furenlid (1990) used an 80 Å range of a degraded solar spectrum. Although both these techniques are valid the use of arbitrary line positions avoids the possibility of other forms of contamination or activity sensitive deviations and ensures that we know all the significant details inherent in the synthetic data. Our tests have shown that an arbitrary choice of line positions has no significant effect on the results of the analysis. Gaussian-shaped lines which do not accurately represent axial broadening are used in order to remain consistent with our mathematical formalism. By using some multiple factor of the template spectrum we form the spectra of the primary and secondary components of the synthetic binary and broaden and shift these by convolving with a suitable function. The binary spectrum is then formed as a weighted sum of the two components. An RV standard spectrum is also formed by convolving the template with another broadening and shifting function. Although our observational RV standards are not broadened for the analysis this procedure represents the broadening in the standard beyond the instrumental profile.

For all spectra the data are normalised to unity continuum and then have synthetic photon noise added. To do this we assume that the distribution of noise is gaussian with a dispersion of $\frac{1}{S}$ (where S is the signal-to-noise ratio) and a mean equal to the data value. The noisy data are then represented by,

$$y' = y_o \left\{ 1 + \frac{r}{S} \right\} \tag{20}$$

where r is a normally-distributed pseudo-random number with a mean of zero and a σ of 1. Every synthetic spectrum has a pseudo-random number generation which is different so that the added noise is never correlated between spectra.

Figure 15 shows an example of synthetic data. Panel a) shows arbitrarily scaled spectra for the primary and secondary components, the weighted combination forming the binary spectrum, with noise added, and the synthetic standard spectrum. The resulting CCF for these data is shown in panel b) where the positions of each component are clearly measurable. Figure 15 also shows schematically the parameters related by Equations 17, 18 an 19. Although several procedures employed in the construction of our synthetic data and some of our assumptions are perhaps not appropriate for real binary systems

Fig. 16. A combined grey-scale and contour plot of the CCF with increasing velocity separation between the synthetic binary components. Constant parameters used in the synthetic data were α_p=0.9, β_p=0.6, α_s=0.8, β_s=0.4, τ=2.0 and δ_t=5.0 (the initial shift of the RV standard). The signal to noise ratios were 120 and the binary components velocity difference was successively shifted by one pixel. The primary and secondary widths were σ_p=5.0 and σ_s=2.0 respectively which gives a resolution limit of 4.34 pixels (solid white line). The components were shifted so as to reveal the reflection effects in the CCF. The amplitudes have been normalised by the mean continuum level.

Fig. 17. A combined grey-scale and contour plot of the CCF with increasing velocity separation similar to Figure 16. This plot is for a synthetic binary with widths $\sigma_p = 10$, $\sigma_s = 5$ and $\tau = 2$ giving a resolution limit of 8.07 (solid white line). The increase in component widths is clearly visible as is the shift in the resolution limit.

we have assumed that systematic effects so produced are small compared to those arising in the CCF method and from the synthetic binaries themselves. We intend to further investigate the sources of error involved in CCF techniques by extending our preliminary analysis to *real* spectral data obtained for a large sample of RV standards and well-studied binaries.

Using our synthetic data maker (an IDL procedure) we have synthesised spectra where the separation of the binary components varies linearly from 0 to 40 pixels. Knowing the width parameters on input we can compare the resulting CCFs to

Fig. 18. The effects of varying the component rotational broadening on the measured position errors with increasing velocity separation. See the text for a full explanation.

our resolution limit. Figure 16 shows a combined grey-scale and contour plot of the CCF image for one of our data sets. The image consists of the CCF amplitude plotted for varying separation and relative lag. In this case the component widths were $\sigma_p = 5$, $\sigma_s = 2$ and $\tau = 2$ pixels giving a resolution limit of 4.34 pixels. Other input parameters were $\alpha_p = 0.9$, $\beta_p = 0.6$, $\alpha_s = 0.8$, $\beta_s = 0.4$ and the signal-to-noise was set at 120 in all spectra. Note that the binary components have been shifted significantly from the CCF centroid in order to reveal reflection effects in the CCF. The amplitude scale for each CCF has been normalised by the mean continuum level in the CCF.

Figure 16 clearly shows both components of the binary shifting in velocity. The broader component on the right is the primary component which demonstrates that the brighter or hotter component does not necessarily give the greatest peak in the CCF since it is denuded by its width. The resolution limit is marked on the diagram as a solid white line and occurs where the two components become adequately separated. Figure 16 also shows the important effects of reflection in the CCF. We demonstrate below that for some data the reflection effect is one of the most significant source of error in CCF velocity determinations beyond the resolution limit. For comparison Figure 17 shows a similar plot for a synthetic binary with widths $\sigma_p = 10$, $\sigma_s = 5$ and $\tau = 2$ pixels giving a resolution limit of 8.07 pixels. The increase in component widths is clearly visible as is the shift in the resolution limit.

We have investigated the systematic effects in the component positions measured from our synthetic data. Our method for assessing the technique is to derive radial velocities using the synthetic data and compare these to the known values used to create the data. We can then investigate the errors induced by, for example, low signal-to-noise or small binary separation. This is achieved with the same fitting procedure used for our observational data (see §3). A remaining source of error which is not possible to model is the user-supplied positions for the double or single gaussian fitting procedure. These errors are inherent in the results just as with the SSS radial velocities. Our investigations have shown however that independent determinations agree well within any tolerances we wish to adopt. However a full treatment of fitting errors would be helpful.

Figure 18 shows the results of our measurements where we plot the error in the position (actual minus measured) against the separation of the components. Where the two CCF components are blended the data are plotted as an open circle; in this case the actual position of the *larger* peak is used to determine the error. This is invariably the slower rotator, ie. the secondary. Points plotted as filled circles are for the secondary component and filled squares are the primary. The four panels a) to d) of Figure 18 show the results for resolution limits of 3.74, 4.34, 8.07 and 12.87 pixels respectively. The parameters of each component are displayed in each panel and the resolution limit is displayed as a vertical dashed line. The other parameters used in the formation of these data were $\alpha_p = 0.9$, $\beta_p = 0.6$, $\alpha_s = 0.8$, $\beta_s = 0.4$ and the signal-to-noise in all spectra was 120.

Figure 18 shows many interesting features. Firstly our definition of the resolution limit appears to be consistent. Although it is sometimes possible to determine the positions of two CCF components below this limit these are likely to be significantly in error. The RMS position of the CCF peak at zero separation for these parameters is 0.066 pixels but the error rapidly increases as the peak is skewed by the shifting

Fig. 19. The effects of varying the spectral signal-to-noise on the measured position errors with increasing velocity separation. See the text for a full explanation.

of blended components. The rms error over all measurements lying above the resolution limit in panels a) to d) are 0.08, 0.18, 0.22 and 0.54 pixels respectively. Clearly there are periodic variations in the errors whose amplitude increases with the resolution limit. These systematic variations are in anti-phase between primary and secondary components. This important effect is caused by the reflection peaks in the CCF moving through the component peaks as the separation increases. Although such reflection peaks can be very small their skewing effect on the actual peaks is sufficient to induce significant errors. Large errors at large separations in panel d) are caused by the very wide CCF peak lying close to the edge of the CCF window used in the analysis thus giving rise to fitting errors. Figure 18 demonstrates that one of the most significant effects in the measurement of velocities with the CCF technique is the confusion by reflection peaks. This effect rapidly increases with resolution limit but may be negligible in comparison with instrumental or other systematic effects (as with the SSS). Although Popper & Jeong (1994) make no reference to this effect we believe their synthetic data also shows periodic variations due to reflection peaks (their Figures 5-9).

We have also investigated the effects of signal-to-noise ratio, luminosity weight and spectral type mismatch on the derived velocities. Figure 19 shows the position errors for binary spectra signal-to-noise ratios of 140, 100, 60, and 20. The other parameters used in the formation of these data were $\alpha_p = 0.9$, $\beta_p = 0.6$, $\alpha_s = 0.8$, $\beta_s = 0.4$, $\sigma_p = 10.0$, $\sigma_s = 5.0$, $\tau = 2.0$ and the signal-to-noise was set at 120 in all spectra. The periodic effect in the errors is clearly visible but as expected the dispersion in the values is greater for low signal-to-noise. For this data set the signal-to-noise becomes dominant between 60 and 100. The rms of all errors lying above the resolution limit in panels a) to d) are 0.40, 0.35, 0.85 and 1.45 pixels respectively. Figure 19 shows that the measured velocities are systematically slightly smaller than the true ones and this effect seems to be insensitive to the separation. Popper & Jeong (1994) also noticed this effect in their synthetic results and attributed this to the CCF procedure.

Figure 20 shows position errors for different multiple factors which gives an *ad hoc* representation of changes in spectral type over a narrow range by varying the initial line depths in the binary spectrum. We synthesised data sets for multiple factors $\alpha_p = 2.0$ and $\alpha_s = 1.5$, $\alpha_p = 1.5$ and $\alpha_s = 1.0$, $\alpha_p = 1.0$ and $\alpha_s = 0.5$ and $\alpha_p = 0.5$ and $\alpha_s = 0.2$. The other parameters were $\beta_p = 0.6$, $\beta_s = 0.4$, $\sigma_p = 10.0$, $\sigma_s = 5.0$, $\tau = 2.0$ and the signal-to-noise was set at 120 in all spectra. Clearly the errors increase with decreasing line depth. The rms of errors beyond the resolution limit (8.07) for panels a) to d) are 0.31, 0.29, 0.35 and 0.53 pixels respectively. It appears that over a small range of spectral types where no changes in the pattern of lines occurs a mismatch in spectral types does not induce large errors providing that the line depths are sufficient, when broadened, to produce a correlation well above the continuum in the CCF. Providing lines are deeper or of similar depth as the RV standard the spectral type mismatch will not be the dominating source of error.

Finally we investigated the systematic errors in the measured positions for varying luminosity ratios. We synthesised data for luminosity weights of $\beta_p = 0.8$ and $\beta_s = 0.2$, $\beta_p = 0.66$ and $\beta_s = 0.33$, $\beta_p = 0.5$ and $\beta_s = 0.5$ and $\beta_p = 0.33$ and $\beta_s = 0.66$. These then correspond to luminosity ratios of 4/1, 2/1, 1/1, and 1/2 respectively. The other parameters used

Fig. 20. The effects of varying the multiple factor on the measured position errors with increasing velocity separation. See the text for a full explanation.

Fig. 21. The effects of varying the luminosity ratio on the measured position errors with increasing velocity separation. See the text for a full explanation.

were $\alpha_p = 0.9$, $\alpha_s = 0.8$, $\sigma_p = 10.0$, $\sigma_s = 5.0$, $\tau = 2.0$ and the signal-to-noise was set at 120 in all spectra. Figure 21 shows the results of this analysis. Errors are less serious for luminosity ratios of 1 and increase with the difference in luminosity. The rms of all errors above the resolution limit in panels a) to d) are 0.37, 0.35, 0.36 and 0.63 respectively. Although the combined rms values are similar for luminosity ratios 4, 2 and 1 the errors for secondary and primary components scale well with their relative weights. The large error for luminosity ratio 0.5 is due mainly to the faint primary; the rms error for the secondary is only 0.09 pixels for values above the resolution limit. Panel d) shows well the effects of both a large rotational velocity and low luminosity on the primary component and the correspondingly small errors for the secondary.

In Figures 3 to 14 where we present our observational results we have depicted as dashed lines the positions of our resolution limits centred on the systemic velocities. These are calculated using Equation 19 when the necessary rotational velocities are known and taking the maximum RV standard rotational velocity of 15 km s^{-1}. It can be seen from these diagrams that all blended velocities lie within or slightly above these limits. Our preliminary analytical results therefore seem to be reliable for this limited set of data. We plan to extend this study of CCF resolution limits, the effects of various stellar parameters and the induction of errors by the CCF technique itself by analysing a large sample of medium-resolution spectra for RV standards and well-studied binary systems.

7. Conclusions

We have presented observational radial velocity results for a small sample of chromospherically active binary systems. The data were collected with the medium-resolution Solar Stellar Spectrograph on the John Hall 1.1-meter telescope located near Flagstaff, Arizona. We have employed a cross-correlation technique for radial velocity determinations and compared our results to published ephemerides and orbital parameters. We have excellent agreement for TY Pyx and GK Hya, very good agreement for LX Per, MM Her, Z Her and ER Vul, reasonable agreement for V471 Tau, OU Gem, V772 Her and AR Lac. V772 Her has demonstrated the induced errors due to the primary component CCF peak skewing the fitting procedure for the small secondary peak. The system EI Eri has given extremely confused results perhaps due to the high rotational velocity of this system. Although results for RT And appear to be consistent these do not appear to agree well with any published values for the system. EZ Peg has also given confused results possibly due to higher rotation rates than previously published. The system BD-004234 appears to be approximately 0.25 out of phase with the published ephemeris. SZ Psc and KZ And have also given individual measurements which conflict with predicted values.

We have also presented a rigourous derivation of the relationship between binary component rotation rates, RV standard rotation rates and the width of features in the CCF. An *ad hoc* resolution limit for binary star cross-correlation radial velocities is also given. Using synthetic spectral data we have investigated the errors induced by rotational broadening, signal-to-noise ratio, spectral-type mismatch and luminosity ratio and identified some sources of confusion in the technique.

In summary, although we have not formally derived an error equation for CCF analysis of binary orbits nor given a quantitative treatment we have demonstrated the relative importance of rotation, signal-to-noise, spectral type mismatch and luminosity ratio on the determination of component velocities. For data with good signal-to-noise and medium resolution the primary sources of error seem to be reflection effects in the CCF, spectral type mismatch and low luminosity ratio. In a future paper we will examine in more detail the aspects of CCF radial velocity methods described in the present paper.

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References

Aitken R. G., 1923, Bull. Lick Obs., 11, 85
Alden H. L., 1958, AJ, 63, 358
Andersen J., Popper D. M., 1975, A&A, 39, 131
Andersen J., Clausen J. V., Nordström B., Reipurth B., 1981, A&A, 101, 7
Batten A. H., Morbey C. L., Fekel F. C., Tomkin J., 1979, PASP, 91, 304
Bopp B. W., Hall D. S., Henry G. W., Noah P., Klimke A., 1981, PASP, 93, 504
Bracewell R. N., 1965, *The Fourier Transform and Its Applications*, McGraw-Hill, New York
Brault J. W., White O. R., 1971, A & A, 13, 169
Furenlid I. & Furelid L., 1990, PASP, 102, 592
Gordon K., 1955, AJ, 60, 422
Griffin R. F. & Emerson B., 1975, *The Observatory*, 95, 23
Griffin R. F., 1985, *The Observatory*, 105, 81
Guarnieri A., Bonifazi A., Battistini P., 1975, A&AS, 20, 199
Hall D. S., Richardson T. R., Chambliss C. R., 1976, AJ, 81, 1138
Hall D. S., Kreiner J. M., 1980, Acta Astr, 30, 387
Hall D. S., Kreiner J. M., Shore S. N., 1980, in Close Binary Stars: Observations and Interpretation, M. J. Plavec, D. M. Popper and R. K. Ulrich (eds), IAU Symposium No. 8, p.383
Hall J. C., Fulton E. E., Huenemoerder D. P., Welty A. D., Neff J. E., 1994, PASP, 106, 315
Hill G., Fisher W. A., Holmgren D., 1990, A&A, 238, 145
Howell F. B. & Bopp B. W., 1985, PASP, 97, 72
Kalimeris A., Mitrou C. K., Doyle J. G., Antonopoulou E., Rovithis-Livaniou H., 1995, A&A, 293, 371
Kreiner J. M., 1971, Acta Astr, 21, 365
Kristenson H., 1967, BAC, 18, 261
McLean B. J., 1982, MNRAS, 201, 421
Ore C. D., Faber S. M., Jesús J., Stoughton R., Burstein D., 1991, ApJ, 366, 38
Payne-Gaposchkin C. H., 1946, ApJ, 103, 291
Peterson R. C., Willmarth D. W., Carney B. W., Chaffee F. H., 1980, ApJ, 239, 928

Philip A. G., Latham D. W. (eds), 1985, IAU Coll. 88, *Stellar Radial Velocities*, Schenectady NY: L. Davis Press
Popper D. M., 1988, AJ, 95, 1242
Popper D. M., 1990, AJ, 100, 247
Popper D. M., Hill G., 1991, AJ, 101, 600
Popper D. M., 1994, AJ, 108, 1091
Popper D. M., Jeong Y.-C., 1994, PASP, 106, 189
Reglero V., Fernández-Figueroa M. J., Giménez A., De Castro E., Fabregat J., Cornide M., Armentia J. E., 1991, A&AS, 88, 545
Sanford R. F., 1951, ApJ, 113, 299
Sargent W. L. W., Schechter P. L., Boksenberg A., Shortridge K., 1977, ApJ, 212, 326
Scargle J. D., 1989, ApJ, 343, 874
Schlesinger F., Barney I. & Gesler C., 1934, *Yale Trans.*, 10, 170
Simkin S. M., 1974, A & A, 31, 129
Soderblom D. R., Pendleton J., Pallavicini R., 1989, AJ, 97. 539
Spite M., Maillard J. P., Spite F., 1984, A&A, 141, 56
Strassmeier K. G., 1990, ApJ, 348, 682
Strassmeier K. G., Fekel F. C., 1990, A&A, 230, 389
Strassmeier K. G., Hall D. S., Fekel F. C., Scheck M., 1993, A&AS, 100, 173
Tomkin J., 1980, AJ, 85, 294
Tonry J., Davis M., 1979, AJ, 84, 1511
Vyssotsky A. N. & Balz A. G. A., 1958, *Leander McCormick Publ.*, 13, 79
Wang X., Lu W., 1993, in New Frontiers in Binary Star Research, K.-C. Leung & I.-S. Nha (eds), ASP Conference Series 38 (ASP, San Francisco), p. 374
Weiss W. W., Jenkner H., Wood H. J., 1978, A&A, 63, 247
Williamson R. M., 1974, PASP, 86, 924
Young A., 1976, ApJ, 205, 182

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